25-11-2017

BILKENT UNIVERSITY

Department of Electrical and Electronics Engineering

EEE313 Electronic Circuit Design

Midterm Exam II

SOLUTION

Surname: ____________________

Name: ____________________

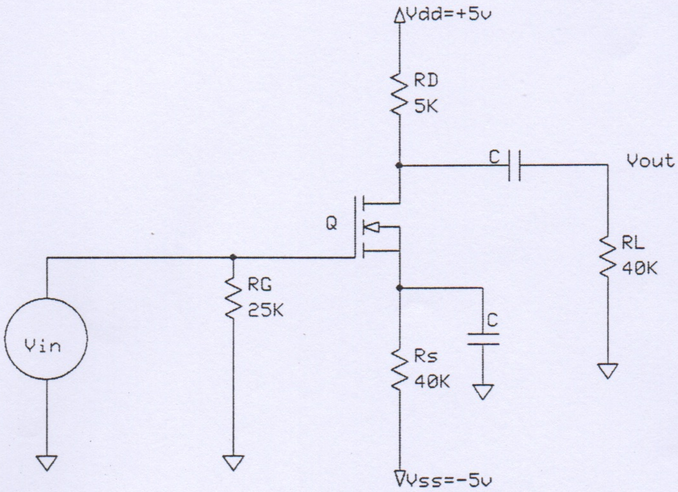
ID-Number: ____________________

Signature: ____________________

Duration is 110 minutes. Solve all 4 questions. Show all your work.
No books or notes.

Q1 (25 points)	
Q2 (25 points)	
Q3 (25 points)	
Q4 (25 points)	
Total (100 points)	

Q1. In the circuit shown below, the NMOS transistor has parameters V_{TN}=0.8 V, K_n=0.85 mA/V^2 and λ=0.02 V^{-1}. Find the small signal voltage gain. Draw the small signal equivalent circuit and derive your results.



DC analysis $V_G = 0$ $\quad V_{GS} = 0 - I_D \times R_s - (-5) = 5 - I_D R_s = 5 - 40 I_D$

Assume TR is SAT: $I_D = 0.85 (V_{GS} - 0.8)^2$

$$\frac{5 - V_{GS}}{40} = 0.85(V_{GS}^2 - 1.6 V_{GS} + 0.64)$$

$$5 - V_{GS} = 34 V_{GS}^2 - 54.4 V_{GS} + 21.76$$

$$34 V_{GS}^2 - 53.4 V_{GS} + 16.76 = 0$$

$$V_{GS} = \begin{cases} 1.137\,V > 0.8 ✓ \\ 0.4335\,V < 0.8 ✗ \end{cases} \qquad \boxed{V_{GS} = 1.137\,V}$$

$$\boxed{I_{DQ} = \frac{5 - 1.137}{40} = 0.0966\,mA}$$

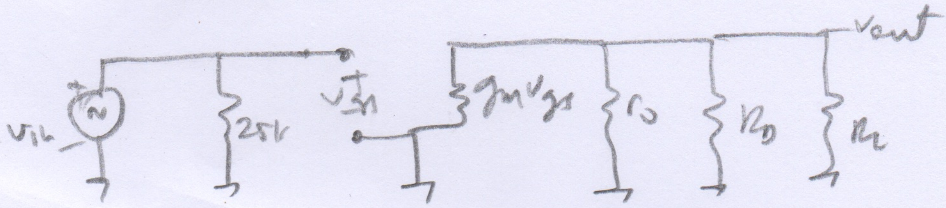
$V_{DSQ} = 5 - I_{DQ} \times 5 - I_{DQ} \times 40 + 5$

$= 10 - 0.0966 \times 45 = 5.65\,V$

$> 1.137 - 0.8$ ✓

∴ TR is SAT.

