EE-202 Circuit Theory

Lab5 -Bandpass Filter

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Introduction:

Purpose of this lab is to design a bandpass filter with load 50 Ohm load resistance. Specifications are as follows:

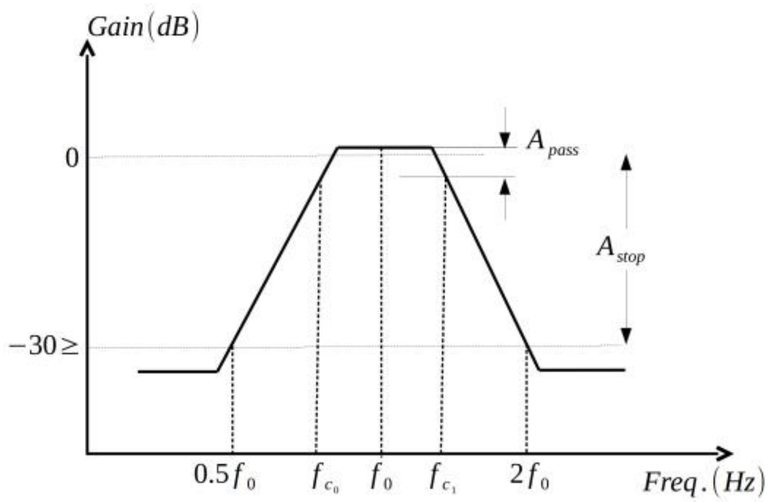

Figure 1: Frequency response of the filter

Central frequency: $3Mhz \leq f_0 \leq 6Mhz$
Passband width: $f_{c_1} - f_{c_0} = 0.05f_0$
Gain variation in the passband: $A_{pass} \leq 3dB$
Stopband attenuation: $A_{stop} \geq 30dB$

Fig 1: Specifications

Chosen signal is AC amplitude 2

Chosen frequency for this lab was **4.4MHz.**

Hence the bandwidth (Δf) becomes **220kHz.**

In such case **f_{c1}=4.510MHz** and **f_{c0}=4.290MHz**

For the design of bandpass filters, a Butterworth Filter of second order was used.

Methodology:

To be able to design the wanted Butterworth Filter, one must first decide on the order of the filter. The equation to be able to decide on the order comes from the transducer power gain of the filter. The calculations can be seen in figure 2.

$$\frac{P_L}{P_A} = \frac{1}{1+(f_0/\Delta f)^{2n}(f/f_0 - f_0/f)^{2n}} \quad (1)$$

We know $f_0 = 4.4$ Megahertz

$\Delta f = 220$ kHz

At $f = 8.8$ Mhz we expect less than $-30dB$

• Turning (1) into its decibel equation and putting into values,

$$10 \log \frac{P_L}{P_A} = 10 \log \frac{1}{1+(f_0/\Delta f)^{2n}(f/f_0 - f_0/f)^{2n}} \leq -30dB$$

$$= \log \frac{1}{1+(20)^{2n}\left(\frac{3}{2}\right)^{2n}} \leq -3dB$$

$$\frac{1}{1+(30)^{2n}} \leq 10^{-3}$$

$n > 1.0155$

we need therefore <u>a second order filter</u>

Fig 2: Butterworth Filter Order Calculation

After figuring out the order of the wanted bandpass filter, lookup table can be used to be able to calculate values of the capacitors and inductors.

		a_1	a_2	a_3	a_4	a_5	a_6	a_7
1	1.0	2.0000						
2	1.0	1.4142	1.4142					
3	1.0	1.0000	2.0000	1.0000				
4	1.0	0.7654	1.8478	1.8478	0.7654			
5	1.0	0.6180	1.6180	2.0000	1.6180	0.6180		
6	1.0	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
7	1.0	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450

Fig 3: Table of Prototype element values in Butterworth low-pass-filters

Since we are using a second order bandpass filter the element values are **a_1=a_2=1.4142**. Then the design process is:

