

Name: SOLUTIONS

Section: _____

Signature: _____

EEE 313 Spring 2010

Bilkent University
Department of Electrical and Electronics Engineering
EEE 313 Electronic Circuit Design

Final Exam
23 May 2010, 09:00
(5 questions, 150 minutes)

- This is a **closed book**, closed notes exam. No cheat sheet allowed.
- All cell-phones should be completely **turned off**.
- Use a calculator for numerical computations. Carry at least **4 significant digits** during calculations. Your final answer should be at least **3 significant digits**.
- Be sure to write the **units** of all numerical results.
- **Show** all work clearly.
- Please put your **final answer** for each part inside a box for easy identification. Do not give multiple answers, they will not be graded.
- Do not remove the **staple** from the exam sheets or separate pages of the exam. All extra pages must be stamped to your exam.
- You may leave the exam room when you are done. However, please do not leave during the **last five minutes** of the exam.
- At the end of the exam, please stay seated until **all** exam papers are collected.

FET equations:

n-channel MOSFET

$$i_D = K_n (v_{GS} - V_{Tn})^2 \quad \text{SAT}$$

$$i_D = K_n [2(v_{GS} - V_{Tn})v_{DS} - v_{DS}^2] \quad \text{NON-SAT}$$

p-channel MOSFET

$$i_D = K_p (v_{SG} + V_{Tp})^2 \quad \text{SAT}$$

$$i_D = K_p [2(v_{SG} + V_{Tp})v_{SD} - v_{SD}^2] \quad \text{NON-SAT}$$

n-channel JFET

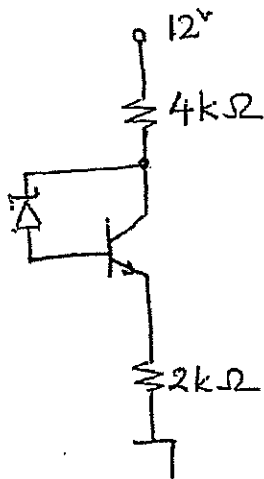
$$i_D = \frac{I_{DSS}}{V_p^2} (v_{GS} - V_p)^2 \quad \text{SAT}$$

$$i_D = \frac{I_{DSS}}{V_p^2} [2(v_{GS} - V_p)v_{DS} - v_{DS}^2] \quad \text{NON-SAT}$$

Please do not write below this line

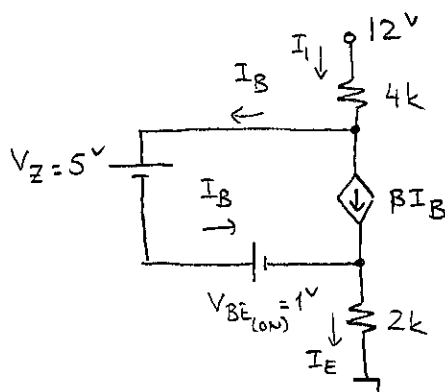
1. 10 pts.	
2. 30 pts.	
3. 20 pts.	
4. 15 pts.	
5. 25 pts.	
Total 100 pts.	

1. (10 points) For the circuit given below $\beta=200$, $\beta_R=10$, $V_{BE(ON)}=1V$, $V_{CE(SAT)}=0V$, $V_\gamma=0.8V$, and $V_Z=5V$. Determine the state of the transistor and find I_E and V_{CE} .



assume TR is in ACT and Zener in breakdown

DC equivalent:



$$I_E = I_1 = (\beta+1)I_B$$

$$\textcircled{1} V_E = 2k \cdot (\beta+1)I_B$$

$$\textcircled{2} V_C = V_E + 6V$$

$$\textcircled{3} V_C = 12V - 4k(\beta+1)I_B$$

$$\textcircled{2} + \textcircled{3} \quad V_E + 6V = 12V - 4k(\beta+1)I_B$$

$$\textcircled{1} \quad 2k(\beta+1)I_B + 6V = 12V - 4k(\beta+1)I_B$$

$$6k(201)I_B = 6V$$

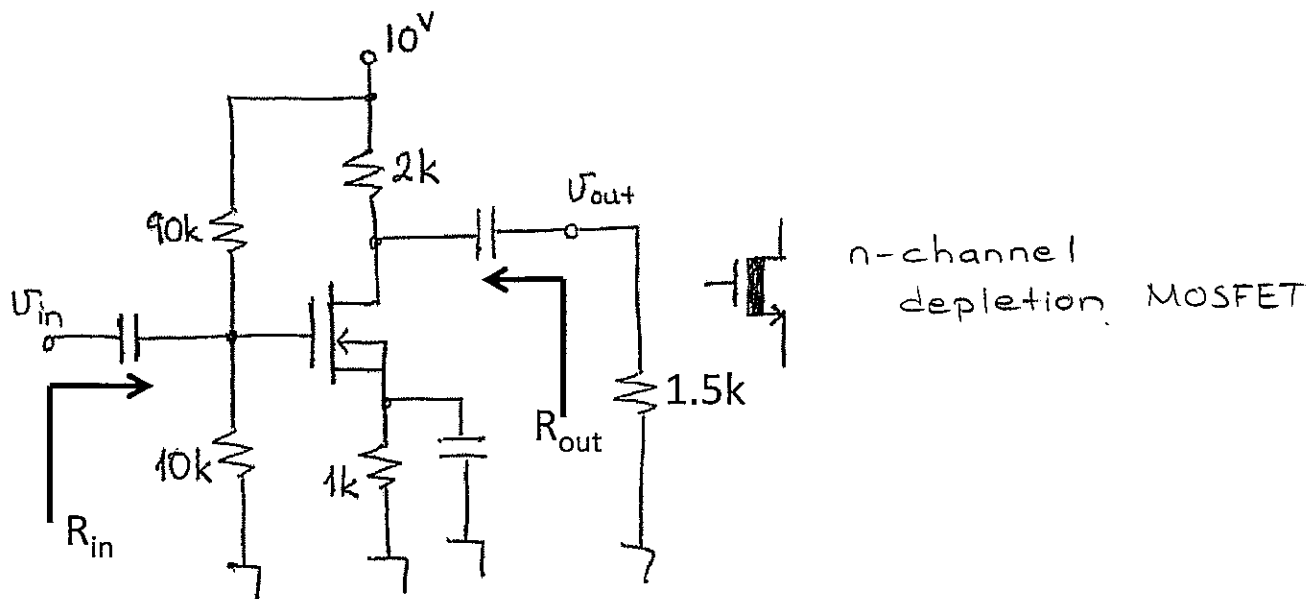
$$I_B = 4.975\mu A$$

$$V_{CE} = 6V \quad \checkmark \quad \text{since } V_{CE} = 6V > V_{CE(SAT)} = 0V$$

$$I_E = (201)I_B = 1mA$$

2. (30 points) For the circuit given below $V_{TN} = -2V$, $K_N = 1mA/V^2$

- (05 points) Find the DC drain current.
- (05 points) Draw the small signal equivalent circuit.
- (05 points) Determine the voltage gain, v_o/v_{in} .
- (10 points) Find the input and output impedance.
- (05 points) Calculate the maximum symmetric undistorted peak-to-peak swing.



i) $V_G = 1V$ $V_S = 1k \cdot I_D$
 assume SAT \Rightarrow ① $I_D = K_N (V_{GS} - V_{Th})^2$ (Transistor relation)
 ② $10V = 3k I_D + V_{DS}$ (Load line, DC)

$$I_D = 1mA/V^2 \left[(1 - 1k I_D) - (-2) \right]^2$$

$$\xrightarrow{\text{(in mA)}} I_D = (1 - I_D + 2)^2 \Rightarrow I_D = (3 - I_D)^2$$

