Spring 2016-17 **EEE 202: Circuit Theory** **Instructors: Ö. Morgül & E.Ü. Sarıtaş**

23/03/2017 Midterm 1 Examination 120 minutes

Name:

Student ID:

Signature:

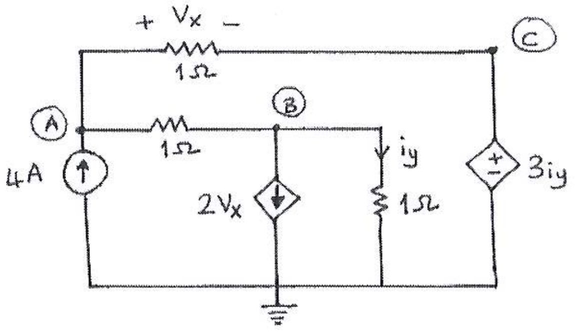
Instructions:

1. Attempt all questions.
2. No partial credits to unjustified answers.
3. Show all the steps of your work clearly.
4. Specify all node, mesh labels (A, B, C, etc.) clearly on the circuit diagrams.
5. Clearly indicate the unit of any answer you provide.
6. Extra paper is available if you need it.

Question	Points	Your Score
Q1	24	
Q2	26	
Q3	26	
Q4	24	
TOTAL	100	

Question 1. [24 points]

i) **[6 points]** Consider the following circuit. Find V_A, node voltage of node A, by using **node analysis.**



$i_y = V_B$

$V_x = V_A - 3i_y = V_A - 3V_B$

KCL at A : $-4 + V_A - 3V_B + V_A - V_B = 0$

$2V_A - 4V_B = 4$

$\boxed{V_A - 2V_B = 2}$

KCL at B : $V_B - V_A + 2(V_A - 3V_B) + V_B = 0$

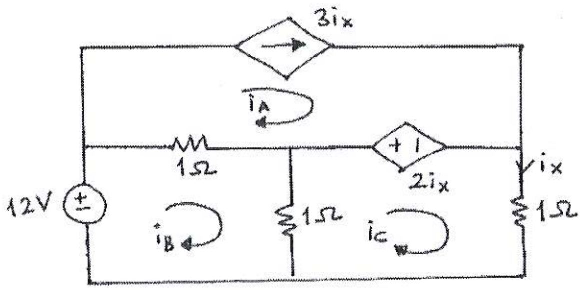
$V_A - 4V_B = 0$

$\boxed{V_A = 4V_B}$

* $4V_B - 2V_B = 2 \Rightarrow V_B = 1V$

$V_A = 4V_B = \boxed{4V}$

ii) **[6 points]** Consider the following circuit. Find the mesh current i_c by using **mesh analysis.**



$i_x = i_c$

$i_A = 3i_x = 3i_c$

KVL at B : $-12 + i_B - i_A + i_B - i_c = 0$

$2i_B - i_A - i_c = 12$

$\boxed{2i_B - 4i_c = 12}$

KVL at C : $i_c - i_B + 2\underbrace{i_x}_{i_c} + i_c = 0$

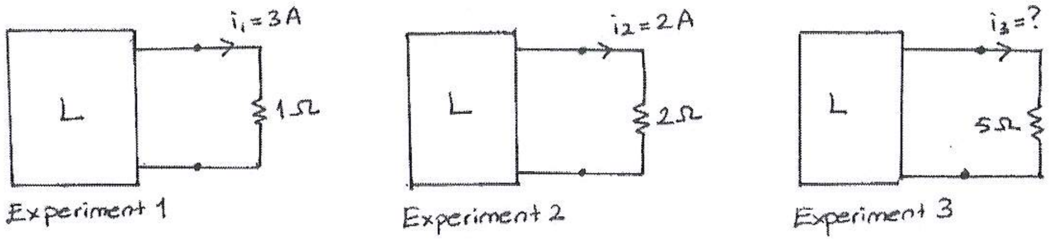
$\boxed{4i_c = i_B}$ $\longrightarrow$ $8i_c - 4i_c = 12$

$\boxed{i_c = 3A}$

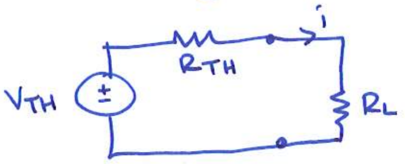
$\boxed{i_B = 4i_c = 12A}$

$\boxed{i_A = 3i_c = 9A}$

iii) **[6 points]** Consider the following 3 experimental results. Here L represents the same linear circuit, which contains ideal sources, dependent sources, and resistors. Find the current i_3 in the third experiment.