1. A low pass 2nd order low pass should be design using Δf as the cutoff frequency, the corresponding capacitor and inductor values are,

$$L_1 = \frac{a_1 R_L}{2\pi * \Delta f}$$

$$C_2 = \frac{a_2}{2\pi * \Delta f * R_L}$$

2. After the low pass filter design, every series inductor needs to be paired with a series capacitor and every shunt capacitor needs to be paired with a shunt inductor which are resonant in the center frequency. Therefore, corresponding values would be:

$$C_1 = \frac{1}{(2\pi f_0)^2 L_1}$$

$$L_2 = \frac{1}{(2\pi f_0)^2 C_2}$$

Now since all the equations needed for filter is ready, the values of filter elements are as follows

L_1	51.1 uH
C_2	20.4 nF
C_1	25.6 pF
L_2	64 nH

Table 1: Bandpass filter component values

Since all the information for the bandpass filter is ready the circuit can be designed and tested on software.

Software Lab

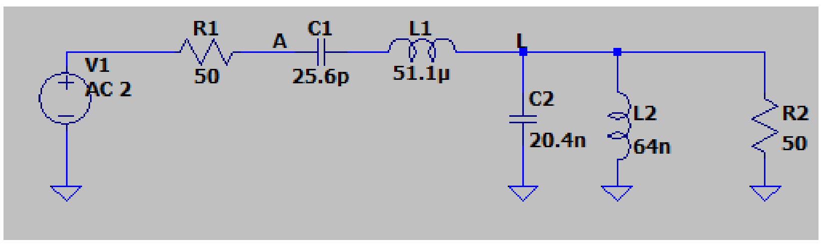

Fig 4: Designed LTspice Schematic

Values according to lab specifications can be seen below.

