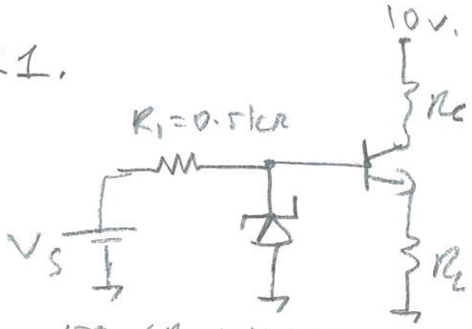
EE313 Solution of Final Exam 20-5-2021

Q1.



$50 < R_L < 100\,\Omega$

$9 < V_S < 12\,V.$

$3\,mA < I_Z < 11\,mA$

$V_{Z0} = 6\,v.$

wrong combination of 12v, 9v, 50Ω, 100Ω ⇒ 0 points

a) $I_{Z\,max} = \frac{12-6}{0.5} - \frac{1}{\beta+1}\,\frac{6-0.7}{0.1}$

$= 12 - \frac{53}{\beta+1} < 11 \quad (= \frac{66}{6} = 11\,mA)$

$\frac{53}{\beta+1} > 1 \quad \beta+1 < 53 \quad \beta < 52.$

$I_{Z\,min} = \frac{9-6}{0.5} - \frac{1}{\beta+1}\,\frac{6-0.7}{0.05}$

$= 6 - \frac{106}{\beta+1} > 0.1, \; 5.9 > \frac{106}{\beta+1} \qquad \beta+1 > 17.97$

$\beta > 17$

$\boxed{17 < \beta < 52}$ (8p)

b) we need to check for the worst case: $V_E = 6 - 0.7 = 5.3$

$V_{CE} = 10 - \frac{5.3}{R_L} \times \frac{\beta}{\beta+1} \times R_c - 5.3 > 0.2$

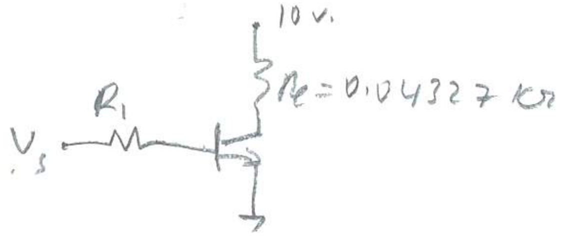
$4.5 > \frac{5.3}{R_L} \times \frac{\beta}{\beta+1} R_c \quad$ worst case $4.5 > \frac{5.3}{0.05} \times \frac{52}{53} \times R_c$

$\Rightarrow R_c < 4.5 \times \frac{53 \times 0.05}{5.3 \times 52} = 0.04327\,k\Omega = 43.27\,\Omega.$

(6p)

c) $R_L = 0$. If the TR is ON then $V_Z = 0.7$ and so Z is OFF.

$I_B = \frac{V_S - 0.7}{0.5} \Rightarrow \frac{9-0.7}{0.5} < I_B < \frac{12-0.7}{0.5}$



$\Rightarrow 16.6 < I_B < 22.6\,mA.$

Assume TR is SAT

$I_C = \frac{10-0.2}{R_c} = \frac{9.8}{0.04327} = 226.5\,mA.$

(6p)

$16.6 \times 17 = 282\,mA$ is wrong

In all cases $\beta I_B < I_C$ so SAT.

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b) approx. $V_{CE} = 10 - \frac{5.3}{R_L} \times R_c - 5.3 > 0.2$

$4.5 > \frac{5.3}{R_L} \times R_c \qquad R_c < \frac{4.5 \times R_L}{5.3} = \frac{4.5 \times 0.05}{5.3} = 0.0425$

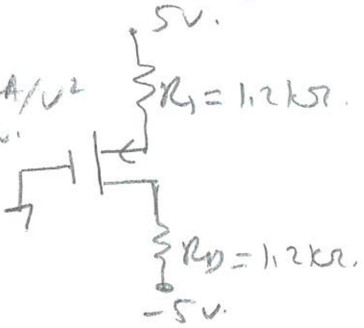
a) 6p
b) 8p
c) 8p
d) 8p

(VSG, VSD, ID, check) −2p for no check −2 for wr VSD, ID
(1 for gm)

Q2.

a)

$K_p = 1\,mA/V^2$
$V_{Tp} = -1.5V$
$\lambda = 0$



Assume SAT.

