Name: Tarik Reyhan

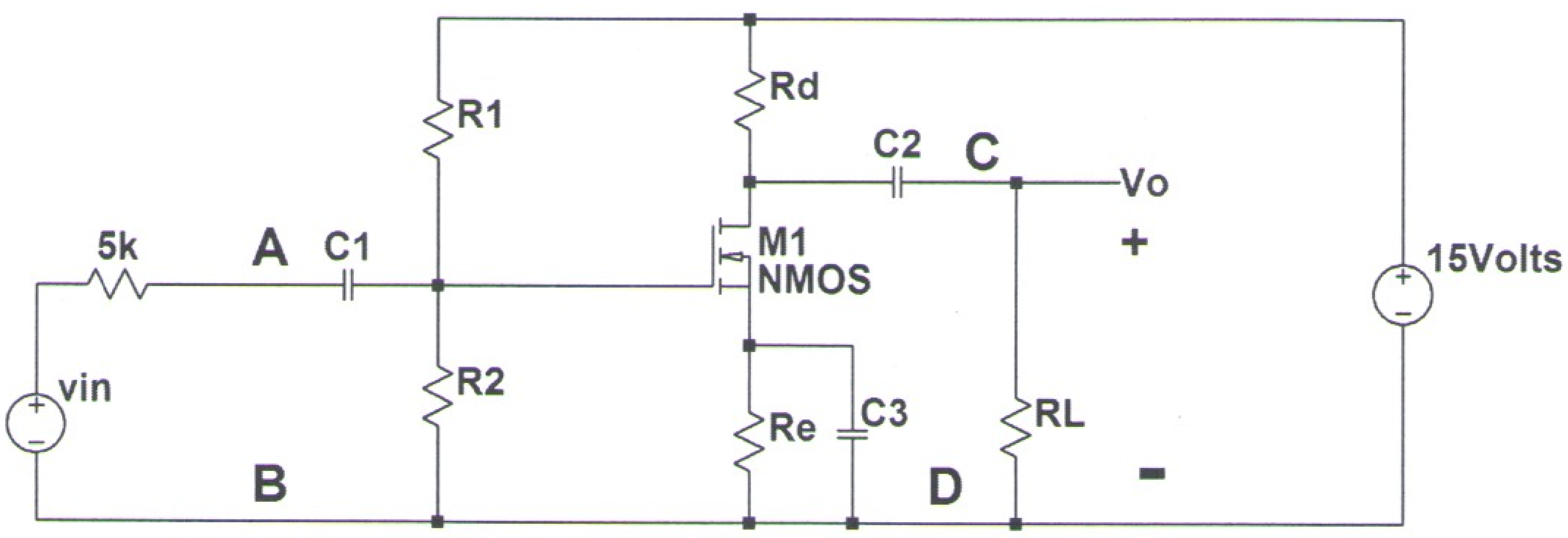
Signature: ________________ **EEE 313 Spring 2015**

Bilkent University
Department of Electrical and Electronics Engineering
EEE 313 Electronic Circuit Design
Midterm 2
20 April 2015, 17:50
(4 questions, 120 minutes)

- This is a **closed book**, closed notes exam. No cheat sheet allowed.
- All cell-phones should be completely **turned off**.
- Use a calculator for numerical computations. Carry at least **4 significant digits** during calculations. Your final answer should be at least **3 significant digits**.
- Be sure to write the **units** of all numerical results.
- **Show** all work clearly.
- Please put your **final answer** for each part inside a box for easy identification. Do not give multiple answers, they will not be graded.
- Do not remove the **staple** from the exam sheets or separate pages of the exam. All extra pages must be stamped to your exam.
- You may leave the exam room when you are done. However, please do not leave during the **last five minutes** of the exam.
- At the end of the exam, please stay seated unitl **all** exam papers are collected.

Please do not write below this line

1. 25 pts.	
2. 25 pts.	
3. 20 pts.	
4. 30 pts.	
Total 100 pts.	



1) (25 points) At the circuit given above, $K_n=10\times10^{-3}$ A/V^2, V_{TN}=2Volts. Assume that the DC solution was calculated before and the drain current, I_D was found to be equal to 10mA and the state of the transistor was found to be saturation. The amplifier is going to be used as microphone amplifier with a bandwidth of 20Hz-20kHz. The values of R1 and R2 are set to 12kΩ, R_E=300Ω, Rd=500Ω and R_L=1kΩ.

   a) (7 points) Assuming that the rest of the capacitors are very large, find the value of C_1 in order to have 3dB lower cut-off frequency.

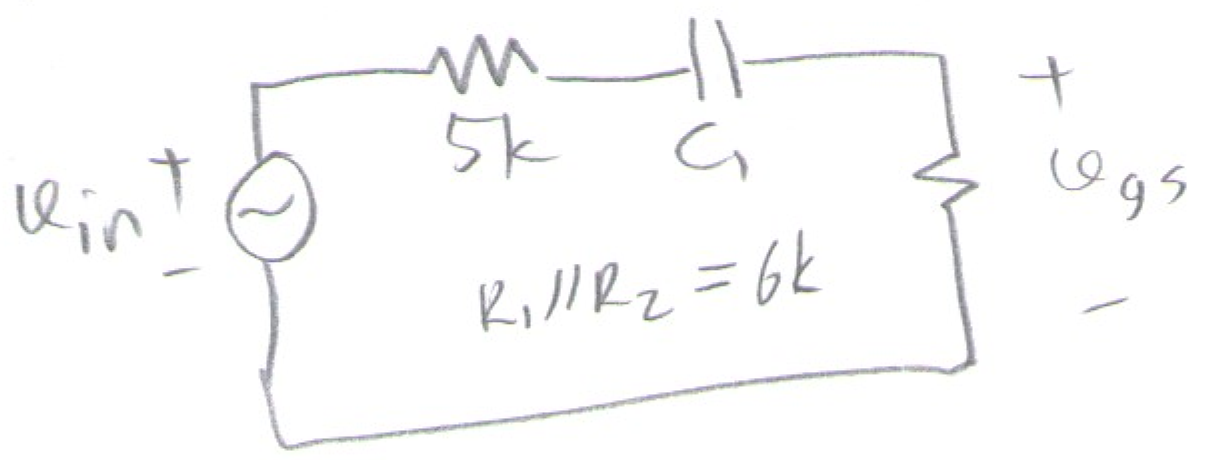
   b) (8 points) Assuming that the rest of the capacitors are very large, find the value of C_2 in order to have 3dB lower cut-off frequency.

   c) (10 points) Assuming that the rest of the capacitors are very large, find the value of C_3 in order to have 3dB lower cut-off frequency.

Solutionss

a-) Drawing the equivalent circuit of the input circuit;

