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EEE 313 Spring 2014-2015

Bilkent University

Department of Electrical and Electronics Engineering

EEE 313 Electronic Circuit Design

Final Exam

May 22, 2015 18:30 (4 questions, 180 minutes)

- This is a **closed book**, closed notes exam. No cheat sheet allowed.
- All cell-phones should be completely **turned off**.
- Use a calculator for numerical computations. Carry at least **3 significant digits**. Double check your numerical calculations.
- Be sure to write the **units** of all numerical results.
- **Show all work clearly. Show all the intermediate steps including formulas. Never, just carry out calculations without formulas and explanation.**
- Please put your **final answer** for each part inside a box for easy identification.
  Do not give multiple answers, they will not be graded.
- Do not remove the **staple** from the exam sheets or separate pages of the exam. All extra pages must be stamped to your exam.
- You may leave the exam room when you are done.
  However, please do not leave during the **last five minutes** of the exam.
- At the end of the exam, please stay seated unitl **all** exam papers are collected.

Please do not write below this line

1. 20 pts.	
2. 30 pts.	
3. 25 pts.	
4. 25 pts.	
Total 100 pts.	

Useful constants and formulas:

Boltzmann's constant: 86×10^{-6} eV/K°
Electron charge: 1.6×10^{-19} Coulombs

Drain currrent equations for n-channel MOSFET:

$I_D = K_n(V_{GS} - V_{TN})^2$ for $V_{GS} - V_{TN} \le V_{DS}$

$I_D = K_n(2(V_{GS} - V_{TN})V_{DS} - {V_{DS}}^2)$ for $V_{GS} - V_{TN} \ge V_{DS}$

Drain currrent equations for p-channel MOSFET:

$I_D = K_p(V_{SG} + V_{TP})^2$ for $V_{SG} + V_{TP} \le V_{SD}$

$I_D = K_p(2(V_{SG} + V_{TP})V_{SD} - {V_{SD}}^2)$ for $V_{SD} + V_{TP} \ge V_{SD}$

Forward active current of BJT is given by:

$$I_C = I_S\left(e^{\frac{V_{BE}}{nV_t}} - 1\right)$$

Drain currrent equations for JFET:

$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_p}\right)^2$ for $V_{DS} \ge (V_{GS} - V_P)$ in saturation region

$I_D = \frac{2I_{DSS}}{V_P^2}\left(V_{GS} - V_P - \frac{V_{DS}}{2}\right)V_{DS}$ for $V_{DS} < (V_{GS} - V_P)$ in nonsaturation region

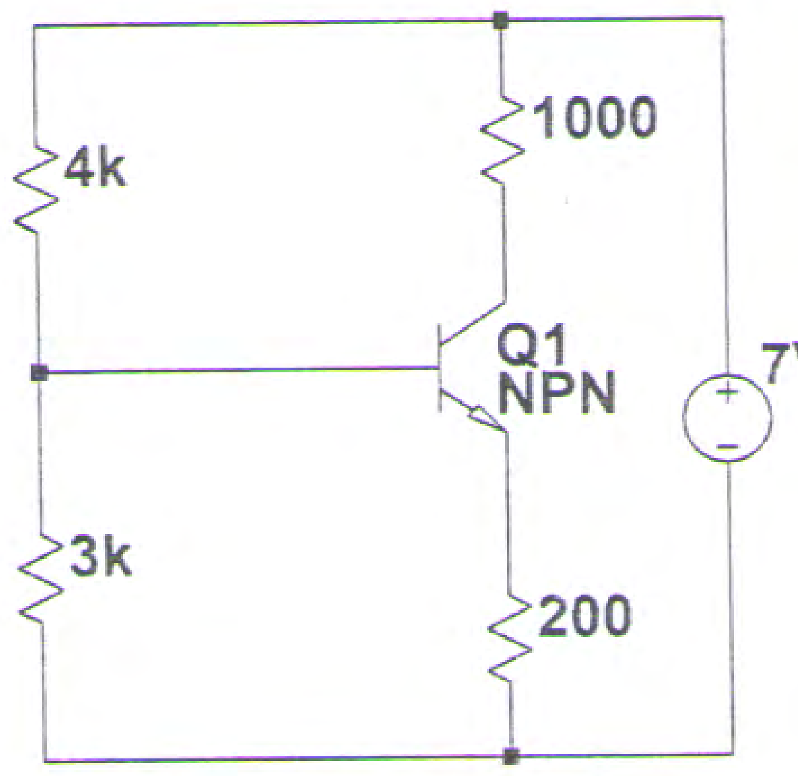
Other equations must be deduced by the students.

1. (20 points) Determine the states and biases of the transistors in the circutis below. Show all your work for full points. Also verify the states. Put your answer in the designated table.

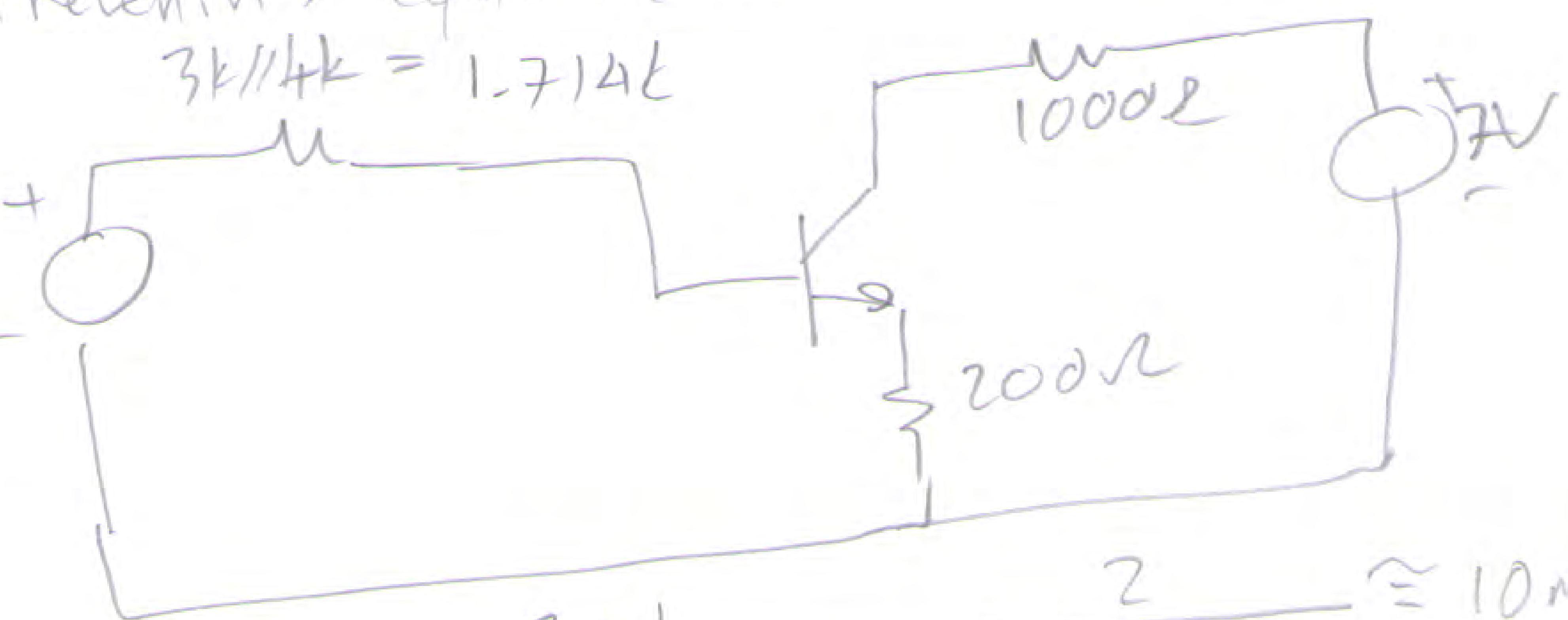
a) (4 points)

