Defne Yaz Kılıç- 22102167
Sait Sarper Özaslan- 22002861

EEE 313 Course Project
Analog Multiplier

Introduction

For this final project, the topic is chosen as the design and implementation of an analog multiplier that would multiply two sine waves according to the following specifications:
• Dual power supplies (+/- VDD) no more than +/-10V.
• Power consumption should be less than 200mW, i.e., <10mA total current per supply.
• Resistors are allowed.
• The circuit should generate its own biasing, if you need different voltages you should generate them from the supplies.
• A double balanced mixing cell is required.

In order to achieve the multiplication, Gilbert Cell design is used.

Gilbert Cell working principles:
Gilbert cell is a cross-coupled differential amplifier meaning that it consists of two differential amplifiers connected in parallel and the output of each amplifier goes to the input of the other one [1]. To understand how a Gilbert Cell works, the working principle of differential amplifiers must be explained. A differential amplifier is used for amplifying the voltage difference between two inputs, both BJTs and FETs can be used for its design. However, for this project BJT based differential amplifiers are used (Fig 1.).

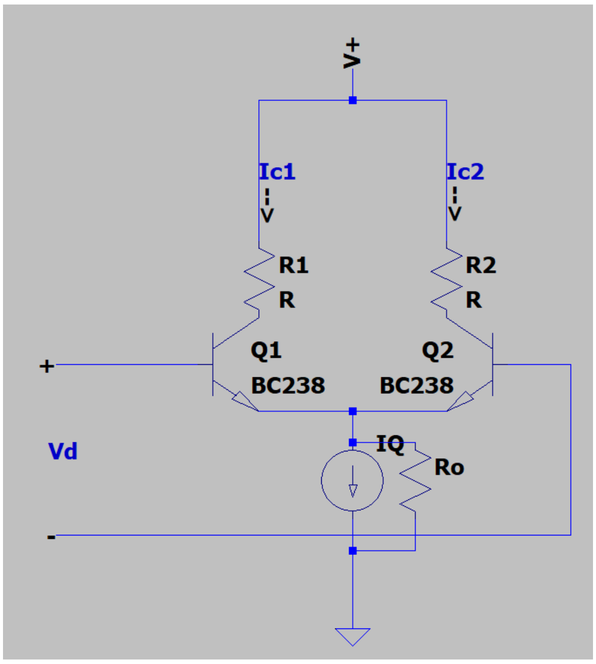

Figure 1: A Differential Amplifier with Differential Input

In Figure 1, IQ is a current source with internal resistance Ro.
The analog multiplier depends on the changing current relations caused by the differential input, hence it is more rational to explain the current relation of the differential amplifier. The current relations of collector currents can be given from BJTs exponential current relation.

$$i_{C1} = I_Q/(1 + e^{-V_d/V_T})$$

$$i_{C2} = I_Q/(1 + e^{+V_d/V_T})$$

As can be seen from the formula the currents have an inversely proportional relation between each other. (Fig.2)

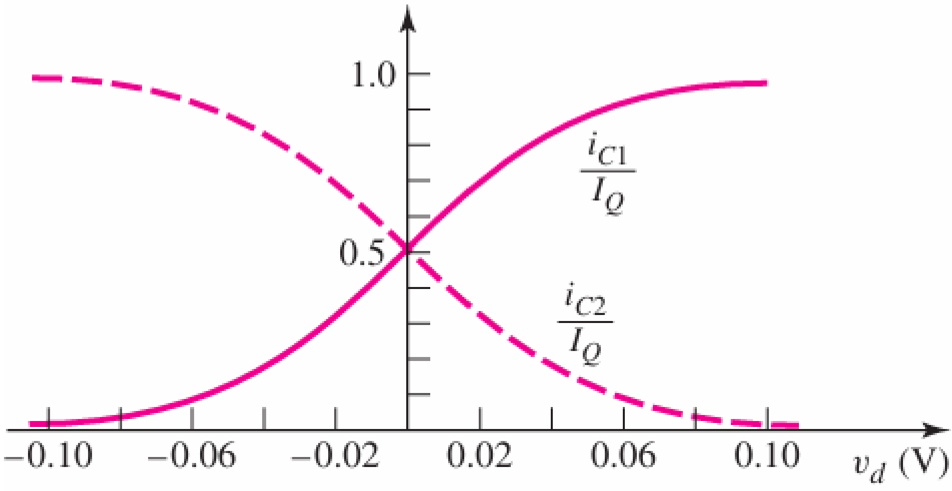

Figure 2: DC transfer characteristics of BJT differential amplifier [2]

Hence one will find the current difference relation as:

$$\Delta Ic = I_{C1} - I_{C2} = I_Q tanh(V_d/2V_T)$$

As there is a need to achieve multiplication, the current source can be changed with another BJT pair which can be seen in Figure 3.

