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Signature: solutions

EEE 313 Spring 2012

Bilkent University
Department of Electrical and Electronics Engineering
EEE 313 Electronic Circuit Design **Final Exam**
25 May 2012, 15.30 (5 questions, 150 minutes)

- This is a **closed book**, closed notes exam. No cheat sheet allowed.
- All cell-phones should be completely **turned off**.
- Use a calculator for numerical computations. Carry at least **3 significant digits**. Double check your numerical calculations.
- Be sure to write the **units** of all numerical results.
- **Show** all work clearly. Show all the intermediate steps including formulas. **Never, just carry out calculations without formulas and explanation.**
- Please put your **final answer** for each part inside a box for easy identification. Do not give multiple answers, they will not be graded.
- Do not remove the **staple** from the exam sheets or separate pages of the exam. All extra pages must be stamped to your exam.
- You may leave the exam room when you are done. However, please do not leave during the **last five minutes** of the exam.
- At the end of the exam, please stay seated unitl **all** exam papers are collected.

Useful constants and formulas:

Boltzmann's constant: 86×10^{-6} eV/K°

Electron charge: 1.6×10^{-19} Coulombs

Drain currrent equation for n-channel MOSFET:

$I_D = K_n(V_{GS} - V_{TN})^2$ for $V_{GS} - V_{TN} \leq V_{DS}$

$I_D = K_n(2(V_{GS} - V_{TN})V_{DS} - {V_{DS}}^2)$ for $V_{GS} - V_{TN} \geq V_{DS}$

Drain currrent equation for p-channel MOSFET:

$I_D = K_p(V_{SG} + V_{TP})^2$ for $V_{SG} + V_{TP} \leq V_{SD}$

$I_D = K_p(2(V_{SG} + V_{TP})V_{SD} - {V_{SD}}^2)$ for $V_{SD} + V_{TP} \geq V_{SD}$

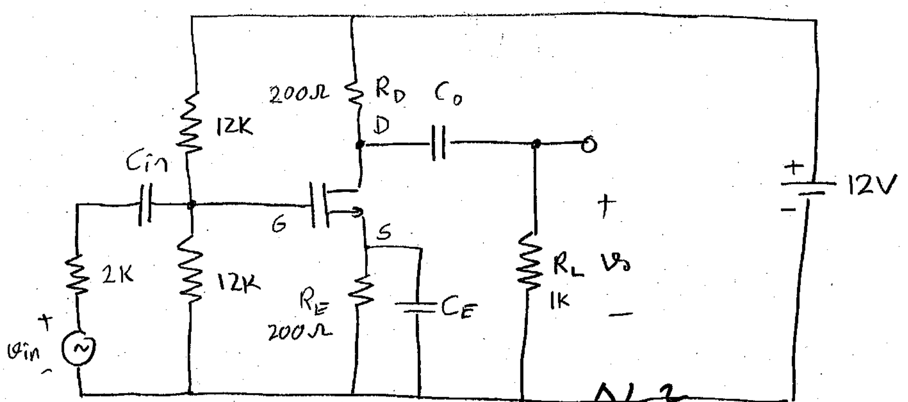
Forward active current of BJT is given by:

$$I_C = I_S\left(e^{\frac{V_{BE}}{nV_t}} - 1\right)$$

Other equations must be deduced by the students.

Please do not write below this line

1. 30 pts.	
2. 30 pts.	
3. 20 pts.	
4. 20 pts.	
Total 100 pts.	



1. **Question:** (30 Pts.) At the circuit given above, the transistor is an n-channel MOSFET with K_n=2.5x10^{-3}~~mhos~~ A/V^2 and V_{TN}=2Volts. Answer the following:
   a. 9 (~~6~~ Pts.) Find the DC bias of the transistor.
   b. 9 (~~7~~ Pts.) Assuming that the capacitors C_{in}, C_E and C_o are very large and the drain current is equal to 10mA, find the voltage gain of the amplifier defined as v_o/v_{in}.
   ~~c. (7 Pts.) Find the output peak-to-peak undistorted voltage swing again assuming that the drain current is equal to 10mA.~~
   d. 12 (~~10~~ Pts.) The amplifier is to be used to amplify music signals. The BW of the music signal is assumed to between 20Hz and 20kHz. Find limiting values for the capacitors C_{in}, C_E and C_o seperately for each them in order to put the 3dB-cutoff frequencies to the appropriate frequency assuming that the others are very large each time and the drain current is equal to 10mA.

