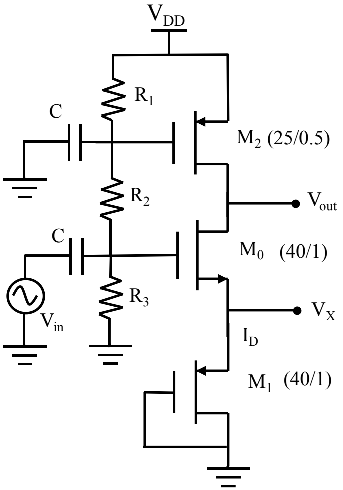
Bilkent University **EE313: Electronic Circuit Design** **Spring 2019-2020, 03.05.2020**

EEE313: Electronic Circuit Design
Midterm# 2

Duration: 120 minutes. Call 0312-290-3193 if your connection fails and you cannot reconnect.

1. (30 points)



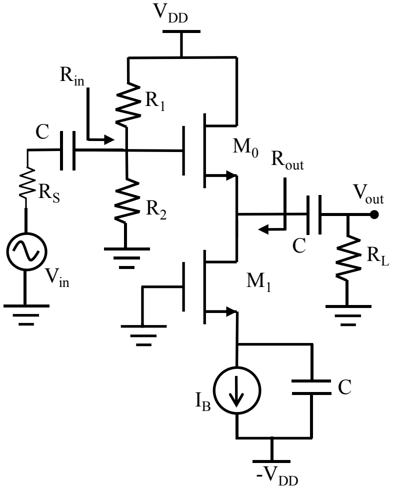
$V_{DD}=5V$
$R_1=54.4k\Omega$
$R_2=25.6k\Omega$
$R_3=120k\Omega$
$k_n`=0.1mA/V^2$
$k_p`=0.1mA/V^2$
$V_{TN}=0.5V$
$V_{TP}=-0.5V$
$\lambda_0=0$
$\lambda_1=0$
$\lambda_2=0.05V^{-1}$
$K_N=k_n`/2(W/L)$
$K_P=k_p`/2(W/L)$

a. Calculate the Q points, small signal parameters for $M_{0\text{-}1\text{-}2}$ and verify that they are in SAT. Consider the channel length modulation ($\lambda \neq 0$) in this part also.

b. Calculate V_{out}/V_{in} first symbolically and then find the numerical value.

Hint: You may consider the impedances looking into M1 from V_X and M2 from V_{out}.

2. (30 points)



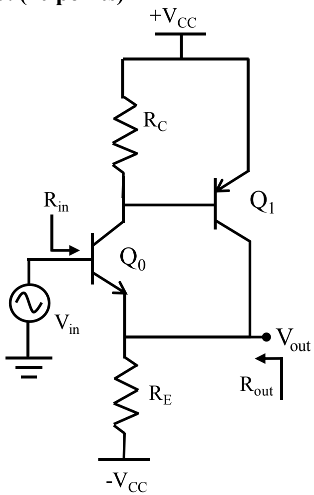
$+V_{DD}=3V$
$-V_{DD}=-3V$
$R_1=100k\Omega$
$R_2=200k\Omega$
$R_L=10k\Omega$
$R_S=5k\Omega$
$I_B=1mA$
$\lambda_0=0.02V^{-1}$
$\lambda_1=0.04V^{-1}$
$K_{N0}=K_{N1}=0.5mA/V^2$
$V_{TN}=0.8V$
for all

a. Calculate the small signal parameters for M1 and M0 assuming that they are in SAT

b. Find V_{out}/V_{in}, R_{out}, and R_{in} first symbolically and then find the numerical value

Hint: You may consider the impedance looking into M1 from V_{out}.

3. (40 points)



$+V_{CC}=5V$
$-V_{CC}=-5V$
$\beta_0=\beta_1=50$
$R_C=10k\Omega$
$R_E=2k\Omega$
$V_A=\infty$
$V_{BEon0}=0.7V$
$V_{CESAT0}=0.2V$
$V_{EBon1}=0.7V$
$V_{ECSAT1}=0.2V$

a. Find the Q points, small signal parameters for Q_0 and Q_1, and verify transistor regions (states).

b. Assuming Q point remains the same for Q_0, find R_{in} and R_{out} symbolically if Q_1 is removed. V_{out} still remains on R_E, provide just expression no numbers.

c. Find V_{out}/V_{in}, R_{in}, and R_{out} for this circuit first symbolically and then find the numerical value. Comment on the effect of Q_1 on R_{in} and R_{out}.