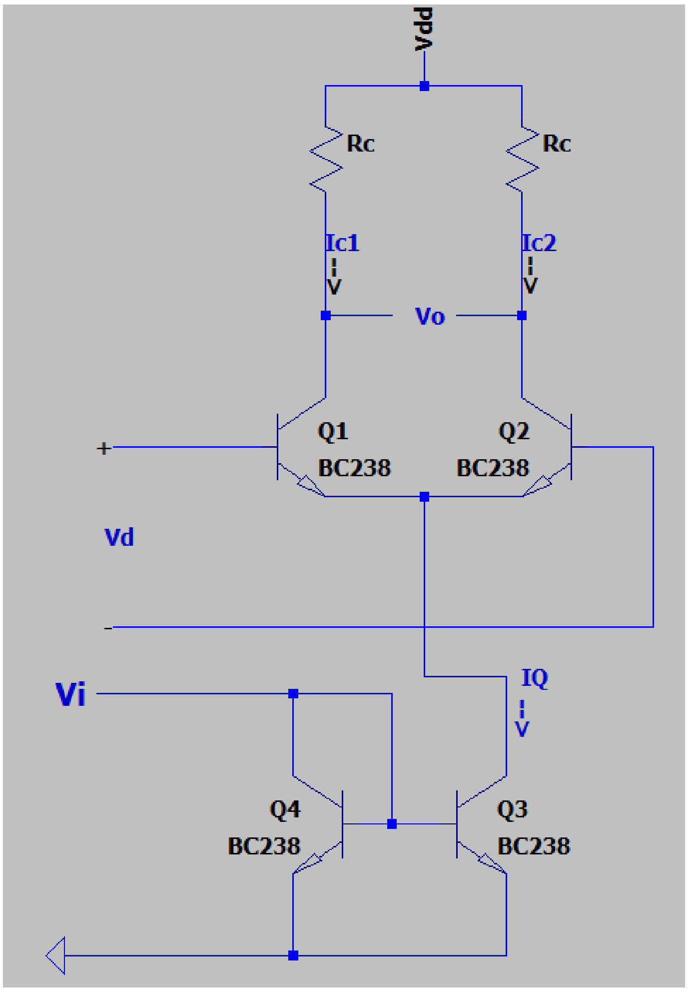

Figure 3: Addition of Current Circuitry

Here the current I_Q is controlled by another voltage source. Whose relationship is as explained:

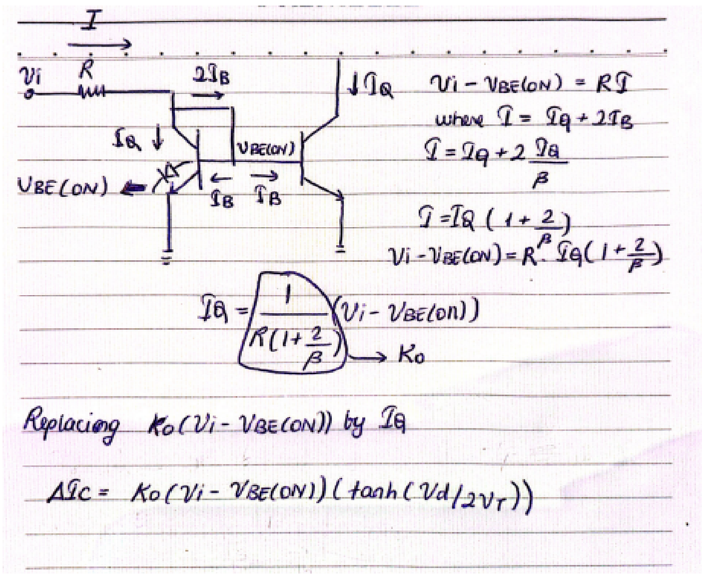

Figure 4: Output Current Relation from the Circuitry

Assuming $V_d \ll 2V_t$, for linearizing the expression one can use the Taylor expansion of tanh around 0. Doing the math, the final expression becomes:

$$\Delta Ic = I_{C1} - I_{C2} = \frac{K_0 \cdot V_d \cdot (V_i - V_{BE(on)})}{2V_T}$$

Thus a scaling and multiplicative expression is obtained using the difference between the two collector currents. This was a two quadrant Gilbert cell consisting of two differential amplifiers. For better performance, a four quadrant Gilbert cell can be used for maximum flexibility of the inputs [1]. Therefore another multiplication pair is added to this design in common emitter configuration.

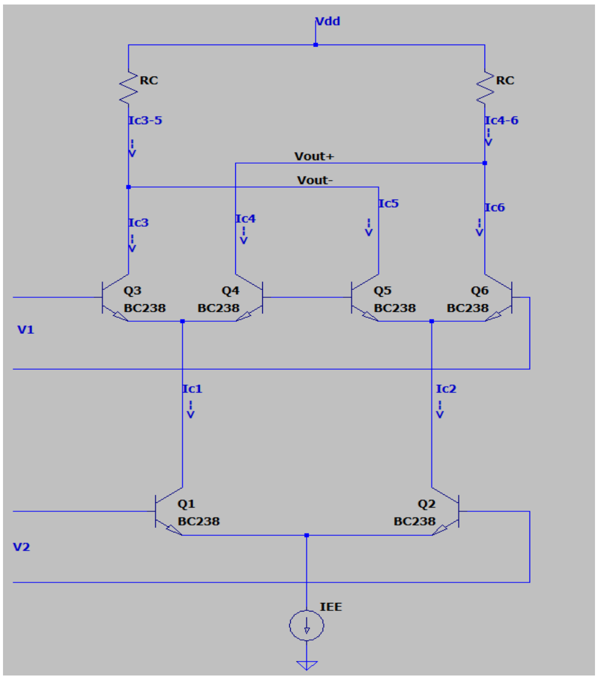

Figure 5: Gilbert Cell

Doing similar calculations but for each differential amplifier, the following results are obtained:

