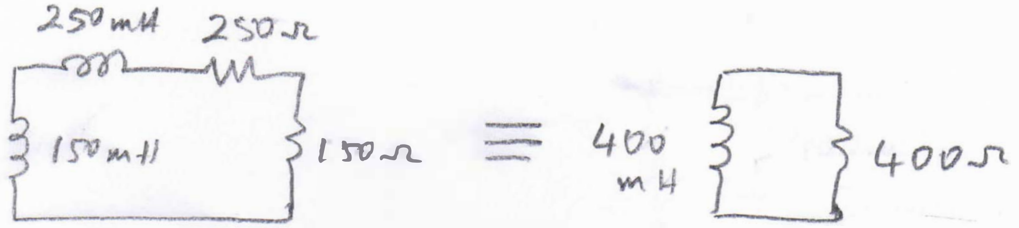
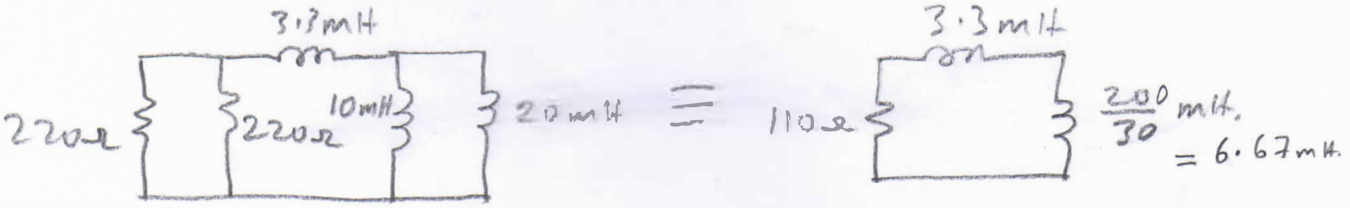
EEE202 Homework 2 solution

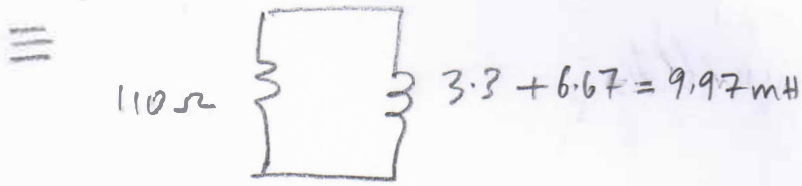
Problems are from 8th edition of textbook, chapter 7.

Q3.



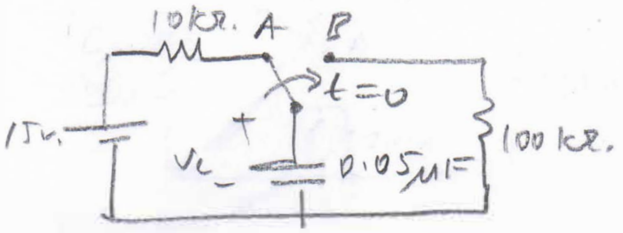
$$\tau = \frac{400 \times 10^{-3}}{400} = 1\,\text{msec.}$$





$$\tau = \frac{9.97 \times 10^{-3}}{110} = 90.6\,\mu\text{sec.}$$

Q10.



ch has been at position A for a long time and thus $v_c(0) = 15\,\text{V}$.

for $t \geq 0$ $\tau = RC = 100 \times 10^{3} \times 0.05 \times 10^{-6} = 5\,\text{msec}$

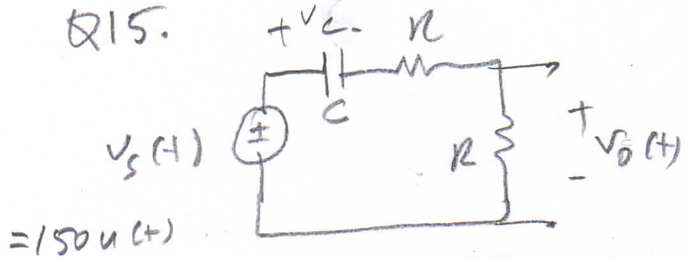
$v_c(\infty) = 0$

$$\therefore v_c(t) = 0 + (15 - 0)\, e^{-\frac{t}{5}}$$

$$= 15 e^{-\frac{t}{5}} \quad t \text{ in msec.}$$

$$\text{or} = 15 e^{-0.2t} \quad t \text{ in msec.}$$

Q15.



$C = 0.022 \mu F \quad R = 82 k\Omega$.