On your 1st solution page write and sign:
I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

Submission process:
Solve each question on a new page. Scan your solutions into **a single pdf file**, upload it to Moodle using "**Midterm Exam 2 upload**" tab, and email to "**eee313.spring19@gmail.com**". The subject line and the name of the file should be "**Midterm 2 -Your ID#-Name Surname**".

BJT: in Forward Active region, B-E forward B-C reverse biased: $I_C = I_S(e^{\frac{V_{BE}}{V_T}} - 1)$

$$\alpha I_E = I_C, \qquad I_C = \beta I_B, \qquad \alpha = \beta/(\beta+1)$$

$$r_\pi = \frac{V_T}{I_{BQ}}, g_m = \frac{I_{CQ}}{V_T}, r_o = \frac{V_A}{I_{CQ}}, V_T = 26mV$$

nMOS: Non-Saturation ($V_{DS}<V_{GS}-V_{TN}$)

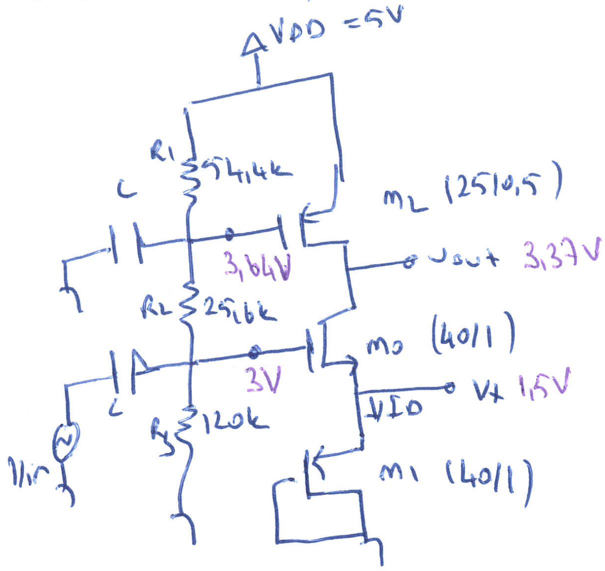
$$I_D = K_N[2(V_{GS}-V_{TN})V_{DS} - V_{DS}^2](1+\lambda V_{DS})$$

SAT ($V_{DS}>V_{GS}-V_{TN}$): $I_D = K_N(V_{GS}-V_{TN})^2(1+\lambda V_{DS})$

for pMOS replace V_{GS} with V_{SG}, $-V_{TN}$ with V_{TP}, and V_{DS} with V_{SD}

$$g_m = 2K_N(V_{GSQ} - V_{th}) = 2\sqrt{K_N I_{DQ}}, r_0 = {}^{1}\!/_{\lambda I_{DQ}}$$

Q.1. (30 points)



$K'_n = 0,1\,mA/V^2$ $V_{tn} = 0,5V$

$K'_p = 0,1\,mA/V^2$ $V_{tp} = 0,5V$

$\lambda_2 = 0,05V^{-1}$, $\lambda_0 = \lambda_1 = 0$

a) 9 points. (15 points)

$V_{G0} = \frac{120}{200} \cdot 5 = 3V$ 02

$V_{G2} = \frac{145,6}{200} \cdot 5 = 3,64V$

$\lambda_0 = \lambda_1 = 0$

M0 - M1 SAT

$\frac{0,1}{2} \cdot \frac{40}{1} \cdot (3 - V_x - 0,5)^2 = \frac{0,1}{2} \frac{40}{1} (V_x - 0,5)^2$

$(2,5 - V_x)^2 = (V_x - 0,5)^2$

$2,5 - V_x = V_x - 0,5$ ~~$2,5 - V_x = 0,5 - V_x$~~ ?

$\boxed{V_x = 1,5V}$ 03

$I_0 = \frac{0,1}{2} \frac{40}{1} (3 - 1,5 - 0,5)^2 = 2mA$ $\boxed{I_0 = 2mA}$ 02

for M2

$2 = \frac{0,1}{2} \cdot \frac{25}{0,5} \cdot (5 - 3,64 - 0,5)^2 \cdot \left(1 + 0,05 \cdot \underbrace{(5 - V_{out})}_{V_{SD}}\right)$ 03

~~$2 = 2,5 \cdot t$~~ $2 = 1,85 \cdot (1 + 0,05\, V_{SD})$

$V_{SD_2} = 1,63V$ $\Rightarrow$ $V_{out} = 5 - 1,63 \Rightarrow$ $\boxed{V_{out} = 3,37V}$

M1

already in SAT.

