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NAME *FAMILYNAME* *SECTION*

EEE 202 CIRCUIT THEORY

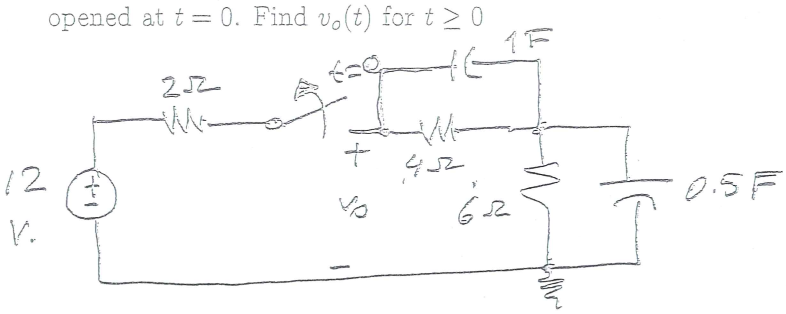
Final, Spring 2014-15

No credits will be given for unjustified answers. Good luck.

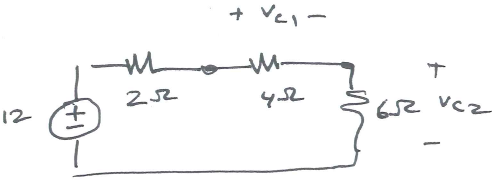
Problem 1	Problem 2	Problem 3	Problem 4	TOTAL

Prob. 1 : (20 pt.s)

i : (8 pt.s) Consider the following circuit. Switch is closed for a long time for $t < 0$ and then is opened at $t = 0$. Find $v_o(t)$ for $t \geq 0$



$t<0 \Rightarrow$ capacitors open



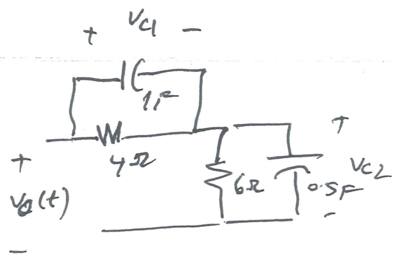
01

$v_{c1} = \frac{4}{12} \cdot 12 = 4V$

$v_{c2} = \frac{6}{12} \cdot 12 = 6V$

01

$t>0$



$v_o(t) = v_{c1}(t) + v_{c2}(t)$

$v_{c1}(t) = 4 \cdot e^{-t/4 \cdot 1} = 4e^{-t/4}$ 02

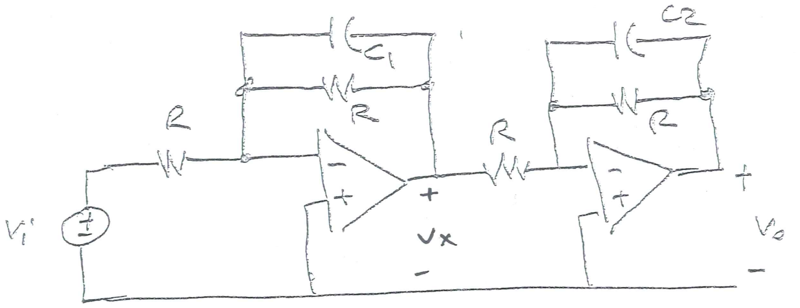
$v_{c2}(t) = 6 \cdot e^{-t/0.5 \cdot 6} = 6e^{-t/3}$ 02

$$\boxed{v_o(t) = 4e^{-t/4} + 6e^{-t/3} \quad t \geq 0}$$ 02

ii : (12 pt.s) Consider the following circuit. Assume that op-amps are linear and operate in the linear region. The input voltage $v_i(t)$ is unit-step function. For simplicity, assume that all initial conditions are zero.

ii-1 Is it possible to have a damped sinusoid term (i.e. a term $e^{-\alpha t}\cos(\omega t+\phi)$) at the output voltage $v_o(t)$?

ii-2 Let $R = 1\ \Omega$, $C_1 = 1\ F$, $C_2 = 0.5\ F$. Find $v_o(t)$.