$\beta=99$, $V_{BE(ON)}=1V$, $V_{CE(SAT)}=0.5V$,

Q_1 state	I_E	V_{CE}	I_C	I_B
sat	5.82mA	0.5V	5.33mA	0.4875 mA



The thevenin's equivalent circuit is given by

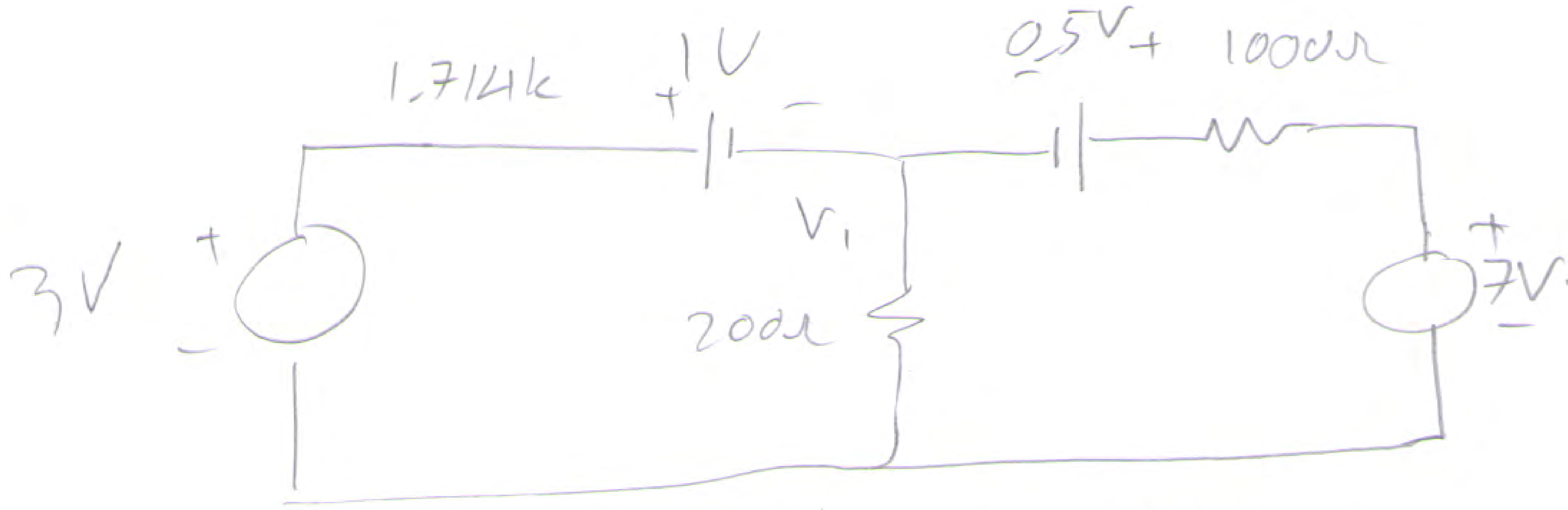


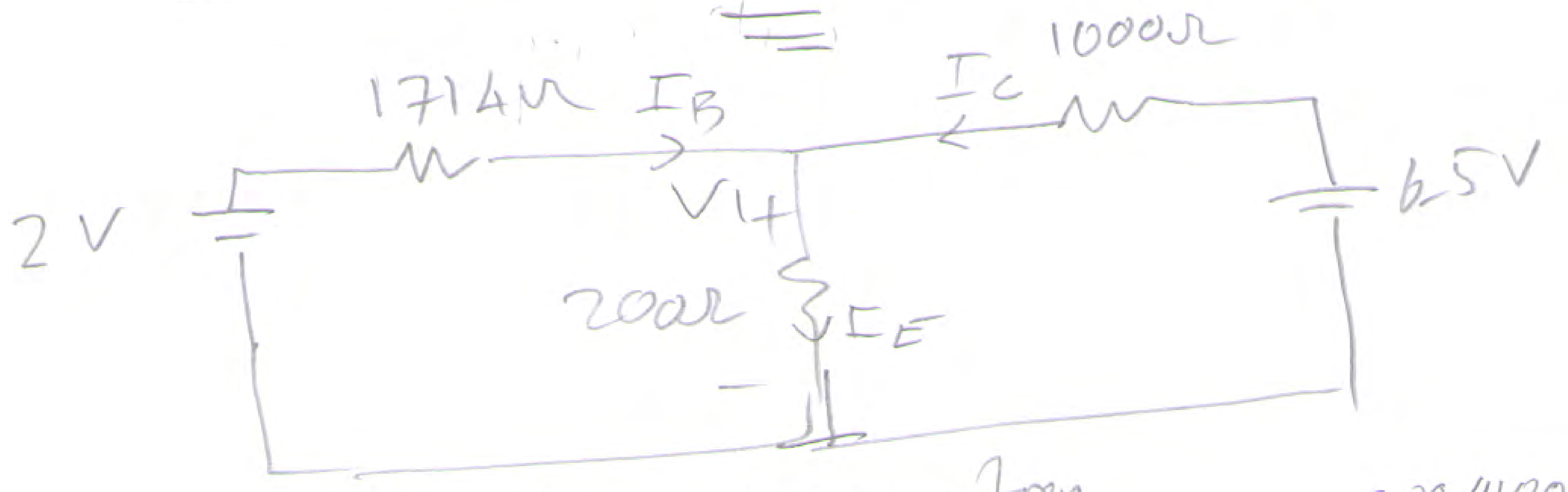
$7V \times \frac{3}{3+4} = 3V$

Assuming F.A. $I_E = \frac{3-1}{200 + \frac{1.714}{100}} = \frac{2}{217.4\,\Omega} \cong 10mA$

$V_{CE} = 7V - 10\times10^{-3} \times 1.2\times10^{3} = 7-12 = -5V \Rightarrow$ not F.A.

Assuming SAT → back page





Using superposition

$200 // 1000 = \frac{1 \times 0.2}{1.2} = 0.1666 k\Omega = 166.6\Omega$

$V_1 = 2V \times \frac{166.6}{1714 + 166.6} + 6.5V \times \frac{179.1}{1000 + 179.1}$

$1714 // 200 = \frac{1714 \times 200}{1914} = 179.1$

$= 0.1772V + 0.9873$

$= 1.1645V \Rightarrow$

$V_{CE} = 0.5V$ (sat)

$I_E = \frac{1.1645V}{200\Omega} = 0.00582A = 5.82mA$

$I_B = \frac{2 - 1.1645}{1714\Omega} = 4.875 \times 10^{-4} A = 0.4875 mA$

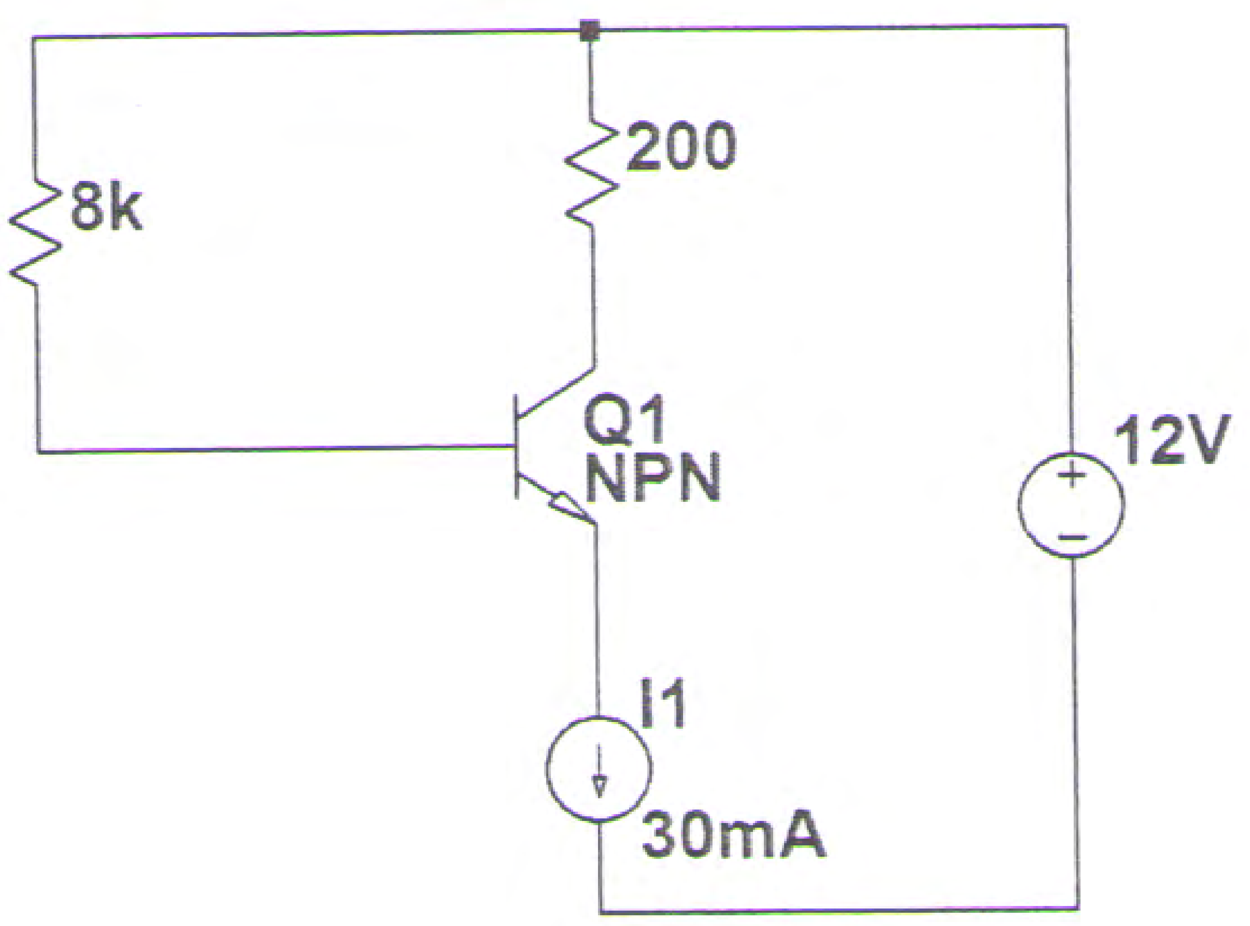
$I_C = -I_B + I_E = 5.82 - 0.4875 = 5.3325 mA$

$\beta I_B = 0.4875 \times 99 = 48.26 mA > I_C = 5.3325 mA \Rightarrow$ SAT ✓

b) (4 points)

β=29, $V_{BE(ON)}$=0.7V, $V_{CE(SAT)}$=0.3V

Q_1 state	I_B	V_{CE}	I_C
F-A.	1mA	2.9V	29mA



Assuming F-A.

