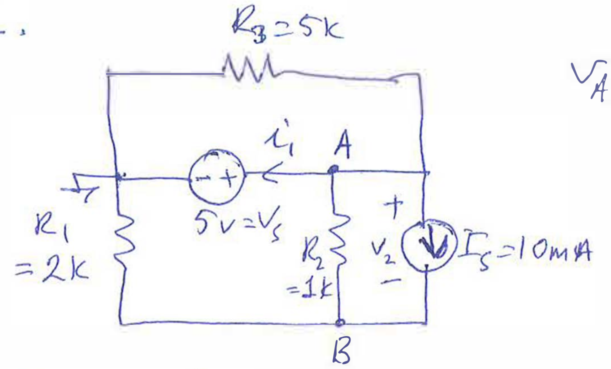
EEE202

Homework 1 solution

Q1.



$V_A = 5v.$

$$\frac{V_B - V_A}{R_2} - I_s + \frac{V_B}{R_1} = 0$$

$$\frac{V_B - 5}{1} - 10 + \frac{V_B}{2} = 0$$

$$2V_B - 10 - 20 + V_B = 0$$

$$3V_B = 30$$

$$V_B = 10\text{ v.}$$

To find i_1:

$$i_1 + \frac{5-10}{1} + 10 + \frac{5}{5} = 0$$

$$i_1 = -6\text{ mA}.$$

$\therefore$ power of voltage source $= -6 \times 5 = -30\,mW < 0$

$\therefore$ voltage source is supplying power.

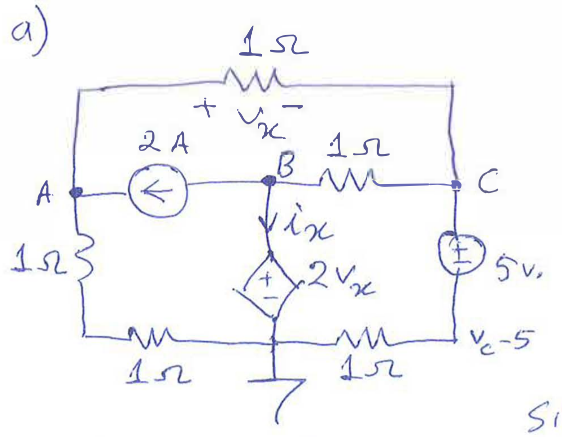
To find v_2:

$$V_2 = V_A - V_B = 5 - 10 = -5\,v.$$

$\therefore$ power of current source $= -5 \times 10 = -50\,mW < 0$

$\therefore$ Current source is supplying power.

Q2. a)