$$I_D^2 - 7I_D + 9 = 0$$

$$I_D = \begin{cases} 5.3mA \\ 1.697mA \end{cases}$$

$$\Rightarrow V_{DS} = \begin{cases} -5.9V \\ 4.91V \end{cases}$$

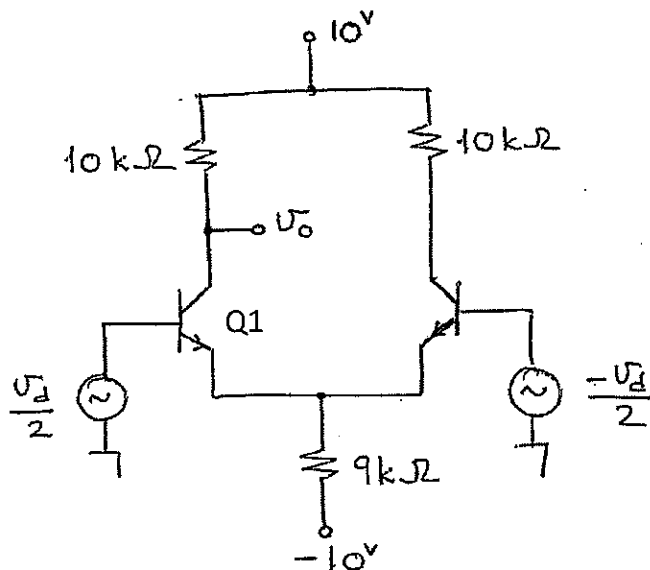
$$\Rightarrow V_{DS} = 4.91V > V_{GS} - V_{Th} = (1 - 1.697) + 2$$

$$V_{DS} = 4.91V > 1.303V$$

$$\boxed{I_D = 1.697mA \quad V_{DS} = 4.91V}$$

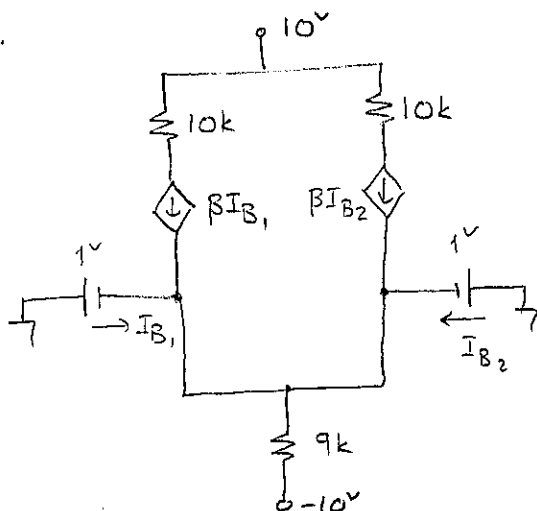
3. (20 points) For the circuit given below, the transistors are identical and $\beta=99$, $V_{BE(ON)}=1V$, $V_{CESAT}=0V$, $r_o=\infty$.

- (05 points) Determine the DC base current of Q1
- (05 points) Draw the ac equivalent circuit
- (10 points) Find the single-sided voltage gain, v_o/v_d .



a) TR are in F.A.

Dc eq.



since identical $I_{B1} = I_{B2} = I_B$
 $I_{E1} = I_{E2} = I_E$

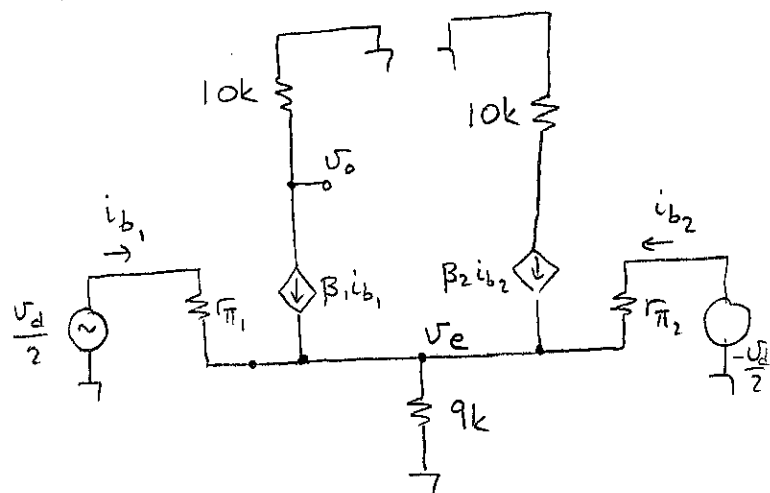
$$2I_E = \frac{(0 - 1V) - (-10V)}{9k}$$

$$I_E = 0.5 \text{ mA}$$

$$I_B = 5 \mu A$$

$$I_{B1} = I_{B2} \Rightarrow r_{\pi 1} = r_{\pi 2} = 26 \text{ mV} / 5 \mu A = 5.2 \text{ k}$$

b) ac eq.



$$c) v_o = -\beta i_{b1} \cdot 10k = -\beta i_b 10k$$

$$\text{emitter node: } (\beta+1)i_{b1} + (\beta+1)i_{b2} = \frac{v_e}{9k}$$

$$\frac{v_d}{2} - r_{\pi} i_{b1} = v_e = -\frac{v_d}{2} - r_{\pi} i_{b2}$$

$$\left. \begin{aligned} \frac{v_d}{2} - r_{\pi} i_{b1} &= v_e \\ \frac{v_d}{2} + r_{\pi} i_{b2} &= v_e \end{aligned} \right\} \Rightarrow i_{b1} = -i_{b2} = i_b$$

