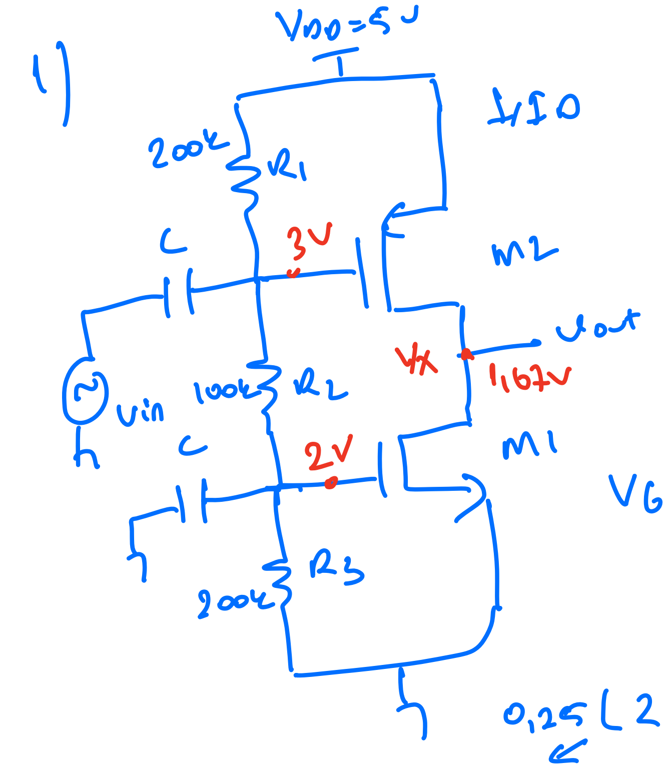
1)



$V_{tn} = |V_{tp}| = 1V \quad K_N = K_P = 0{,}25\ mA/V^2$

$\lambda_1 = 0{,}02\ V^{-1} \quad \lambda_2 = 0{,}01\ V^{-1}$

a.) we need to use λ's of m1 and m2, otherwise we cannot calculate the DC value of Vout. (V_x)

$V_{G1} = \frac{R_3}{R_1+R_2+R_3} \cdot 5V = 2V \quad V_{G2} = \frac{R_2+R_3}{R_1+R_2+R_3} \cdot 5V = 3V$

$I_{D1} = I_{D2}.$

$0{,}25(2-1)^2(1+\lambda_1 V_x) = 0{,}25(5-3-1)^2 \cdot [1+\lambda_2(5-V_x)]$

$1+\lambda_1 V_x = 1+\lambda_2(5-V_x)$

$\lambda_1 V_x = 5\lambda_2 - \lambda_2 V_x$

$V_x = 5 \cdot \frac{\lambda_2}{\lambda_1+\lambda_2} = 5 \cdot \frac{1}{3} = 1{,}67V$

$= 0{,}25 \cdot 1 \cdot (1+0{,}02 \cdot 1{,}67) =$

$I_D = I_{D1} = I_{D2} = 0{,}25 \cdot 1 \cdot (1+0{,}02 \cdot 1{,}67)$

$\boxed{I_D = 0{,}258\,mA}$ $\quad V_{DS_1} = 1{,}67V,\ V_{SD_2} = 3{,}33V$

$g_{m1} = g_{m2} = 2\sqrt{K_N I_D} = 2\sqrt{0{,}25 \cdot 0{,}258} = 0{,}508\ \frac{mA}{V}$

$r_{o1} = \frac{1}{\lambda_1 I_D} = \frac{1}{0{,}02 \cdot 0{,}258} = 193{,}8\ k\Omega$

$r_{o2} = \frac{1}{\lambda_2 I_D} = \frac{1}{0{,}01 \cdot 0{,}258} = 387{,}6\ k\Omega$

m1

$V_{GS} - V_{tn} < V_{DS}$

$2-1 < 1{,}67$

SAT ✓

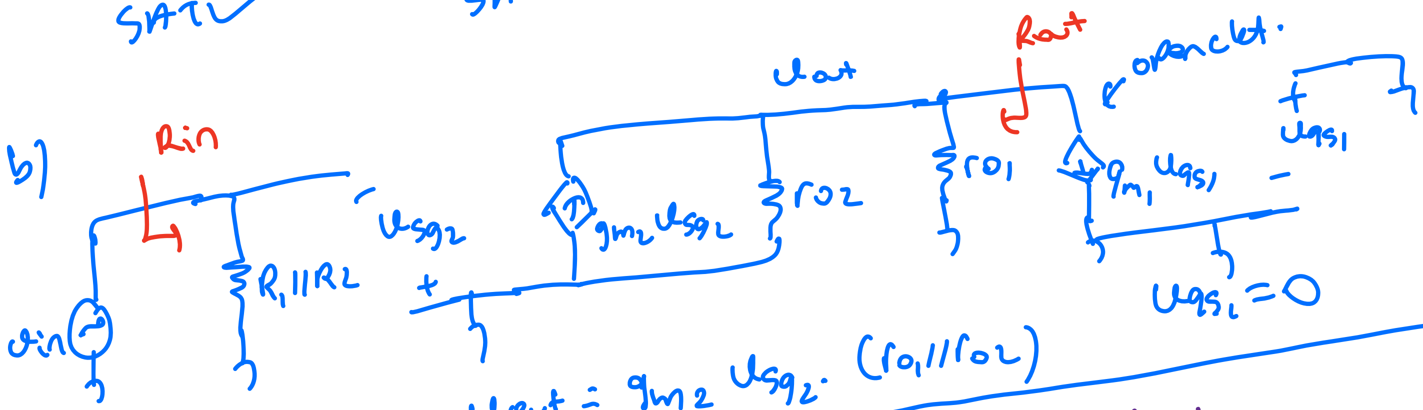
m2

$V_{SG} - |V_{tp}| < V_{SD}$

$2-1 < 3{,}33$

SAT ✓

b)



$v_{gs1} = 0$

$v_{in} = -v_{sg2}$, $v_{out} = g_{m2} v_{sg2} \cdot (r_{o1} \| r_{o2})$

$A_v = \frac{v_{out}}{v_{in}} = -g_{m2}(r_{o1} \| r_{o2})$

$A_v = -0{,}508 \cdot (193{,}8 \| 387{,}6)$

$\boxed{A_v = -65{,}6\ \frac{V}{V}}$

c) $R_{in} = R_1 \| R_2 = 66{,}7\ k\Omega$

Rout kill vin $v_{sg2} = 0$, $g_{m2} v_{sg2} \to$ open ckt.