ii-1 $\quad Y_1 = \frac{1}{R} + sC_1 = \frac{sC_1R+1}{R} \qquad Y_2 = \frac{1}{R} + sC_2 = \frac{sC_2R+1}{R}$

$$\frac{V_x}{V_i} = -\frac{Z_1}{R} = -\frac{1}{sC_1R+1} = -\frac{1/C_1R}{s+1/C_1R} \qquad \frac{V_o}{V_x} = -\frac{Z_2}{R} = -\frac{1/C_2R}{s+1/C_2R}$$

04

$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{C_1R}\cdot\frac{1}{C_2R}}{(s+1/C_1R)\,(s+1/C_2R)}$$

04

$+ V_i(s) = \frac{1}{s} \Rightarrow$ ALL POLES ARE REAL! NO COMPLEX POLES

$\Rightarrow$ DAMPED SINUSOIDS CANNOT EXIST !

ii-2

04

$$V_o = \frac{2}{(s+1)(s+2)}\cdot\frac{1}{s} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+2}$$

$$k_1 = sV_o\Big|_{s=0} = \frac{2}{(s+1)(s+2)}\Big|_{s=0} = 1 \quad 01$$

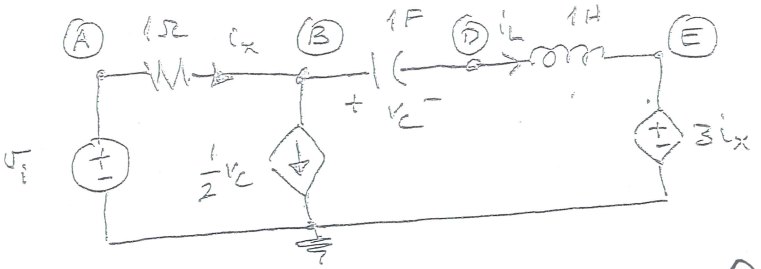
$$k_2 = (s+1)V_o\Big|_{s=-1} = \frac{2}{s(s+2)}\Big|_{s=-1} = -2 \quad 01$$

$$k_3 = (s+2)V_o\Big|_{s=-2} = \frac{2}{s(s+1)}\Big|_{s=-2} = 1 \quad 01$$

01

$$\Rightarrow v_o(t) = 1 - 2e^{-t} + e^{-2t} \qquad t>0$$

Prob. 2 : (25 pt.s) Consider the following circuit. Let $v_C(0) = V_0$ and $i_L(0) = I_0$ be given.
i : Find $I_x(s)$ (in terms of $V_i(s), V_0; I_0$).
ii : Let $v_i(t)$ be a unit step function, $V_0 = 0$ and $I_0 = 0$. Find $i_x(t)$.
iii : Find the node voltages of nodes B, D and E as $t \to \infty$ (i.e. at the steady state).



i) (A) $V_A = V_i$

(B) $-V_A + (s+1)V_B - V_0 + \frac{1}{2}V_C - sV_D = 0$

(D) $-sV_B + (s+\frac{1}{s})V_D - \frac{1}{s}V_E + V_0 + \frac{I_0}{s} = 0$

(E) $V_E = 3I_x = 3(V_A - V_B)$ (03)

$V_C = V_B - V_D$

$$\Rightarrow \boxed{\begin{aligned} (s+\tfrac{3}{2})V_B - (s+\tfrac{1}{2})V_D &= V_i + V_0 \\ (\tfrac{3}{s} - s)V_B + (s+\tfrac{1}{s})V_D &= \tfrac{3}{s}V_i - V_0 - \tfrac{I_0}{s} \end{aligned}}$$ (03)

$$\Rightarrow \left[\left(\frac{s^2+1}{s}\right)\frac{2s+3}{2} + \frac{2s+1}{2}\cdot\frac{3-s^2}{s}\right]V_B = \frac{s^2+1}{s}(V_i+V_0) + \frac{2s+1}{2}\left(\frac{3}{s}V_i - V_0 - \frac{I_0}{s}\right)$$

$$\Rightarrow \left(\frac{2s^2+8s+6}{2s}\right)V_B = \frac{2s^2+6s+5}{2s}V_i + \frac{2-s}{2s}V_0 - \frac{2s+1}{2s}I_0$$ (03)

$$\Rightarrow \boxed{V_B = \frac{2s^2+6s+5}{2s^2+8s+6}V_i + \frac{2-s}{2s^2+8s+6}V_0 - \frac{2s+1}{2s^2+8s+6}I_0}$$

