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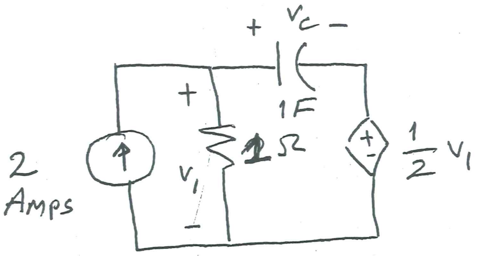
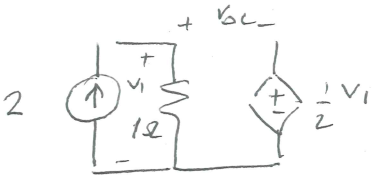
EEE 202 CIRCUIT THEORY
Final, Spring 2012-13

No credits will be given for unjustified answers.

Prob. 1 : (25 pt.s)

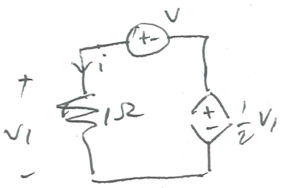
i : (6 pt.s) In the following circuit, $v_C(0) = 0$ Volts. Find $v_C(t)$.





$v_1 = 2V$ $\frac{1}{2}v_1 = 1V$

$\boxed{V_{oc} = 2-1 = 1V}$ 02

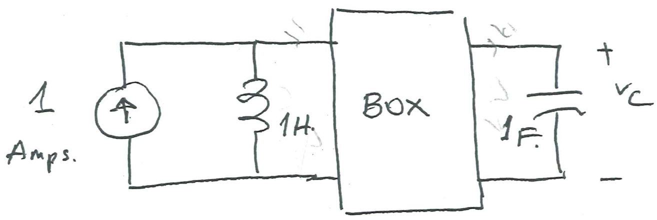


$v = v_1 - \frac{1}{2}v_1 = \frac{1}{2}v_1$, $v_1 = i$ } $v = \frac{1}{2}i$ $\Rightarrow$ $R_{eq} = \frac{1}{2}\Omega$ 02

$\Rightarrow \tau = \frac{1}{2}$ sec.

$v_C(t) - 1 = [0-1]e^{-t/\tau}$ $\Rightarrow$ $\boxed{v_C(t) = 1 - e^{-2t}}$ 02

ii : (4 pt.s) In the following circuit, the box contains only resistors and dependent sources. A student claims that $v_C(t)$ is found as $v_C(t) = 1 + 2e^{-t}\cos t + e^{-2t}\sin 3t$ Volts. Do you believe this result, and why.



$v_C(t) = V_F + V_H$

V_F : forced response $\Rightarrow$ due to unit step $\Rightarrow$ constant part 01

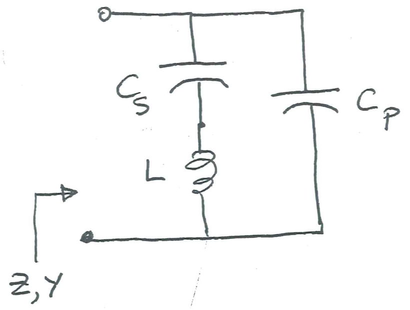
V_H $\Rightarrow$ homogeneous part $\Rightarrow$ due to the roots of characteristic polynomial $\Rightarrow$ poles of the system 01

$e^{-t}\cos t \rightarrow -1 \mp j$
$e^{-2t}\sin 3t \rightarrow -2 \mp j3$ } 4 roots (4 poles) are required.

But this is a SECOND ORDER SYSTEM 02

$\Rightarrow$ IMPOSSIBLE

iii : (5 pt.s) The following circuit is an equivalent of a piezoelectric crystal. Assume that the circuit is in sinusoidal steady state. Find a frequency ω_s such that $Z(\jmath\omega_s) = 0$. (This is called the series resonance frequency). Find a frequency ω_p such that $Y(\jmath\omega_p) = 0$. (This is called the parallel resonance frequency). Here Z and Y represent the impedance and admittance, as usual.



