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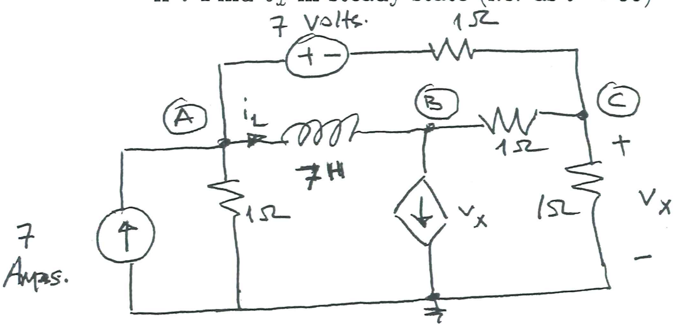
EEE 202 CIRCUIT THEORY
Second Midterm, Spring 2012-13

No credits will be given for unjustified answers.

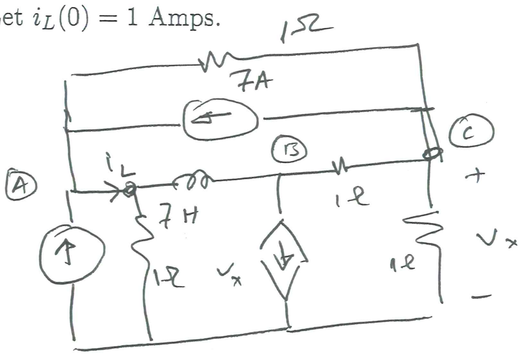
Prob. 1 : (20 pt.s) Consider the following circuit. Let $i_L(0) = 1$ Amps.

i : Find $i_L(t)$.

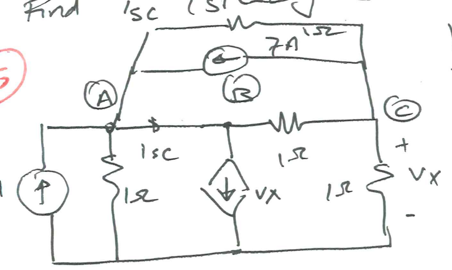
ii : Find v_x in steady state (i.e. as $t \to \infty$)



$\Rightarrow$



Find i_{sc} (steady state)



$\boxed{V_A = V_B}$

Ⓐ $\to$ $2V_A - V_B + i_{sc} = 14$

Ⓑ $- i_{sc} + v_x + V_B - V_C = 0$

Ⓒ $3V_C - V_B - V_A = -7$

$v_x = V_C$

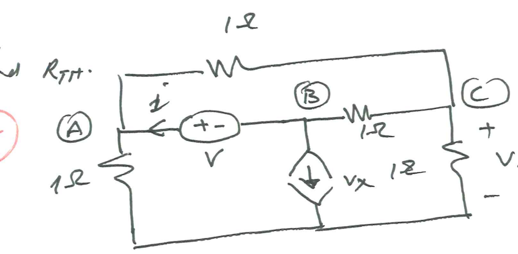
$\Rightarrow \boxed{i_{sc} = V_B}$

$\boxed{\begin{aligned} 3V_A - V_C &= 14 \\ 3V_C - 2V_A &= -7 \end{aligned}}$ (03)

$\Rightarrow \boxed{\begin{aligned} V_A = V_B &= 5V. \\ V_C &= 1V. \end{aligned}}$

$\Rightarrow \boxed{i_{sc} = V_B = 5A}$ (02)

Find R_{TH}.



Ⓐ : $2V_A - V_C = i$

Ⓑ $v_x + V_B - V_C = -i$

Ⓒ $3V_C - V_B - V_A = 0$

$v_x = V_C$

$\boxed{V_B = -i}$

$\boxed{\begin{aligned} 2V_A - V_C &= +i \\ 3V_C - V_A &= -i \end{aligned}}$ (03)

$\Rightarrow \boxed{V_A = \frac{2}{5} i}$

$v = V_A - V_B = \frac{2}{5} i + i = \frac{7}{5} i \Rightarrow \boxed{R_{TH} = \frac{7}{5} \Omega}$ (02)

$\tau = \frac{L}{R} = \frac{7}{7/5} = 5$ sec. (01)

$\Rightarrow \boxed{i_L(t) - 5 = [1 - 5] e^{-t/5}}$

$\boxed{i_L(t) = 5 - 4 e^{-t/5}}$ (04)