M0

$1,5 - 0,5 < 3,37 - 1,5$ ✓

M2

$5 - 3,64 - 0,5 < 1,63$ ✓

02 transistor operation verification.

$g_m = 2\sqrt{k_n I_0}$

$= 2\sqrt{\frac{0,1}{2} \cdot 40 \cdot 2}$

$g_{m_1} = 4\frac{mA}{V}$

$g_m = 2\sqrt{\frac{0,1}{2} \cdot 40 \cdot 2}$

$g_{m_0} = 4\frac{mA}{V}$

03 → S.S. parameter calculation.

$g_{m2} = 2\sqrt{\frac{0,1}{2} \cdot 50 \cdot 2}$

$g_{m2} = 4,47\frac{mA}{V}$

$r_{o2} = \frac{1}{\lambda I_0} = \frac{1}{0,05 \cdot 2} = 10k\Omega$

1. a.) λ_2 ignored in DC analysis. max 9 points.

m2

$I_D =$

$V_{G0} = 3V$

$V_{G2} = 3{,}64V$ 02

m2

$I_D = \frac{0{,}1}{2} \cdot \frac{25}{0{,}5} (5 - 3{,}64 - 0{,}5)^2 = 1{,}85 mA$ 02

m0

$1{,}85mA = \frac{0{,}1}{2} \cdot \frac{40}{1} \cdot (3 - V_x - 0{,}5)^2$ 01

$1{,}85 = 2 \cdot (2{,}5 - V_x)^2$

$2{,}5 - V_x = \pm 0{,}96$

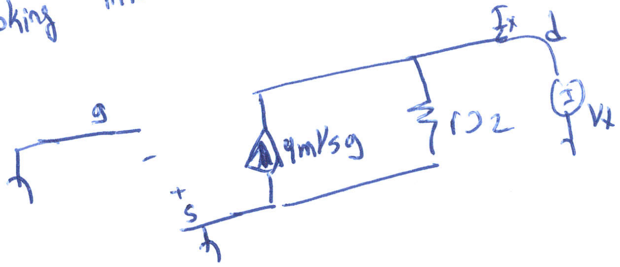
$V_x = 1{,}54V$ or $V_x = 3{,}46V$ } which one to pick is not obvious here.

M1 01	M0	M2
already in SAT	state cannot be verified	state cannot be verified
$g_m = 2\sqrt{\frac{0{,}1}{2} \cdot 40 \cdot 1{,}85}$	$g_{m0} = g_m = 3{,}8 \frac{mA}{V}$	$g_{m2} = 2\sqrt{\frac{0{,}1}{2} \cdot 50 \cdot 1{,}85}$
$g_{m1} = 3{,}8 \frac{mA}{V}$	03 s.s. parameters.	$g_{m2} = 4{,}3 \frac{mA}{V}$
		$r_{o2} = \frac{1}{0{,}05 \cdot 1{,}85} = 10{,}8 k\Omega$

Since the DC potential cannot be calculated @ Vout with $\lambda_2 = 0$ states of m_2 and m_0 cannot be verified. Also which solution to pick for Vx DC point is not obvious.

b) Calculate $\frac{V_{out}}{V_{in}}$

looking into the drain of m_2.