$\boxed{I_x = V_i - V_B}$ (03)

$$\Rightarrow \boxed{I_x = \frac{s+1/2}{s^2+4s+3}V_i + \frac{1/2\, s - 1}{s^2+4s+3}V_0 + \frac{s+1/2}{s^2+4s+3}I_0}$$

ii) $$I_x = \frac{s+1/2}{(s^2+4s+3)}\cdot\frac{1}{s} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+3}$$ (01)

$k_1 = sI_x\Big|_{s=0} = \frac{s+1/2}{(s+1)(s+3)}\Big|_{s=0} = 1/6$ (01)

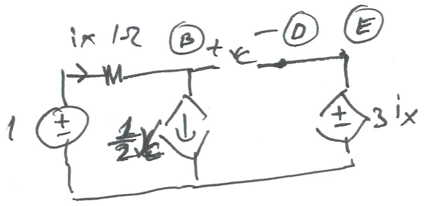
$k_2 = (s+1)I_x\Big|_{s=-1} = \frac{s+1/2}{s(s+3)}\Big|_{s=-1} = 1/4$ (01)

$k_3 = (s+3)I_x\Big|_{s=-3} = \frac{s+1/2}{s(s+1)}\Big|_{s=-3} = -5/12$ (01)

$$\boxed{i_x(t) = \frac{1}{6} + \frac{1}{4}e^{-t} - 5/12\, e^{-3t} \quad t \geq 0}$$ (03)

07

iii) Capacitor → open
inductor → short



$\boxed{I_x(\infty) = \frac{1}{6}}$ (from above) (07)

$i_x = \frac{1}{2}V_C \Rightarrow V_C = 2i_x = \frac{2}{6}$

$\boxed{V_E = 3i_x = \frac{3}{6}}$ (01)

$\boxed{V_D = V_E = 3/6}$ (02)

$V_C = V_B - V_E \Rightarrow$ $\boxed{\begin{aligned} V_B &= V_C + V_E \\ &= \tfrac{2}{6} + \tfrac{3}{6} = 5/6 \end{aligned}}$ (02)

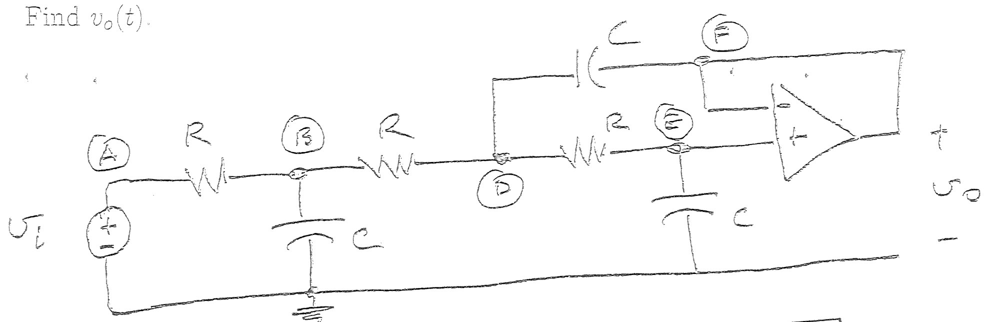
06

Prob. 3 : (25 pt.s) Consider the following circuit. Assume that the op-amp is ideal and operates in the linear region. Assume zero initial conditions.

i : Find the transfer function $H(s) = \frac{V_o(s)}{V_i(s)}$ in terms of R and C.

ii : Find and sketch $|H(j\omega)|$. If this circuit acts as a filter, determine its type.

iii : Let $R = 1\,\Omega$, $C = 1\,F$ and $v_i(t) = \sin t\ V$. Assume that the circuit is in sinusoidal steady-state. Find $v_o(t)$.



i) $\boxed{V_E = V_F = V_o}$

$\boxed{V_A = V_i}$

E $\Rightarrow$ $-\frac{V_D}{R} + \left(\frac{1}{R} + sC\right)V_o = 0 \Rightarrow \boxed{V_D = (sCR+1)\,V_o}$

