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Signature: ______________

EEE 313 Spring 2012
(Corrected)

Bilkent University
Department of Electrical and Electronics Engineering
EEE 313 Electronic Circuit Design **Midterm Examination #2**
18 April 2012, 17.40 (4 questions, 120 minutes)

- This is a **closed book**, closed notes exam. No cheat sheet allowed.
- All cell-phones should be completely **turned off.**
- Use a calculator for numerical computations. Carry at least **3 significant digits.** Double check your numerical calculations.
- Be sure to write the **units** of all numerical results.
- **Show** all work clearly. Show all the intermediate steps including formulas. **Never, just carry out calculations without formulas and explanation.**
- Please put your **final answer** for each part inside a box for easy identification. Do not give multiple answers, they will not be graded.
- Do not remove the **staple** from the exam sheets or separate pages of the exam. All extra pages must be stamped to your exam.
- You may leave the exam room when you are done. However, please do not leave during the **last five minutes** of the exam.
- At the end of the exam, please stay seated unitl **all** exam papers are collected.

Useful constants and formulas:

Boltzmann's constant: 86×10^{-6} eV/K°

Electron charge: 1.6×10^{-19} Coulombs

Drain currrent equation for n-channel MOSFET:

$I_D = K_n(V_{GS} - V_{TN})^2$ for $V_{GS} - V_{TN} \leq V_{DS}$

$I_D = K_n(2(V_{GS} - V_{TN})V_{DS} - {V_{DS}}^2)$ for $V_{GS} - V_{TN} \geq V_{DS}$

Drain currrent equation for p-channel MOSFET:

$I_D = K_p(V_{SG} + V_{TP})^2$ for $V_{SG} + V_{TP} \leq V_{SD}$

$I_D = K_p(2(V_{SG} + V_{TP})V_{SD} - {V_{SD}}^2)$ for $V_{SD} + V_{TP} \geq V_{SD}$

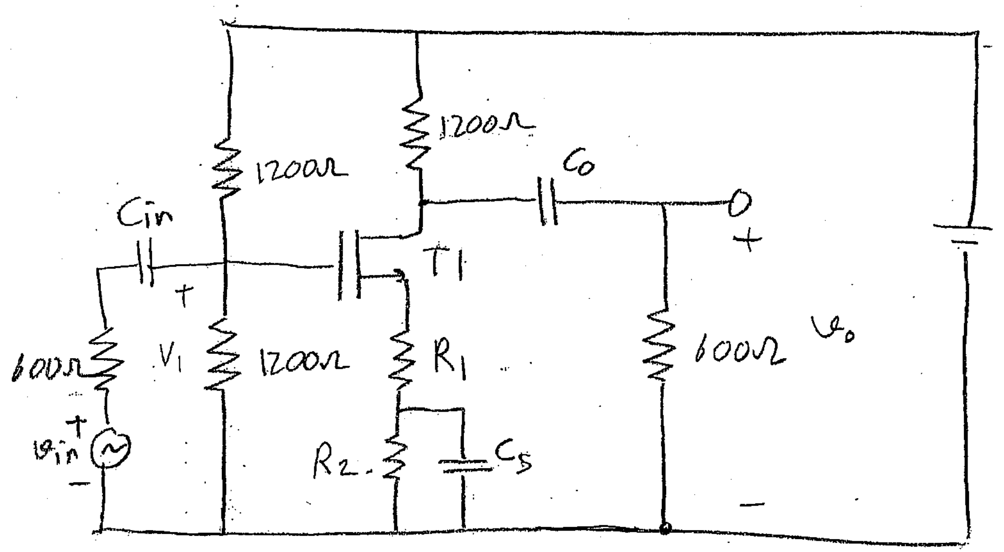
Forward active current of BJT is given by:

$$I_C = I_S\left(e^{\frac{V_{BE}}{nV_t}} - 1\right)$$

Other equations must be deduced by the students.

Please do not write below this line

1. 30 pts.	
2. 20 pts.	
3. 25 pts.	
4. 25 pts.	
Total 100 pts.	



