ASSIGNMENT-2

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Introduction :

Gödel's incompleteness theorems :

These are two theorems of mathematical logic that are concerned with the limits of provability in formal axiomatic theories. And the results of these are important in mathematical logic and the philosophy of mathematics . these theorems are widely , but not universally , interpreted as showing that Hilbert's program to find a complete and consistent set of axioms for all mathematics is impossible .

THE FIRST INCOMPLETENESS THEOREM states that no consistent system of axioms whose theorems can be listed by an effective procedure is capable of proving all truths about the arithmetic of natural numbers. For any such consistent formal system, there will always be statements about natural numbers that are true, but that are unprovable within the system.

THE SECOND INCOMPLETENESS THEOREM an extension of the first, shows that the system cannot demonstrate its own consistency.

This theorem is stronger than first incompleteness theorem because the statement constructed in the first incompleteness theorem does not directly express the consistency of the system .

Employing a diagonal argument, Gödel's incompleteness theorems were the first of several closely related theorems on the limitations of formal systems. They were followed by Tarski's undefinability theorem on the formal undefinability of truth, Church's proof that Hilbert's Entscheidungsproblem is unsolvable, and Turing's theorem that there is no algorithm to solve the halting problem.

Formal systems : completeness , consistency and effective axiomatization

This incompleteness is applicable to formal systems that are of sufficient complexity to express the basic arithmetic of natural numbers and which are consistent and effective axiomatization .

In general, a formal system is a deductive apparatus that consists of a particular set of axioms along with rules of symbolic manipulation (or rules of inference) that allow for the derivation of new theorems from the axioms.

Effective axiomatization :

It is recursively enumerable set .which means that there is a computer program that, in principle, could enumerate all the theorems of the system without listing any statements that are not theorems. Examples of effectively generated theories include Peano arithmetic and Zermelo–Fraenkel set theory (ZFC).

Consistency :

A set of axioms is consistent if there is no statement such that both the statement and its negation are provable from the axioms, and inconsistent otherwise.

Relation with computability :

The incompleteness theorem is closely related to several results about undecidable sets in recursion theory.

Kleene presented a proof of Gödel's incompleteness theorem using basic results of computability theory. One such result shows that the halting problem is undecidable: there is no computer program that can correctly determine, given any program *P* as input, whether *P* eventually halts when run with a particular given input. Kleene showed that the existence of a complete effective system of arithmetic with certain consistency properties would force the halting problem to be decidable, a contradiction .

The incompleteness results affect the philosophy of mathematics, particularly version of formalism ,which use a single system of formal logic to define their principles. The incompleteness theorem is sometimes thought to have severe sequences for the program of localism.