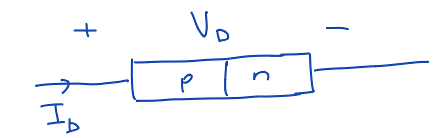
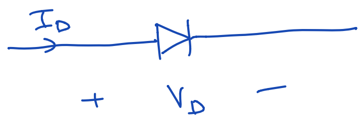
DIODE CIRCUITS

pn Junction Diode





$$I_D = I_S\left(e^{\frac{V_D}{nV_T}} - 1\right)$$: Shockley equation

I_S : reverse-bias saturation current
- depends on doping concentration
- proportional to cross-sectional area

For silicon pn junction: $10^{-18} - 10^{-12}$ A

V_T : thermal voltage , $V_T = \frac{k.T}{e_q} \approx 26\,mV$ @ 300 K

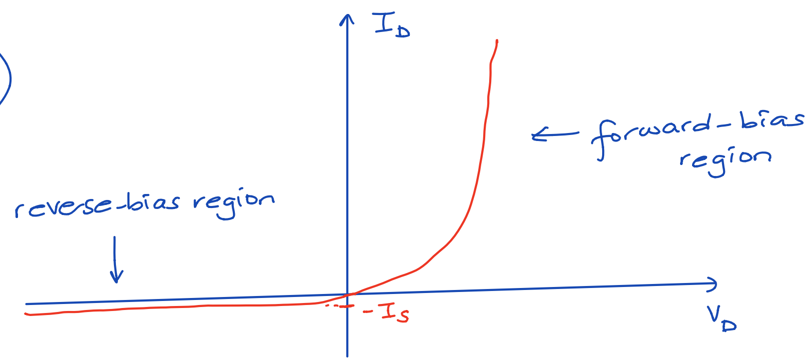
n : emission coefficient, $1 \leq n \leq 2$

@ low current levels, $n \approx 2$

@ high current levels, $n \approx 1$

Assume $n \approx 1$ unless otherwise stated.

* $I_D = I_S\left(e^{\frac{V_D}{nV_T}} - 1\right)$



Passive device : $P_D = I_D . V_D \geq 0$ always

Example: pn junction @ 300K , $n = 1$, $V_T = 26\,mV$, $I_S = 10^{-14}$ A

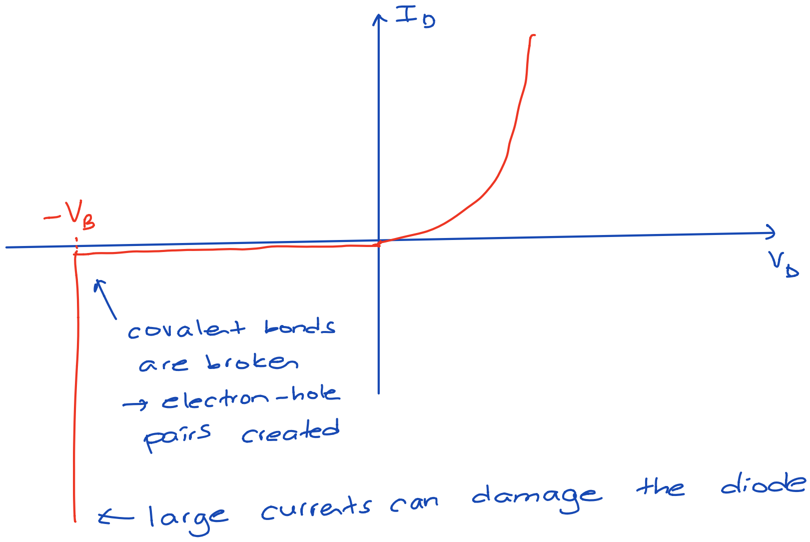
* when $V_D = 0.7\,V$, $I_D = I_S\left(e^{\frac{V_D}{nV_T}} - 1\right) = 10^{-14}\left(e^{\frac{700mV}{26mV}} - 1\right)$

$\simeq 4.93\ mA$ $\Rightarrow$ a small forward bias voltage can induce moderate current

* when $V_D = -0.7V$, $I_D = 10^{-14}\left(\cancel{e^{-\frac{700mV}{26mV}}} - 1\right) \approx -10^{-14}$ A

$\Rightarrow$ in reverse bias, the current is virtually zero.

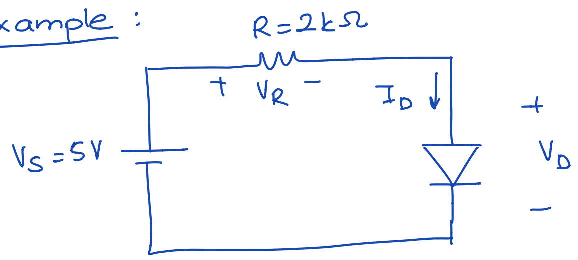
* In reality, there is also breakdown:



$V_B \sim 50-200V$
→ can be made smaller with larger doping (Zener diodes)

Diode Circuits

Example:



For the diode, assume $I_S = 10^{-13} A$, $n = 1$, $V_T = 26\ mV$.

KVL: $V_S = V_R + V_D$

$$V_S = I_D \cdot 2k\Omega + V_D$$

$$V_S = R \cdot I_S \left[e^{\frac{V_D}{nV_T}} - 1 \right] + V_D$$

* solving this equation is not easy.

Method 1: Iterative approach

* Try $V_D = 0.6V$

$$V_S \stackrel{?}{=} 2.10^3 . 10^{-13} \left[e^{\frac{600mV}{26mV}} - 1 \right] + 0.6\ V$$

$$5 \neq 2.7V$$

* Try $V_D = 0.65\ V$

$$V_S \stackrel{?}{=} 2.10^3 . 10^{-13} \left[e^{\frac{650mV}{26mV}} - 1 \right] + 0.65\ V$$

$$5 \neq 15.05V$$

V_D	Right-Hand Side
0.6 V	2.7 V
0.65 V	15.05 V
⋮	⋮
0.619 V	4.99 V

after some iterations

Then, $V_D = 0.619\,V$

$$I_D = \frac{5-0.619}{2k} = 2.19\,mA \quad \leftarrow \text{much easier}$$

OR, $$I_D = 10^{-13}\left[e^{\frac{619\,mV}{26\,mV}} - 1\right] = 2.19\,mA$$

Method 1 : Alternative : logaritmic iteration approach

$$V_S = R \cdot I_S\left[e^{\frac{V_D}{nV_T}} - 1\right] + V_D$$

$$V_D = nV_T \cdot \ln\left(\frac{V_S - V_D}{R \cdot I_S} + 1\right)$$

* Try $V_D = 0.6\,V$

$$V_D = 0.026 \times \ln\left(\frac{5-0.6}{2.10^3 \cdot 10^{-13}} + 1\right) = 0.619\,V \Rightarrow$$ converges a lot more quickly!

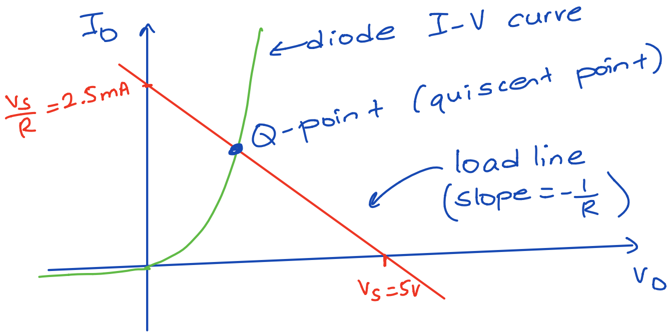
Method 2 : Graphical approach

$$V_S = I_D \cdot R + V_D \Rightarrow \underbrace{I_D = \frac{V_S}{R} - \frac{V_D}{R} = \frac{5 - V_D}{2\,k\Omega}}$$ ← linear relationship between V_D, I_D

$V_D = 0 \Rightarrow I_D = 2.5\,mA$

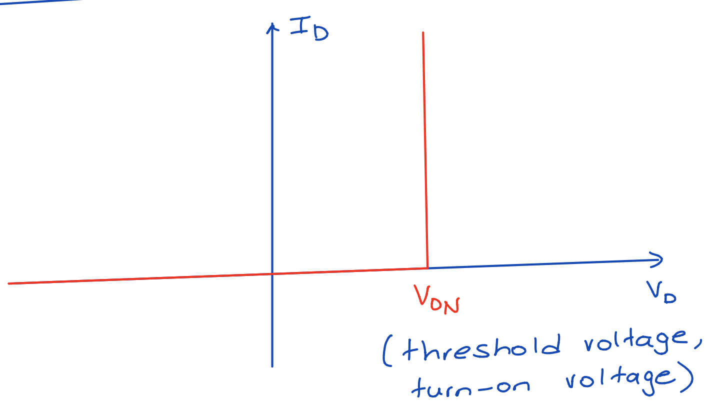
$I_D = 0 \Rightarrow V_D = 5\,V$

slope : $-\frac{1}{R}$



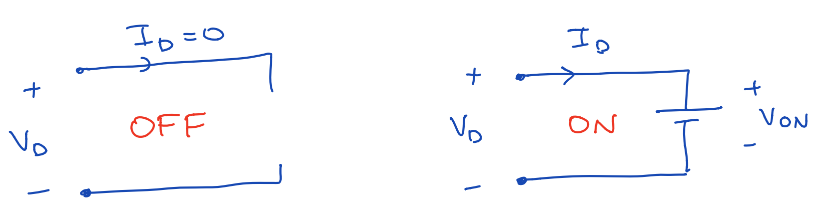
* This method is convenient and very accurate if you have a plotting tool (e.g., MATLAB)

