FINAL EXAM

- Find the symbolic expression first and then plug in the numerical values for all your answers

- Show all the derivation for your answers, no points will be given to direct answers with no explanation even though they are correct

- Call 0312-290-3193 in case of emergency or connection failure

FINAL EXAM

Formula Page

BJT

- Forward active region, B-E forward, B-C reverse biased

$$I_C = I_S(e^{\frac{V_{BE}}{V_T}} - 1)$$

$$\alpha I_E = I_C, \qquad I_C = \beta I_B, \qquad \alpha = \beta/(\beta + 1)$$

$$r_\pi = \frac{V_T}{I_{BQ}}, g_m = \frac{I_{CQ}}{V_T}, r_o = \frac{V_A}{I_{CQ}}, V_T = 26mV\ at\ 300K$$

Diode

- VT=26mV at 300K

$$I_D = I_S\left(e^{\frac{V_D}{V_T}} - 1\right)$$

nMOS

- Non-saturation ($V_{DS}<V_{GS}-V_{TN}$)

$$I_D = K_N\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right](1 + \lambda V_{DS})$$

- Saturation ($V_{DS}>V_{GS}-V_{TN}$)

$$I_D = K_N(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$$

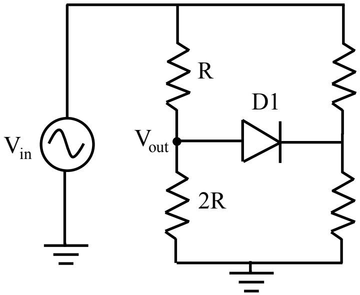
- for pMOS replace V_{GS} with V_{SG}, $-V_{TN}$ with V_{TP} , and V_{DS} with V_{SD}

$$g_m = 2K_N\left(V_{GSQ} - V_{th}\right) = 2\sqrt{K_N I_{DQ}}, r_0 = {}^{1}/_{\lambda I_{DQ}}$$

Bilkent University **EEE313: Electronic Circuit Design** **Spring 2019-20, 01.06.2020**

FINAL EXAM – Part 1, 60 minutes

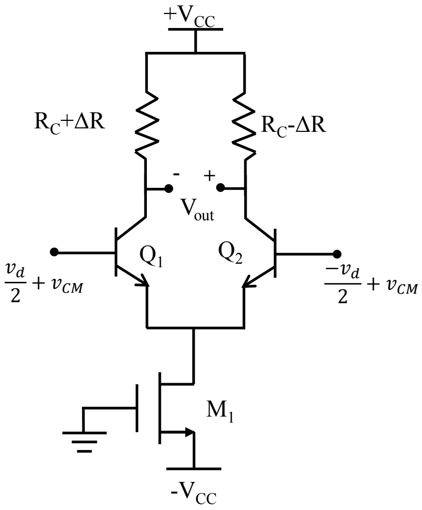
1. (15 points)



V_{on}=0.7V for D1
Plot V_{out} vs V_{in} for the above circuit. Clearly label all the critical values and slopes.

Hint: Start by assuming D1 is OFF and finding the corresponding V_{in} range

2. (25 points)



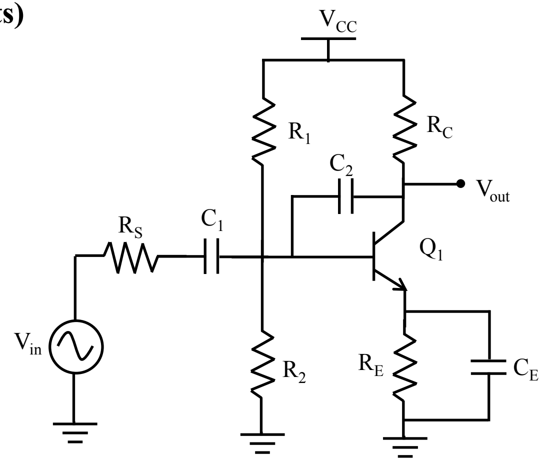
V_{CC}=2.5V
β=100
V_{BEon}=0.7V
V_{CEsat}=0. 2V
$V_A=\infty$
V_{TN}=1V
K_N=1mA/V^2
R_C=2kΩ
ΔR=100Ω
λ=0.04 1/V

Note that there is a mismatch between the collector resistances

a. Calculate the small signal parameters (Take v_d=0, v_{CM}=0 for this purpose), and find the range of v_{CM} to keep the circuit functional.
b. Find the small signal differential and common mode gain of this circuit first symbolically and then numerically.

