

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Signature: \_\_\_\_\_

EEE 313 Fall 2010

Bilkent University  
Department of Electrical and Electronics Engineering  
EEE 313 Electronic Circuit Design

**Midterm 1**

23 October 2010, 10:00  
(4 questions, 120 minutes)

- This is a **closed book**, closed notes exam. No cheat sheet allowed.
- All cell-phones should be completely **turned off**.
- Use a calculator for numerical computations. Carry at least **4 significant digits** during calculations. Your final answer should be at least **3 significant digits**.
- Be sure to write the **units** of all numerical results.
- **Show** all work clearly.
- Please put your **final answer** for each part inside a box for easy identification. Do not give multiple answers, they will not be graded.
- Do not remove the **staple** from the exam sheets or separate pages of the exam. All extra pages must be stamped to your exam.
- You may leave the exam room when you are done. However, please do not leave during the **last five minutes** of the exam.
- At the end of the exam, please stay seated until **all** exam papers are collected.

**FET equations:**

**n-channel MOSFET**

$$i_D = K_n (v_{GS} - V_{Tn})^2 \quad \text{SAT}$$

$$i_D = K_n [2(v_{GS} - V_{Tn})v_{DS} - v_{DS}^2] \quad \text{NON-SAT}$$

**p-channel MOSFET**

$$i_D = K_p (v_{SG} + V_{Tp})^2 \quad \text{SAT}$$

$$i_D = K_p [2(v_{SG} + V_{Tp})v_{SD} - v_{SD}^2] \quad \text{NON-SAT}$$

**n-channel JFET**

$$i_D = \frac{I_{DSS}}{V_p^2} (v_{GS} - V_p)^2 \quad \text{SAT}$$

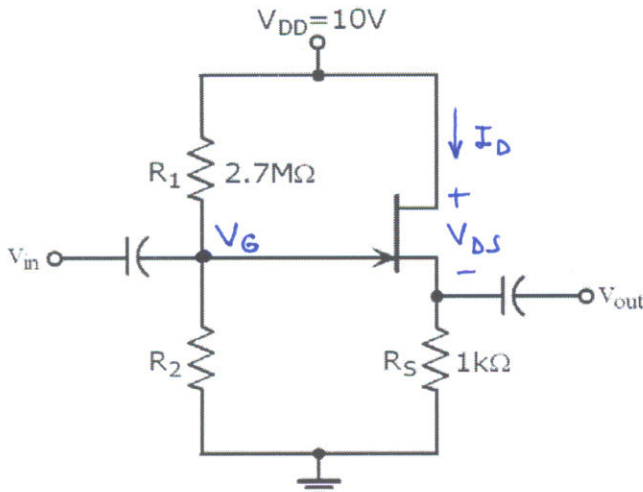
$$i_D = \frac{I_{DSS}}{V_p^2} [2(v_{GS} - V_p)v_{DS} - v_{DS}^2] \quad \text{NON-SAT}$$

Please do not write below this line

1. 30 pts.	
2. 15 pts.	
3. 25 pts.	
4. 30 pts.	
Total 100 pts.	

1. (30 points) Given  $I_{DSS} = 15\text{mA}$  and  $V_P = -5\text{V}$ . Assume that the capacitor values are very large. Design the following FET amplifier circuit.

- Find the value of  $R_2$  such that the transistor operates at the edge of the SAT and NON-SAT regions.
- Find the value of  $R_2$  such that the undistorted symmetric output voltage swing is as large as possible. And determine the value of the undistorted symmetric output voltage swing ( $V_{\text{peak-to-peak}}$ ).



SOLUTION:  
(a) At the edge of the SAT and NON-SAT regions:

$$V_{DST} = V_{GST} - V_P \quad +3$$

$$+2 \quad V_{DST} = V_{DD} - I_{DT} R_S \quad (\text{KVL})$$

$$V_{DST} = V_{DD} - \frac{I_{DSS} R_S}{V_P^2} (V_{GST} - V_P)^2$$

$$V_{DST} = V_{DD} - \frac{I_{DSS} R_S}{V_P^2} V_{DST}^2$$

$$0.6 V_{DST}^2 + V_{DST} - 10 = 0$$

$$+5 \quad \boxed{V_{DST} = \frac{10}{3} \text{V}} \quad \text{must be positive}$$