1. (30 points) At the circuit given above, C_{in}, C_o and C_s are very large. $K_n=1\times10^{-2}A/V^2$. $V_{TN}=2V$

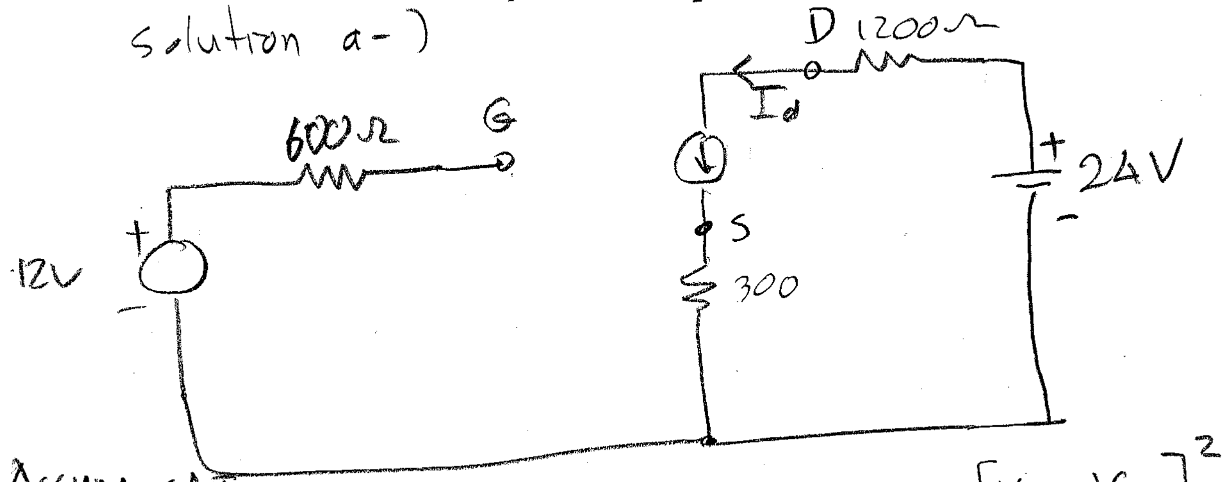
a) (7 points) Find the DC bias of the transistor, $R_1+R_2=300\Omega$

b) (7 points) Draw the AC equivalent circuit,

c) (8 points) Find the maximum undistorted peak-to-peak output swing of the circuit assuming that the drain current is equal to 10mA and the transistor is at nonsaturation with $R_1=0$ and $R_2=300\Omega$.

d) (8 points) Find the maximum undistorted peak-to-peak output swing of the circuit assuming that the drain current is equal to 10mA and the transistor is at nonsaturation with $R_1=100\ \Omega$ and $R_2=200\Omega$.

Solution a-)



Assume SAT

$$12V = V_{GS} + I_d\,300 = V_{GS} + K_n\left[V_{GS}-V_{TN}\right]^2$$

$$\cancel{12V} = V_{GS} + 300\times10^{-2}\left[V_{GS}^2-4V_{GS}+4\right] = V_{GS}+3V_{GS}^2-12V_{GS}+\cancel{12}$$

$$3V_{GS}^2-11V_{GS}=0 \Rightarrow V_{GS_1}=0 \quad V_{GS_2}=\frac{11}{3}=3.666\,V$$