Piecewise Linear model : a simplified model



OFF : $V_D < V_{ON}$, $I_D = 0$

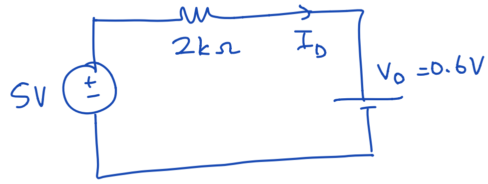
ON : $I_D > 0$, $V_D = V_{ON}$



For previous example:

* Assuming ON state for diode, $V_{ON} = 0.6\,V$

$V_D = V_{ON} = 0.6\,V$



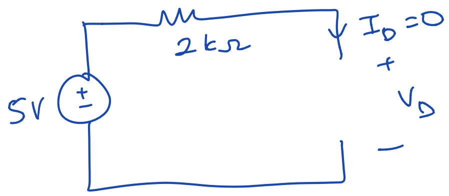
Then, $I_D = \frac{5 - 0.6}{2k} = 2.2\,mA$

$\Downarrow$

very close to previous result

* If we assumed OFF state:

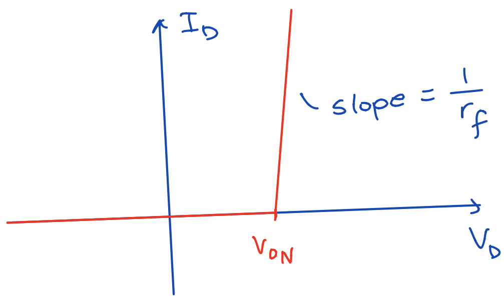
$I_D = 0$, we assume $V_D < V_{ON}$



$\times$ contradiction.

$\Rightarrow$ it cannot be the solution.

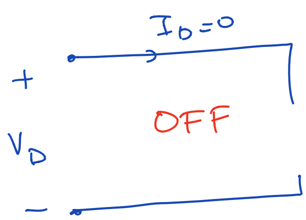
Improved model:

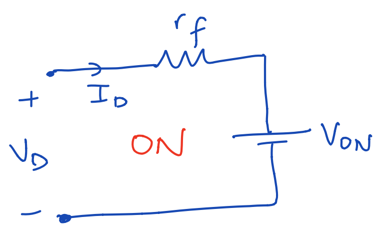


OFF: $V_D < V_{ON}$, $I_D = 0$

ON: $I_D > 0$, $V_D = V_{ON} + r_f \cdot I_D$

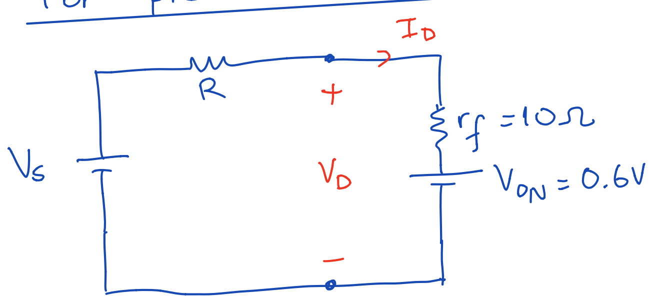
r_f: Forward diode resistance





For previous example:

Assume ON operation, $V_{ON} = 0.6V$, $r_f = 10\Omega$



$$I_D = \frac{V_S - V_{ON}}{R + r_f} = \frac{5 - 0.6}{2.10^3 + 10} = 2.19\,mA$$

$$V_D = I_D \cdot r_f + V_{ON} = 0.622\,V$$

much closer!!

since $R \gg r_f$, using r_f in the model does not make a huge difference.

In general for diode circuits:

* Assume the diode is ON or OFF, solve circuit, and check your assumptions. In most cases, we ignore r_f, since it is typically small when compared to the resistances in the rest of the circuit

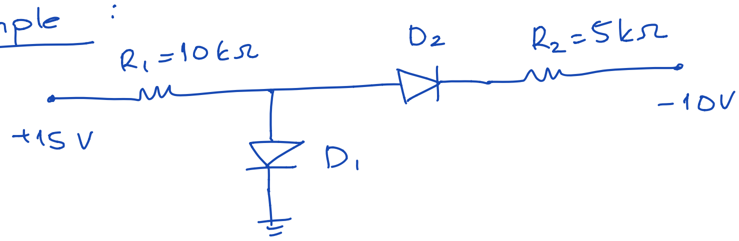
* So, we typically use the following model:

  OFF : $V_D < V_{ON}$, $I_D = 0$

  ON : $I_D > 0$, $V_D = V_{ON}$

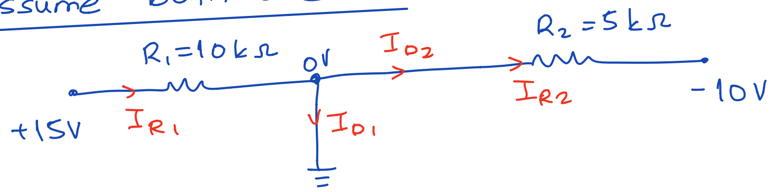
where $V_{ON} \sim 0.6 - 0.7\,V$ for most diodes.

Example :



Assume ideal diodes, $V_{ON} = 0$

* Assume both are ON :



$V_{D1} = 0$
$V_{D2} = 0$ } short circuit

$$I_{R_1} = \frac{15-0}{10k} = 1.5\,mA$$

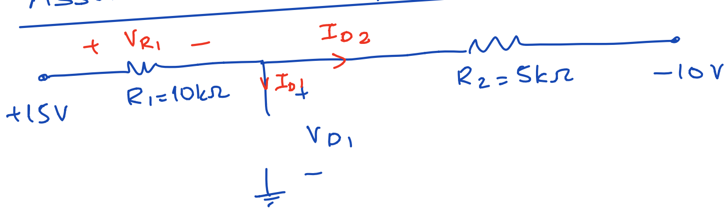
$$I_{R_2} = I_{D2} = \frac{0-(-10)}{5k} = 2\,mA > 0 \quad \checkmark \quad I_{D2} > 0, \text{ o.k.}$$

KCL : $-I_{R_1} + I_{D_1} + I_{D2} = 0$

$$I_{D_1} = -0.5\,mA < 0 \quad \times$$

contradiction with D_1 ON. we need $I_{D_1} > 0$

* Assume D_1 OFF, D_2 ON :



$$I_{D_1} = 0$$

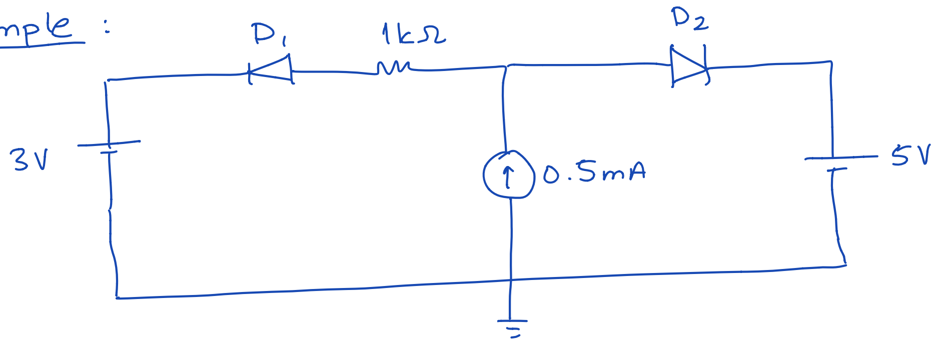
$$I_{D2} = \frac{15-(-10)}{10k+5k} = 1.67\,mA > 0 \quad \checkmark \quad \text{o.k for } D_2$$

KVL : $15 = V_{R_1} + V_{D_1} = R_1 \cdot I_{D2} + V_{D_1}$

$$V_{D_1} = 15 - (10k \cdot 1.67\,mA) = -1.67\,V < V_{ON} = 0 \quad \checkmark \text{ o.k.}$$

$\Rightarrow$ so, this is the solution.