$v_{gs1} = 0$ $\quad R_{out} = r_{o1} \| r_{o2}$

$\boxed{R_{out} = 129{,}2\ k\Omega}$

A Note on Calculating g_m and r_o with λ included in DC analysis

$$I_D = K_n (V_{GS} - V_{tn})^2 (1 + \lambda V_{DS})$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2 K_n (V_{GS} - V_{tn}) (1 + \lambda V_{DS}) \text{ --- } 1$$

$g_m = 2\sqrt{K_n I_D}$ is not exactly equal to 1

$g_m = 2\sqrt{K_n . I_D . (1 + \lambda V_{DS})}$ is exact.

But for simlicity we accept the error and use

$g_m = 2\sqrt{K_n I_D}$ in the solution.

* you do not have to worry about this, unless you are told to do so.

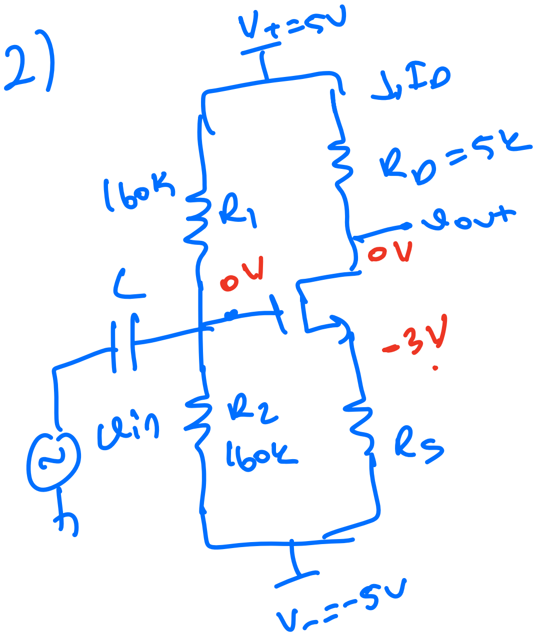
Similarly for r_o

$$\frac{1}{r_o} = \frac{\partial I_D}{\partial V_{DS}} = \lambda K_n (V_{GS} - V_{tn})^2 \Rightarrow r_o = \frac{1}{\lambda I_D}$$

is not accurate either, in this case.

* But you can use the expressions derived in the class. as we did in the solution.

2)



$K_N = 0.25\ \frac{mA}{V^2}$ $V_{tN} = 1V$, $\lambda = 0.01\ V^{-1}$

a) If we want 0V DC @ v_{out}

then $I_D = 1mA$

$V_G = 0$ note $R_1 = R_2$.

$V_S = -5 + R_S$

$1 = 0.25(5 - R_S - 1)^2$

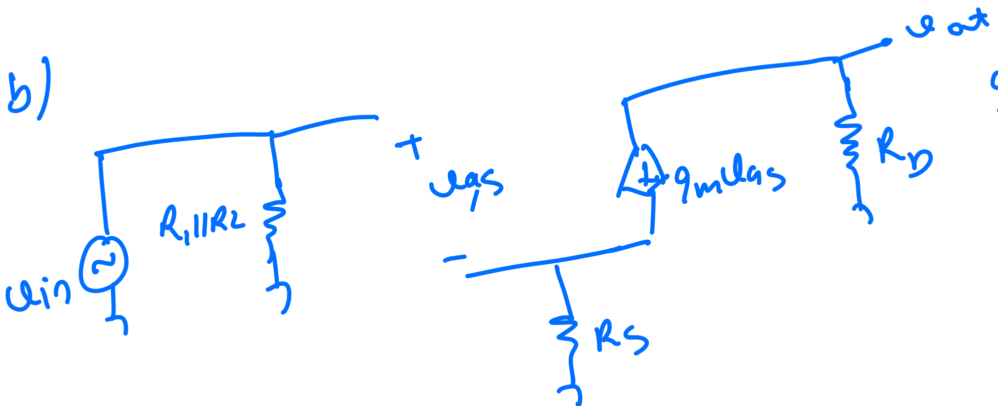
$4 = (4 - R_S)^2$

$4 = 16 - 8R_S + R_S^2$ $R_S = 6k$ × $V_{GS} < 0$ $\boxed{R_S = 2k}$

$V_S = 3V$, $V_{GS} - V_{tn} < V_{DS}$

$3 - 1 < 3$ SAT ✓

b)



$g_m = 2\sqrt{K_N I_D}$

$g_m = 2\sqrt{0.25 \cdot 1} = 1\ \frac{mA}{V}$

$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 \cdot 1} = 100k\Omega$

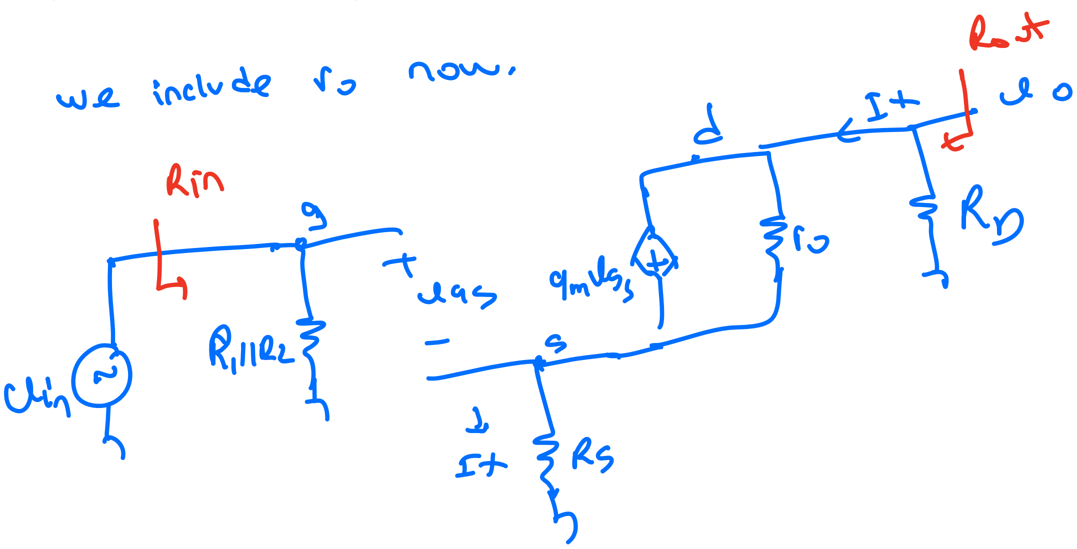
$v_{out} = -g_m v_{gs} R_D$ → $\frac{v_{out}}{v_{in}} = \frac{-g_m R_D}{1 + g_m R_S}$