FINAL EXAM – Part 2, 40 minutes

3. (25 points)



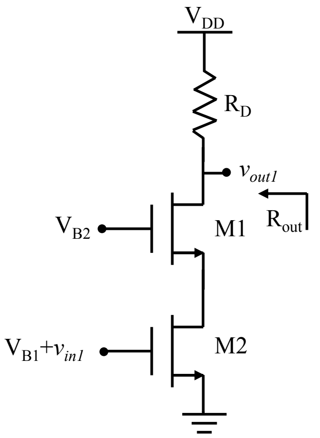
V_{CC}=5V
β=100
$V_A=\infty$
V_{BEon}=0.7V
V_{CEsat}=0.2V
$R_1=R_2$=200kΩ
R_E=0.6kΩ
R_C=2.5kΩ
R_S=5kΩ
C_1=1μF
C_2=10pF
C_E very large

a. Verify the state of Q_1 at the Q-point and find the small signal parameters.
b. Calculate $A_V(j\,\omega)=V_{out}(j\omega)/V_{in}(j\omega)$, and provide the basic bode plot. Express $A_V(j\omega)=A_0*H_1(j\omega)*H_2(j\omega)*\ldots$ where A_0 is the mid-band gain. $H_i(j\omega)$ should be in the form of (jf/fc)/(1+jf/fc), or 1/(1+jf/fc), or multiplication of such factors. Indicate all the breaking point frequencies, slopes and mid-band gain in the bode plot.

Hint: How many poles does C_2 introduce?

FINAL EXAM – Part 3, 40 minutes

4.a. (20 points)

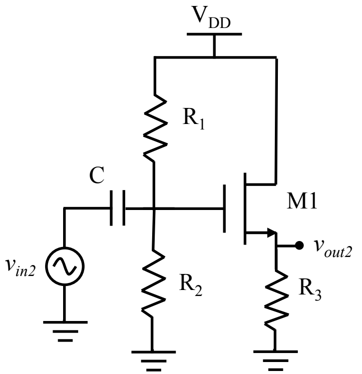


V_{DD}=5V
R_D=4kΩ
V_{TN}=1V
V_{B1}=2V
K_N=0.5mA/V^2
λ=0.04 1/V

a. M1 and M2 are matched transistors. Find the DC biasing voltage V_{B2} such that M1 and M2 have the same V_{DS}.
b. Find $A_{V1}=v_{out1}/v_{in1}$ (Assume λ=0 in this part)
c. Find R_{out} (Note that it is NOT simply R_D).

Reminder: λ=0 only for part b.

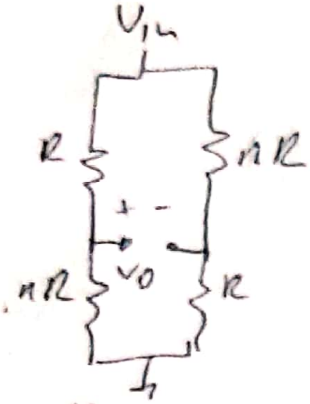
4.b. (15 points)



V_{DD}=5V
$R_1=R_2$=20kΩ
R_3=2kΩ
K_N=1mA/V^2
V_{TN}=0.5V
λ=0

a. Calculate the small signal parameters and verify the transistor state.
b. Find $A_{V2}=v_{out2}/v_{in2}$
c. Instead of v_{in2}, v_{out1} is connected to the left side of C, calculate the overall gain $A_{overall}=v_{out2}/v_{in1}$ using the already found A_{V1} and A_{V2}.

Q1. Assume diode is OFF $n = 2, 3,$ or 5

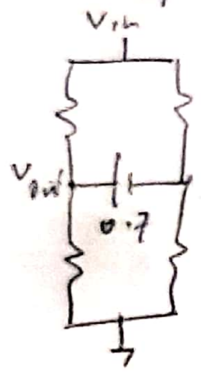


$$V_O = V_{in}\frac{nR}{nR+R} - V_{in}\frac{R}{nR+R} = \frac{n-1}{n+1}V_{in} < 0.7$$

$$\Rightarrow V_{in} < 0.7 \times \frac{n+1}{n-1} \triangleq V_1 \quad \text{and } V_{out} = \frac{n}{n+1}V_{in}$$

$= S\,V_{in}$ where $S = \frac{n}{n+1}$

Otherwise D is ON



Using superposition

i) kill 0.7, $V_{out} = \frac{V_{in}}{2}$

ii) kill V_{in}, $V_{out} = C$ (a constant)