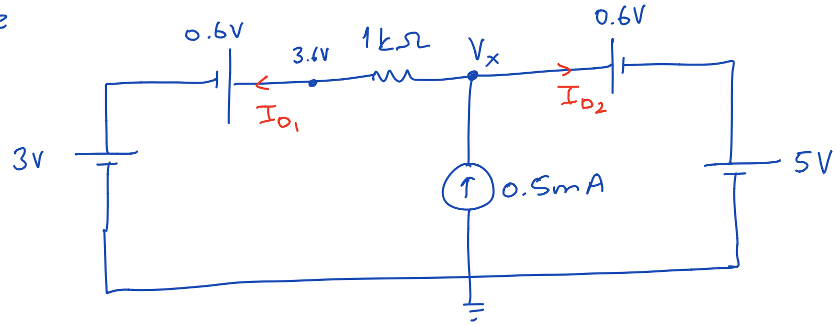
Example :



$V_{ON} = 0.6V$

* Both diodes cannot be OFF, since 0.5 mA current has to flow somewhere

Assume both ON :



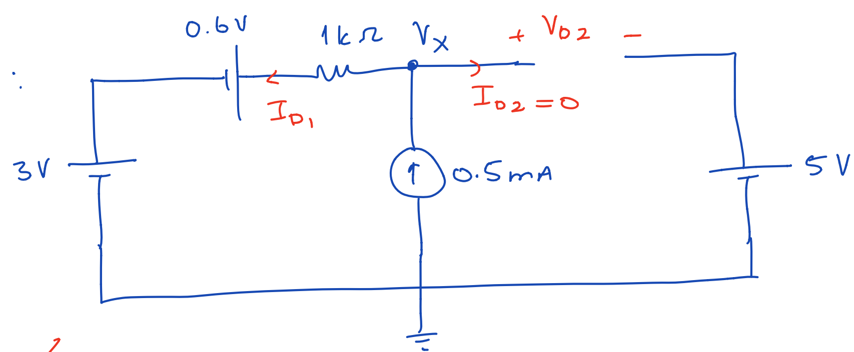
$V_x = 5 + 0.6 = 5.6V$

$I_{D1} = \dfrac{V_x - 3.6}{1k} = \dfrac{5.6 - 3.6}{1k} = 2mA > 0$ ✓ o.K.

KCL at V_x : $I_{D1} + I_{D2} = 0.5mA \Rightarrow I_{D2} = -1.5\ mA < 0$ ✗

conflict

Assume D_1 ON, D_2 OFF :



$I_{D1} = 0.5\ mA > 0$ ✓

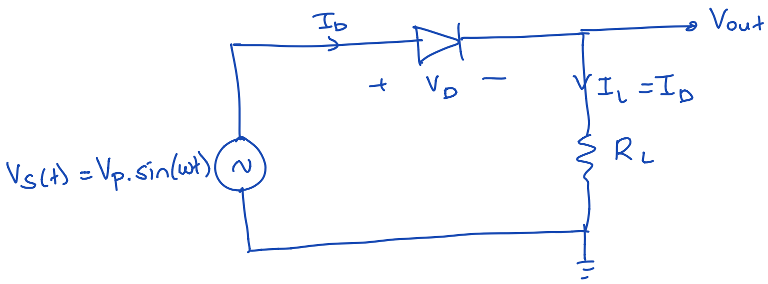
$V_x = 3 + 0.6V + 1k \cdot I_{D1} = 3.6 + 0.5 = 4.1V$

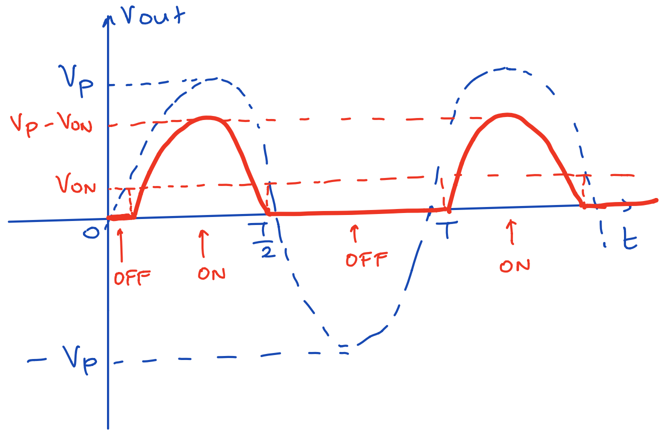
$V_{D2} = V_x - 5 = 4.1 - 5 = -0.9V < V_{ON}$ ✓

$\Rightarrow$ so, this is the solution

Half-wave Rectifier :

* A rectifier is the first stage of a DC power supply. used in converting the AC signal to DC.





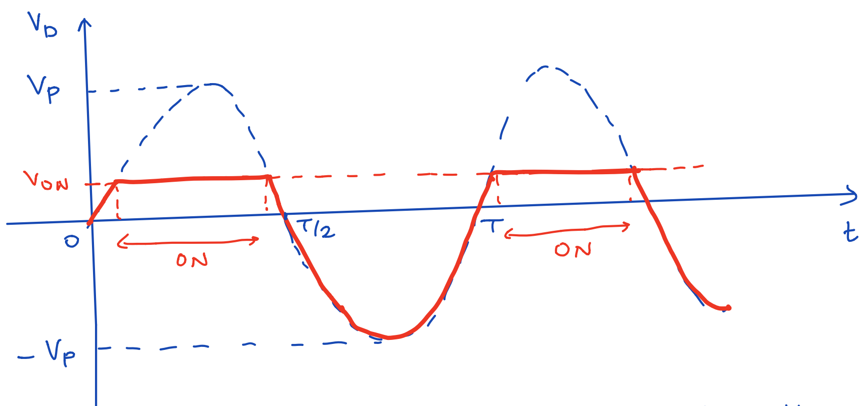
* When $V_S < V_{ON}$, the diode is OFF $\Rightarrow$ $I_D = 0$,
$V_{out} = R_L . I_D = 0$

* When $V_S > V_{ON}$, the diode is ON $\Rightarrow$ $V_D = V_{ON}$
$V_{out} \simeq V_S - V_{ON}$
$$I_D = \frac{V_S - V_{ON}}{R_L}$$

* $V_D = V_S - V_{out}$

$$= \begin{cases} V_S , & \text{when diode OFF} \\ V_{ON} , & \text{when diode ON} \end{cases}$$

$$= \begin{cases} V_S , & \text{when } V_S < V_{ON} \\ V_{ON} , & \text{when } V_S > V_{ON} \end{cases}$$



* The diode must be able to handle :
  - the peak current : $I_{D,max} = \dfrac{V_{out,max}}{R_L} = \dfrac{V_p - V_{ON}}{R_L}$
  - $(-V_p)$ voltage in the reverse bias case.

* Assume $V_{ON} = 0$,
Then, the diode conducts during half of the period, and the average V_{out} is not zero.

$$V_{out,avg} = \frac{1}{T}\int_0^T V_{out}(t)\,dt = \frac{1}{T}\int_0^{T/2} V_p \cdot \sin(\omega t)\,dt$$

$$= \frac{1}{T}\left(-\frac{V_p}{\omega}\right)\cos(\omega t)\Big|_0^{T/2} = -\frac{V_p}{T.\omega}\left[\cos\left(\omega \cdot \frac{T}{2}\right) - 1\right]$$

$\omega = 2\pi f = \frac{2\pi}{T}$

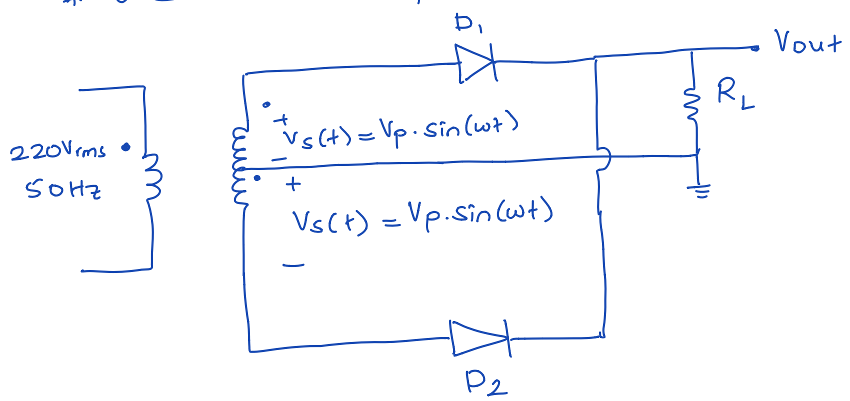
$$= -\frac{V_p}{\cancel{T}\cdot\frac{2\pi}{\cancel{T}}}\cdot\left[\underbrace{\cos\Big(\underbrace{\frac{2\pi}{T}\cdot\frac{T}{2}}_{\pi}\Big)}_{-1} - 1\right]$$

$$= -\frac{V_p}{2\pi}(-2) = \boxed{\frac{V_p}{\pi}}$$

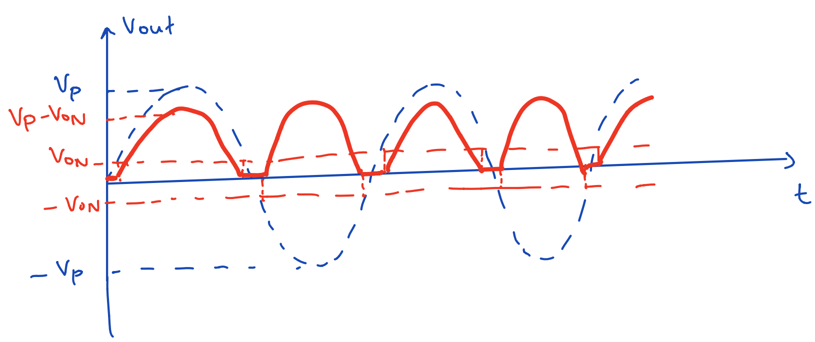
← non-zeo average voltage at the output

Full-wave Rectifier:

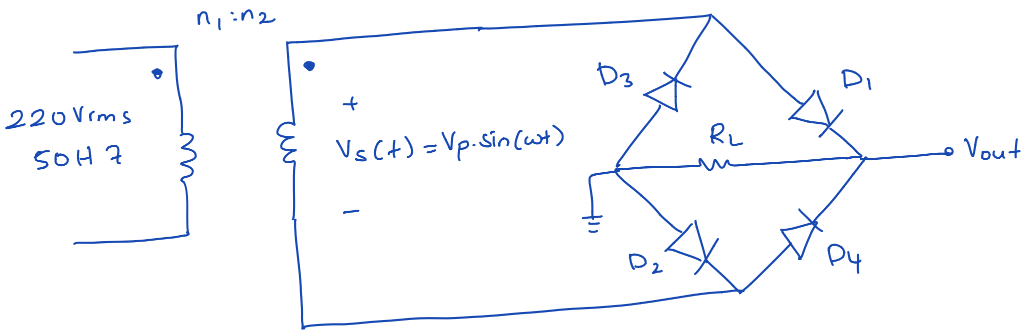
* We lose half of the sinewave in half-wave rectifier.