$v_{in} = v_{gs} + g_m v_{gs} R_S$

$v_{in} = v_{gs}(1 + g_m R_S)$

$A_V = \frac{-1 \cdot 5}{1 + 1 \cdot 2} = 1.67\ V/V$

c) we include r_o now.



$v_{gs} = v_{in} - v_s$

KCL @ the source.

$$\frac{v_s}{R_S} + \frac{v_s - v_o}{r_o} - g_m v_{gs} = 0$$

$$\frac{v_s}{R_S} + \frac{v_s - v_o}{r_o} - g_m(v_i - v_s) = 0$$

$$v_s\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right) = g_m v_i + \frac{v_o}{r_o}$$

$$v_s = \left(g_m v_i + \frac{v_o}{r_o}\right) \cdot \left[R_S \| r_o \| \frac{1}{g_m}\right]$$

KCL @ the drain

$$g_m v_{gs} + \frac{v_o - v_s}{r_o} + \frac{v_o}{R_D} = 0$$

$$g_m(v_{in} - v_s) + \frac{v_o - v_s}{r_o} + \frac{v_o}{R_D} = 0$$

$$g_m v_{in} + v_o\left(\frac{1}{r_o} + \frac{1}{R_D}\right) = v_s\left(g_m + \frac{1}{r_o}\right)$$

$$g_m v_{in} + v_o\left(\frac{1}{r_o} + \frac{1}{R_D}\right) = \frac{1}{r_o \| \frac{1}{g_m}} \cdot \left[R_S \| r_o \| \frac{1}{g_m}\right] \cdot \left(g_m v_i + \frac{v_o}{r_o}\right)$$

$$= \frac{1}{r_o \| \frac{1}{g_m}} \cdot \frac{R_S \left\{r_o \| \frac{1}{g_m}\right)}{R_S + \left[r_o \| \frac{1}{g_m}\right)}$$

$$g_m v_{in} + v_o\left(\frac{1}{r_o} + \frac{1}{R_D}\right) = \frac{R_S}{R_S + \left[r_o \| \frac{1}{g_m}\right]} \cdot \left(g_m v_i + \frac{v_o}{r_o}\right)$$

$$g_m v_{in}\left[1 - \frac{R_S}{R_S + \left\{r_o \| \frac{1}{g_m}\right)}\right] = v_o\left[\frac{1}{r_o} \cdot \frac{R_S}{R_S + \left[r_o \| \frac{1}{g_m}\right]} - \frac{1}{r_o} - \frac{1}{R_D}\right]$$

$$g_m v_{in}\left[\frac{r_o \| \frac{1}{g_m}}{R_S + r_o \| \frac{1}{g_m}}\right] = v_o\left[\frac{1}{r_o} \cdot \frac{-r_o \| \frac{1}{g_m}}{R_S + r_o \| \frac{1}{g_m}} - \frac{1}{R_D}\right]$$

$$A_V = \frac{v_o}{v_{in}} = \frac{\dfrac{g_m\left[r_o \| \frac{1}{g_m}\right)}{R_S + r_o \| \frac{1}{g_m}}}{\dfrac{1}{r_o} \cdot \dfrac{-r_o \| \frac{1}{g_m}}{R_S + r_o \| \frac{1}{g_m}} - \dfrac{1}{R_D}} \cdot$$

$r_o \to \infty$

$$\frac{\dfrac{g_m \cdot \frac{1}{g_m}}{R_S + \frac{1}{g_m}}}{-\dfrac{1}{R_D}}$$

$$\boxed{A_V = \frac{-g_m R_D}{1 + g_m R_S}}$$

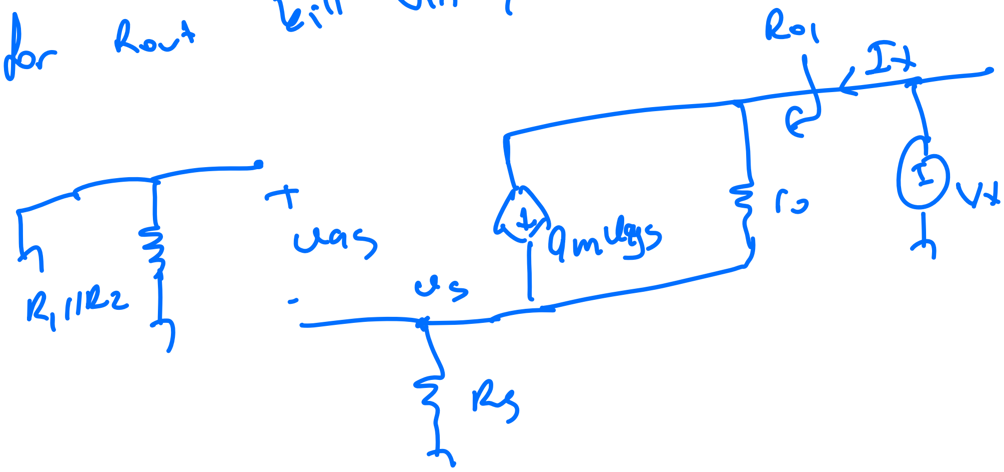
Same as part b.

with some reasonable simplification if we assume $r_o \| \frac{1}{g_m} \sim \frac{1}{g_m}$ $\quad r_o \gg \frac{1}{g_m}$

$$A_v = \frac{v_o}{v_{in}} \cong \frac{\dfrac{g_m \cdot \frac{1}{g_m}}{R_s + \frac{1}{g_m}}}{\dfrac{1}{r_o}\cdot\dfrac{\frac{1}{g_m}}{R_s+\frac{1}{g_m}} - \dfrac{1}{R_D}} = \frac{\dfrac{g_m}{1+g_mR_s}}{\underbrace{\dfrac{1}{r_o}\cdot\dfrac{1}{1+g_mR_s}}_{\text{reduces the gain}} - \dfrac{1}{R_D}}$$

d) $R_{in} = R_1 \| R_2 = 80k\Omega$.

for R_{out} kill vin, Let's add R_D at the end.