Solution:

a-) $V_G = 6V$, assuming sat, $6V = V_{GS} + 200 I_D = V_{GS} + 200\times 2.5\times10^{-3}(V_{GS}-V_{TN})^2$

$6V = V_{GS} + 200\times 2.5\times10^{-3}[V_{GS}-2]^2 = 0.5[V_{GS}^2 - 4V_{GS} + 4] + V_{GS}$

$6V = V_{GS} + 0.5[V_{GS}^2 - 4V_{GS} + 4] \Rightarrow 6 = V_{GS} + 0.5V_{GS}^2 - 2V_{GS} + 2$

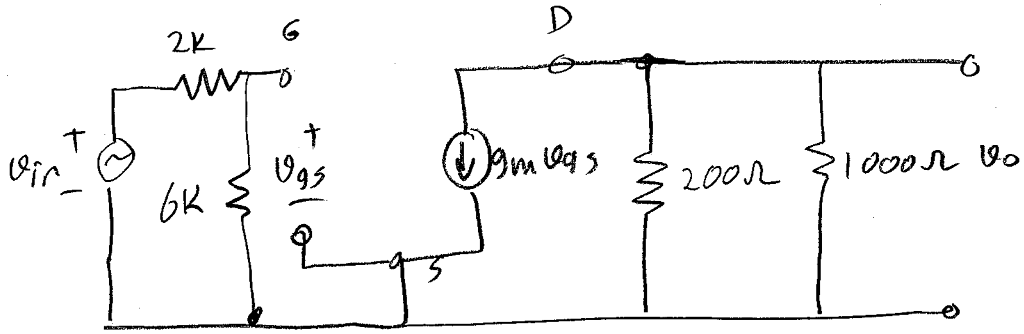
$+0.5V_{GS}^2 - V_{GS} - 4 \Rightarrow V_{GS}^2 - 2V_{GS} - 8 = 0$

$V_{GS_{1,2}} = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{2}{2} \pm \frac{\sqrt{4+32}}{2} = 1 \pm 3V$

$V_{GS} = -2$ is not feasible as it makes drain current zero.

$\Rightarrow V_{GS} = 4V \quad I_D = K_n(V_{GS}-V_{TN})^2 = 2.5\times10^{-3}[4-2]^2 = 10^{-2}A = 10mA$

$V_{DS} = 12V - 400\Omega\times10^{-2} = 12-4 = 8V > V_{GS}-V_{TN} = 4-2 = 2 \Rightarrow$ SAT b-) First draw the AC equivalent circuit



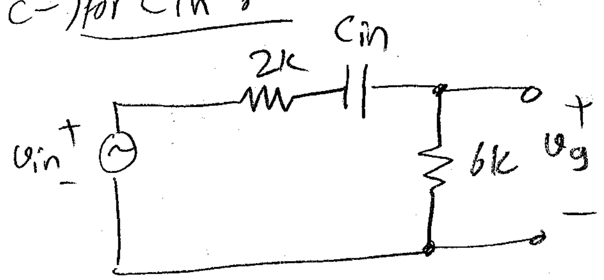
$$g_m = 2(V_{GS}-V_{TN})K_n = 2\times 2.5\times10^{-3}[4-2] = 5\times10^{-3}\times2 = 10^{-2} \text{ mhos}$$

$$v_o = \frac{6K}{2+6K}\cdot g_m (200//1000\Omega) v_{in} = \frac{6}{8}\cdot 10^{-2}\times 166.6\Omega\, v_{in}$$

$$200//1000 = 166.\bar{6}\Omega$$

$$\boxed{A_V = \frac{v_o}{v_{in}} = 1.25}$$

c-) for Cin :