$I_D = 1(V_{SG} - 1.5)^2 = \frac{5 - V_{SG}}{1.2}$

$1.2(V_{SG}^2 - 3V_{SG} + 2.25) = 5 - V_{SG}$

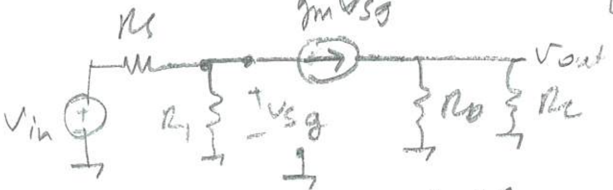
$1.2V_{SG}^2 - 2.6V_{SG} - 2.3 = 0$

$V_{SG} = \begin{cases} 2.84 > 1.5V ✓ \\ -0.67 \;\times \end{cases} \Rightarrow I_D = \frac{5 - 2.84}{1.2} = 1.8\,mA.$

$V_{SD} = 5 - 1.2 \times 1.8 - 1.2 \times 1.8 - (-5)$
$= 10 - 2.4 \times 1.8 = 5.68V > 2.84 - 1.5$ ✓

b) $g_m = 2\sqrt{1 \times 1.8} = 2.683\,mA/V$

$r_o = \infty$



$\frac{v_{sg}}{R_1} + \frac{v_{sg} - v_{in}}{R_S} + g_m v_{sg} = 0$

$v_{sg}\left(\frac{1}{R_1} + \frac{1}{R_S} + g_m\right) = \frac{v_{in}}{R_S}$

$v_{sg}\left(\frac{1}{R_1 // \frac{1}{g_m}} + \frac{1}{R_S}\right) = \frac{v_{in}}{R_S}$

$v_{sg}\left(\frac{R_S}{R_1 // \frac{1}{g_m}} + 1\right) = v_{in}$

$\frac{v_{sg}}{v_{in}} = \frac{1}{1 + \frac{R_S}{R_1 // \frac{1}{g_m}}} = \frac{R_1 // \frac{1}{g_m}}{R_S + R_1 // \frac{1}{g_m}}$

without $\frac{1}{g_m}$, 0p.

$R_D // R_L = 1.2 // 50 = 1.17\,k\Omega.$

$\frac{v_{out}}{v_{sg}} = g_m R_D // R_L$

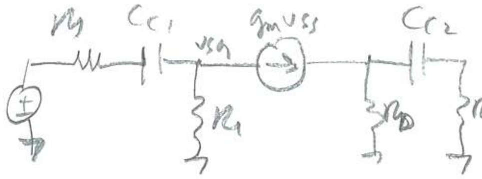
$\therefore A_{MB} = \frac{v_{out}}{v_{in}} = \frac{R_1 // \frac{1}{g_m}}{R_1 // \frac{1}{g_m} + R_S} \times g_m R_D // R_L$

$\frac{1}{g_m} = \frac{1}{2.683} = 0.373$

$\frac{1}{g_m} // R_1 = 0.373 // 1.2 = 0.285$

$A_{MB} = \frac{0.285}{0.285 + 0.2} \times 2.683 \times 1.17 = 0.5876 \times 3.139 = 1.845\,V/V$

c)



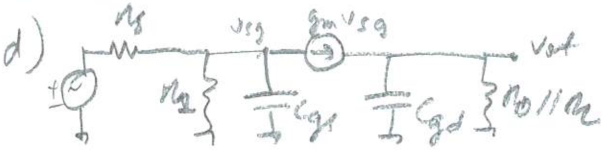
$f_1 = \frac{1}{2\pi \times C_{C1} \times (R_S + R_1 // \frac{1}{g_m})}$