$RC = 0.022 \times 10^{-6} \times 164 \times 10^{3}$

$= 3.6$ msec.

Let us first find $v_c(t)$: $v_c(0) = 0 \quad v_c(\infty) = 150v$.

$\therefore \quad v_c(t) = 150 + (0 - 150) e^{-\frac{t}{3.6}} = 150\left(1 - e^{-\frac{t}{3.6}}\right)$ t in msec.

$\therefore \quad v_0(t) = [150 - v_c(t)] \times \frac{R}{R+R} = \left[150 - 150\left(1 - e^{-\frac{t}{3.6}}\right)\right] \times \frac{1}{2}$

$= \underbrace{75 e^{-t/3.6}}_{\text{is Natural response}} \quad t \geq 0$ t in msec.

is Natural response. Forced response is zero.

Q19. $\frac{dv(t)}{dt} + 50\, v(t) = v_s(t) \quad v(0) = 0v \quad v_s(t) = 10\cos(250t)$

$v(t) = A\cos(250t) + B\sin(250t)$ is the particular soln.

$\frac{dv}{dt}(t) = -250 A \sin(250t) + 250 B\cos(250t)$

$-250 A\sin(250t) + 250 B\cos(250t) + 50 A\cos 250t + 50 B\sin 250t$
$= 10\cos 250t$

$$\left.\begin{array}{l} -250A + 50B = 0 \\ 250B + 50A = 10 \end{array}\right\} \quad \begin{array}{l} -5A + B = 0 \Rightarrow B = +5A \\ 5B + A = 0.2 \Rightarrow A = 0.2 - 5B = 10 - 25A \end{array}$$

$\Rightarrow A = \frac{0.2}{26} = \frac{1}{130}$

$\Rightarrow B = \frac{5}{130}$

$v(t) = K e^{-50t} + \frac{1}{130} \times \cos 250t + \frac{5}{130} \times \sin 250t$

$v(0) = 0 \quad \Rightarrow \quad K + \frac{1}{130} = 0 \Rightarrow K = \frac{-1}{130}$

$\therefore \quad v(t) = -\frac{1}{130} e^{-50t} + \frac{1}{130}\cos 250t + \frac{5}{130}\sin \omega t$

Q25. At $t=0$ $i_L(0) = \frac{15}{10k} = 1.5\times10^{-3} = 1.5\,mA.$

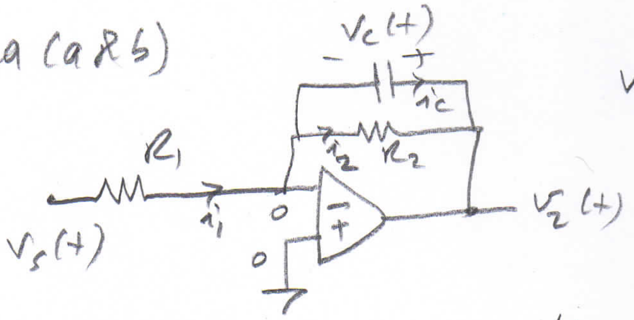
For $t \geq 0$ $i_L(\infty) = \frac{15}{100k\Omega} = 0.15\,mA,$

$\tau = \frac{L}{R} = \frac{100\,mH}{100\,k\Omega} = 1\,\mu sec.$

$\therefore\ i_L(t) = 0.15 + (1.5 - 0.15)e^{-t}$ t in μsec.

$= 0.15 + 1.35e^{-t}$ mA, t in μsec.

Q29 (a & b)



$v_s(t) = 2u(t)$ $R_1 = 10\,k\Omega$ $R_2 = 20\,k\Omega$
$C = 0.1\,\mu F$

$v_c(0) = 4v$ is given

a) $i_1 = i_2 + i_c \Rightarrow \frac{v_s}{R_1} = \frac{0 - v_2}{R_2} - C\frac{dv_c}{dt}$

$v_c(t) = v_2(t) \Rightarrow \frac{v_s}{R_1} = \frac{-v_2}{R_2} - C\frac{dv_2}{dt}$

$C\frac{dv_2}{dt} + \frac{v_2}{R_2} = \frac{-v_s}{R_1} \Rightarrow \frac{dv_2}{dt} + \frac{1}{R_2C}v_2 = -\frac{v_s}{R_1C}$

$R_2C = 20\times10^3 \times 0.1\times10^{-6} = 0.002$ sec.

$R_1C = 10\times10^3 \times 0.1\times10^{-6} = 0.001$ sec.