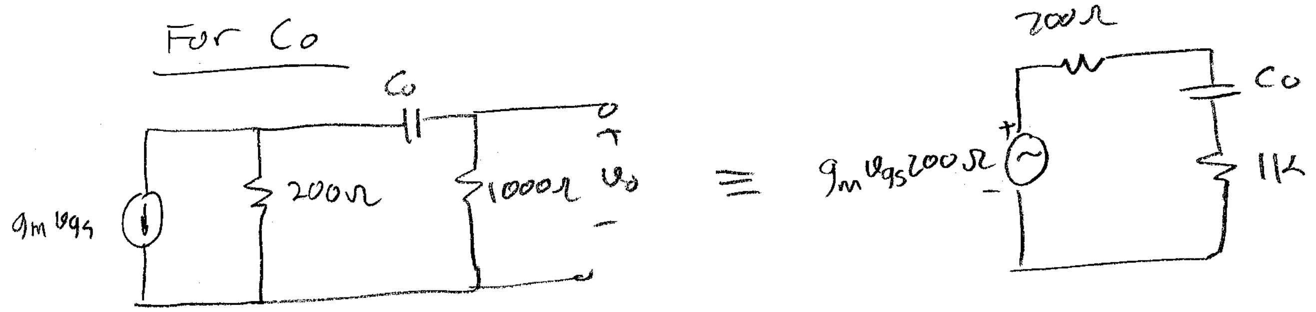
$$\frac{v_G}{v_{in}} = \frac{6K}{2K+6K+\frac{1}{j\omega C_{in}}}$$

$$W_o = \frac{1}{8000\,C} \Rightarrow C = \frac{1}{w_o\, C}$$

$w_o = 2\pi\times 20Hz$ as the formula is for the lower cut-off

$$C_{in} = \frac{1}{2\pi\times20\times8000} = 9.95\times10^{-7}F \cong 10\times10^{-7}F = 1\ \mu F$$

For Co

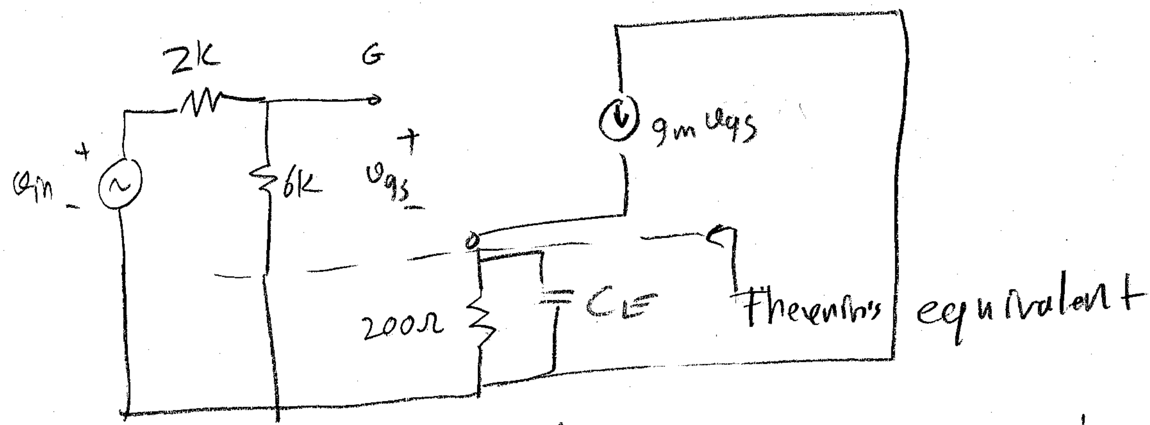


$$\frac{v_o}{g_m v_{gs} 200} = \frac{1000\Omega}{200\Omega + 1000\Omega + \frac{1}{j\omega C_o}} \Rightarrow$$