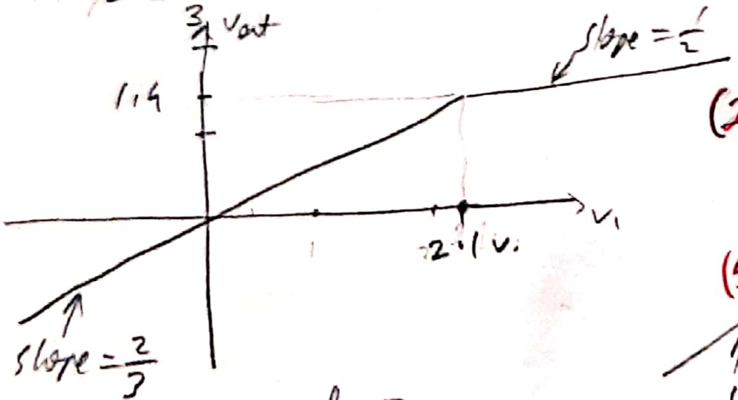
So $V_{out} = \frac{1}{2}V_{in} + C$

C is not important because the line must be continuous

$n = 2$

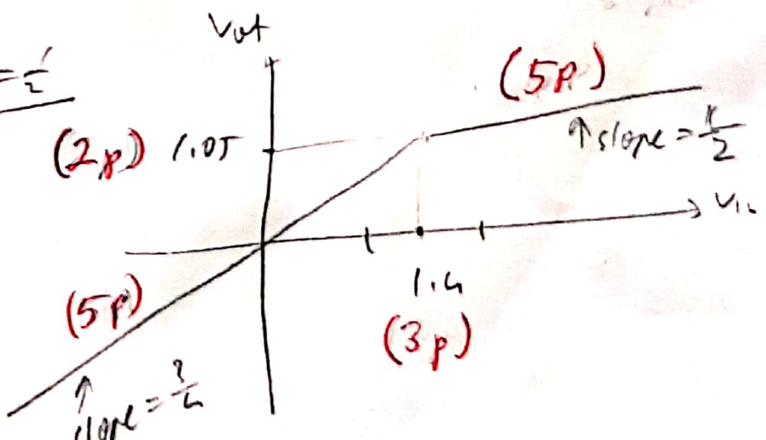
$V_1 = 0.7 \times \frac{3}{1} = 2.1\,V.$

$S = \frac{2}{3}$



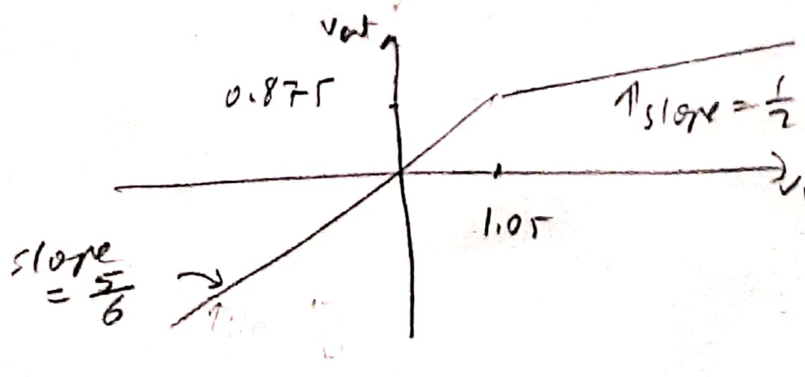
$n = 3$

$V_1 = 0.7 \times \frac{4}{2} = 1.4$ $\quad S = \frac{3}{4}$



$n = 5$

$V_1 = 0.7 \times \frac{6}{4} = 1.05$, $S = \frac{5}{6}$

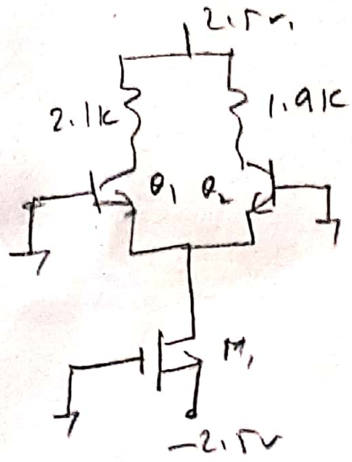


(−2p) for not drawing the line for $V_{in} < 0$.

All equations correct but no plot −7p.

Q2.

a)



Assume FA and SAT.

$I_D = 1(2.5-1)^2(1+0.04(-0.7-(-2.5)))$

$= 1.5^2(1+0.072) = 2.412\ mA$ (3p)

$V_{DS} = -0.7-(-2.5) = 1.8 > 2.5-1$ ✓ (1p)

$I_{E_1} = I_{E_2} = \frac{2.412}{2} = 1.206\ mA$

$I_{C_1} = I_{C_2} = \frac{100}{101} \times 1.206 = 1.194\ mA$. (2p)

$V_{CE_1} < V_{CE_2}$ $\quad V_{CE_1} = (2.5 - 1.194\times 2.1) - (-0.7) = 0.69v > 0.2$ ✓ (1p)

To find a range for V_{cm}:

$V_{DS} = V_{cm} - 0.7 - (-2.5) = V_{cm} + 1.8 > 2.5 - 1 \Rightarrow V_{cm} > -0.3$

* $V_{CE_1} = (2.5 - 1.194\times 2.1) - (V_{cm} - 0.7) > 0.2$

$0.69 - V_{cm} > 0.2 \Rightarrow V_{cm} < 0.49\ v.$

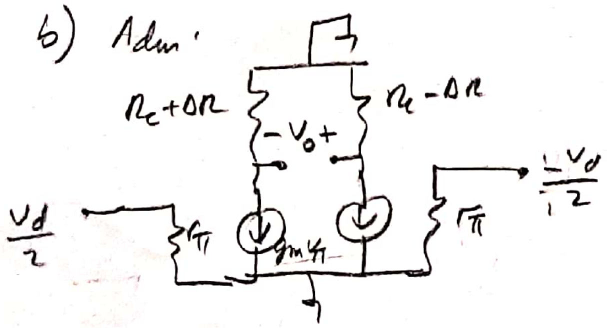
If the above checks are not made but range found the 6p.

