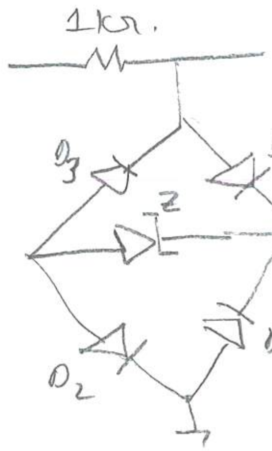
Solution of MT1 EEE313 Spring 19-3-2021

Q1 solution

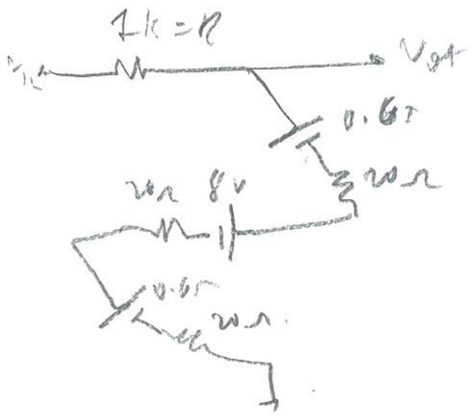
$V_Z = V_{Z_0} + I_Z r_Z \Rightarrow 8.2 = V_{Z_0} + 10\times10^{-3}\times 20 = V_{Z_0} + 0.2$

$\Rightarrow \boxed{V_{Z_0} = 8\text{ V.}}$ (5p)



$v_i > 2\times0.65 + 8 = 9.3$ v then

D_1, D_2 and Z are ON. D_3, D_4 OFF

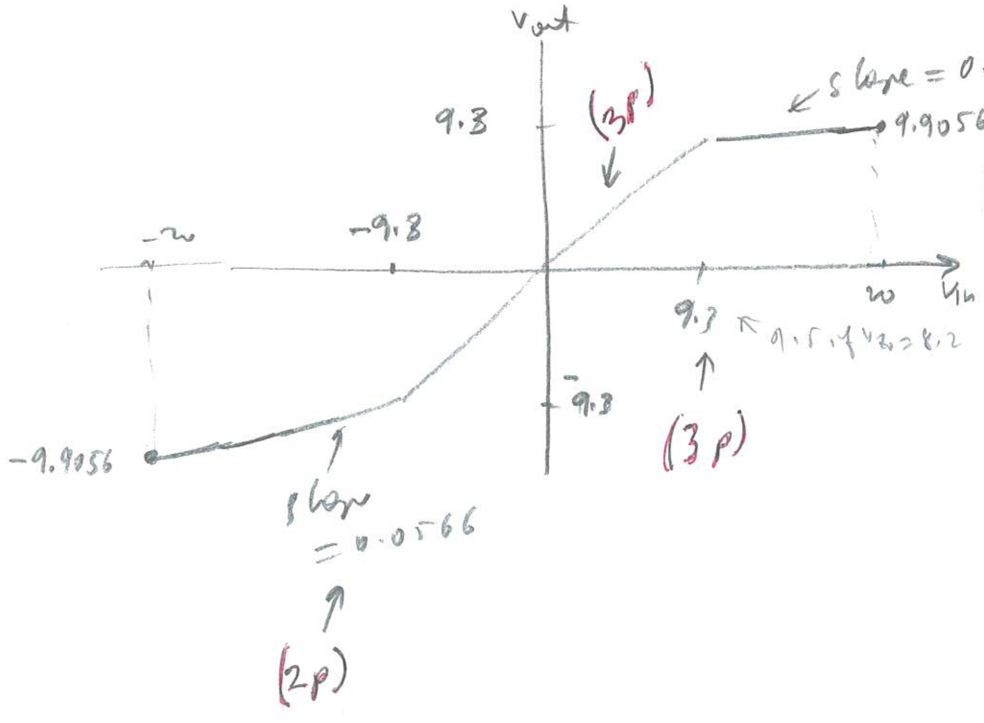


$$V_{out} = \frac{v_{in} - 9.3}{R + 2\times r_d + r_z}\times(2r_d + r_z) + 9.3$$

$$= \frac{v_{in} - 9.3}{1060}\times 60 + 9.3$$

$$= 0.0566\,(v_{in} - 9.3) + 9.3$$

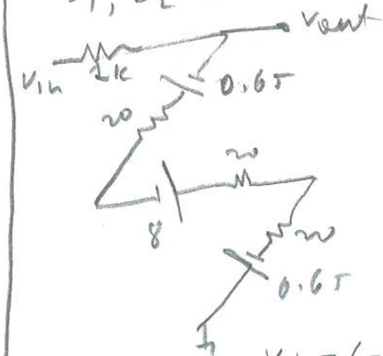
$-9.3 < v_i < 9.3$ all OFF $\Rightarrow V_{out} = v_{in}$.