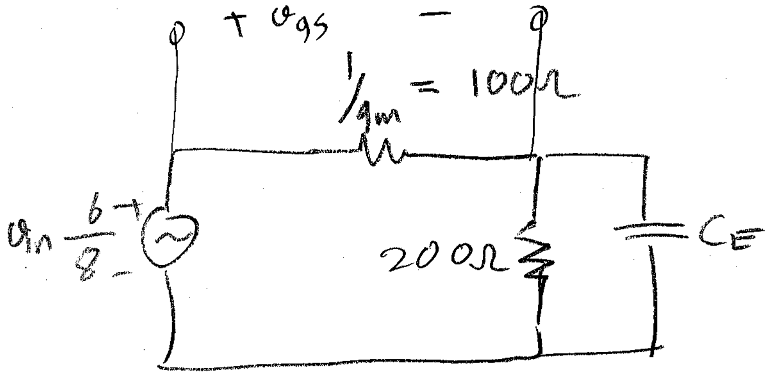
$$\omega_o = \frac{1}{1200\Omega\, C_o} \Rightarrow C_o = \frac{1}{2\pi \times 20 \times 1200} = 6.63 \times 10^{-6}\,F$$

$$\boxed{C_o = 6.6\mu F}$$

for C_E



if R_o is calculated it turns out to be $\frac{1}{g_m}$ with v_{in} short-circuited, and open circuit voltage is $v_{in} \times \frac{6}{8} \Rightarrow$ Thevenin's equivalent becomes



$$v_{gs} = v_{in} \frac{100\Omega}{200 /\!/ \frac{1}{j\omega C_E} + 100} \frac{6}{8}$$

$$\frac{v_{gs}}{v_{in}} = \frac{6}{8} \frac{100}{100 + \frac{200 \frac{1}{j\omega C_E}}{200 + \frac{1}{j\omega C_E}}} \frac{6}{8} \quad \frac{100}{100 + \frac{200}{1 + 200 j\omega C_E}} = .75 \frac{1 + 200 j\omega C_E}{100 + 20000 j\omega C_E + 200}$$

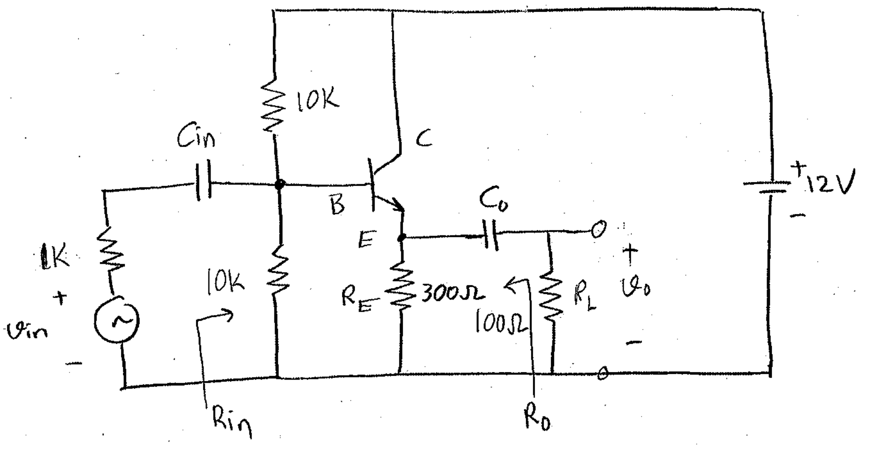
$$\frac{v_{gs}}{v_{in}} = 75\,\frac{1+200j\omega C_E}{300+20000j\omega C_E} = \frac{75}{300}\,\frac{1+200j\omega C_E}{1+\frac{200\cancel{00}}{3\cancel{00}}j\omega C_E}$$

$$\frac{v_{gs}}{v_{in}} = \frac{1+j\omega 200 C_E}{1+j\omega 66.6 C_E}$$

$\Rightarrow$ The lower cutoff frequency w_0 is given by

$$w_0 = \frac{1}{66.6 C_E} \Rightarrow C_E = \frac{1}{2\pi \times 20 \times 66.6} = 1.194 \times 10^{-4}\ \mathrm{F}$$