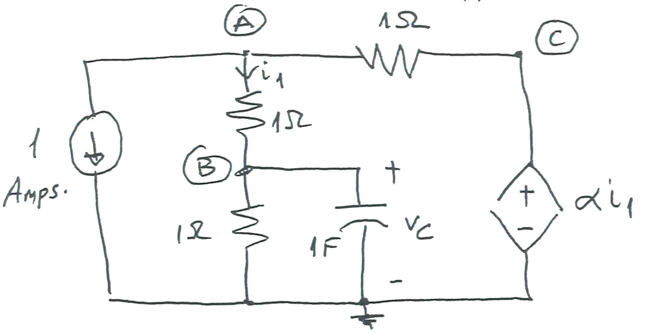
ii) $v_x(\infty) = V_C$ found above $\Rightarrow$ (when inductor is short) (02)

$\Rightarrow \boxed{v_x(\infty) = 1V.}$ (05)

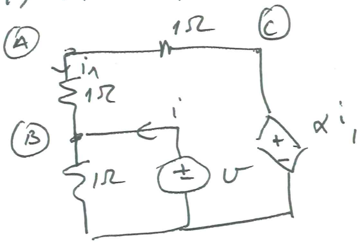
Prob. 2 : (20 pt.s) Consider the following circuit. Let $v_C(0) = 3$ Volts.

i : Find the value of α so that the time constant τ of the circuit is $\tau = 2$ *sec.* (If this is not possible, explain why).

ii : Let $\alpha = 4$. Find $v_C(t)$.



i) Find R_{TH}



$\boxed{v_B = v}$ $\boxed{v_C = \alpha i_1 = \alpha(v_A - v_B)}$

B $\boxed{2v_B - v_A = i}$

A $\boxed{2v_A - v_B - v_C = 0}$ 05

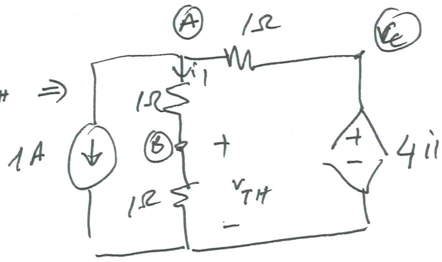
$\Rightarrow$ $\boxed{2v - v_A = i}$

$\boxed{(2-\alpha)v_A + (\alpha-1)v_B = 0}$ 02

$v_A = \frac{\alpha-1}{\alpha-2} v_B$ $\Rightarrow$ $\left[2 - \frac{\alpha-1}{\alpha-2}\right]v = \boxed{\frac{\alpha-3}{\alpha-2}}$ $v = i$ $\Rightarrow$ $\boxed{R_{TH} = \frac{\alpha-2}{\alpha-3}\ \Omega}$ 03

$\Rightarrow$ $\tau = R_{TH}C = \frac{\alpha-2}{\alpha-3} = 2$ $\Rightarrow$ $\alpha-2 = 2\alpha-6$ $\Rightarrow$ $\boxed{\alpha = 4}$

ii) $\boxed{R_{TH} = 2\ \Omega}$ For V_{TH} $\Rightarrow$



$i_1 = \frac{V_A}{2}$ $V_C = 4i_1 = \frac{4V_A}{2}$

A $\Rightarrow$ $1 + \frac{V_A}{2} + \left(V_A - 4\cdot\frac{V_A}{2}\right) = 0$

$1 + \frac{V_A}{2} - V_A = 0$ $\Rightarrow$ $\boxed{V_A = 2V.}$ 06

$\Rightarrow$ $\boxed{V_B = V_{TH} = 1\ V.}$

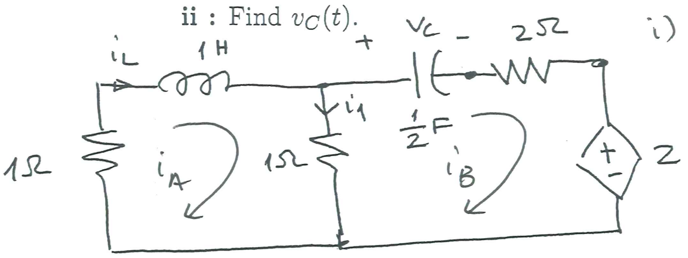
$\tau = R_{TH}C = 2$ sec. $\Rightarrow$ $v_C - 1 = [3-1]e^{-t/2}$

$\boxed{v_C(t) = 1 + 2e^{-t/2}\ V.}$ 04

Prob. 3 : (20 pt.s) Consider the following circuit. Let $v_C(0) = 1\ V$ and $i_L(0) = 2\ A$.