AC analysis



$g_m = 2\sqrt{0.85 \times 0.0966}$
$= 0.573\,mA/V$

$r_o = \frac{1}{0.02 \times 0.0966} = 517.6\,k$

$V_{out} = -g_m v_{gs} \times (r_o // R_D // R_L)$

$V_{gs} = V_{in}$

$\Rightarrow A_v = -g_m (r_o // R_D // R_L) = -0.573 \times (517.6 // 5 // 40)$

$= -0.573 \times (517.6 // 4.44)$

$= -0.573 \times 4.402$

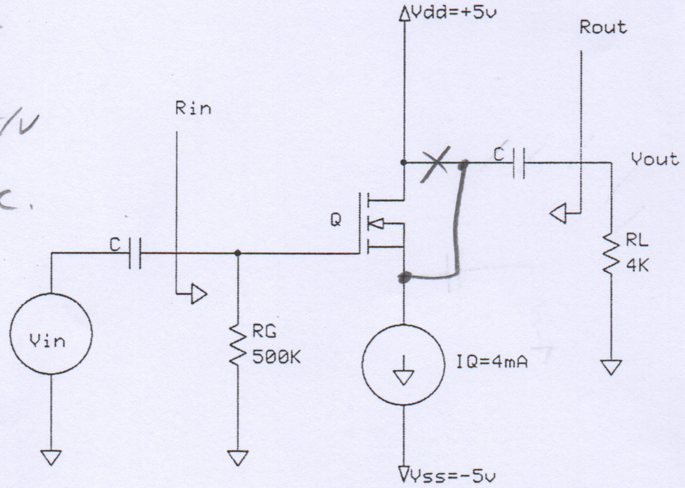
$= -2.52$

Q2. In the circuit shown below, the NMOS transistor has parameters V_{TN}=0.8 V, k_n'=0.1 mA/V^2, W/L=20 and λ=0.02 V^{-1}. Find the small signal voltage gain, the input impedance R_{in}, and the output impedance R_{out}. Draw the small signal equivalent circuit and derive your results.

$K_N = \frac{0.1}{2}\times 20 = 1\,mA/V^2$

$g_m = 2\sqrt{1\times 4} = 4\,mA/V$

$r_o = \frac{1}{0.02\times 4} = \frac{1}{0.08} = 12.5K.$

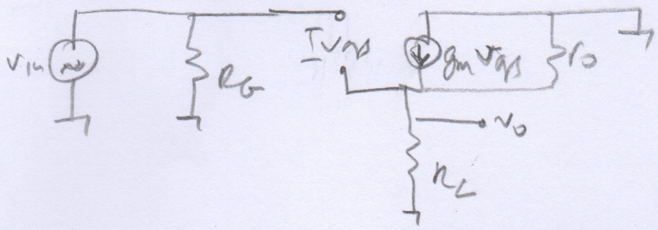


$\frac{1}{g_m} = \frac{1}{4} = 0.25$

$\frac{0.25\times 12.5}{0.25+12.5}$

$= \frac{3.125}{12.75}$

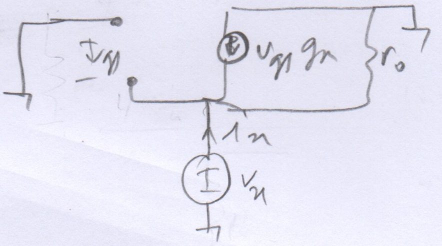
$R_{in} = R_G = 500K$



$v_o = (g_m v_{gs} - \frac{v_o}{r_o}) R_L \Rightarrow v_o(1+\frac{R_L}{r_o}) = g_m v_{gs} R_L \quad v_o = \frac{g_m v_{gs}}{\frac{1}{R_L}+\frac{1}{r_o}}$

$v_{in} = v_{gs} + v_o \Rightarrow v_{in} = v_{gs} + \frac{g_m v_{gs}}{\frac{1}{R_L}+\frac{1}{r_o}} = v_{gs}\left(1+\frac{g_m}{\frac{1}{R_L}+\frac{1}{r_o}}\right)$

$\frac{v_o}{v_{in}} = \frac{\frac{g_m}{\frac{1}{R_L}+\frac{1}{r_o}}}{1+\frac{g_m}{\frac{1}{R_L}+\frac{1}{r_o}}} = \frac{g_m}{\frac{1}{R_L}+\frac{1}{r_o}+g_m} = \frac{4}{\frac{1}{12.5}+\frac{1}{4}+4} = \frac{4}{4.33} = 0.92$