$v_{in} < -9.3$

D_3, D_4, Z are ON

D_1, D_2 OFF



$$V_{out} = -9.3 + \frac{v_{in} - (-9.3)}{1060}\times 60$$

$$= 0.0566\,(v_{in} + 9.3) - 9.3$$

Without sketch no 3+3+2+2.

Q2 solution

$V_{TN} = 0.6$ v.
$K_n = 0.5$ mA/v²

$I_D = 12.5$ mA

[$V_{DS} = V_{GS} \Rightarrow V_{DS} > V_{GS} - V_{tn} \Rightarrow$ SAT or OFF
Since $I_D > 0 \Rightarrow$ SAT
2 points for this replacing the other checks]

a) $\lambda = 0$ $\quad 12.5 = 0.5\,(V_{GS} - 0.6)^2$

$$V_{GS} = 0.6 \pm \sqrt{25} = 0.6 \pm 5 = \begin{cases} 5.6 > 0.6 \checkmark \\ -4.4 > 0.6 \times \end{cases}$$

$V_{GS} = 5.6 = V_{DS}$ $\quad (5.6 > 5.6 - 0.6 \checkmark)$ (1p)

$$R = \frac{10 - 5.6}{12.5} = \frac{4.4}{12.5} = 0.352\,\text{k}\Omega = \boxed{352\,\Omega = R} \quad \text{(4p)}$$

b) $\lambda = 0.02$ $\quad 12.5 = 0.5\,(V_{GS} - 0.6)^2\,(1 + 0.02\,V_{GS})$ (1p)

$$25 = (V_{GS}^2 - 1.2V_{GS} + 0.36)(1 + 0.02V_{GS})$$

$$= V_{GS}^2 - 1.2V_{GS} + 0.36 + 0.02V_{GS}^3 - 0.024V_{GS}^2 + 0.0072V_{GS}$$

$$0.02V_{GS}^3 + 0.976 \times V_{GS}^2 - 1.1928\,V_{GS} - 24.64 = 0$$ (1p)

$$V_{GS} = \begin{cases} -49.5 \times \\ 5.35 > 0.6 \checkmark \\ -4.65 \times \end{cases}$$ ☆

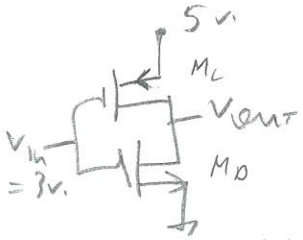
$V_{GS} = V_{DS} = 5.35 > 5.35 - 0.6 \checkmark$

$$R = \frac{10 - 5.35}{12.5} = 0.372\,\text{k}\Omega$$

$\boxed{R = 372\,\Omega}$ (8p)

numerically wrong solution −5p.
wrong approach −8p

Q3 solution



$K_n = K_p = 1$ mA/v²
$V_{TN} = 1$v $\quad V_{TP} = -1$v
$\lambda = 0$

Assume M_D is NONSAT, M_L = SAT.

$$1\,[2\,(3-1)V_O - V_O^2] = 1\,(5-3-1)^2$$

$4V_O - V_O^2 = 1 \qquad V_O^2 - 4V_O + 1 = 0$

$$V_O = V_{DS_D} = \begin{cases} 3.732 < 3-1 \times \\ 0.268 < 3-1 \checkmark \end{cases} \Rightarrow \boxed{V_O = 0.268\,\text{V}}$$

$V_{SD_L} = 5 - 0.268 > (5-3) - 1 \checkmark$ ☆☆

Checks: 1p + 1p Trans. Eqn 4p, soln 4p. wrong transistor equation (−5p) fundamentally wrong tr. eqn (−8p)

☆ Iterative solution per Q2 part b).
Initially take $V_{GS} = 5.6$. $\quad F = 25 - (V_{GS} - 0.6)^2(1 + 0.02\,V_{GS}) = 0$

V_{GS}	F
5.6	−2.8
5.0	3.7
5.3	0.568
5.4	−0.528
5.35	0.023
✓ 5.352	0.0014

☆☆ Check the other case M_D SAT, M_L NONSAT

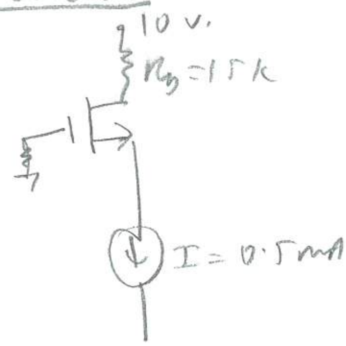
$$(3-1)^2 = 2\,(5-3-1)V_{SD_L} - V_{SD_L}^2$$