$= \frac{1}{2\pi \times 4.7 \times 10^{-6} \times 0.485 \times 10^3} = 69.8\,Hz.$

$f_2 = \frac{1}{2\pi \times C_{C2} \times (R_D + R_L)} = \frac{1}{2\pi \times 10^{-6} \times 51.2 \times 10^3}$

$= 3.11\,Hz. \Rightarrow f_L = 69.8\,Hz.$

d)



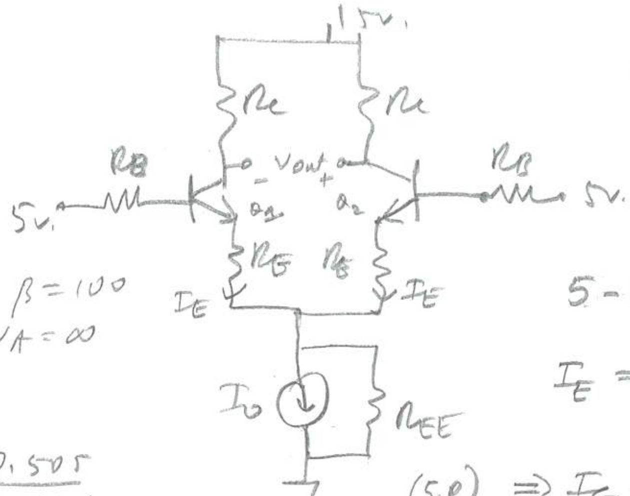
$f_1 = \frac{1}{2\pi \times C_{gs} \times (R_S // R_1 // \frac{1}{g_m})} = \frac{1}{2\pi \times 10 \times 10^{-12} \times \underbrace{(0.2 // 0.285)}_{0.117} \times 10^3} = 136\,MHz.$

$f_2 = \frac{1}{2\pi \times C_{gd} \times R_D // R_L}$

$= \frac{1}{2\pi \times 4 \times 10^{-12} \times 1.17 \times 10^3} = 34\,MHz$

$\Rightarrow f_H \cong 34\,MHz.$

Q3. a)



a) 8p
b) 7p
c) 7p
d) 4p
e) 4p

$\beta = 100$
$V_A = \infty$

Assume TRs are F.A.

$$5 = \frac{I_E}{\beta+1} \times R_B + 0.7 + I_E R_E + (2 I_E - I_0) R_{EE}$$

$$5 - 0.7 + I_0 R_{EE} = I_E \left(\frac{R_B}{\beta+1} + R_E + 2 R_{EE} \right)$$

$$I_E = \frac{5 - 0.7 + 200}{\frac{5}{101} + 0.15 + 400} = \frac{204.3}{400.2} = 0.51\,mA.$$

(5p) $\Rightarrow I_{C_1} = I_{C_2} = I_C = \frac{100}{101} \times 0.51 = 0.505\,mA$

$$V_{CE} = 15 - I_C \times R_C - (5 - I_B R_B - 0.7)$$
$$= 15 - 0.505 \times 10 - 5 + 0.7 + \frac{0.51}{101} \times 5$$
$$= 5.675\,V > 0.2 \checkmark \text{ (2p)}$$

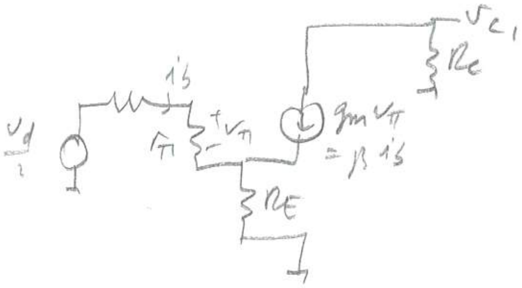
$$g_m = \frac{0.505}{0.026} = 19.42\,mA/V$$