$v_x = -v_{gs}$

$i_x = -v_{gs} g_m + \frac{v_x}{r_o} = +v_x g_m + \frac{v_x}{r_o}$

$= v_x\left(\frac{1}{\frac{1}{g_m}} + \frac{1}{r_o}\right)$

$\frac{v_x}{i_x} = \frac{1}{g_m} // r_o$

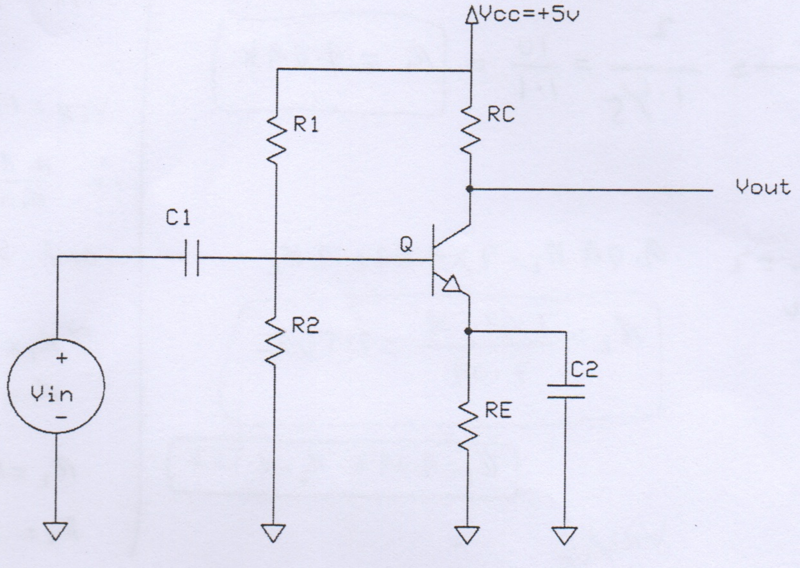
$= \frac{1}{4} // 12.5 = \frac{\frac{12.5}{4}}{\frac{1}{4}+12.5} = \frac{12.5}{1+4\times 12.5} = \frac{12.5}{51}$

$= 0.245\,K.$

Q3. Design the below Common Emitter (CE) stage, i.e. find the values of components such that I_{CQ}=1 mA, voltage drop across R_E resistor is 400 mV, small signal AC voltage gain magnitude is 20, and small signal AC input impedance is 2 Kohm. Check and verify the state of the transistor.

(18p) a) Assume for the transistor that β=100, $V_{BE(ON)}$=0.7V, $V_{CE(SAT)}$=0.2 V and $V_A=\infty$

(7p) b) Assume for the transistor that β=100, $I_S=5\times10^{-16}$A, and $V_A=\infty$



C1 and C2 are very large

(3p)

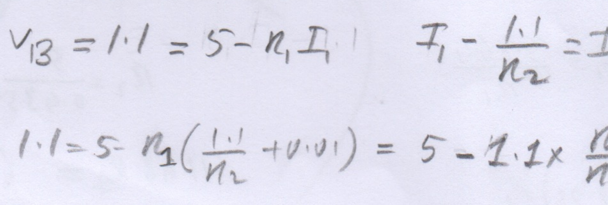
a) $V_{RE} = I_{CQ}\left(\frac{\beta+1}{\beta}\right) R_E = 1 \times \frac{101}{100} \times R_E \qquad R_E = \frac{100}{101} \times 0.4 = 0.396K = 396\,\Omega$

$r_\pi = \frac{V_T}{I_{BQ}} = \frac{0.026}{1/101} = 2.626\ k\Omega \leftarrow$ (1p)

$g_m = \frac{I_{CQ}}{V_T} = \frac{1}{0.026} = 38.5\ mA/V, \leftarrow$ (1p)

Check TR SAT

$V_{CE} = 5 - 1\times0.52 - 0.4$
$= 4.08 > 0.02$ ✓
(2p)



$A_v = -g_m R_C \Rightarrow g_m R_C = 20$

$R_C = \frac{20}{38.5} = 0.52\ K.$

(4p)

$I_B = \frac{1}{100} = 0.01\ mA \qquad V_B = 0.4 + 0.7 = 1.1\ V.$

If I_B is not considered then (3p)

$V_B = 1.1 = 5 - R_1 I_1 \qquad I_1 - \frac{1.1}{R_2} = I_B = \frac{1}{100} = 0.01\ mA. \qquad I_1 = \frac{1.1}{R_2} + 0.01$

$1.1 = 5 - R_1\left(\frac{1.1}{R_2} + 0.01\right) = 5 - 1.1\times\frac{R_1}{R_2} - 0.01 R_1 \Rightarrow \boxed{3.9 = 1.1\frac{R_1}{R_2} + 0.01 R_1}$

I_B considered but R_{in}=2k is taken (4p)

$R_{in} = R_1 // R_2 // r_\pi = 2K$

$3.9 = 1.1\left(\frac{R_1}{8.67} - 1\right) + 0.01 R_1$

$\frac{(R_1//R_2)\times 2.6}{R_1//R_2 + 2.6} = 2 \qquad 2.6(R_1//R_2) = 2(R_1//R_2) + 5.2$