$\frac{dv_2}{dt} + 500v_2 = -1000v_s$ or $v_2(\infty) = \frac{-1000\times2}{500} = -4$

$v_c(\infty) = v_2(\infty) = -\frac{R_2}{R_1}\times 2 = -4\,v,$

$\therefore\ v_o(t) = -4 + (+4 - (-4))e^{-500t} = -4 + 8e^{-500t}$

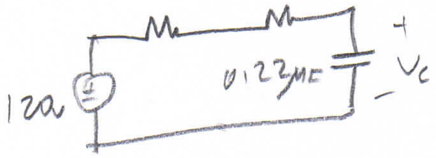
b) $v_o(1.386\,ms) = -4 + 8e^{-500\times1.386\times10^{-3}} = -4 + 8e^{-0.693}$

$\simeq 0.000589$ v, $\simeq 0$ v

$\simeq 0$

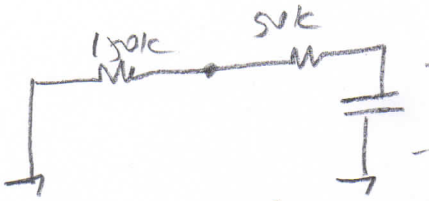
Q31.

$t = 0^-$



$V_c(0) = 120\,v.$

$0 \le t < 20$

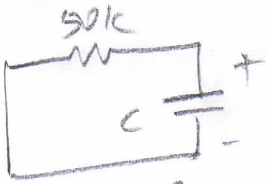


$$v_c(t) = 0 + (120 - 0)e^{-\frac{t}{44}}$$
$$= 120\, e^{-\frac{t}{44}} \quad t \text{ in msec.}$$

$\tau = 200 \times 10^3 \times 0.22 \times 10^{-6}$
$= 44$ msec.

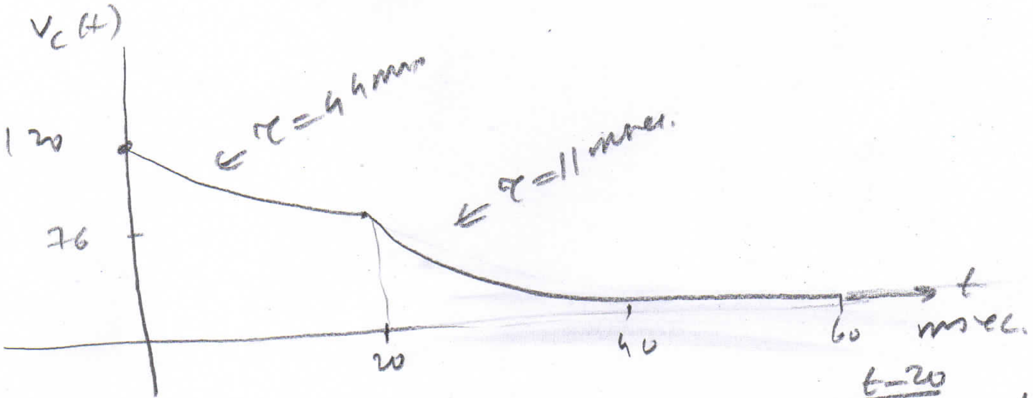
at $t = 20^-$ $\quad v_c(t) = 120\, e^{-\frac{20}{44}} = 76.17\,v.$

$20 \le t$



$$v_c(t) = 0 + (76.17 - 0)\, e^{-\frac{t-20}{11}}$$
$$= 76.17\, e^{-\frac{t-20}{11}}$$
t in msec.

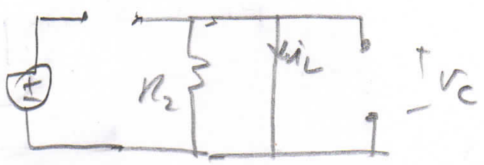
$\tau = 50 \times 10^3 \times 0.22 \times 10^{-6}$
$= 11$ msec.



$$v_c(t) = 120\, e^{-\frac{t}{44}}\left(u(t) - u(t-20)\right) + 76.17\, e^{-\frac{t-20}{11}}\, u(t-20)$$
t in msec.