D $\rightarrow$ $-\frac{V_B}{R} + \left(\frac{2}{R} + sC\right)V_D - \frac{V_o}{R} - sCV_o = 0 \Rightarrow V_B = (2+sCR)V_D - V_o(1+sCR)$

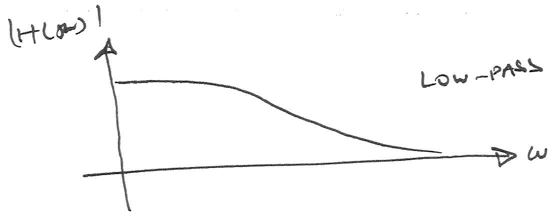
$V_B = \left[(2+sCR)(sCR+1) - (1+sCR)\right].V_o = (1+sCR)^2 V_o$

B $\Rightarrow$ $-\frac{V_i}{R} + \left(\frac{2}{R} + sC\right)V_B - \frac{V_D}{R} = 0 \Rightarrow V_i = (2+sCR)V_B - V_D = \left[(2+sCR)(1+sCR)^2 - (1+sCR)\right]V_o$

$$\Rightarrow H(s) = \frac{V_o}{V_i} = \frac{1}{(sCR)^3 + 4(sCR)^2 + 4sCR + 1} = \frac{1/(CR)^3}{s^3 + \frac{4}{(CR)^2}s^2 + \frac{4}{CR}s + \frac{1}{(CR)^3}}$$

ii) $$|H(j\omega)| = \frac{1/(CR)^3}{\left|\frac{1}{(CR)^3} - \frac{4}{(CR)^2}\omega^2 + j\omega\left(\frac{4}{CR} - \omega^2\right)\right|} = \frac{\frac{1}{(CR)^3}}{\sqrt{\frac{1}{(CR)^4}\left(\frac{1}{CR} - 4\omega^2\right)^2 + \omega^2\left(\frac{4}{CR} - \omega^2\right)^2}}$$

$|H(j0)| = 1$ $\quad |H(j\infty)| = 0$ $\Rightarrow$



LOW-PASS FILTER

iii) $RC = 1 \Rightarrow H(s) = \frac{1}{s^3 + 4s^2 + 4s + 1}$ $\qquad V_i = 1.e^{-j\pi/2}$ (phasor

$\omega = 1$ $\qquad H(j1) = \frac{1}{-j - 4 + 4j + 1} = \frac{1}{-3+3j} = \frac{1}{3\sqrt{2}}e^{-j135°}$

$V_o = \frac{1}{3\sqrt{2}}e^{-j135°}\,1\,e^{-j90°} = \frac{1}{3\sqrt{2}}e^{-j225°}$

$$\boxed{v_o(t) = \frac{1}{3\sqrt{2}}\cos(t - 225°)}$$

Prob. 4 : (30 pt.s) A Wien bridge oscillator is a type of electronic oscillator that generates sine waves.The modern circuit is based on William Hewlett's 1939 Stanford University master's degree thesis.Hewlett eventually co-founded Hewlett-Packard, the first product of which was the HP200A, which is a precision Wien bridge oscillator. Note that there is no input signal (it is an oscillator, so not needed). Assume that the op-amp is ideal.

i : Let V_+ and V_- denote the op-amp non-inverting and inverting terminal voltages, respectively. Find the transfer functions $H_+(s) = \frac{V_+(s)}{V_o(s)}$ and $H_-(s) = \frac{V_-(s)}{V_o(s)}$ in terms of resistor and capacitor values.

ii : Now assuming that the op-amp is in the linear region, by using the tesult obtained in i above, find a condition in terms of resistor and capacitor values such that this circuit shows a pure oscillation of the form $v_o(t) = V_0 \cos \omega_0 t$.

iii : For the case in ii, find the oscillation frequency ω_0 in terms of resistor and capacitor values.

iv : This circuit is a second order circuit. By using the results obtained in i and assuming that the op-amp operates in the linear region, obtain a second order ode for v_o in the form $a\ddot{v}_o + b\dot{v}_o + cv_o = 0$, where a, b, c depends on resistor and capacitor values.