i : Find coupled ordinary differential equations for v_C and $i_L(t)$ in the form $\dot{v}_C = a_{11}v_C + a_{12}i_L$, $\dot{i}_L = a_{21}v_C + a_{22}i_L$, here $\dot{()}$ represents time derivative, a_{ij} are appropriate constants.

ii : Find $v_C(t)$.



i)

Ⓐ $V_L + (i_A - i_B) + i_A = 0 \Rightarrow \boxed{V_L + 2i_A - i_B = 0}$ (1) 02

Ⓑ $V_C + 2i_B + 2i_1 + (i_B - i_A) = 0 \Rightarrow$

$i_1 = i_A - i_B \Rightarrow \boxed{V_C + i_A + i_B = 0}$ (2) 02

$\boxed{\begin{matrix} i_A = i_L \\ i_B = i_C \end{matrix}} \Rightarrow$ (2) $\Rightarrow \boxed{i_C = -i_L - V_C}$ 01 $\Rightarrow$ $V_L = -2i_L + i_C \Rightarrow \boxed{V_L = -3i_L - V_C}$ 01

$\Rightarrow \frac{1}{2}\dot{V}_C = -i_L - V_C \Rightarrow \boxed{\begin{matrix} \dot{V}_C = -2V_C - 2i_L \\ \dot{i}_L = -V_C - 3i_L \end{matrix}}$ 04/04

ii) $\boxed{\ddot{V}_C - T.\dot{V}_C + \Delta V_C = 0}$ $\quad T = -5$, $\Delta = 6 - 2 = 4$ $\Big\}$ $\boxed{\ddot{V}_C + 5\dot{V}_C + 4V_C = 0}$ 02

$\Rightarrow s^2 + 5s + 4 = 0 \Rightarrow \begin{matrix} s_1 = -4 \\ s_2 = -1 \end{matrix}$ 02 $\Rightarrow \boxed{V_C(t) = C_1 e^{-t} + C_2 e^{-4t}}$ 02

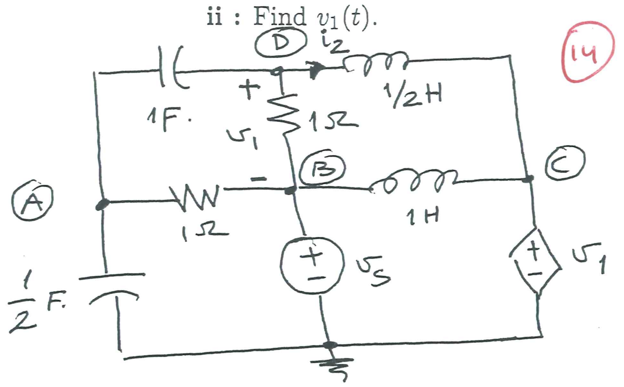
$V_C(0) = 1$, $\dot{V}_C(0) = -2V_C(0) - 2i_L(0) = -6$ $\Rightarrow \begin{matrix} 1 = C_1 + C_2 \\ -6 = -C_1 - 4C_2 \end{matrix}\Big\}$ $\begin{matrix} C_1 = -\frac{2}{3} \\ C_2 = 5/3 \end{matrix}$ 02

$\Rightarrow \boxed{V_C(t) = -\frac{2}{3}e^{-t} + \frac{5}{3}e^{-4t}}$ 02

Prob. 4 : (20 pt.s) Consider the following circuit. Let $v_s(t) = 2\cos(2t+30^\circ)$ Volts. Assume that the circuit is in sinusoidal steady state.

i : Find the phasors V_1 and I_2 (corresponding to $v_1(t)$ and $i_2(t)$).

ii : Find $v_1(t)$.



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i) $\boxed{V_B = V_s}$ $\quad \boxed{V_C = V_1 = V_D - V_B}$ $\quad \boxed{\omega = 2 \text{ rad/sec.}}$

(A) $\boxed{(1+3j)V_A - V_B - j2V_D = 0}$ 02

(B) $-j2V_A + (1+j2+\frac{1}{j})V_D - \frac{1}{j}V_C - V_B = 0$

$\Rightarrow \boxed{-j2V_A + (1+j)V_D + jV_C = V_s}$ 02

02

$(1+3j)V_A - j2V_D = V_s$

$-j2V_A + (1+j)V_D + j(V_D - V_B) = V_s$

$\boxed{\begin{array}{l}(1+3j)V_A - j2V_D = V_s \\ -j2V_A + (1+2j)V_D = (1+j)V_s\end{array}}$ 02