$V_S = 12 - V_{GS} = 12 - 3.666 = 8.333V$

$I_D = \frac{8.333V}{300\Omega} = 27.77mA$

$V_{DS} = V_{CC} - I_D [R_S + R_D] = 24 - 27.77\times10^{-3}[300+1200]$

$= -17.66V \Rightarrow$ nonplausable $\Rightarrow$ not a solution $\Rightarrow$

Transistor is at non-sat

Assuming non-sat

$12V = V_{GS} + 300K_n [2[V_{GS} - \overset{V_{TN}}{2}]V_{DS} - V_{DS}^2]$

$24V = (300+1200)K_n[2[V_{GS}-2]V_{DS} - V_{DS}^2] + V_{DS}$

$24 = 1500\times10^{-2}[2[V_{GS}-2]V_{DS} - V_{DS}^2] + V_{DS}$

$12 = V_{GS} + 3\{2[V_{GS}-2]V_{DS} - V_{DS}^2]$

$-60 = -5V_{GS} + 15[2[V_{GS}-2]V_{DS} - V_{DS}^2]$

+

$24 - 60 = -5V_{GS} + V_{DS}$

$-36 = -5V_{GS} + V_{DS} \Rightarrow 5V_{GS} = V_{DS} + 36$

$\Rightarrow \boxed{V_{DS} = 5V_{GS} - 36}$

$12 = V_{GS} + 3[2[V_{GS}-2][5V_{GS}-36] - [5V_{GS}-36]^2]$

$12 = V_{GS} + 3[2[5V_{GS}^2 - 46V_{GS} + 72] - [25V_{GS}^2 - 360V_{GS} + 1296]]$

$12 = V_{GS} + 3[10V_{GS}^2 - 92V_{GS} + 144 - 25V_{GS}^2 + 360V_{GS} - 1296]$

$12 = V_{GS} - 45V_{GS}^2 + 804V_{GS} - 3456$

$\underbrace{+45}_{a}V_{GS}^2 \underbrace{-805}_{b}V_{GS} + \underbrace{3468}_{c} = 0$

$V_{GS} = 7.23V$, $10.65804\,V$ too large to make $V_{gs} < V_{TN}$ *

$I_D = 10^{-2}\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right] = 0.015897\,A = 15.97\,mA$

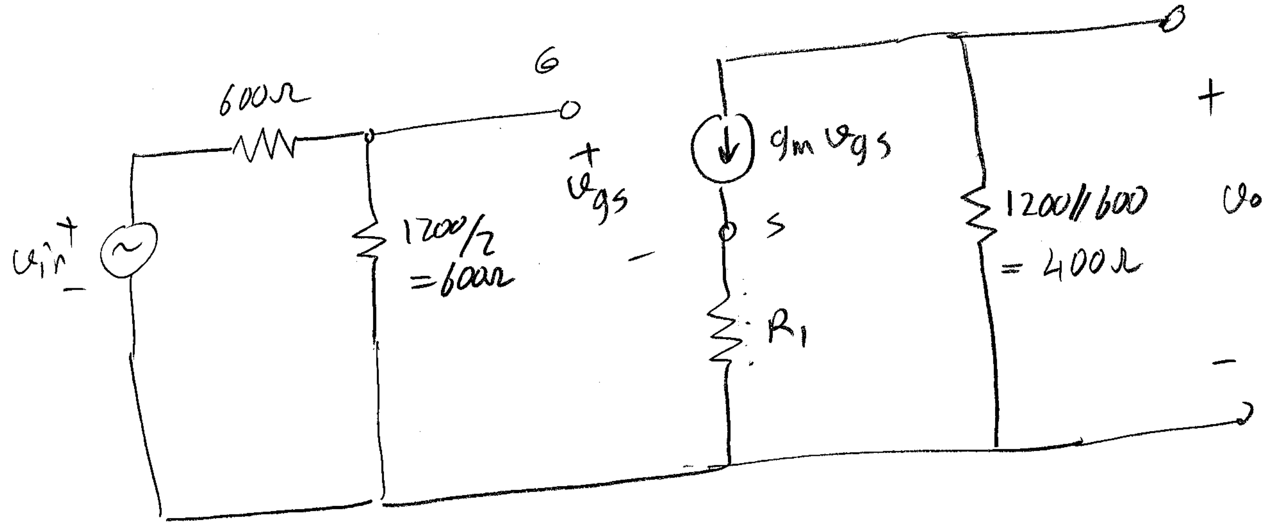
$V_{DS} = 24 - I_D(300 + 1200) = 24 - 1500 I_D = 0.15423\,V$