$0.6(R_1//R_2) = 5.2$

$R_1//R_2 = 8.67K.$

$5 = \left(\frac{1.1}{8.67} + 0.01\right) R_1 \Rightarrow \boxed{R_1 = 36.5K}$

$\frac{36.5 R_2}{36.5 + R_2} = 8.67 \qquad 36.5 R_2 = 8.67 R_2 + 36.5\times8.67$

$\frac{R_1 R_2}{R_1 + R_2} = 8.67 \qquad \frac{R_1}{\frac{R_1}{R_2} + 1} = 8.67 \qquad \frac{R_1}{R_2} = \frac{R_1}{8.67} - 1$

(7p)* $\boxed{R_2 = 11.37K.}$

Alternative solutions for part (a)

Yet another soln: $I_b \neq 0$ but $R_1 // R_2 = 2$

$V_{TH} = 1.1 + 2\times 0.01 = 1.12\,v = \frac{R_2}{R_1+R_2}\times 5 = \frac{1}{R_1}(R_1//R_2)\,5 = \frac{2}{R_1}\times 5 = \frac{10}{R_1}$ $\quad R_1 = 8.92\,k.$

$R_1R_2 = 2R_1 + 2R_2 \quad R_2 = \frac{2R_1}{R_1-2} = 2.58\,k.$ $\quad \boxed{R_1 = 8.92\,k \quad R_2 = 2.58\,k}$

If I_B is neglected

$$\frac{R_1R_2}{R_1+R_2} = 2 \qquad 1.1 = \frac{R_2}{R_1+R_2}\times 5$$

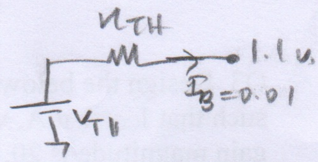
$$\frac{\frac{R_1R_2}{R_1+R_2}}{\frac{R_2}{R_1+R_2}} = \frac{2}{1.1/5} = \frac{10}{1.1} = \boxed{R_1 = 9.09\,k}$$

$$\frac{9.09\,R_2}{9.09+R_2} = 2 \qquad 9.09\,R_2 = 2\times 9.09 + 2R_2$$

$$\boxed{R_2 = \frac{2\times 9.09}{7.09} = 2.56\,k}$$

$$\boxed{R_1 = 9.09\,k \quad R_2 = 2.56\,k}$$

Another solution



$$R_{TH} = R_1//R_2 = \frac{R_1R_2}{R_1+R_2} \qquad V_{TH} = 5\times\frac{R_2}{R_1+R_2}$$

$$R_{TH}//r_\pi = 2 \qquad \frac{2.6\,R_{TH}}{2.6+R_{TH}} = 2 \Rightarrow R_{TH} = 8.67\,k$$

$$V_{TH} = 1.1 + R_{TH}\times 0.01 = 1.1867\,v.$$

$$\therefore \frac{R_1R_2}{R_1+R_2} = 8.67$$

and $5\frac{R_2}{R_1+R_2} = 1.1867 \Rightarrow \frac{R_2}{R_1+R_2} = \frac{1.1867}{5} = 0.23734$

$$\Rightarrow R_1\times 0.23734 = 8.67$$

$$R_1 = \frac{8.67}{0.23734} = 36.5\,k$$

$$R_2 = 1.1867R_1 + 1.1867R_2$$

$$R_2 = \frac{1.1867\times 36.5}{1-0.23734} = 11.36\,K.$$

$$\boxed{R_1 = 36.5\,k \quad R_2 = 11.36\,k}$$

(7p) b)

$$I_C = I_S\, e^{V_{BE}/V_T}$$

$$1 = 5\times 10^{-13}\, e^{V_{BE}/V_T}$$

$$e^{V_{BE}/V_T} = 0.2\times 10^{13}$$

$$\frac{V_{BE}}{V_T} = 28.32$$

$$V_{BE} = 28.32\times 0.026 = 0.736\,V.$$ only! 3

All other values are the same except that R_1 and R_2 must be different.