$R_1 // R_2 = 6k$

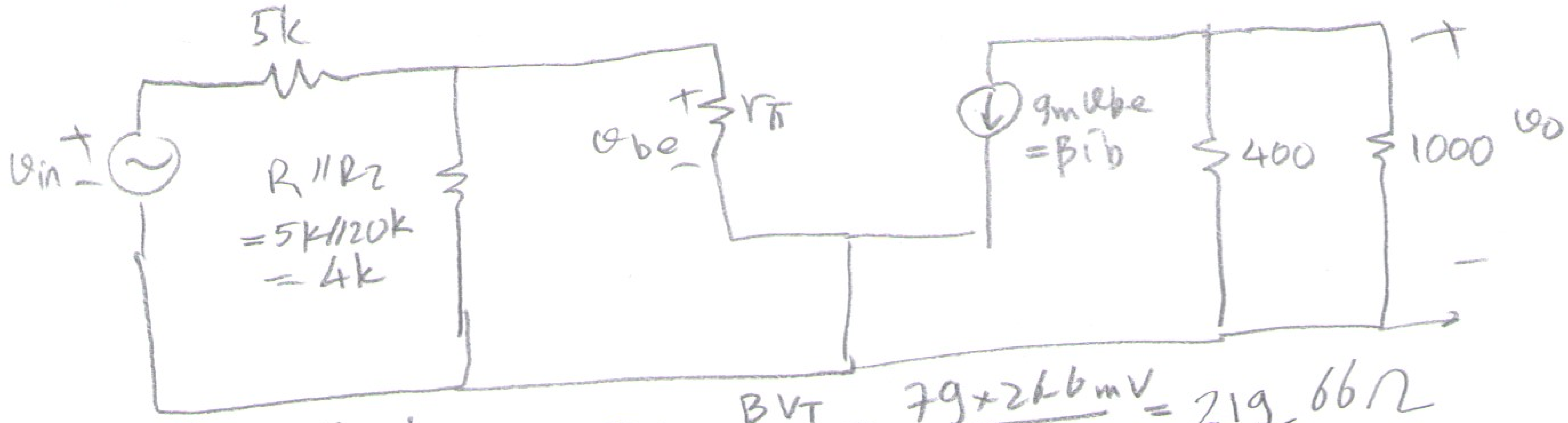


$$\frac{v_{gs}}{v_{in}} = \frac{6000}{6000+500+\frac{1}{j\omega C_1}} \Rightarrow \omega_0 = 2\pi\times 20 = \frac{1}{11000\, C_1} \Rightarrow$$

$$= 125.66$$

$$C_1 = \frac{1}{126.66\times 11000} = 7.23\times10^{-7}\,F = 0.723\,\mu F$$

b-) Drawing the AC equivalent circuit:



$g_m = \frac{I_C}{V_T} = \frac{10mA}{26mV} =$
$= 0.376$ S

$r_\pi = \frac{\beta V_T}{I_C} = \frac{79 \times 26mV}{10mA} = 219.66\Omega$

$v_o = -(400 // 1000\Omega) g_m \cdot \frac{r_\pi // 4k}{(r_\pi // 4k) + 5k} v_{in} \Rightarrow$ $r_\pi // 4000 = \frac{219.66 \times 4000}{4219.66}$
$= 208.2\Omega$

$\frac{v_o}{v_{in}} = -285.7\Omega \times 0.3768 \times \frac{208.2}{5208.2} = -4.295$

c-) $r_\pi // R_1 // R_2 = R_{in} = 208.2\Omega$

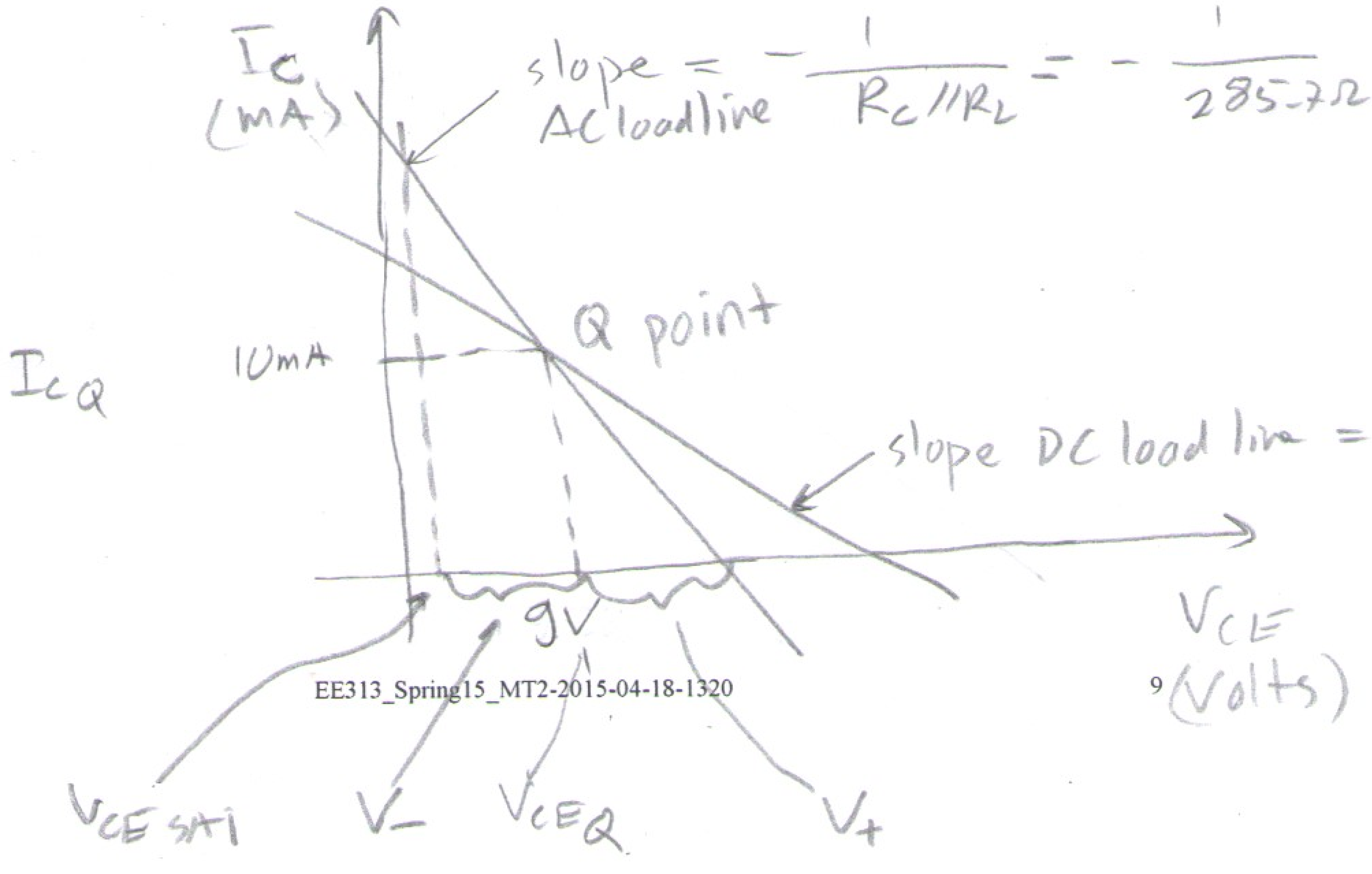
d-) $R_o = R_C = 400\Omega$

e-) $V_+ = I_{CQ} \times R_C // R_L = 10mA \times 285.7\Omega = 2.857V$

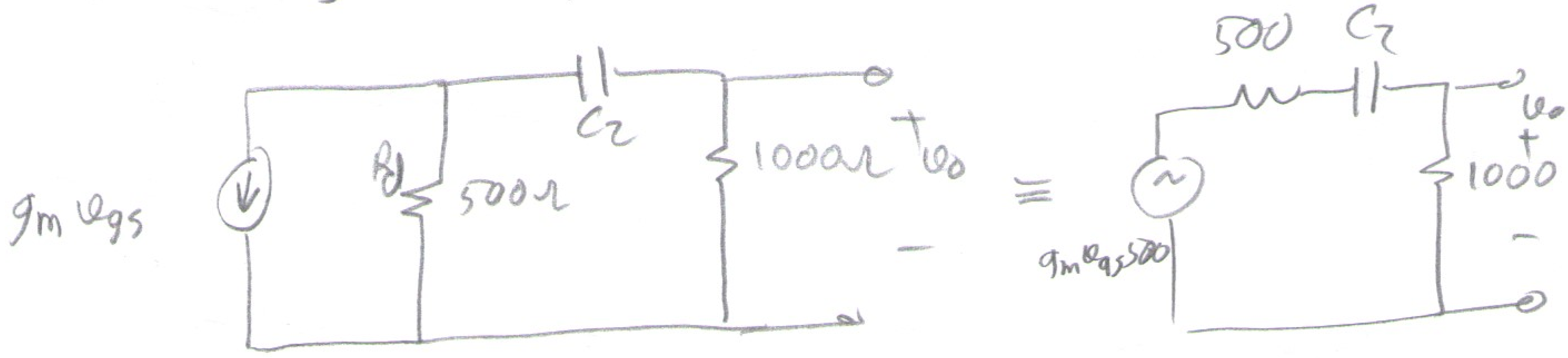
$V_- = V_{CEQ} - V_{CE_{SAT}} = 9.0V - 0.4V = 8.6V$