$\boxed{V_{DS} = 5V_{GS} - 36 = 0.15423V}$ solution is correct

$\boxed{V_{GS} = 7.23\,V}$

$\boxed{I_D = 15.97\,mA}$

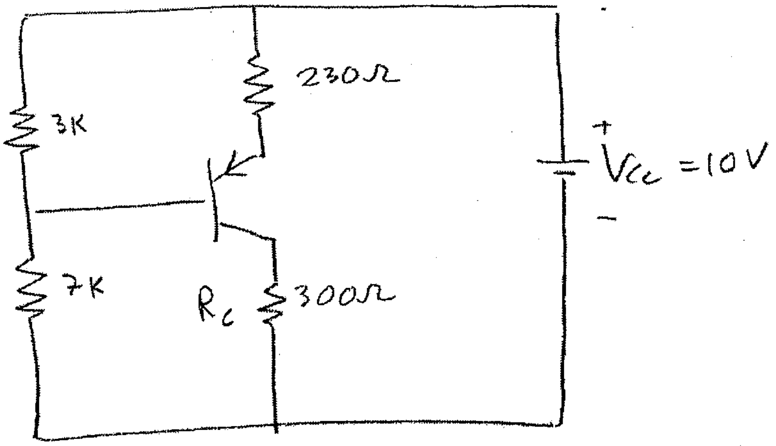
b-) The AC equivalent circuit



$$g_m = \frac{d}{dV_{GS}}\left\{K_n\left[2(V_{GS}-2)V_{DS} - V_{DS}^2\right]\right\} = 10^{-2}\left[2V_{DS}\right]$$

$$g_m = 2\times10^{-2}\times0.15423 = 0.0030846\,S = 3.08\times10^{-3}\,S$$

c-) & d-) are cancelled because of misspelling of "saturation" as "nonsaturation"



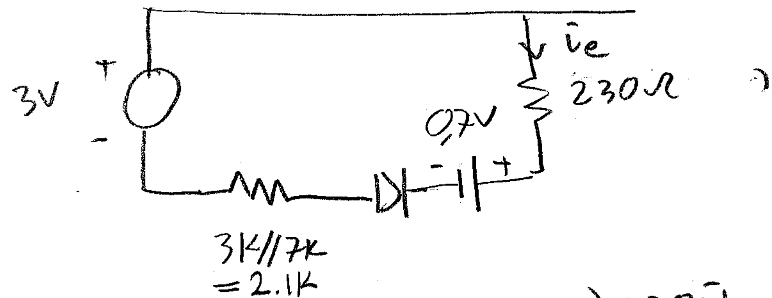
2. (20 points) At the circuit given above, β=99, $V_{\cancel{BE}SAT}$=0.7Volts and $V_{\cancel{CE}SAT}$=0.3Volts.
Please answer the following: EB EC

a) (10 points) Find the bias (Voltages and currents of the transistor) of the transistor and verify it.

b) (10 points) What should be the value of the collector resistor (R_c) in order to bias the transistor at just the borderline of saturation?

Assuming forward active

a-)



$$3V = i_b 2100 + 0.7 + (99+1)230 i_b \Rightarrow$$

$$\frac{2.3V}{(23000+2100)\Omega} = i_b = \frac{2.3V}{25100\Omega} = 9.16\times10^{-5}A = 91.6\mu A$$

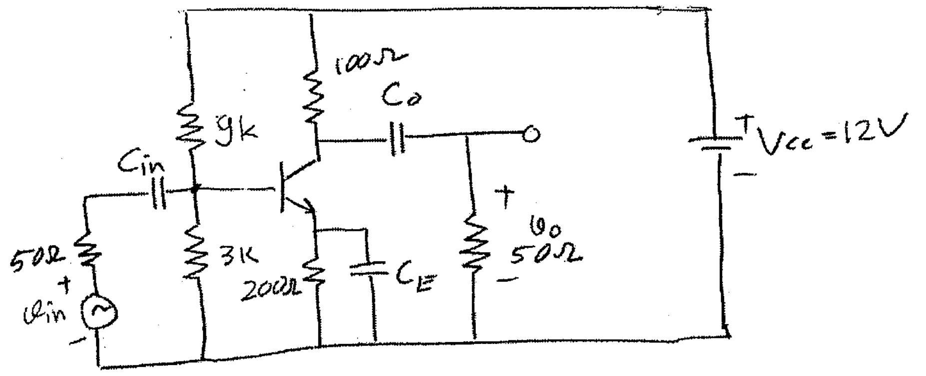
$$i_c = 99 i_b = 9.07mA \qquad i_e = (99+1) i_b = 9.16\,mA$$