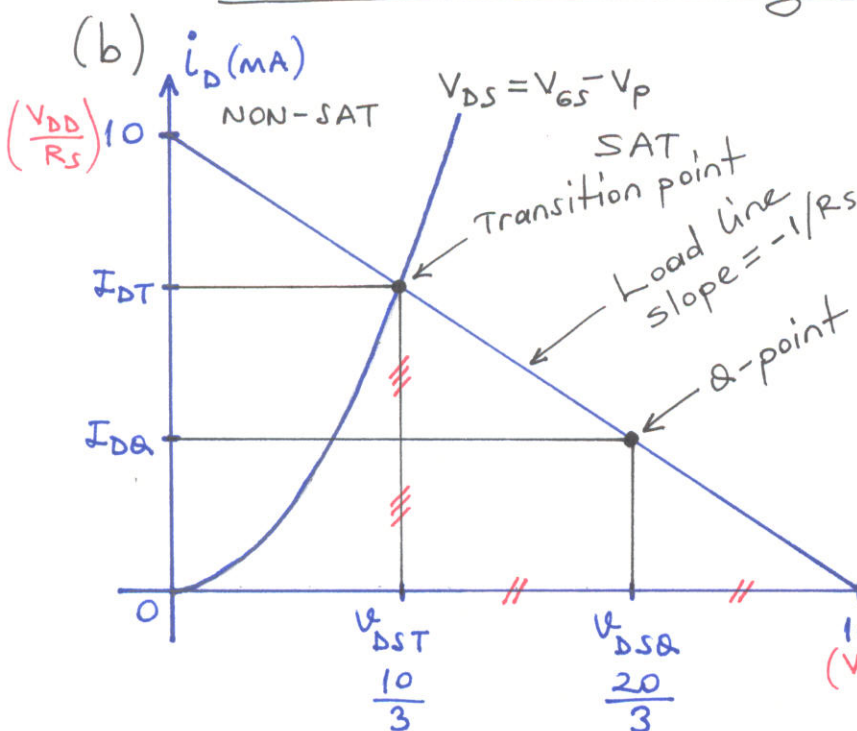
$$\text{Then } V_{GST} = V_{DST} + V_P = \frac{10}{3} - 5$$

$$\boxed{V_{GST} = -\frac{5}{3} \text{V}}$$

$$I_{DT} = \frac{V_{DD} - V_{DST}}{R_S} = \frac{10 - 10/3}{1\text{k}\Omega} \Rightarrow \boxed{I_{DT} = \frac{20}{3} \text{mA}}$$

$$V_G = V_{GST} + I_{DT} R_S = -\frac{5}{3} + \left(\frac{20}{3} \text{mA}\right)(1\text{k}\Omega) \Rightarrow \boxed{V_G = 5\text{V}}$$

$$\text{Therefore, } R_1 = R_2 \text{ and } \boxed{R_2 = 2.7\text{M}\Omega} \quad +5$$



To obtain max. undistorted symmetric output voltage swing, Q-point must be in the middle of SAT region.

$$\text{Then } I_{DQ} = \frac{I_{DT}}{2} = \boxed{\frac{10}{3} \text{mA}}$$

$$V_{DSQ} = V_{DD} - I_{DQ} R_S = \boxed{\frac{20}{3} \text{V}} \quad +4$$

$$V_{GSQ} = V_P \left(1 - \sqrt{\frac{I_{DQ}}{I_{DSS}}}\right) = \boxed{-2.643\text{V}}$$

$$V_G = V_{GSQ} + I_{DQ} R_S = 0.6903\text{V}$$

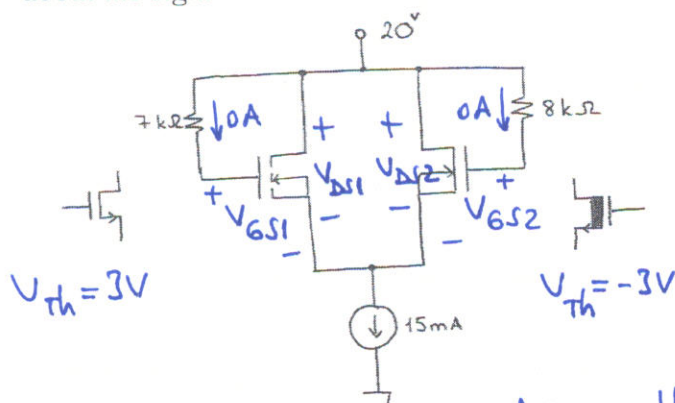
$$V_G = \frac{R_2}{R_1 + R_2} V_{DD}$$

$$\boxed{R_2 = 200.2\text{k}\Omega} \quad +6$$

$$+5 \quad V_{PP} = 2 \left(\frac{10}{3}\right) = \boxed{\frac{20}{3} \text{V}} \quad \text{maximum.}^2$$