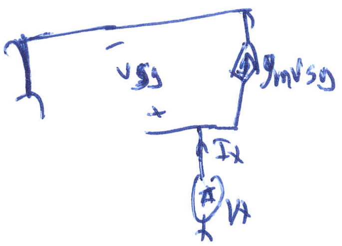


$V_{sg} = 0$

$$\boxed{\frac{V_x}{I_x} = r_{o2}}$$

03

looking in the source of $M1$



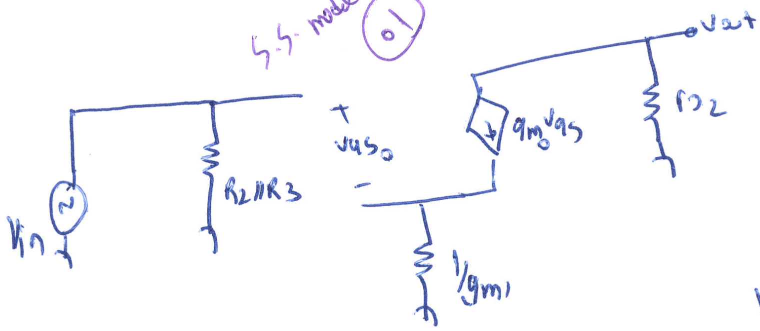
$V_x = V_{sg}$

$I_x = g_m V_{sg}$

$$\Rightarrow \boxed{\frac{V_x}{I_x} = \frac{1}{g_{m1}}}$$

03

s.s. model 01



$V_{out} = -g_{m_o} v_{gs} \, r_{o2}$ 01

$V_{in} = V_{gs} + g_{mo} V_{gso} \cdot \frac{1}{g_{m1}}$

$V_{in} = V_{gs}\left(1 + \frac{g_{mo}}{g_{m1}}\right)$ 02

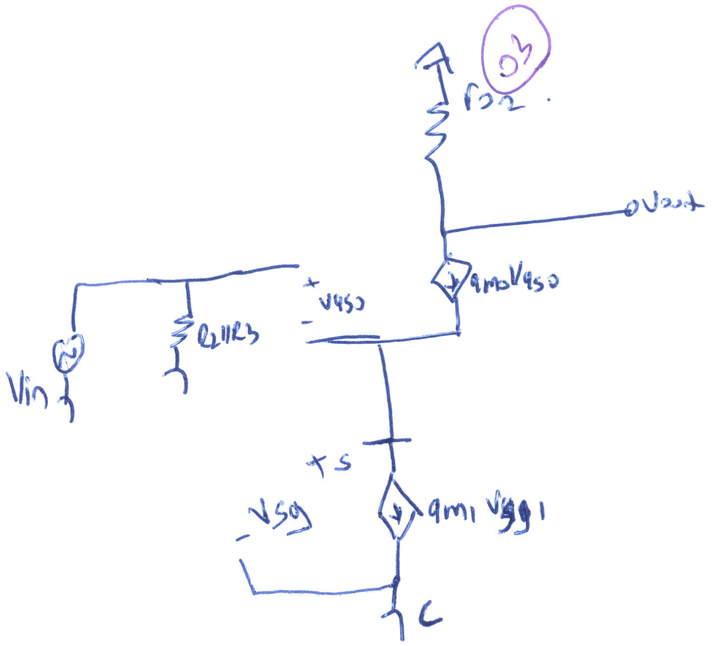
$\cancel{V_{out}} = \; V_{gs} = \frac{V_{in}}{1 + \frac{g_{mo}}{g_{m1}}}$

$$V_{out} = -g_{mo} \cdot \frac{V_{in}}{1 + \frac{g_{mo}}{g_{m1}}} \cdot r_{o2} \Rightarrow A_V = \frac{V_{out}}{V_{in}} = \frac{-g_{mo} r_{o2}}{1 + \frac{g_{mo}}{g_{m1}}} = \frac{-4 \cdot 10}{1+1} \Rightarrow \boxed{A_V = -20} \; \frac{V}{V}$$

03

02 plugging in numbers.

b) alternative way.



03 model

$V_{out} = -g_{mo} V_{gso} \; r_{o2}$ 01

$g_{mo} V_{gso} = g_{m1} V_{sg1}$ 03

$V_{gso} = V_{sg1}$

$V_{in} = 2 V_{gso} = V_{gso} + V_{sg1}$

$V_{gso} = \frac{1}{2} V_{in}$ 02

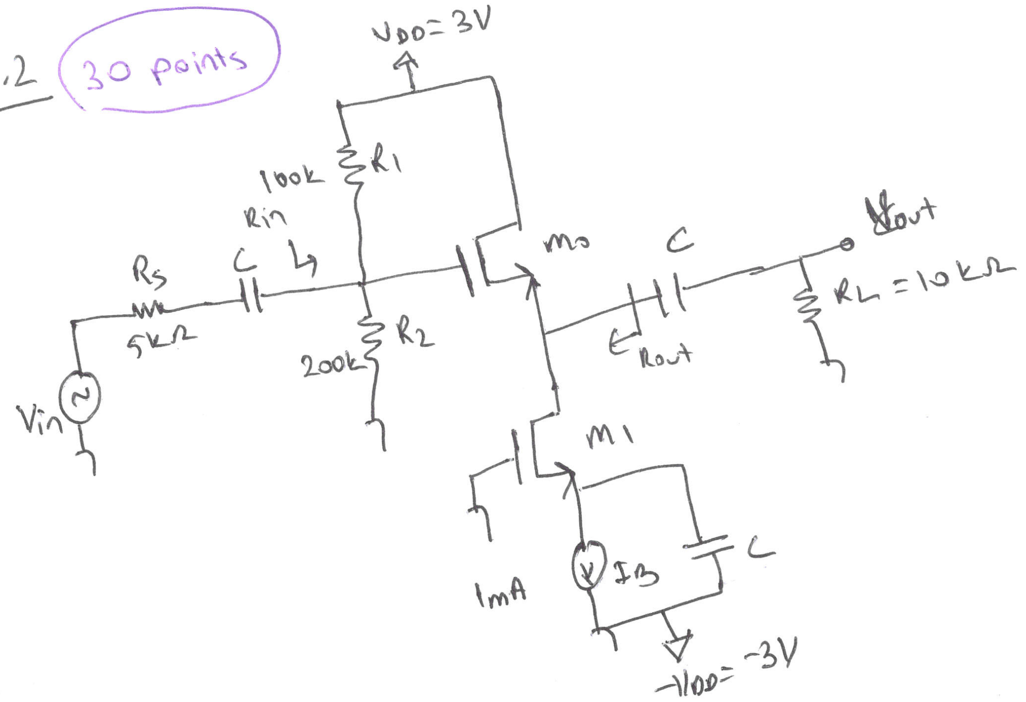
$V_{out} = -g_{mo} \cdot \frac{1}{2} V_{in} \; r_{o2}$ 01