$$V_{EC} = V_{CC} - I_c(230+300) = 10 - 9.1\times10^{-3}\,530 = 5.14V$$

$$V_{EC} = V_{EC_{SAT}} = 0.3 = 10V - I_c(230+R) \Rightarrow$$

$$\frac{10-0.3V}{I_c} = 230+R \Rightarrow \frac{9.7}{9.1\times10^{-3}} = 1065.9 = 230+R \Rightarrow$$

$$R = 1066 - 230 = 836\Omega$$



3. (25 points) At the circuit given above, C_{in}, C_o and C_s are very large and β=79, V_{BESAT}=0.6Volts and V_{CESAT}=0.2Volts. Please answer the following:

a) (7 points) Find the bias of the transistor,

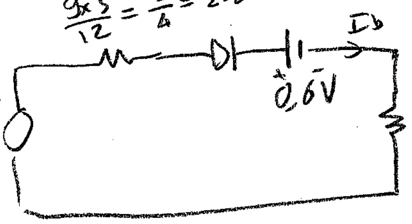
b) (8 points) Find the voltage gain of the circuit defined as Vo/Vin assuming that the quiescent collector current is equal to 10.5mA,

c) (10 points) Find the output undistorted voltage swing of the circuit assuming that the quiescent collector current is equal to 10.5mA,

a-)

$\frac{9\times 3}{12} = \frac{9}{4} = 2.25k$

$12 \times \frac{3}{9+3} = 3V$



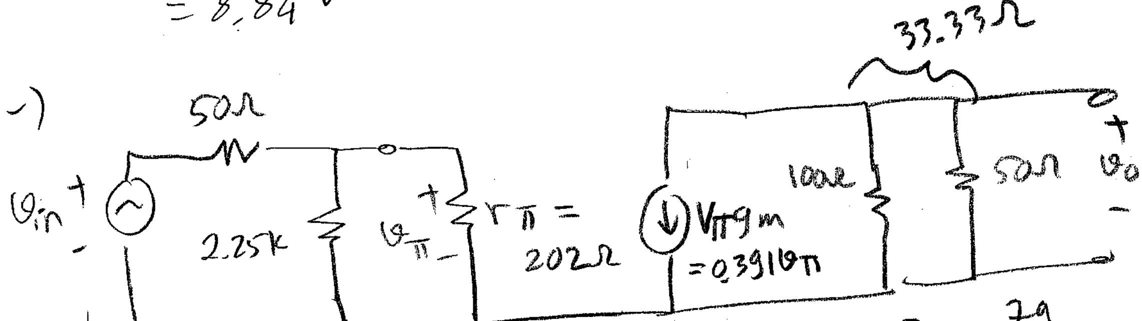
$\rightarrow \frac{3-0.6}{2250+80\times 200} = 1.315\times 10^{-4} A = 131.5\,\mu A = I_B$

$I_C = 79 I_B = 10.4\,mA$

$I_E = 80 I_B = 10.5\,mA$

$V_{CE} = 12V - (10.5\times 10^{-3})(100+200)$
$= 8.84\,V$

b-)