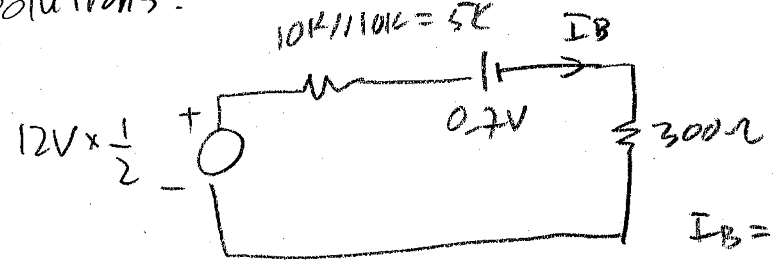
$$\boxed{C_E = 119.4\ \mu\mathrm{F}}$$



2. **Question:** (30 Pts.) For the circuit given above β=120, V_{CESAT}=0.4Volts and V_{BESAT}=0.7Volts. Answer the following:
   a. (6 Pts.) Find the DC bias of the transistor.
   b. (7 Pts.) Find the voltage gain of the amplifier defined as v_o/v_{in} assuming that the Capacitors C_{in} and C_o are very large and the collector current is 15mA.
   c. (10 Pts.) Find the input and output resistances as shown at the figures assuming that the collector current is 15mA.
   d. (7 Pts.) Find the output peak-to-peak undistorted voltage swing assuming that the collector current is 15mA.

Solutions:

a-)



$I_B = \dfrac{6-0.7}{5000+300\times(120+1)}$

$I_B = 1.283\times10^{-4} = 128.3\ \mu A$

$I_C = 15.4\,mA$, $I_E = 15.4\,mA$ $V_{CE} = 12V - 300\times15.4\times10^{-3}$
$= 12 - 7.38V$ ✓
$\Rightarrow$ F.A.

b-) $\dfrac{v_B}{v_{in}} = \dfrac{5K//[(300//100)(\beta+1)] + r_\pi}{5K//[(300//100)(\beta+1)] + 1000 + r_\pi}$ $300//100 = 75\Omega$

$(300//100)(\beta+1) = 75\times121 = 9075\Omega$ $9075//5000 = 3224\Omega$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{\beta\times26.6\times10^{-3}}{15mA} = \dfrac{120\times26.6}{15} = 212.8\Omega$

$\dfrac{v_b}{v_{in}} = \dfrac{3224+212.8}{3224+212.8+1000} = 0.7746$

$$\frac{v_o}{v_b} = \frac{(300 // 100)}{(300 // 100) + \frac{1}{g_m}}$$

$$\frac{1}{g_m} = \frac{26.6\,mV}{15\,mA} = 1.773\,\Omega$$

$$g_m = \frac{1}{1.773} = 0.5642\,\mho$$

$$\frac{v_o}{v_b} = \frac{75}{75 + 1.773} = 0.977$$

$$\boxed{A_v = \frac{v_o}{v_{in}} = \frac{v_o}{v_b} \cdot \frac{v_b}{v_{in}} = 0.7746 \times 0.977 = 0.7568}$$

c-)

$$R_{in} = \left((R_E // R_L)(\beta+1) + r_\pi\right) // R_1 // R_2$$

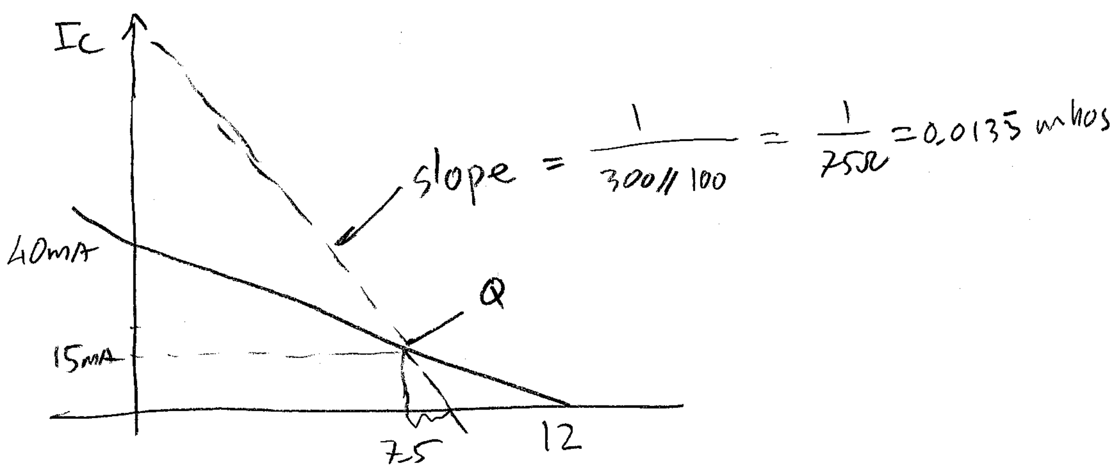
$$(75 \times 121 + 212.8) // 5000 = (9075 + 212.8) // 5000 = 3250\,\Omega$$