$v_{gs} = v_g - v_s = -v_s$

$v_s = I_x R_s$

$V_x = (I_x - g_m v_{gs}) r_o + I_x R_s$

$V_x = [I_x - g_m(-I_x R_s)] r_o + I_x R_s$

$V_x = I_x [(1+g_m R_s) r_o + R_s]$

$\frac{V_x}{I_x} = (1+g_m R_s) r_o + R_s = R_{o1}$

$R_{o1} = (1+1.2)\cdot 100 + 2$

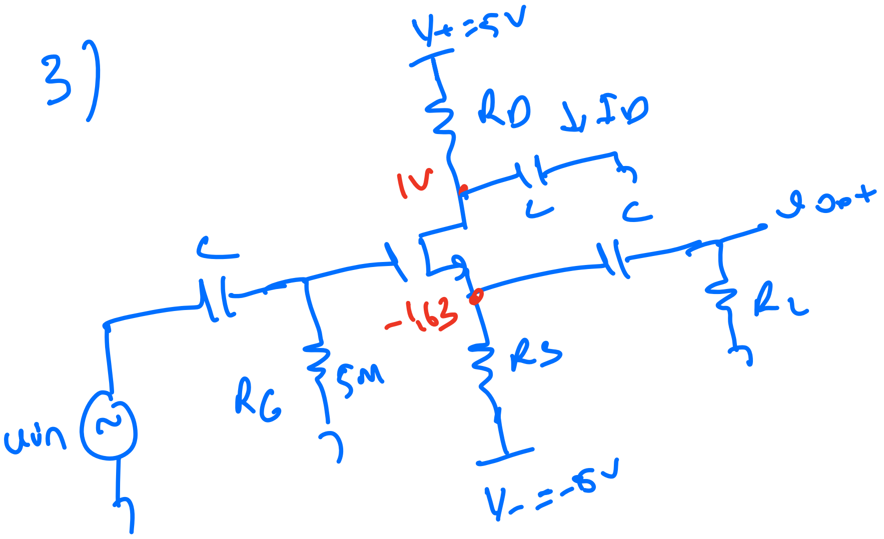
$R_{o1} = 302k$

$R_o = R_{o1} \| R_D$

$R_o = 302 \| 5$

$\boxed{R_o = 4{,}92\,k\Omega}$

3)



$K_N = 0{,}5\,\frac{mA}{V^2}$

$V_{tN} = 1V$, $\lambda = 0{,}01\,V^{-1}$

a) $I_{DQ} = 0{,}2\,mA$

$V_{DQ} = 1V$

$V_D = 5 - R_D I_D$

$1 = 5 - R_D \cdot 0{,}2$

$\boxed{R_D = 20k\Omega}$

$0{,}2 = 0{,}5\,(V_{GS} - 1)^2$

$V_{GS} - 1 = \pm\, 0{,}63 \Rightarrow V_{GS} = 1{,}63V$

$V_G = 0$, so $V_S = -1{,}63V$

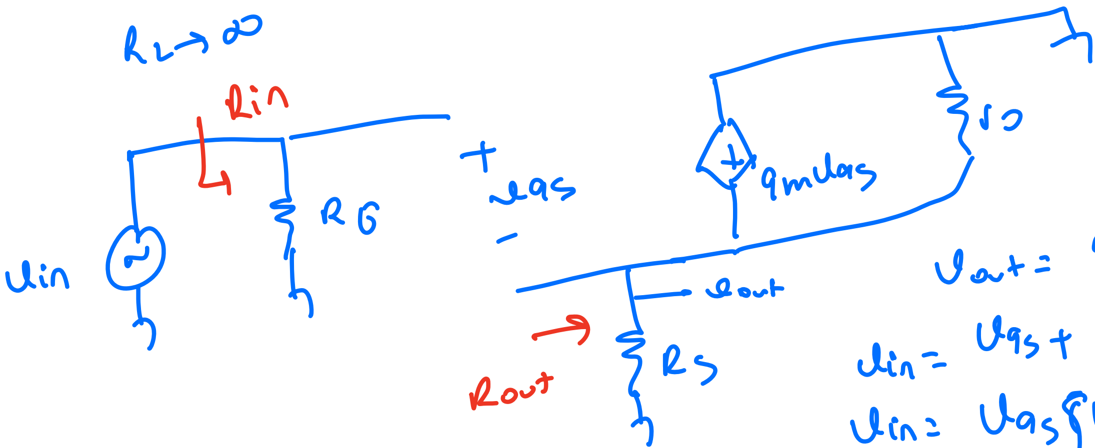
$-5 + R_S \cdot 0{,}2 = -1{,}63$ $\quad \boxed{R_S = 16{,}85\,k\Omega}$

$V_{GS} - V_{tn} < V_{DS}$ SAT ✓

$0{,}63 \quad\quad 2{,}63$

b) $g_m = 2\sqrt{0{,}5 \cdot 0{,}2} = 0{,}63\,\frac{mA}{V}$ $\quad r_o = \frac{1}{0{,}01 \cdot 0{,}2} = 500k$

$R_L \to \infty$



note that r_o and R_s are in parallel.