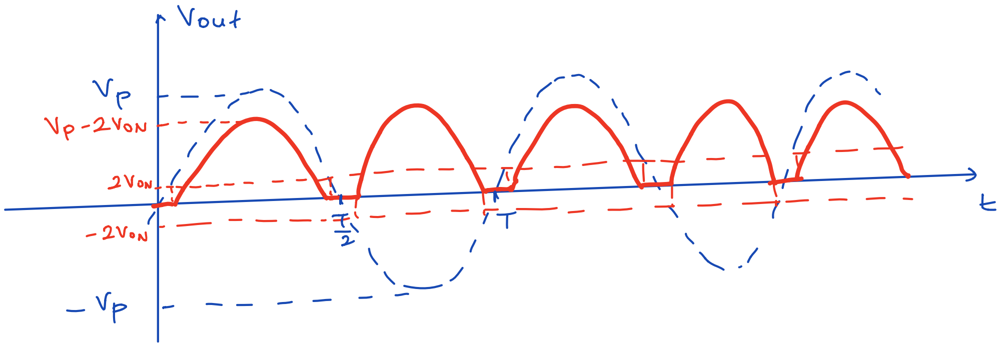


* a center-tapped transformer
* D_1 will conduct during the positive half cycle.
* D_2 will conduct during the negative half cycle.

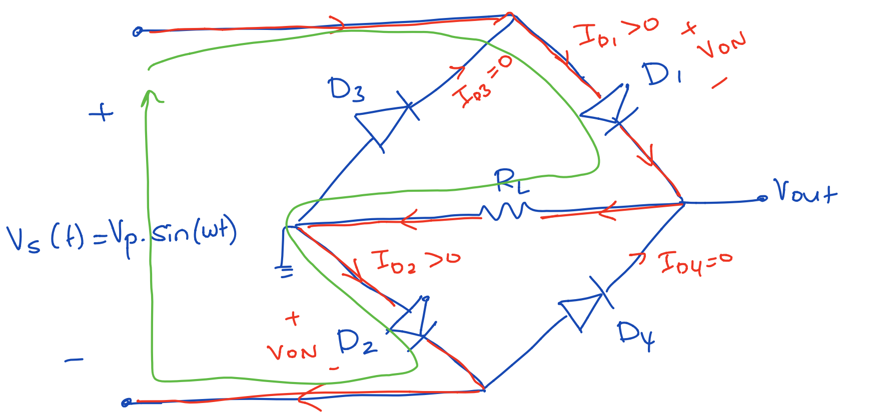


Another full-wave rectifier:





* when V_s is positive and sufficiently large (i.e., $V_s > 2V_{ON}$):



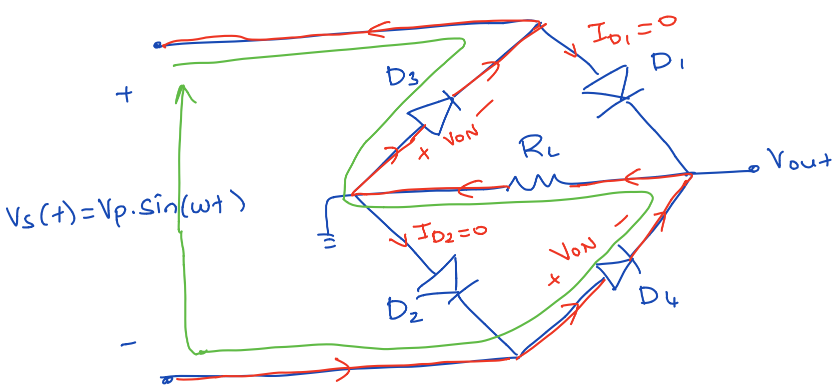
D_1, D_2 will be ON

D_3, D_4 will be OFF

KVL: $V_{ON} + V_{out} + V_{ON} - V_s = 0$

$$\boxed{V_{out} = V_s - 2V_{ON}}$$

* when V_s is negative and sufficiently large (i.e., $V_s < -2V_{ON}$)



D_1, D_2 will be OFF

D_3, D_4 will be ON

KVL: $-V_{ON} - V_{out} - V_{ON} - V_s = 0$

$$\boxed{V_{out} = -V_s - 2V_{ON}}$$

* So, overall,

$$V_{out} = \begin{cases} |V_s| - 2V_{ON}, & \text{when } |V_s| > 2V_{ON} \\ 0, & \text{otherwise} \end{cases}$$

* Turns ratio needed for the transformer:

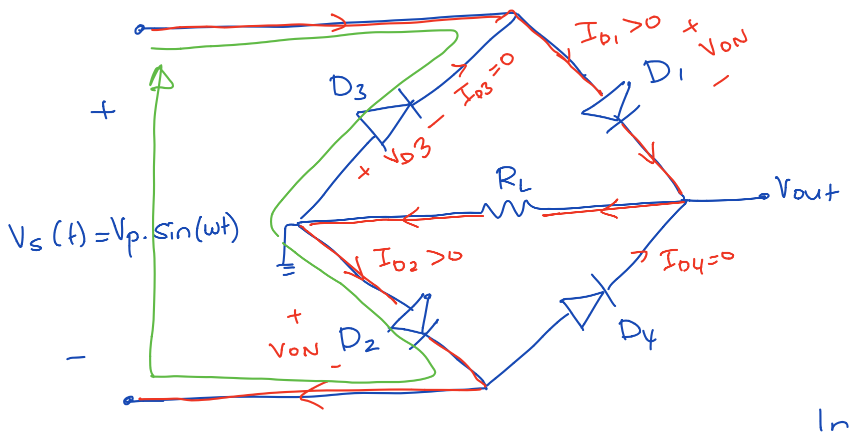
If we want 9V peak at the output, with $V_{ON} = 0.6V$

$9V = V_p - 2V_{ON} \Rightarrow V_p = 9 + 2 \times 0.6 = 10.2V$

On the primary side: $220\ V_{rms} \Rightarrow$ peak voltage $= 220 \times \sqrt{2}$
$= 311\ V_{peak}$

Then, the turns ratio: $\frac{n_1}{n_2} = \frac{311}{10.2} = 30.5$

* The peak reverse diode voltage:



$\underline{KVL}$: $-V_{D3} + V_{ON} - V_s = 0$

$V_{D3} = -V_s + V_{ON}$

Peak reverse diode voltage:

$|V_{D3,peak}| = V_p - V_{ON}$

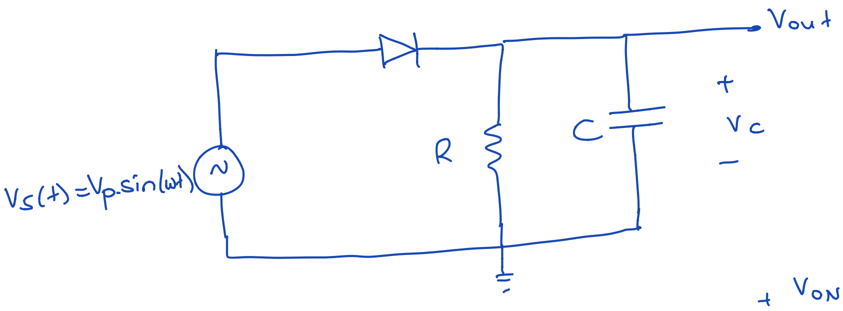
↳ same for other diodes

In our case,

$|V_{D3,peak}| = 10.2 - 0.6 = 9.6V$

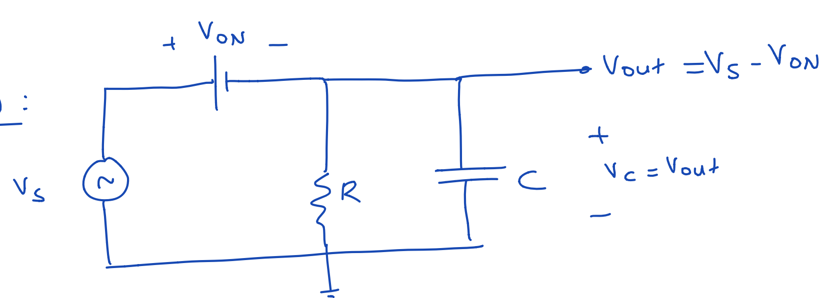
In the book, this is called peak inverse voltage (PIV).