$\therefore -0.3 < V_{cm} < 0.49$ (4p)

$Q: \ g_m = \frac{1.194}{0.026} = 45.9\ mA/v \Rightarrow r_\pi = \frac{100}{45.9} = 2.18\ k\Omega.$ (2p)

$M: \ r_o = \frac{1}{0.04\times 2.412} = 10.36\ k\Omega. \quad g_m = 2\sqrt{1\times 2.412} = 3.1\ mA/v$

b) A_{dm}:

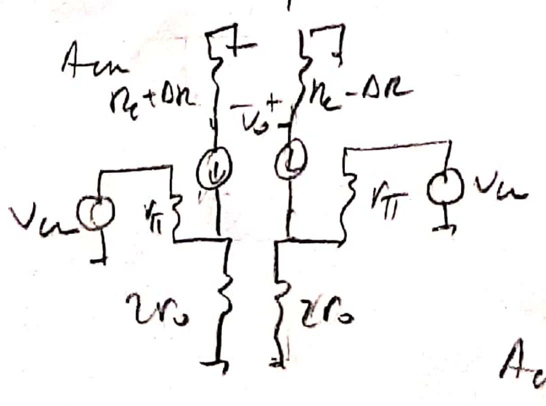


$V_o = g_m \frac{V_d}{2}(R_c - \Delta R) + g_m \frac{V_d}{2}(R_c + \Delta R)$

$= g_m V_d R_c \quad A_{dm} = \frac{V_o}{V_d} = g_m R_c$

(6 p) (5+1)

$= 45.9 \times 2 = 91.8$



$V_{out} = \frac{-V_{cm}}{r_\pi + (\beta+1)2r_o} \times \beta \times (R_c - \Delta R)$

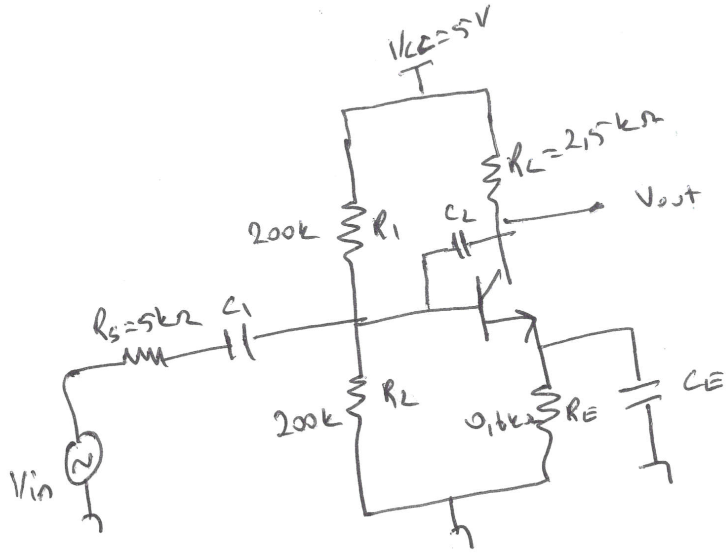
$+ \frac{V_{cm}}{r_\pi + (\beta+1)2r_o} \times \beta(R_c + \Delta R)$

$= \frac{V_{cm}\times\beta\times\Delta R\times 2}{r_\pi + 2(\beta+1)r_o}$

(6p) (5+1)

$A_{cm} = \frac{\beta\times\Delta R\times 2}{r_\pi + 2(\beta+1)r_o} = \frac{100\times 0.1\times 2}{2.18\times 202\times 10.36} = 0.0095$

Part 2 – Question #3 (25 points)

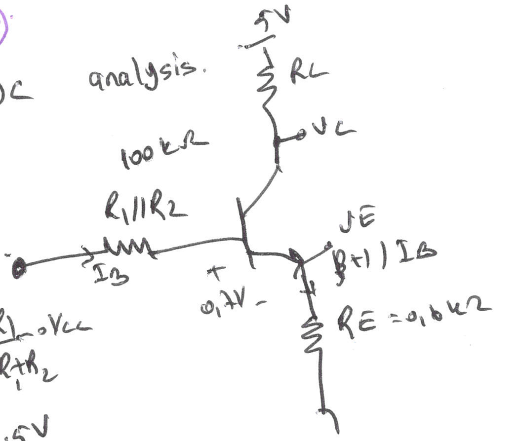


There were 3 different C_{1-2} pairs.

	Set 1	Set 2	Set 3
C_1	1µF	2µF	1,5µF
C_2	10pF	8pF	7pF

Very large C_E means it acts like open circuit in DC and short circuit in AC analysis. No contribution to frequency response.

a) (06 points) DC analysis.