* using Thevenin eq.



$\underline{Exp. 1}$: $V_{TH} = 3(R_{TH} + 1)$

$\underline{Exp 2}$: $V_{TH} = 2(R_{TH} + 2)$

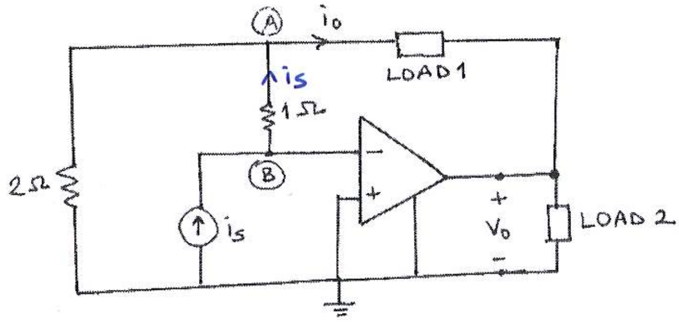
$3(R_{TH} + 1) = 2(R_{TH} + 2)$

$\boxed{R_{TH} = 1\Omega}$

$V_{TH} = 3(R_{TH} + 1) = \boxed{6V}$

Then, for exp 3: $i = \dfrac{V_{TH}}{R_{TH} + 5} = \dfrac{6}{1+5} = \boxed{1\ A}$

iv) **[6 points]** Consider the following circuit. Here, the OPAMP is ideal and operates in the linear region. Load 1 and Load 2 represent arbitrary circuits. Find i_O in terms of i_S.



$V_B = 0$

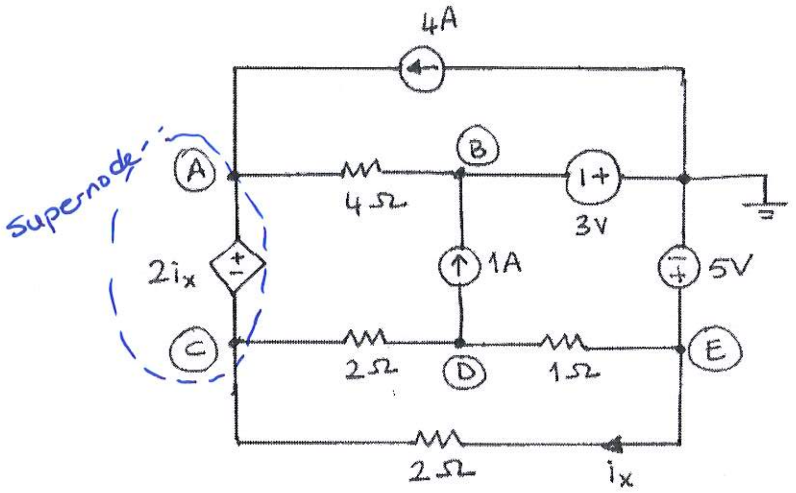
$i_s = \dfrac{V_B - V_A}{1} = -V_A \Rightarrow V_A = -i_s$

$\underline{KCL\ at\ A}$: $\dfrac{V_A}{2} + \dfrac{V_A}{1} + i_o = 0$

$i_o = -\dfrac{3}{2} V_A = \boxed{\dfrac{3}{2} i_s}$

Question 2. [**26 points**] Note that this question has two parts; both parts contain the same circuit. But both parts should be solved independently, i.e., the results obtained in one part cannot be used in the other part.

i) [**13 points**] Consider the following circuit. By using **node analysis** find the node voltages and i_x. For the node convention, use the labeling indicated in the figure.



$$\boxed{\begin{aligned} V_E &= 5V \\ V_B &= -3V \end{aligned}}$$

$$i_x = \frac{V_E - V_C}{2} = \frac{5 - V_C}{2}$$

KCL at supernode: $-4 + \frac{V_A + 3}{4} + \frac{V_C - V_D}{2} + \frac{V_C - 5}{2} = 0$