Q3 & Q4 pair

$$I_{C3} = \frac{I_{C1}}{1+\exp(-V_1/V_T)}, \quad I_{C4} = \frac{I_{C1}}{1+\exp(V_1/V_T)}$$

Q5 & Q6 pair

$$I_{C5} = \frac{I_{C2}}{1+\exp(V_1/V_T)}, \quad I_{C6} = \frac{I_{C2}}{1+\exp(-V_1/V_T)}$$

I_{C1}, I_{C2} relation to 2nd input voltage V_2

$$I_{C1} = \frac{I_{EE}}{1+\exp(-V_2/V_T)}, \quad I_{C2} = \frac{I_{EE}}{1+\exp(V_2/V_T)}$$

Substituting I_{C1} & I_{C2} we get:

- $I_{C3} = \dfrac{I_{EE}}{[1+\exp(-V_1/V_T)][1+\exp(-V_2/V_T)]}$
- $I_{C4} = \dfrac{I_{EE}}{[1+\exp(V_1/V_T)][1+\exp(-V_2/V_T)]}$
- $I_{C5} = \dfrac{I_{EE}}{[1+\exp(V_1/V_T)][1+\exp(V_2/V_T)]}$
- $I_{C6} = \dfrac{I_{EE}}{[1+\exp(-V_1/V_T)][1+\exp(V_2/V_T)]}$

Figure 6: Collector Current Relations of BJTs [1]

ΔI_{out} : the differential output $= I_{C3\text{-}5} - I_{C4\text{-}6}$

$$= I_{C3} + I_{C5} - (I_{C4} + I_{C6})$$
$$= (I_{C3} - I_{C6}) - (I_{C4} - I_{C5})$$
$$= I_{EE} \tanh(V_1/2V_T) \tanh(V_2/2V_T)$$

assuming $V_1 \ll 2V_T$, $V_2 \ll 2V_T$ and using taylor approximation around 0:

$$\Delta I_{out} = I_{EE} \cdot \frac{V_1 . V_2}{4V_T^2}$$

Replacing this

$$V_0 = \underbrace{V_{out+} - V_{out-}}_{\Delta V_{out}} = R_C \Delta I_{out} = \frac{R_C I_{EE} V_1 V_2}{4V_T^2}$$