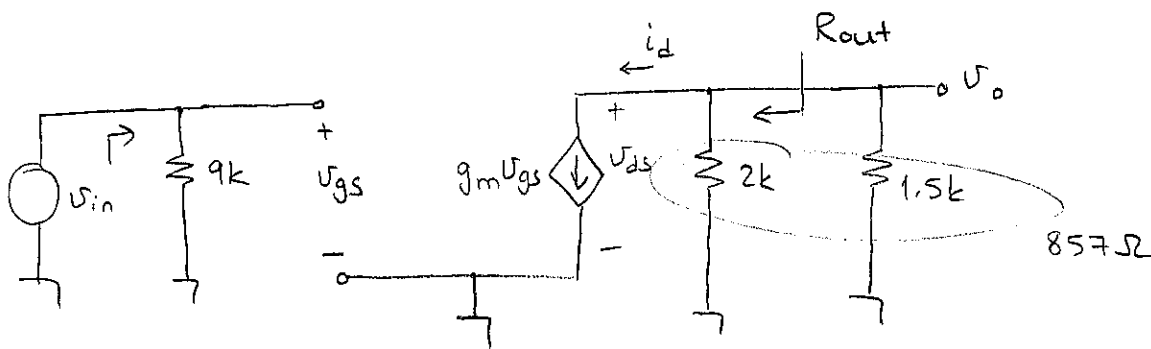
$$v_e = 0$$

$$\frac{v_d}{2} = r_{\pi} i_b$$

$$\frac{v_o}{v_d} = \frac{-\beta \cdot 10k}{r_{\pi}} = -95.192$$

ac eq.

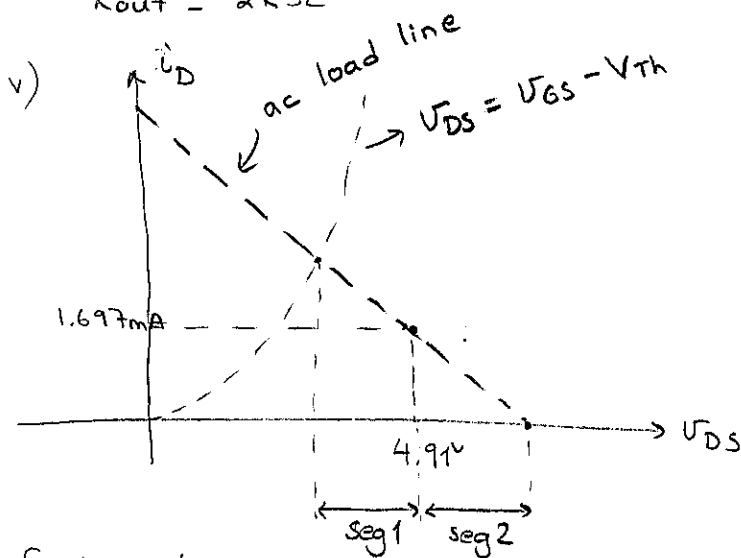
$$ii) \quad g_m = 2 \sqrt{K_n I_D} = 2 \sqrt{\frac{1 \text{ mA}}{V^2} \cdot 1.697 \text{ mA}} = 2.606 \text{ mA/V}$$



$$iii) \quad \left. \begin{aligned} v_o &= -g_m v_{gs} (2k // 1.5k) \\ v_{in} &= v_{gs} \end{aligned} \right\} \quad v_o/v_{in} = -g_m \cdot 857 \Omega = \boxed{-2.23 = A_v}$$

$$iv) \quad R_{in} = 9k \Omega$$

$$R_{out} = 2k \Omega$$



slope of ac load line

$$v_{ds} = -i_d (2k // 1.5k)$$

$$\Rightarrow R_{ac} = 2k // 1.5k$$

$$\text{slope} = -\frac{1}{R_{ac}}$$

$$\text{seg2} = 1.697 \text{ mA} \cdot 857 \Omega$$

$$\boxed{\text{seg2} = 1.454 \text{ V}}$$

to find seg1

ac load line eq.

