Data mining and machine learning

Assignment - 5

(7) (i) standard 1-NN distance

$h(x) = y(x_t)$ where $x_t = \arg\min_{x \in D} \|x - x_t\|_2^2$

kernelized 1-NN : Replace the euclidean distance $\|x - x_t\|_2^2$ with a distance based on kernel function $k(x, x')$

$$\|x - x_t\|_2^2 = x^T x - 2x^T x_t + x_t^T x_t$$

In kernel space :

$$\|\phi(x) - \phi(x_t)\|^2 = k(x, x) - 2k(x, x_t) + k(x_t, x_t)$$

The predicted label is determined by finding the training point x_t that minimizes the kernel distance

$$x_t = \arg\min_{x \in D} (k(x, x) - 2k(x, x_t) + k(x_t, x_t))$$

The output prediction : $h(x) = y(x_t)$

(8) (ii) Update α values

kernel function : $k(x_i, x_j) = \exp(-\|x_i - x_j\|^2)$

Initial $\alpha_i = 0.1$ for all i

$X = [1, 2, 3]$, $Y = [1, 4, 9]$

the steps to update α involve

Computing the kernel matrix k

$$k_{ij} = \exp(-\|x_i - x_j\|^2)$$

where $\|x_i - x_j\|^2 = (x_i - x_j)^2$

Using the update rule :

$$\alpha = (k + \lambda I)^{-1} y$$

After calculating the kernel matrix k is

kernel matrix k :

$$k = \begin{bmatrix} 1.0 & 0.3679 & 0.0183 \\ 0.3679 & 1.0 & 0.3679 \\ 0.0183 & 0.3679 & 1.0 \end{bmatrix}$$

the entries are computed using $k(x_i, x_j)$

$= \exp(-\|x_i - x_j\|^2)$

Updated α values:

$$\alpha = [0.2461, 1.1669, 4.2831]$$