$\cancel{4}$

$A_v = \frac{V_{out}}{V_{in}} = -4 \cdot \frac{1}{2} \cdot 10$ 02

$\boxed{A_v = -20 \ \frac{V}{V}}$

Q.2 (30 points)



$\lambda_0 = 0{,}02\ V^{-1}$

$\lambda_1 = 0{,}04\ V^{-1}$

$K_{n0} = K_{n1} = 0{,}5\ \frac{mA}{V^2}$

$V_{TN} = 0{,}8\ V$

a) 05

$I_{D\,M0} = I_{D\,M1} = I_B = 1mA$ 01

$g_{m0} = 2\sqrt{0{,}5 \cdot 1} = 1{,}41\ \frac{mA}{V}$ 01

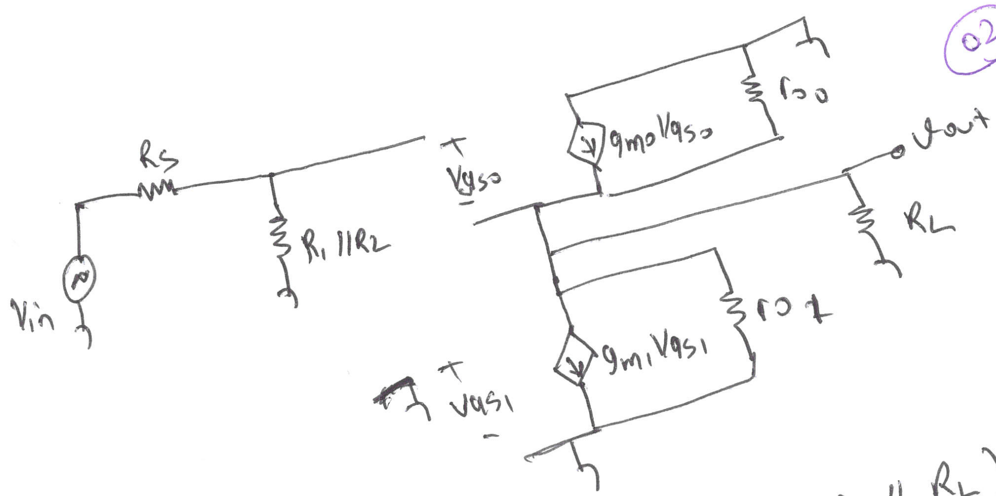
$g_{m1} = 2\sqrt{0{,}5 \cdot 1} = 1{,}41\ \frac{mA}{V}$ 01

$r_{o0} = \frac{1}{0{,}02 \cdot 1} = 50\ k\Omega$ 01

$r_{o1} = \frac{1}{0{,}04 \cdot 1} = 25\ k\Omega$. 01

02 s.s. model

b)



$V_{gs1} = 0$

$g_{m1}v_{gs1} \rightarrow$ open circuit.

only r_{o1} is seen looking into M1 02

$v_{out} = g_{m0} V_{gs0} (r_{o0} // r_{o1} // R_L)$ 02

$V_{gs0} = \frac{R_1 // R_2}{R_1 // R_2 + R_S} \cdot V_{in} - g_{m0} V_{gs0} (r_{o0} // r_{o1} // R_L)$ 02

$V_{gs0} (1 + g_{m0} (r_{o0} // r_{o1} // R_L)) = \frac{R_1 // R_2}{R_1 // R_2 + R_S} \cdot v_{in}$

$V_{gs0} = \frac{R_1 // R_2}{R_1 // R_2 + R_s} \cdot \frac{1}{1 + g_{m0} (r_{o0} // r_{o1} // R_L)} \cdot V_{in}$. 01

