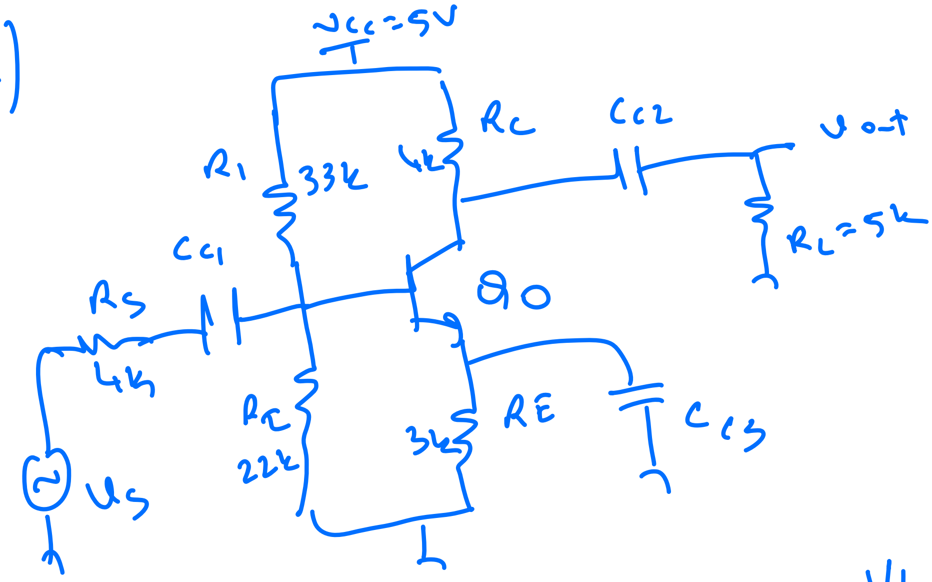
1)



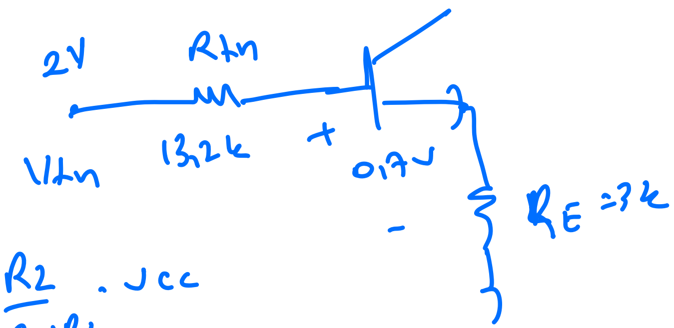
$V_{BE(on)} = 0{,}7V$

$V_{CE(SAT)} = 0{,}2V$

$\beta_0 = 100$

$r_o = 200k.$

a)



$V_{th} = \frac{R_2}{R_1+R_2} \cdot V_{CC}$

$V_{th} = \frac{2}{5} \cdot 5 = 2V$

$R_{th} = R_1 \| R_2 = 33 \| 22 = 13{,}2k$

$V_{th} = R_{th} \cdot I_B + 0{,}7V + (\beta+1) I_B R_E$

$2 = 13{,}2 I_B + 0{,}7 + 101 I_B\, 3$

$1{,}3 = 316{,}2 I_B$

$I_B = 4{,}1\ \mu A$

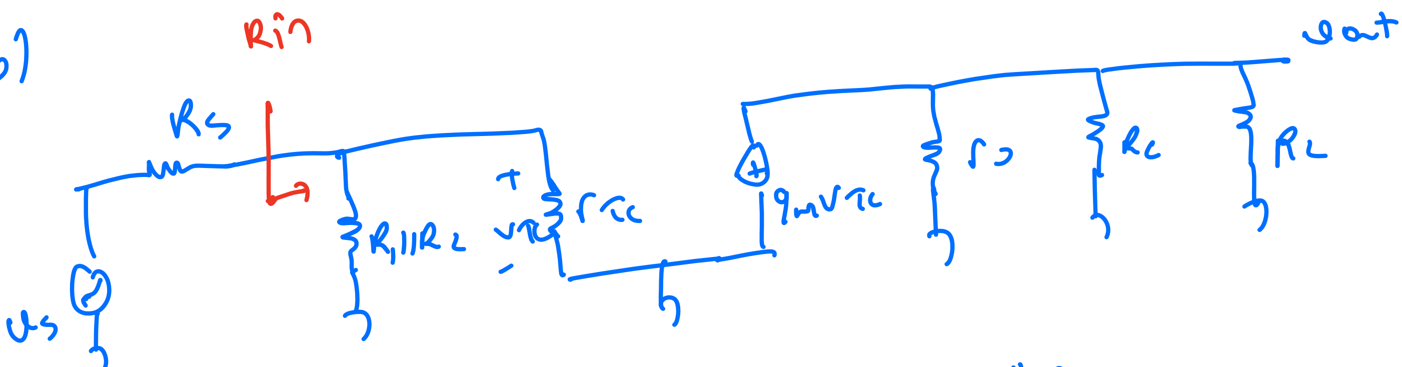
$I_C = 0{,}41 mA$

$I_E = 0{,}415 mA$

$V_{CE} = 5 - 4 \cdot 0{,}41 - 3 \cdot 0{,}415$

$\boxed{V_{CE} = 2{,}115V > 0{,}2V}$ f-A. ✓

b)



$R_{in} = R_1 \| R_2 \| r_\pi$

$R_{in} = 13{,}2k \| 6{,}1k = 4{,}17k\Omega$

$g_m = \frac{I_C}{V_T} = \frac{0{,}41}{0{,}025} = 16{,}4 mA/V$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{16{,}4} = 6{,}1 k\Omega$

$$v_\pi = \frac{R_{in}}{R_{in}+R_s} \cdot v_s \qquad \frac{v_{out}}{v_s} = A_v = -\frac{R_{in}}{R_{in}+R_s} \cdot g_m (r_o \| R_C \| R_L)$$