$(4) \qquad (2) \qquad (2)$

$$\boxed{V_A + 4V_C - 2V_D = 23} \qquad \text{Eq. 1}$$

KCL at D: $\frac{V_D - V_C}{2} + \frac{V_D - 5}{1} + 1 = 0$

$$\boxed{3V_D - V_C = 8}$$

Also from supernode: $V_A - V_C = 2i_x = 5 - V_C \Rightarrow \boxed{V_A = 5V}$

Eq. 1 $\Rightarrow$ $4V_C - 2V_D = 18$

Eq. 2 $\Rightarrow$ $+\ 12V_D - 4V_C = 32$

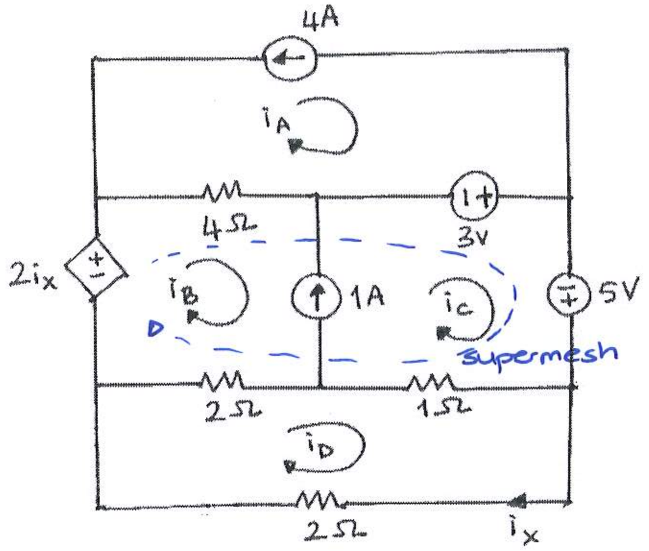
$10V_D = 50 \Rightarrow \boxed{V_D = 5V}$

$V_C = 3V_D - 8 = \boxed{7V}$

Then,

$$i_x = \frac{5 - V_C}{2} = \frac{5-7}{2} = \boxed{-1A}$$

ii) [**13 points**] Consider the following circuit below. By using **mesh analysis** find the mesh currents. For the mesh currents, use the labeling indicated in the figure. Find i_x, as well.



$\boxed{i_A = -4A}$

$i_x = i_D$

$i_C - i_B = 1$

$\boxed{i_C = i_B + 1}$

KVL at supermesh: $-2\underbrace{i_x}_{i_D} + 4(i_B - \underbrace{i_A}_{-4A}) - 3 - 5 + (i_C - i_D) + 2(i_B - i_D) = 0$

$$\boxed{6i_B + i_C - 5i_D = -8}$$

using $i_C = i_B + 1$ $\longrightarrow$ $\boxed{7i_B - 5i_D = -9}$ Eq. 1

KVL at mesh D: $2(i_D - i_B) + (i_D - i_C) + 2i_D = 0$

$$\boxed{5i_D - 2i_B - i_C = 0}$$

using $i_C = i_B + 1$ $\longrightarrow$ $\boxed{5i_D - 3i_B = 1}$ Eq. 2

$3 \times$ Eq. 1 $\Rightarrow$ $21 i_B - 15 i_D = -27$

$7 \times$ Eq. 2 $\Rightarrow$ $+\ 35 i_D - 21 i_B = 7$

$20 i_D = -20$ $\Rightarrow$ $\boxed{i_D = -1A}$

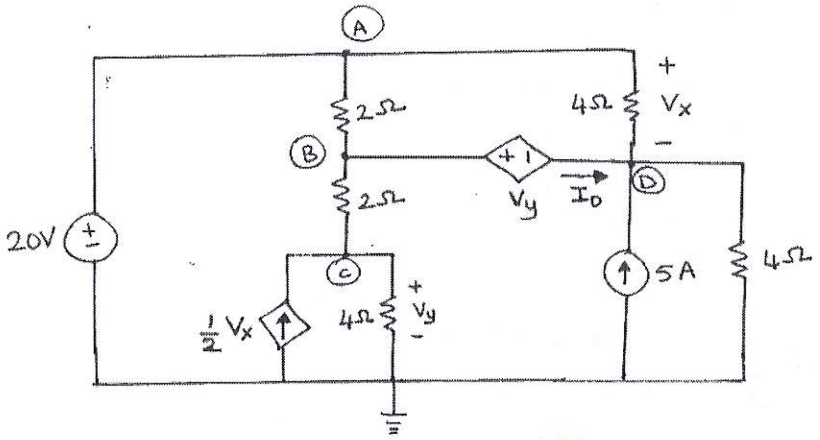
$i_B = \dfrac{5i_D - 1}{3} = \boxed{-2A}$