A: $\dfrac{V_A - V_C}{1} + \dfrac{V_A}{1+1} - 2 = 0$

$$3V_A - 2V_c = 4 \quad (\text{eqn. 1})$$

C: $\dfrac{V_C - V_A}{1} + \dfrac{V_C - V_B}{1} + \dfrac{V_C - 5}{1} = 0$

$$3V_C - V_A - V_B = 5$$

Since $V_B = 2V_x = 2(V_A - V_C)$

$$3V_C - V_A - 2(V_A - V_C) = 5$$

$$5V_C - 3V_A = 5 \quad (\text{eqn. 2})$$

Add eqn 1 and eqn 2

$$3V_c = 9 \Rightarrow \boxed{V_c = 3}$$

To find i_x: KCL at B

$$i_x + 2 + \frac{V_B - V_C}{1} = 0$$

$$i_x + 2 + \left(\frac{2}{3} - 3\right) = 0$$

$$\boxed{i_x = +\frac{1}{3} A.}$$

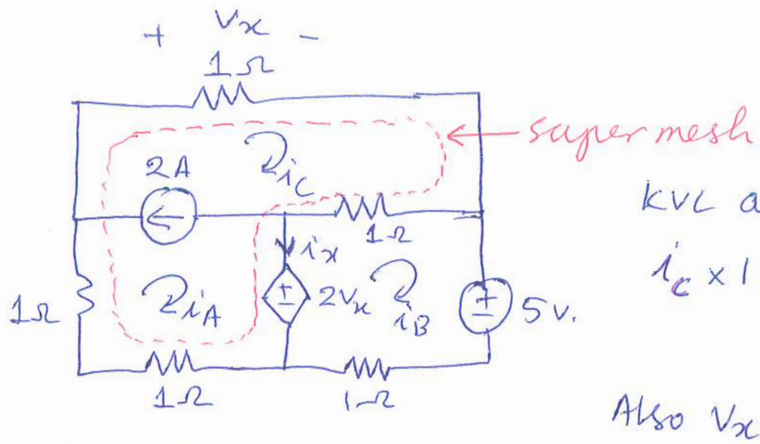
To find V_x:

$$V_x = V_A - V_C = \frac{10}{3} - 3 = \boxed{\frac{1}{3}\,v = V_x}$$

From eqn. 1 $V_A = \dfrac{4 + 2V_C}{3} = \dfrac{4+6}{3} = \boxed{\dfrac{10}{3}\,v = V_A}$

$$V_B = 2V_x = 2(V_A - V_C) = 2\left(\frac{10}{3} - 3\right) = \boxed{\frac{2}{3}\,v = V_B}$$

b)



KVL at supermesh

$i_C \times 1 + (i_C - i_B) \times 1 + 2V_x$

$+ i_A \times 1 + i_A \times 1 = 0$

Also $V_x = 1 \times i_C = i_C$

$\therefore \ i_C + i_C - i_B + 2i_C + 2i_A = 0$

$2i_A - i_B + 4i_C = 0$

Also $i_C - i_A = 2A. \Rightarrow i_A = i_C - 2$

$\therefore \ 2(i_C - 2) - i_B + 4i_C = 0$

$\boxed{-i_B + 6i_C = 4 \quad \text{eqn. 1}}$

KVL at mesh B:

$(i_B - i_C) \times 1 + 5 + i_B \times 1 - 2V_x = 0$

Since $V_x = i_C$

$\boxed{2i_B - 3i_C = -5 \quad \text{eqn. 2}}$

Multiply eqn. 2 by 2 and add to eqn. 1.

$3i_B = -10 + 4 = -6 \Rightarrow \boxed{i_B = -2\ A}$

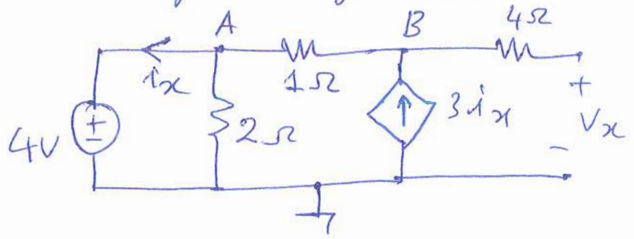
From eqn. 1 $\quad i_C = \dfrac{4 + i_B}{6} = \dfrac{4-2}{6} = \boxed{\dfrac{1}{3} A = i_C}$

$i_A = i_C - 2 = \dfrac{1}{3} - 2 = \boxed{-\dfrac{5}{3} A = i_A}$

$i_x = i_A - i_B = -\dfrac{5}{3} - (-2) = \dfrac{-5}{3} + 2 = \dfrac{-5+6}{3} = \boxed{\dfrac{1}{3} A = i_x}$

$V_x = i_C \times 1 = \dfrac{1}{3} \times 1 = \boxed{\dfrac{1}{3} V = V_x}$

Q3. a) step 1: Keep voltage source and kill current source



$V_A = 4V.$

KCL at B : $\frac{V_B - 4}{1} - 3i_x = 0 \Rightarrow V_B - 3i_x = 4$ eqn.1

KCL at A : $\frac{4 - V_B}{1} + \frac{4}{2} + i_x = 0 \Rightarrow -V_B + i_x = -6$ eqn.2