Rectifier with Filter:



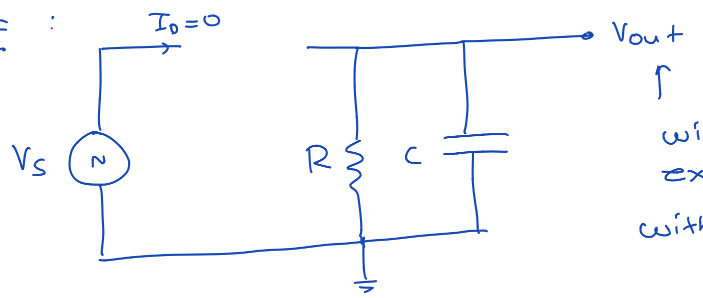
* Assume initial capacitor voltage is 0.

* when diode is ON:

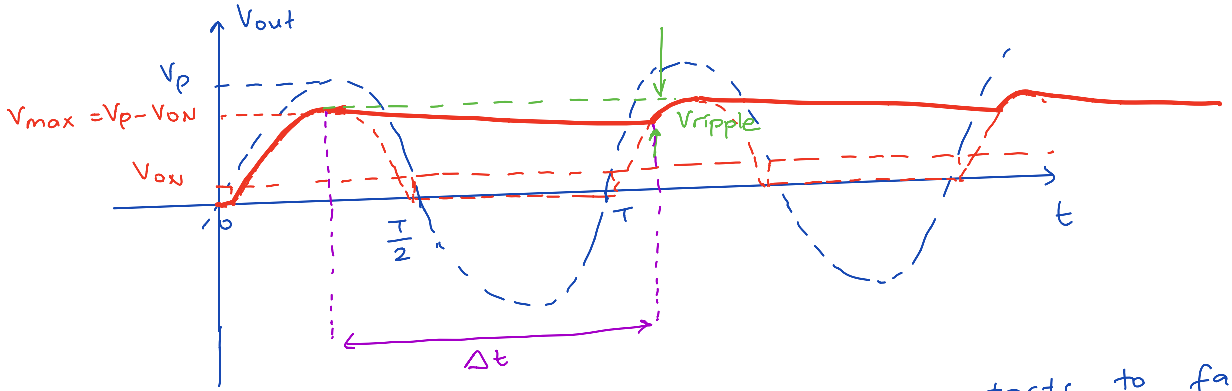


V_{out} will rise with V_s initially.

* when diode is OFF :



V_{out} will decrease exponentially, with $\tau = R.C$



* when Vs reaches its max and starts to fall, C should discharge and the only path is through R. So, diode turns OFF. If $\tau = RC >> T$, then V_{out} $(=V_c)$ remains relatively constant.

* when diode turns back ON, capacitor charges again.

* This action converts AC to DC, and C is the filter capacitor.

* The ripple voltage :

At the end of discharging: $V_{out} = V_{max} \cdot e^{-\frac{\Delta t}{\tau}}$, $\tau = RC$ and $\tau >> T$

$$V_{ripple} = V_{max} - V_{max} \cdot e^{-\frac{\Delta t}{\tau}}$$
$$= V_{max}\left(1 - e^{-\frac{\Delta t}{\tau}}\right)$$

* Since $\tau >> T$, $\Delta t \approx T << \tau$

So, $e^{-\Delta t/\tau} = 1 - \frac{\Delta t}{\tau}$ (Taylor series expansion for $e^{-x} = 1 - x$, if $x << 1$)

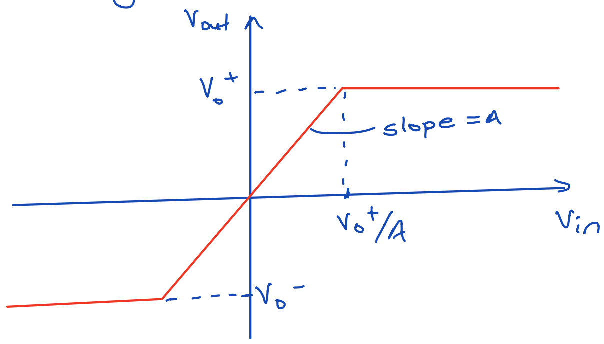
* So, $V_{ripple} = V_{max}\left(1 - \left(1 - \frac{\Delta t}{\tau}\right)\right) = \frac{V_{max}.\Delta t}{\tau} \simeq \frac{V_{max}.T}{\tau}$ $T = \frac{1}{f}$

$$\boxed{V_{ripple} = \frac{V_{max}}{f.RC}}$$ for half-wave rectifier

$$\boxed{V_{ripple} = \frac{V_{max}}{2fRC}}$$ for full-wave rectifer $\left(\text{since } \Delta t \approx \frac{T}{2}\right)$

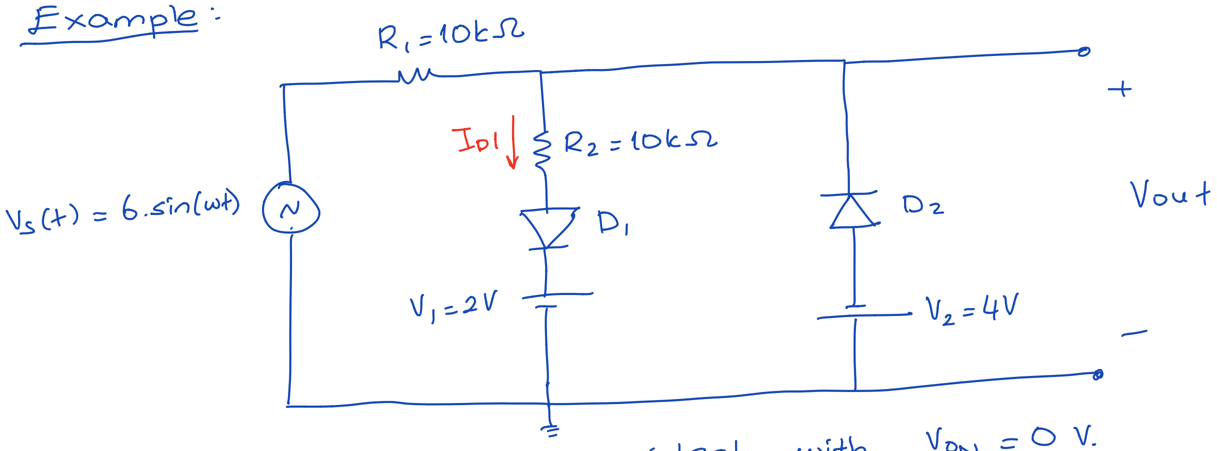
Clippers

* Limiter circuits used to eliminate (saturate) portions of a signal above or below specified values.



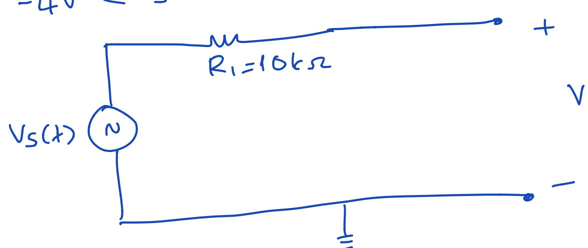
* These circuits are used to prevent over-voltage on circuit elements.

Example:



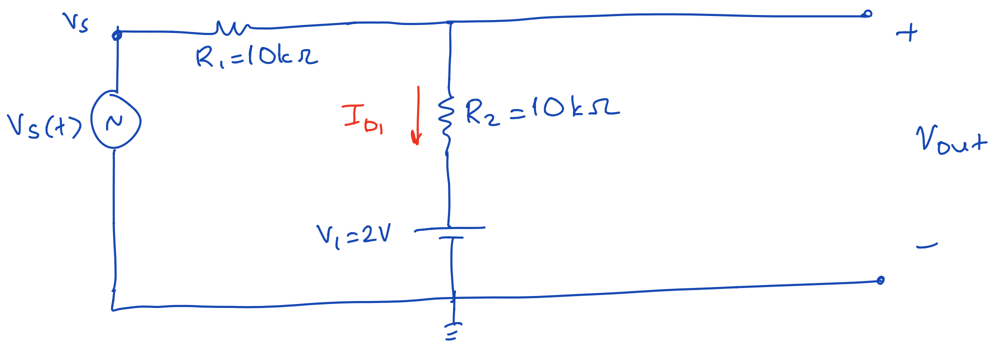
Assume diodes are ideal with $V_{ON} = 0$ V.