$$i_D = a v_{DS} + b$$

$$a = -1/R_{ac} \Rightarrow i_D = -\frac{1}{857 \Omega} v_{DS} + b$$

$$1.697 \text{ mA} = -\frac{4.91 \text{ V}}{857} + b$$

$$\Rightarrow b = 7.43 \text{ mA}$$

$$\Rightarrow i_D = \frac{-v_{DS}}{0.857k} + 7.43 \text{ mA} \quad \text{ac load line}$$

$$i_D = K_n v_{DS}^2 = 1 \text{ mA/V}^2 v_{DS}^2 \quad \text{parabolic curve}$$

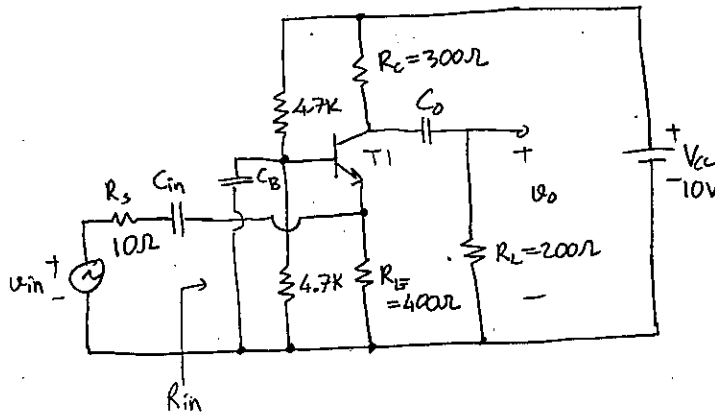
$$\left. \begin{aligned} &\text{intersection} \\ &v_{DS}^2 + \frac{v_{DS}}{0.857} - 7.43 = 0 \\ &v_{DS} = \begin{cases} -3.37 \\ 2.204 \end{cases} \end{aligned} \right\}$$

$$\text{seg1} = 4.91 \text{ V} - 2.204 \text{ V}$$

$$\boxed{\text{seg1} = 2.7 \text{ V}}$$

$$\boxed{\text{Swing } V_{pp} = 2.91 \text{ V}}$$

4. (15 points)



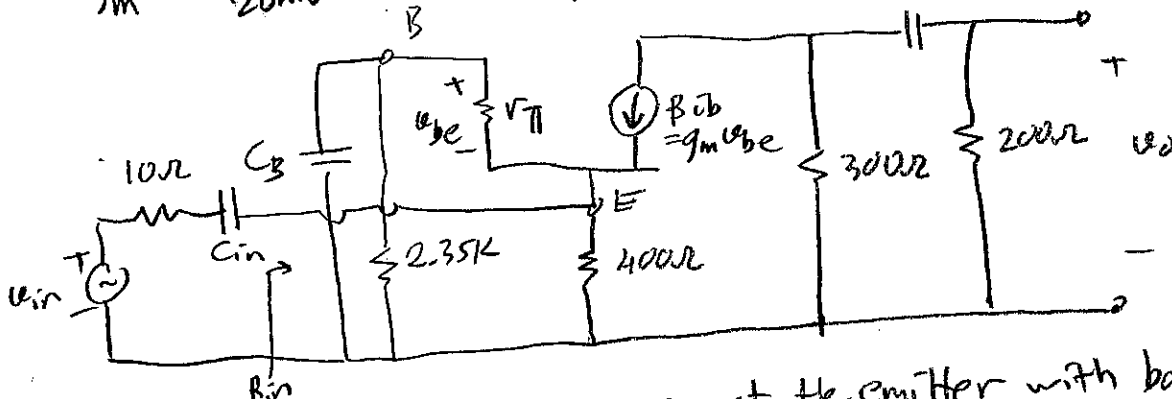
At the circuit given above, $\beta=100$, $V_{CESAT}=0.5$ Volt, $V_{BEON}=0.7$ Volt., $R_E=400\Omega$, $R_C=300\Omega$ and $R_S=10\Omega$. Assume that the transistor is in active mode and $I_C=10$ mA. Please answer the following:

- (5 points) Assuming that all the capacitors are very large, find the input resistance R_{in} .
- (5 points) Again assuming that all the capacitors are very large, find the midband gain.
- (5 points) Assuming that C_B and C_o are very large, find the value of C_{in} in order to make the 3-dB lower cut-off frequency equal to 300 Hz.