$V_{th}=\frac{R_1}{R_1+R_2}\cdot V_{CC}$

$=2{,}5V$

$V_{th}=(R_1\|R_2)\,I_B+0{,}7+(\beta+1)\,I_B\,R_E$ (02)

$2{,}5=100 I_B+0{,}7+101 I_B\,0{,}6k$

$1{,}8=160{,}6\,I_B$

$I_B=11{,}21\ \mu A$

$I_C=1{,}12\ mA$ (01) $I_C\sim I_E$

$I_E=1{,}13\ mA$ (01)

$V_{CE}=1{,}52V > 0{,}2V$ F.A ✓ (01)

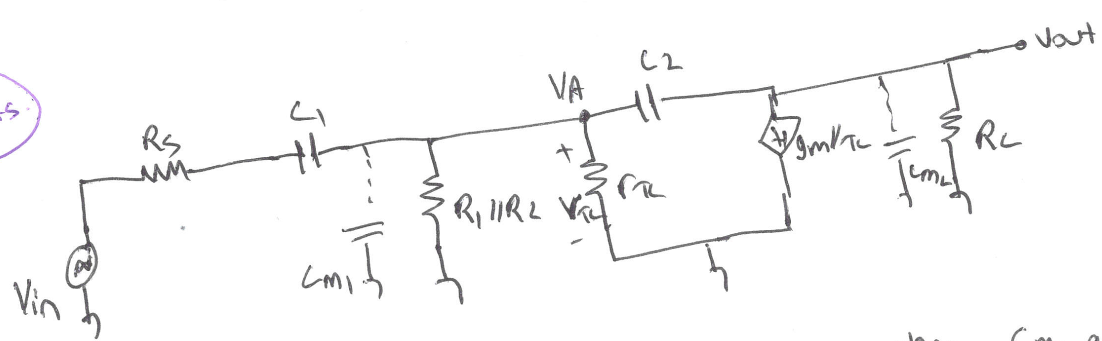
$V_E=0{,}6\cdot 1{,}13=0{,}68V$

$V_C=5-R_C I_C=5-2{,}5\cdot 1{,}12=2{,}2V$

$g_m=\frac{I_C}{V_T}=\frac{1{,}12}{0{,}026}=43{,}1$ (1)

$r_\pi=\frac{\beta}{g_m}=2{,}32\ k\Omega$.

b) (19 points)



due to miller Effect C_2 will split into two capacitors C_{m1} and C_{m2}

(miller effect + small signal model (4))

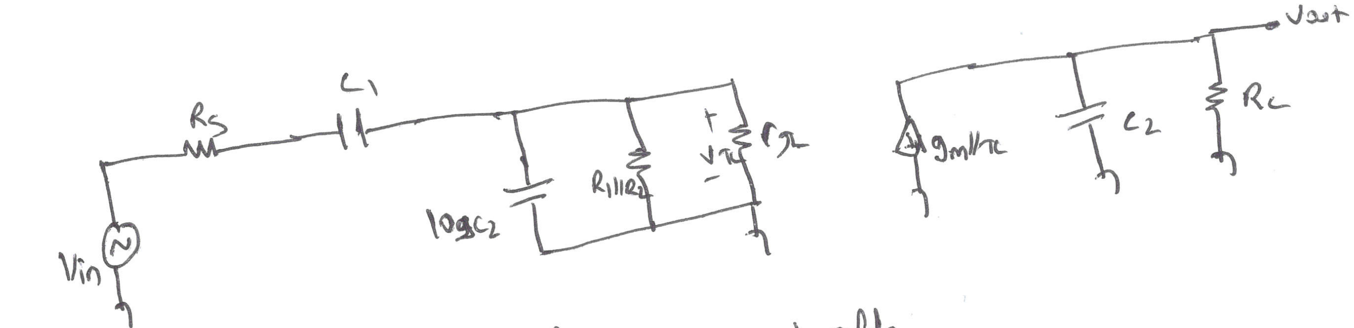
$C_{m1}=(1+A)\cdot C_2$ $C_{m2}=(1+\frac{1}{A})\cdot C_2$

$-A=\frac{V_{out}}{V_a}=-g_m R_C=-43{,}1\cdot 2{,}5=-107{,}75$

$A=107{,}75\sim 108$

$C_{m1}=109\,C_2$ $C_{m2}\cong C_2\cdot\left(\frac{1}{A}\ll 1\right)$

The new ckt with miller Effect



C_2 introduces two high frequency cut offs.

C_1 introduces one low frequency cut off.

$C_1 \gg 10 g C_2$ so at low frequencies we assume C_2 is open and

$\cdot 1\mu F$ $\sim 1nF$

at high frequencies we assume C_1 is short.

$$A_o = \frac{\overbrace{R_1 \| R_2 \| r_\pi}^{R_{in}}}{R_1 \| R_2 \| r_\pi + R_S} \cdot -g_m R_C = -\frac{2{,}27}{2{,}27+5} \cdot 43{,}1 \cdot 2{,}5 = -33{,}6$$

