MT1 solution 7-11-2021

Fall 2021-22 ~~EEE~~313

Q1.

a) Assume all transistors are SAT.

$V_{GS_4} + V_{GS_3} = 5$ Since $I_{D_3} = I_{D_4}$ we have

$K_{n_3}(V_{GS_3}-1)^2 = K_{n_4}(V_{GS_4}-1)^2$

$\frac{K_{n3}}{K_{n4}}(V_{GS_3}-1)^2 = (V_{GS_4}-1)^2 = (5-V_{GS_3}-1)^2 = (4-V_{GS_3})^2$

$4(V_{GS_3}^2 - 2V_{GS_3} + 1) = 16 - 8V_{GS_3} + V_{GS_3}^2$

$4V_{GS_3}^2 - 8V_{GS_3} + 4 - 16 + 8V_{GS_3} - V_{GS_3}^2 = 0$

$3V_{GS_3}^2 - 12 = 0$ $V_{GS_3}^2 = 4$ $V_{GS_3} = \begin{cases} +2 > 1 \checkmark \\ -2 \overset{?}{>} 1 \times \end{cases}$

$\boxed{V_{GS_3} = 2}$

$0.2 = K_{n_3}(2-1)^2 = K_{n_3} \Rightarrow \boxed{K_{n_3} = 0.2\ mA/V^2}$

$\frac{K_{n3}}{K_{n_4}} = 4$ $K_{n_4} = \frac{K_{N3}}{4} = \frac{0.2}{4} \Rightarrow \boxed{K_{n_4} = 0.05\ mA/V^2}$

$V_{D_3} = V_{S_4} = -5+2 = -3\,V.$

$V_{DS_3} = V_{GS_3} = 2 > 2-1 \checkmark$

$V_{DS_4} = 0 - V_{S_4} = 0-(-3) = 3 > 2-1 \checkmark$

$0.1 = K_{n_2}(V_{GS_2}-1)^2 = K_{n_2}(V_{GS_3}-1)^2 = K_{n_2}(2-1)^2 = K_{n_2}$

M_1 and M_2 are matched $\therefore$ $\boxed{K_{n_2} = K_{n_1} = 0.1\ mA/V^2}$

Also $I_{D_1} = I_{D_2} \Rightarrow V_{GS_1} = V_{GS_2} = V_{GS_3} = 2\,V.$

$V_{DS_1} = (5 - 0.1 \times 20) - (-2) = 5-2+2 = 5 > 2-1 \checkmark$

$V_{DS_2} = -V_{GS_2} - (-5) = -(+2)+5 = 3\,V > 2-1 \checkmark$

b) $V_{DS_1} = (5 - 0.1 R_D) - (-2) > 2-1$

$5 - 0.1 R_D + 2 > 2-1 \Rightarrow 5 - 0.1 R_D > -1$

$5+1 > 0.1 R_D$

$\boxed{R_D < 60\ k\Omega}$

Q2.

a) $\lambda = 0$ Assume SAT.

$$0.4\,mA = 0.2(V_{SG} - 0.8)^2$$

$$2 = (V_{SG} - 0.8)^2$$

$$V_{SG} = 0.8 \pm \sqrt{2} = 0.8 \pm 1.41 = \begin{cases} 2.21 > 0.8 \checkmark \\ -0.61 > 0.8 \times \end{cases}$$

$\boxed{V_{SG} = 2.21V}$ $\Rightarrow V_S = 2.21V.$

$$V_{SD} = V_S - V_D = 2.21 - (0.4 \times 5 - 5)$$

$$= 2.21 - 2 + 5 = 5.21 > 2.21 - 0.8 \checkmark$$

$\boxed{V_{SD} = 5.21}$

b) $\lambda = 0.02 \frac{1}{V}.$

Assume SAT.