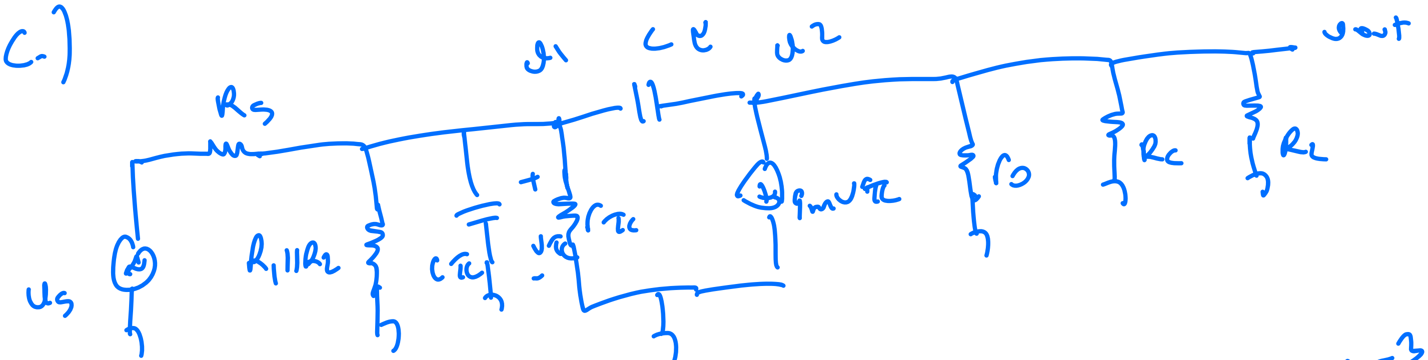
$$v_{out} = -g_m v_\pi (r_o \| R_C \| R_L)$$

$$A_v = -\frac{4.17}{4.17+4} \cdot (164) \cdot (200 \| 445)$$

$$\boxed{A_v = -18.4 \ \frac{V}{V}}$$

midband gain
(capacitors have no effect)

c.)



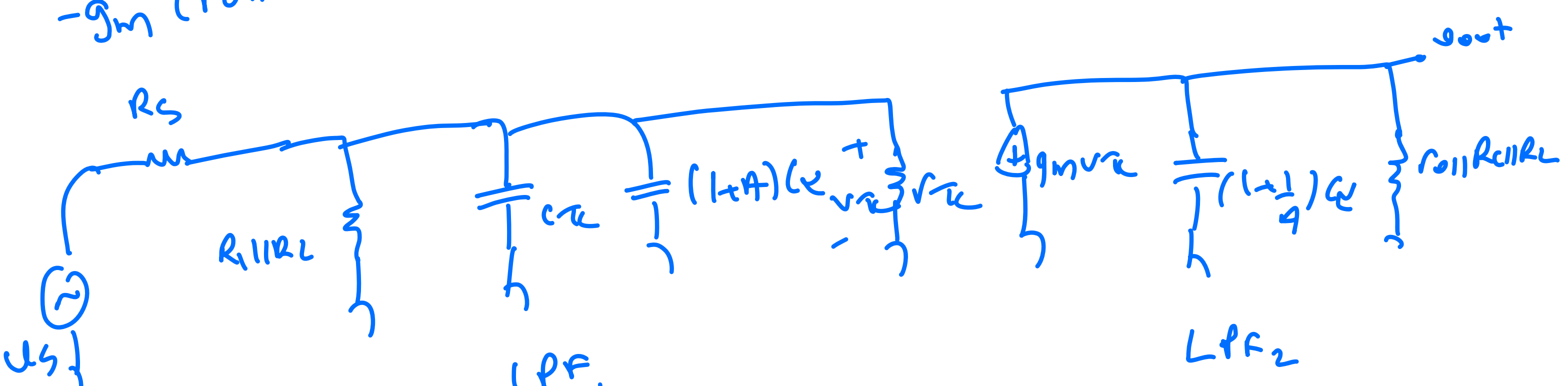
from lecture notes $f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)}$ $\Rightarrow$ $700 \cdot 10^6 = \frac{164 \cdot 10^{-3}}{2\pi (C_\pi + C_\mu)}$

$C_\pi + C_\mu = 3.73 \text{ pF}$

$C_\mu = 1 \text{pF}$ ← given

$C_\pi = 2.73 \text{ pF}$

gain from v_1 to v_2
$-g_m (r_o \| R_C \| R_L) = -36 \ \frac{V}{V} = -A$



LPF$_1$

LPF$_2$

$C_T = C_\pi + (1+A) C_\mu = 2.73 + 37.1 =$
$= 39.73 \text{ pF}$

$(1+\frac{1}{36}) C_\mu$
$\sim 1 \text{pF}$

There is a LPF @ the input side and → $\frac{1}{1+j\frac{f}{f_{c1}}}$

There " " " output side. ↪ $\frac{1}{1+j\frac{f}{f_{c2}}}$

$$f_{c1} = \frac{1}{2\pi\, C_T\,(R_S \parallel \underbrace{R_1 \parallel R_2 \parallel r_\pi}_{Rin})} = \frac{1}{2\pi \cdot 39{,}73 \cdot 10^{-12} \cdot (4 \parallel 4{,}17\ 8k}$$