A_o 03

$R_{in} = 100 \| 2{,}32 = 2{,}27 k\Omega$

Low freq. pole due to C_1 $\quad C_1 \times (\underset{5k}{R_S} + \underbrace{R_1 \| R_2 \| r_\pi}_{2{,}27}) = C_1 \cdot 7{,}27k$

f_{c1} 02

$$f_{c1} = \frac{1}{2\pi \; C_1 \; 7{,}27k} =$$

	f_{c1}
$C_1 = 1\mu F$	21,9 Hz
$C_1 = 2\mu F$	10,9 Hz
$C_1 = 1{,}5\mu F$	14,6 Hz

High freq. pole due to $10 g C_2$

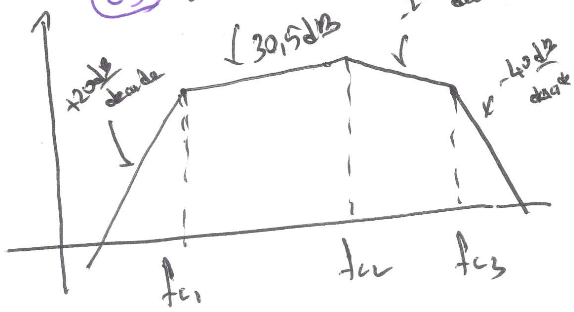
f_{c2} 02 $\quad f_{c2} = \dfrac{1}{2\pi \; 10 g C_2 \, (\underbrace{\underbrace{R_1 \| R_2 \| r_\pi}_{2{,}27} \| R_S}_{1{,}56k\Omega})}$

C_2	f_{c2}	f_{c3}
10pF	93,6 kHz	6,4 MHz
8 pF	117 kHz	7,95 MHz
7pF	133,7 kHz	9,1 MHz

High freq. pole due to C_2 $\quad f_{c3}$ 02

$$f_{c3} = \frac{1}{2\pi \; C_2 R_C} =$$

Bode plot 03 $20\log(33{,}6)$



Bode plot for magnitude.

$A_{j\omega}$ 03

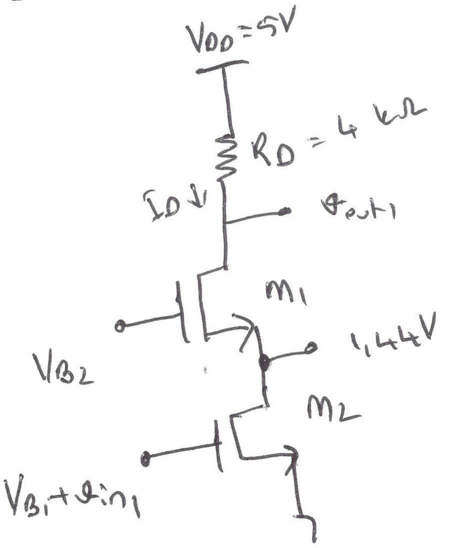
$$A_V(j\omega) = \underset{-33{,}6}{A_o} \cdot \underbrace{\frac{jf/f_{c1}}{1 + j\frac{f}{f_{c1}}}}_{\text{high pass filter}} \cdot \underbrace{\frac{1}{1 + j\frac{f}{f_{c2}}}}_{\text{Low pass filter}} \cdot \underbrace{\frac{1}{1 + j\frac{f}{f_{c3}}}}_{\text{Low pass filter.}}$$

~~$A_V(j\omega) = -33{,}6 \ldots$~~

f_{c1}, f_{c2}, f_{c3} calculated above

Part 3.

Question 4. a (20 points)



$V_{TN} = 1V$ $\quad K_N = 0{,}5\ \frac{mA}{V^2}$

$V_{B1} = 2V$ $\quad \lambda = 0{,}04\ \frac{1}{V}$

a.) (6 points) $m_1 - m_2$ matched. $V_{DS1} = V_{DS2} = V_{DS}$

$V_{DD} - I_D R_D = V_{DS1} + V_{DS2} = 2V_{DS}$ 02

$V_{DS} \cong \frac{V_{DD} - I_D R_D}{2}$ 02

$I_D = K_N (V_{B1} - V_{TN})^2 \cdot (1 + \lambda V_{DS})$

$I_D = 0{,}5\ (2-1)^2 \cdot \left(1 + 0{,}04 \cdot \frac{5 - I_D 4}{2}\right)$

$I_D = 0{,}5 + 0{,}05 - 0{,}04 I_D$

$1{,}04 I_D = 0{,}55 \Rightarrow I_D = 0{,}53 mA$ 01

$V_{DS} = \frac{5 - 4 \cdot 0{,}53}{2} = 1{,}44 V$

$V_{GS2} = V_{GS1}$ (for same V_{DS})

$V_{B1} - 1 = V_{B2} - 1{,}44 - 1$

$\underbrace{V_{B1} - 1}_{1} = V_{B2} - 2{,}44 \Rightarrow \boxed{V_{B2} = 3{,}44 V}$

$m_1 - m_2$ have same V_{GS}, V_{DS} so check SAT. cond. for m_2. (optional)