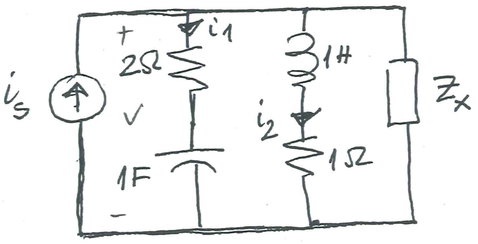
$$Y = j\omega C_p + \frac{1}{j\omega L + \frac{1}{j\omega C_s}} = j\omega C_p + \frac{j\omega C_s}{1-\omega^2 L C_s} = \frac{j\omega\left[C_p(1-\omega^2 L C_s) + C_s\right]}{1-\omega^2 L C_s}$$ 01

$$Z(j\omega_s) = 0 \Rightarrow 1-\omega_s^2 L C_s = 0 \Rightarrow \boxed{\omega_s = \sqrt{\frac{1}{L C_s}}}$$ 02

$$Y(j\omega_p) = 0 \Rightarrow C_p(1-\omega^2 L C_s) + C_s = 0$$

$$C_p + C_s = \omega_p^2 L C_s C_p \Rightarrow \boxed{\omega_p = \frac{1}{\sqrt{L\frac{C_s C_p}{C_s + C_p}}}}$$ 02

iv : (5 pt.s) Consider the following circuit, which is in sinusoidal steady state. Here Z_x is an unknown impedance, which contains only linear elements. If $i_1(t) = \cos t$, find $i_2(t)$.



01

$I_1 = 1$ A

$\omega = 1$ rad/sec.

$$V = \left(2 + \frac{1}{j}\right) I_1 = (2-j)$$

$$I_2 = \frac{V}{1+j} = \frac{2-j}{1+j} = \frac{\sqrt{5}\, e^{-j\tan^{-1}1/2}}{\sqrt{2}\, e^{+j45°}}$$ 02

$$\boxed{i_2(t) = \frac{\sqrt{5}}{\sqrt{2}}\cos\left(t - \tan^{-1}1/2 - 45°\right)}$$ 02

v : (5 pt.s) Consider the circuit shown above (problem **iv**). Let $i_s(t) = 3\cos t$, $i_1(t) = \cos t$. Find the average power dissipated in Z_x.

$$I_s = I_1 + I_2 + I_x \Rightarrow I_x = 3 - 1 - \frac{2-j}{1+j} = 2 - \frac{2-j}{1+j} = \frac{3j}{1+j}$$ 01

$$P_x = \frac{1}{2} V I_x^* = \frac{1}{2}(2-j)\frac{-3j}{1-j} = \frac{1}{2}\cdot\frac{-3-6j}{1-j} = \frac{1}{4}\cdot(-3-6j)(1+j)$$ 02

$$= \frac{1}{4}(6-3-9j) = \frac{1}{4}(3-9j)$$

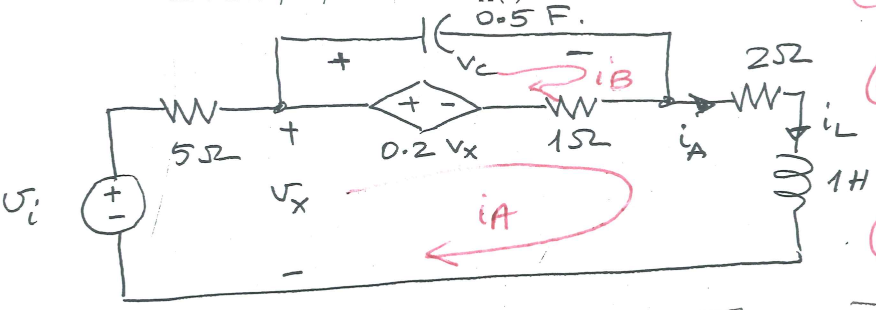
$$\boxed{P_{ave} = Re\{P_x\} = \frac{3}{4} \text{ Watts.}}$$ 02