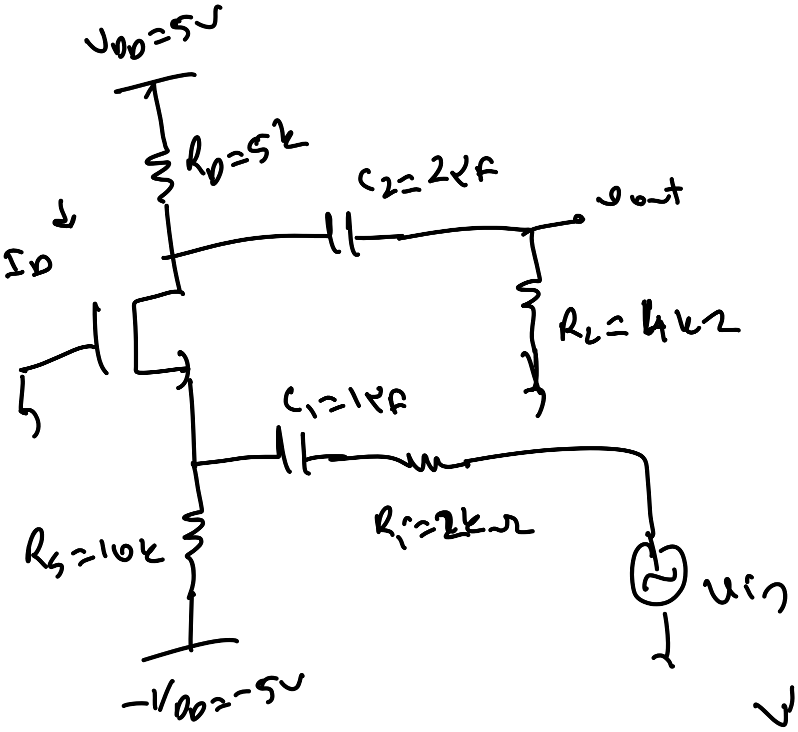
$$f_{c1} = \frac{1}{2\pi\, 39{,}73 \cdot 10^{-12} \cdot 2.04 \cdot 10^3} = 1{,}96\ \text{mHz}$$

$$f_{c2} = \frac{1}{2\pi\, C_N\,(r_o \parallel R_C \parallel R_L)} = \frac{1}{2\pi \cdot 1 \cdot 10^{-12} \cdot 2{,}2k} = 72{,}3\ \text{mHz}$$

$$\boxed{f_H = f_{c1} = 1{,}96\ \text{mHz}}$$

The input side dominates the response because of the miller effect.

2)



$V_{TN} = 1V$

$K_N = 2 \frac{mA}{V^2}$

$\lambda = 0$

$C_{gs} = 12pF$

$C_{gd} = 3pF$

a) $V_G = 0V$

$V_S = -5 + 10 I_D$

$V_D = 5 - 5 I_D$

$V_{GS} = 5 - 10 I_D$

$V_{GS} = 1{,}4V$

$V_{DS} = 10 - 15 I_D$

$V_{DS} = 4{,}6V$

$V_{DS} > V_{GS} - V_{TN}$ SAT ✓

$4{,}6 \quad 1{,}4 - 1$

$I_D = 2 . (\underbrace{0 - (-5 + 10 I_D)}^{V_{GS}} - 1)^2$

$I_D = 2 (4 - 10 I_D)^2$

$I_D = 2 . 2^2 . (2 - 5 I_D)^2$

$I_D = 8 . (4 - 20 I_D + 25 I_D^2)$

$0{,}125 I_D = 4 - 20 I_D + 25 I_D^2$

$I_D = 0{,}45 mA$ ✗ $V_{GS} < V_{th}$

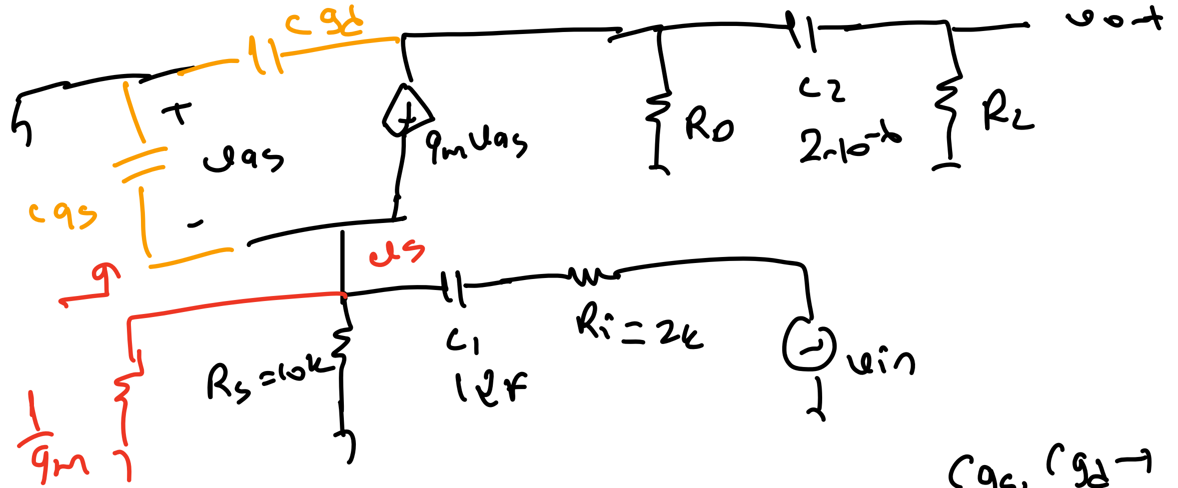
$\boxed{I_D = 0{,}36 mA}$

$g_m = 2 \sqrt{K_N I_D}$

$g_m = 2 \sqrt{2 . 0{,}36}$

$\boxed{g_m = 1{,}7 \frac{mA}{V}}$

b)



$C_{gs}, C_{gd} \to$ open ckt.

for mid band gain $C_1, C_2 \to$ short ckt.

$$v_s = \frac{\frac{1}{g_m} \parallel R_s}{\frac{1}{g_m} \parallel R_s + R_i} \cdot v_{in} =$$

$v_{gs} = v_g - v_s$

$v_{gs} = -v_s$

or you can write a KCL @ the source.

$$v_{out} = -g_m v_{gs} \cdot (R_O \parallel R_L)$$

$$A_v = \frac{v_{out}}{v_{in}} = g_m \cdot \frac{\frac{1}{g_m} \parallel R_s}{\frac{1}{g_m} \parallel R_s + R_i} \cdot (R_O \parallel R_L)$$

$$A_v = 1{,}7 \cdot \frac{0{,}59 \parallel 10}{0{,}59 \parallel 10 + 2} \cdot (5 \parallel 4) \Rightarrow \boxed{A_v = 0{,}82 \ \frac{V}{V}}$$

c) f_L is determined by C_1 and C_2.