$I_B = \frac{30mA}{29+1} = 1mA$

$I_C = I_E - I_B = 30-1 = 29mA$

$V_C = 12 - 29\times\cancel{10^{-3}}\times 0.2\times\cancel{10^{3}}$
$= 6.2V$

$V_B = 12 - 8\times\cancel{10^{3}}\Omega\times 1\times\cancel{10^{-3}}A$
$= 12 - 8 = 4V$

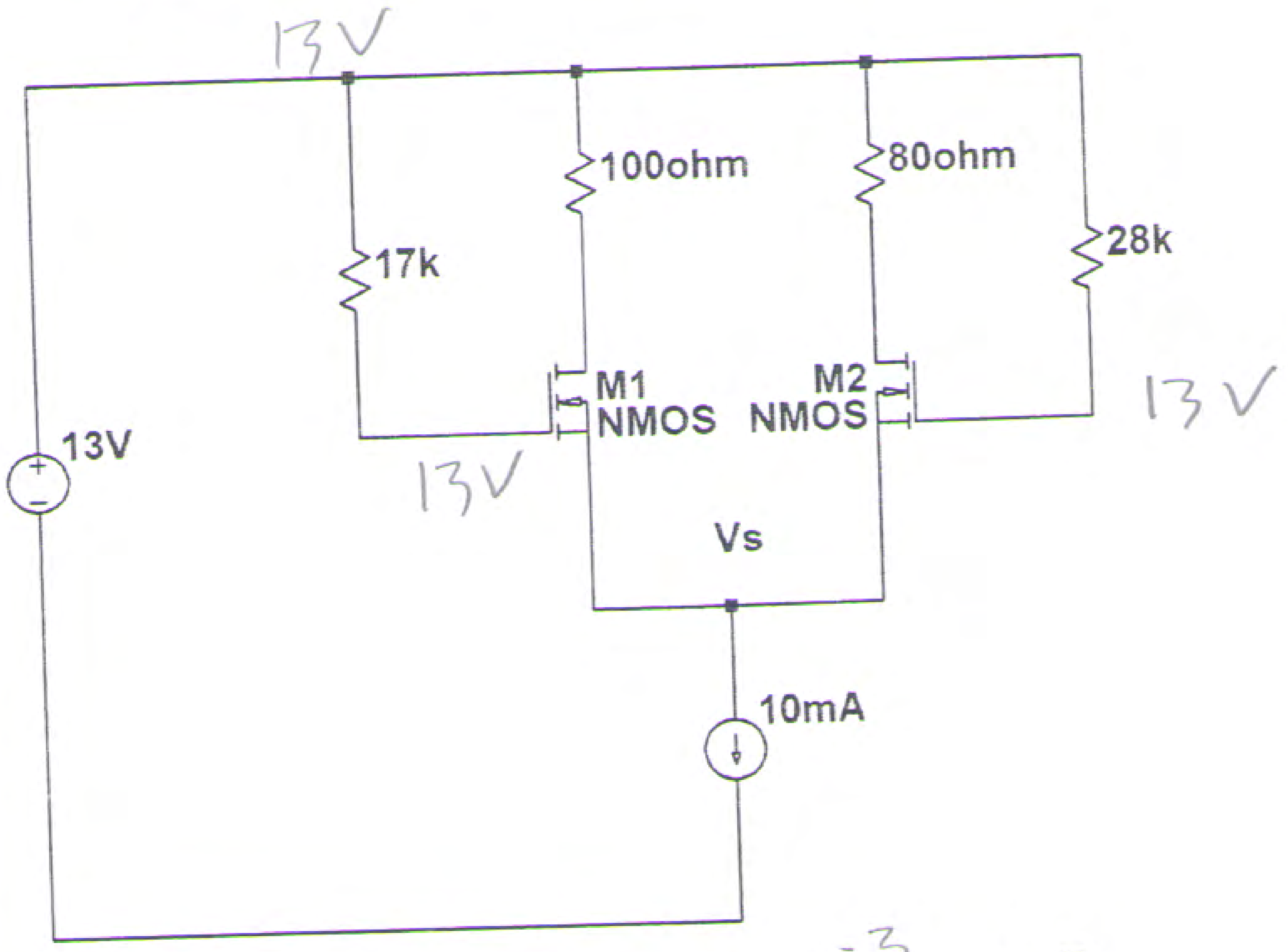
$V_{CB} = 6.2V - 4V = 2.2V$
$\Rightarrow$ F-A

$V_{CE} = V_{CB} + V_{BE} = 2.2V + 0.7V$
$= 2.9V$

c) (7 points)

$V_{Tn1} = V_{Tn2} = 2V$, $K_{n1} = \cancel{4}K_{n2} = 4mA/V^2$

Q_1 state	Q_2 state	I_{D1}	I_{D2}	V_S	
SAT	SAT	5mA	5mA	9.882V	$K_{n1} = K_{n2}$
SAT	SAT	8mA	2mA	9.59V	$K_{n1} = 4K_{n2}$



Solution with $K_{n1} = K_{n2} = 4 \times 10^{-3}\, A/V^2$

Assuming non-sat: $V_{GS1} = V_{GS2} \Rightarrow I_{D1} = I_{D2} = \frac{10mA}{2} = 5mA$

$5mA = 4\times10^{-3}[V_{GS} - V_{TN}]^2 = 4\times10^{-3}[V_{GS} - 2]^2 \Rightarrow$

$V_{GS} - 2 = \left[\frac{5\times10^{-3}}{4\times10^{-3}}\right]^{1/2} = (1.25)^{1/2} = 1.118 \Rightarrow$

$V_{GS} = 2 + 1.118 = 3.118V$

$V_S = V_G - V_{GS} = 13 - 3.118 = 9.882V$

$V_{D1} = 13V - 5mA \times 100\Omega = 13 - 0.5 = 12.5V$

$V_{D2} = 13 - 5mA \times 80\Omega = 13 - 5\times10^{-3} \times 0.08\times10^{3} = 13 - 0.4 = 12.6V$

$V_{DS1} = 12.5 - 9.882 = 2.618V > V_{GS} - V_{TN} = 1.18$ ✓ SAT

$V_{DS2} = 12.6 - 9.882 = 27.18V > V_{GS} - V_{TN} = 1.18$ ✓ SAT

Solution with

$K_{n_1} = 4K_{n_2} = 4\times10^{-3}\,A/V^2 \Rightarrow K_{n_1} = 4\times10^{-3}\,A/V^2$

assuming sat $\quad K_{n_2} = 10^{-3}\,A/V^2$

$(V_{GS1}-2)^2\,4\times\cancel{10^{-3}} + (V_{GS2}-2)^2\times\cancel{10^{-3}} = 10\times\cancel{10^{-3}}$

but $\quad V_{GS1} = V_{GS2} = V_{GS} \Rightarrow$

$(V_{GS}-2)^2\,4 + (V_{GS}-2)^2 = 10 \Rightarrow$

$5\,(V_{GS}-2)^2 = 10 \Rightarrow (V_{GS}-2)^2 = 2$

$V_{GS} - 2 = \sqrt{2} = 1.41 \Rightarrow$

$V_{GS} = 2 + 1.41 = 3.41\,V$

$$\left.\begin{aligned} I_{D_1} &= 4\times10^{-3}(3.41-2)^2 = 4\times10^{-3}\times2 = 8\,mA \\ I_{D_2} &= 1\times10^{-3}(3.41-2)^2 = 10^{-3}\times2 = 2\,mA \end{aligned}\right\}\ \text{total of } 10\,mA$$