$$0.4 = 0.2(V_{SG} - 0.8)^2(1 + 0.02\, V_{SD})$$

$$V_{SD} = V_S - V_D = V_{SG} - (5 \times 0.4 - 5) = V_{SG} + 3$$

$$\therefore\ 2 = (V_{SG} - 0.8)^2(1 + 0.02 V_{SG} + 0.02 \times 3)$$

$$2 = (V_{SG}^2 - 1.6 V_{SG} + 0.64)(1.06 + 0.02 V_{SG})$$

$$2 = 1.06 V_{SG}^2 - 1.6 \times 1.06 V_{SG} + 0.64 \times 1.06$$
$$+ 0.02 V_{SG}^3 - 1.6 \times 0.02 V_{SG}^2 + 0.64 \times 0.02 V_{SG}$$

$$2 = 0.02 V_{SG}^3 + V_{SG}^2(1.06 - 1.6 \times 0.02) + V_{SG}(-1.6 \times 1.06 + 0.64 \times 0.02)$$
$$+ 0.64 \times 1.06$$

$$0 = 0.02 V_{SG}^3 + 1.028 V_{SG}^2 - 1.6832 V_{SG} - 1.3216$$

$$V_{SG} = \begin{cases} -52.96 > 0.8 \times \\ 2.1466 > 0.8 \checkmark \\ -0.58 > 0.8 \times \end{cases} \Rightarrow \boxed{V_{SG} = 2.1466V}$$

$$V_{SD} = 2.1466 - (5 \times 0.4 - 5) = 2.1466 + 3$$

$\boxed{V_{SD} = 5.1466V}$ $V_{SD} > 2.1466 - 0.8 \checkmark$

Q3. Assume SAT.

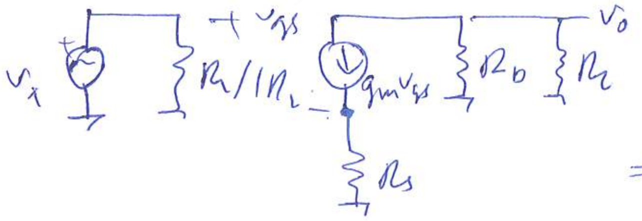
$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{K_n \times 6} = 2.2 \qquad 4 \times 6 \times K_n = 2.2^2$$

$$K_n = \frac{2.2^2}{24} = \boxed{0.2017\ mA/V = K_n}$$

$$6 = K_n(2.8 - V_{TN})^2 = 0.2017(2.8 - V_{TN})^2$$

$$V_{TN} = 2.8 \pm \sqrt{\frac{6}{0.2017}} = \begin{cases} 8.2541\ V < 2.8 & ✗ \\ -2.6541\ V < 2.8 & ✓ \end{cases}$$

$$\therefore \boxed{V_{TN} = -2.6541} \qquad 10 > 2.8 - (-2.6541)\ ✓$$



$$v_i = v_{gs} + g_m v_{gs} R_S = v_{gs}(1 + g_m R_S)$$

$$v_o = -g_m v_{gs} R_D // R_L$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{-g_m R_D // R_L}{1 + g_m R_S} = -1$$

$$\frac{2.2 \times R_D // R_L}{1 + 2.2 \times R_S} = 1$$

$$2.2 \times (R_D // 1) = 1 + 2.2 \times R_S$$

$$\frac{2.2 R_D}{1 + R_D} = 1 + 2.2 R_S = 1 + 2.2\left(\frac{4}{3} - R_D\right) = 3.933 - 2.2 R_D$$

$$2.2 R_D = 3.933 + 3.933 R_D - 2.2 R_D - 2.2 R_D^2$$

$$2.2 R_D^2 + 0.467 R_D - 3.933 = 0$$

$$R_D = \begin{cases} 1.2351 > 0 & ✓ \\ -1.4474 > 0 & ✗ \end{cases}$$

$$\therefore \boxed{R_D = 1.2351\ k\Omega}$$

$$\Rightarrow R_S = 1.333 - 1.2351$$

$$\boxed{R_S = 0.0982\ k\Omega}$$

$$18 = 6 R_D + 10 + 6 R_S$$

$$8 = 6(R_D + R_S)$$

$$R_D + R_S = \frac{4}{3}$$

$$\frac{R_1 R_2}{R_1 + R_2} = 100$$

$$V_G = 2.8 + 6 \times 0.0982 = 3.3892\ V.$$

$$18 \times \frac{R_2}{R_1 + R_2} = 3.3892$$

$$R_1 \times \frac{R_2}{R_1 + R_2} = 100 \qquad R_1 \times \frac{3.3892}{18} = 100$$