$$f_{LC_1} = \frac{1}{2\pi C_1 \underbrace{\left(\frac{1}{g_m} \parallel R_s + R_i\right)}_{R \text{ seen by } C_1 \text{ (kill } v_{in})}} = \frac{1}{2\pi \cdot 10^{-6} \cdot (0{,}59 \parallel 10 + 2)k}$$

$$f_{LC_1} = 62 \text{ Hz}$$

the form is for low freq.

$$A_0 \cdot \frac{j\frac{f}{f_{LC_1}}}{1 + j\frac{f}{f_{LC_1}}} \cdot \frac{j\frac{f}{f_{LC_2}}}{1 + j\frac{f}{f_{LC_2}}}$$

$$f_{Lc_2} = \frac{1}{2\pi\, C_2 (R_D + R_L)} = \frac{1}{2\pi \cdot 2\cdot 10^{-6} \cdot (9k)} = 8{,}84\ Hz$$

$(R_D + R_L) \to$ R seen by C_2

$$\boxed{f_L = 62\ Hz}$$

d) f_H is determined by C_{gs} and C_{gd}

R seen by C_{gs} $\left(\frac{1}{g_m} \parallel R_S \parallel R_i\right)$, C_{gs} is from Source to gnd. gnd vin to find R.

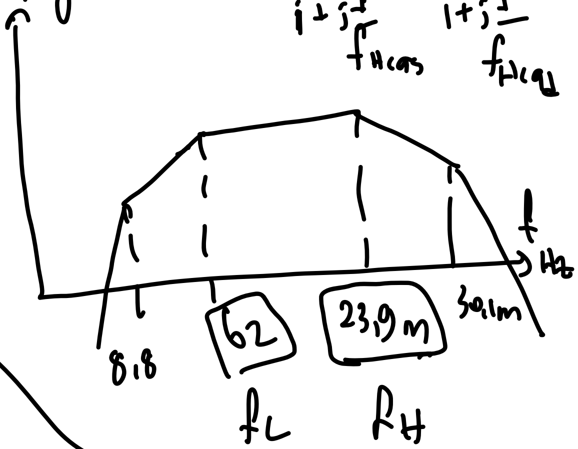
$$f_{Hc_{gs}} = \frac{1}{2\pi\, C_{gs} \left(\frac{1}{g_m} \parallel R_S \parallel R_i\right)} = \frac{1}{2\pi \cdot 12\cdot 10^{-12} \cdot (\underbrace{0{,}59 \parallel 10 \parallel 2}_{0{,}44k})k}$$

$= 30{,}1\ mHz$

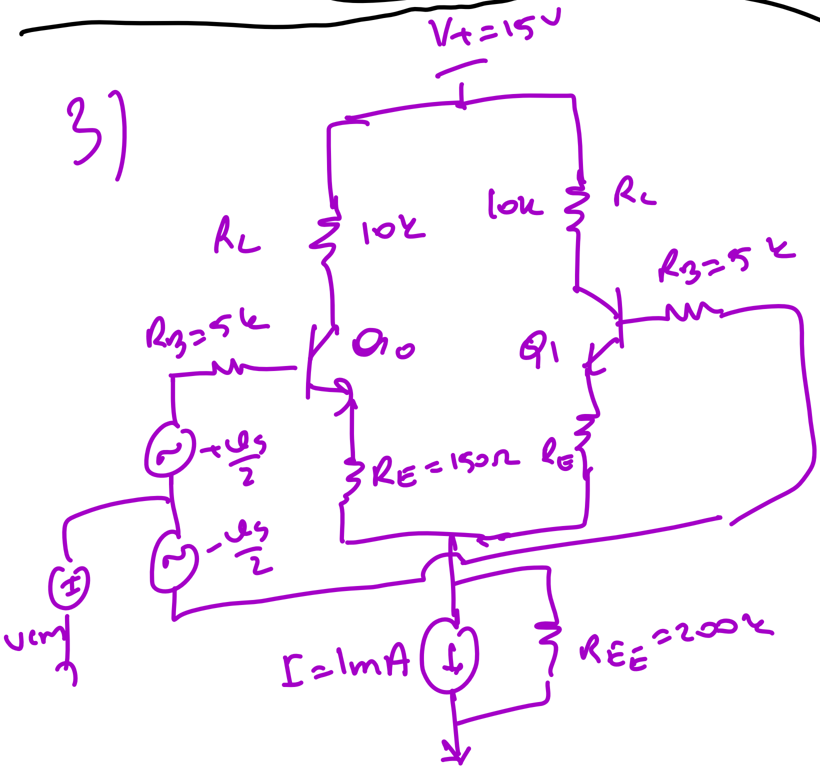
$$f_{Hc_{gd}} = \frac{1}{2\pi\, C_{gd} (R_D \parallel R_L)} = \frac{1}{2\pi\, 3\cdot 10^{-12} \cdot (4 \parallel 5)k} = 23{,}9\ mHz$$

$$\boxed{f_H = 23{,}9\ mHz}$$

for high freq. $\frac{1}{1+j\frac{f}{f_{Hc_{gs}}}} \cdot \frac{1}{1+j\frac{f}{f_{Hc_{gd}}}}$



3)



$\beta = 100$

$V_A = \infty$

$V_{BE(on)} = 0{,}7V$

a) input differential Resistance, R_{id}.

Let's find DC operating point 1st.