Q58 (a & b)

a) At $t=0^-$ C is open and L is short

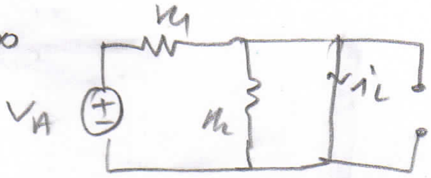


$v_C(0^-)=0$

$i_L(0^-)=0$

$C=0.05\mu F$, $R_1 = 33\,k\Omega$ $R_2 = 33\,k\Omega$ $V_A = 20\,V$, $L=1.25\,H$

For $t=\infty$



$v_C(\infty)=0$ $\quad i_L(\infty)=\frac{V_A}{R_1}=\frac{20}{33k}=606\,\mu A$

For $t \geq 0$

$$\frac{V_C - V_A}{R_1} + \frac{V_C}{R_2} + i_L + i_C = 0$$

$$V_C\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + i_L + i_C = \frac{V_A}{R_1}$$

$$V_L(t) = L\frac{di_L}{dt} = V_C(t)$$

$$L\frac{d^2 i_L}{dt^2} = \frac{dV_C}{dt} \Longrightarrow$$

$$L\frac{di_L}{dt}\times\frac{1}{R_1 // R_2} + i_L + C\frac{dV_C}{dt} = \frac{V_A}{R_1}$$

$$L\frac{di_L}{dt}\cdot\frac{1}{R_1 // R_2} + i_L + LC\frac{d^2 i_L}{dt^2} = \frac{V_A}{R_1}$$

$$\Longleftarrow$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{C\times R_1//R_2}\frac{di_L}{dt} + \frac{1}{LC}i_L = \frac{V_A}{LCR_1}$$

$$R_1//R_2 = \frac{33\times 33}{66} = \frac{33}{2} = 16.5k.$$

$$\frac{1}{C\times R_1//R_2} = \frac{1}{0.05\times 10^{-6}\times 16.5\times 10^3}$$

$$= 1.2121\times 10^3 = 1212.1$$

$$\frac{d^2 i_L}{dt^2} + 1.21\times 10^3\frac{di_L}{dt} + 16\times 10^6 i_L = 9697$$

$$\frac{1}{LC} = \frac{1}{1.25\times 0.05\times 10^{-6}} = 16\times 10^6$$

$$s^2 + 1212.1\,s + 16\times 10^6 = 0$$

$$s_{1,2} = -606.1 \pm j\,3953.8$$

$$\frac{V_A}{LCR_1} = \frac{20}{1.25\times 0.05\times 10^{-6}\times 33\times 10^3}$$

$$= 9697$$

$$i_L(t) = x_N(t) + x_F(t)$$

$$x_F(t) = A \qquad A = \frac{9697}{16\times 10^6} = 6.06\times 10^{-4}$$

$$= 0.606\,mA.$$

$$i_L(t) = 6.06\times 10^{-4} + Ke^{-606.1t}\cos(3953.8t+\phi)$$

$$i_L(0) = 0 = 6.06\times 10^{-4} + K\cos\phi$$

$$\frac{di_L}{dt} = -606.1\,K e^{-606.1}\cos(3953.8t+\phi) - 3953.8\,Ke^{-606.1t}\sin(3953.8t+\phi)$$

$$\frac{di_L}{dt}(0) = 0 = -606.1\,K\cos\phi - 3953.8\,K\sin\phi$$

$$K\cos\phi = -6.06\times10^{-4}$$

$$0 = -606.1\times(-6.06\times10^{-4}) - 3953.8\ K\sin\phi$$

$$K\sin\phi = +\frac{606.1\times6.06\times10^{-4}}{3953.8} = 9.2897\times10^{-5}$$

$$K^2 = (6.06\times10^{-4})^2 + (9.2897\times10^{-5})^2$$

$$K = 6.1308\times10^{-4}$$

$$\phi = \text{atan2}(9.2897\times10^{-5}, -6.06\times10^{-4}) = 2.9895 \text{ radia}$$

$$= 171.285^{\circ}$$

$$\therefore\ i_L(t) = 6.06\times10^{-4} + 6.1308\times10^{-4}\, e^{-606.1t} \times\cos(3953.8t + 2.9895) \quad *$$

$$\text{or; } i_L(t) = 6.06\times10^{-4} + e^{-606.1t}\left[-6.06\times10^{-4}\cos(3953.8t) - 9.2886\times10^{-5}\times\sin(3953.8t)\right]$$

in mAmperes $i_L(t) = 0.606 - e^{-606.1t}\left[0.606\cos(3953.8t) + 0.09288\sin(3953.8t)\right]$

$$v_C(t) = v_L(t) = L\frac{di_L}{dt} = 1.125\times606.1\, e^{-606.1t}\left[0.606\cos + 0.09288\sin\right) - 1.125\times e^{-606.1t}\left[0.606\times3953.8\sin + 0.09288\times3953.8\cos\right]$$

$$= e^{-606.1t}\Big[(1.125\times606.1\times0.606 - 1.125\times0.09288\times3953.8)\cos(3953.8t) + (1.125\times606.1\times0.09288 + 1.125\times0.606\times3953.8)\sin(3953.8t)\Big]$$

$$= e^{-606.1t}\left[0.0846\cos(3953.8t) + 3.0654\times10^{3}\sin(3953.8t)\right] \text{ (mV)}$$

