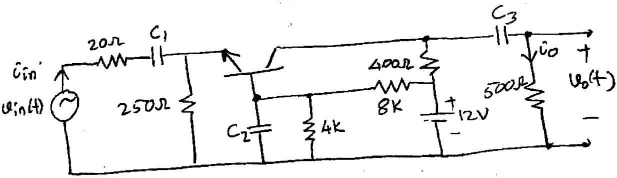
Question 2 (20 points)



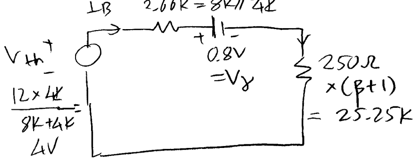
At the circuit given above, all the capacitors are very large, β=100, Vγ=0.8 Volts and n=1. Temperature of the transistor junction is 300 °K. Please answer the following:

a) (6 points) Find the state and the bias of the transistor (Voltages and currents of the transistor),

b) (7 points) Find the current gain of the amplifier measured as i_o/i_{in} as shown at the figure assuming that the transistor is at forward active state.

c) (7 points) Find the voltage gain of the transistor in dB measured as v_o/vin again assuming that the transistor is at forward active state.

a-) Alternatively for (a)



$$I_B = \frac{4 - 0.8}{27.91666k} = 114.63 \mu A$$

$$I_C = \beta I_B = 11.46 mA$$

$$I_E = (\beta + 1) I_B = 11.58 mA$$

$$V_{CE} = 12 - 0.65 \times 11.46 = 4.475V$$

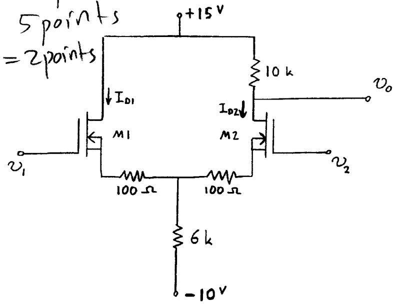
Question 3 (20 points)

Consider the differential amplifier shown in the figure. The two n-channel depletion-mode MOSFETs are identical with $K_n = 2mA/V^2$, $V_{TH} = -2$ Volts and $\lambda=0$.

a) (10 points) Determine the Q-point and find V_{GS1Q}, V_{GS2Q}, I_{D1Q}, I_{D2Q}, V_{DS1Q} and V_{DS2Q} at the common-mode input equal to 0 Volts.

b) (10 points) Determine the differential gain A_d (in dB) and the common-mode rejection ratio (CMMR, in dB)

$A_{cm} \rightarrow$ 3 points
$A_d \Rightarrow$ 5 points
CMMR = 2 points



(a)	$V_{GS1Q} =$	$V_{GS2Q} =$	$I_{D1Q} =$	$I_{D2Q} =$
	$V_{DS1Q} =$	$V_{DS2Q} =$		
(b)	$A_d(dB) =$	$CMRR(dB) =$		

Q-3 - D.C. solution

$$V_{GS} + 100 I_D + 6000 \times 2 I_D = 10V$$

$$V_{GS} + 12100 I_D = 10V$$

$$V_{GS} + 12100 K_n [V_{GS} + 2]^2 = 10$$

$$V_{GS} + 12100 \times 2\times10^{-3} [V_{GS}^2 + 4V_{GS} + 4] = 10$$

$$V_{GS} + 24.2 [V_{GS}^2 + 4V_{GS} + 4] = 10$$

$$V_{GS} + 24.2 V_{GS}^2 + 96.8 V_{GS} + 96.8 - 10 = 0$$

$$\underset{a}{24.2} V_{GS}^2 + \underset{b}{97.8} V_{GS} + \underset{c}{86.8} = 0$$

$$V_{GS_{1,2}} = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = -2.02066 \pm 0.70448$$

$$= -1.31618V - 2.72514V$$ beyond cut-off therefore discard

$$\cong -1.3162$$

but if you neglect $R = 100\Omega$ in the DC analysis

$$V_{GS} + 12000 I_D = 10 \Rightarrow V_{GS} + 24 [V_{GS} + 2]^2 = 10$$

$$V_{GS} + 24 [V_{GS}^2 + 4V_{GS} + 4] - 10 = 0$$

$$\underset{a}{24} V_{GS}^2 + \underset{b}{97} V_{GS} + \underset{c}{86} = 0$$

$$V_{GS_{1,2}} = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = -2.02083 \pm 0.70741 \Rightarrow$$