$$R_1 = \frac{100 \times 18}{3.3892} = 531\ k\Omega.$$

$$\frac{531 R_2}{531 + R_2} = 100 \qquad 531 R_2 = 53100 + 100 R_2$$

$$R_2 = \frac{53100}{431} = 123.2\ k\Omega$$

Q4,

a) Assume SAT.

$K_n = \frac{0.1}{2} \times 20 = 1\,mA/V^2$

$5 = 1(V_{GS} - 0.8)^2$

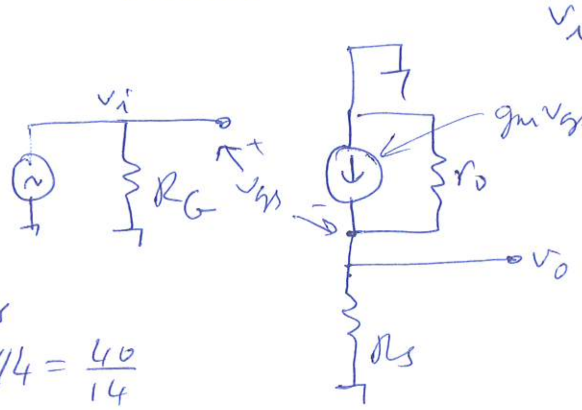
$V_{GS} = 0.8 \pm \sqrt{5} = \begin{cases} 3.036 > 0.8\,V \checkmark \\ -1.436 > 0.8 \times \end{cases} \Rightarrow \boxed{V_{GS} = 3.036\,V}$

$V_{DS} = 5 - (-3.036) = 8.036 > 3.036 - 0.8 \checkmark$ $\boxed{V_{DS} = 8.036\,V}$

$\boxed{I_D = 5\,mA}$

b) $g_m = 2\sqrt{1 \times 5} = 4.472\,mA/V.$

$r_o = \frac{1}{0.02 \times 5} = 10\,k\Omega.$



$r_o // R_s = 10//4 = \frac{40}{14} = 2.857\,k\Omega.$

$v_i = v_{gs} + g_m v_{gs}(r_o // R_s)$

$\frac{v_{gs}}{v_i} = \frac{1}{1 + g_m(r_o // R_s)}$

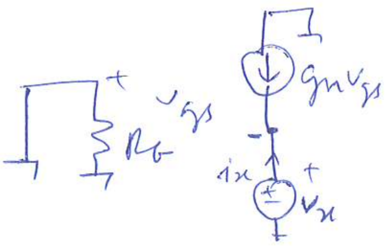
$v_o = g_m v_{gs}(r_o // R_s)$

$\therefore A_v = \frac{g_m(r_o // R_s)}{1 + g_m(r_o // R_s)}$

$= \frac{4.472 \times 2.857}{1 + 4.472 \times 2.857} = \frac{12.777}{13.777}$

$= 0.9274\,V/V$

c)



$\left.\begin{array}{l} v_x = -v_{gs} \\ i_x = -g_m v_{gs} \end{array}\right\} \frac{v_x}{i_x} = \frac{1}{g_m}$

$\therefore R_o = r_o // \frac{1}{g_m} = 10 // \frac{1}{4.472}$

$= 10 // 0.2236 = 0.2187\,k\Omega$

$= 218.7\,\Omega$

d) $A_v = \frac{v_{in}}{v_i + v_{in}} \times \frac{v_o}{v_{in}} = \frac{500}{10 + 500} \times 0.9274$

$= 0.98 \times 0.9274 = 0.909$

e) when $R_L = \infty$ $A_{oc} = \frac{g_m r_o}{1 + g_m r_o} = \frac{4.472 \times 10}{1 + 4.472 \times 10} = \frac{44.72}{45.72} = 0.978$