Prob. 2 : (25 pt.s) Consider the following circuit. Assume that the initial conditions are given as $v_C(0) = \gamma$ Volts, and $i_L(0) = \beta$ Amps.

i : Find $I_A(s)$ in terms of $V_i(s)$, γ and β, i.e. like $I_A(s) = H_1(s)V_i(s) + H_2(s)\gamma + H_3(s)\beta$.

ii : Let $v_i(t) = 5\ u(t)$ (here $u(t)$ is the unit-step function). By choosing initial conditions γ and β appropriately, is it possible to have $i_A(t) = I_0\ u(t)$, where I_0 is a constant and $u(t)$ is the unit step function?(i.e. i_A does not contain any transients). If possible, find such initial conditions γ and β. (If possible, this is called "to set up a **mode**")

iii : Let $\gamma = \beta = 0$. Find $i_A(t)$.



10/10 i) Write mesh eqn.s

$7I_A + (I_A - I_B) + sI_A - \beta = -0.2V_x + V_i$

02 $\boxed{(s+8)I_A - I_B = -0.2V_x + V_i + \beta}$

$\frac{2}{s}I_B + \frac{1}{s}\gamma + (I_B - I_A) = 0.2V_x$

02 $\boxed{-I_A + \left(\frac{2}{s}+1\right)I_B = 0.2V_x - \frac{1}{s}\gamma}$

$V_x = V_i - 5I_A \Rightarrow \boxed{(s+7)I_A - I_B = 0.8V_i + \beta}$ (1) $\boxed{\left(\frac{2}{s}+1\right)I_B = 0.2V_i - \frac{1}{s}\gamma}$

$\Rightarrow \boxed{I_B = \frac{0.2s}{s+2}V_i - \frac{1}{s+2}\gamma}$ 02 $\Rightarrow \boxed{I_A = \frac{s+1.6}{(s+2)(s+7)}V_i - \frac{1}{(s+2)(s+7)}\gamma + \frac{1}{s+7}\beta}$ 04

ii) $v_i = 5u(t) \Rightarrow \boxed{I_A = \frac{5s+8}{s(s+2)(s+7)} - \frac{1}{(s+2)(s+7)}\gamma + \frac{1}{s+7}\beta}$

08/08

$\boxed{I_A = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+7}}$ we need $\boxed{\begin{matrix}K_2 = 0\\ K_3 = 0\end{matrix}}$ 02

$\boxed{\begin{matrix}K_2 = (s+2)I_A\Big|_{s=-2} = \frac{1}{5} - \frac{\gamma}{5} \\ K_3 = (s+7)I_A\Big|_{s=-7} = -\frac{27}{35} + \frac{\gamma}{5} + \beta\end{matrix}}$ 02 02

$\Rightarrow \boxed{\gamma = 1}$ 01 $\boxed{\beta = +\frac{24}{35}}$ 01 $\Rightarrow \boxed{I_A = \frac{K_1}{s}} \rightarrow \boxed{i_A(t) = K_1\,u(t)}$

iii) $\gamma = \beta = 0 \Rightarrow I_A = \frac{5s+8}{s(s+2)(s+7)} = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+7}$ 01

07/07

01 $\boxed{K_1 = sI_A\Big|_{s=0} = \frac{8}{14}}$ 01 $\boxed{K_2 = (s+2)I_A\Big|_{s=-2} = \frac{1}{5}}$ 01 $\boxed{k_3 = (s+7)I_A\Big|_{s=-7} = -\frac{27}{35}}$ 01