$i_C = i_B + 1 = \boxed{-1A}$

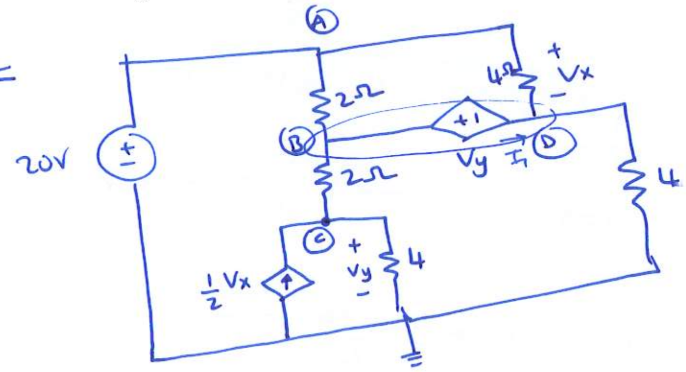
Finally, $\boxed{i_x = i_D = -1A}$ (same as before)

Question 3. **[26 points]**

i) **[13 points]** Consider the circuit below. Find I_o by using superposition.



From voltage source



$V_x = 20 - V_D$

$V_y = V_C$

Supernode: $\dfrac{V_B - V_C}{2} + \dfrac{V_B - 20}{2} + \underbrace{\dfrac{V_D - 20}{4} + \dfrac{V_D}{4}}_{\frac{V_D - 10}{2}} = 0$

$$\boxed{2V_B - V_C + V_D = 30}$$

Also: $\boxed{V_B - V_D = V_y = V_C} \longrightarrow \boxed{V_B + 2V_D = 30}$

KCL at C: $\dfrac{V_C}{4} + \dfrac{V_C - V_B}{2} - \dfrac{V_x}{2} = 0$

$3V_C - 2V_B - 2(20 - V_D) = 0$

$3V_C - 2V_B + 2V_D = 40$

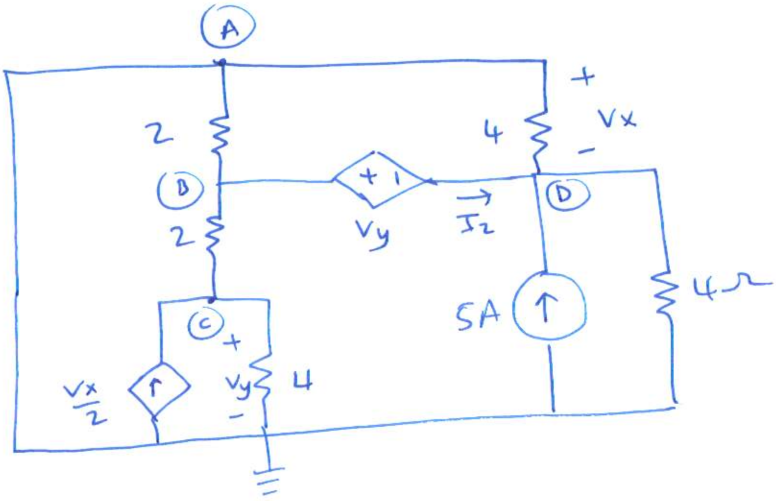
$\Rightarrow \boxed{V_B - V_D = 40}$

$3V_D = -10V$

$\boxed{V_D = -\dfrac{10}{3} V}$

Then, $I_1 = \dfrac{V_D - V_A}{4} + \dfrac{V_D}{4} = \dfrac{V_D - 10}{2} = \dfrac{-40/3}{2} = \boxed{-\dfrac{20}{3} A}$

* From current source :