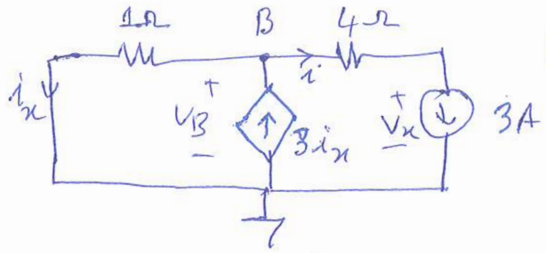
From eqn.2, $i_x = V_B - 6$. Put this in eqn 1 $V_B - 3(V_B - 6) = 4$

$-2V_B + 18 = 4$

$\therefore V_{x,1} = V_B = 7v.$ $(i_x = 7-1 = 1A)$ $-2V_B = -14$

$\boxed{V_B = 7v}$

Step 2: Keep the current source and kill the voltage source



B: $i_x - 3i_x + 3 = 0$

$\Rightarrow i_x = \frac{3}{2} A.$

$V_B = i_x \times 1 = \frac{3}{2} v.$

$i = 3i_x - i_x = 2i_x = 2 \times \frac{3}{2} = 3A.$

$\therefore 4i + V_x - V_B = 0 \Rightarrow V_x = V_B - 4i = \frac{3}{2} - 4 \times 3$

$\Rightarrow V_{x,2} = 1.5 - 12 = -10.5 v.$

Overall result is the sum of results in step 1 and step 2.

$V_x = V_{x,1} + V_{x,2} = 7 - 10.5 = -3.5 v.$

b) 4V voltage source: $i_x = i_{x,1} + i_{x,2} = 1 + \frac{3}{2} = 2.5 A.$

P of 4V voltage source $= 4 \times 2.5 A = 10W$ receiving power

3A current source: $P = 3 \times V_x = 3 \times (-3.5) = -10.5W$ supplying power

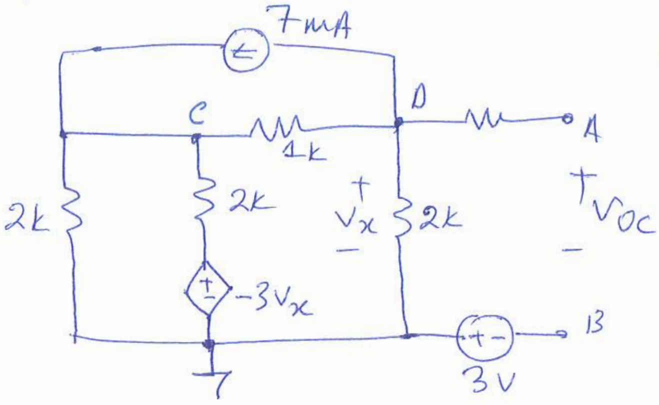
$3i_x$ current source: $V_B = V_{B,1} + V_{B,2} = 7 + \frac{3}{2} = 8.5 V.$

$P = 8.5 \times (-3i_x) = 8.5 \times (-3 \times 2.5) = -63.75 W$ supplying power

c) From linearity

$V_x = \frac{8}{4} \times V_{x,1} + \frac{-5}{3} \times V_{x,2} = 2 \times 7 - \frac{5}{3} \times (-10.5) = 14 + 17.5 = 31.5 v.$

Q4. To find the Thevenin equivalent circuit let us first find the open-circuit voltage