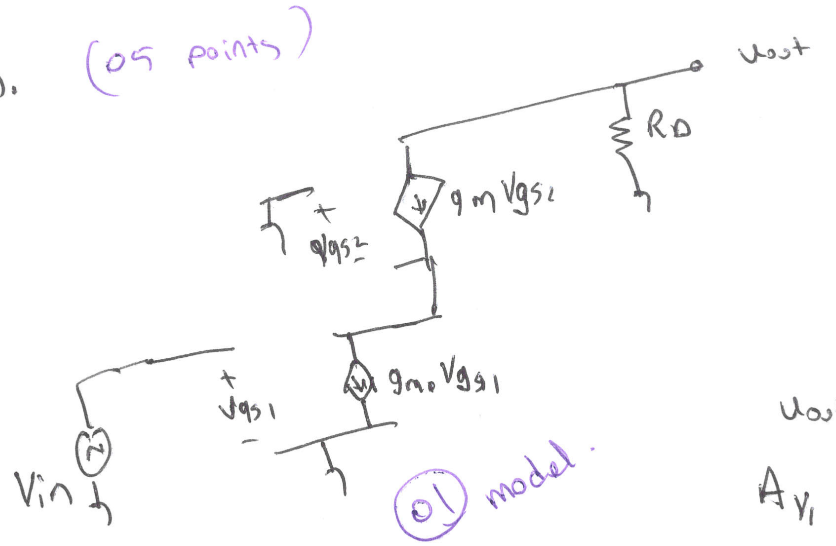
$\underbrace{V_{GS} - V_{TN}}_{2-1} < \underbrace{V_{DS}}_{1{,}44}$ ✓ verified

b.) S. S. parameters.

$g_{m1} = g_{m2} = g_m = 2\sqrt{K_N I_D} = 2\sqrt{0{,}5 \cdot 0{,}53} = 1{,}03\ \frac{mA}{V}$ 01

$r_{o1} = r_{o2} = r_o = \frac{1}{\lambda I_D} = \frac{1}{0{,}04 \cdot 0{,}53} = 47{,}2\ k\Omega$

b. (05 points)



01 model.

No r_o or $r_o = \infty$ since $\lambda = 0$

$g_m V_{gs1} = g_m V_{gs2}$ 02

$V_{gs1} = V_{gs2} = v_{in}$

$v_{out} = -g_m v_{in} R_D$ 01

$A_{V1} = -g_m R_D =$

$A_{V1} = -1{,}03{,}4 \Rightarrow \boxed{A_{V1} = -4{,}12 \ \frac{V}{V}}$

01

(max 4 points)

4. a.a. alternative solution (λ ignored)

$I_D = 0{,}5\,(V_{B1} - V_{TN})^2 = 0{,}5\,(2-1)^2 = 0{,}5 \text{ mA}$ 01

$V_{DS1} + V_{DS2} = 2V_{DS} = \underset{V_{DD}-}{5} - \underset{4RD}{4 \cdot 0{,}5} = 3\text{ V} \Rightarrow V_{DS} = 1{,}5\text{ V}$ 02

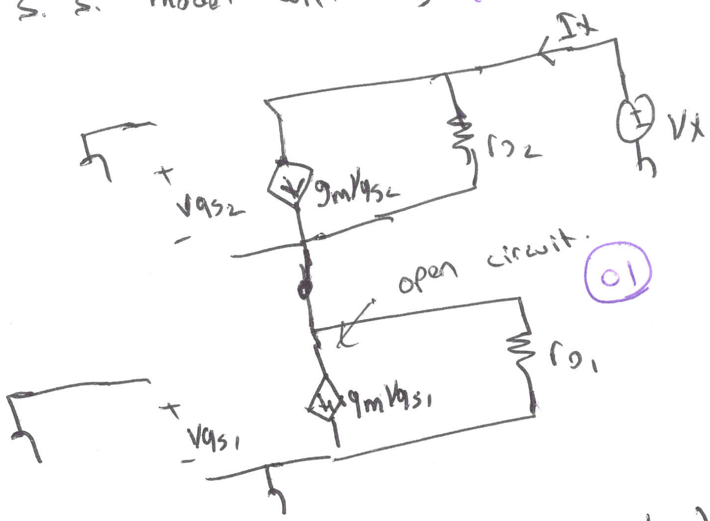
$V_{B1} - V_{TN} = V_{B2} - V_{TN} - \underset{V_{DS2}}{1{,}5} \Rightarrow \underline{V_{B2} = 3{,}5\text{ V}}$

$g_{m1} = g_{m2} = g_m = 2\sqrt{0{,}5 \cdot 0{,}5} = 1 \frac{mA}{V}$ 01

$r_{o1} = r_{o2} = r_o = \frac{1}{0{,}04 \cdot 0{,}5} = 50\text{ k}\Omega.$

The numerical values for part 4a.b and 4a.c would be pretty similar.