2. (15 points) In the circuit shown below, determine the states of the transistors and the corresponding bias ( $I_{DS}$  and  $V_{DS}$ ). Justify your answer clearly.  $K_n = 1 \text{ mA/V}^2$  and  $|V_{Th}| = 3 \text{ V}$ .

Hint: The absolute value of the threshold values of the transistors are given, be careful about the sign.



### SOLUTION:

Since gate current is zero

$$V_{GS1} = V_{GS2} = V_{DS1} = V_{DS2}$$

For enhancement-mode MOSFET

$$V_{GS1} = V_{DS1} \Rightarrow V_{DS1} > V_{GS1} - V_{Th}$$

Assume that it is in the SAT region.

For depletion-mode MOSFET

$$V_{DS2} = V_{GS2} \Rightarrow V_{DS2} < V_{GS2} - V_{Th} = V_{GS} + 3 \text{ V}$$

Assume that it is in the NON-SAT region.

Then  $I_{D1} + I_{D2} = 15 \text{ mA}$

$$K_n (V_{GS1} - 3)^2 + K_n [2(V_{GS2} + 3)V_{DS2} - V_{DS2}^2] = 15$$

$$(1)(V_{GS} - 3)^2 + (1)[2(V_{GS} + 3)V_{GS} - V_{GS}^2] = 15$$

$$V_{GS}^2 - 6V_{GS} + 9 + [2V_{GS}^2 + 6V_{GS} - V_{GS}^2] = 15$$

$$2V_{GS}^2 = 6 \Rightarrow V_{GS} = \sqrt{3} = 1.732 \text{ V}$$

For enhancement-mode transistor

$V_{GS} < V_{Th} = 3 \text{ V}$ . Therefore it must be **CUTOFF** ( $I_{D1} = 0 \text{ A}$ )

Then  $I_{D2} = 15 \text{ mA}$

$$K_n [2(V_{GS2} + 3)V_{DS2} - V_{DS2}^2] = 15$$

$$(1)[2(V_{GS} + 3)V_{GS} - V_{GS}^2] = 15$$

$$2V_{GS}^2 + 6V_{GS} - V_{GS}^2 = 15 \Rightarrow V_{GS}^2 + 6V_{GS} - 15 = 0$$

$$\Rightarrow V_{GS} = 1.899 \text{ V and } I_{D2} = 15 \text{ mA}$$

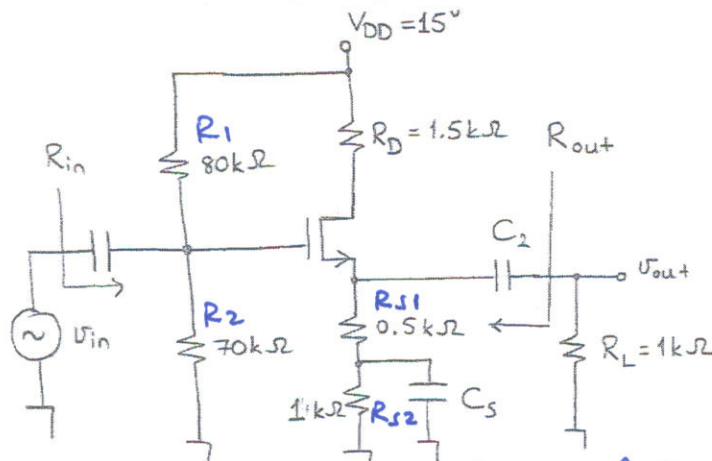
Finally

$$I_{D1} = 0, V_{DS1} = 1.899 \text{ V and } I_{D2} = 15 \text{ mA, } V_{DS2} = 1.899 \text{ V}$$



3. (25 points) Consider the single stage MOSFET amplifier shown in the figure.  $V_{th} = 2V$ ,  $K_n = 0.5 \text{ mA/V}^2$ . Neglect  $r_o$ . Assume the capacitors are very large. Also assume that  $I_{DQ} = 2 \text{ mA}$ . Determine