In Volts $v_C(t) = e^{-606.1t}\left[0.0000846\cos(3953.8t) + 3.065\sin(3953.8t)\right]$ (V)

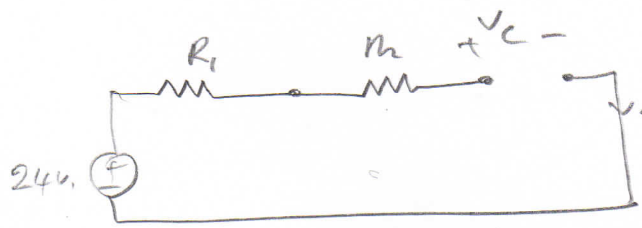
or from $*$ $v_C(t) = 1.125\left[-606.1\times6.1308\times10^{-4}\cos(3953.8t + 2.9895) - 6.1308\times10^{-4}\times3953.8\sin(3953.8t+2.9895)\right]e^{-606.1t}$

$$= e^{-606.1t}\left[-4645\times10^{-4}\cos(3953.8t + 2.9895) - 30300\times10^{-4}\sin(3953.8t+2.9895)\right]$$

b) Roots are complex conjugate and reals are negative $\Longrightarrow$ system is underdamped.

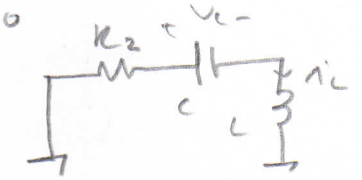
Q62 (a&b)

a) At $t=0^-$



$v_C(0^-) = 24V$
$i_L(0) = 0$

For $t \geq 0$



$R_1 = 1k\Omega$. $L = 250mH$
$R_2 = 100\Omega$ $C = 3.3\mu F$

$$0 = R_2 i_L + v_C + v_L = R_2 C \frac{dv_C}{dt} + v_C + L\frac{di_L}{dt}$$
$$= R_2 C \frac{dv_C}{dt} + v_C + L \frac{d}{dt}\left(C\frac{dv_C}{dt}\right)$$
$$= R_2 C \frac{dv_C}{dt} + v_C + LC\frac{d^2v_C}{dt^2}$$

So

$$\frac{d^2v_C}{dt^2} + \frac{R_2}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = 0$$
$$\frac{d^2v_C}{dt^2} + \frac{100}{250\times10^{-3}}\frac{dv_C}{dt} + \frac{1}{250\times10^{-3}\times3.3\times10^{-6}}v_C = 0$$
$$\frac{d^2v_C}{dt^2} + 400\frac{dv_C}{dt} + 1.2\overline{12}\times10^{6}\, v_C = 0$$

$w = 1082.6$

$$s^2 + 400s + 1.2\overline{12} = 0$$
$$s_{1,2} = -200 \pm j\,1082.6$$

$$v_C(t) = K_1 e^{-200t}\cos(1082.6t) + K_2 e^{-200t}\sin(1082.6t)$$

$$v_C(0) = 24 = K_1$$

$$\frac{dv_C}{dt} = -200K_1 e^{-200t}\cos wt - K_1 e^{-200t} w\sin wt$$
$$-200K_2 e^{-200t}\sin wt + K_2 e^{-200t}\times w\cos wt$$

$$\frac{dv_C}{dt}(0) = \frac{1}{C}i_L(0) = 0 \Rightarrow -200K_1 + wK_2 = 0$$
$$K_2 = \frac{200K_1}{1082.6} = \frac{200\times24}{1082.6} = 4.434$$

$$\therefore v_C(t) = e^{-200t}\left[24\cos(1082.6t) + 4.434\sin(1082.6t)\right]$$