where $V_{CEQ} = 15 - 10mA \times 600\Omega = 9V$

V_{p-p} undistorted $= 2 \times \min\{V_+, V_-\} = 2 \times 2.857 = 5.714V$



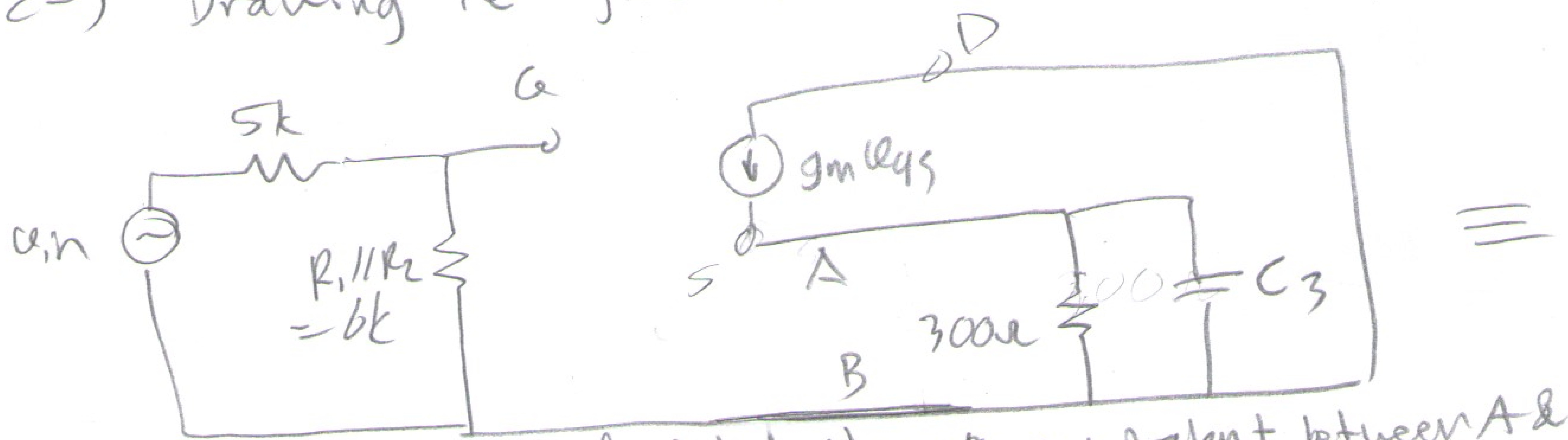
b-) Drawing the output circuit



$$\frac{v_o}{g_m v_{gs} 500} = \frac{1000}{1000+500+\frac{1}{j\omega C_2}} \Rightarrow \omega_0 = 126.66 \text{ rad/sec} = \frac{1}{1500 C_2}$$

$$\Rightarrow C_2 = \frac{1}{125.66\times 1500} = 5.305\times10^{-6} \text{ F} \cong 5.3\,\mu\text{F}$$

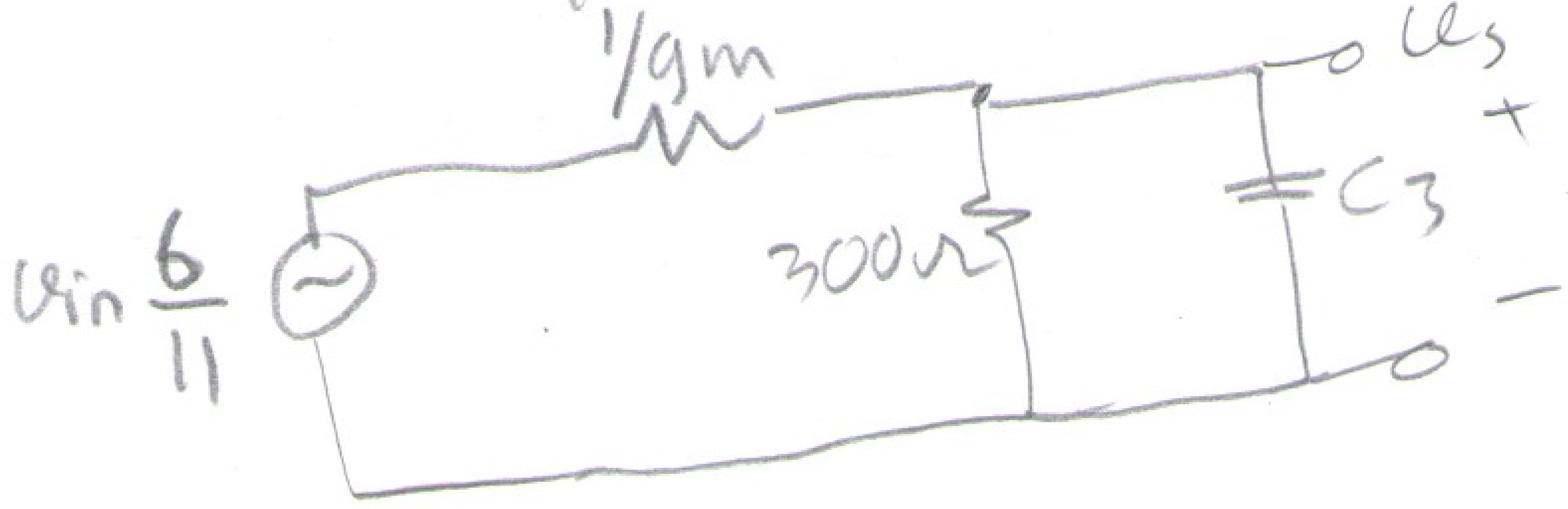
c-) Drawing the gate-to-source circuit



in order to find the thevenin's equivalent between A & B find the open circuit voltage between A & B with 300Ω and C_3 removed ⇒

$v_{\text{open circuit}} = v_{in} \times \frac{6k}{5k+6k}$ as source voltage will be equal to the gate voltage if no current passes i.e. $g_m v_{gs} = 0 \Rightarrow v_{gs} = 0$

R_o between A & B is equal to $1/g_m$ ⇒ the equivalent circuit becomes

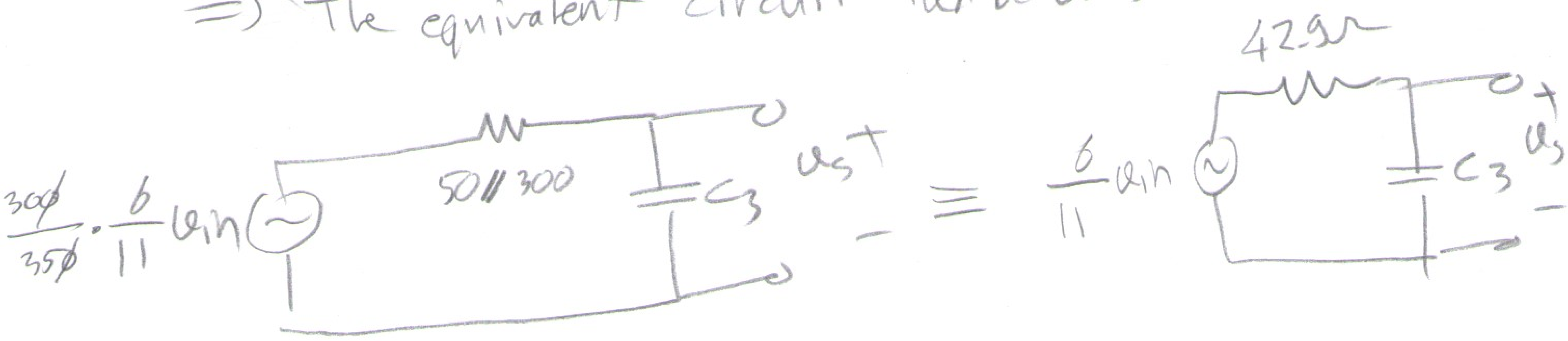


with $g_m = 2\sqrt{I_D K_n'} = 2\sqrt{10^{-2}\times10^{-2}} = 2\times10^{-2}$ ℧

$\Rightarrow \frac{1}{g_m} = \frac{1}{2\times10^{-2}} = 50\Omega$

continued →

⇒ The equivalent circuit then becomes



the output current is $g_m v_{gs}$ ⇒ $i_o = g_m(v_g - v_s)$ ⇒

$$i_o = g_m\left[\frac{6}{11}v_{in} - \frac{6}{11}v_{in}\left(\frac{1/j\omega C_3}{42.9 + \frac{1}{j\omega C_3}}\right)\cdot\frac{6}{7}\right]$$