$v_{out} = g_{m0} v_{gs0} (r_{o0} \| r_{o1} \| R_L)$

$$\frac{v_{out}}{v_{in}} = \frac{R_1 \| R_2}{R_1 \| R_2 + R_S} \cdot \frac{g_{m0} (r_{o0} \| r_{o1} \| R_L)}{1 + g_{m0} (r_{o0} \| r_{o1} \| R_L)}$$ 03

$R_1 \| R_2 = 100 \| 200 = 66{,}7\ k\Omega$

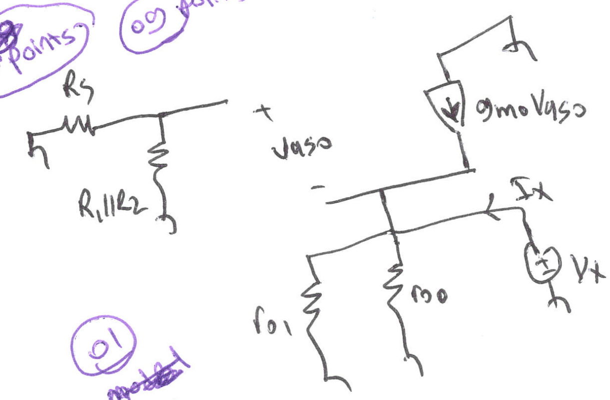
$r_{o0} \| r_{o1} \| R_L = 50 \| 25 \| 10 = 6{,}25k$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{66{,}7}{66{,}7 + 5} \cdot \frac{1{,}41 \cdot 6{,}25}{1 + 1{,}41 \cdot 6{,}25} = 0{,}93 \cdot 0{,}898$$

$\boxed{A_v = 0{,}84\ \frac{V}{V}}$ 02

$R_{in} = R_1 \| R_2 = 66{,}7\ k\Omega$ 02

R_{out} (~~10~~ 08 points) (09 points)



01
~~model~~
s.s. model.

$V_{gs0} = 0 - V_S = -V_S$ 02

$V_X = +V_S$ ~~02~~

$-I_X + \frac{V_X}{r_{o0}} + \frac{V_X}{r_{o1}} - g_{m0} V_{gs0}$

02

$I_X = \frac{V_X}{r_{o0}} + \frac{V_X}{r_{o1}} + g_{m0} V_S$
V_X

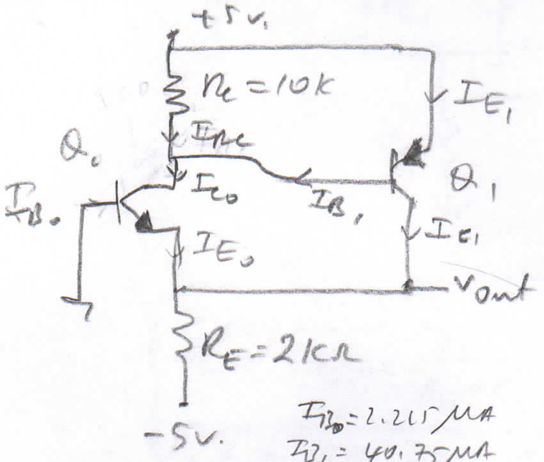
$$R_{out} = \frac{V_X}{I_X} = \left(\frac{1}{\frac{1}{r_{o0}} + \frac{1}{r_{o1}} + \frac{1}{g_{m0}}} \right)$$

$\boxed{R_{out} = r_{o0} \| r_{o1} \| \frac{1}{g_{m0}}} =$ 02

$\boxed{R_{out} = 50 \| 25 \| \frac{1}{1{,}41} = 0{,}68\ k\Omega = 680\ \Omega.}$ 02

Q3 Solution

a) Assume both transistors are F.A.



$I_{B_0} = 2.215 \mu A$
$I_{B_1} = 40.75 \mu A$
$I_{C_0} = 0.11 mA$
$I_{C_1} = 2.038 mA$

$$I_{R_C} = \frac{0.7}{10} = 0.07 mA$$

$$I_{R_C} + I_{B_1} = I_{C_0}$$

$$0.07 + I_{B_1} = 50 I_{B_0}$$

$$\underbrace{51 I_{B_0}}_{I_{E_0}} + \underbrace{50 I_{B_1}}_{I_{C_1}} = \frac{-0.7-(-5)}{2} = 2.15$$