$I_{E_2} = I_{E_1} = \frac{1mA}{2} = 0{,}5mA$ $\quad$ β large enough $\quad I_C = I_E$

$I_B = \frac{0{,}5}{100} = 5\mu A$ $\quad$ $V_B = -5 \cdot 5\mu A = 0{,}025V \sim 0$

$V_C = 15 - 10 \cdot 0{,}5 = 10V$

$V_E = -0{,}7V$

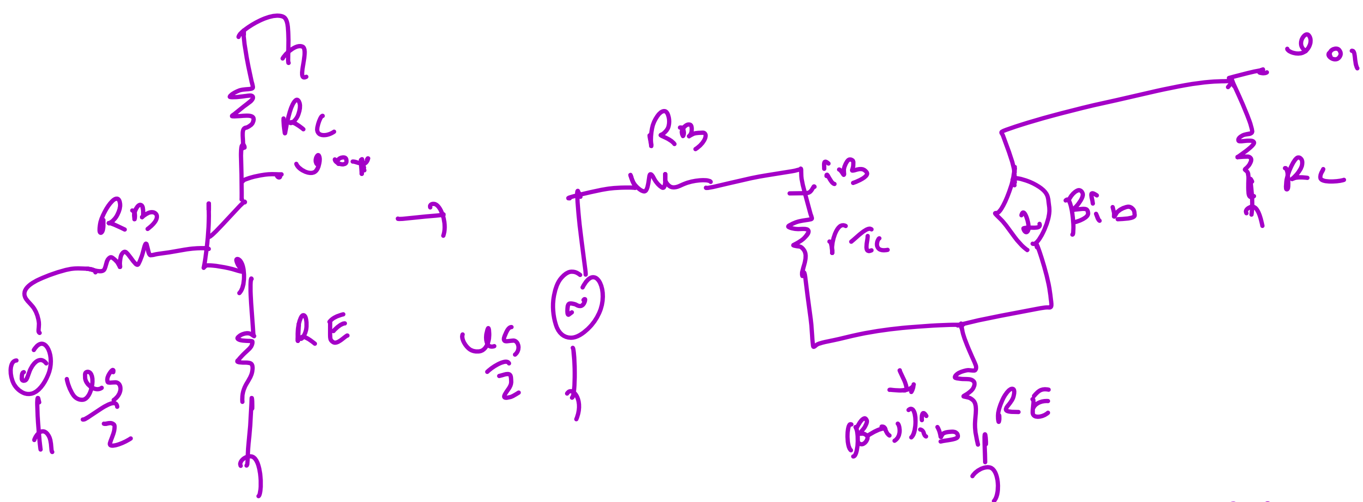
$V_{CE} = 10{,}7V > 0{,}2V$ F.A. ✓

$g_m = \frac{I_C}{V_T} = \frac{0{,}5}{0{,}026} = 19{,}2mA/V$ $\quad$ $r_\pi = \frac{\beta}{g_m} = 5{,}2k\Omega$

$R_{id} = (R_B + r_\pi + (\beta+1) R_E) \times 2$

$R_{id} = (5 + 5{,}2 + 101 \cdot 0{,}5) \times 2 \Rightarrow \boxed{R_{id} = 50{,}7 k\Omega}$

b) Differential half ckt. $\quad$ S.S. model.



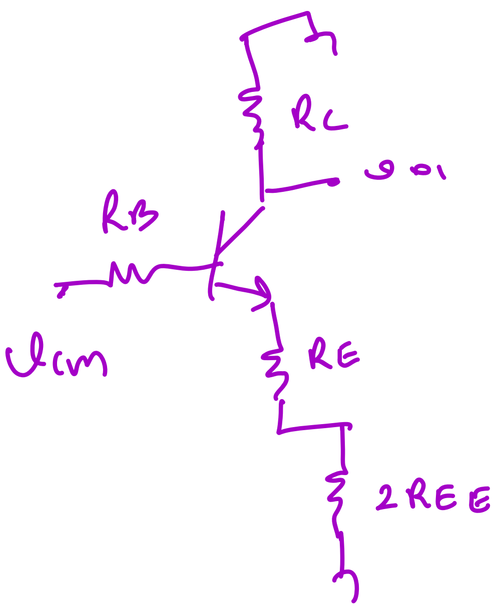
$$\left.\begin{aligned} v_{o1} &= -\beta i_b R_C \\ \frac{v_s}{2} &= (R_B + r_\pi) i_b + (\beta+1) i_b R_E \end{aligned}\right\} \Rightarrow \frac{v_{o1}}{\frac{v_s}{2}} = \frac{-\beta R_C}{(R_B + r_\pi) + (\beta+1) R_E}$$

$$\frac{v_{o1}}{v_s} = \frac{-\beta R_C}{2.\left[R_B + r_\pi + (\beta+1)R_E\right]}$$

$$\frac{v_{o2}-v_{o1}}{v_s} = A_{dm} = \frac{\beta R_C}{R_B + r_\pi + (\beta+1)R_E} = \frac{100.10}{5+5,2+101.0,15}$$

$$\boxed{A_{dm} = 39,5 \ \frac{V}{V}}$$

c) the common mode gain.



the s.s. model is the same as (b)

with $R_E \to R_E + 2R_{EE}$

$$\frac{v_{o1}}{v_{cm}} = \frac{-\beta R_C}{(R_B + r_\pi) + (\beta+1).(R_E + 2R_{EE})}$$

$$\frac{v_{o2}}{v_{cm}} = \frac{-\beta R_C}{(R_B + r_\pi) + (\beta+1)(R_E + 2R_{EE})}$$

Ideally if everything matches $A_{cm} = \frac{v_{o2}-v_{o1}}{v_{cm}} = 0$

v_{cm} has the same gain. in both branches.