$v_{out} = g_m v_{gs} (r_o \| R_s)$

$v_{in} = v_{gs} + g_m v_{gs} (r_o \| R_s)$

$v_{in} = v_{gs} \{1 + g_m (r_o \| R_s)\}$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{g_m (r_o \| R_s)}{1 + g_m (r_o \| R_s)}$$

$$A_v = \frac{0{,}63 \cdot (500 \,//\, 20)}{1 + 0{,}63 \cdot (500 \,//\, 20)} = \frac{0{,}63 \cdot 19{,}23}{1 + 0{,}63 \cdot 19{,}23} = 0{,}92\,\frac{V}{V}$$

* this is a common drain amplifier and the gain should be <1

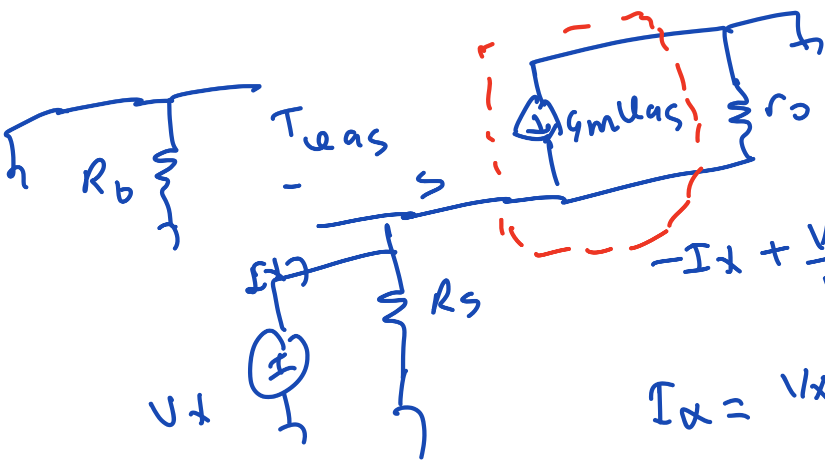
c) $R_{in} = R_G = 5M\Omega$

$R_{out} = \frac{1}{g_m} \parallel R_S \parallel r_o$ (short cut)

current source $= \frac{1}{g_m}$

$v_{gs} = v_g - v_s$

$v_{gs} = -v_s$ $\quad V_s = V_x$



KCL @ the source

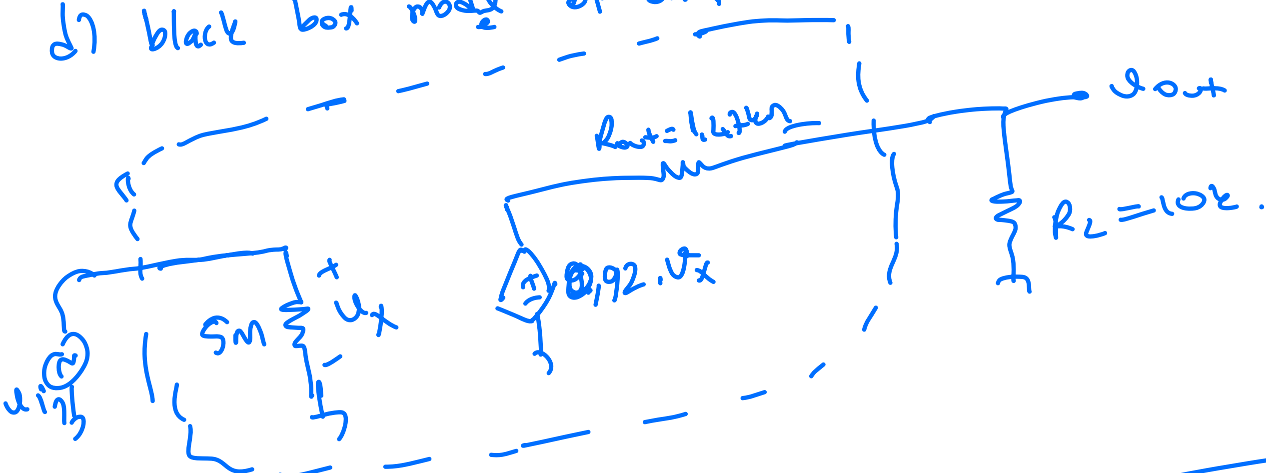
$$-I_x + \frac{V_x}{R_S} - g_m v_{as} + \frac{V_x}{r_o} = 0$$
$$+ g_m v_s$$

$$I_x = V_x \left(\frac{1}{R_S} + \frac{1}{r_o} + g_m \right)$$

$$R_{out} = \frac{V_x}{I_x} = \frac{1}{\frac{1}{R_S} + \frac{1}{r_o} + g_m} = R_S \parallel r_o \parallel \frac{1}{g_m}$$

$R_{out} = 500 \parallel 20 \parallel 1{,}59 = 19{,}23 \parallel 1{,}59 \Rightarrow \boxed{R_{out} = 1{,}47\,k\Omega}$

d) black box model of ampl.



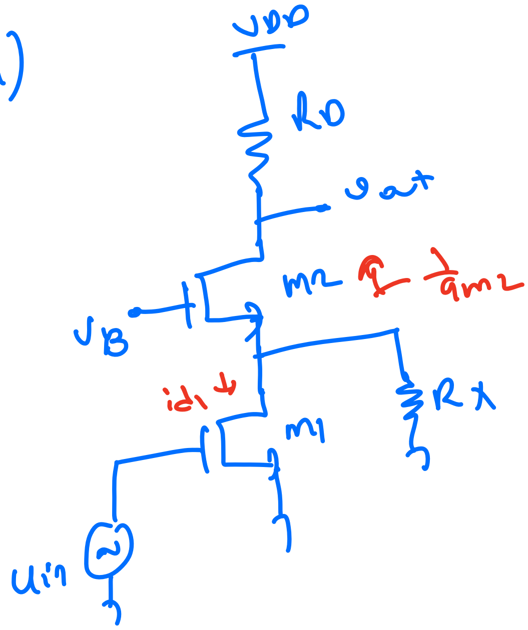
no source resistance $V_{in} = V_x$

$$A_V = \frac{v_{out}}{v_{in}} = \frac{R_L}{R_L + R_{out}} \cdot 0{,}92 = \frac{10}{10 + 1{,}47} \cdot 0{,}92 \Rightarrow \boxed{A_V = 0{,}8 \ \frac{V}{V}}$$