1p g_m and/or r_π

$$r_o = \infty$$

$$r_\pi = \frac{100}{19.42} = 5.15\,k\Omega.$$

b) Differential half circuit



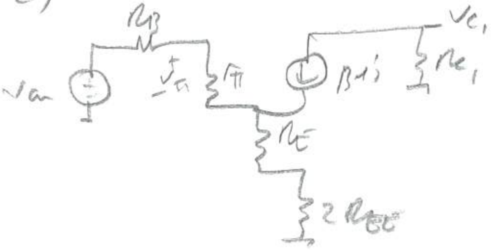
$$\frac{v_{c_1}}{\frac{v_d}{2}} = -\frac{\beta R_C}{R_B + r_\pi + (\beta+1) R_E} = -\frac{100 \times 10}{5 + 5.15 + 101 \times 0.15}$$

$$= -\frac{1000}{25.3} = -39.5$$

$$\frac{v_{c_1}}{v_d} = \frac{-39.5}{2} = -19.75$$

$$\frac{v_{c_2}}{v_d} = +19.75 \qquad A_{dm} = \frac{v_{c_2} - v_{c_1}}{v_d} = 39.5.$$

c)



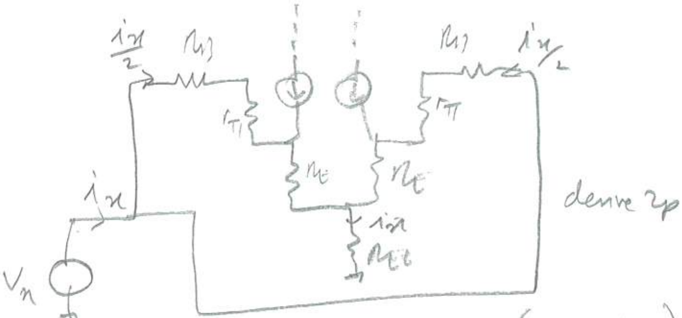
$$\frac{v_{c_1}}{v_{cm}} = \frac{-\beta \times R_{c_1}}{R_B + r_\pi + (\beta+1)(R_E + 2R_{EE})}$$

$$= \frac{-100 \times R_{c_1}}{5 + 5.15 + 101 \times 400.15} = \frac{-100 \times R_{c_1}}{40425}$$

$$\frac{v_{c_2}}{v_{cm}} = -\frac{100 \times R_{c_2}}{40425}$$

$$A_{cm\,worst} = \pm \frac{100 \times (R_{c_2} - R_{c_1})}{40425} = \frac{\pm 100 \times 0.02 \times 10}{40425}$$
$$= \pm 4.95 \times 10^{-4}$$

d)

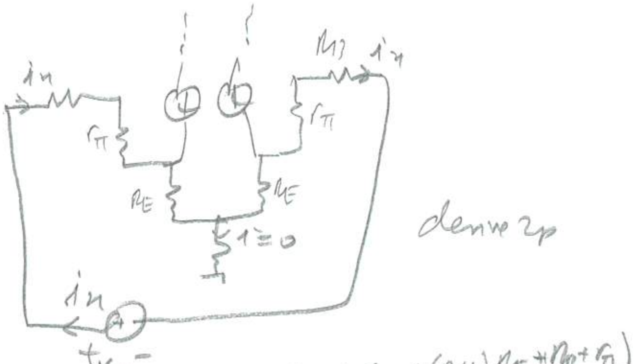


derive 2p

$$v_x = \frac{i_x}{2}(R_B + r_\pi) + \frac{i_x}{2}(\beta+1)(R_E + 2R_{EE})$$

$$R_{icm} = \frac{v_x}{i_x} = \frac{1}{2}\left[R_B + r_\pi + (\beta+1)(R_E + 2R_{EE})\right]$$
$$= \frac{1}{2} \times 40425 = 20.2\,M\Omega.$$