$R_C \to$ +/- 1%   worst case $\to 1,01 R_C$, $\to 0,99 R_C$

$$\frac{v_{o2}-v_{o1}}{v_{cm}} = \frac{\beta . 0,02 . R_C}{(R_B + r_\pi) + (\beta+1)(R_E + 2R_{EE})}$$

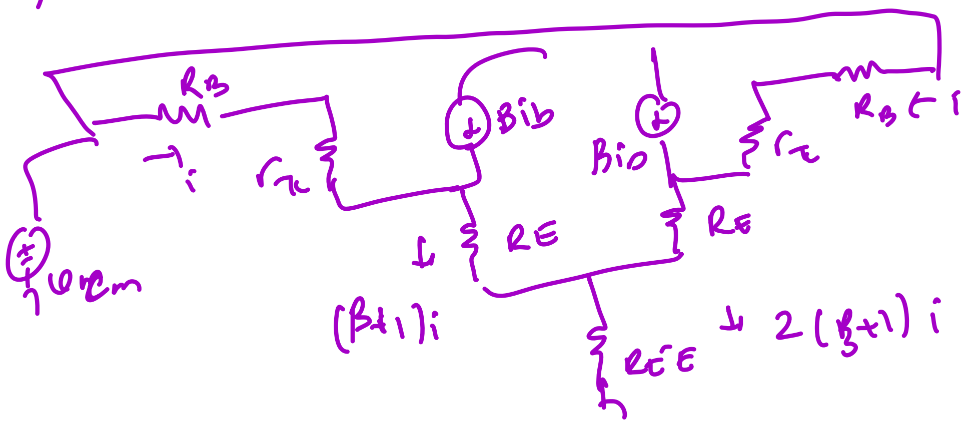
$\leftarrow$ absolute value.

$$A_{cm} = \frac{100 \cdot 0{,}02 \cdot 10}{(5+5{,}2) + 101 \cdot (0{,}15 + 400)} = \boxed{4{,}9 \cdot 10^{-4}}$$ worst case.

d) $$CMRR = \frac{A_{dm}}{A_{cm}} = \frac{39{,}5}{4{,}9 \cdot 10^{-4}} = 79840$$

$$CMRR = 20 \log (79840) = \boxed{98 dB}$$

e-) Common mode input resistance R_{ic}



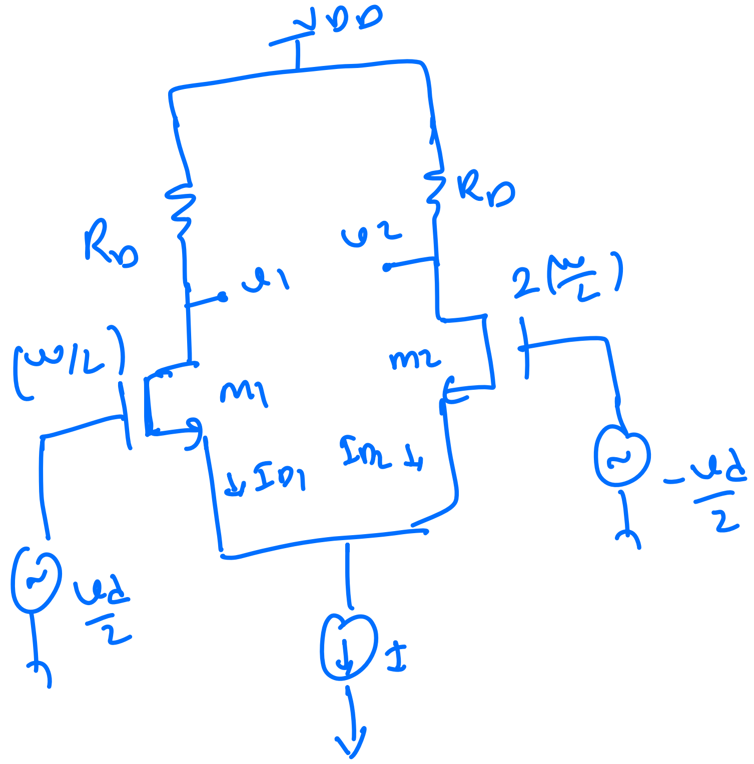
$$v_{cm} = i(R_B + r_\pi) + (\beta+1) i R_E + 2(\beta+1) i R_{EE}$$

$$R_{icm} = \frac{v_{cm}}{2i} = \frac{1}{2}\left[R_B + r_\pi + (\beta+1) R_E\right] + (\beta+1) R_{EE}$$ dominates

$$R_{icm} = \frac{1}{2}\left[5 + 5{,}2 + 101 \cdot 0{,}15\right] + 101 \cdot 200k.$$

$$\boxed{R_{icm} = 20{,}2 \ M\Omega}$$

4)



The DC bias on the gate of m1 and m2 are same.

M1-M2 have same V_{GS}

then $I_{D2} = 2I_{D1}$

$I_{D1} = \frac{I}{3}$

$K_N = \frac{K_N'}{2}\left(\frac{W}{L}\right)$

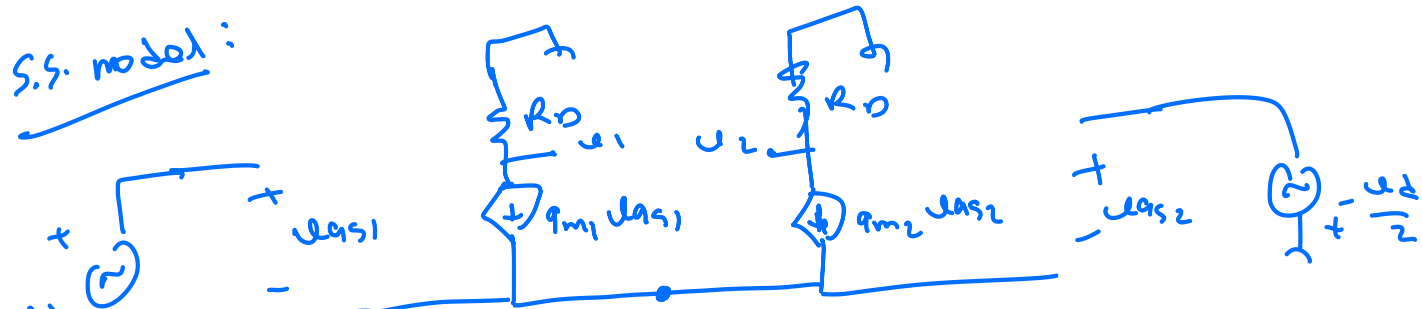
$I_{D2} = \frac{2I}{3}$

$g_{m1} = 2\sqrt{K_{N1} I_{D1}}$ $\quad g_{m2} = 2\sqrt{K_{N2} I_{D2}} = 2g_{m1}$

$K_{N2} = 2K_{N1}$

$I_{D2} = 2I_{D1}$

S.S. model:



this point is not virtual gnd any more.