$$R_o = \left[\frac{(R_s // R_1 // R_2)}{\beta + 1} + \frac{1}{g_m}\right] // R_E = \left(\frac{1000 // 5000}{121} + 1.773\right) // 300$$

$$= \left(\frac{833}{121} + 1.773\right) // 300 = (6.887 + 1.773) // 300$$

$$= 8.66 // 300 = 8.417 \approx 8.4\,\Omega$$

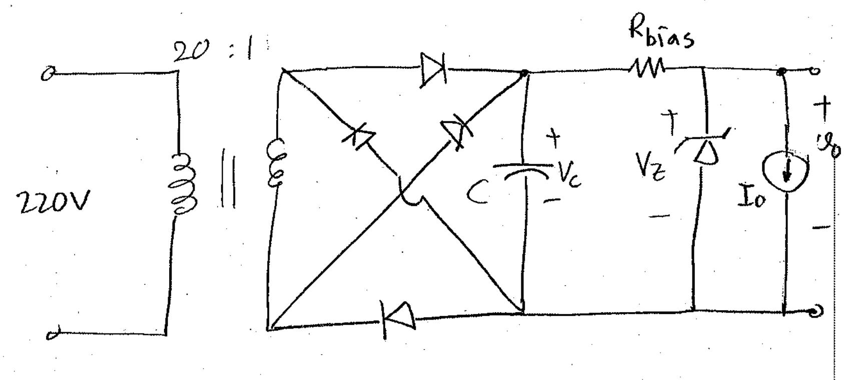
d-)



$V_+ = 15mA \times 75\Omega = 1.125V$

$V_- = 7.5V - V_{CE_{SAT}} = 7.5 - 0.4 = 7.1V$

$V_{o_{p-p}} = 2 \times \min\{1.125, 7.1\} = 2.25V$



3. **Question:** (20 Pts.) At the circuit given above, V_γ=1Volt, V_z=7Volts, I_{zmin}=20mA, I_o=200mA, the transformer input is 220V_{rms}, the transformer is ideal and r_o (the output resistance) of the zener diode is equal to 0. Answer the following:
   a. (10 Pts.) Find the ~~minimum~~ maximum value of R_{bias} which will work with 2V_{p-p} ripple voltage, $V_C(t)$
   b. (10 Pts.) Find the approximate value of the capacitance of the capacitor C which will limit the ripple voltage to 2V_{p-p} if the value of R_{bias} were 15Ω. ~~and the maximum current is drawn from the power supply~~

Solution

a-) $$V_{c_{peak}} = \frac{220 \times \sqrt{2}}{20} - 2V_\gamma = 13.51V$$

$$V_{min} = V_{c_p} - V_{ripple} = 13.51 - 2 = 11.51V$$

$$V_{R_{bias_{min}}} = 11.51V - 7V = 4.51V$$

$$I_{R_{bias_{max}}} = I_o + I_{zmin} = 200 + 20 = 220mA$$

$$R_{bias_{max}} = \frac{V_{R_{bias\,min}}}{I_{R_{bias\,max}}} = \frac{4.51V}{220mA} = 20.5\Omega$$

b-)