$$\Rightarrow i_o = \frac{6 g_m v_{in}}{11}\left[1 - \frac{6}{7}\,\frac{1}{1 + j42.9\,\omega C_3}\right] \Rightarrow$$

$$\frac{i_o}{v_{in}} = \frac{6 g_m}{11}\left[\frac{1}{7} + \frac{6}{7}\left[1 - \frac{1}{1 + j42.9\,\omega C_3}\right]\right]$$

$$\frac{i_o}{v_{in}} = \frac{6 g_m}{11}\left[\frac{1}{7} + \frac{6}{7}\left[\frac{\cancel{1} + j42.9\,\omega C_3 - \cancel{1}}{1 + j42.9\,\omega C_3}\right]\right] \Rightarrow$$

$$\frac{i_o}{v_{in}} = \frac{6\times 0.02}{11\times 7}\left[1 + 6\,\frac{j\omega 42.9 C_3}{1 + j\omega 42.9 C_3}\right]$$ ⇒ The 3dB point is defined by

$$\omega_0 = 2\pi\times 20 = 126.66 = \frac{1}{42.9 C_3} \Rightarrow$$

$$C_3 = \frac{1}{42.9\times 126.66} = 1.855\times 10^{-4}\,F = 185.5\,\mu F$$

or one can simply say that ω_0 is defined by

$$\omega_0 = \frac{1}{\left(\frac{1}{g_m} \parallel R_E\right) C_3}$$ as mentioned in the classroom

To be more precise:

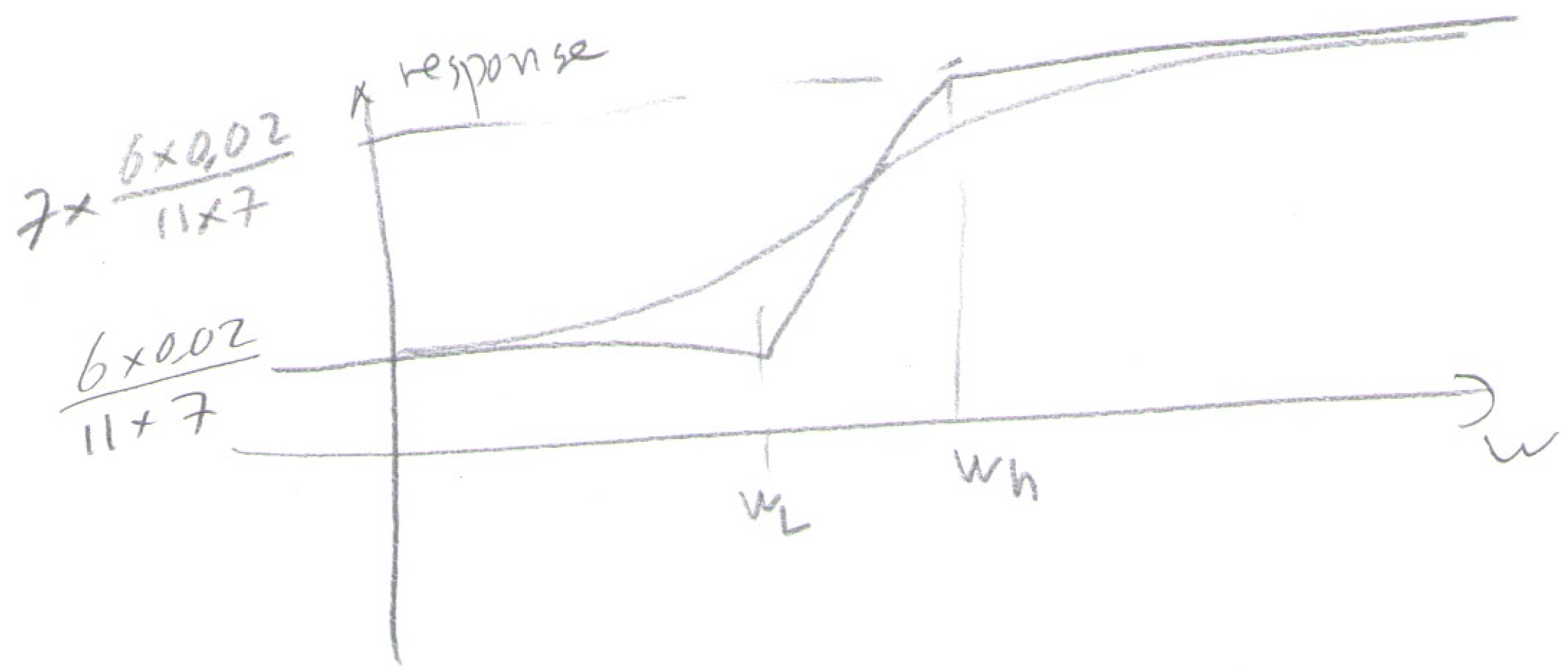
$$\frac{i_o}{v_{in}} = \frac{6 \times 0.02}{11 \times 7} \left[1 + 6 \, \frac{j\omega 42.9 C_3}{1 + j\omega 42.9 C_3} \right]$$

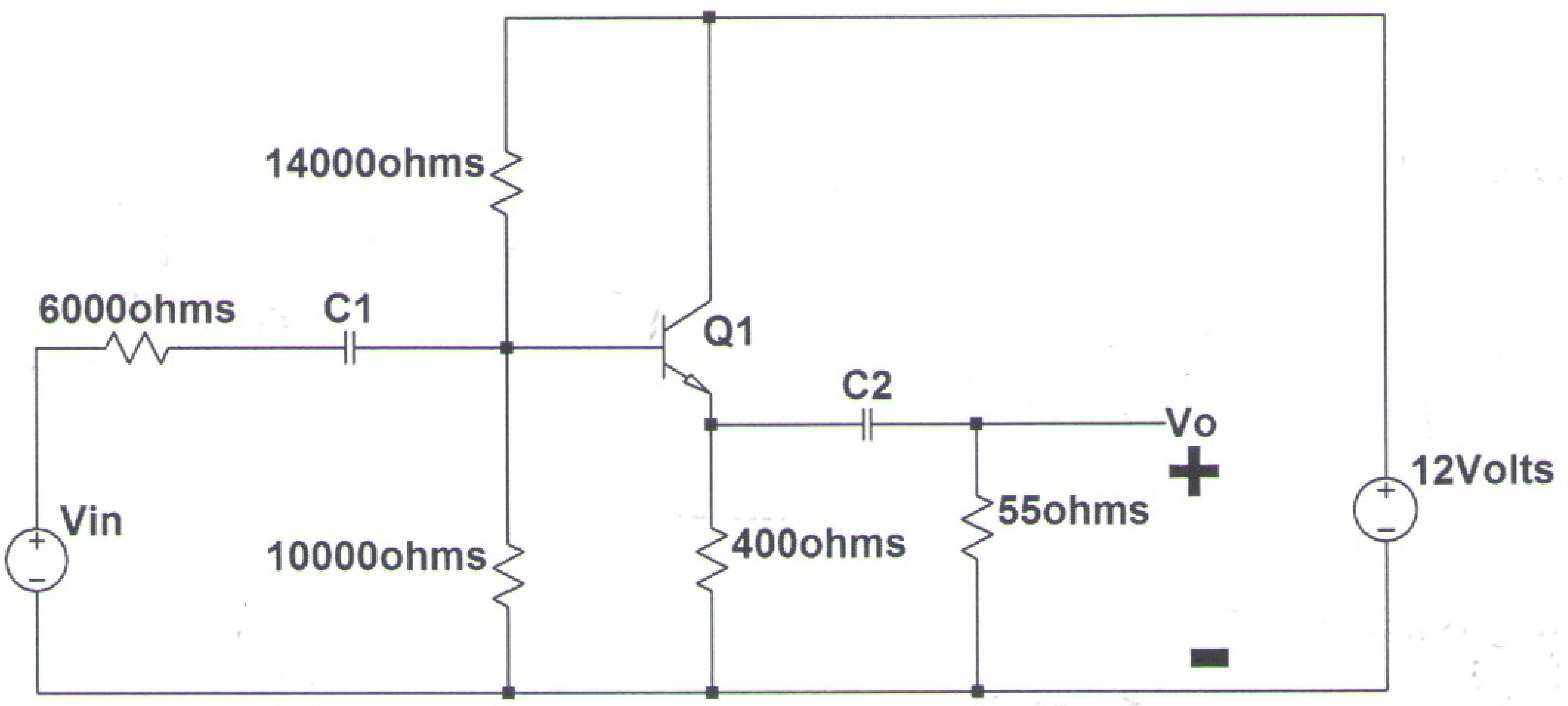
$$\frac{i_o}{v_{in}} = \frac{6 \times 0.02}{11 \times 7} \left[\frac{1 + j\omega 42.9 C_3 + 6 j\omega 42.9 C_3}{1 + j\omega 42.9 C_3} \right]$$