$$V_{GS_{1,2}} = -1.3, \cancel{-2.7} \Rightarrow V_{GS} = 1.3$$

$$\Rightarrow g_m = 2K_n (V_{GS}+2) = 0.02828 = 0.0283 \text{ siemens}$$

$$I_D = 2\times10^{-3} (-1.3+2)^2 = 1mA$$ or more precisely if needed

$$I_D = 2\times10^{-3} (-1.3162+2)^2 = 0.9352 \text{ mA}$$

$$\Rightarrow g_m = 0.002735 \text{ siemens}$$

Q_3

A.C. analysis (formal)

$$V_1 = V_{cm} + \frac{V_d}{2} \qquad V_2 = V_{cm} - \frac{V_d}{2}$$

$$V_d = V_1 - V_2 \qquad V_{cm} = \frac{V_1 + V_2}{2}$$

$$g_m v_{gs1} + g_m v_{gs2} = V_x / R_o \Rightarrow g_m (v_{gs1} + v_{gs2}) = \frac{V_x}{R_o}$$

$$v_{gs1} + v_{gs2} = \frac{V_x}{g_m R_o}$$

$$g_m R (v_{gs1} + v_{gs2}) = \frac{V_x R}{R_o}$$

$$V_1 - (1 + g_m R) v_{gs1} - V_x = 0$$

1 $$V_2 - (1 + g_m R) v_{gs2} - V_x = 0$$

$$V_1 + V_2 - (1 + g_m R)(v_{gs1} + v_{gs2}) - 2V_x = 0$$

$$V_1 + V_2 - (v_{gs1} + v_{gs2}) - g_m R (v_{gs1} + v_{gs2}) - 2V_x = 0$$

$$V_1 + V_2 - \frac{V_x}{g_m R_o} - \frac{V_x R}{R_o} = 2V_x$$

$$V_1 + V_2 = V_x \left[2 + \frac{1}{g_m R_o} + \frac{R}{R_o} \right] \Rightarrow$$

$$V_x = \frac{V_1 + V_2}{2 + \frac{1}{g_m R_o} + \frac{R}{R_o}}$$

$$V_2 - V_x = V_{cm} - \frac{V_d}{2} - V_x = V_{cm} - \frac{V_d}{2} - \frac{V_1 + V_2}{2} \cdot \frac{2}{2 + \frac{1}{g_m R_o} + \frac{R}{R_o}}$$

$$V_2 - V_x = V_{cm} \left[1 - \frac{2}{2 + \frac{1}{g_m R_o} + \frac{R}{R_o}} \right] - \frac{V_d}{2}$$

$$V_2 - V_x = V_{cm}\left[\frac{\cancel{2} + \frac{1}{g_m R_0} + \frac{R}{R_0} - \cancel{2}}{2 + \frac{1}{g_m R_0} + \frac{R}{R_0}}\right] - \frac{V_d}{2}$$

$$V_2 - V_x = V_{cm}\,\frac{1}{R_0}\cdot\frac{\frac{1}{g_m} + R}{2 + \frac{1}{g_m R_0} + \frac{R}{R_0}} - \frac{V_d}{2}$$

1 $$v_{gs2} = \frac{V_2 - V_x}{1 + g_m R} = \frac{1}{(1+g_m R)}\left[\frac{V_{cm}}{R_0}\,\frac{\frac{1}{g_m} + R}{2 + \frac{1}{g_m R_0} + \frac{R}{R_0}} - \frac{V_d}{2}\right]$$

$$v_{gs2} = V_{cm}\cdot\frac{\frac{1}{g_m}\left[\cancel{1 + g_m R}\right]}{\left[\cancel{1 + g_m R}\right] R_0}\cdot\frac{1}{2 + \frac{1}{g_m R_0} + \frac{R}{R_0}} - v_d\,\frac{1}{2(1+g_m R)}$$

$$v_{gs2} = V_{cm}\,\frac{1}{g_m R_0}\,\frac{1}{2 + \frac{1}{g_m R_0} + \frac{R}{R_0}} - v_d\,\frac{1}{2(1+g_m R)}$$

$$v_{gs2} = V_{cm}\,\frac{1}{2 g_m R_0 + 1 + g_m R} - v_d\,\frac{1}{2(1+g_m R)}$$