$$\boxed{\Delta V_{out} = \frac{R_C I_{EE}}{4V_T^2} V_1 V_2}$$

Figure 7: Output Voltage Relation from Gilbert Cell

Software Implementation

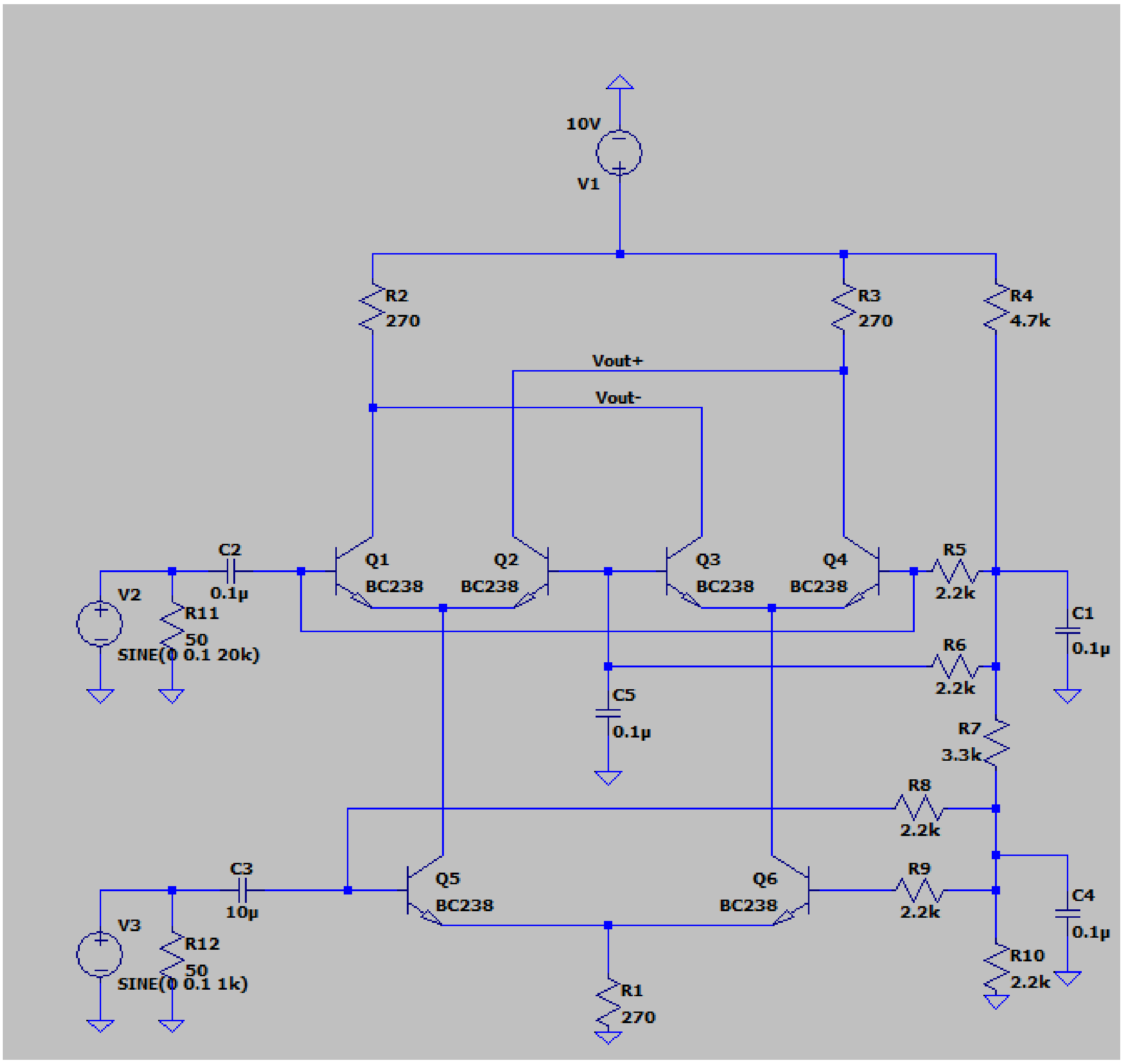

Figure 8: Software Implementation Circuit

The software implementation is properly biased with the available resistance values in the lab. The calculations of this part were too complex so biasing was done through trial and error web search.

Afterwards one can move onto the software results, $\Delta V_{out} = Vout_{+} - Vout_{-}$