$$= \frac{6 \times 0.02}{11 \times 7} \cdot \frac{1 + 7 \times 42.9 j\omega C_3}{1 + 42.9 j\omega C_3}$$

← higher cut-off w_L

← lower cut-off w_h





2. (25 points) The transistor in the emitter follower circuit shown below has the following parameters:

β=99, $V_{BE_{\cancel{ON}}}$ SAT = 0.7 Volts, $V_{CE_{SAT}}$ = 0.4 Volts

The amplifier is used as a preamplifier in a music set. Therefore the frequency range is equal to 20Hz-20kHz. Answer the following:

a) (7 points) Find the state and the bias of the transistor and verify it.

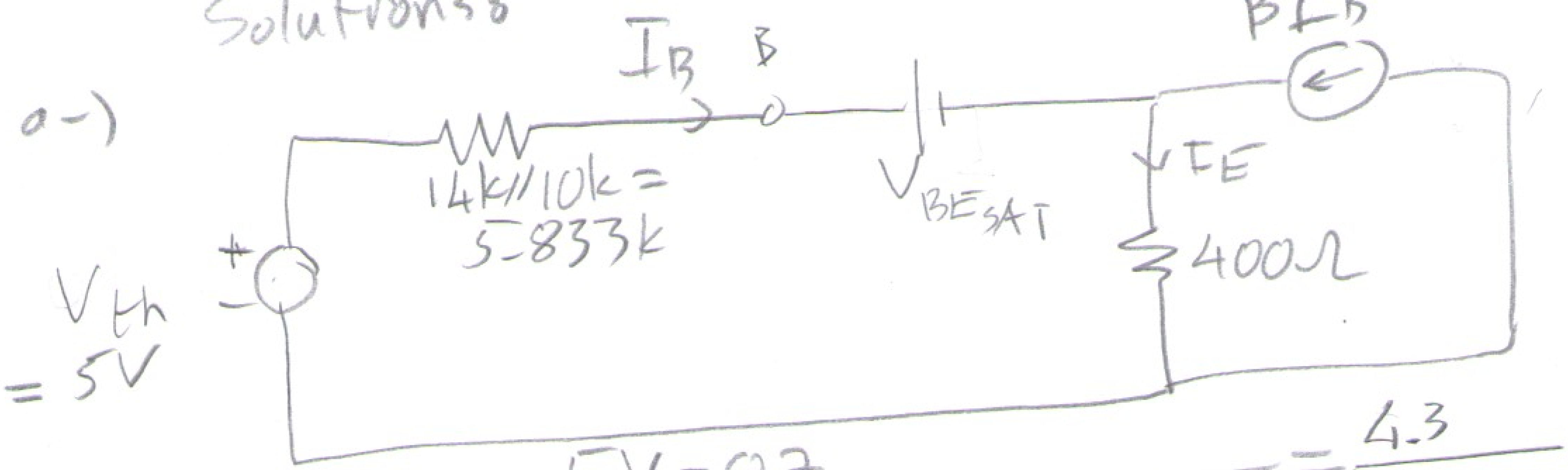
For the items of question (2) below, please take I_E as 10mA.

b) (8 points) Assuming that C_2 is very large, find the value of C_1 in order to have 3dB lower cut-off frequency.

c) (10 points) Assuming that C_1 is very large, find the value of C_2 in order to have 3dB lower cut-off frequency.

Solutions:

a-)



$$\Rightarrow I_B = \frac{5V - 0.7}{5.833\times10^3 + (99+1)400} = \frac{4.3}{45893} = 9.37\times10^{-5} A$$

$\Rightarrow I_B = 93.7 \mu A \Rightarrow I_C = 9.28 mA, I_E = 9.37 mA$

The transistor is at FA because of the circuit configuration

$V_{CE} = V_{CC} - I_E R_E = 12 - 9.37 \times 400\Omega = 12 - 3.748V = 8.252V$

b-) Drawing the AC equivalent circuit:

$$r_\pi = \frac{\beta}{g_m} = \frac{99 \times V_T}{I_C} = \frac{99 \times 26.6mV}{10mA} = 263.34\Omega$$

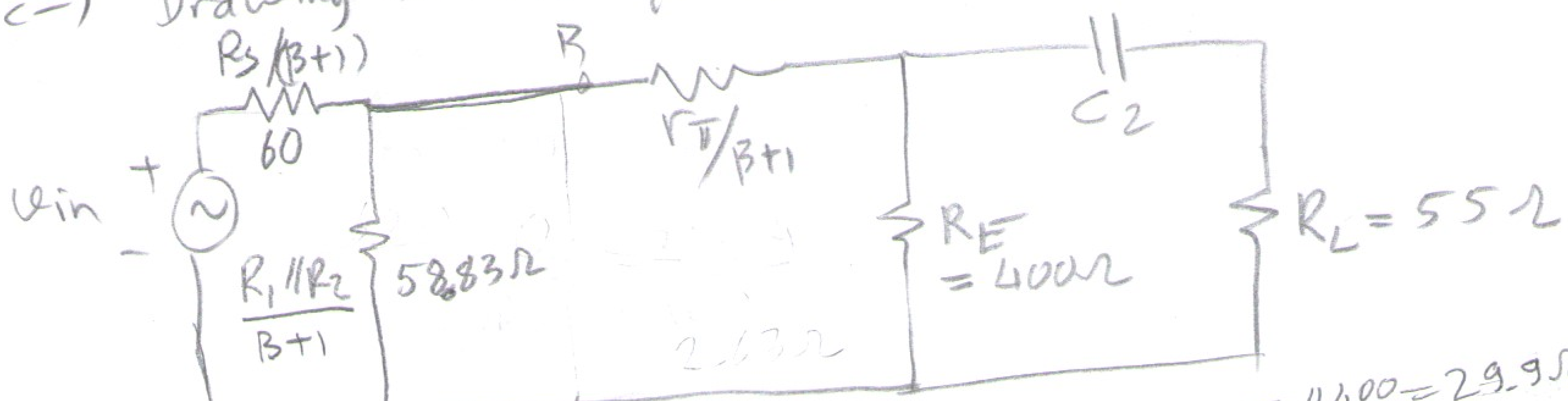
$$\frac{v_{be}}{v_{in}} = \frac{5883//(263+40000//5500)}{6000+(5883//40000//5500)+\frac{1}{j\omega C_1}} = \frac{(5883//(263+4835)}{6000+(5883//(263+4835))+\frac{1}{j\omega C_1}}$$

$$\frac{v_{be}}{v_{in}} = \frac{5883//5098}{6000+(5883//5098)+\frac{1}{j\omega C_1}} = \frac{2731.2}{6000+2731.2+\frac{1}{j\omega C_1}}$$