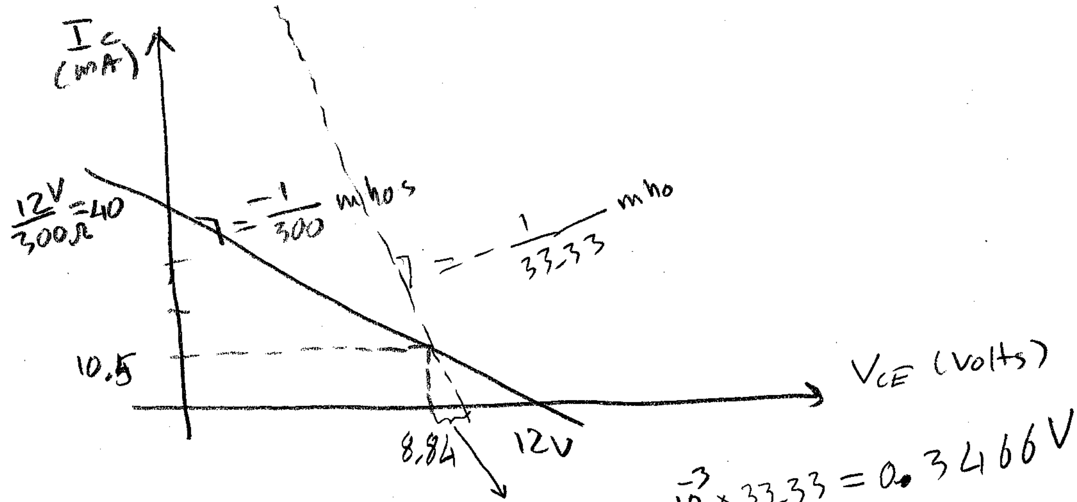


$g_m = \frac{10.4\,mA}{26.6\,mV} = 0.3918$ $\quad r_\pi = \frac{\beta}{g_m} = \frac{79}{0.391} = 202.05\,\Omega$

$100 // 50\,\Omega = \frac{100}{3} = 33.33\,\Omega$ $\quad 2.25k // 0.202k = 185\,\Omega$

$$\frac{v_o}{v_{in}} = \frac{-185}{185+50} \cdot 0.391 \times 33.33 = -10.26 = A_v$$

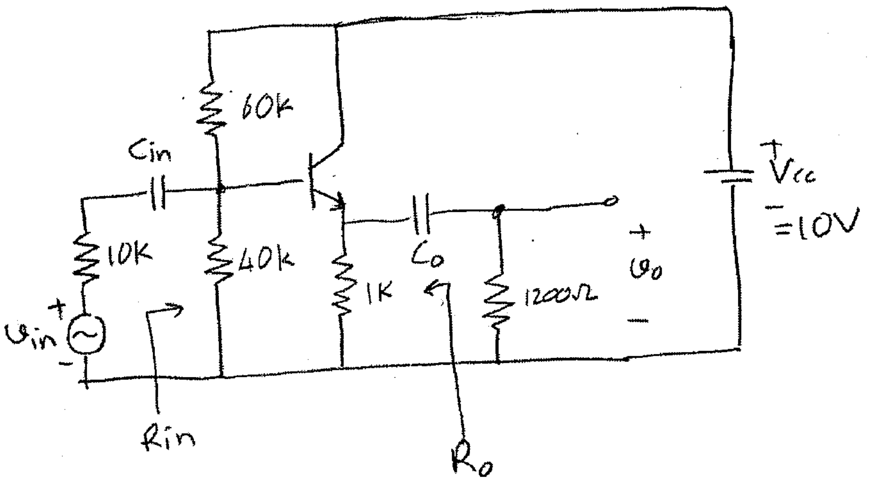
(c)



$$V_+ = 10.4 \times 10^{-3} \times 33.33 = 0.3466V$$

$$V_- = \sim 8.84V \Rightarrow$$

$$V_{p\text{-}p\ \text{swing}} = 2 \min\{V_-, V_+\} = 2 \times 0.3466 = 0.693V$$

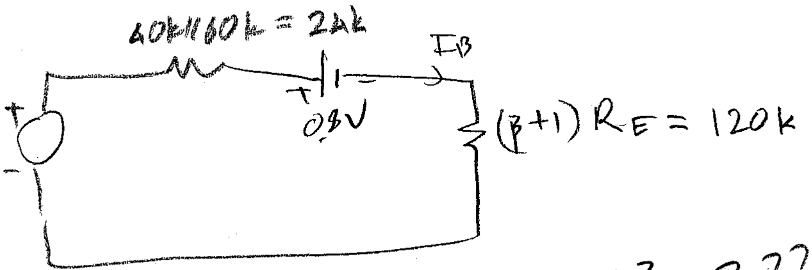


4. (25 points) At the circuit given above, C_{in} and C_o are very large and β=119, V_{BESAT}=0.8Volts and V_{CESAT}=0.4Volts. Please answer the following:

a) (7 points) Find the DC bias of the transistor,
b) (8 points) Find the voltage gain of the amplifier defined as Vo/Vin assuming that C_{in} and C_o are very large and the collector current is equal to 3mA.
c) (10points) Find the input and output impedances of the circuit as shown above assuming that C_{in} and C_o are very large and the collector current is equal to 3mA.