* if $-4V < V_s < 2V$, D1 and D2 OFF



$V_{out} = V_s$

* if $V_s > 2V$, D1 ON, D2 OFF

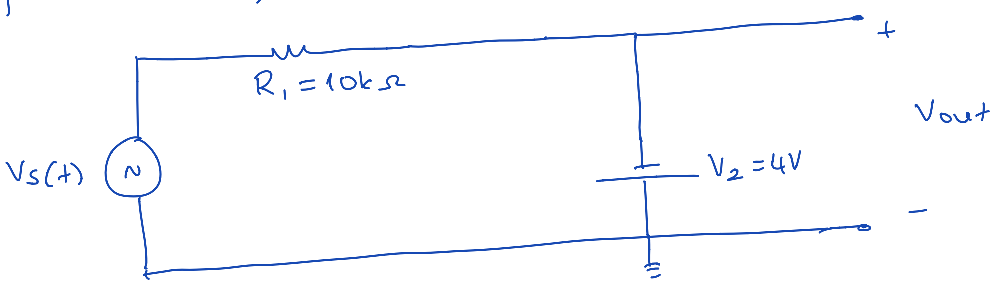


$$I_{D1} = \frac{V_s - V_1}{10k + 10k} = \frac{V_s - V_1}{20k}$$

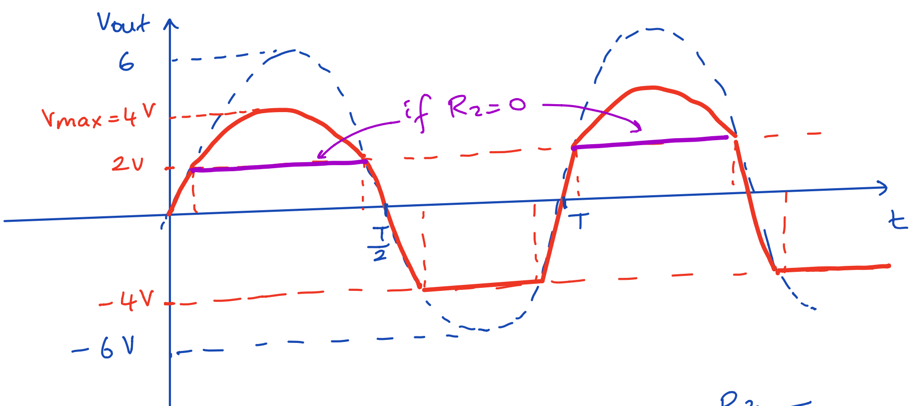
$$V_{out} = I_{D1} \cdot 10k + V_1 = \frac{V_s - V_1}{2} + V_1 = \frac{V_s + V_1}{2} = \frac{V_s}{2} + 1 \; V$$

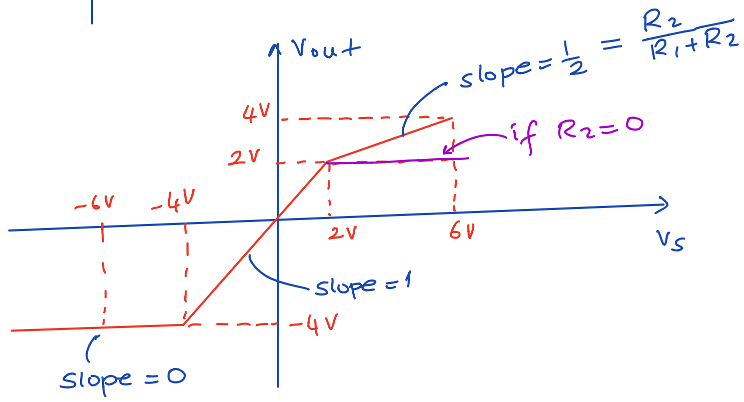
So, $V_{max} = \frac{6}{2} + 1 = 4V$

* if $V_s < -4V$, D_2 ON, D1 OFF :

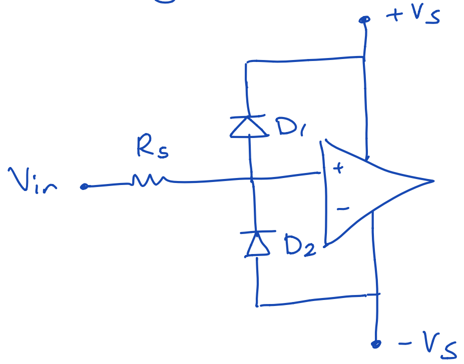


$$V_{out} = -4V$$





* Electrostatic Discharge (ESD) protection (over-voltage protection)



$V_+ = V_{in}$ in the normal setting

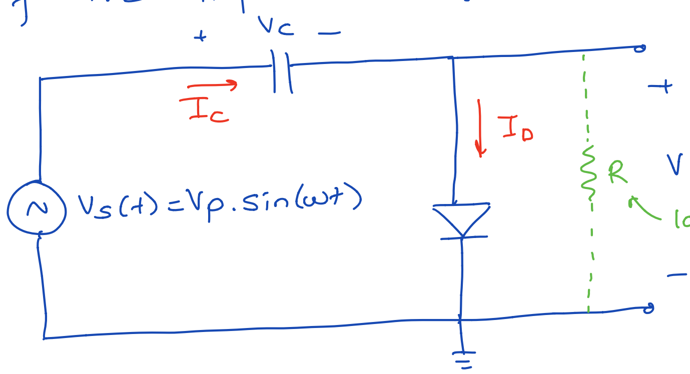
* if $V_{in} > V_s + V_{ON}$, D_1 turns ON.
  then $V_+ = V_s + V_{ON}$

* if $V_{in} < -V_s - V_{ON}$, D_2 turns ON.
  then $V_+ = -V_s - V_{ON}$

* otherwise, both diodes are OFF, so they have no effect on the operation of the OPAMP.

<u>Clampers</u>: (a.k.a. Level Shifters, DC voltage restorers)

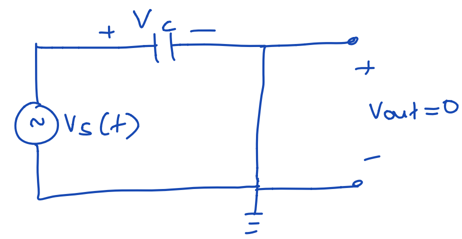
* shift the entire signal by a DC level.
* Otherwise, the output waveform is an exact replica of the input waveform.



large resistance (in reality)
capacitor can discharge only minimally (large $\tau = RC$)

* Assume $V_{ON} = 0\,V$
* Assume capacitor is initially not charged.

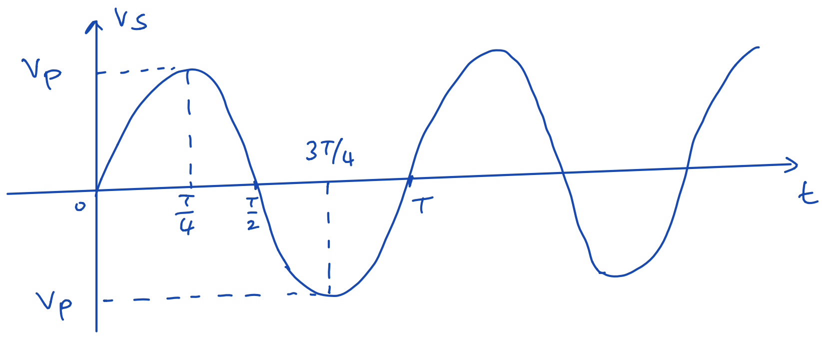
* During $0 < t < \frac{T}{4}$, diode is ON.

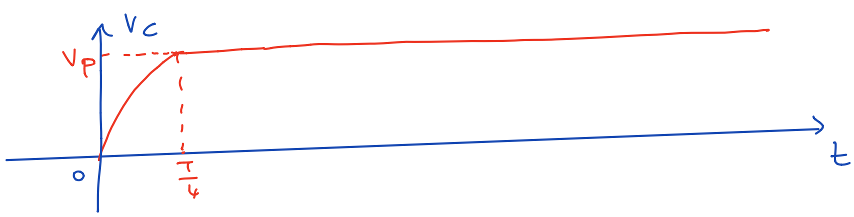


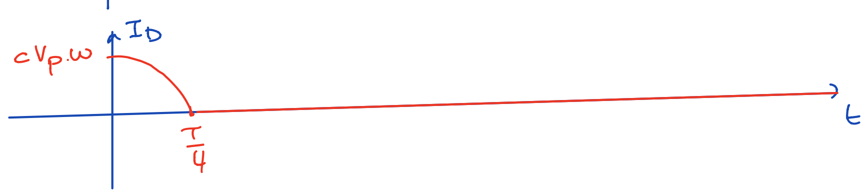
$V_{out} = V_{ON} = 0$

$V_C = V_s - V_{out} = V_s$

at $t = \frac{T}{4}$, $V_C = V_p$







* Once V_s starts to fall from its peak, diode turns OFF. The capacitor cannot discharge (there is no path.) $\Rightarrow V_C = V_p$

* During this time $(t > \frac{T}{4})$:

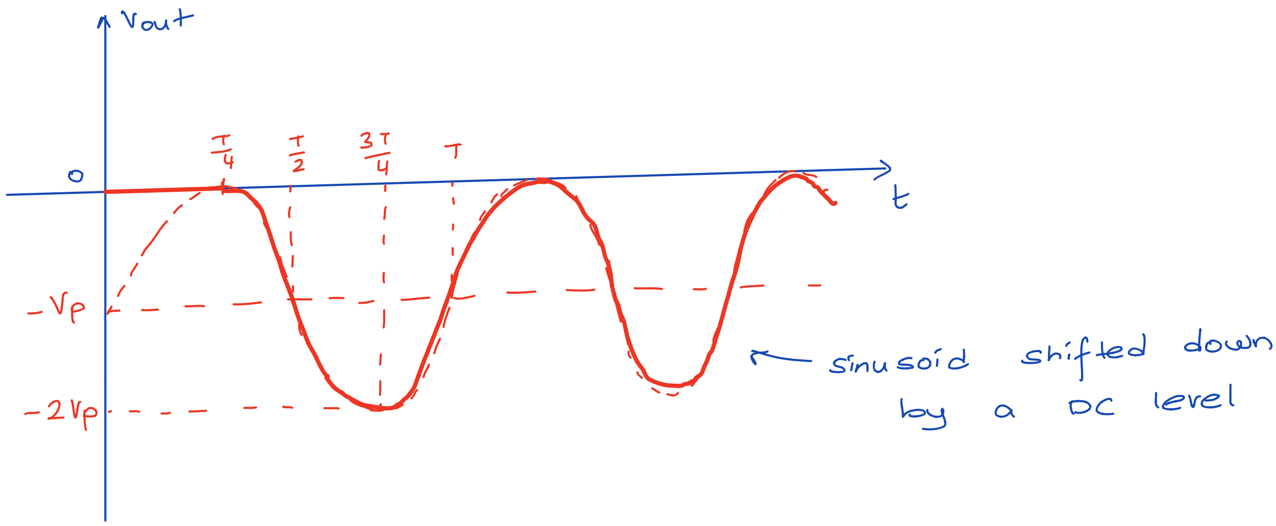
$$V_{out} = V_s - V_c$$
$$= V_p \cdot \sin(\omega t) - V_p \quad = V_p\,(\sin(\omega t) - 1)$$

Also,

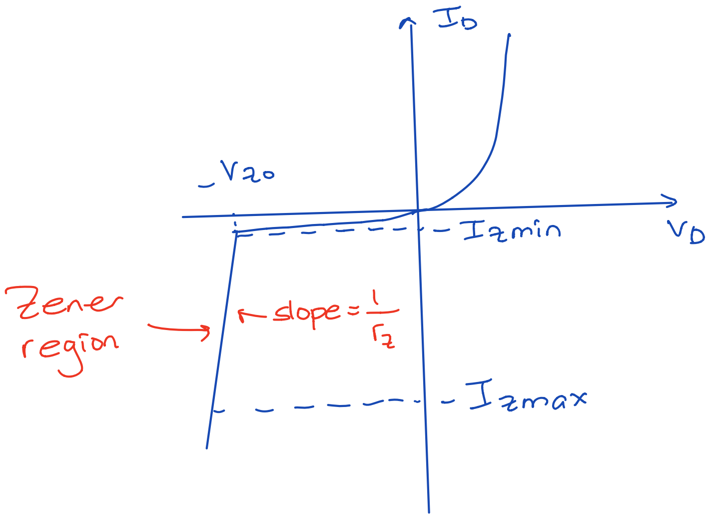
$$I_c = I_D = C\frac{dV_c}{dt} = \begin{cases} C\,\frac{d}{dt}\left(V_p \cdot \sin(\omega t)\right) & , \quad 0 < t < \frac{T}{4} \\ C \cdot \frac{d}{dt}\,(V_p) & , \quad t > \frac{T}{4} \end{cases}$$

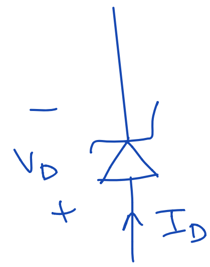
$$= \begin{cases} C\,V_p\,\omega \cdot \cos(\omega t) & , \quad 0 < t < \frac{T}{4} \quad \rightarrow \text{diode ON} \\ 0 & , \quad t > \frac{T}{4} \quad \rightarrow \text{diode OFF} \end{cases}$$

$$I_{D,peak} = C\,V_p \cdot \omega$$

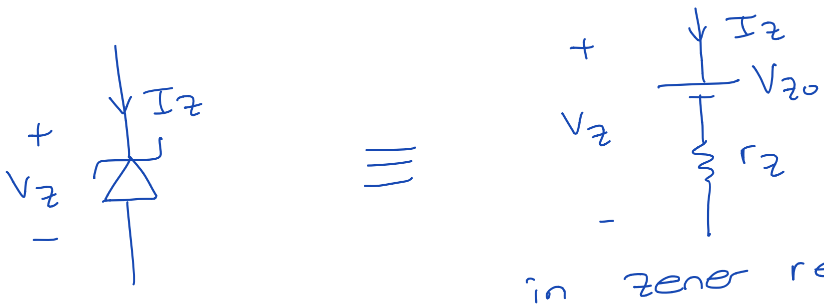


Zener Diode Circuits:



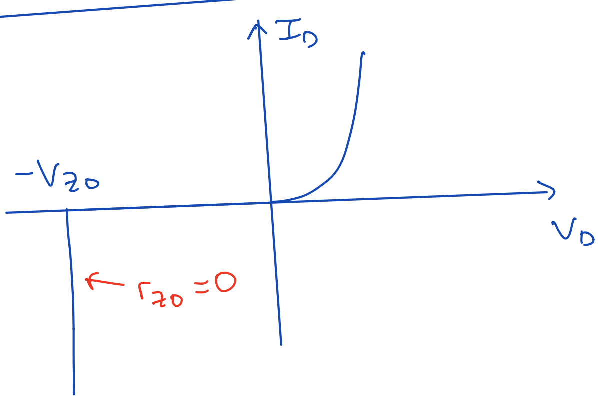


* We almost always use Zener diodes in Zener region.

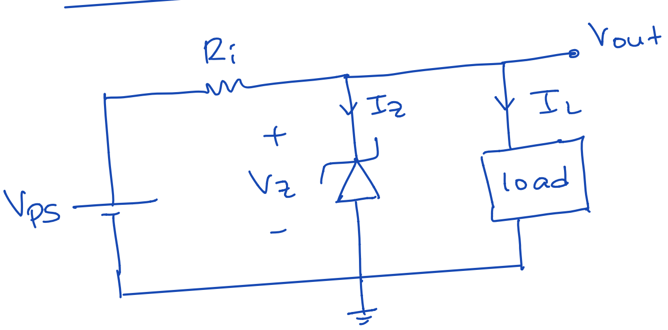


in zener region

Ideal Zener Diode



Simple Voltage Regulator



Depending on the application, R_L and/or I_L has some limits.
Also, V_{PS} has some limits (max and min).
We need to find R_i so that Zener limitations are not exceeded.

* Zener limitations :

I_{zmax} : maximum allowable Zener current. Related to max. Zener power dissipation $\simeq V_{z0} \times I_{zmax} = P_{zmax}$

I_{zmin} : to guarantee that we are in Zener region

Example : Consider a car radio operating from the car battery V_{ps}.
We know that V_{ps} has a certain range :

$$11\text{ V} < V_{ps} < 13.6\text{ V}$$

Load current I_L depends on how loud the radio is :

$$0 < I_L < 100\text{ mA}$$

Zener properties : $V_{z0} = 9V$. Assume $r_z = 0$

$I_{zmin} = 3mA = 0.003\text{ A}$

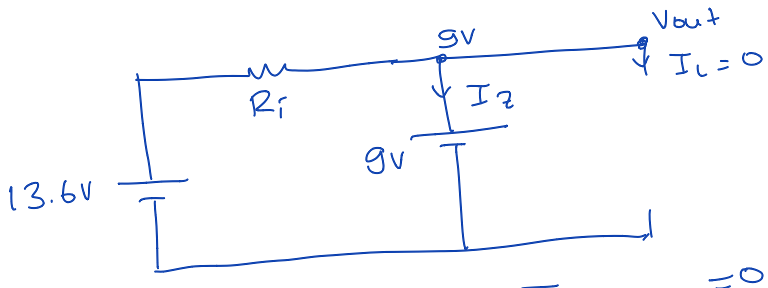
$P_{zmax} = 3W$

Find R_i so that Zener limitations are not violated.

* Not to exceed I_{zmax} :

$$I_{zmax} = \frac{P_{zmax}}{V_{z0}} = \frac{3W}{9V} = 0.333\text{ A} = 333\text{ mA}$$

I_z is max. when V_{ps} is max. and I_L is min :
$V_{ps} = 13.6V$, $I_L = 0$ A



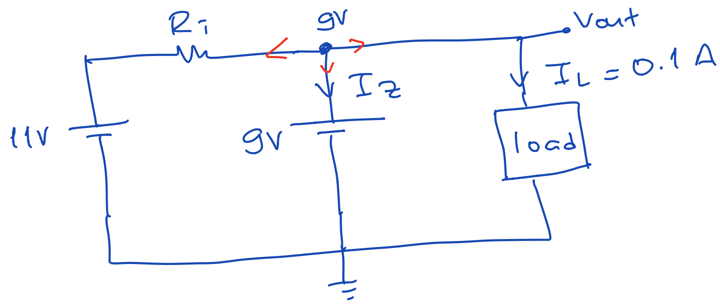
$$I_z = \frac{13.6 - 9}{R_i} < I_{zmax} = 0.33A$$

$$R_i > \frac{13.6 - 9}{0.333} = 13.8\Omega$$

* Not to violate I_{zmin} :

I_z is min. when V_{ps} is min. and I_L is max :

$V_{ps} = 11V$, $I_L = 100\text{ mA} = 0.1A$



KCL: $I_Z + \frac{9-11}{R_i} + 0.1 = 0$

$I_Z = \frac{11-9}{R_i} - 0.1 = \frac{2}{R_i} - 0.1 > I_{Zmin} = 0.003$ A

$\frac{2}{R_i} > 0.103 \implies R_i < \frac{2}{0.103} = 19.4\,\Omega$

* So, the acceptable range for R_i is:
$13.8\,\Omega < R_i < 19.4\,\Omega$

For example, we can choose $R_i = 15\,\Omega$.
Then, despite variations in V_{ps} and I_L, we will always be in Zener region, and $V_{out} = 9$V.