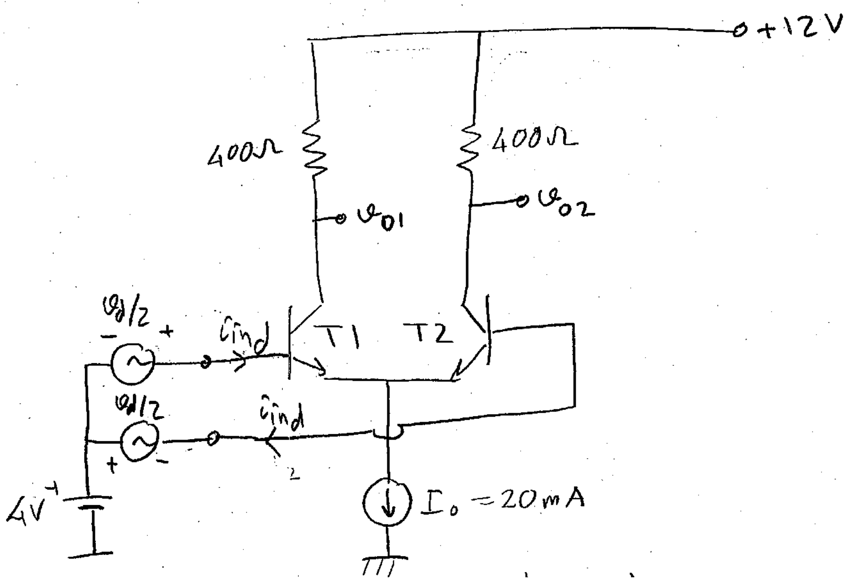
$$I_{av} = \frac{I_{max} + I_{min}}{2} = \left(\frac{13.51 - 7V}{15\Omega} + \frac{11.51 - 7}{15\Omega} \right) \frac{1}{2}$$

$$= \left[\frac{6.51}{15} + \frac{4.51}{15} \right] \frac{1}{2} = \frac{11.02}{2 \times 15} = 0.3673A$$

$$I \Delta t = C \Delta V \Rightarrow \quad 0.3673 \times 10^{-2} = C \times 2V$$

$$C = \frac{0.3673 \times 10^{-2}}{2} = 0.001836F$$

$$\boxed{C = 1836 \mu F}$$



4. **Question :** (20 Pts.) For the circuit given above β=100, V_{CESAT}=0.4Volts and V_{BESAT}=0.7Volts. answer the following:
   a. (6 Pts.) Find the states of the transistors and transistor currents assuming $v_d=0$,
   b. (7 Pts.) Find the small signal voltage gain defined as $(v_{o1}-v_{o2})/v_d$,
   c. (7 Pts.) Find the differential input resistance defined as $R_{ind}=v_d/i_{ind}$.

v_{o1}, v_{o2}, v_d are AC voltages,
v_d is the differential input voltage,
R_{ind} is the differential input resistance,
$i_{ind}$is the differential input current.

Solution:

a-) $I_{C_1} = I_{C_2} = \frac{20}{2} = 10mA$

$V_{O_2} = V_{O_1} = 12V - 10mA \times 400\Omega = 12 - 4 = 8V$

$V_{B_1} = V_{B_2} = 4V \rightarrow V_{CB} = 8-4 = 4V \Rightarrow$ F.A.

b-) $v_{o_1} = -\frac{v_d}{2} g_m 400 \qquad g_m = \frac{10mA}{26.6mV} = 0.376$ mhos

$v_{o_2} = -\left(-\frac{v_d}{2} g_m 400\right)$

$v_{o_1} - v_{o_2} = -\frac{v_d}{2} g_m 400 - \frac{v_d}{2} g_m 400 = -v_d\, g_m\, 400$

$\frac{v_{o_1} - v_{o_2}}{v_d} = -0.376 \times 400 = -150.4$

c-) $i_{ind} = \frac{v_d}{2} / r_\pi = \frac{v_d}{2r_\pi}$

$R_{ind} = \frac{v_d}{i_{ind}} = 2r_\pi$ $= 2 \times 100 / 0.376$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{g_m} = \frac{100}{0.375} = 266.6\ \Omega$

$R_{ind} = \frac{200}{0.376} = 531.9\ \Omega$