- (12 points) The voltage gain,  $v_o/v_{in}$
- (5 points) The input impedance,  $R_{in}$
- (8 points) The output impedance  $R_{out}$



### SOLUTION:

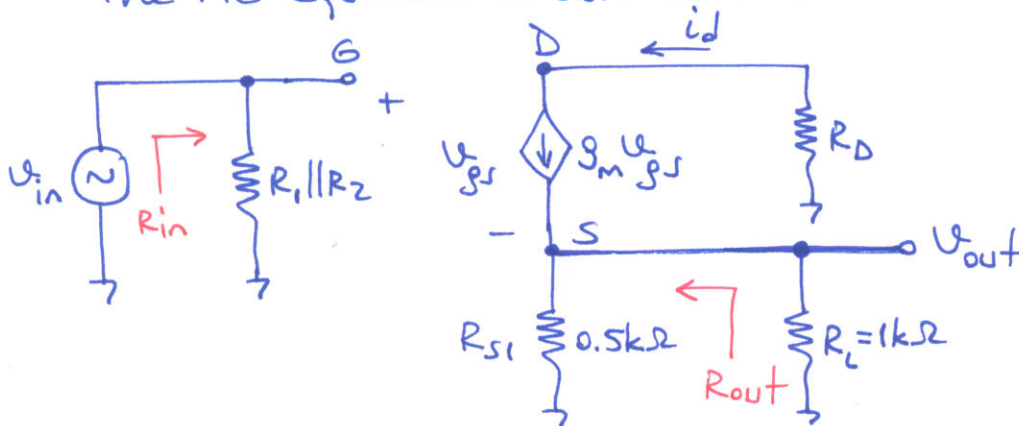
Given that  $I_{DQ} = 2 \text{ mA}$

$$V_{GSQ} = \sqrt{\frac{I_{DQ}}{K_n}} + V_{TN} = 4 \text{ V}$$

$$V_{DSQ} = V_{DD} - I_{DQ}(R_D) = 9 \text{ V}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2 \text{ mA/V}$$

The AC equivalent circuit is as follows.



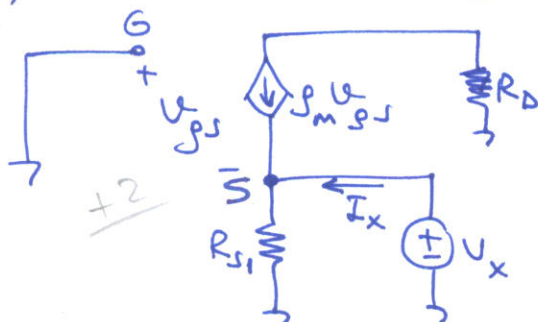
$$(a) \quad v_{in} = v_{gs} + v_{out} \Rightarrow v_{gs} = v_{in} - v_{out} \quad +3$$

$$v_{out} = g_m v_{gs} (R_{S1} \parallel R_L) = g_m (v_{in} - v_{out}) (R_{S1} \parallel R_L) \quad +3$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{g_m (R_{S1} \parallel R_L)}{1 + g_m (R_{S1} \parallel R_L)} = \boxed{0.4} \quad +2$$

$$(b) \quad R_{in} = R_1 \parallel R_2 = (70 \text{ k} \parallel 80 \text{ k}) = \boxed{37.333 \text{ k}\Omega}$$

(c) To find  $R_{out}$ , set  $v_{in} = 0$  and apply a test voltage at the output.



$$v_{gs} = -V_x$$

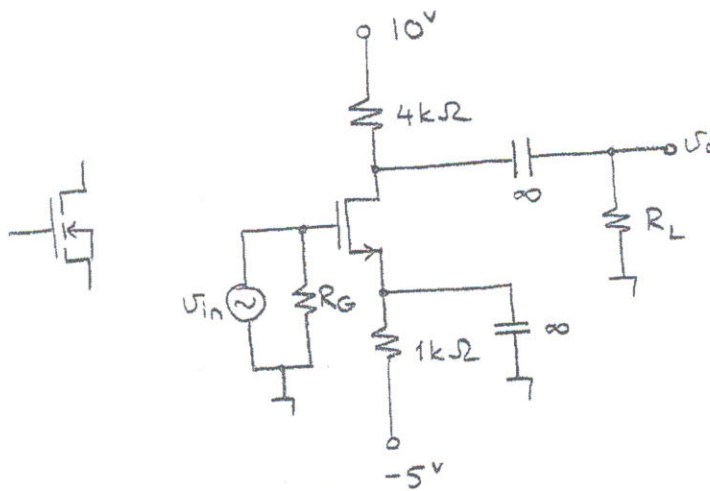
Apply KCL at source node.