note you would get the same result if R_L was included in part b

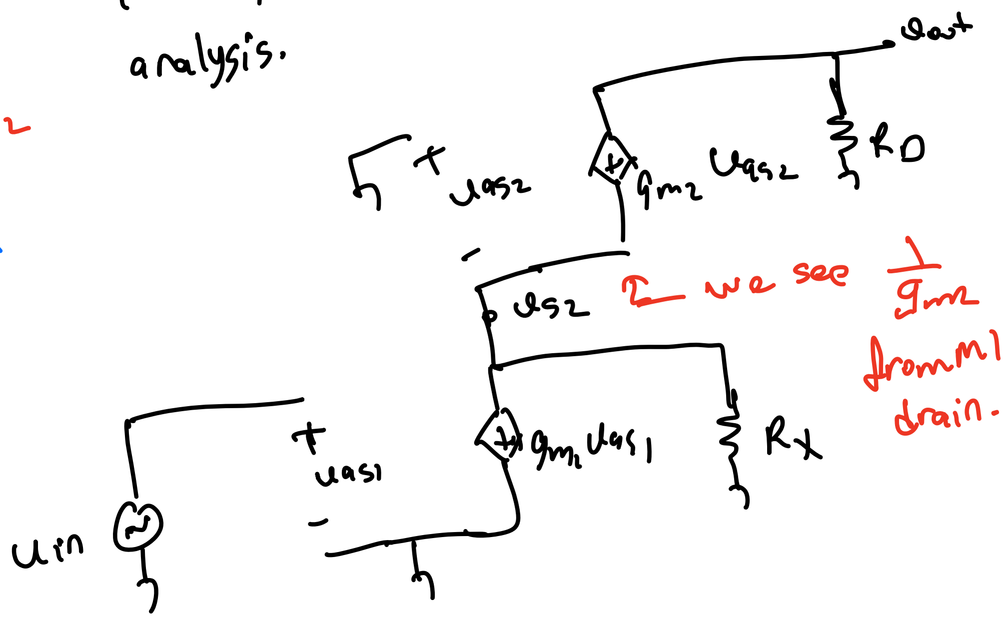
then gain would be. $A_V = \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)}$

4)



m1 and m2 are assumed to be in SAT.

$\lambda = 0$, no values here just symbolic analysis.



$v_{in} = v_{gs1}$

$v_{out} = -g_{m2} v_{gs2} R_D$

$v_{gs2} = v_g - v_{s2} = -v_{s2}$

KCL @ V_{s2}

$$g_{m1} v_{gs1} + \frac{v_{s2}}{R_x} - g_{m2} v_{gs2} = 0$$

$$g_{m1} v_{in} + \frac{v_{s2}}{R_x} + g_{m2} v_{s2} = 0$$

$$g_{m1} v_{in} = -V_{s2}\left(\frac{1}{R_x} + g_{m2}\right)$$

$$v_{s2} = \frac{-g_{m1}}{\frac{1}{R_x} + g_{m2}} v_{in} \quad \text{---} \; (*)$$

$v_{out} = g_{m2} v_{s2} R_D$

using (*)

$$A_v = \frac{v_{out}}{v_{in}} = -g_{m2} R_D \cdot \frac{g_{m1}}{\frac{1}{R_x} + g_{m2}}$$

$$A_v = -g_{m2} R_D \cdot \frac{g_{m1} R_x}{1 + g_{m2} R_x}$$

Short solution.

$i_{d1} = g_{m1} v_{in}$

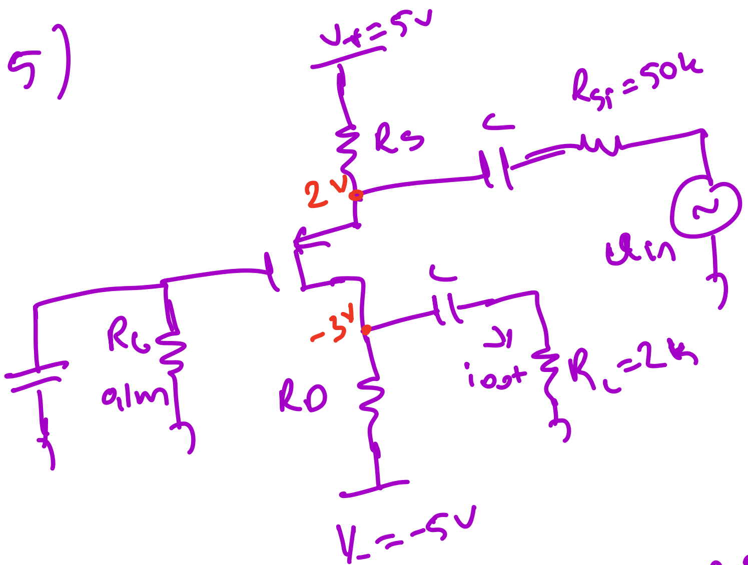
i_{d1} is divided between $\frac{1}{g_{m2}}$ and R_x and converted into voltage through R_D.

$$v_{out} = -g_{m1} v_{in} \frac{R_x}{R_x + \frac{1}{g_{m2}}} \cdot R_D \quad \leftarrow \text{current division.}$$

$$\frac{v_{out}}{v_{in}} = -g_{m1} R_D \frac{g_{m2} R_x}{1 + g_{m2} R_x} \rightarrow \text{same result.}$$

the gain is (-) since when $v_{in}\uparrow$ $i_d\uparrow$ and $v_{out}\downarrow$ due to inversion.