$$V_B = 0.4 + 0.736 = 1.136\,v.$$

$$1.136 = 5 - I_1R_1 \Rightarrow I_1 = \frac{3.864}{R_1}$$

$$1.136 = (I_1 - 0.01)R_2$$

$$\Rightarrow 1.136 = \left(\frac{3.864}{R_1} - 0.01\right)R_2$$

$$1.136 = 3.864\frac{R_2}{R_1} - 0.01R_2 \quad *$$

Since $\frac{R_1R_2}{R_1+R_2} = 8.67 \Rightarrow \frac{R_2}{1+\frac{R_2}{R_1}} = 8.67 \Rightarrow \frac{R_2}{R_1} = \frac{R_2}{8.67} - 1$

$$* \Rightarrow 1.136 = 3.864\times\left(\frac{R_2}{8.67} - 1\right) - 0.01R_2$$

$$1.136 + 3.864 = \left(\frac{3.864}{8.67} - 0.01\right)R_2$$

$$R_2 = \frac{5}{0.4357} = 11.48\,K.$$

$$11.48\,R_1 = (R_1 + 11.48)\,8.67$$

$$R_1 = \frac{11.48\times 8.67}{11.48 - 8.67} = 35.42\,K.$$

$$\boxed{R_1 = 35.42\,K \quad R_2 = 11.48\,K}$$

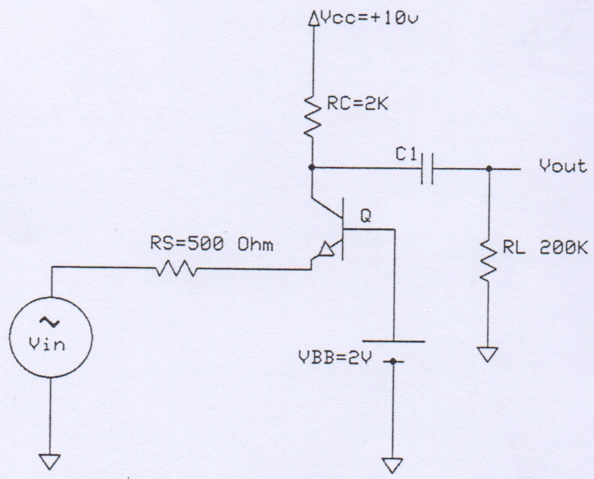
Q4. In the circuit shown below (for parts a,b,c assume that $\beta=100$, $V_{BE(ON)}=0.7V$, $V_{CE(SAT)}=0.2$ V and $V_A=\infty$),

(5) a) Find the quiscent voltage and current values (I_{CQ} and V_{CEQ}), verify transistor state.

(7) b) Find small signal ac input impedance.

(6) c) Find the small signal voltage gain.

(7) d) Find the small signal ac output impedance if $\beta=100$, $V_{BE(ON)}=0.7V$, $V_{CE(SAT)}=0.2$ V and $V_A=100$ V.



$R_s // r_\pi = 500//1010 = 334$

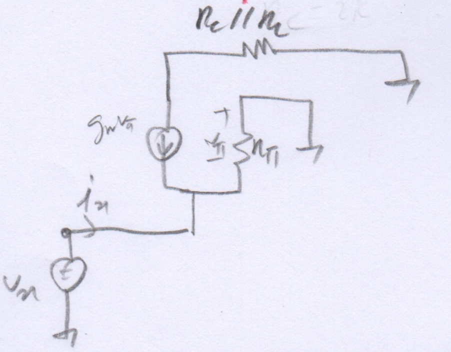
$R_{o2} = 38.9(1+99\times 0.334) + 0.334$

a) $2 = 0.7 + I_E \times 0.5 \quad I_E = \dfrac{1.3}{0.5} = 2.6\,mA$ (2p)

$I_C = \dfrac{100}{101}\times 2.6 = 2.57\,mA.$

$V_{CEQ} = 10 - 2\times 2.57 - (2-0.7) = 8.7 - 5.14 = 3.56V. > 0.2 \Rightarrow SAT.$ (2p) (1p)

b)



$r_\pi = \dfrac{V_T}{I_{BQ}} = \dfrac{0.026\times 101}{2.6} = 1.01K = 1010\Omega.$ (1p)

$i_{in} = \dfrac{v_{in}}{r_\pi} - g_m v_\pi = \dfrac{v_{in}}{r_\pi} + g_m v_{in}$

$R_{in} = \dfrac{1}{g_m} // r_\pi = \dfrac{\frac{1}{g_m} r_\pi}{\frac{1}{g_m} + r_\pi} = \dfrac{r_\pi}{1+\beta} = \dfrac{1.01}{101}$

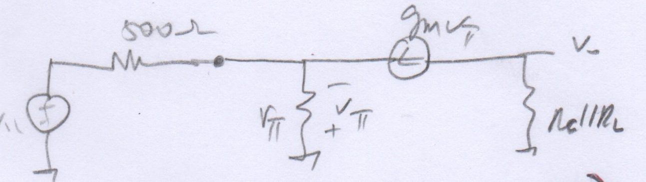
(5p) $= 10\Omega.$

If R_s is included into the R_{in} definition no points are cancelled.
$\Rightarrow R_{in} = 500 + 10 = 510\,\Omega$.