$V_A = 0$

$V_x = -V_D$

$V_y = V_C$

Supernode : $\frac{V_B}{2} + \frac{V_B - V_C}{2} + \underbrace{\frac{V_D}{4} + \frac{V_D}{4}}_{\frac{V_D}{2}} - 5 = 0$

$$\boxed{2V_B - V_C + V_D = 10}$$

$$\boxed{V_B + 2V_D = 10}$$

Also : $V_B - V_D = V_y = V_C$

KCL at C : $-\frac{V_x}{2} + \frac{V_C}{4} + \frac{V_C - V_B}{2} = 0$

$$3V_C - 2V_B - 2(-V_D) = 0$$

$$3V_C - 2V_B + 2V_D = 0 \Rightarrow V_B - V_D = 0 \Rightarrow \boxed{V_B = V_D}$$

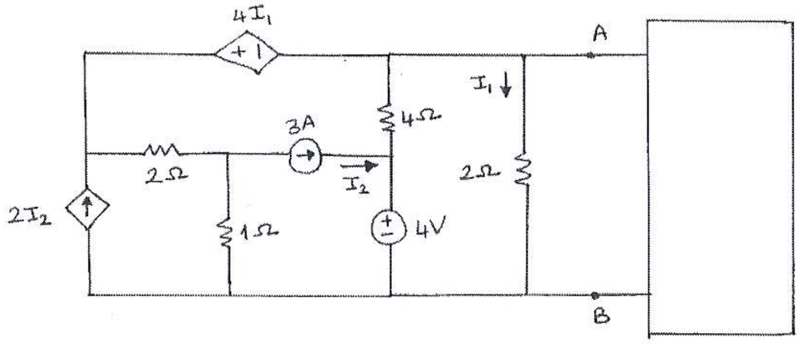
$$3V_D = 10 \Rightarrow \boxed{V_D = \frac{10}{3} V}$$

Then, $I_2 = \frac{V_D}{4} + \frac{V_D}{4} - 5 = \frac{V_D}{2} - 5$

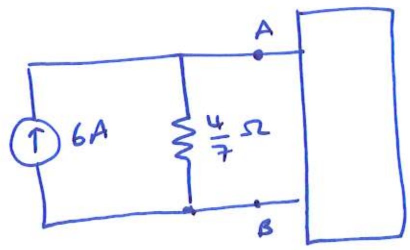
$$I_2 = \frac{5}{3} - 5 = \boxed{-\frac{10}{3} A}$$

Finally, $I_0 = I_1 + I_2 = -\frac{20}{3} - \frac{10}{3} = \boxed{-10 A}$

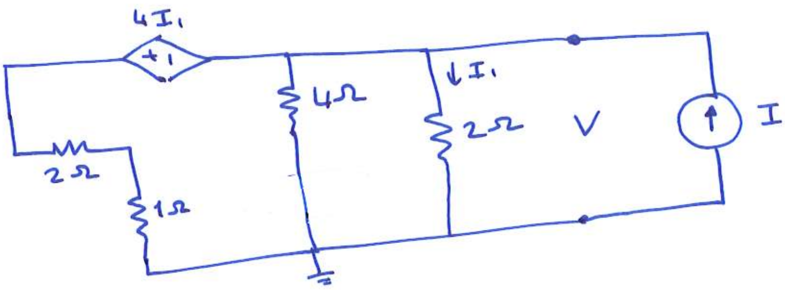
ii) [13 points] Consider the circuit below. Find the Norton Equivalent of the circuit between the terminals A-B. Draw the equivalent circuit.



$\Rightarrow$



* Req : Kill all independent sources.
Then $I_2 = 0$.



$R_{eq} = \frac{V}{I}$

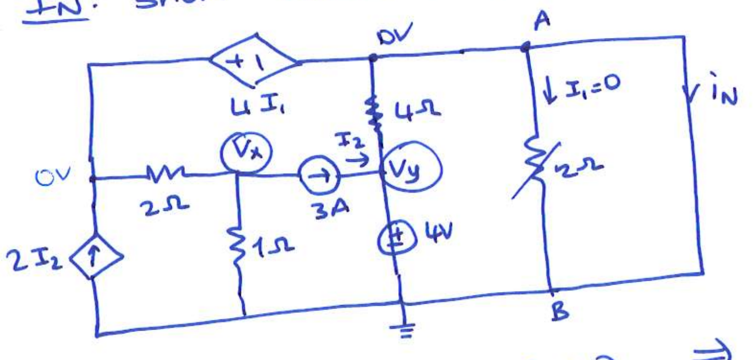
$I_1 = \frac{V}{2}$

from KCL : $\frac{V + 4I_1}{3} + \frac{V}{4} + \frac{V}{2} = I$

$V + \frac{V}{4} + \frac{V}{2} = I$

$\frac{7V}{4} = I \Rightarrow R_{eq} = \frac{V}{I} = \boxed{\frac{4}{7}\,\Omega}$