Node equation at C:

$$\frac{V_C}{2} + \frac{V_C-(-3V_x)}{2} + \frac{V_C-V_D}{1} - 7 = 0$$

Since $V_x = V_D$

$$\frac{V_C}{2} + \frac{V_C+3V_D}{2} + \frac{V_C-V_D}{1} = 7$$

$$V_C + V_C + 3V_D + 2V_C - 2V_D = 14$$

$$\boxed{4V_C + V_D = 14} \quad \text{eqn. 1}$$

Node equation at D:

$$\frac{V_D}{2} + 7 + \frac{V_D - V_C}{1} = 0$$

$$V_D + 14 + 2V_D - 2V_C = 0$$

$$\boxed{3V_D - 2V_C = -14} \quad \text{eqn. 2}$$

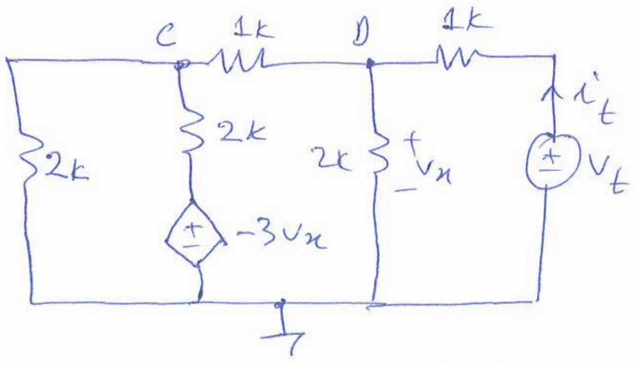
multiply eqn. 2 by 2 and add to eqn. 1:

$$7V_D = -14 \qquad V_D = -2\text{ V.}$$

$$\therefore V_{OC} = V_D + 3 = -2 + 3 = \boxed{1\text{ V.} = V_{TH}}$$

To find R_{TH}, let us kill the independent sources and apply a test source between A and B:

Note: We may apply a voltage test source or a current test source. Both are acceptable.



$V_D = V_x$

$$C: \quad \frac{V_C}{2} + \frac{V_C+3V_D}{2} + \frac{V_C-V_D}{1} = 0$$

$$V_C + V_C + 3V_D + 2V_C - 2V_D = 0$$

$$4V_C + V_D = 0 \quad (*)$$

$$D: \quad \frac{V_D-V_C}{1} + \frac{V_D}{2} + \frac{V_D - V_t}{1} = 0$$

$$2V_D - 2V_C + V_D + 2V_D = 2V_t$$

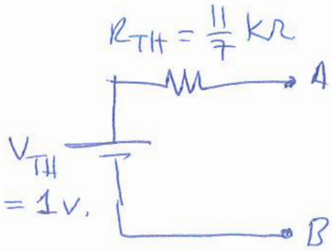
$$5V_D - 2V_C = 2V_t \quad (**)$$

Multiply (**) by 2 and add to (*)

$$11V_D = 4V_t \Rightarrow V_D = \frac{4}{11}V_t$$

$$i_t = \frac{V_t - V_D}{1} = V_t - \frac{4}{11}V_t = \frac{7}{11}V_t$$

$$R_{TH} = \frac{V_t}{i_t} = \frac{V_t}{\frac{7}{11}V_t} = \frac{11}{7}\text{ k}\Omega.$$



Q5. a) $n = 4$
$b = 6$

$n-1 = 3$ KCL equations

$$i_1 + i_2 - i_5 = 0$$
$$-i_2 + i_3 + i_6 = 0$$
$$-i_3 + i_4 + i_5 = 0$$

$b - n + 1 = 6 - 4 + 1 = 3$ KVL equations

$$v_2 + v_6 - v_1 = 0$$
$$v_3 + v_4 - v_6 = 0$$
$$-v_5 - v_3 - v_2 = 0$$

b element equations

$$v_1 - R_1 i_1 = 0$$
$$v_2 - R_2 i_2 = 0$$
$$v_3 - R_3 i_3 = 0$$
$$v_4 - R_4 i_4 = 0$$
$$i_5 = 10$$
$$v_6 = 5$$