$V_{D_1} = 13 - 100\times8\times10^{-3} = 12.2\,V$

$V_{D2} = 13 - 80\times2\times10^{-3} = 13 - 0.16 = 12.84\,V$

$V_S = V_G - V_{GS} = 13 - 3.41 = 9.59\,V$

$V_{DS_1} = 12.2 - 9.59 = 2.61\,V > V_{GS} - V_{TN} = 1.41\,V \Rightarrow SAT$

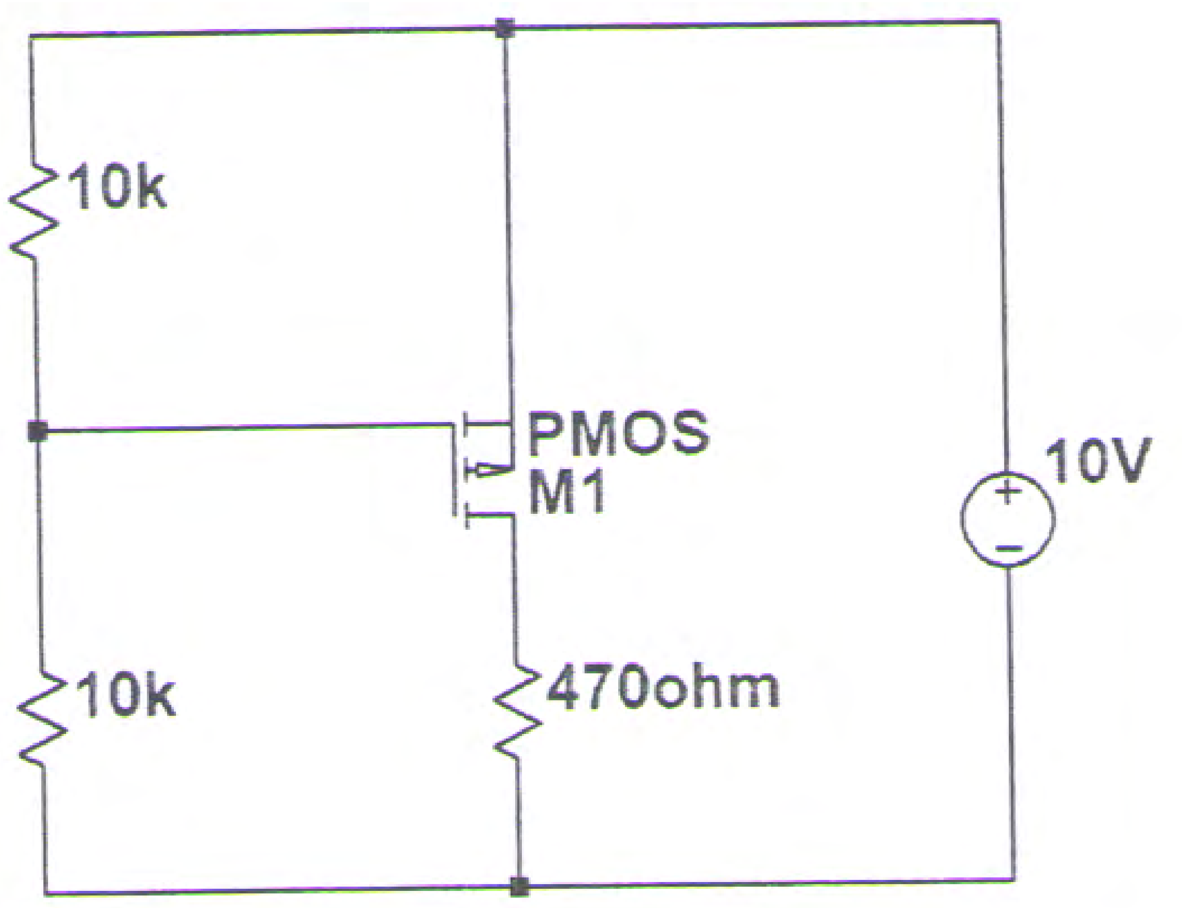
$V_{DS_2} = 12.84 - 9.54 = 33 > V_{GS} - V_{TN} = 1.41\,V \Rightarrow SAT$

d) (5 points)

V_{TP}= -2V, K_p=1mA/V^2

Q_1 state	I_D	I_S	V_{SD}
SAT	9mA	9mA	5.77V

$V_G = 5V$



Assuming SAT

$I_D = K_P[V_{SG} + V_{TP}]^2$

$= 10^{-3}[5+(-2)]^2$

$= 3^2 \times 10^{-3} = 9mA$

$V_{SG} = 10-5 = 5V$

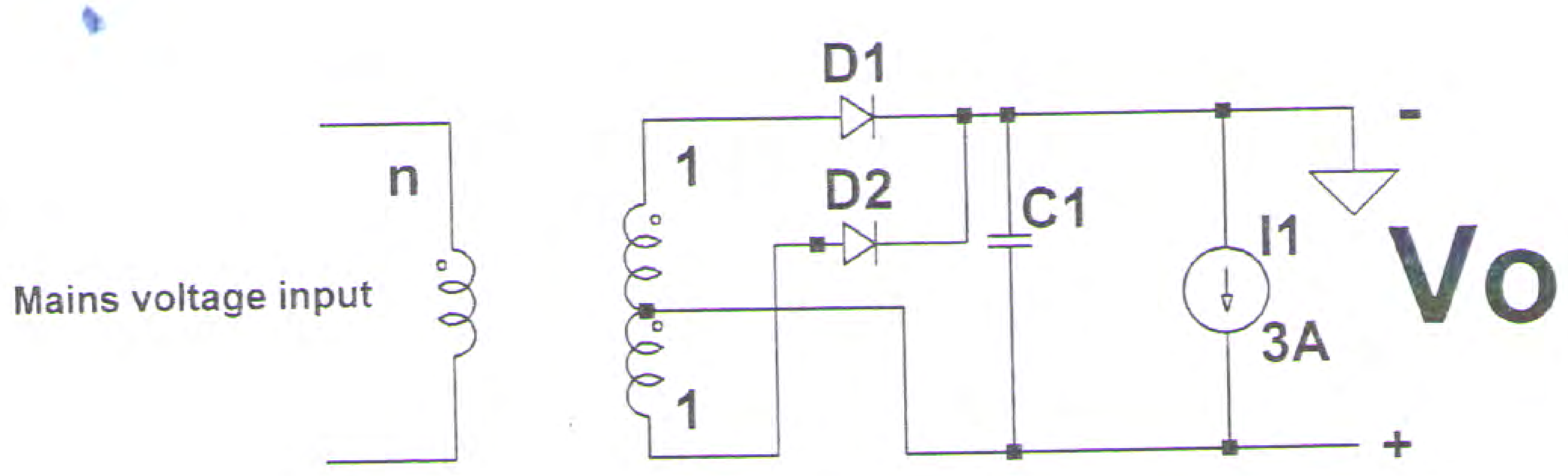
$V_{SD} = 10 - 470\Omega \times 9mA = 10 - 0.47 \times \cancel{10^3} \times 9 \times \cancel{10^{-3}}$