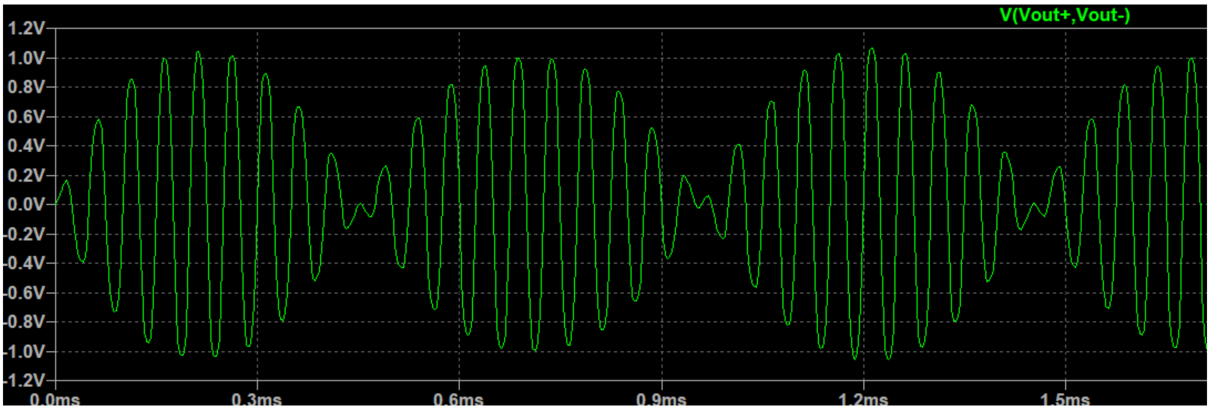

Figure 9: Amplitude Modulated Output

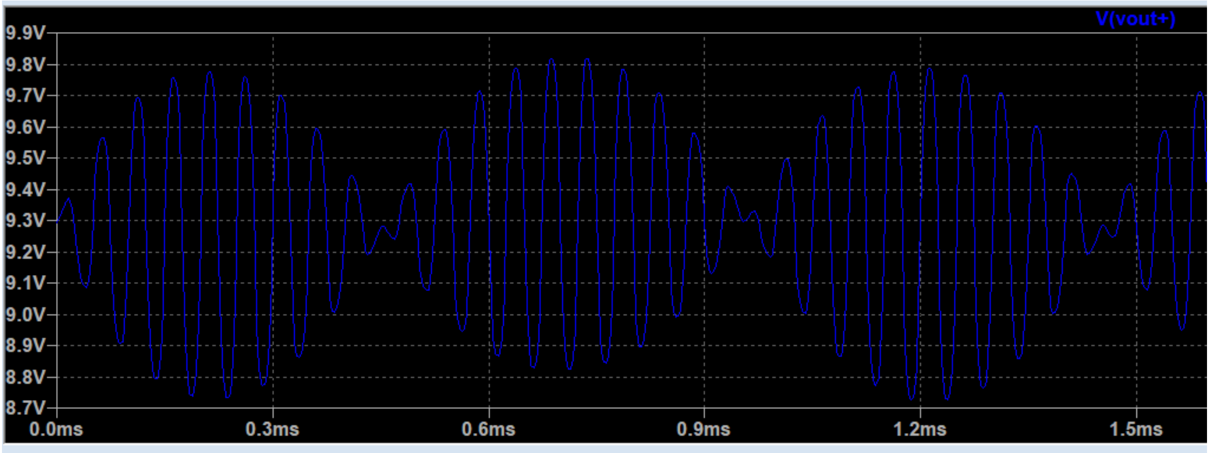

Figure 10: Output from $Vout_+$

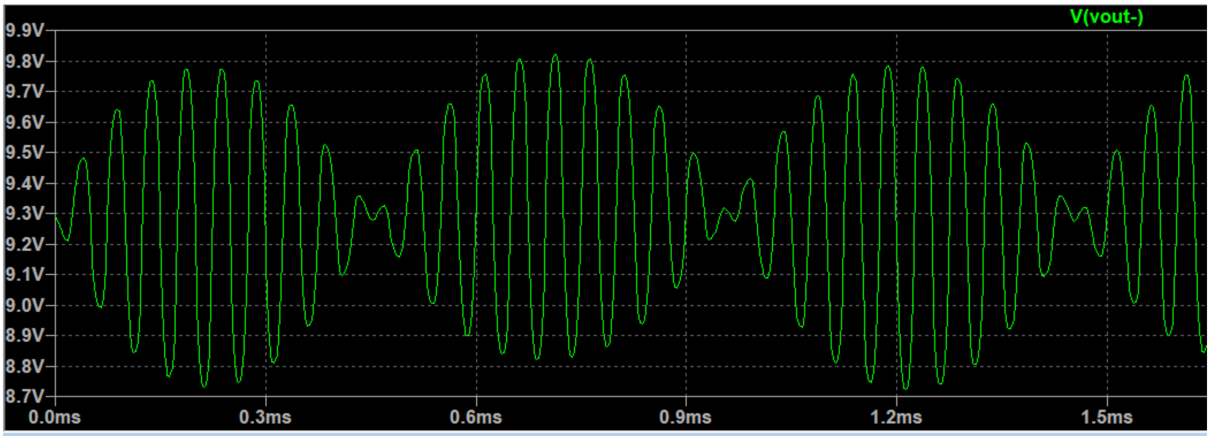

Figure 11: Output From $Vout_-$

As can be seen from Figure 9 the output is a successful amplitude modulated signal. This indicates that the Gilbert cell is multiplying and modulating correctly.

As the input voltages are increased, one has to take into account that the BJTs may switch to OFF state. At this state, the read voltage value will relatively stay stable. In terms of modulation, this will correspond to a state where AM signal's modulation will not work appropriately. In other words, rather than 1kHz creating a proper envelope on top of 20kHz, the envelope will seem to have a constant value. It should also be observable that there are constant values on each peak and low, which would be from the 20kHz signal.

In software, though there were distortions starting from 0.25V, the input voltage that was found to cause visible OFF conditions was 0.3 Volts.

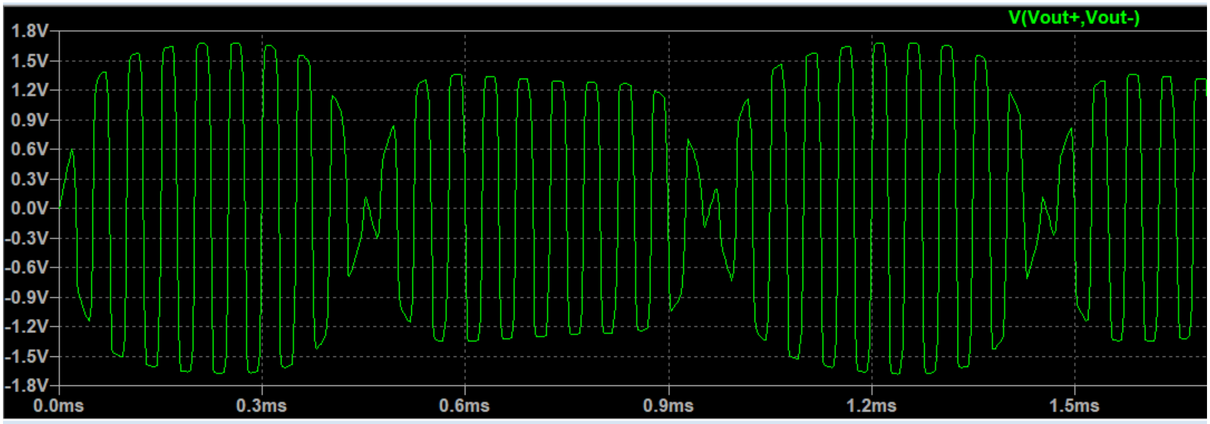

Figure 12: Distorted Output

One can easily see from the Figure 12 that at every half cycle of 1kHz there seems to be a visible constant value on the envelope made by 1kHz. To show the OFF state, showing the current behavior off $Vout_+$ *or* $Vout_-$ is more rational. This can be seen in the figure below.