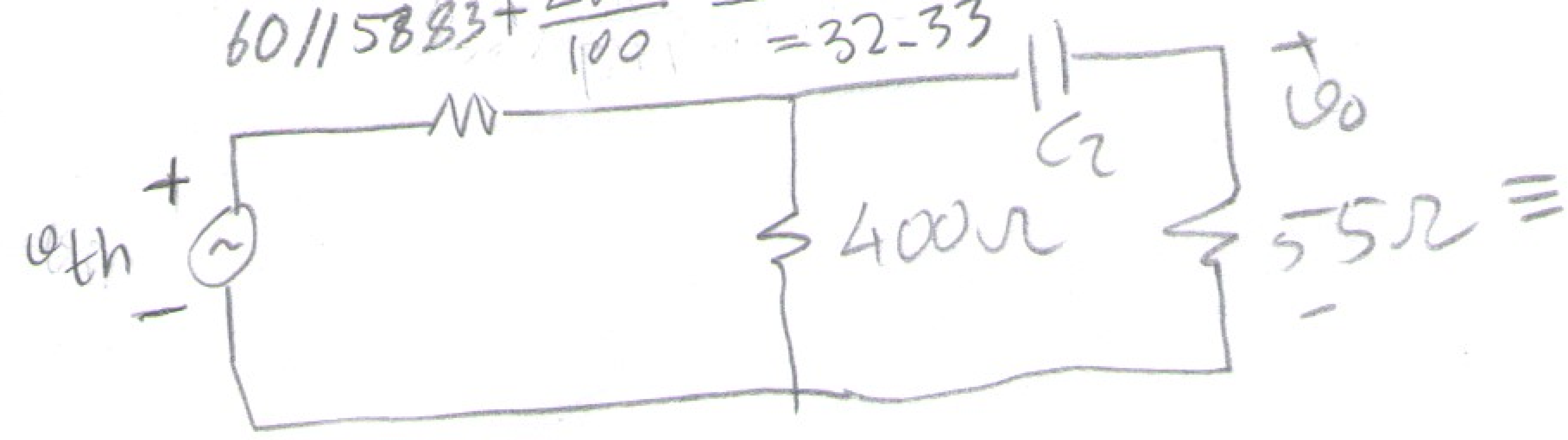
$$w_0 = 2\pi \times 20 = 125.66 = \frac{1}{8231.2C_1} \Rightarrow$$

$$C_1 = \frac{1}{125.66 \times 8231.2} = 9.668 \times 10^{-7} F = 0.9668 \mu F$$

c-) Drawing the AC equivalent circuit of the output

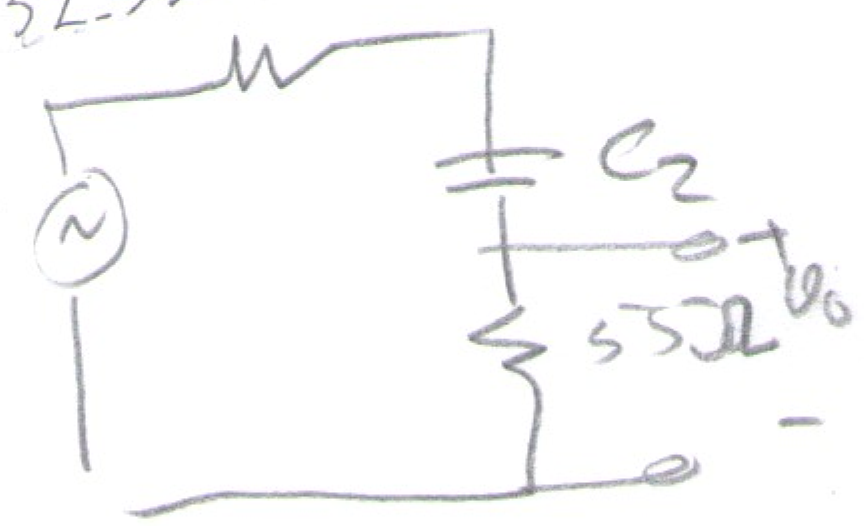


$$60//5883 + \frac{263}{100} = 29.7+2.63 = 32.33$$



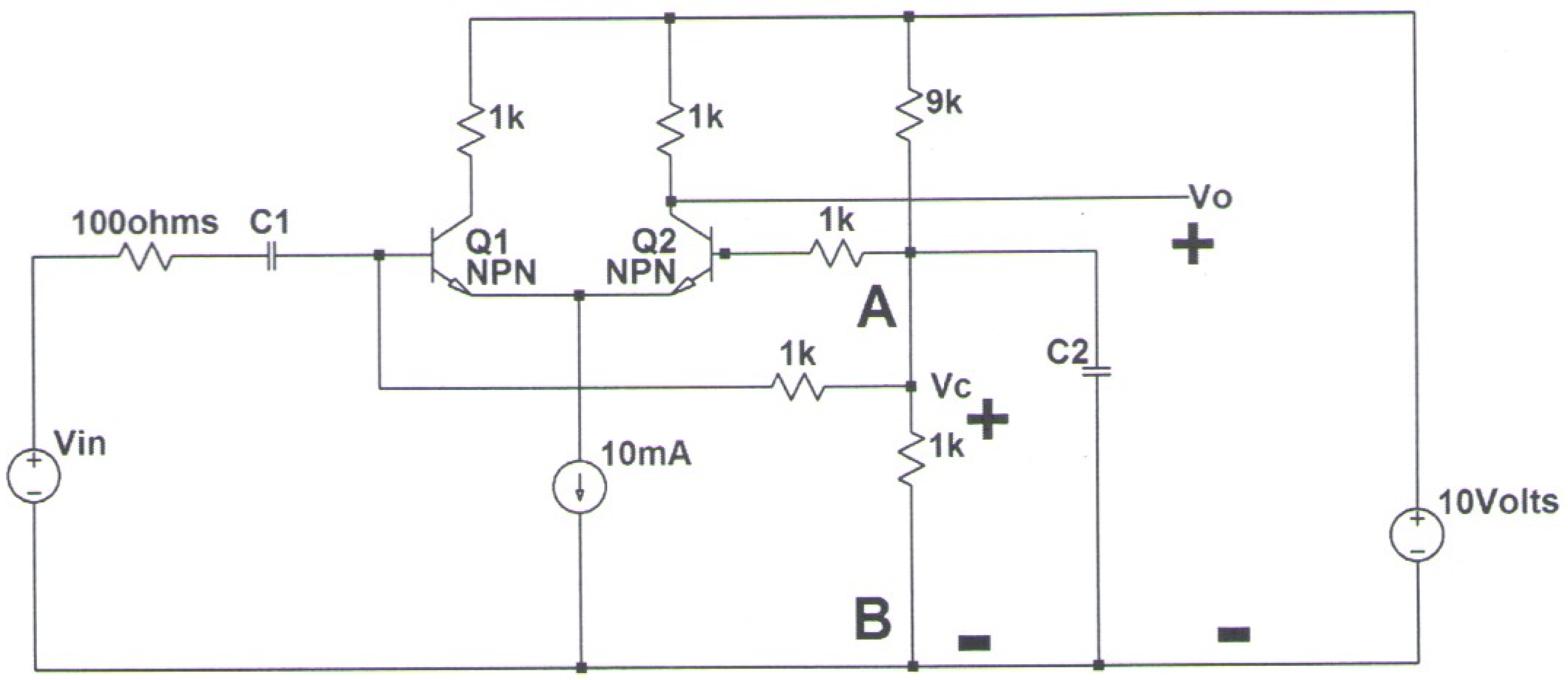
$\equiv$

$$32.33//400 = 29.9\Omega$$



$$\Rightarrow \frac{v_0}{v_{in}} = v_{th}\frac{55\Omega}{29.9\Omega+55\Omega+\frac{1}{j\omega C_2}} \Rightarrow W_0 = 125.66 = \frac{1}{84.91C_2}$$