$$v_o = -g_m v_{gs2} R_D = v_d\,\overbrace{\frac{g_m R_D}{2(1+g_m R)}}^{A_d} - \overbrace{\frac{g_m R_D}{1 + 2 g_m R_0 + g_m R}}^{A_{cm}}\,v_{cm}$$

$$\Rightarrow \text{CMMR} = \frac{1 + 2 g_m R_0 + g_m R}{2(1 + g_m R)}$$

$$=$$

$$A_d = \frac{g_m R_D}{2(1+g_m R)} = \frac{0.002735 \times 10^4}{2(1+0.2735)} = 10.7389$$

$$= 20.62\,dB$$

$$|A_{cm}| = \frac{g_m R_D}{1+g_m(2R_o+R)} = \frac{2.735\times10^{-3} \times 10^4}{1+2.735\times10^{-3}[12000+100]} = \quad 27$$

$$= \frac{27.35}{34.093} = 0.8022 \quad -1.91\,dB$$

$$CMMR = \frac{1+2g_m R_o + g_m R}{2(1+g_m R)} = \frac{1+g_m(2R_o+R)}{2(1+g_m R)}$$

$$= \frac{27.35}{2.547} = 10.73 = 20.62\,dB$$

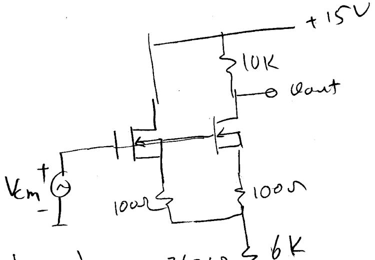
alternatnelly

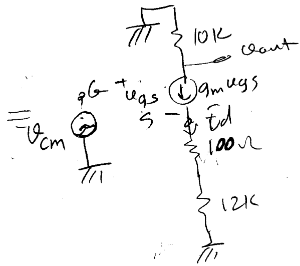
$$CMMR = \frac{|A_d|}{|A_c|} = 20.62-(-1.92) = 22.53\,dB$$

Q_3 ①/AS

Alternative solution =

$A_{common\ mode}$ ⇒ the equivalent circuit becomes





$$\frac{1}{g_m} = \frac{1}{2.735\times10^{-3}} = 365.6\Omega$$

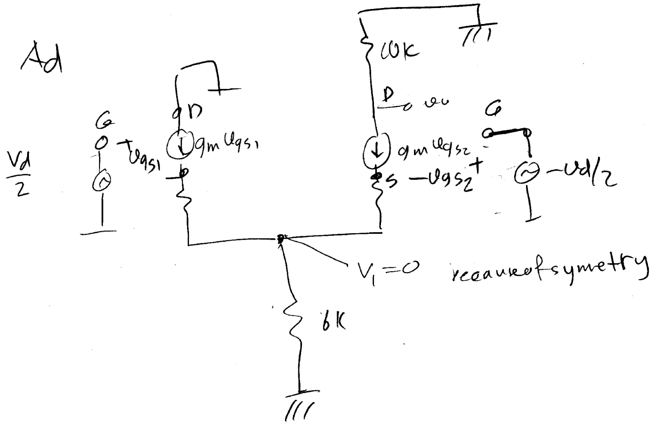
$$i_d = v_{cm}\frac{1}{\frac{1}{g_m} + 12.1K} \Rightarrow A_{cm} = \frac{v_{out}}{v_{cm}} = \frac{10000}{365.6 + 12100} \simeq \frac{10}{12.4656}$$

$A_{cm} = 0.8022 \Rightarrow -1.91\ dB$

$\frac{②}{AS}$

Ad

$\frac{V_d}{2}$



$v_o = -(-g_m v_{gs}) = g_m \, v_d/2$

$I_{d_2} = \frac{v_d}{2} \cdot \frac{1}{1/g_m + 100\Omega} = \frac{v_d}{2\,(365.6+100)} = \frac{v_d \, g_m}{(1+g_m 100)2}$

$\frac{v_o}{v_d} = \frac{g_m R_D}{(1+g_m R)2}$.

$\frac{v_o}{v_d} = \frac{10000}{2(465.6)} = 10.7388 = 20.619 \text{ dB}$

$CMMR = \frac{Ad}{Acm} = 20.619 - (-1.91) = 22.53 \text{ dB}$