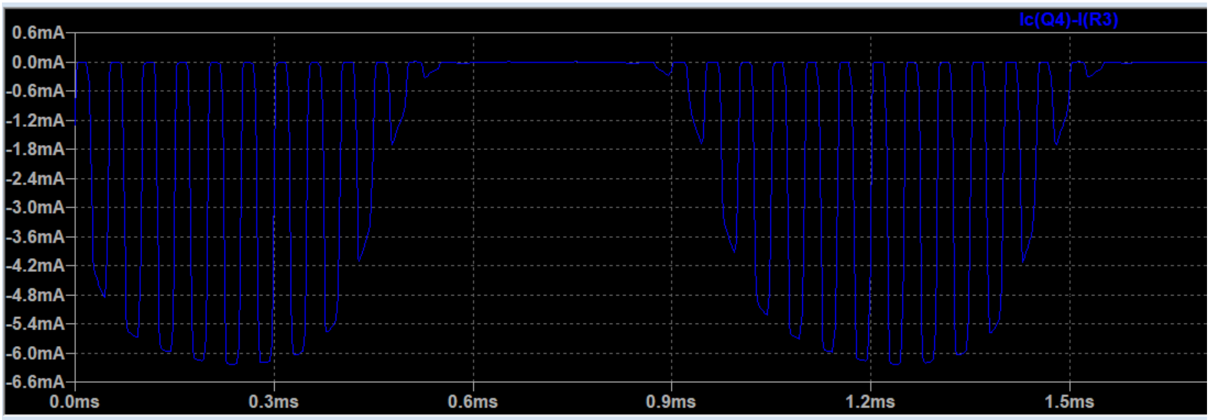

Figure 13: Current Behavior of $Vout_+$

The current relation proves the expectation of OFF state in BJTs. As the BJT is off at some point at each cycle for a bit the envelope will give constant voltage values.

Hardware Implementation

Here is the real-life implementation of the circuit:

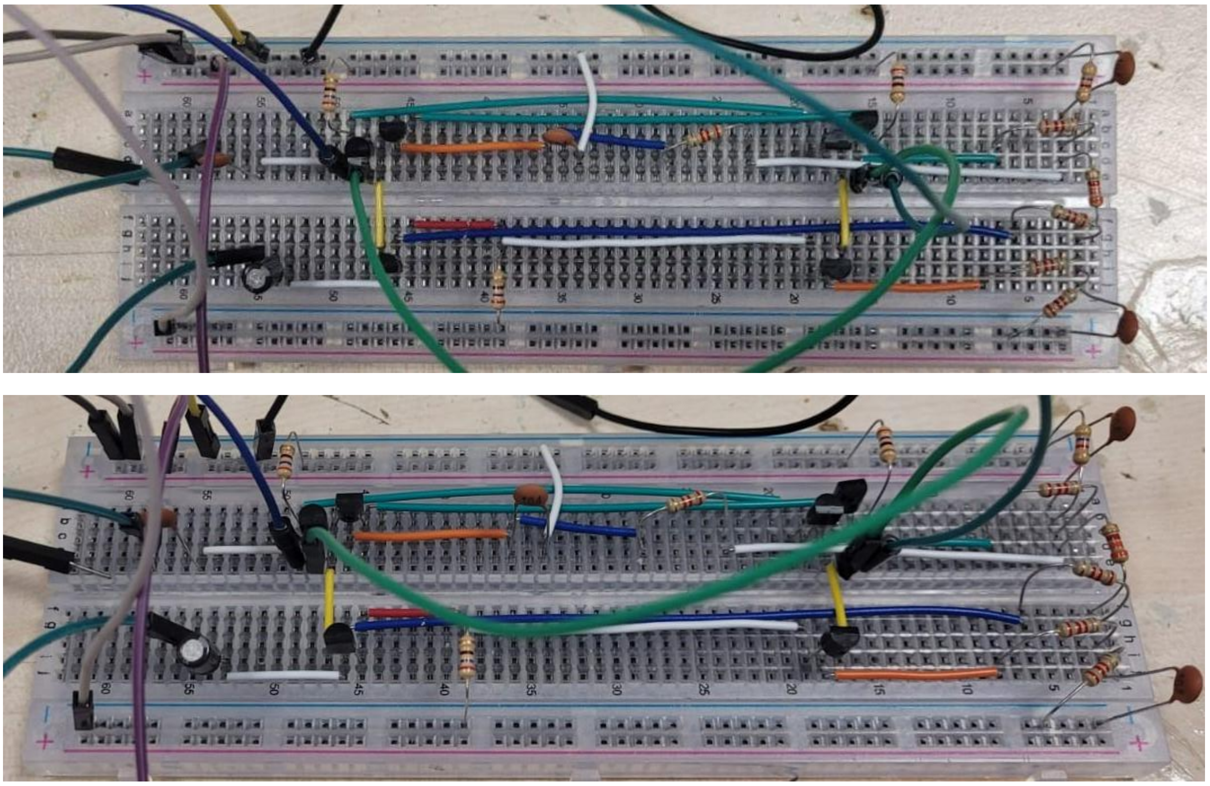

Figure 14: Breadboard design

According to the software results, applied input voltage and frequency values for each input of the differential amplifiers are shown in the following figures.