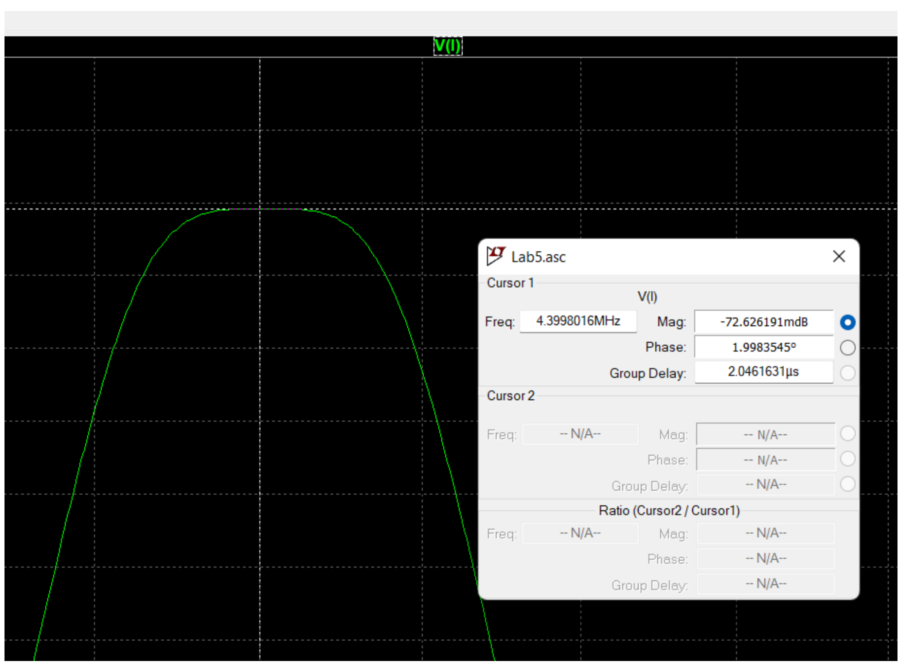

Fig 5: Center Frequency, f_0 = 4.4 MHz

Center frequency is as expected

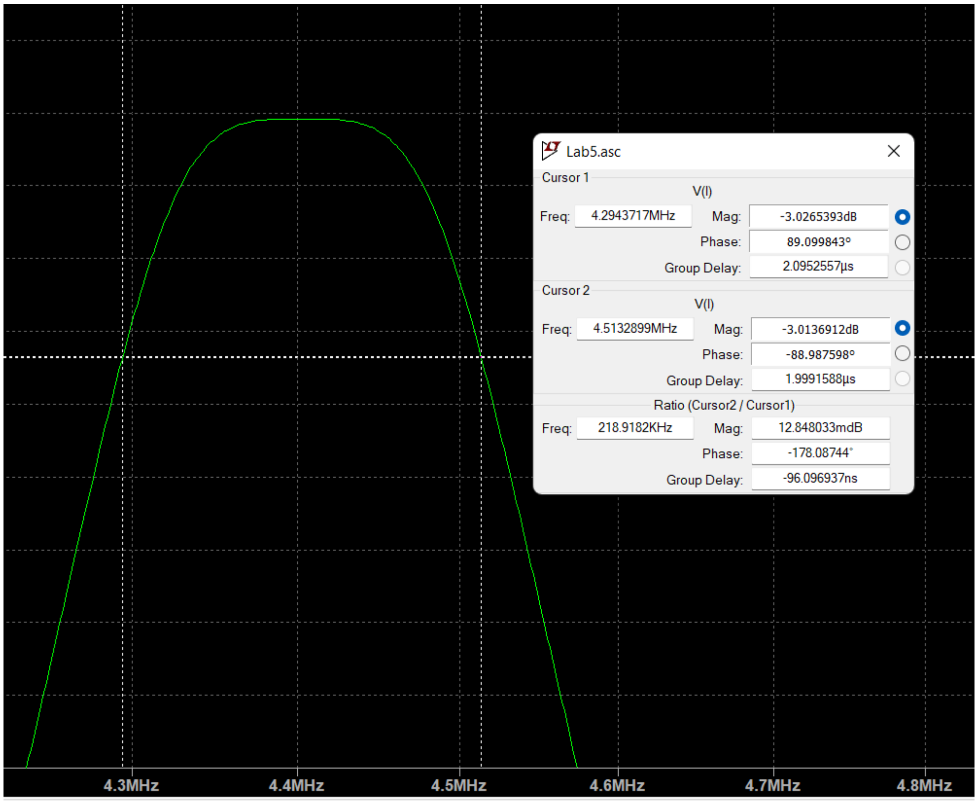

Fig 6: Bandpass width = 218.9 kHz

Passband width has a negligible error meaning it is within expectations. As gain variations occur in the bandpass width and 3db (Cut-off) is not surpassed, it can be said that, gain variations is also within the wanted range.

As f_0 is 4.4 MHz, $f_0/2$ and 2 f_0 are 2.2 MHz and 8.8 MHz respectively. In these points the expected dB is lower than -30 dB as can be seen in figure 7.