$10 - 4.23 = 5.77V > V_{SG} + V_{TP} = 5-2 = 3$ ✓ SAT

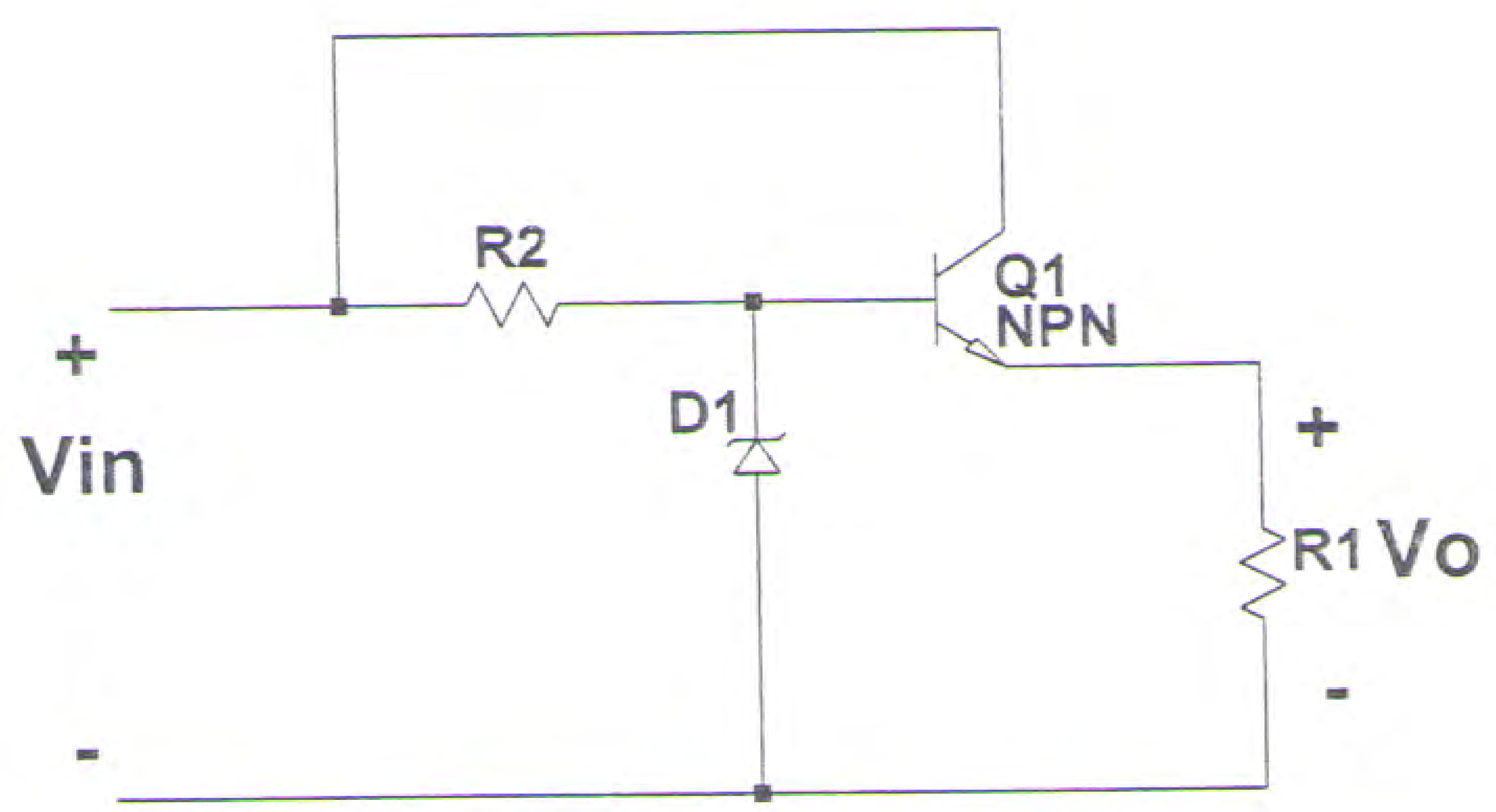
$I_D = I_D = 9mA$

2. (30 points) At the circuit given below, V_γ=1.0Volts, I_1=3A, the transformer is ideal and r_o (the output resistance) of the zener diode is equal to 0. The BJT parameters are; β=99, $V_{BE(ON)}$=0.8V, $V_{CE(SAT)}$=0.4V,

Answer the following:



a. (12 points) The input is 110Volts rms +/-10% and 400Hz. In order to have a ripple voltage equal to 4Volts and V_{max} (the peak value of the output voltage) equal to at least 18Volts at the output of the power supply for all input voltages, what must be the values of n and C_1? Please also plot the output voltage waveform for steady state, $V_o(t)$. *Note the polarity of the output voltage! Please note that the phase of the input voltage is not important.*



V_Z=9Volts, I_{Zmin}=7mA, R_1=8Ω

b. (10 points) If $V_{in}(t)$=16+cos(500t) Volts, what must be the maximum value of R_2 which will let the output regulation to continue? (Output does not change)

c. (8 points) Find the maximum current that you can draw from the voltage regulator shown at (b) without destroying the regulation if the input voltage is 17Volts and ~~R_1~~ R_2 is equal to 150Ω?

Solutions:

(a) $\dfrac{V_{rms} \times \sqrt{2} \times 0.9}{n} - V_{\gamma} = 18$

$\dfrac{110 \times 1.41 \times 0.9}{n} - 1 = 18 \Rightarrow n = \dfrac{110 \times 1.41 \times 0.9}{19}$

$\boxed{n = 7.347}$

$I \Delta t = C \Delta V \Rightarrow 3 \times 1.25 \times 10^{-3} = C \times 4V \Rightarrow$

$\dfrac{1}{400}/2$

$C = \dfrac{3 \times 1.25 \times 10^{-3}}{4} = 9.375 \times 10^{-4} = \boxed{937.5 \mu F = C}$

b-) $Vin_{min} = 16 - 1 = 15V \quad V_Z = 9V \quad V_{BEON} = 0.8V$

$\Rightarrow V_o = 9V - 0.8V = 8.2V$

$I_{R_1} = \dfrac{8.2V}{8\Omega} = 1.025A \quad I_B = \dfrac{1.025}{\beta + 1} = \dfrac{1.025}{100} = 10.25mA$

$\dfrac{Vin_{min} - V_Z}{R_2} = I_B + I_{Z_{min}} \Rightarrow \dfrac{(15-9)V}{(10.25+7)mA} = R_2$

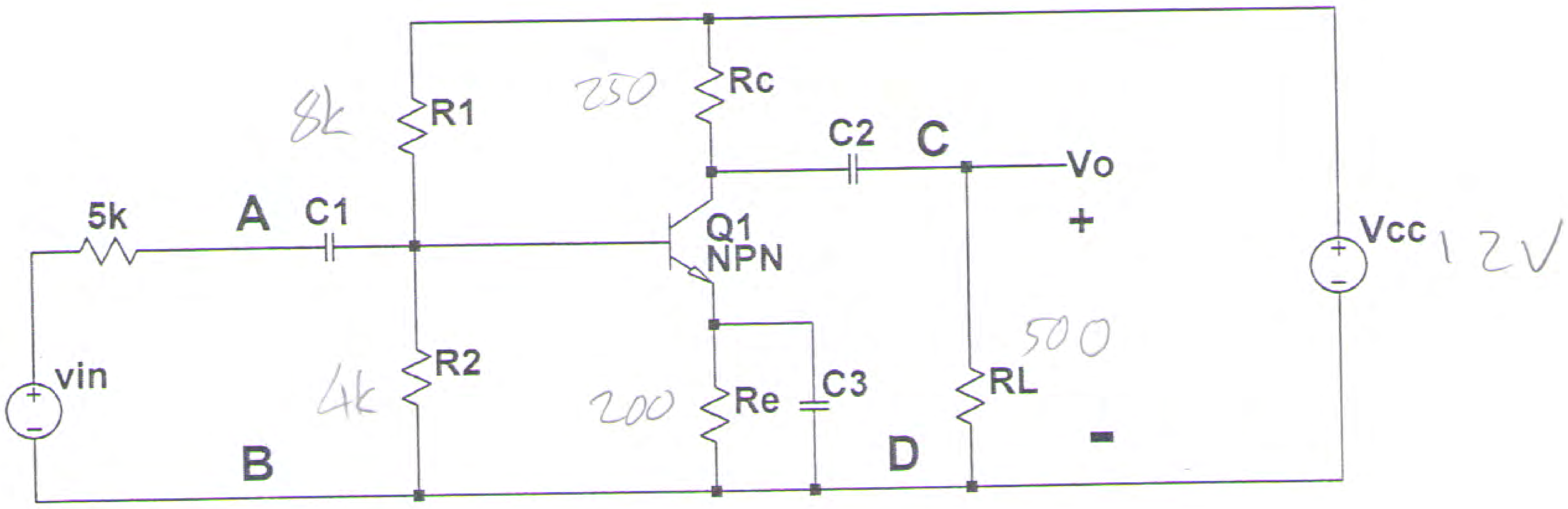
$R_2 = \dfrac{6}{17.25 \times 10^{-3}} = 347.83\,\Omega$

c-) $\dfrac{(17-9)V}{150\Omega} - I_{Z_{min}} = I_{B_{max}}$

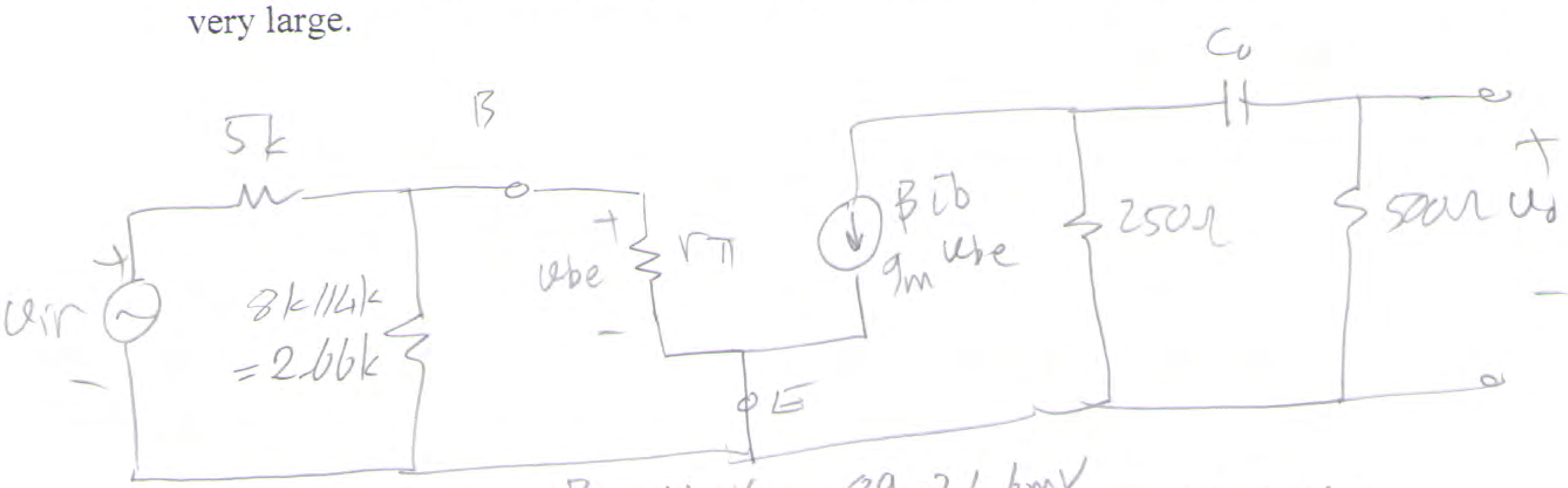
$\dfrac{8}{150} - 7mA = I_{B_{max}} \Rightarrow 0.05\overline{33} - 7 = 0.046\overline{3}A$