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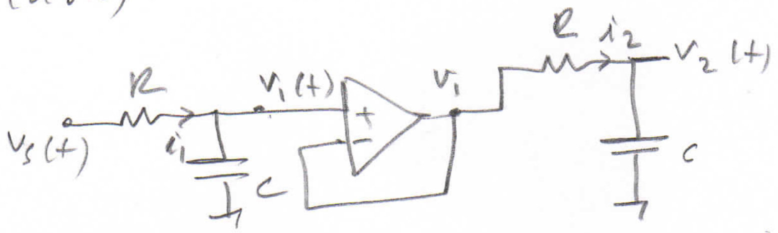
$$i_L(t) = i_C(t) = C\frac{dv_C}{dt} = C\Big[-200e^{-200t}(24\cos wt + 4.434\sin wt)$$
$$+ e^{-200t}(-24\times1082.6\sin wt + 4.434\times1082.6\times\cos wt)\Big]$$
$$= Ce^{-200t}\left[0.2484\cos wt - 26869\sin wt\right]$$
$$= e^{-200t}\left[8.1972\times10^{-7}\cos(1082.6t) - 0.0887\sin(1082.6t)\right]\ (A)$$
$$\approx -88.7e^{-200t}\sin(1082.6t)\ (mA)$$

b) system is underdamped.

Q68 (a & b)

a)



$R = 10\,k\Omega$,
$C = 0.1\,\mu F$
$v_s(t) = 5u(t)$

$v_s = Ri_1 + v_1 = RC\frac{dv_1}{dt} + v_1$

$v_1 = Ri_2 + v_2 = RC\frac{dv_2}{dt} + v_2 \Rightarrow \frac{dv_1}{dt} = RC\frac{d^2v_2}{dt^2} + \frac{dv_2}{dt}$

$\therefore\ v_s = RC\left(RC\frac{d^2v_2}{dt^2} + \frac{dv_2}{dt}\right) + RC\frac{dv_2}{dt} + v_2$

$R^2C^2\frac{d^2v_2}{dt^2} + 2RC\frac{dv_2}{dt} + v_2 = v_s$

$\frac{d^2v_2}{dt^2} + \frac{2}{RC}\frac{dv_2}{dt} + \frac{1}{R^2C^2}v_2 = \frac{1}{R^2C^2}v_s$

$\frac{2}{RC} = \frac{2}{10\times10^3\times0.1\times10^{-6}} = 2\times10^3 = 2000$

$\frac{1}{R^2C^2} = \left(\frac{1}{10\times10^3\times0.1\times10^{-6}}\right)^2 = 10^6$

$s^2 + 2000s + 10^6 = 0 \Rightarrow s_1 = s_2 = -1000$

$v_2(t) = K_1e^{-1000t} + K_2te^{-1000t} + 5$

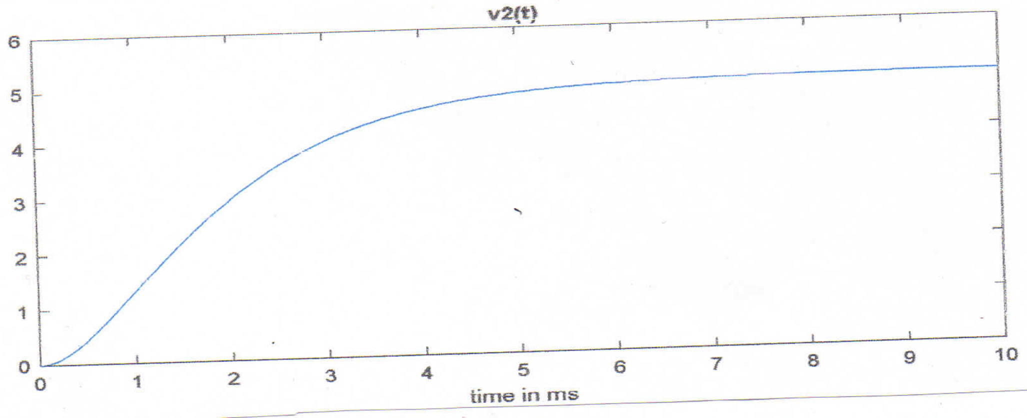
$0 = K_1 + 5 \Rightarrow K_1 = -5$

$\frac{dv_2}{dt} = -1000K_1e^{-1000t} + K_2e^{-1000t} + K_2t(-1000)e^{-1000t}$

$\frac{dv_2}{dt}(0) = -1000K_1 + K_2 = 0 \qquad K_2 = 1000K_1 = -5000$

$\therefore\ v_2(t) = -5e^{-1000t} - 5000te^{-1000t} + 5 \qquad t \geq 0$

b)



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