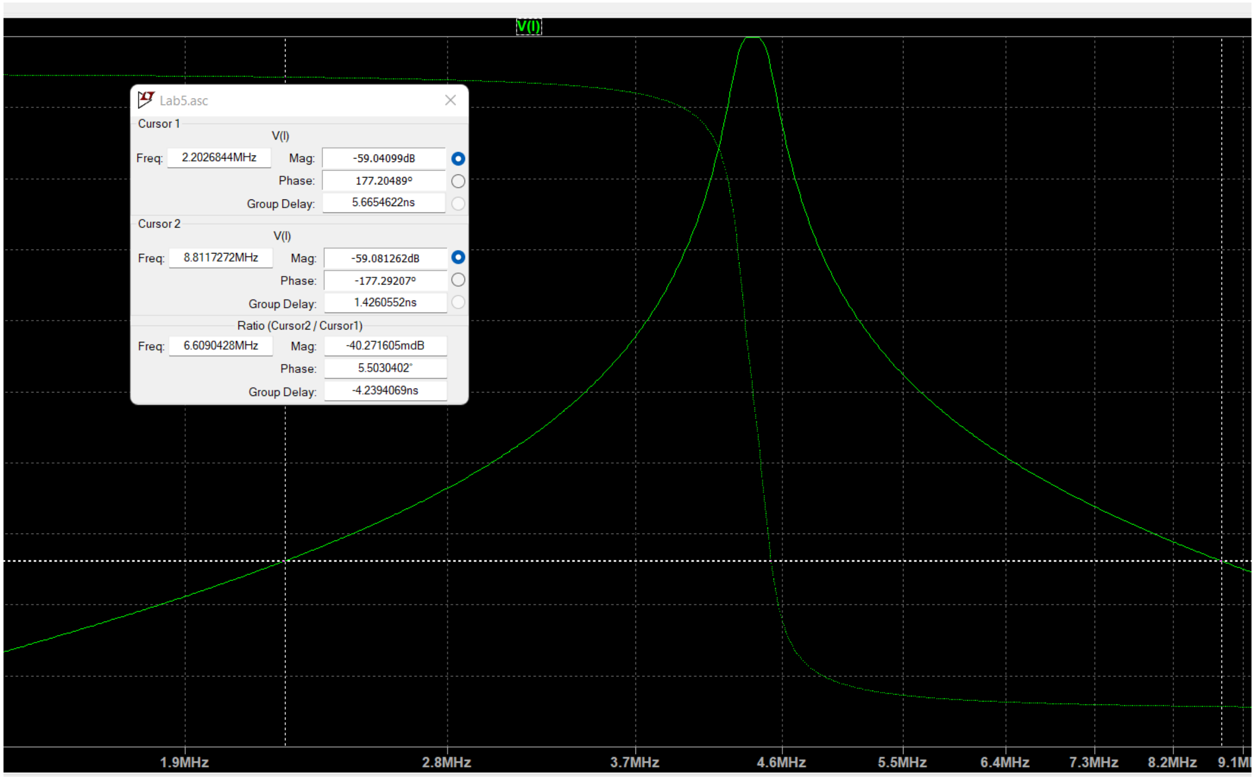

Fig 7: Stopband Attenuation of $f_0/2$ = -59dB and 2 $f_{0=}$ -59db

As can be seen from figure 7, stopband attenuation is much higher than 30dB indicating that BPF is working adequately considering also the center and bandpass width.

The table regarding software values can be seen at the end of hardware part as it was used as a means of result comparison.

Hardware Results:

Here is the designed circuit and its explanation:

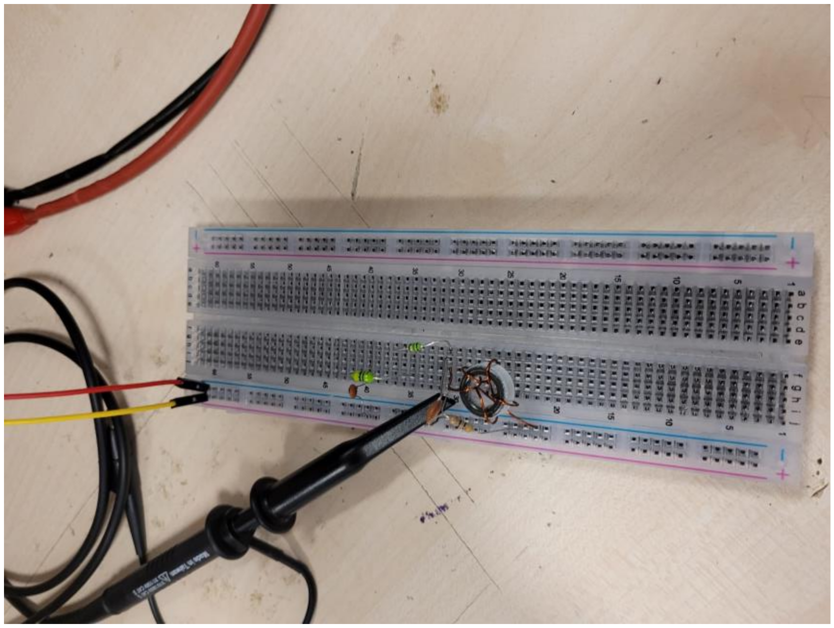

Fig 8: BPF Circuit

Component	Software Value	Hardware Value
L_1	51.1uH	(47+6.8 = 53.8) uH
C_2	20.4 nF	22nF
C_1	25.6 pF	27pF
L_2	64 nH	Toroidal Core T50-7

Table 2: Hardware Component Values

As toroidal core T50-7 has inductance value of **4.3nH/N^2,** with wanted value of 64 nH

$$64/4.3 = 14.88$$

$$n^2 = 14.88$$

$$n = 3.85$$

4 windings were considered acceptable as it should normally give around 68.8 nH. However, after trial and error the winding was increased by one as the leakage due to toroid was higher than expected.