Assuming that C_B and C_{in} are very large, find the value of C_o in order to make the 3dB lower cut-off frequency equal to 300 Hz.

Solutions: First draw the AC equivalent circuit

$$g_m = \frac{10\text{mA}}{26\text{mV}} = 0.385\text{S} \quad \frac{1}{g_m} = \frac{26\text{mV}}{10\text{mA}} = 2.6\Omega$$



- a-) The i/p resistance seen at the emitter with base grounded (C_B is AC short) is
- $$\frac{\beta}{g_m(\beta+1)} \approx \frac{1}{g_m} = 2.6\Omega \Rightarrow R_{in} = 2.6\Omega \parallel 400\Omega = \frac{2.6 \times 400}{402.6} = 2.583\Omega$$

$$\frac{4-b)}{v_{be} = -v_{in} \frac{2.583}{10+2.583} = \frac{2.583}{12.583} v_{in}}$$

$$v_o = -g_m (R_L \parallel R_C) v_{be} = \frac{200 \times 300}{500} \frac{1}{2.583} = \frac{120}{2.583}$$

$$\frac{v_o}{v_{in}} = \frac{2.583}{12.583} \cdot \frac{120}{2.583} = \frac{120}{12.583} = 9.536$$

4-g) The input circuitry is



The rest of the gain is proportional

$$\frac{v_{be}}{v_{in}} = \frac{2.583}{2.583 + 10 + \frac{1}{j\omega C_{in}}} = \frac{j\omega C_{in}}{1 + j12.583\omega C_{in}}$$

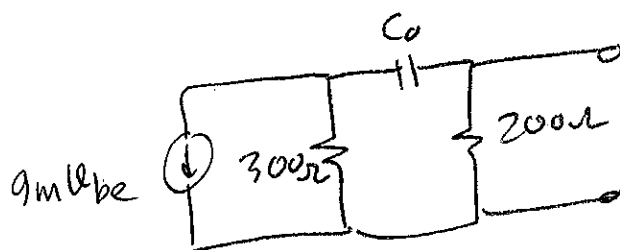
The 3dB point of the high-pass circuit represented by the above equation is

$$2\pi f_0 = \omega_0 = \frac{1}{RC_{in}} = \frac{1}{12.583 \times C_{in}} \Rightarrow$$

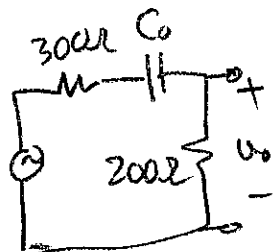
$$C_{in} = \frac{1}{12.583 \times 2 \times \pi \times 300} = 4.216 \times 10^{-6} = 42.16 \times 10^{-6} F = 42.16 \mu F$$

3 points

The output circuitry is



$v_o \equiv g_m v_{be} 300$
Thevenin's equiv.