* I_N: short circuit A-B



$I_1 = 0$

$I_2 = 3A$

$V_y = 4V$

KCL at Vx : $\frac{V_x}{2} + \frac{V_x}{1} + 3 = 0 \Rightarrow 3V_x = -6 \Rightarrow \boxed{V_x = -2V}$

KCL at A : $-\frac{V_y}{4} - \frac{V_x}{2} - 2I_2 + i_N = 0$

$i_N = \frac{V_y}{4} + \frac{V_x}{2} + 2I_2 = 1 - 1 + 6 = \boxed{6\,A}$

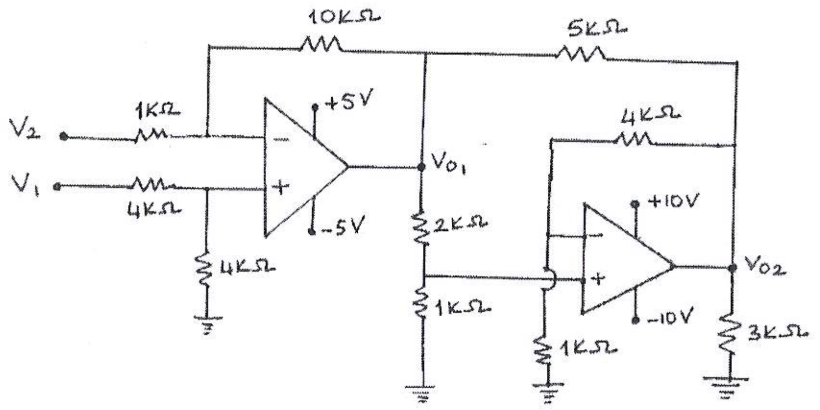
Question 4. [24 points]

Consider the following circuit. All OPAMPs are ideal. The positive/negative saturation voltages are equal to the supply voltages.

a) Express the voltages V_{o1} and V_{o2} as a function of the input voltages V_1 and V_2, assuming that the OPAMPs are in linear region.

b) Find the value of v_o if $v_1 = 3$ V and $v_2 = 2$ V.

c) Find the value of v_o if $v_1 = 1$ V and $v_2 = 2$ V.



a) OPAMP 1 : $V_+ = \frac{V_1}{2} = V_-$

$$\frac{V_2 - \frac{V_1}{2}}{1} = \frac{\frac{V_1}{2} - V_{o1}}{10} \Rightarrow 10V_2 - 5V_1 = \frac{V_1}{2} - V_{o1}$$

$$\boxed{V_{o1} = \frac{11V_1 - 20V_2}{2}}$$

OPOAMP2: $V_+ = \frac{V_{o1}}{3} = V_-$

$$\frac{V_{o2}}{5} = V_- = \frac{V_{o1}}{3} \Rightarrow \boxed{V_{o2} = \frac{5}{3}V_{o1} = \frac{55V_1 - 100V_2}{6}}$$

b) if $V_1 = 3V$, $V_2 = 2V$, $V_{o1} = \frac{33-40}{2} = \boxed{-3.5V}$ ✓ not saturated

$$V_{o2} = \frac{5}{3}V_{o1} = \boxed{-5.83V}$$ ✓ not saturated

c) if $V_1 = 1V$, $V_2 = 2V$, $V_{o1} = \frac{11-40}{2} = -\frac{29}{2} = -14.5V$ ✗ not possible

OPAMP1 is saturated. $\Rightarrow \boxed{V_{o1} = -5V}$

Then, $V_{o2} = \frac{5}{3}V_{o1} = -\frac{25}{3} = \boxed{-8.33V}$ ✓ not saturated (in linear region)