$I_{B_{max}} \cdot (\beta + 1) = \boxed{I_{o_{max}} = 0.046\overline{3}A \times 100 = 4.6\overline{3}A}$

3. (25 points) Consider the BJT amplifier show below.
The BJT parameters are β=89, $V_{BE(ON)}$=0.7V, $V_{CE(SAT)}$=0.4V.
The resistors are; Vcc=12V, Re=200Ω, R_2=4kΩ, R_1=8kΩ, Rc=250Ω, R_L=500Ω
The BW of the amplifier is 300Hz-3kHz
Please answer the below questions assuming the collector DC current is **15mA and the transistor is in FA.**



a) (3 points) Draw the small signal ac equivalent circuit assuming the capacitors are very large.



$$r_\pi = \frac{\beta}{g_m} = \frac{\beta \times 26.6mV}{I_{CQ}} = \frac{89\times 26.6mV}{15\,mA} = 157.8\Omega$$

$$g_m = \frac{26.6mV}{15mA} = 0.564\ S \qquad \frac{1}{g_m} = 1.773\Omega$$

b) (2 points) Find input and output impedance, R_{in} (Between A and B) and R_{out}, (Between Cand D) of the amplifier.

R_{in}	R_{out}
149Ω	$R_C = 250\Omega$

$R_{in} = r_\pi \,/\!/\, R_1 /\!/ R_2 = 157.8 /\!/ 2666\Omega$

$= 148.98 = 149\Omega$

c) (3 points) Find the midband voltage gain, $A_v = v_o/v_{in}$ in dB units

$A_v = v_o/v_{in}$ (dB)
8.41dB

$V_0 = (R_c /\!/ R_L) g_m \cdot \frac{R_1 /\!/ R_2 /\!/ r_\pi}{R_1 /\!/ R_2 /\!/ r_\pi + 5000} v_{in}$

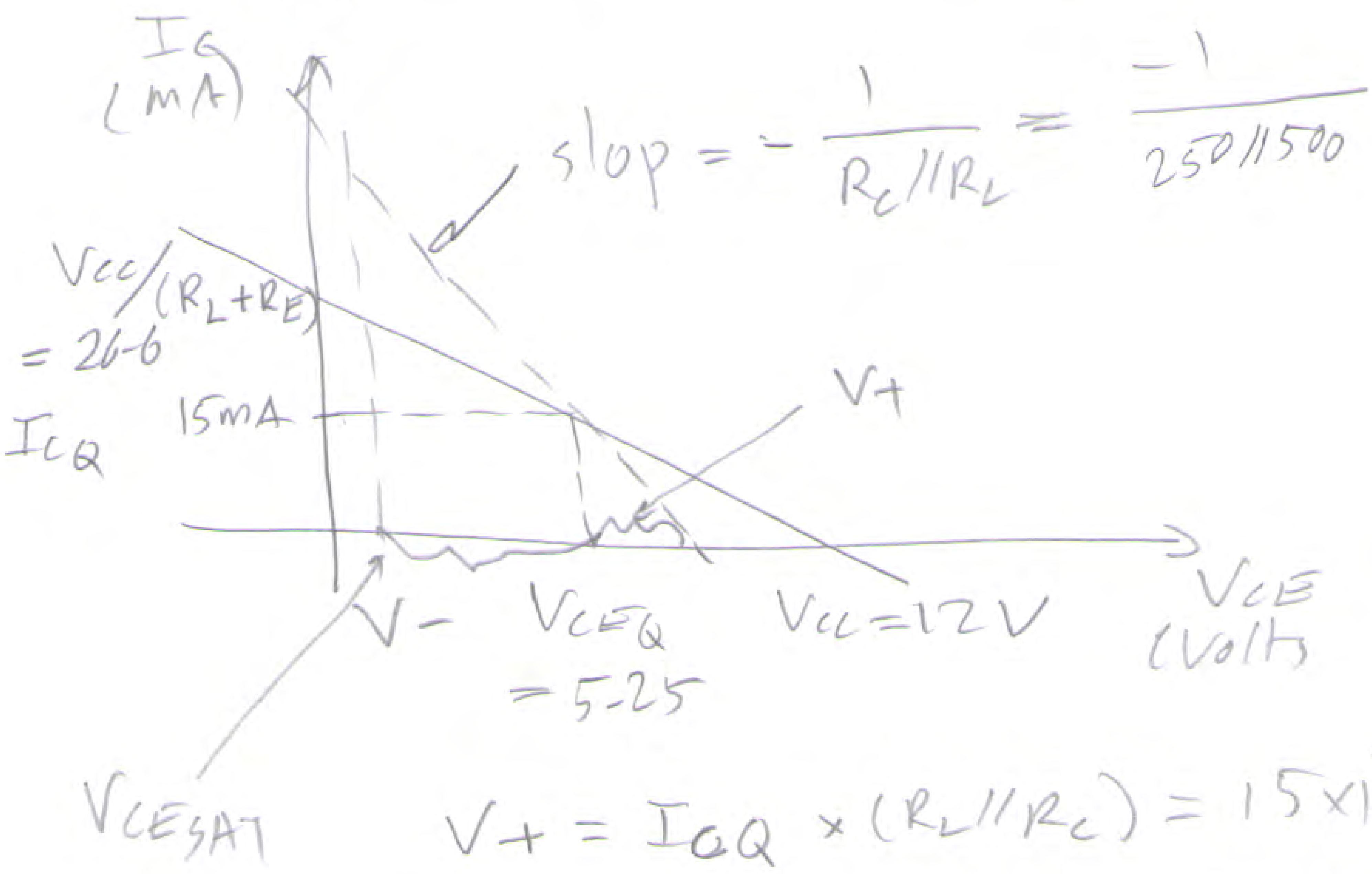
$\frac{v_o}{v_{in}} = \frac{149 \times 0.546 \times 166.7}{5149} = 2.634 \quad = 20 \times \log 2.634$

$= 20 \times 0.4205 = 8.41 dB$

d) (5 points) Determine the maximum peak-to-peak symmetric undistorted output voltage swing

v_o peak-to-peak
5V

$slop = -\frac{1}{R_c /\!/ R_L} = \frac{-1}{250 /\!/ 500} = \frac{-1}{166.7}$



$V_{CEQ} = 12V - 15mA\,(R_C + R_E)$

$= 12V - 0.015 \times 450$

$= 5.25V$

$V+ = I_{CQ} \times (R_L /\!/ R_c) = 15 \times 10^{-3} \times 166.7 = 2.5V$

$V_- = 5.25 - 0.4 = 4.85V$

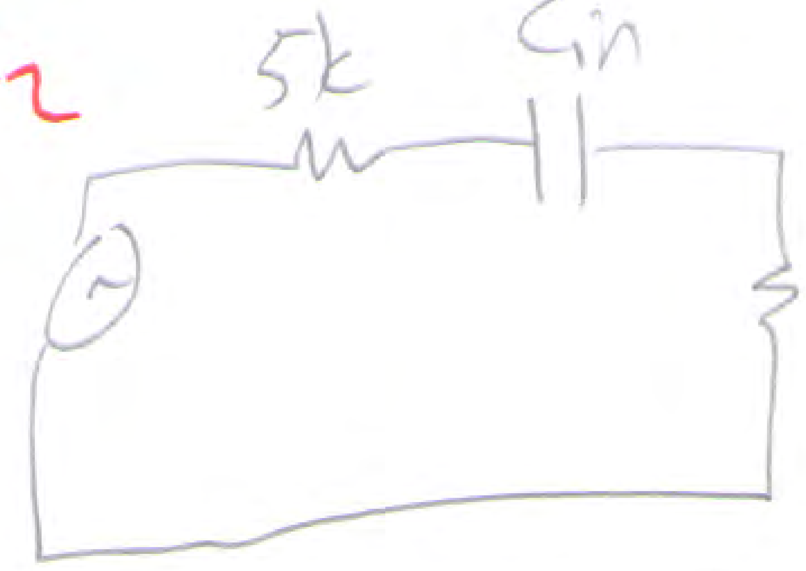
$V_{p-p} = 2 \times \min\{4.85, 2.5\} = 5V$

e) (5 points) Determine the values of C_{in} and C_{out} so that the 3dB corner frequencies due to each of the input and output coupling capacitors are the same and equal to 300Hz. Please assume that the other capacitor and the bypass capacitor C_3 are very large and they can be considered short circuit at this frequency.