$$I_x + g_m v_{gs} = \frac{V_x}{R_{S1}} \quad +2$$

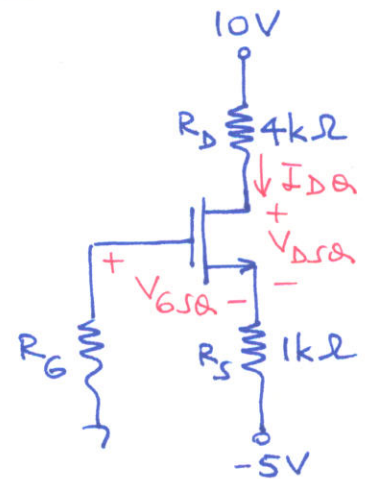
$$I_x - g_m V_x = \frac{V_x}{R_{S1}} \quad +4$$

$$R_{out} = \frac{V_x}{I_x} = \frac{1}{1/R_{S1} + g_m} = \frac{1}{g_m} \parallel R_{S1} = \boxed{250 \Omega}$$

4. (30 points) For the amplifier circuit shown below, determine the value of the load resistance,  $R_L$ , in order to obtain an undistorted output voltage swing of 3V peak-to-peak.  $V_{Th}=1V$ ,  $K_n=0.25mA/V^2$ .  $v_{in}$  is a purely ac voltage signal with zero volts average. Neglect  $r_o$ .



SOLUTION:



DC equivalent circuit

DC Analysis:

Apply KVL around gate-to-source loop.

$$V_{GSQ} + I_{DQ} R_S - 5 = 0 \text{ where } I_{DQ} = K_n (V_{GSQ} - V_{Th})^2$$

$$V_{GSQ} + K_n R_S (V_{GSQ} - V_{Th})^2 - 5 = 0 \Rightarrow 0.25 V_{GSQ}^2 + 0.5 V_{GSQ} - 4.75 = 0$$

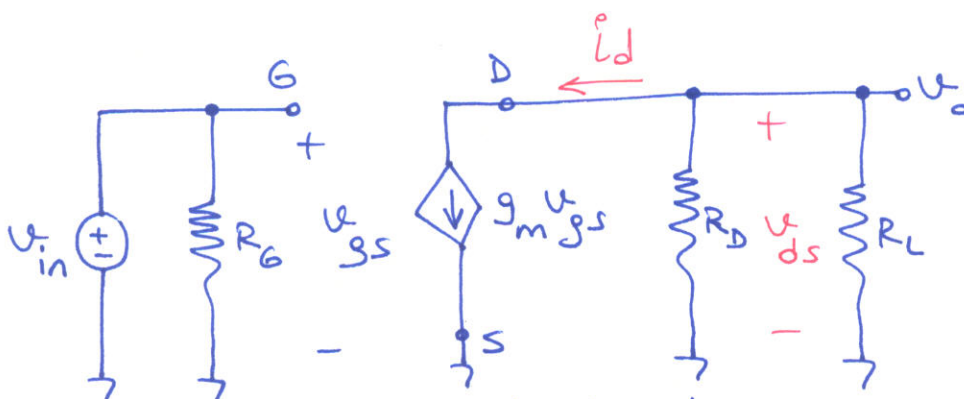
Thus,  $V_{GSQ} = 3.4721V$  and  $I_{DQ} = 1.5279mA$

$$V_{DSQ} = 15 - I_{DQ} (R_D + R_S) = 15 - (1.5279)(5)$$

$$V_{DSQ} = 7.3605V$$

$$V_{DS} = 15 - I_D (R_D + R_S) \text{ DC load line eqn.}$$

AC Analysis:



AC equivalent circuit

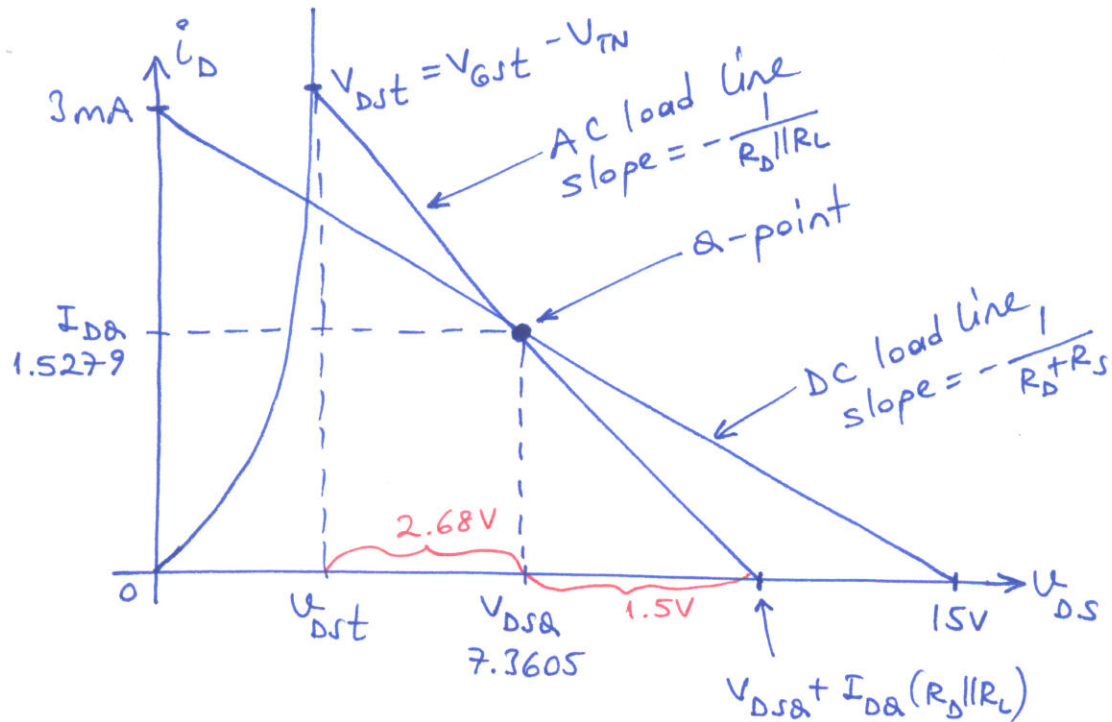
$$v_{DS} = -i_d (R_D || R_L)$$

$$i_d = -\frac{v_{DS}}{R_D || R_L}$$

AC load line eqn.

Note that AC load line must pass through the DC operating point (Q-point).

$$g_m = 2K_n (V_{GSQ} - V_{Th}) = 1.2361mA/V$$



$I_{DQ}(R_D || R_L) = 1.5V$  is given.

$$R_L = \frac{1.5R_D}{R_D I_{DQ} - 1.5} = \frac{1.5}{I_{DQ} - \frac{1.5}{R_D}} = \boxed{1.3011k\Omega}$$

$$R_D || R_L = 981.76\Omega$$

At the transition point:  $V_{DSt} = V_{GSt} - V_{TN}$

where  $U_{DS} = V_{DSQ} + u_{ds}$  and  $U_{GS} = V_{GSQ} + u_{gs}$

$$U_{DSt} = U_{GSt} - V_{TN}$$

$$V_{DSQ} + u_{ds} = V_{GSQ} + u_{gs} - V_{TN}$$

$$V_{DSQ} - i_d(R_D || R_L) = V_{GSQ} + u_{gs} - V_{TN} \text{ where } i_d = g_m u_{gs}$$

$$V_{DSQ} - g_m u_{gs}(R_D || R_L) = V_{GSQ} + u_{gs} - V_{TN}$$

$$u_{gs} = \frac{V_{DSQ} - V_{GSQ} + V_{TN}}{1 + g_m(R_D || R_L)} = 2.2084V \rightarrow U_{GSt} = 5.6805V$$

$$i_d = g_m u_{gs} = 2.7298mA \rightarrow i_{Dt} = 4.2577mA$$

$$u_{ds} = -i_d(R_D || R_L) = -2.68V$$

$$U_{DSt} = V_{DSQ} + u_{ds} = \boxed{4.6805V}$$

Then  $\boxed{V_{pp} = 3.0V}$