a -)

$V_{th} = 10 \times \frac{40}{40+60} = 4V$

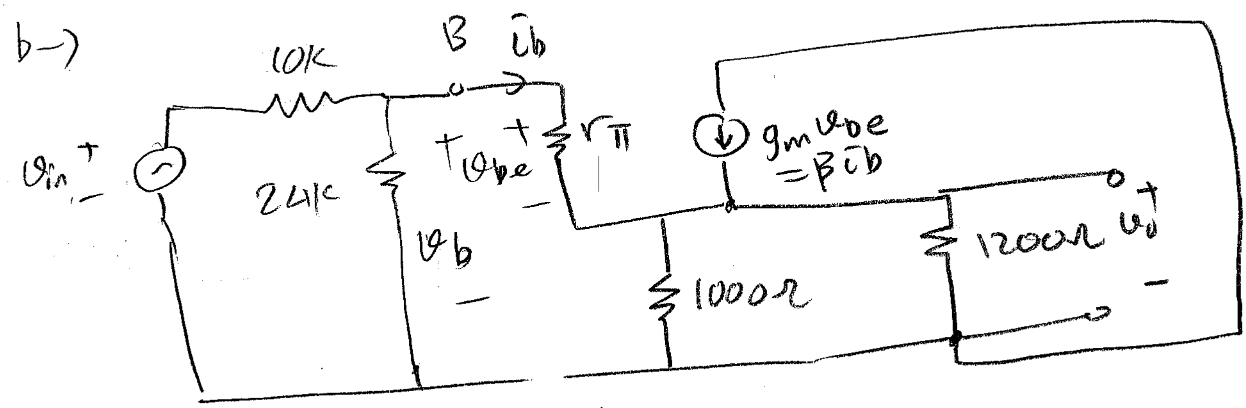


$I_B = (4V - 0.8V)/((120+24)\times 10^3) = 2.22\times 10^{-5} A = 22.2\mu A$

$I_C = \beta I_B = 119 \times 22.2\mu A = 2.644\,mA$ $\quad I_E = (\beta+1) I_B = 2.666\,mA$

$V_{CE} = V_{CC} - I_E R_E = 10 - 2.666mA \times 1000 = 7.33V$

b-)



$g_m = \frac{3.\ \ mA}{26.6mV} = 0.11288$ $\qquad r_\pi = \frac{119}{\ } = 1055\Omega$

$R_C // R_L = 545.5\Omega$

$$v_b = v_{in} \cdot \frac{((\beta+1)(R_C // R_L) + r_\pi) // 24k}{((\beta+1)(R_C // R_L) + r_\pi) // 24k + 10K}$$

$$[(\beta+1) R_C // R_L + r_\pi] = 545.4 \times 120 + 1055 = 66509\,\Omega$$

$$[(\beta+1)(R_C // R_L + r_\pi] // 24000 = 17636\,\Omega \Rightarrow$$

$$v_b = \frac{17636}{17636 + 10000} v_{in} = 0.638\, v_{in}$$

$$v_o = v_b \cdot \frac{(R_L // R_C)(\beta+1)}{(R_C // R_L)(\beta+1) + r_\pi} = \frac{R_L // R_C}{R_L // R_C + \frac{r_\pi}{\beta+1}}$$