5)



$V_{tp} = -1V \quad K_p = 0,5\ mA/V^2, \lambda = 0$

a) $I_{DQ} = 0,5mA \quad V_{SDQ} = 5V$

$V_{SD} + (R_S + R_D) I_D = 10V$

$(R_S + R_D) \cdot 0,5 = 5$

$R_S + R_D = 10k$

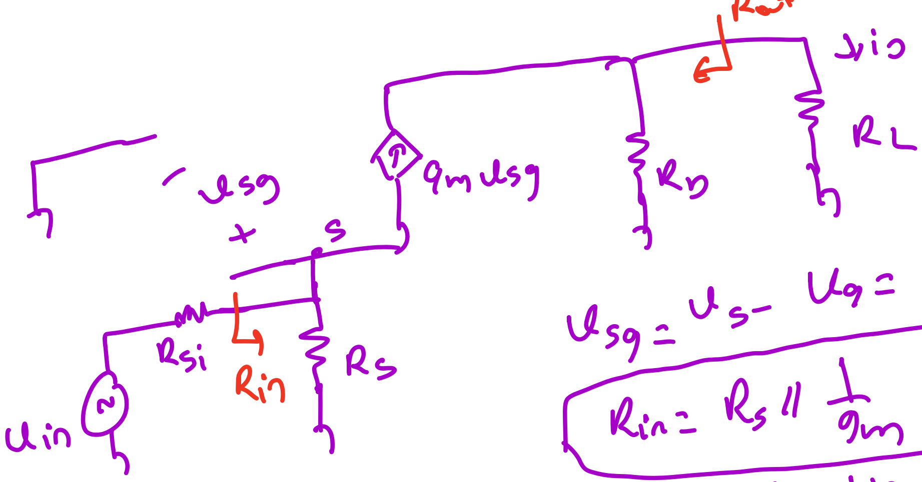
$0,5 = 0,5 (\underbrace{5 - R_S \cdot 0,5}_{V_S} - 0 - 1)^2$

$V_S - 1 = \pm 1 \quad \underline{V_S = 2V}$

$\boxed{R_S = 6k \quad R_D = 4k}$

$\left.\begin{array}{l} V_{SG} = 2V \\ V_{SD} = 5V \end{array}\right\} \begin{array}{l} V_{SG} - 1 < V_{SD} \\ \text{SAT} \checkmark \end{array}$

b)



$g_m = 2\sqrt{0,5 \cdot 0,5}$

$g_m = 1 \frac{mA}{V}$

$v_{sg} = v_s - v_g = \underline{v_s}$

$R_{in} = R_S \,//\, \frac{1}{g_m} = 6//1 = \frac{6}{7} k\Omega.$

looking towards the source we see $\frac{1}{g_m}$

$\underline{R_{out}}$ kill v_{in}.

$v_s = -g_m v_{sg} (R_S // R_{si})$

$v_{sg} = v_s$

$v_s = -g_m v_s (R_S // R_{si})$

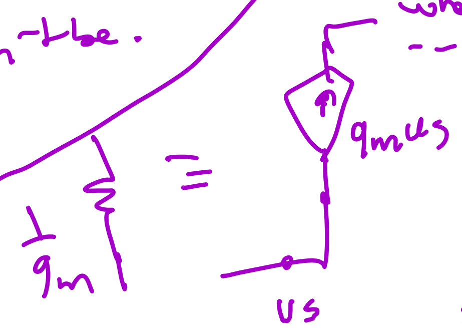
$v_s = 0$

or $g_m = -\frac{1}{R_S // R_{si}}$ ✗ can't be.

so $g_m v_{sg} = 0$ open ckt.

$\boxed{R_{out} = R_D = 4k\Omega}$

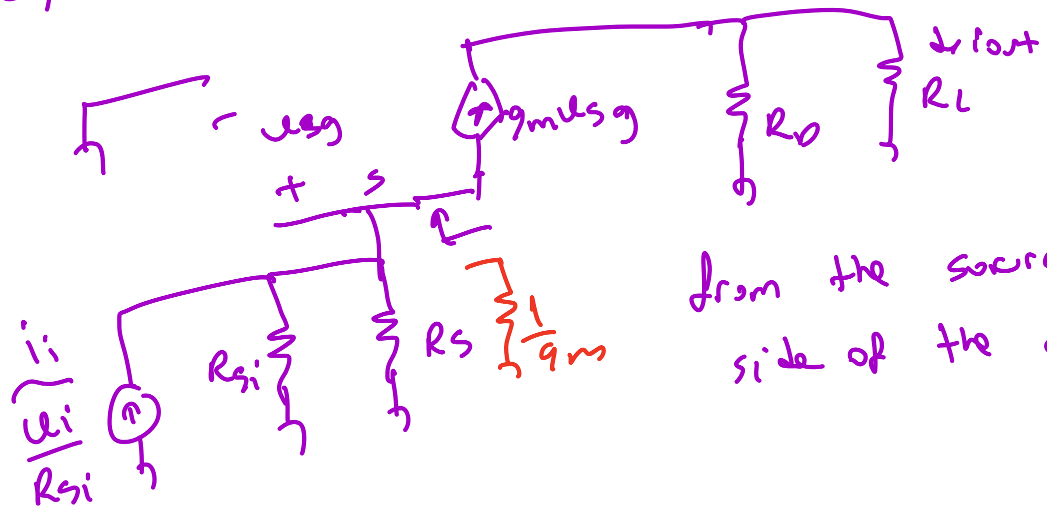
doesn't matter what is connected in here ---



$= \frac{v_s}{g_m v_s} = \frac{1}{g_m}$

=

c) since the output is current let's work in terms of current.



from the source side the drain side of the ckt seems like $\frac{1}{g_m}$.

$$\frac{i_o}{i_i} = \underbrace{\frac{R_{si} \parallel R_S}{R_{si} \parallel R_S + \frac{1}{g_m}}}_{\text{current on } \frac{1}{g_m} \text{ flowing to the drain}} \cdot \underbrace{\frac{R_D}{R_D + R_L}}_{\text{current division at the output}}$$

$$i_o = \frac{R_{si} \parallel R_S}{R_{si} \parallel R_S + \frac{1}{g_m}} \cdot \frac{R_D}{R_D + R_L} \cdot \underbrace{\frac{v_i}{R_{si}}}_{i_i}$$ in terms of

$$i_o = \frac{50 \parallel 6}{50 \parallel 6 + 1} \cdot \frac{4}{4+2} \cdot i_i$$

$$i_o = \frac{5{,}35}{5{,}35+1} \cdot \frac{2}{3} \cdot i_i \Rightarrow i_o = 0{,}56\, i_i$$

$$i_o = 0{,}56 \cdot \frac{0{,}5 \cos wt}{50\text{k}}$$

$$\boxed{i_o = 5{,}6 \cdot \cos wt \ \text{nA}}$$

$v_{out} = g_m v_s (R_D \parallel R_L)$

$v_s = \frac{1/g_m \parallel R_S}{\frac{1}{g_m} \parallel R_S + R_{si}} \cdot v_{in}$ or