C_{in}	C_{out}
103nF	707nF

$C_{out} = C_2$

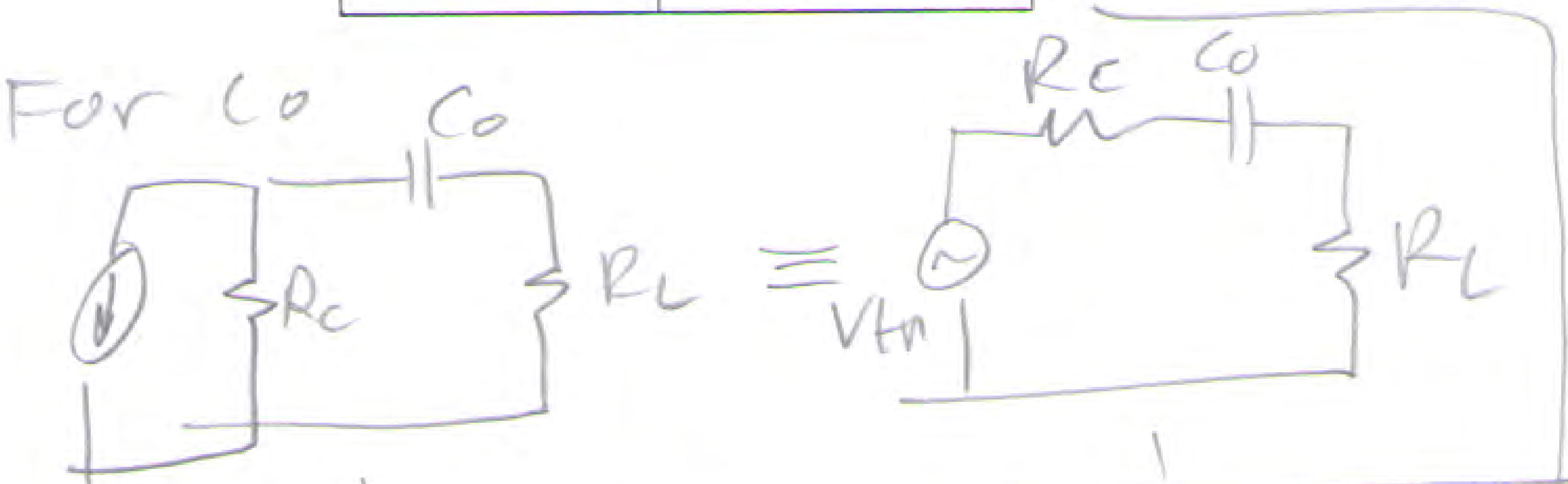
For Cin



$r_\pi // R_1 // R_2 = 149\,\Omega$

$w_0 = 2\pi \times 300 = \dfrac{1}{5149 C_{in}}$

For Co



$C_{in} = \dfrac{1}{2\pi \times 300 \times 5149} = 1.03 \times 10^{-7} F$

$= 103\,nF$

$C_{out} = \dfrac{1}{2\pi \times 300 \times (R_L + R_C)} = \dfrac{1}{2\pi \times 300 \times 750}$

$= 7.07 \times 10^{-7} F = 0.707\,\mu F$

f) (3 points) Using the capacitor values found in part (e), find the value of the magnitude of the voltage gain (in dB units) at 3kHz (Upper cut-off frequency) considering the combined effect of both coupling capacitors, C_{in} and C_{out}.

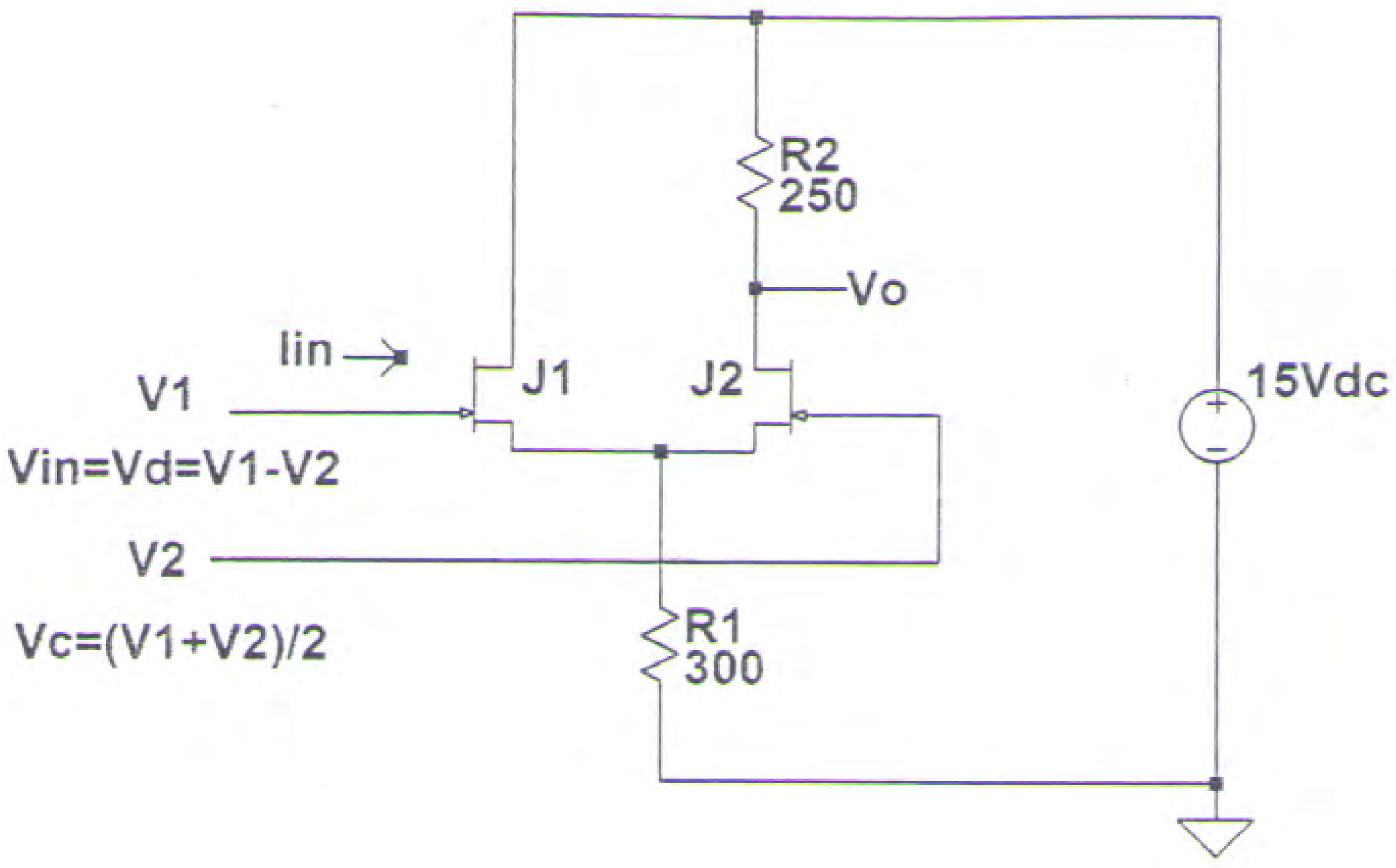
$A_v = v_o/v_{in}$ (dB) at 3kHz
same as (c) 8.41dB

g) (4 points) Using the capacitor values found in part (e), find the value of the magnitude of the voltage gain (in dB units) at 300Hz (Lower cut-off frequency) considering the combined effect of both coupling capacitors, C_{in} and C_{out}.