Here are the results of the Hardware circuit with a table of wanted values in the end:

We use AC Amplitude 2(V_{pp} = 4V):

First looking at the center frequency:

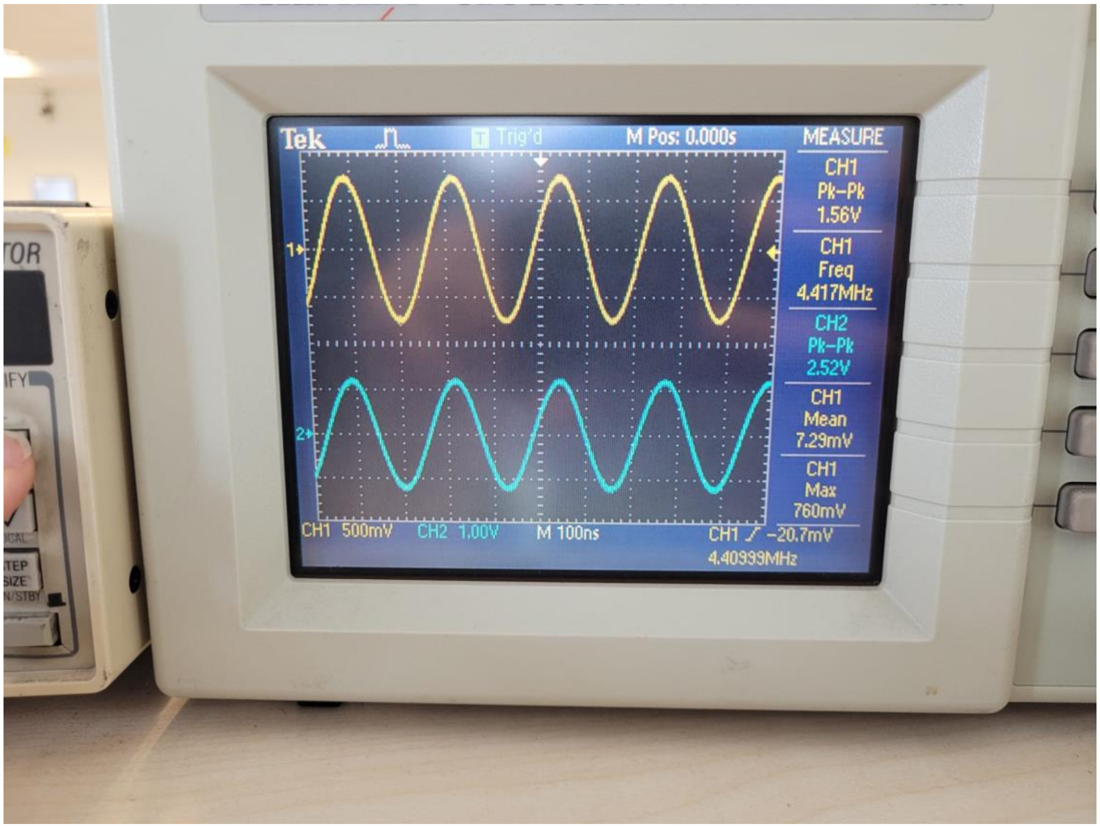

Fig 9: f_0 = 4.417 MHz, CH1= Vout, CH2= Vin

The center frequency is of **by 0.017MHz which is an error of 0.38%, can be negligible**. To be able to calculate gain,

$$20\log\left(\frac{Vout}{Vin}\right) = Gain\ in\ dB$$

From here the gain can be calculated as

$$20\log\left(\frac{1.56}{2.52}\right) = \ -4.1dB$$

In software, the gain at center was 0 dB, however in **Hardware it is -4.1dB** which is lower. Though it is not far off, the reason off loss will be explained in the conclusion. Moving on to cut off frequency, that is to say bandpass width with gain variation of 3dB:

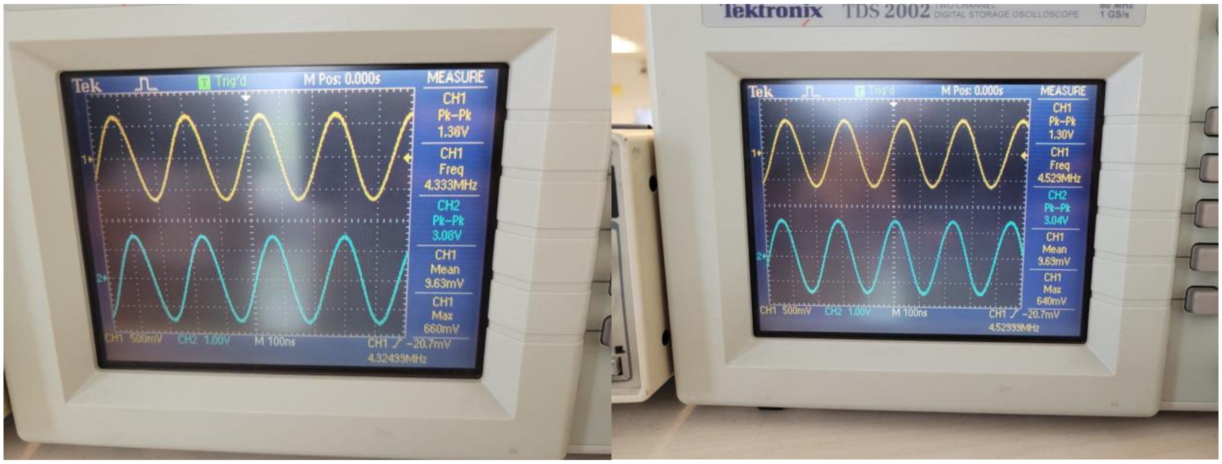

Fig 10 a&b: f_{c0} = 4.333MHz and f_{c1} = 4.529MHz

Using the gain formula for both**, dB values -7.1 dB and -7.21 d**B is acquired, respectively. Therefore, bandpass width is:

$$f_{c1} - f_{c0} = \mathbf{196\mathit{kHz}}$$

Considering the expected value was 220kHz, passband width of hardware circuit has **10.9%** error percentage. It can be seen that 3dB gain variation is functioning adequately on the circuit.

Moving on to stop variations:

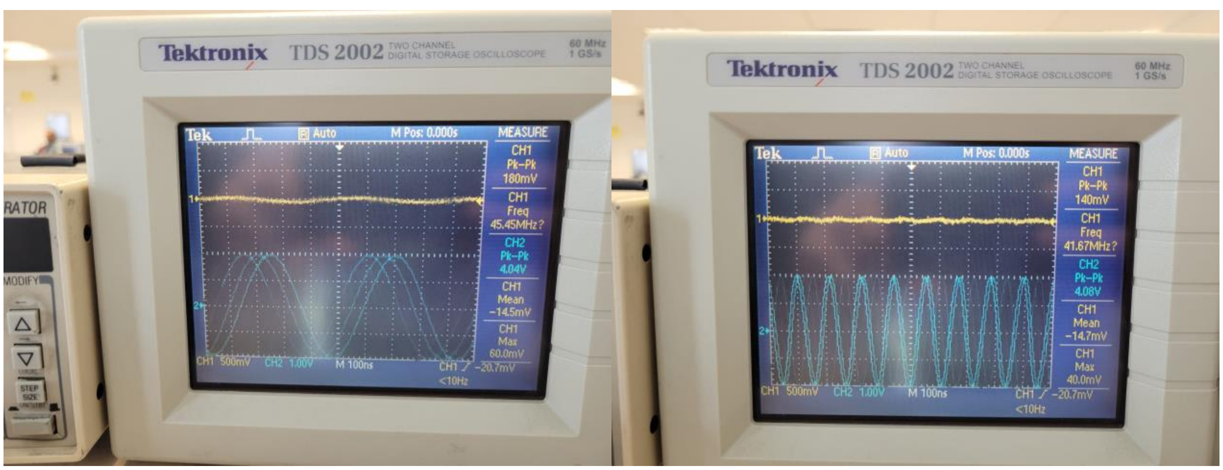

Fig 11 a&b: $f_0/2$ = 2.21MHz and $2f_0$ = 8.83MHz

Requirement for stopband attenuation was at least 30dB for this lab. After using, gain formula on both of these cases we get:

$f_0/2$-> -27.1 dB

$2f_0$-> -29.29 dB

In terms of requirements, found values are quite close to 30db with **error rates of 9.6% and 2.3%.** However, it should be noted th**%.** at these values found in software was -59dB which means BPF's effectiveness in high and low frequencies are weaker compared to the software values.

Here is the error rate table according to lab specifications:

Requirement	Expectation	Results	Error rate
Central Freq	4.4MHz	4.417Mhz	0.38%
Cutoff Freq (F_{c0})	4.29Mhz	4.333Mhz	1%
Cutoff Freq (F_{c1})	4.51Mhz	4.529MHz	0.42%
Bandwidth	220kHz	196kHz	10.9%
Central Gain	0dB	-4.1dB	N/A
Cutoff Gain (F_{c0})	-3db	-7.1dB	N/A
Cutoff Gain (F_{c1})	-3db	-7.21dB	N/A
Stop band Gain ($F_0/2$)	<-30db	-27.21dB	9.6%
Stop band Gain ($2F_0$)	<-30db	-29.29dB	2.3%

Table 3: Error Table

Error is not calculated for central frequency gain and cutoff frequency gain as it is simply impossible to be calculate for central gain. Moreover cutoff gains values depend upon the central gain in such case it is smarter to compute the error from the bandwidth.

More figures regarding the hardware lab can be found in appendix , they are not put in here to not disturb the flow of the report. Apart from cutoff frequencies and stopband points, the hardware table was made as 17kHz shifted right version of software table.

Software Freq.	Gain	Hardware Freq	Gain
2.20MHz ($f_0/2$)	**-59dB**	**2.21MHz ($f_0/2$)**	**-27.1dB**
4.150MHz	-15dB	4.167MHz	-13.3dB
4.192MHz	-11.9dB	4.209MHz	-11.46dB
4.257MHz	-6.3dB	4.274MHz	-9.27dB
4.294MHz (f_{c0})	**-3.05dB**	**4.333MHz (f_{c0})**	**-7.1dB**
4.336MHz	-2.22dB	4.353MHz	-6.4dB
4.40MHz (f_0)	**-72mdB**	**4.417MHz(f_0)**	**-4.16dB**
4.431MHz	-106mddB	4.448MHz	-4.43dB
4.447Mz	-246mdB	4.464MHz	-5.7dB
4.513MHz (f_{c1})	**-3.01dB**	**4.529MHz (f_{c1})**	**-7.21dB**
4.613MHz	-11.2dB	4.630MHz	-10.2dB
4.655MHz	-14.12dB	4.672MHz	-11.35dB
8.80MHz ($2f_0$)	**-59dB**	**8.83MHz ($2f_0$)**	**-29.29dB**

Table 4: Software vs Hardware Results

Comparing the general change of values in software and hardware, software results seem to give a steeper rise and fall in values, as the general value change in 505kHz is more. This can visibly be seen in value changes in top 3 values of the table (excluding 2.2MHz). The decibel change is 8.7dB for software in 107kHz. However, for hardware the decibel change is 4.03dB in 107kHz which is much less compared to software results. Around bandpass width, the results are more consistent even though hardware results 4dB less than software.