$4 = 2V_{SD_L} - V_{SD_L}^2 \qquad V_{SD_L}^2 - 2V_{SD_L} + 4 = 0$

V_{SD_L} = complex

Q4 solution

a)



Assume SAT ($\lambda = 0$)

$I_D = 0.5mA \quad 0.5 = 0.5(V_{GS} - 1.5)^2$

$\Rightarrow V_{GS} = 1.5 \pm 1 <$ $2.5 > 1.5$ ✓ ; $0.5 > 1.5$ ✗

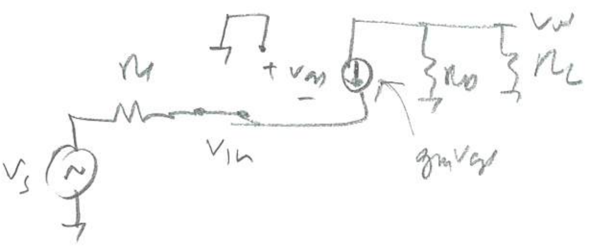
$V_{GS} = 2.5V$

$V_{DS} = V_D - V_S = 10 - 15 \times 0.5 - (-2.5)$

$= 10 - 7.5 + 2.5 = 5V. > 2.5 - 1.5$ ✓

(2p)

$g_m = 2\sqrt{0.5 \times 0.5} = 1 mA/V$ (1p)



$v_s = R_S(-g_m v_{gs}) - v_{gs}$

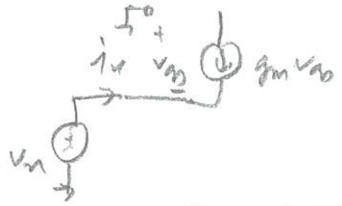
$= -v_{gs}(1 + g_m R_S)$

$\frac{v_{gs}}{v_s} = -\frac{1}{1 + g_m R_S}$

$v_{out} = -g_m v_{gs}(R_D // R_L) \qquad \frac{v_{out}}{v_{gs}} = -g_m(R_D // R_L)$ (6p)

$A_v = \frac{v_{out}}{v_s} = \frac{v_{gs}}{v_s} \cdot \frac{v_{out}}{v_{gs}} = \frac{g_m(R_D // R_L)}{1 + g_m R_S} = \frac{1 \times (15 // 15)}{1 + 1 \times 0.05} = \frac{7.5}{1.05} = \boxed{7.143 \, V/V}$

R_{in}:

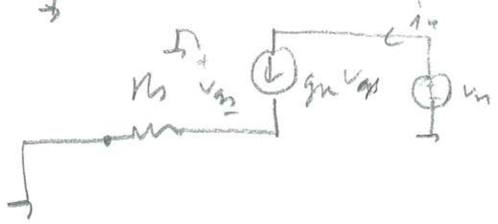


$v_x = -v_{gs}$

$i_x = -g_m v_{gs} \Rightarrow R_{in} = \frac{v_x}{i_x} = \frac{1}{g_m} = \boxed{1 k\Omega = R_{in}}$ (5p)

R_{out}:

No derivation $\Rightarrow \times \frac{1}{2}$

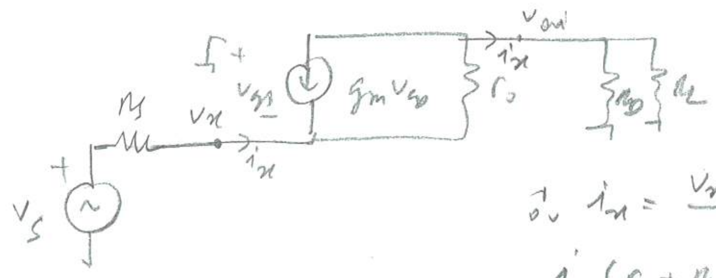


$-v_{gs} = g_m v_{gs} R_S \Rightarrow v_{gs} = 0$

$\Rightarrow i_x = 0 \Rightarrow \frac{v_x}{i_x} = \infty$ (4p)

$\therefore R_{out} = R_D // \infty = \boxed{15 k\Omega = R_{out}}$

b) $\lambda = \frac{1}{75}$ DC values same $\Rightarrow g_m = 1mA/V \quad r_o = \frac{1}{\frac{1}{75} \times 0.5} = 150 k\Omega$ (1p)



$i_x = \frac{v_x - v_{out}}{r_o} - g_m v_{gs}$

$v_{gs} = -v_x \qquad v_{out} = i_x(R_D // R_L)$

$\therefore i_x = \frac{v_x - i_x(R_D // R_L)}{r_o} + g_m v_x$

$i_x(r_o + R_D // R_L) = v_x(1 + g_m r_o)$

$\frac{v_x}{i_x} = \frac{r_o + R_D // R_L}{1 + g_m r_o} = \frac{150 + 7.5}{1 + 1 \times 150} = \frac{157.5}{151} = 1.043 k\Omega.$