$A_v = v_o/v_{in}$ (dB) at 300Hz
2.41dB

each capacitor loses 3dB ⇒

gain = 8.41dB − 3dB − 3dB = 2.41dB



4. (25 points) For the circuit given above I_{DSS}=52mA, V_p= - 5Volts. Answer the following:
   a. (5 points) Find the states of the transistors and transistor currents for V_d=0 and V_C=2Volts.
   b. (5 points) Find the small signal differential voltage gain defined as v_o/v_d when Vc = 0Volts assuming that the the current through R_1 is equal to 11.2mA and the transistors are in saturation.
   c. (5 points) Find the common-mode small signal gain defined as v_o/v_c when Vc=0Volts and Vd= 0Volts assuming that the the current through R_1 is equal to 11.2mA and the transistors are in saturation.
   d. (5 points) State the formula for CMRR
   e. (5 points) Find the differential input resistance defined as $R_{ind}=v_d/i_{ind}$ assuming that the the current through R_1 is equal to 11.2mA and the transistors are in saturation.

v_1, v_2, v_d, v_c and v_o are AC voltages,
v_d is the differential input voltage, $v_1 - v_2$
v_c is the common input voltage, $\frac{v_1 + v_2}{2}$
V_d and V_C are DC voltages
R_{INd} is the differential input resistance (for V_c=0)
i_{INd} is the differential input current (for V_c=0)

Solutions

Assuming saturation

a-) $2\text{ Volts} = V_{GS} + I_{DSS}\left[1 - \frac{V_{GS}}{V_p}\right]^2 \times 300\Omega$ if we had a single transistor. Having 2 parallel transistors, I will use $I_{DSS} = 2\times 52$ mA and have the voltages as the original one.

$$2 = V_{GS} + 0.104\left[1 - \frac{V_{GS}}{-5}\right]^2 300 \Rightarrow 2 = V_{GS} + 0.104\left[1 + \frac{V_{GS}}{5}\right]^2 \times 300$$

$$\Rightarrow 2 = V_{GS} + 0.104\left[5 + V_{GS}\right]^2 300/25 \Rightarrow 2 = V_{GS} + 0.104\times 12\left[5 + V_{GS}\right]^2$$

$$2 = V_{GS} + 1.248\left[25 + 10V_{GS} + V_{GS}^2\right]$$

$$\frac{2}{1.248} = \frac{V_{GS}}{1.248} + 25 + 10V_{GS} + V_{GS}^2 \Rightarrow$$

$$V_{GS}^2 + 10.8V_{GS} + 23.4 = 0$$

$a=1 \quad b=10.8 \quad c=23.4$

$$V_{GS_{1,2}} = \frac{-b}{2a} +/- \frac{\sqrt{b^2-4ac}}{2a} = -5.4 +/- \frac{\sqrt{23.04}}{2}$$

$$= -5.4 \pm \frac{4.8}{2} = -5.4 \pm 2.4 = -3, \cancel{-5.8}$$

too low

$V_{GS} = -3V$

$$I_{D_T} = 0.104\left[1 - \frac{-3}{-5}\right]^2 = 0.104 \times 0.4^2 = 0.01664\,A$$

total current on 300Ω

$V_{GS} = 2 - 300Ω \times 0.01664A = -2.992$ Volts ✓

solution verified

$$I_{D_1} = I_{D_2} = \frac{I_{DT}}{2} = \frac{16.64mA}{2} = 8.32mA$$

For J_2 $\quad V_{DS_2} = 15V - 8.32mA \times 250 - 16.64mA \times 300 = 7.928V$

$V_{DS_2} = 7.928V > V_{GS_2} - V_p = -3 - (-5) = +2$ ✓

$V_{DS_1} > V_{DS_2} \Rightarrow$ also sat

b) $g_m = \frac{dI_D}{dV_{GS}} = \frac{d}{dV_{GS}} I_{DSS}\left[1-\frac{V_{GS}}{V_P}\right]^2$

$= I_{DSS}\, 2\left[1-\frac{V_{GS}}{V_P}\right] \times \frac{-1}{V_P}$ —

$= \frac{-2I_{DSS}}{V_P}\left[1-\frac{V_{GS}}{V_P}\right]$ but $\underbrace{5.6\times10^{-3}}_{I_D} = I_{DSS}\left[1-\frac{V_{GS}}{V_P}\right]^2$

$\Rightarrow \left(\frac{5.6\times10^{-3}}{52\times10^{-3}}\right)^{1/2} = 0.328 = \left[1-\frac{V_{GS}}{V_P}\right]$

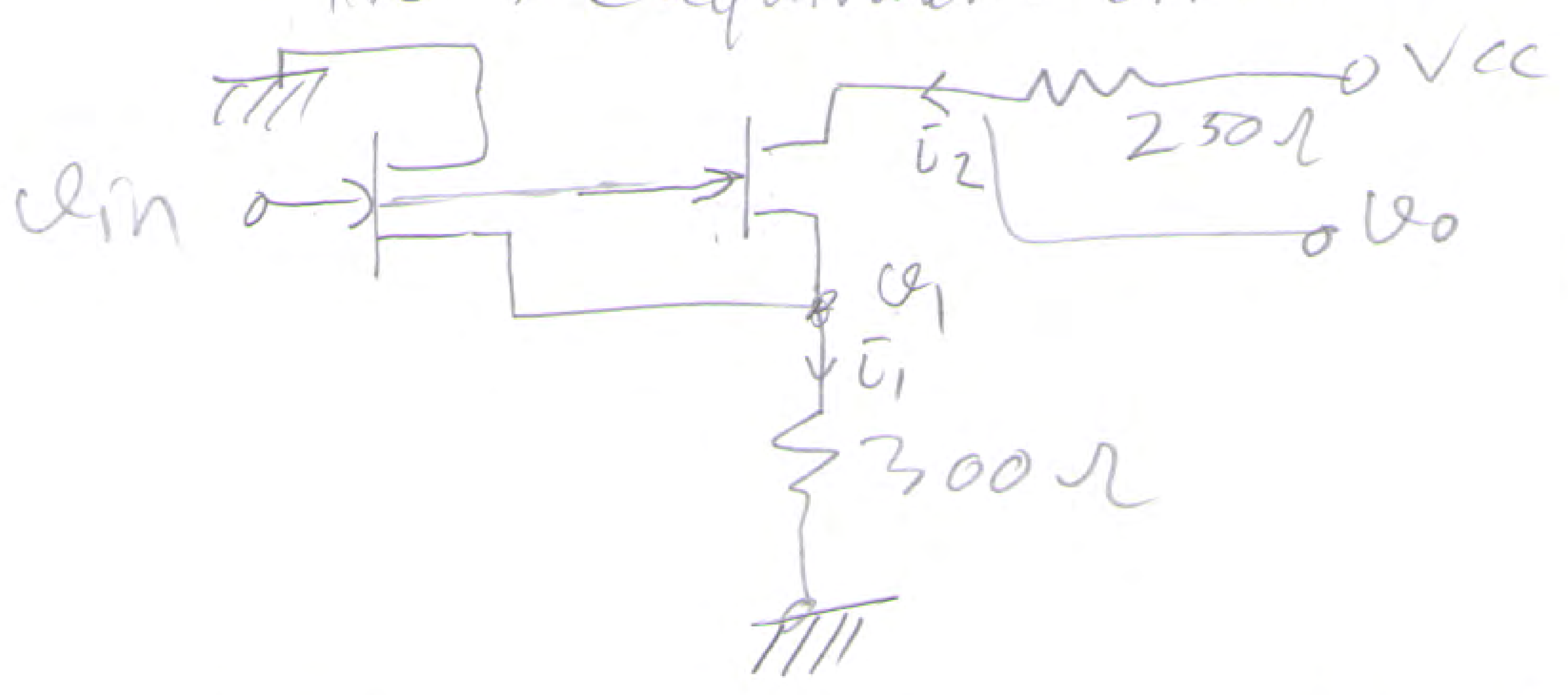
$g_m = \frac{+2\times52\times10^{-3}}{5} 0.328 = 0.00682\ S$

$v_{gs_1}\, g_m \times 250 = v_o \Rightarrow v_o = g_m 250 \times \frac{v_d}{2}$

$\Rightarrow \text{gain} = \frac{v_o}{v_d} = \frac{6.82\times10^{-3}\times0.25\times10^{3}}{2} = 0.853$

c) The total g_m of the two transistors in parallel is $g_{m_1}\times2 = 6.82\times10^{-3}\times2 = 13.64\times10^{-3}\ S$

The AC equivalent circuit for common-mode is



$v_1 = \frac{v_{in}\times300}{300+\frac{1}{g_{m_T}}}$

$v_1 = v_1 \frac{300}{300+73.31} = 0.804$; Half of the current through 300Ω passes through 250Ω $\Rightarrow i_2 = \frac{i_1}{2} \Rightarrow v_o = i_2 250 = \frac{i_1}{2}\times250$

$v_o = \frac{v_1}{300}\cdot\frac{250}{2} = 0.804 v_1 \frac{250}{2\times300}$

$\Rightarrow \frac{v_o}{v_{in}} = \frac{0.804\times250}{600} = 0.335 = A_c$

d-) $CMRR = \frac{A_d}{A_c}$

e-) $R_{in_d} = \frac{v_{ind}}{i_{ind}} = \frac{v_{in}/2}{0} \rightarrow \infty$