$$51 I_{B_0} + 50(50 I_{B_0} - 0.07) = 2.15$$

$$(51 + 2500) I_{B_0} = 50 \times 0.07 + 2.15$$

$$I_{B_0} = \frac{3.5 + 2.15}{2551} = 0.002215 mA = 2.215 \mu A$$

$$\Rightarrow \underline{\underline{I_{C_0} = 0.11 mA}}$$

$$I_{B_1} = 50 I_{B_0} - 0.07 \Leftarrow$$
$$= 50 \times 0.002215 - 0.07$$
$$= 0.04075 mA = 40.75 \mu A.$$

$$\Rightarrow \underline{I_{C_1} = 2.038 mA}$$

$$V_{CE_0} = V_{C_0} - V_{E_0} = (5 - 0.7) - (-0.7) = \underline{\underline{5 v}} > 0.2 \checkmark$$

$$V_{EC_1} = V_{E_1} - V_{C_1} = 5 - V_{E_0} = 5 - (-0.7) = \underline{\underline{5.7}} > 0.2 \checkmark$$

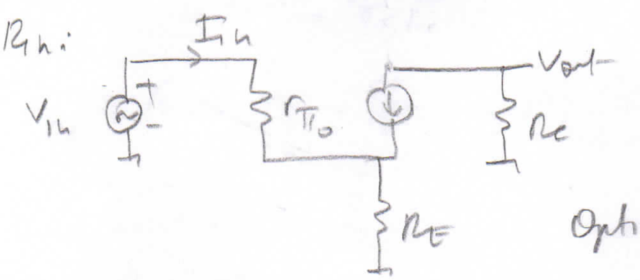
$$\boxed{\begin{array}{l} I_{C_0} \ 4p \\ I_{C_1} \ 4p \\ V_{CE_0} \ 2+1 \\ V_{CE_1} \ 2+1 \end{array}} \; 14p$$

$$r_{\pi_1} = \frac{0.026}{0.04075} = 0.638 \Omega \qquad r_{\pi_0} = \frac{0.026}{0.002215} = 11.74 k\Omega.$$

$$g_{m_1} = \frac{50}{0.638} = 79.6 mA/v \qquad g_{m_0} = \frac{50}{11.74} = 4.26 mA/v$$

$\boxed{\text{2p for either } r_\pi\text{'s or } g_m\text{'s}}$

b) R_{in}:

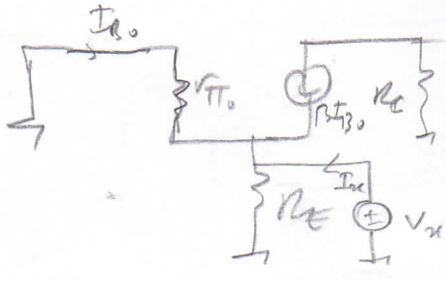


$$V_{in} = I_{in} r_{\pi_0} + (\beta+1) I_{in} R_E$$

$$\Rightarrow R_{in} = r_{\pi_0} + (\beta+1) R_E \quad \boxed{(4p)}$$

Optional: $R_{in} = 11.74 + 51 \times 2 = 113.7 k\Omega.$

R_{out}:



$$I_x = \frac{V_x}{R_E} - (\beta_0 + 1) I_{B_0}$$

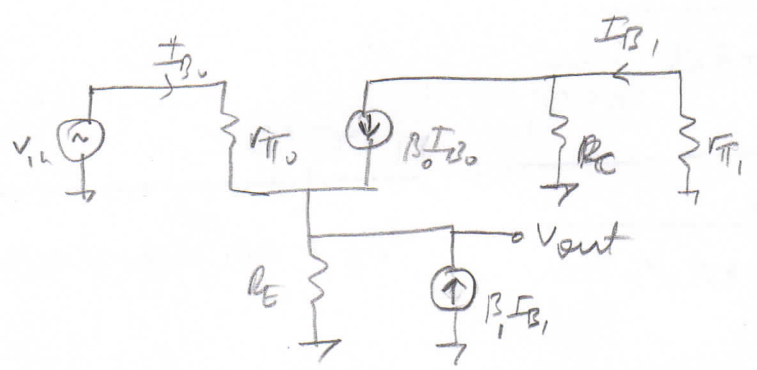
$$= \frac{V_x}{R_E} + (\beta_0 + 1)\frac{V_x}{r_{\pi_0}}$$

$$= V_x \left(\frac{1}{R_E} + \frac{1}{\frac{r_{\pi_0}}{\beta_0 + 1}} \right) \quad \boxed{(4p)}$$