Load Regulation:

* L.R. = Load Regulation $\triangleq \frac{V_{out,no\text{-}load} - V_{out,full\text{-}load}}{V_{out,full\text{-}load}} \times 100\ \%$

No-load means $I_L = 0$
Full-load means I_L is max.

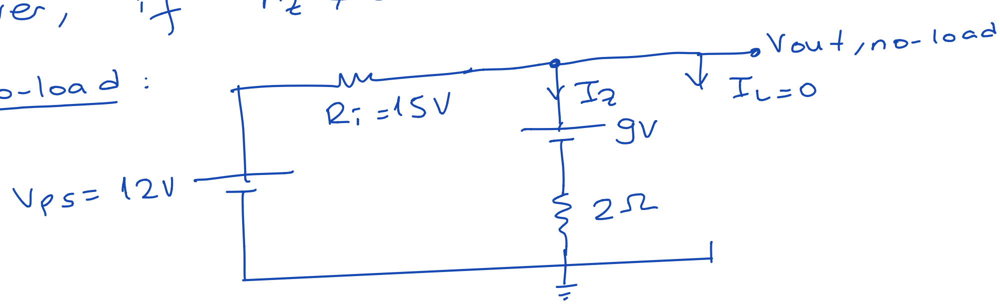
* Assume that $V_{ps} = 12$V and $R_i = 15\,\Omega$. ($0 < I_L < 100$ mA)
In this case, no matter what I_L is, we are always in Zener region, $V_{out} = 9$V, because

$V_{out,no\text{-}load} = V_{out,full\text{-}load} = 9$V

$\implies$ L.R. $= 0\ \%$

* However, if $r_Z \neq 0$, let's say $r_Z = 2\,\Omega$.

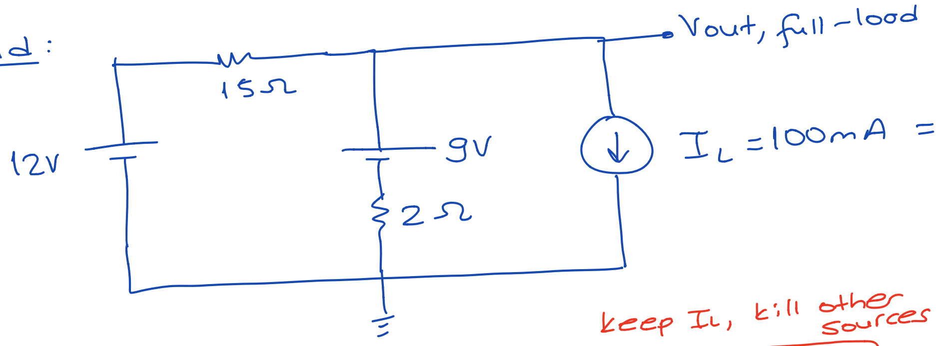
Vout, no-load:



$$I_z = \frac{12-9}{15+2}$$

$$V_{out,\,no\text{-}load} = 9 + I_z \times 2 = 9 + \frac{12-9}{15+2} \times 2 = 9 + 0.35 = 9.35\,V$$

<u>Vout, full-load</u>:



$I_L = 100mA = 0.1\,A$

$$V_{out,\,full\text{-}lood} = \underbrace{9 + \frac{12-9}{15+2} \times 2}_{\text{kill } I_L,\ \text{keep other sources}} \underbrace{- 0.1 \times (2 /\!/ 15)}_{\text{keep } I_L,\ \text{kill other sources}}$$

$$= 9.35 - 0.1 \times \frac{30}{17} = 9.17\,V$$

* So,

$$L.R. = \frac{9.35 - 9.17}{9.17} \times 100 = 1.96\,\% \approx 2\,\%$$

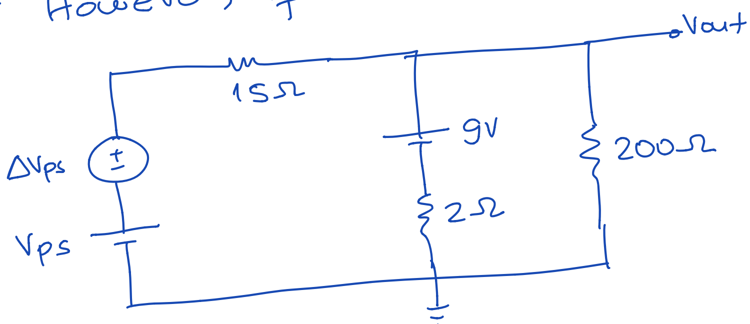
* As I_L changes between 0 mA and 100 mA, Vout changes by only 2%.

<u>Source Regulation</u>:

$$S.R. = \text{Source Regulation} = \frac{\Delta V_{out}}{\Delta V_{ps}} \times 100\,\%$$

* If $r_z = 0$, as long as we are in zener region, Vout = 9V, $\Rightarrow$ S.R. = 0 %

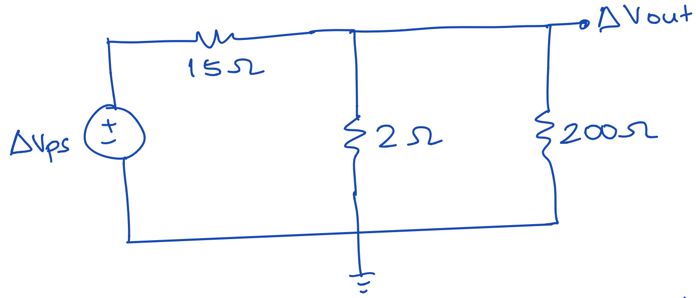
* However, if $r_z \neq 0$, let's say $r_z = 2\,\Omega$:



Let's model the load as a 200 Ω resistor ($R_L = 200\,\Omega$)

Vout has a part due to Vps, 9V, and ΔVps. We are interested in the part ΔVout, which is due to ΔVps.

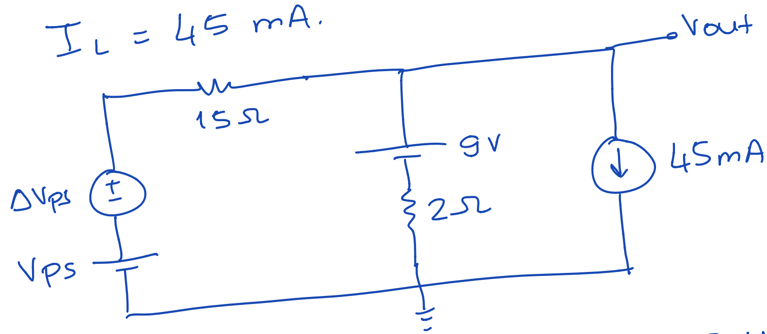
Kill Vps and 9V sources:



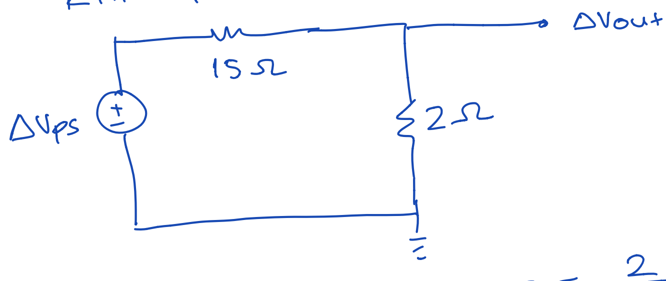
$$\Delta V_{out} = \frac{2 /\!/ 200}{15 + 2/\!/200} \cdot \Delta V_{ps} = 0.1167\ \Delta V_{ps}$$

So, $$S.R. = \frac{\Delta V_{out}}{\Delta V_{ps}} \times 100\ \% = 0.1167 \times 100 = 11.67\ \%$$

* Let's consider another case when load is fixed to $I_L = 45\ mA$.



Kill Vps, 9V and 45 mA sources:



$$\Delta V_{out} = \frac{2}{15+2} \cdot \Delta V_{ps} = \frac{2}{17}\ \Delta V_{ps}$$

$$S.R. = \frac{\Delta V_{out}}{\Delta V_{ps}} = \frac{2}{17} \times 100 = 11.76\ \%$$

What did we learn this semester?

- KCL, KVL, branch relations
- Node analysis, mesh analysis
- Linearity: Thevenin / Norton eq. circuits, superposition
- OPAMPS
- Dependent sources
- Signal waveforms (impulse, unit step, ramp func., exponentials, sinusoids, damped sinusoids)
- First and second-order circuits, natural response, forced response.
- Sinusoidal steady-state, phasor analysis
- Complex power
- Laplace transform
- s-domain analyses
- zero-input response, zero-state response
- initial condition transformation
- transfer function, impedance
- impulse response, step response, convolution
- Diode circuits, rectifiers, clippers, clampers
- Zener diode circuits, voltage regulator.