$$C_2 = \frac{1}{125.66 \times 84.91} = 9.37 \times 10^{-5} F = 93.7 \mu F$$



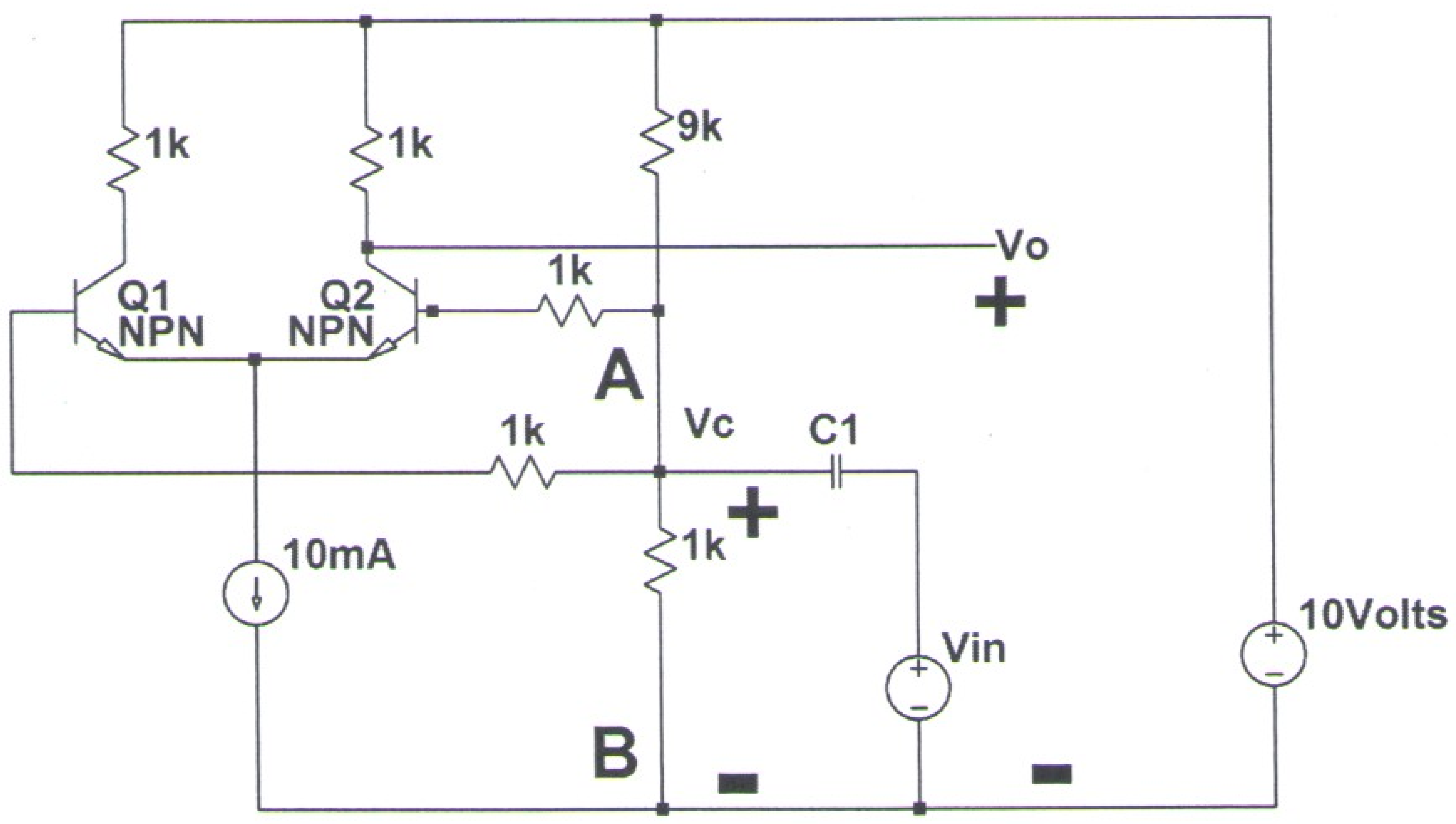
3. (20 points) At the circuit given above:

β=74, $V_{BE_{\cancel{ON}}}$ = 0.7 Volts, $V_{CE_{SAT}}$ = 0.3 Volts, C_1 and C_2 are very large.

SAT

Please answer the following:

a) (4 points) Find the DC bias of the transistors,
b) (4 points) Draw the AC equivalent circuit,
c) (4 points) Find the gain of the circuit defined as Vo/Vin assuming that the transistors are in forward active state.



d) (4 points) The circuit is modified to measure the common-mode voltage. Find the common-mode gain defined as Vo/Vc and assuming that the transistors are in forward active state. Please note that C_1 is very large again. The transistor parameters are still the same.
e) (4 points) Write the definition of CMMR (Common-Mode Rejection Ratio).

Solutions:

a-) As the transistor biases are the same $I_{E_1} = I_{E_2} = \frac{10mA}{2} = 5mA$

$$I_{B_1} = I_{B_2} = \frac{I_E}{(\beta+1)} = \frac{5mA}{75} = 0.0\overline{66}\,mA = 66.6\,\mu A$$

The base voltages are

$$10V \times \frac{1k}{1k+9k} - I_B R_B = 10 \times \frac{1}{10} - 66.6\times 10^{-6} A \times 1000\,\Omega$$

$$= 1 - 0.066\,V = 0.933\,V$$

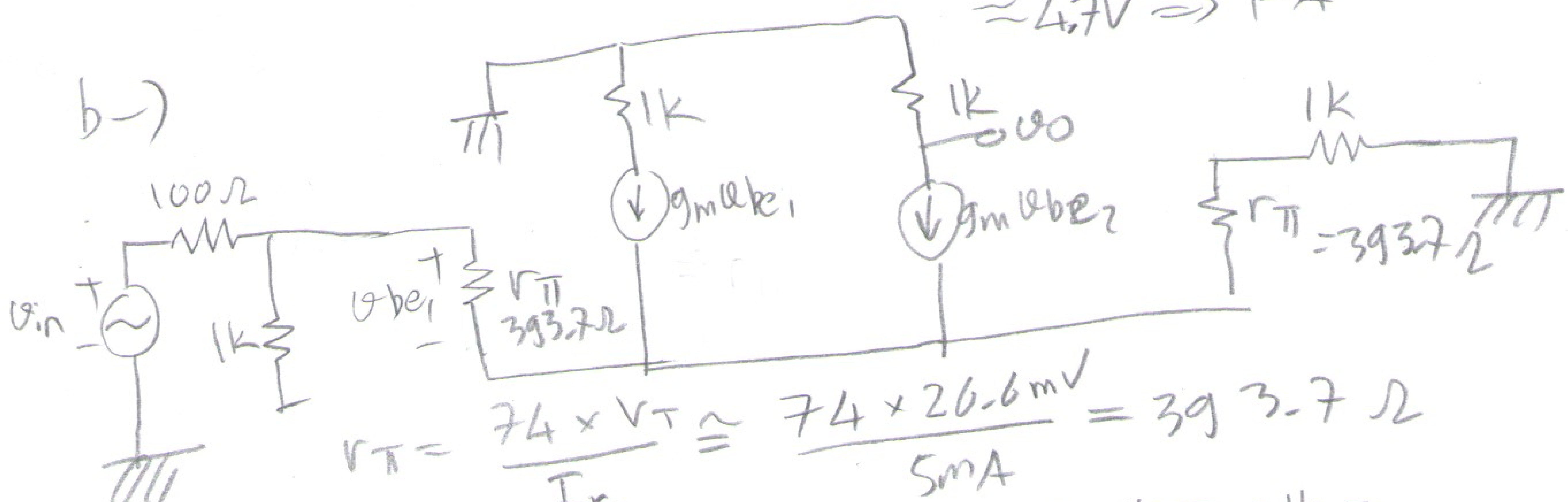
The collector voltages are

$$10V - 5mA \times 1k = 10V - 5\times 10^{-3} \times 10^{3} = 10 - 5 = 5V$$

$$\Rightarrow V_{CE} = 5V - (V_B - V_{BE\,SAT}) = 5 - (0.933) + 0.7 =$$

$$\simeq 4.7V \Rightarrow FA$$

b-)