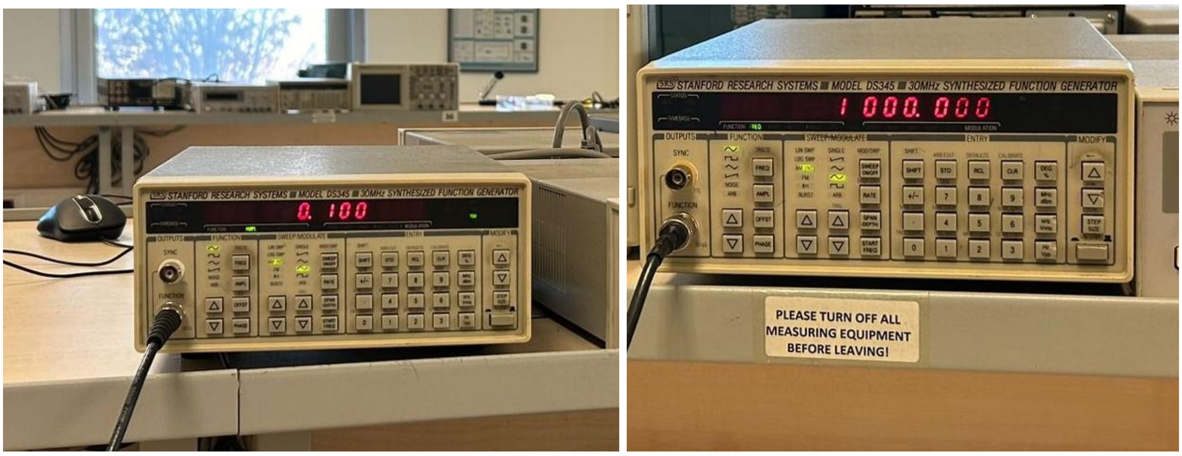

Figure 15: Tail Current Biasing Inputs

For the tail current biasing differential amplifier a sine wave with 0.1 Vpp at 1KHz frequency is applied.

Figure 16: Main Amplifier Input

For the main amplifiers, a sine wave with 0.1 Vpp at 20KHz frequency is applied.

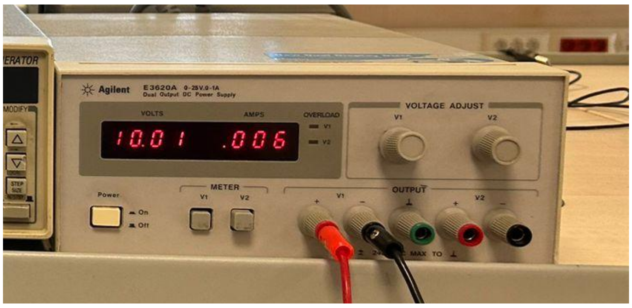

Figure 17: 10V DC Biasing

According to the lab specifications, the current should be less than 10 mA per supply, as can be seen above the supply current is 6 mA which satisfies the condition. Additionally, the power consumption should be less than 200 mW which is also satisfied. Using the power relation:

$$P = I * V$$

$$6\,mA * 10V$$

$$60\,mW < 200mW$$

In order to see the multiplication effect on the oscilloscope both outputs of the differential amplifiers as well as their difference are measured as follows:

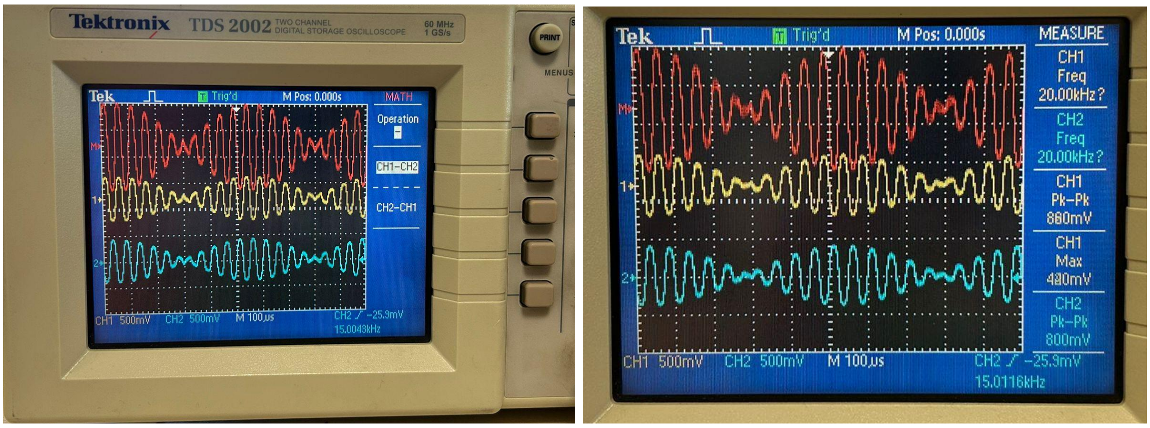

Figure 18: Multiplication results. Red: Multiplied output signal, Yellow: Output of the first differential amplifier, Blue: Output of the second differential amplifier