$$\underbrace{\left[\begin{array}{cccccc|cccccc} 0&0&0&0&0&0&1&1&0&0&-1&0\\ 0&0&0&0&0&0&0&-1&1&0&0&1\\ 0&0&0&0&0&0&0&0&-1&1&1&0\\ \hline -1&1&0&0&0&1&0&0&0&0&0&0\\ 0&0&1&1&0&-1&0&0&0&0&0&0\\ 0&-1&-1&0&-1&0&0&0&0&0&0&0\\ \hline 1&0&0&0&0&0&10&0&0&0&0&0\\ 0&1&0&0&0&0&0&10&0&0&0&0\\ 0&0&1&0&0&0&0&0&10&0&0&0\\ 0&0&0&1&0&0&0&0&0&10&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0&0&0&0&0 \end{array}\right]}_{A} \underbrace{\begin{bmatrix} v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ i_1\\ i_2\\ i_3\\ i_4\\ i_5\\ i_6 \end{bmatrix}}_{x} = \underbrace{\left[\begin{array}{c} 0\\ 0\\ 0\\ \hline 0\\ 0\\ 0\\ \hline 0\\ 0\\ 0\\ 0\\ 10\\ 5 \end{array}\right]}_{b}$$

$$x = A^{-1} b = \left[\begin{array}{c} 52.5\\ 47.5\\ 52.5\\ -47.5\\ -100\\ 5\\ \hline 5.25\\ 4.75\\ 5.25\\ -4.75\\ 10\\ -0.5 \end{array}\right]$$

$$Ax = b$$

Specifically

$v_5 = -100$ V.

$i_6 = -0.5$ mA

b) $V_B = 5V.$

$$\frac{V_A - 5}{10} + \frac{V_A}{10} - 10 = 0 \Rightarrow V_A - 5 + V_A = 100 \Rightarrow V_A = \frac{105}{2} = 52.5V.$$

$$\frac{V_C - 5}{10} + \frac{V_C}{10} + 10 = 0 \Rightarrow V_C - 5 + V_C = -100 \Rightarrow V_C = \frac{-95}{2} = -47.5$$

$$\therefore \quad V_5 = V_C - V_A = -47.5 - (52.5) = -100\,V. \checkmark$$

$$i_6 = i_2 - i_3 = \frac{V_A - V_B}{10} - \frac{V_B - V_C}{10} = \frac{52.5 - 5}{10} - \frac{5 - (-47.5)}{10}$$

$$= \frac{52.5}{10} - \frac{5}{10} - \frac{5}{10} - \frac{47.5}{10} = \frac{5}{10} - \frac{5}{10} - \frac{5}{10} = -0.5 \checkmark$$

c) $\boxed{i_C = -10\,mA}$

$$10(i_A - i_C) + 5 + i_A \times 10 = 0 \Rightarrow 20 i_A - 10 i_C + 5 = 0$$

$$20 i_A + 100 + 5 = 0$$

$$\boxed{i_A = \frac{-105}{20} = -5.25\,mA.}$$

$$10(i_B - i_C) + 10\, i_B - 5 = 0$$

$$20 i_B - 10 i_C - 5 = 0$$

$$20 i_B + 100 - 5 = 0 \Rightarrow 20 i_b = -95 \quad \boxed{i_b = \frac{-95}{20} = -4.75\,mA.}$$

$$V_5 = -V_2 - V_3 = -10(i_A - i_C) - 10(i_B - i_C)$$

$$= -10 i_A + 10 i_C - 10 i_b + 10 i_C$$

$$= -10(i_A + i_B - 2 i_C) = -10(-5.25 - 4.75 + 20)$$

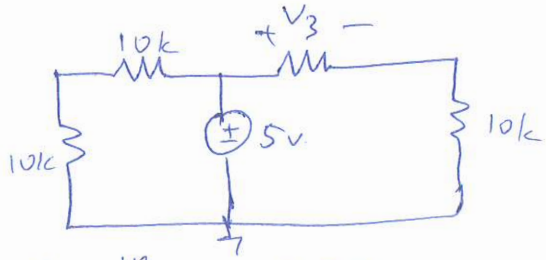
$$= -10(-10 + 20) = -10 \times 10 = -100\,V \checkmark$$