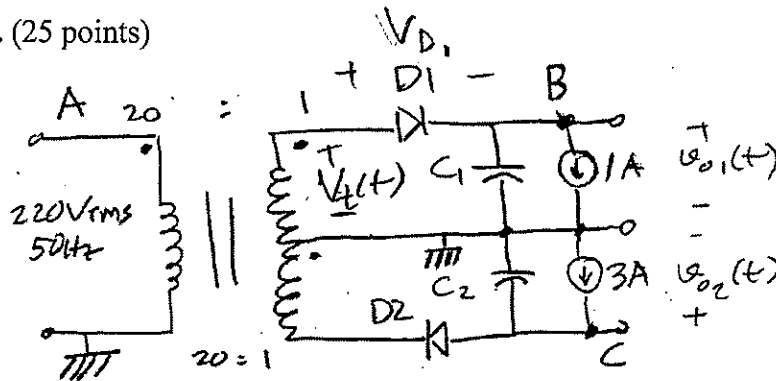
\Rightarrow As in the i/p circuitry

$$2\pi f_0 = \omega_0 = \frac{1}{RC_o} = \frac{1}{500 C_o} \Rightarrow$$

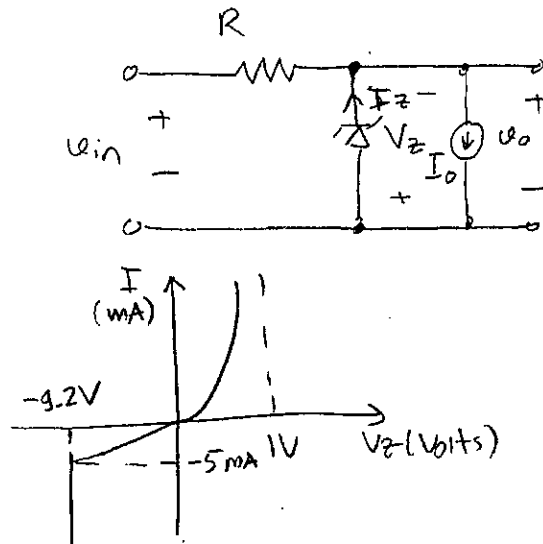
$$f_0 = \frac{1}{2\pi \times 500 \times 300} = 1.06 \times 10^{-6} F = 1.06 \mu F$$

2 points

5. (25 points)



(I)



(II)

At the circuit given at (I), $C_1=10000\mu\text{F}$, $V_2=15000\mu\text{F}$, $V_\gamma=1$ Volts. Each of the secondary winding of the transformer has the transformer ratio of 20:1. Please answer the following:

- (5 points) Find the ripple voltage on $V_{O1}(t)$ and $V_{O2}(t)$.
- (5 points) Find the peak reverse voltages on diodes D_1 and D_2 .
- (5 points) Plot $V_{O1}(t)$ and $V_{O2}(t)$ on the same graph showing the voltages and timing properly.
- (5 points) Would you get electric shock if you touch the points marked A, B and C. Please answer and explain each one separately.
- (5 points) At the circuit given at (II), a regulator circuit and the V-I characteristics of the zener diode used at the regulator is given. The ranges of v_{in} and I_o are given as follows; $12\text{V} < v_{in} < 20\text{V}$ and $10\text{mA} < I_o < 30\text{mA}$. Find the value of R which preserves regulation and which minimizes the zener diode power dissipation at the same time.

Solution of Q5

a-) $I\Delta T \approx C\Delta V$ $\Delta V = V_{\text{ripple}}$, $\Delta T = \frac{1}{50\text{Hz}} = 20\text{ms}$

$$V_{\text{ripple}} = \frac{I\Delta T}{C}$$

at Q_1 $V_r = \frac{1\text{A} \times 20 \times 10^{-3}}{1 \times 10^{-2}} = 2\text{V}$

at Q_2 $V_r = \frac{3\text{A} \times 20 \times 10^{-3}}{1.5 \times 10^{-2}} = 4\text{V}$

0 points for
full wave
calculation

b-) Peak voltage V_p on the capacitors are

$$V_p = \frac{220 V_{\text{rms}} \times \sqrt{2}}{n} - V_g = 14.51 \approx 14.5\text{V}$$

$$V_{D_1}(t) = V_T(t) - V_{D_1}(t) \equiv \frac{220\sqrt{2}}{20} \cos \omega_0 t - V_p$$