e)



derive 2p

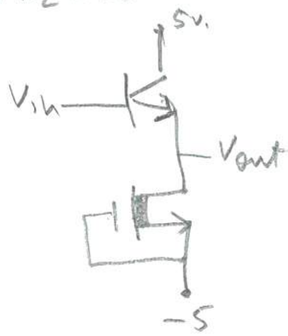
$$v_x = i_x\left(R_B + r_\pi + (\beta+1)R_E + (\beta+1)R_E + R_B + r_\pi\right)$$

$$R_{id} = \frac{v_x}{i_x} = 2\left(R_B + r_\pi + (\beta+1)R_E\right)$$
$$= 2 \times 25.3 = 50.6\,k\Omega.$$

Q4.

a) $R_L = \infty$

(6p)



$V_{GS} = 0 > -1.8 \Rightarrow$ nmos is not OFF. $I_D = 0.2(0+1.8)^2 = 0.648\,mA$.

Assume both active

$V_{DS} = V_{in} - 0.7 - (-5) = V_{in} + 4.3 > 0 - (-1.8)$

$V_{in} + 4.3 > 1.8 \quad \boxed{V_{in} > -2.5\,v}$

$V_{CE} = 5 - (V_{in} - 0.7) = 5 - V_{in} + 0.7 = 5.7 - V_{in} > 0.2$

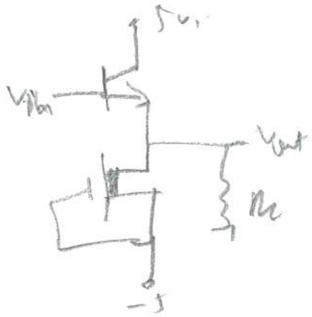
$\Rightarrow \boxed{5.5 > V_{in}}$ Result: $\boxed{-2.5 < V_{in} < 5.5}$

(3p) (3p)

$I_E = 38.88\,mA > 0.$

b) $V_{out} = 2\sin wt.$

(6p)



$V_{DS} = V_{out} - (-5) = 2\sin wt + 5 > 0 - (-1.8)$

$2\sin wt > -5 + 1.8 = -3.2 \quad \checkmark \quad \forall t.$

$V_{CE} = 5 - 2\sin wt > 0.2 \quad \checkmark \quad \forall t.$

$I_E = \frac{V_{out}}{R_L} + I_D = \frac{2\sin wt}{R_L} + 0.2(0+1.8)^2$

$= \frac{2\sin wt}{R_L} + 0.648 \qquad I_{E\,min} = \frac{-2}{R_L} + 0.648 > 0$

$\frac{-2}{R_L} > -0.648 \qquad R_L > \frac{2}{0.648} = 3.086\,k\Omega.$

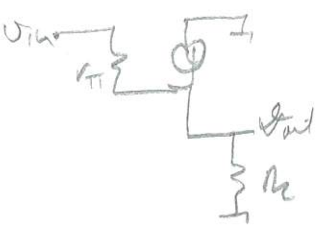
c) $R_L = 500$

(8p)

Q-point: $V_{in} = 0.7 \Rightarrow V_{out} = 0 \Rightarrow I_{R_L} = 0 \Rightarrow I_E = 0.648\,mA$

$\therefore I_C = \frac{100}{101} \times 0.648\,mA = 0.642\,mA$

$r_\pi = \frac{0.026}{0.642} \times 100 = 4.05\,k\Omega.$ (2p)



$i_e = \frac{v_{in}}{r_\pi + (\beta+1)R_L} \times (\beta+1) \qquad v_{out} = i_e R_L$

$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{i_e} \cdot \frac{i_e}{v_{in}} = R_L \times \frac{\beta+1}{r_\pi + (\beta+1)R_L} = \frac{101 \times 0.5}{4.05 + 101 \times 0.5}$

$= \frac{50.5}{4.05 + 50.5} = 0.926$ (6p)