$$i_6 = i_2 - i_3 = (i_A - i_C) - (i_B - i_C) = i_A - i_C - i_B + i_C = i_A - i_B$$

$$= -5.25 - (-4.75)$$

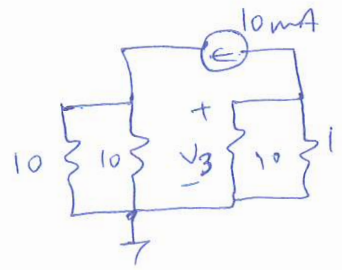
$$= -5.25 + 4.75 = -0.5\,mA \checkmark$$

d) step1 : kill current source



$$V_3 = \frac{10}{10+10} \times 5 = 2.5V.$$

step2 : kill voltage source



$$V_3 = -10 \times (10 // 10)$$

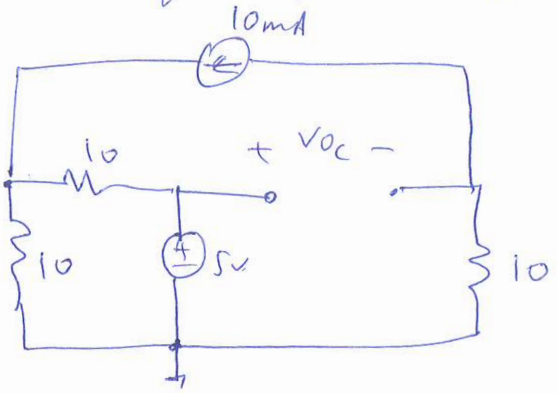
$$= -10 \times 5$$

$$= -50V.$$

Overall result is the sum of solutions in step1 and 2.

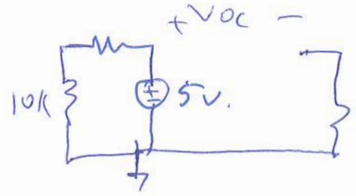
$V_3 = 2.5 + 50 = 52.5V.$ ✓ same as in (a).

e) Let us first find the open-circuit voltage.



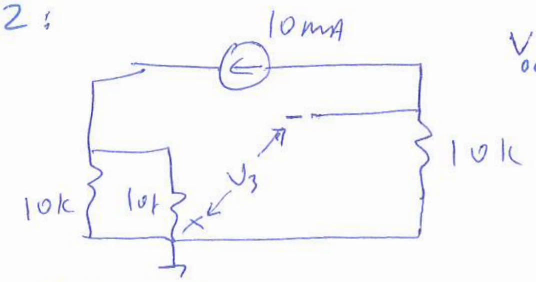
Again superposition can be used!

Step 1:



$V_{oc,1} = 5v.$

Step 2:

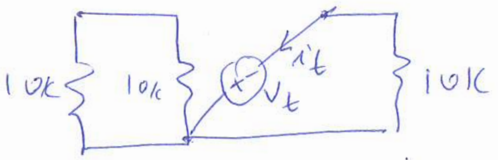


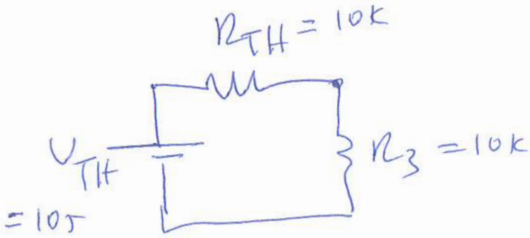
$V_{oc,2} = +100v.$

$V_{oc} = V_{oc,1} + V_{oc,2}$
$= 5+100 = 105 v.$

Now let us find R_{TH}:

$R_{TH} = \frac{v_t}{i_t} = 10k\Omega.$





$V_3 = \frac{10}{10} \times 105 = 52.5 v.$ ✓

Same as in (a) and (d).