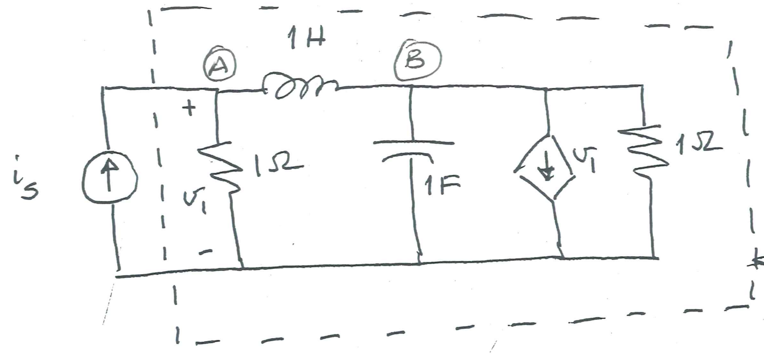
$\boxed{i_A(t) = \frac{4}{7} + \frac{1}{5}e^{-2t} - \frac{27}{35}e^{-7t}}$ 03

Prob. 3 : (25 pt.s) Consider the following circuit. Assume that the initial conditions are zero.

i : Find the transfer function $H(s) = \frac{V_1}{I_s}$.

ii : Assume that the circuit is in sinusoidal steady state and that $i_s(t) = 2\cos t$. Find $v_1(t)$.

iii : Assume that the circuit is in sinusoidal steady state and that $i_s(t) = 2\cos t$. Find the average power dissipated in the circuit N.



Transfer function ⇒ initial cond ≡ 0 !

13/13

i)

03 Ⓐ → $(1+\frac{1}{s})V_A - \frac{1}{s}V_B = I_s$

03 Ⓑ $-\frac{1}{s}V_A + (1+\frac{1}{s}+s)V_B = -V_1$

$V_1 = V_A$

$\frac{s+1}{s}V_A - \frac{1}{s}V_B = I_s$

$\frac{s^2+s+1}{s}V_B + \frac{s-1}{s}V_A = 0$

$\boxed{V_B = \frac{1-s}{s^2+s+1}V_A}$ 03

$\Rightarrow \frac{s+1}{s}V_A - \frac{1}{s}\frac{1-s}{s^2+s+1}V_A = I_s \Rightarrow$ $\boxed{H(s) = \frac{V_A}{I_s} = \frac{s^2+s+1}{s^2+2s+3}}$ 04

ii) $\omega = 1$ rad/sec $\Rightarrow H(j1) = \frac{j}{2+2j} = \frac{1}{2\sqrt{2}}e^{j45°}$ 02 $\quad I_s = 2$

06

$v_1(t) = \frac{2}{2\sqrt{2}}\cos(t+45°) = \frac{1}{\sqrt{2}}\cos(t+45°)$ 04

iii) $I_s = 2 \quad V_1 = \frac{1}{\sqrt{2}}e^{j45°}$ 02

06/06

$P_{ave} = Re\{\frac{1}{2}V_1 \cdot I_s^*\} = \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\cdot\cos 45 \cdot 2$ 02

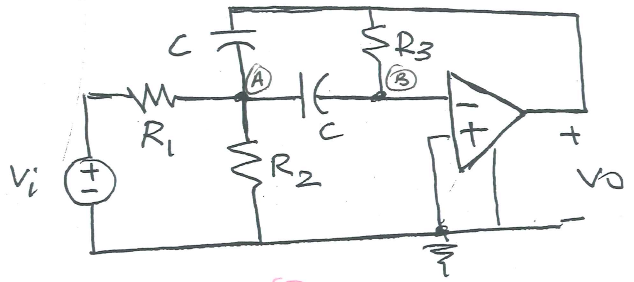
$\boxed{P_{ave} = \frac{1}{2} \text{ watts.}}$ 02

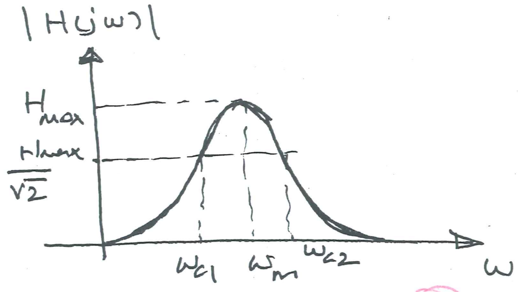
Prob. 4 : (25 pt.s) Consider the following circuit. Assume that the op-amp is ideal and operates in the linear region.