$g_{m1} v_{gs1} + g_{m2} v_{gs2} = 0$

$g_{m1} v_{gs1} + 2g_{m1} v_{gs2} = 0$

$v_{gs2} = -\frac{1}{2} v_{gs1}$

$-\frac{v_d}{2} + v_{gs1} - v_{gs2} - \frac{v_d}{2} = 0$

$v_{gs1} - v_{gs2} = v_d$

$v_{gs1} + \frac{1}{2} v_{gs1} = v_d$

$v_{gs1} = \frac{2}{3} v_d$

$v_{gs2} = -\frac{1}{3} v_d$

$v_1 = -g_{m1} v_{gs1} R_D$

$v_2 = -g_{m2} v_{gs2} R_D$

for M1-M2 matched operation.

$v_{gs1} = \frac{v_d}{2}$

$v_{gs2} = -\frac{v_d}{2}$

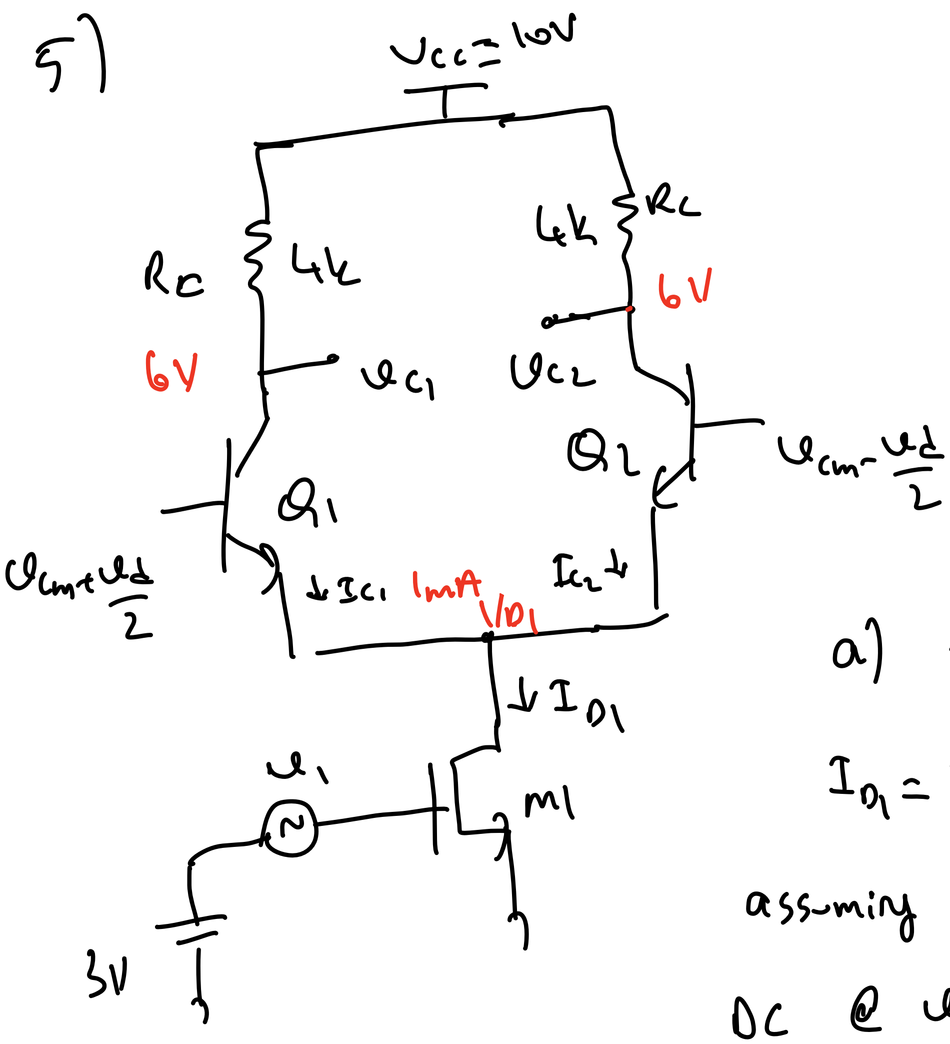
$$v_2 - v_1 = R_D(g_{m_1} v_{gs_1} - g_{m_2} v_{gs_2})$$

$$v_2 - v_1 = R_D\left(g_{m_1} \frac{2}{3} v_d + 2 g_{m_1} \frac{v_d}{3}\right)$$

$$v_2 - v_1 = \frac{4}{3} g_{m_1} R_D v_d$$

$$\boxed{A_{dm} = \frac{v_2 - v_1}{v_d} = \frac{4}{3} g_{m_1} R_D}$$

5)



$\beta = 100$

$K_N = 0.5 \frac{mA}{V^2}$

$V_{TN} = 1V$

a) the range of V_{cm}

$I_{D_1} = 0.5(3-1)^2 = 2mA$

assuming large β $I_{C_1} = I_{C_2} = 1mA$

DC @ v_{C_1} and v_{C_2} = 6V

$$V_{D_1} = V_E = V_{cm} - 0.7$$

for M1 to stay in sat. $V_{DS} > V_{GS} - V_{TN}$

$V_{DS} > 2V$

$V_{cm} - 0.7 > 2V \Rightarrow \boxed{V_{cm} > 2.7V}$

for Q1 and Q2 to stay in f.A.