$$V_{D_1}(t) = (15.5 \cos \omega_0 t - 14.5)\text{V}$$

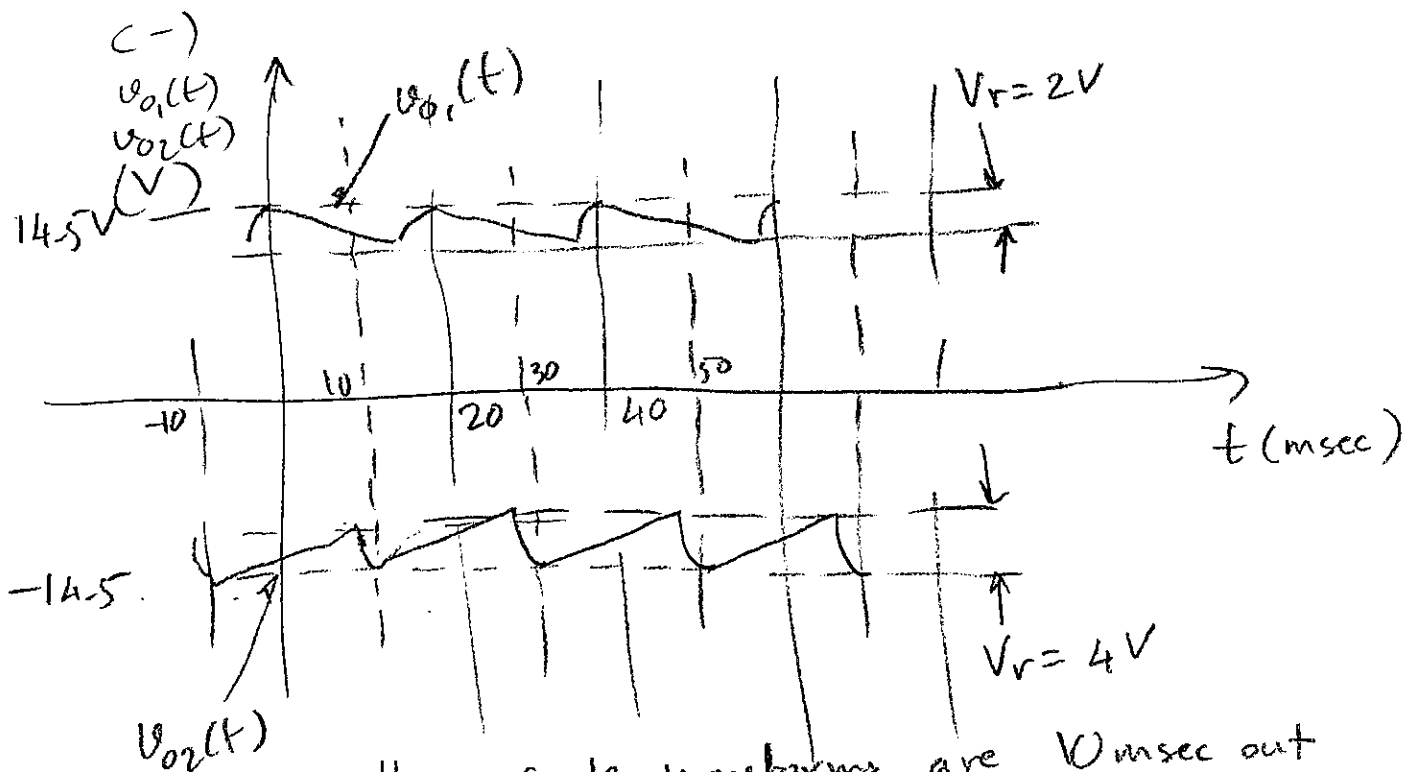
negative maximum of $V_{D_1}(t)$ (peak reverse voltage)

Be equal to

$$-15.5 - 14.5 = -30\text{V}$$

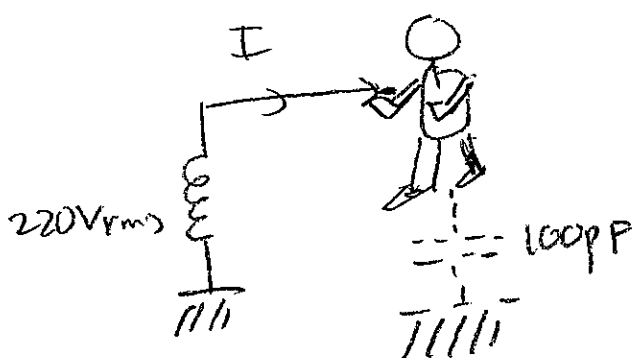
it is the same for D_2

Solution of Q-5 (Continued)



the ripple waveforms are 10msec out of phase

d-) At A, you get electric shock because you touch live phase $220V \times \sqrt{2}$ peak voltage which induces current in you even if you are not grounded. Human body has approximately $100pF$ capacitance to ground. The drawing is as follows



$$I_P \approx \underbrace{2\pi \times 50}_{\omega} \times \underbrace{100 \times 10^{-12}}_C \times \underbrace{220\sqrt{2}}_{V_P}$$

$$\approx 9.745 \mu A \approx 10 \mu A$$

\Rightarrow You get electric shock but current is quite small

Q5 continued

d) cont

But if you are grounded you can be exposed to a large current which can kill you.

at B & C you are exposed to a small DC voltage which will neither harm you nor give you an electric shock because the transformer isolates you from the mains supply.

e-) The minimum input voltage and the maximum o/p current are critical for finding the value of R

$$\Rightarrow R = \frac{V_{inmin} - 9.2V}{I_{max} + I_{zmin}} = \frac{12V - 9.2V}{30mA + 5mA}$$

$$R = \frac{2.8V}{35mA} = 80\Omega$$

because the resistor must also supply the knee current at least