i : Find the transfer function $H(s) = \frac{V_O}{V_i}$. Show that it has the form $H(s) = \frac{K\beta s}{s^2+\beta s+\omega_0^2}$. Find K, β, ω_0 in terms of the resistor and capacitor values.

ii : Evaluate and sketch $|H(\jmath\omega)|$. Show that it has the band-pass characteristics given below. Find H_{max}, ω_m where $|H(\jmath\omega_m)| = H_{max}$ and the bandwidth $BW = \omega_{c2} - \omega_{c1}$ in terms of K, β, ω_0. (This is actually a practical realization of a high quality band-pass filter).

iii : Let $R_1 = R_2 = R_3 = 1\ \Omega$, $C = 1\ F$. Assume that $v_i(t) = 3\cos(2t - 30°)$ and that the circuit is in sinusoidal steady state. Find $v_O(t)$.





i) (B) → $V_A \cdot sC + \frac{V_o}{R_3} = 0 \Rightarrow \boxed{V_A = -\frac{V_o}{sR_3C}}$ 02

(A) → $-\frac{1}{R_1}V_i + \left(\frac{1}{R_2} + 2sC + \frac{1}{R_1}\right)V_A - sCV_o = 0$ 02

$$\Rightarrow -\frac{1}{R_1}V_i - \left[\left(\frac{R_1+R_2}{R_1R_2} + 2sC\right)\cdot\frac{1}{sR_3C} + sC\right]V_o = 0 \Rightarrow \boxed{H(s) = \frac{V_o}{V_i} = \frac{-\frac{1}{R_1C}s}{s^2 + \frac{2}{R_3C} + \frac{1}{R_3\frac{R_1R_2}{R_1+R_2}C^2}}}$$ 02

01 $\boxed{\beta = \frac{2}{R_3C}}$ 01 $\boxed{\omega_0^2 = \frac{1}{\frac{R_1R_2}{R_1+R_2}R_3C^2}}$ 01 $\boxed{K\beta = -\frac{1}{R_1C}} \Rightarrow$ 01 $\boxed{K = -\frac{R_3}{2R_1}}$

ii) $H(j\omega) = \frac{-K\beta j\omega}{\omega_0^2-\omega^2+j\beta\omega}$ $\quad |H(j\omega)| = \frac{K}{\sqrt{1+\left(\frac{\omega_0^2-\omega^2}{\beta\omega}\right)^2}}$ 02

$\omega \to 0$ $|H(j0)| \to 0$
$\omega \to \infty$ $|H(j\infty)| \to 0$ 02
Band-pass.

For max → denominator min $\Rightarrow \boxed{\omega_M = \omega_0}$ 01 $\Rightarrow \boxed{|H(j\omega_m)| = H_{max} = |K|.}$ 01

For ω_{c1}, ω_{c2} → $\frac{\omega_0^2-\omega^2}{\beta\omega} = \mp 1$ 02 $\Rightarrow \boxed{\omega^2 \mp \beta\omega - \omega_0^2 = 0} \Rightarrow \Delta = \beta^2 + 4\omega_0^2$

$\omega_{c1} = \frac{-\beta+\sqrt{\Delta}}{2}$ $\quad \omega_{c2} = \frac{\beta+\sqrt{\Delta}}{2}$ $\Rightarrow \boxed{BW = \omega_{c2} - \omega_{c1} = \beta}$ 02

iii) 05/05

$\frac{V_o}{V_i} = \frac{-s}{s^2+2s+2}$ → $\omega = 2$ $\quad H(j2) = \frac{-j2}{-2+j4} = \frac{-8+j4}{20} = \frac{\sqrt{80}}{20}e^{j(\pi - \tan^{-1}\frac{1}{2})}$ 02

$V_i = 3e^{-j30°}$ 01

$v_o(t) = \frac{3\sqrt{80}}{20}\cos(2t + \underbrace{\pi - \tan^{-1}\tfrac{1}{2} - 30°}_{\phi}) = \frac{3}{\sqrt{5}}\cos(2t+\phi)$ 02