$V_{CE} > 0.2V$, $V_{CE(SAT)}$ $\boxed{2.7V < V_{cm} < 6.5V}$

$V_C - V_E > 0.2V$

$6 - (V_{cm} - 0.7) > 0.2V$, $\boxed{V_{cm} < 6.5V}$

this is equ. to B-C diode off. condition.

b) $v_1 = 0.1\cos(w_1 t)$
$v_d = 0.001\cos(w_2 t)$ $w_2 \gg w_1$

in this case this ckt is an Analog multiplier.

$i_{D_1} = I_{DC} + i_{AC}$

$0.1 \ll 3V$ so we can treat 0.1 as small signal.

$i_{AC} = g_{m_1} \cdot v_1$ $g_{m_1} = 2\sqrt{k_N I_{D1}} = 2\sqrt{0.5 \cdot 2} = 2\frac{mA}{V}$

$i_{AC} = 2 \cdot 0.1\cos(w_1 t) = 0.2\cos w_1 t$ mA

ignore this.

OR. $i_{D_1} = 0.5(3 + 0.1\cos w_1 t - 1)^2 = 0.5[2^2 + 4 \cdot 0.1\cos w_1 t + 0.01\cos^2 w_1 t]$

$= 2 + 0.2\cos w_1 t$ mA

$\boxed{i_{D_1} = 2 + 0.2\cos w_1 t}$ mA

i_{D_1} splits into Q_1 and Q_2 equally.

$$i_{C1} = i_{C2} = 1 + 0{,}1\cos w_1 t \quad mA$$

$$g_{m1} = g_{m2} = \frac{i_C}{V_T} = \frac{1+0{,}1\cos w_1 t}{0{,}026} = [38{,}5 + 3{,}85\cos w_1 t] \ \frac{mA}{V}$$

$$v_{out} = v_{c2} - v_{c1} = g_m R_C v_d. \quad \leftarrow \text{differential operation.}$$

$$v_{out} = [38{,}5 + 3{,}85\cos w_1 t] \cdot 4 \cdot 0{,}001 \cdot \cos(w_2 t)$$

$$\boxed{v_{out} = 0{,}154\cos w_2 t + 0{,}015\cos(w_1 t)\cdot\cos(w_2 t)}$$

@ w_2 freq

@ $w_1 + w_2$ freq. $w_2 - w_1$

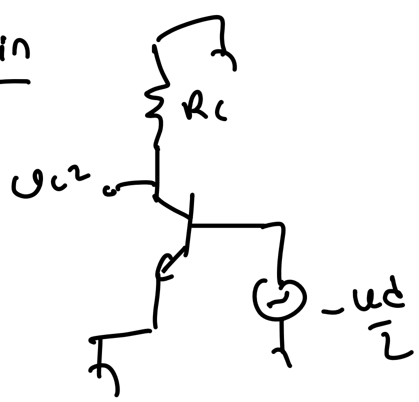
signals are mixed. or multiplied.

* the component @ w_2 freq. is large in this case.

This is a single balanced multiplier, a double balanced multiplier cancels the w_2 component.

c) $v_1 = 0$ and $v_{out} = v_{c2}$, $\lambda = 0{,}02$ $\quad r_o = \frac{1}{\lambda I_D} = \frac{1}{0{,}02 \cdot 2} = 25k\Omega$

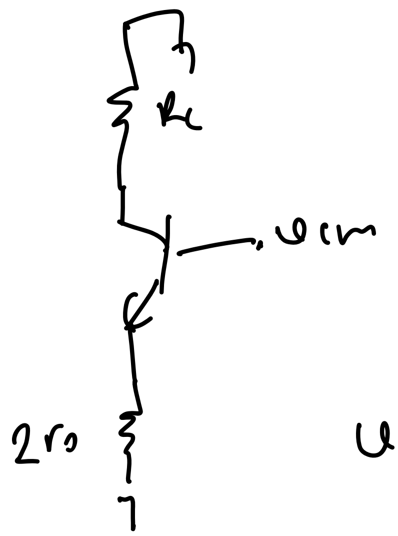
diff. gain

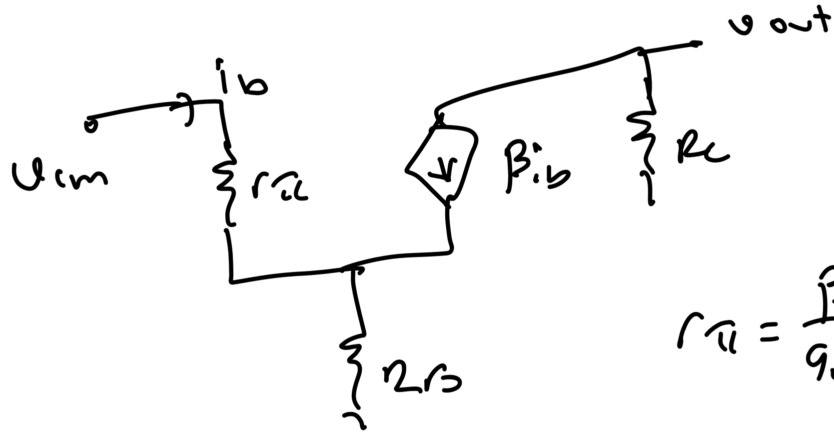


$$v_{c2} = -g_m R_C \cdot -\frac{v_d}{2}.$$

$$A_{dm} = \frac{v_{c2}}{v_d} = \frac{g_m R_C}{2} = \frac{38{,}5 \cdot 4}{2} = 77$$

Common mode gain





$r_\pi = \frac{\beta}{g_m} = 2{,}6k$

$v_{cm} = i_b r_\pi + (\beta+1)\, i_b 2r_o$

$v_{out} = -\beta i_b R_C$

$$A_{cm} = \frac{v_{out}}{v_{cm}} = \frac{-\beta R_C}{r_\pi + (\beta+1)2r_o} = \frac{-100.4}{2{,}6 + 101.50}$$

$A_{cm} = -0{,}08$

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \frac{77}{0{,}08} = 972 \Rightarrow CMRR = 20\log_{10}(972)$$

$\boxed{CMRR = 59{,}8\,dB}$