$g_m = \dfrac{100}{r_\pi}^{*} = 99$ mA/V (1p)

β kullan 1

c)



$v_o = -g_m R_C // R_L v_\pi$

$-v_\pi = \dfrac{10}{510}\times v_{in} \quad v_\pi = \dfrac{v_{in}\times 10}{510}$

$v_o = \dfrac{10}{510}\times g_m \times R_C//R_L = \dfrac{10}{510}\times\dfrac{100}{r_\pi}\times 2//200$

$= 3.84$ (6p)

or

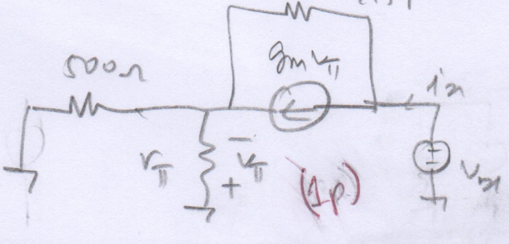
$A_v = \dfrac{r_e}{r_e + R_s}\cdot g_m (R_C // R_L)$

$A_v = \dfrac{r_\pi}{r_\pi + (\beta+1)R_s} g_m (R_C // R_L)$

* If β is used in A_v then 1p for β instead of g_m.

$r_o = \dfrac{100}{2.57} = 38.9K.$ (2p)

d)



(1p)

$v_x = (i_x - g_m v_\pi) r_o + i_x (R_s // r_\pi)$ (2p)

Since $v_\pi = -i_x (R_s // r_\pi)$ (2p)

$v_x = (i_x + g_m (R_s // r_\pi) i_x) r_o + i_x (R_s // r_\pi)$

$R_2 = \dfrac{v_x}{i_x} = r_o (1 + g_m (R_s // r_\pi)) + R_s // r_\pi$

$= 1325K + 0.334 \cong 1325K$

$R_{out} = R_2 // R_C = 1325 // 2$

$= 1.997K.$

Table 3.1 Summary of the MOSFET current–voltage relationships

NMOS	PMOS
Nonsaturation region ($v_{DS} < v_{DS}(\text{sat})$)	Nonsaturation region ($v_{SD} < v_{SD}(\text{sat})$)
$i_D = K_n[2(v_{GS} - V_{TN})v_{DS} - v_{DS}^2]$	$i_D = K_p[2(v_{SG} + V_{TP})v_{SD} - v_{SD}^2]$
Saturation region ($v_{DS} > v_{DS}(\text{sat})$)	Saturation region ($v_{SD} > v_{SD}(\text{sat})$)
$i_D = K_n(v_{GS} - V_{TN})^2$	$i_D = K_p(v_{SG} + V_{TP})^2$
Transition point	Transition point
$v_{DS}(\text{sat}) = v_{GS} - V_{TN}$	$v_{SD}(\text{sat}) = v_{SG} + V_{TP}$
Enhancement mode	Enhancement mode
$V_{TN} > 0$	$V_{TP} < 0$
Depletion mode	Depletion mode
$V_{TN} < 0$	$V_{TP} > 0$

npn transistor	*pnp* transistor	
$i_C = I_S e^{v_{BE}/V_T}$ $i_B = \dfrac{i_C}{\beta} = \dfrac{I_S}{\beta} e^{v_{BE}/V_T}$ $i_E = \dfrac{i_C}{\alpha} = \dfrac{I_S}{\alpha} e^{v_{BE}/V_T}$	$i_C = I_S e^{v_{EB}/V_T}$ $i_B = \dfrac{i_C}{\beta} = \dfrac{I_S}{\beta} e^{v_{EB}/V_T}$ $i_E = \dfrac{i_C}{\alpha} = \dfrac{I_S}{\alpha} e^{v_{EB}/V_T}$	$i_E = i_C + i_B$ $\alpha = \dfrac{\beta}{\beta+1}$ $\beta = \dfrac{\alpha}{1-\alpha}$

$$i_C = I_S e^{v_{BE}/V_T}(1 + v_{CE}/V_A)$$

L=1/V_A

$$r_\pi \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

V_T=26 mV