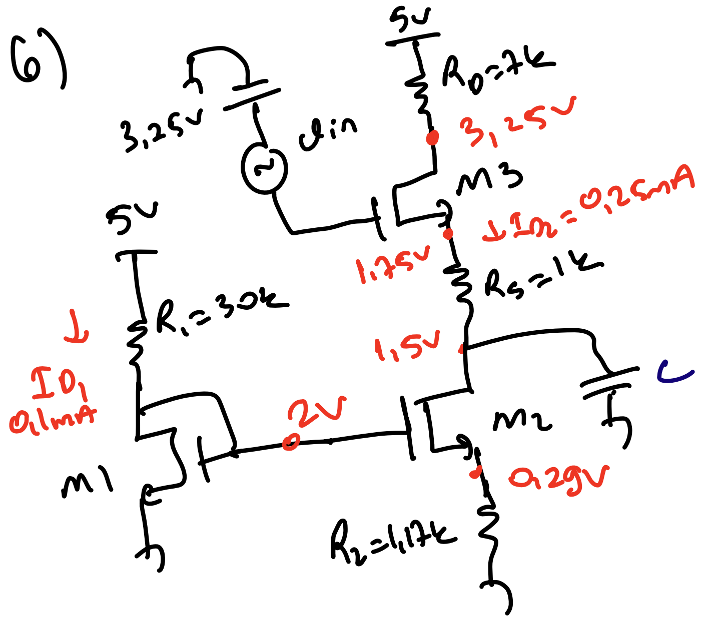
$\frac{1}{g_m}$ 1k, R_{si} 50k

$R_{si} >> \frac{1}{g_m}$ Voltage kills the gain.

$V_o = i_o \cdot R_L = 11{,}2 \cos wt \ \mu V$

$$\boxed{\frac{V_o}{v_i} = 0{,}0224 \ \frac{V}{V}}$$ very small voltage gain.

6)



$k_{n_1} = 0,1\ mA/V^2$ $\quad k_{n_2} = 0,5\ mA/V^2$

$k_{n_3} = 1\ mA/V^2$ $\quad V_{th} = 1V$, $\lambda = 0$

a) M_1 is in SAT. $V_{GS_1} = V_{DS_1}$

$V_{GS_1} = 5 - 30 I_{D_1}$

$I_{D_1} = 0,1 \cdot (5 - 30 I_{D_1} - 1)^2$

$10 I_{D_1} = (4 - 30 I_{D_1})^2$

$10 I_{D_1} = 16 - 240 I_{D_1} + 900 I_{D_1}^2$

$900 I_{D_1}^2 - 250 I_{D_1} + 16 = 0$

$I_{D_1} = 0,18mA$ ✗ $V_{GS_1} < V_{th}$

$\boxed{I_{D_1} = 0,1\ mA}$ $\quad V_{GS_1} = 2V$

M2 $\quad V_{S_2} = 1,17 I_{D_2}$

$I_{D_2} = 0,5 (2 - 1,17 I_{D_2} - 1)^2$

$2 I_{D_2} = (1 - 1,17 I_{D_2})^2$

$2 I_{D_2} = 1 - 2,34 I_{D_2} + 1,17^2 I_{D_2}^2$

$I_{D_2} = 2,92 mA$ ✗ $V_{GS_2} < V_{th}$

$\boxed{I_{D_2} = 0,25 mA}$

$V_{S_2} = 0,25 \cdot 1,17 = 0,29V$

M3: $I_{D_2} = I_{D_3} = 0,25 mA$

$0,25 = 1 (V_{GS_3} - 1)^2$

$V_{GS_3} - 1 = \pm 0,5$

$V_{GS_3} = 1,5V$

$V_{G_3} = 3,25V$

$V_{S_3} = 1,75V$

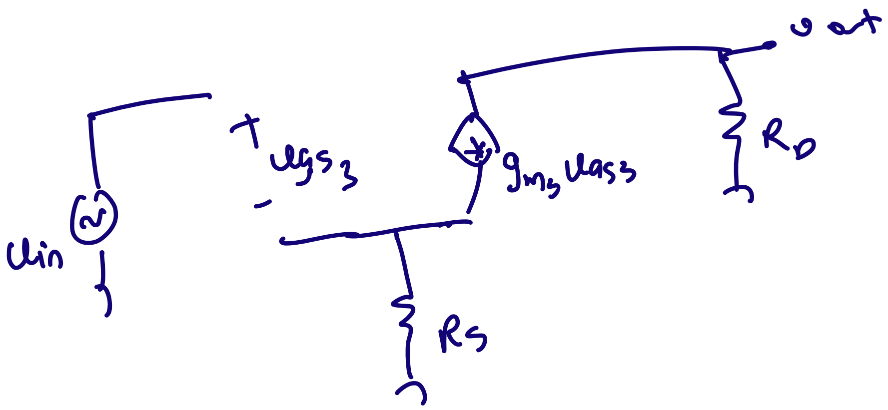
M1 : $V_{DS} = V_{GS}$ $\quad$ SAT ✓

M2 $\quad V_{D_2} > V_{G_2} - 1$ $\quad$ SAT ✓

$\quad 1,5 \quad\quad 2 - 1$

M3 $\quad V_{D_3} = V_{G_3}$ $\quad$ SAT ✓

b) for small signal C grounds the drain of m2 so the ckt simplifies to



$g_m = 2\sqrt{1 \cdot 0{,}25}$

$g_m = 1\ mA/V$

$v_{out} = -g_{m_3} v_{gs3} R_D$

$v_{in} = v_{gs_3} + g_{m_3} v_{gs_3} R_S$

$A_v = \frac{v_{out}}{v_{in}} = \frac{-g_{m_3} R_D}{1 + g_{m_3} R_S}$

$\boxed{A_v = \frac{-1.7}{1 + 1.1} = -3{,}5\ V/V}$

m2 does not matter because C gnds the drain of m2. all the small signal current flows through C to gnd w/o passing through m2.