$\Rightarrow \boxed{V_D = \frac{-2+6j}{-1+5j}V_s}$ 02

$\boxed{\bar{V}_1 = V_D - V_B = \left(\frac{-2+6j}{-1+5j} - 1\right)V_s = \frac{-1+j}{-1+5j}V_s}$ 03

KVL $\Rightarrow$ $V_1 + V_s = jI_2 + V_1$ 03

$\Rightarrow jI_2 = V_s \rightarrow \boxed{I_2 = jV_s}$

ii) 06

$V_1 = \frac{-1+j}{-1+5j}V_s = \frac{(-1+j)(-1-5j)}{(1+25)}V_s = \boxed{\frac{3+2j}{13}V_s}$

$V_s = 2e^{j30^\circ}$ 03

$V_1 = \frac{\sqrt{13}}{13}e^{j\tan^{-1}\frac{2}{3}} \cdot 2e^{j30^\circ} = \frac{2\sqrt{13}}{13}e^{j(30^\circ + \tan^{-1}\frac{2}{3})} = \frac{2\sqrt{13}}{13}e^{j(63.69^\circ)}$

$= 0.554\, e^{j(63.69^\circ)}$

$\boxed{v_1(t) = 0.554\cos(2t + 63.69^\circ)}$ 03

Prob. 5 : (20 pt.s) Consider the following circuit. Let $i_s(t) = I_m \cos(\omega t + \phi)$ Amps. Assume that the circuit is in sinusoidal steady state.

i : Find the transfer function $H = \frac{V_0}{I_s}$ in terms of ω. Here V_0 and I_s are phasors of $v_0(t)$ and $i_s(t)$.

ii : Let $i_s(t) = 2\cos(2t - 45°)$ Amps. Find $v_0(t)$.

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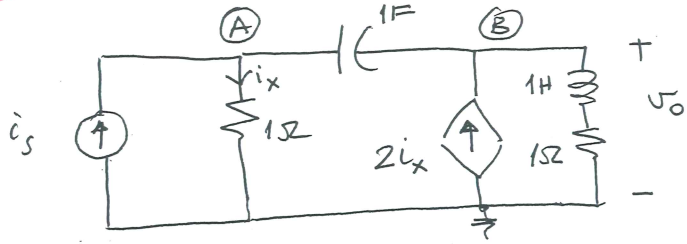

i) $1\Omega + 1H \Rightarrow Z_L = R + j\omega L = \underline{1 + j\omega}$

Ⓐ $I_s = V_A + j\omega(V_A - V_B) = (1+j\omega)V_A - j\omega V_B$ 02

Ⓑ $2I_x = j\omega(V_B - V_A) + \frac{V_B}{1+j\omega}$ 02

$$\rightarrow \left.\begin{aligned} &(1+j\omega)V_A - j\omega V_B = I_s \\ &-j\omega V_A + \left(j\omega + \frac{1}{1+j\omega}\right)V_B = 2I_x \end{aligned}\right\} \Rightarrow \quad I_x = V_A \Rightarrow \quad \begin{aligned} &(1+j\omega)V_A - j\omega V_B = I_s \\ &-(2+j\omega)V_A + \frac{(j\omega)^2 + j\omega + 1}{1+j\omega}V_B = 0 \end{aligned}$$ 02

02

$$\Rightarrow \boxed{V_A = \frac{1-\omega^2 + j\omega}{(2+j\omega)(1+j\omega)} V_B} \Rightarrow \left[\frac{1-\omega^2+j\omega}{2+j\omega} - j\omega\right]V_B = I_s \Rightarrow V_B = \frac{2+j\omega}{1-\omega^2+j\omega-2j\omega+\omega^2} I_s$$

$$\Rightarrow \boxed{H(j\omega) = \frac{V_B}{I_s} = \frac{2+j\omega}{1-j\omega}}$$ 04

ii) $i_s(t) = 2\cos(2t - 45°) \Rightarrow \omega = 2\,\text{rad/sec}. \quad I_s = 2e^{-j45°}$ 04

08/20

$$V_B = \frac{2+j2}{1-j2} \cdot I_s = \frac{2(1+j)}{1-j2} 2e^{-j45} = \frac{2\sqrt{2}e^{j45°}}{\sqrt{5}\, e^{-j\tan^{-1}2}} 2e^{-j45°} = \frac{4\sqrt{2}}{\sqrt{5}} e^{j\tan^{-1}2}$$

$$v_b(t) = v_o(t) = \frac{4\sqrt{2}}{\sqrt{5}} \cos(2t + \tan^{-1}2) = 2.53\cos(2t + 63.44°)$$

04