$\boxed{R_{in} = 1.043 k\Omega}$ (5p)

$i_{in} = \frac{v_s}{R_S + R_{in}} \qquad v_{out} = i_{in} \times R_D // R_L$

$= \frac{v_s}{R_S + R_{in}} R_D // R_L$

$\Rightarrow A_v = \frac{R_D // R_L}{0.05 + 1.043} = \frac{7.5}{1.093} = \boxed{6.86 = A_v}$

$= 0.915 \times 7.5$

(6p)

Or; $\frac{v_{out}}{v_{in}} = \frac{R_D // R_L}{R_{in}} = 7.19$

$\frac{v_{in}}{v_s} = \frac{R_{in}}{R_{in} + R_S} = 0.954$

$\Rightarrow 6.86$

Q5 solution

a) Assume both TRs are SAT.

$$3 = 2(V_{GS_1}-1)^2 \Rightarrow V_{GS_1} = 1 \pm \sqrt{1.5} = \begin{cases} 2.225 & \checkmark > 1.5 \\ -0.225 & \times \end{cases}$$

$V_{G_1} = 2.225 + 3\times 1.2 - 5 = 0.825$ V. $\quad V_{S_1} = 3.6 - 5 = -1.4$ V

$$V_{G_1} = \frac{R_3}{R_1+R_2+R_3}\times 5 = \frac{5R_3}{500} = \frac{R_3}{100} \Rightarrow \boxed{R_3 = 82.5\,k\Omega} \quad (3p)$$

$V_{D_1} = V_{S_1} + V_{DSQ_1} = 3\times 1.2 - 5 + 2.5 = 1.1$ V

$V_{G_2} = 1.1 + 2.225 = 3.325$ V

$$V_{G_2} = \frac{R_2+R_3}{500}\times 5 = \frac{R_2+R_3}{100} = 3.325 \Rightarrow R_2+R_3 = 332.5\,k\Omega$$

$$\Rightarrow \boxed{R_2 = 250\,k\Omega} \quad (3p)$$

$R_1 = 500 - R_2 - R_3 \Rightarrow \boxed{R_1 = 167.5\,k\Omega}$ (3p) $\quad$ check: $2.5 > 2.225 - 1$ ✓

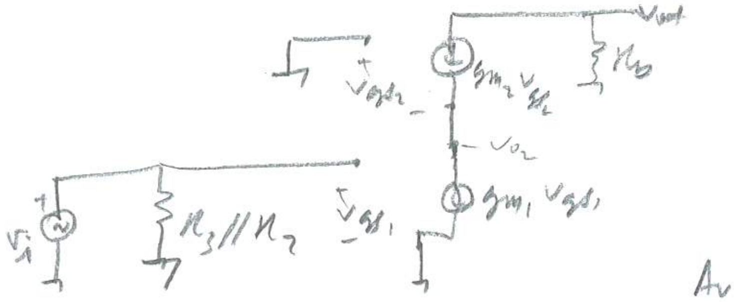
$V_{D_2} = V_{D_1} + V_{DSQ_2} = 1.1 + 2.5 = 3.6$ V

$\Rightarrow R_D = \frac{5-3.6}{3} \Rightarrow \boxed{R_D = 0.467\,k\Omega}$ (3p)

Actually
$5 = 3\times R_D + 2.5 + 2.5 + 3\times 1.2 + (-5)$
$5 - 3.6 = 3R_D \Rightarrow R_D = 467\,\Omega$.

b)

(4p)



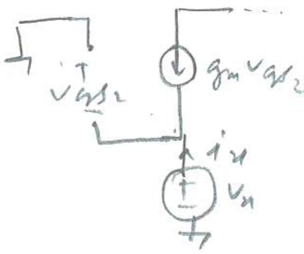
$v_{out} = -g_{m_2} v_{gs_2} R_D$
$= -g_{m_1} v_{gs_1} R_D$
$= -g_{m_1} v_i R_D$

$A_v = \frac{v_o}{v_i} = -g_m R_D$ (1p)

$g_m = 2\sqrt{2\times 3} = 4.9$ mA/V

$A_v = -4.9\times 0.467 = \boxed{-2.29 = A_{v_1}}$ (4p)

c)



$$R_{eq} = \frac{v_x}{i_x} = \frac{-v_{gs_2}}{-g_m v_{gs_2}} = \boxed{\frac{1}{g_m} = R_{eq} = 204\,\Omega} \quad (4p)$$

$v_{o_2} = -g_{m_1} v_{gs_1} \times R_{eq}$

$v_{gs_1} = v_i$

$A_{v_2} = -g_m v_i \frac{1}{g_m} = \boxed{-1 = A_{v_2}}$ (5p)