$$r_\pi = \frac{74 \times V_T}{I_E} \cong \frac{74 \times 26.6mV}{5mA} = 393.7\,\Omega$$

$$g_m = \frac{5mA}{26.6mV} = 0.1888$$

$$1k \,//\, 2r_\pi + 1k =$$

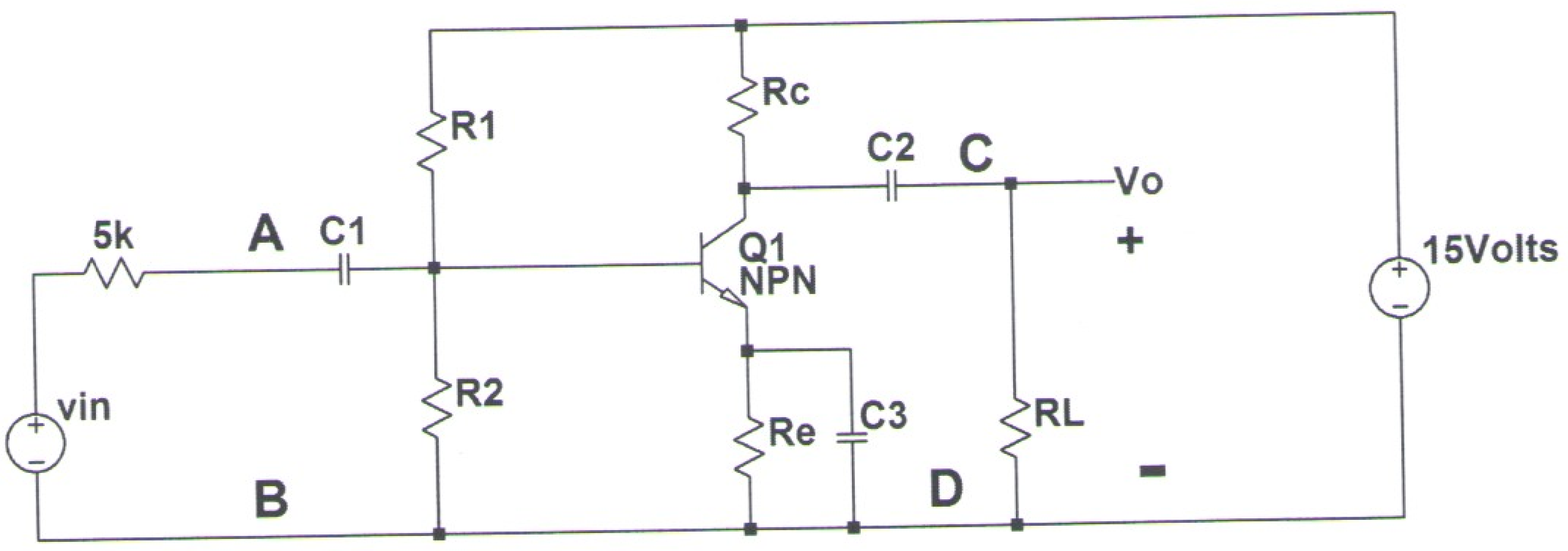
$$1000 \,//\, (787+1000) = 641.2\,\Omega$$

c-) $$\frac{v_o}{v_{in}} = \frac{641.2}{641.2+100} \cdot \frac{393}{1736} \cdot g_m \cdot R_c = 0.8651 \times 0.2264 \times 0.188 \times 1000$$

$$= 36.82$$

d-) As the common-mode voltage does not change the collector current because the emitters are biased through an ideal current source $A_c \equiv \frac{v_o}{v_c} = 0$

e-) $$CMRR = \frac{A_d}{A_c} = \frac{\text{differential mode gain}}{\text{common-mode gain}}$$



$V_{BE_{SAT}}$

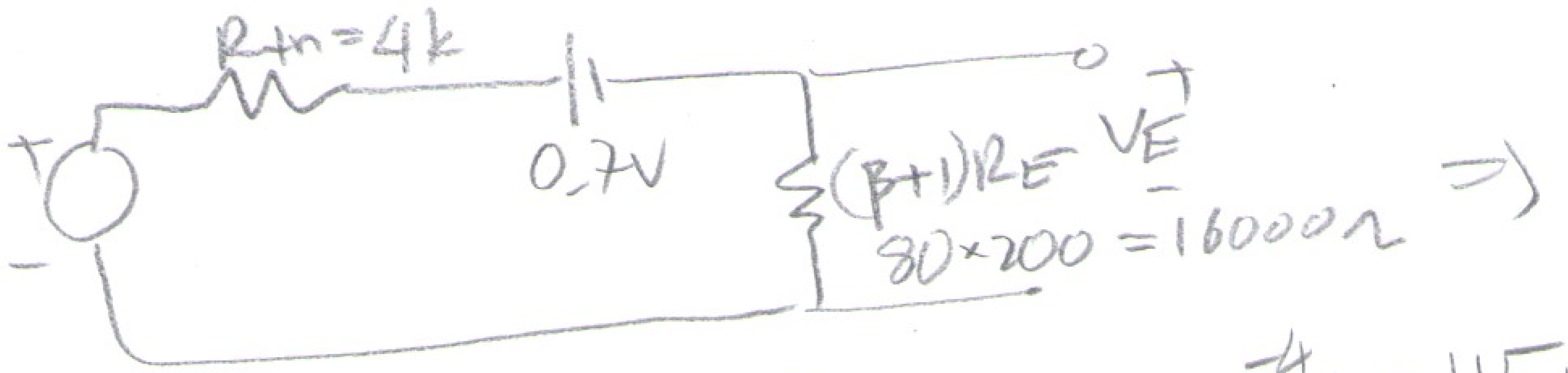
4. (30 points) At the BJT circuit given above:
β=79, $V_{BE_{ON}} = 0.7$ Volts, $V_{CE_{SAT}} = 0.4$ Volts, C_1, C_2 and C_3 are very large, Rc=400Ω, R_L= 1000Ω, R_1=20000Ω, R_2=5000Ω, Re=200Ω. Please answer the following:

a) (6 points) Find the DC bias of the circuit,
b) (6 points) Find the gain of the amplifier defined as Vo/Vin assuming that the transistor is at forward active and the collector current is equal to 10mA.
c) (6 points) Find the input impedance seen between points A and B assuming that the transistor is at forward active and the collector current is equal to 10mA.
d) (6 points) Find the output impedance seen between points C and D assuming that the transistor is at forward active and the collector current is equal to 10mA.
e) (6 points) Find the maximum undistorted peak-to-peak output swing of the amplifier assuming that the transistor is at forward active and the collector current is equal to 10mA.

Solutions:

a-) $15V \times \frac{R_2}{R_1+R_2} = 15 \times \frac{5k}{5k+20k} = 15\frac{5}{25} = \cancel{8}\frac{3}{\cancel{8}} = 3V$

$R_{th} = \frac{5k \times 20k}{5k+20k} = 5k\frac{20}{25} = \frac{20}{5} = 4k \Rightarrow$



$V_{th} = 3V$

$I_B = \frac{3V - 0.7}{4000 + 16000} = \frac{2.3}{20.000} = 1.15 \times 10^{-4} A = 115 \mu A$

$I_{EQ} = (\beta+1) I_B = 80 \times 115 \mu A = 9.2 mA$ $\quad I_c = 79 \times 115 \mu A = 9.085 mA$

$V_{CEQ} = 15V - 400 \times 0.009085A - 200 \times 9.2 \times 10^{-3} A =$

$V_{CEQ} = 15 - 3.634 - 1.84 = 9.526V \Rightarrow FA$