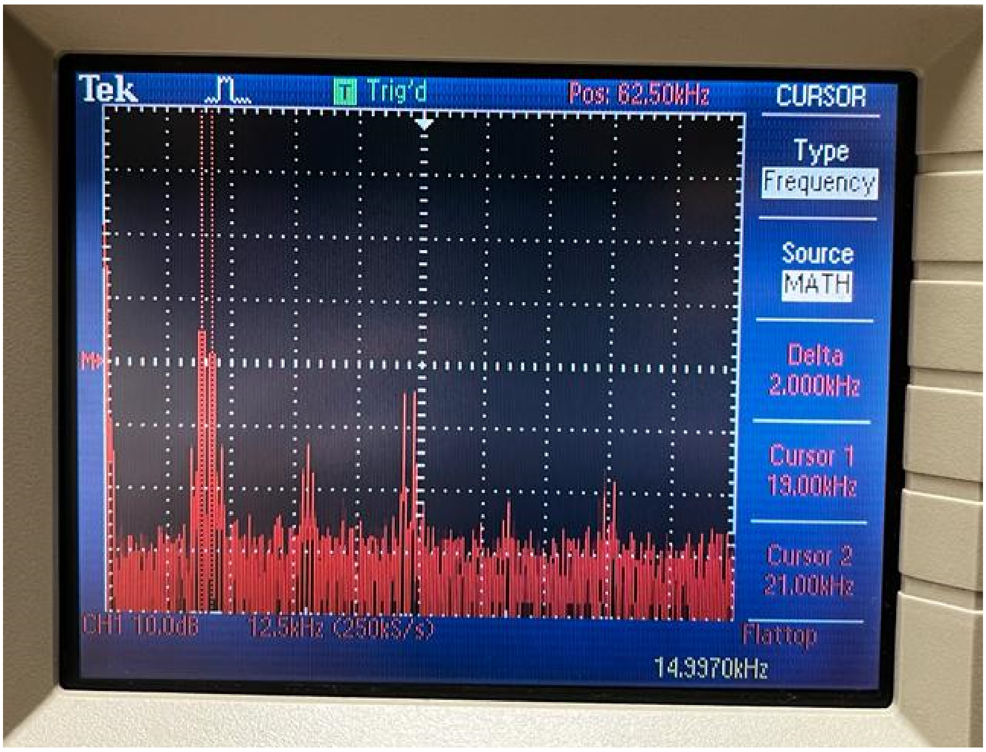

Figure 19: Frequency Response at 0.1Vpp Input

It can be seen in the frequency response that the only peak occurs around the 20kHz signal. As we are modulating two signals the peaks should be around 19kHz and 21kHz, this can be further confirmed from Figure 19.

In hardware the distortions occurred around 0.3 Volts. To solidify the distortion the frequency response was also included. Starting with response at 0.3V.

Figure 20: Distortion at 0.3V Input

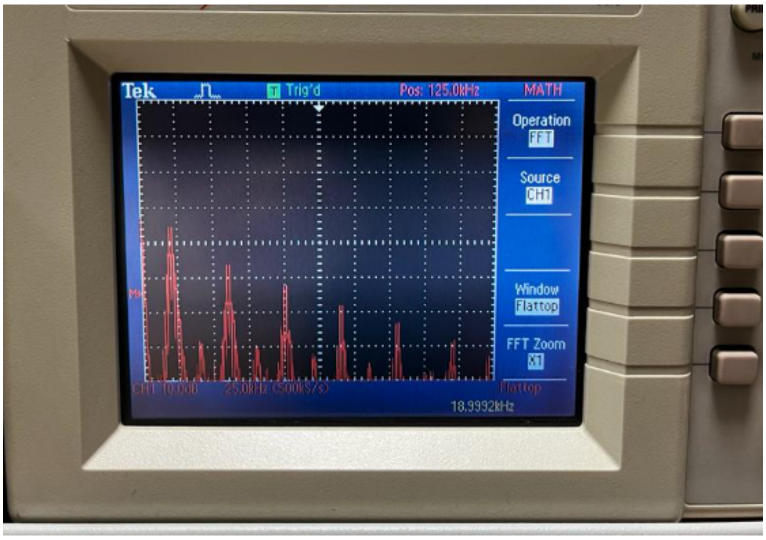

Figure 21: Frequency Response at 0.3V Input

As expected distortions occurred at 0.3V due to the off state of some of the BJTs. Both the envelope and signal at peak had constant values at some points. This is due to the OFF state caused by 1kHz and 20kHz, respectively.

These distortions will create impulses at the harmonics which can be seen in figure 21. This can be given to the periodicity of the rectangular signal having impulses at each harmonic. The decrease of magnitude can also be explained similarly as periodic rectangular functions Fourier Series Coefficients form a sinc function which means the value of the coefficients will decrease as it gets further away from 0.

Conclusion

We believe that this project was quite useful as it allowed us to apply our theoretical knowledge to real life. Considering that the scope of this project was in parallel with the differential amplifiers we learned during the semester was a great opportunity to solidify what has been taught. The most challenging part was the decision process of the resistor and capacitor values. At first, we tried to calculate each resistor value separately but then things got too complicated. Therefore, we decided to use the trial and error method, we also found samples on the internet and modified them according to the lab specifications. For the circuit we needed to use most of the breadboard so we first decided on how we should implement the components effectively rather than jumping into it instantly. Although the hardware implementation of the circuit required great attention and patience, since the criteria wasn't very strict, the circuit worked at the first try which was quite rare for a Bilkent EEE lab session.

References

[1] Gilbert cells, https://user.eng.umd.edu/~neil/EE408D_02/Design_Ex/Mixer/mixer.html (accessed Jan. 3, 2024).

[2] D. Neaman, *Microelectronics Circuit Analysis and Design*. McGraw-Hill Science Engineering, 2007.