2.) S. S. model with r_{os} (9 points)   Lets add R_D later.



$$\frac{V_x}{I_x} = R_{o1}$$

01

$$V_x = (I_x - g_m V_{gs2}) \cdot r_{o2} + (I_x - g_m V_{gs1})\, r_{o1}$$

$V_{gs1} = 0$ 02

$$V_x = (I_x - g_m V_{gs2})\, r_{o2} + I_x r_{o1}$$ 03

$I_x \cdot r_{o1} = V_{S2}$

$-V_{S2} = -I_x r_{o1}$

$V_{gs2} = 0 - V_{S2} = -V_{S2}$

$$V_x = (I_x + g_m I_x r_{o1})\, r_{o2} + I_x r_{o1}$$

$$R_{o1} = \frac{V_x}{I_x} = (1 + g_m r_{o1})\, r_{o2} + r_{o1}$$ 02

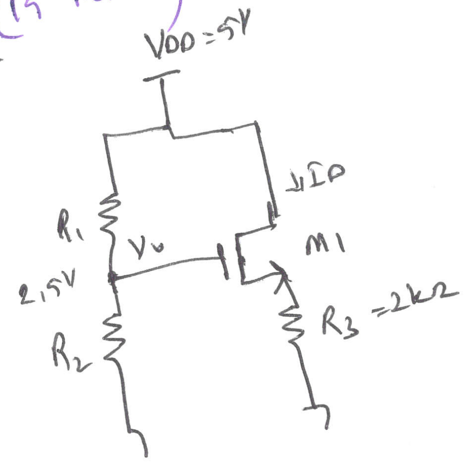
$$R_{o1} = (1 + \cancel{0.95}\ 1.03 \cdot 47.2)\, 47.2 + 47.2 = \boxed{2.39\,M\Omega}$$ (very large!)

$R_o = R_{o1} \parallel R_D = 2.39\,M\Omega \parallel 4\,k\Omega.$

$\boxed{R_o \cong 4\,k\Omega.}$ 01

3.3

4.b (15 points)



$R_1 = R_2 = 20\ k\Omega$ (set1)
$= 16\ k\Omega$ set2
$= 24\ k\Omega$ set3

$K_N = \frac{1mA}{V^2}$ $V_{TN} = 0,5V$ $\lambda = 0$

05 total

a) $V_G = \frac{R_2}{R_1+R_2} \cdot V_{DD} = 2,5V$

$I_D = 1.(2,5 - 2I_D - 0,5)^2$

$I_D = 4(1-I_D)^2$

$I_D = 4.(1 - 2I_D + I_D^2)$

$0,25 I_D = 1 - 2I_D + I_D^2$

$I_D^2 - 2,25 I_D + 1 = 0$

$I_D = 1,64\ mA$ ✗ M_1 will be OFF

$\boxed{I_D = 0,61\ mA}$

03

01

$V_G = 2,5V$

$V_D = 2.0,61 = 1,22V$

$V_{GS} = 2,5 - 1,22 < 5 - 1,22$

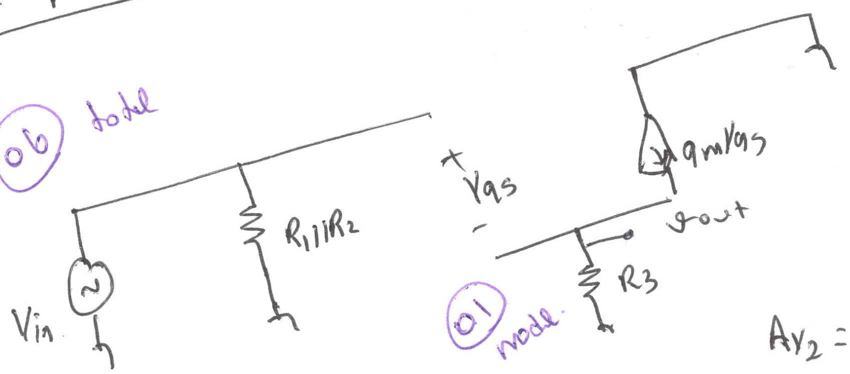
SAT ✓

Drain is connected to VDD TR should be in SAT.

01

s.s. parameters. $g_m = 2\sqrt{K_N I_D} = 2\sqrt{1.0,61} = 1,56\ \frac{mA}{V}$

b) 06 total



01 node

$v_{out} = g_m V_{gs} R_3$ 01

$V_{gs} = V_{in} - g_m V_{gs} R_3$ 02

$V_{gs} = \frac{V_{in}}{1 + g_m R_3}$

$A_{V_2} = \frac{v_{out2}}{v_{in2}} = \frac{g_m R_3}{1 + g_m R_3}$ 01

$A_{V_2} = \frac{1,56 . 2}{1 + 1,56.2} = 0,76\ \frac{V}{V}$ 01

$R_{in_2} = R_1 || R_2$

$R_{in_2} = 10k\Omega$ (set1)
$= 8k\Omega$ (set2)
$= 12\ k\Omega$ (set3)

c) 04 total

01 02

$A_{overall} = A_{V_1} . A_{V_2}$ (Loading effects) $= -4,12 . 0,76 . \frac{R_{in2}}{R_{in2} + R_{out1}}$

R_{out1} → $4k\Omega$.

$A_{overall} = -2,23\frac{V}{V}$ set1 $-2,09$ set2 $-2,35$ set3

3.4