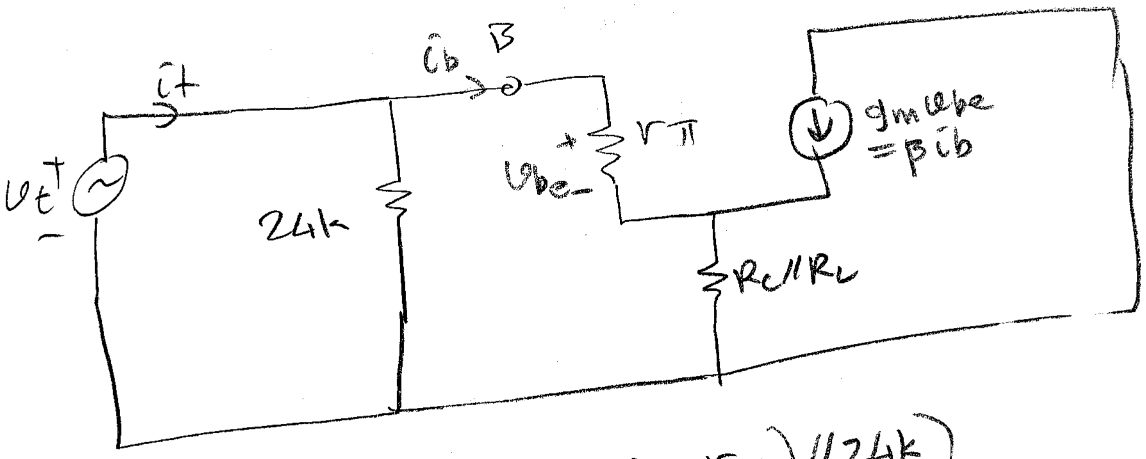
$$v_o = v_b \cdot \frac{R_L // R_C}{R_L // R_C + \frac{\beta}{g_m(\cancel{\beta+1})}} = \frac{R_L // R_C}{R_L // R_C + \frac{1}{g_m}} = \frac{545.4}{545.4 + \frac{1}{0.1158}}$$

$$\frac{v_o}{v_b} = \frac{545.4}{545.4 + 8.6} = \frac{545.4}{554} = 0.984$$

$$\frac{v_o}{v_{in}} = \frac{v_o}{v_b} \cdot \frac{v_b}{v_{in}} = 0.984 \times 0.638 = 0.625$$

Input impedance:

Drawing the AC equivalent circuit again



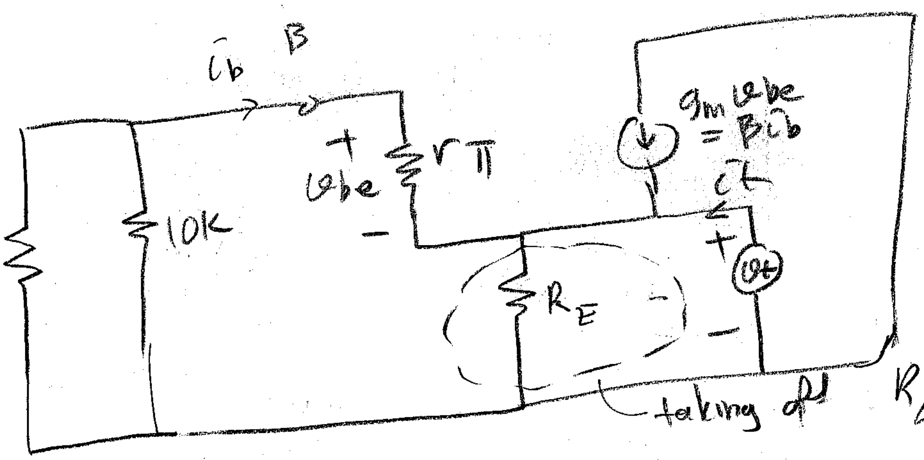
$$R_{in} = \frac{v_t}{i_t} = \left((R_C // R_L)(\beta+1) + r_\pi\right) // 24k)$$

$$= \left[(5455 \times 120 + 1150) // 24000\right]$$

$$(R_C // R_L)(\beta+1) + 1150 = 65460 + 1150 = 66610$$

$$R_{in} = 66610 // 24000 = 17643\,\Omega$$

c-) Output impedance
shorting the i/p source, the AC equivalent circuit becomes



$$i_t' = -(i_b + \beta i_b) = -(\beta+1) i_b \qquad i_b' = -\frac{v_t'}{r_\pi + (24k \parallel 10k)}$$

$$i_b' = -\frac{v_t'}{1055 + 7059} = -\frac{v_t'}{8114\,\Omega} \Rightarrow v_t' = -8114\, i_b$$

$$\frac{v_t'}{i_t'} = \frac{-8114\, i_b}{120\, i_b} = 67.6\,\Omega$$

adding R_E in parallel R_o becomes

$$R_o = 67.6\,\Omega \parallel 1000\,\Omega = 63.32\,\Omega$$