$$R_{out} = R_E // \frac{r_{\pi_0}}{\beta_0 + 1}$$

Optional:
$$R_{out} = 2 // \frac{11.74}{51} = 2 // 0.23 = 206 \Omega$$

c)



ss ac circuit: 2p
Av: 4p
Rin: 4p
Rout: 4p
Comment: 2p
} 16p

$$V_{in} = I_{B_0} r_{\pi_0} + \left[(\beta_0+1) I_{B_0} + \beta_1 I_{B_1}\right] R_E$$

$$\boxed{I_{B_1} = +\beta_0 I_{B_0} \frac{R_C}{R_C + r_{\pi_1}}} \Rightarrow V_{in} = I_{B_0} r_{\pi_0} + \left[(\beta_0+1) I_{B_0} + \beta_1 \beta_0 \frac{R_C}{R_C + r_{\pi_1}} I_{B_0}\right] R_E$$

$$V_{out} = \left[(\beta_0+1) I_{B_0} + \beta_1 I_{B_1}\right] R_E = \left[(\beta_0+1) I_{B_0} + \beta_1 \beta_0 \frac{R_C}{R_C + r_{\pi_1}} I_{B_0}\right] R_E$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{\left[(\beta_0+1) + \beta_1 \beta_0 \frac{R_C}{R_C + r_{\pi_1}}\right] R_E}{r_{\pi_0} + \left[(\beta_0+1) + \beta_1 \beta_0 \frac{R_C}{R_C + r_{\pi_1}}\right] R_E}$$

$r_{\pi_1} = \frac{0.026}{0.04071} = 0.638\ k\Omega$ $\quad r_{\pi_0} = \frac{0.026}{0.002215} = 11.74\ k\Omega$

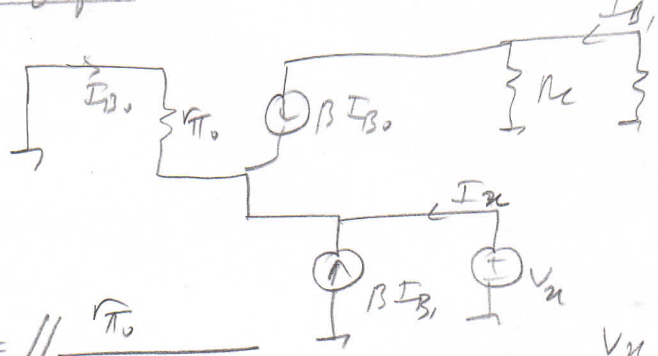
$$A_V = \frac{\left[51 + 2500 \times \frac{10}{10+0.638}\right] \times 2}{11.74 + \left[51 + 2500 \times \frac{10}{10+0.638}\right] \times 2} = \frac{[51 + 2350] \times 2}{11.74 + [51 + 2350] \times 2} = \frac{4802}{4813.7} = 0.9976$$

R_{in}: $\quad v_{in} = I_{B_0} r_{\pi_0} + \left[(\beta+1) I_{B_0} + \beta^2 \frac{R_C}{R_C + r_{\pi_1}} I_{B_0}\right] R_E$

$$\Rightarrow R_{in} = r_{\pi_0} + \left[(\beta+1) + \beta^2 \frac{R_C}{R_C + r_\pi}\right] \times 2 = 4813.7\ k\Omega$$
$$= 4.8137\ M\Omega.$$

R_{out}:

~~Ignore R_E first~~



$$I_x = -(\beta+1) I_{B_0} - \beta I_{B_1}$$

$$I_{B_1} = +\beta I_{B_0} \frac{R_C}{R_C + r_{\pi_1}}$$

$$I_x = -(\beta_0+1) I_{B_0} - \beta_1 \beta_0 \frac{R_C}{R_C + r_{\pi_1}} I_{B_0}$$

$$V_x = -I_{B_0} r_{\pi_0}$$

$$\frac{V_x}{I_x} = \frac{r_{\pi_0}}{(\beta_0+1) + \beta_1 \beta_0 \frac{R_C}{R_C + r_{\pi_1}}} = \frac{11.74}{51 + 2500 \times \frac{10}{10+0.638}}$$

$$= \frac{11.74}{2401} = 0.00489\ k\Omega.$$

$$R_{out} = R_E // \frac{r_{\pi_0}}{(\beta_0+1) + \beta_1 \beta_0 \frac{R_C}{R_C + r_{\pi_1}}}$$

$$= 2 // 0.00489 = 0.00488\ k\Omega$$
$$= 4.88\ \Omega$$

* Q_1 makes R_{in} very very large, and R_{out} very very small, This is due to -ive feedback, compared to results in part b).