Q6. Since the circuit is linear we may write

$$v_0 = K_1 v_{s1} + K_2 v_{s2} + K_3 v_{s3} + K_4 v_{s4}$$

The data given in the Table can be collected into a matrix equation

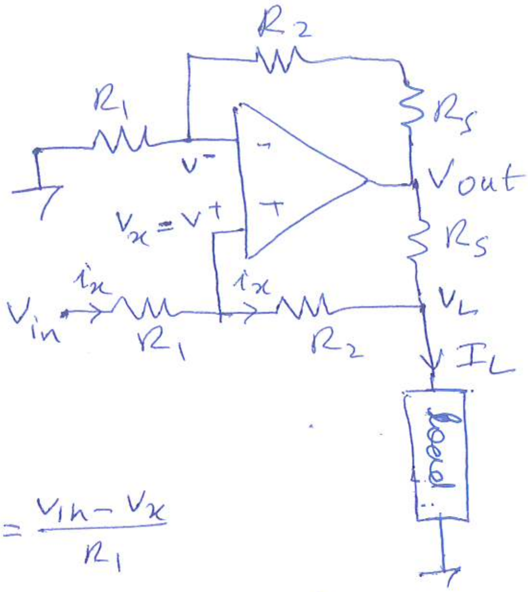
$$\underbrace{\begin{bmatrix} 2 & 4 & -4 & 1 \\ 1 & 2 & 2 & 1.5 \\ 1 & 4 & 2 & 2 \\ 0 & 5 & 3 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix}}_{K} = \underbrace{\begin{bmatrix} 20 \\ -4 \\ -1 \\ 3 \end{bmatrix}}_{b} \qquad AK = b$$

Using Matlab the determinant of the coefficient matrix is $48 \neq 0$. Therefore the system has a unique solution

Again using Matlab $K = A^{-1}b = \begin{bmatrix} 1 \\ 2 \\ -3 \\ -2 \end{bmatrix}$

$$\therefore \quad v_0 = v_{s_1} + 2v_{s_2} - 3v_{s_3} - 2v_{s_4}$$

Q7.



Assuming that the OpAmp is in the linear region (i.e. not saturated)

$v^+ = v^-$, Define $V_x \stackrel{\Delta}{=} v^+ = v^-$

$$\frac{V_{out} - V_x}{R_2 + R_s} = \frac{V_x}{R_1}$$

$$\frac{V_{out}}{R_2 + R_s} = V_x\left(\frac{1}{R_2+R_s} + \frac{1}{R_1}\right)$$

$$V_{out} = V_x\left(1 + \frac{R_2+R_s}{R_1}\right)$$

$$i_x = \frac{V_{in} - V_x}{R_1}$$

$$V_L = V_{in} - R_1 i_x - R_2 i_x$$

$$V_L = V_{in} - (R_1+R_2)\frac{V_{in}-V_x}{R_1}$$

$$V_L = V_{in}\left(1 - \frac{R_1+R_2}{R_1}\right) + \frac{R_1+R_2}{R_1}V_x = -\frac{R_2}{R_1}V_{in} + \left(1+\frac{R_2}{R_1}\right)V_x$$

$$i_L = i_x + \frac{V_{out} - V_L}{R_s} = \frac{V_{in}-V_x}{R_1} + \frac{1}{R_s}\left[V_x\left(1+\frac{R_2+R_s}{R_1}\right) + \frac{R_2}{R_1}V_{in} - \left(1+\frac{R_2}{R_1}\right)V_x\right]$$

$$= \frac{V_{in}-V_x}{R_1} + \frac{1}{R_s}\left[V_x\frac{R_s}{R_1} + \frac{R_2}{R_1}V_{in}\right] = \frac{V_{in}}{R_1} - \frac{V_x}{R_1} + \frac{V_x}{R_1} + \frac{R_2}{R_sR_1}V_{in}$$

$$i_L = V_{in}\left(\frac{1}{R_1} + \frac{R_2}{R_sR_1}\right) = V_{in} \times \frac{R_s+R_2}{R_sR_1}$$