Here is the graph of hardware results:

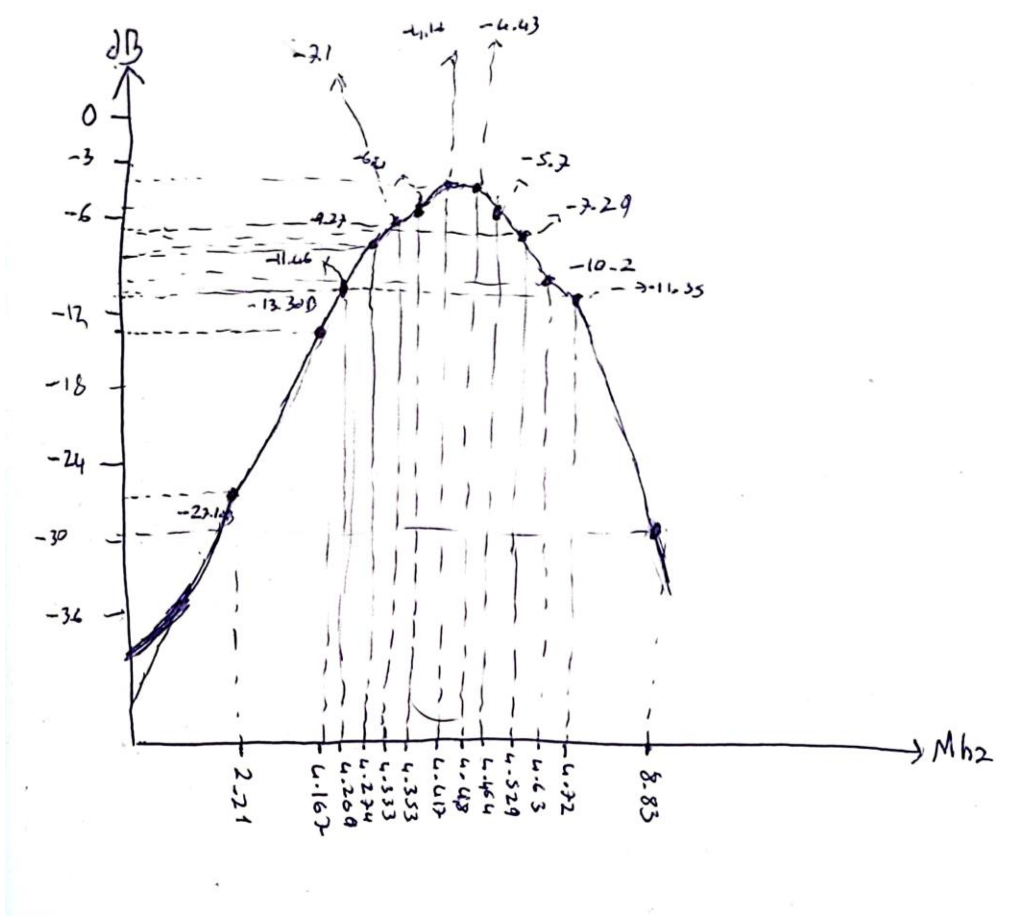

Fig 12: Frequency response plot

Conclusion:

In this lab, we were asked to design a bandpass filter with center frequency in range of 3Mhz to 6 MHz with bandwidth frequency equal to 1/20th of the center frequency. We have chosen 4.4MHz as center frequency indicating a bandwidth of 220kHz. To design this circuit a Butterworth BPF was used. For software the circuit worked nearly perfectly.

Moving onto hardware, though the circuit was still working and almost met up the necessary requirements it carried some errors especially on stop frequencies. Starting off with center frequency, -4.1 dB was lower than the expected value of 0dB. Then bandpass width was 22kHz lower than the expected value of 220kHz. Finally at stop frequencies found decibels did not pass dB and fall short by 2.9 dB and 0.7 dB respectively. All of this can be explained by the components used.

As components value were chosen closest to the that of software lab there are bound to be some errors. Moreover, as we are using physical components there are bound to be some losses on preventing signals as well such as the loss caused by the toroid. Even after adding 1 extra winding there were still some losses due to leakage in the toroid. This can also explain the 17kHz right shift in the center frequency.

This lab was very useful in terms of implementing BPF and understanding that filters' implementation requires step by step build up and several trial-and-error processes.

Appendix – Other values in Hardware lab

Here you can see most of the values that is used for table 3 hardware results.

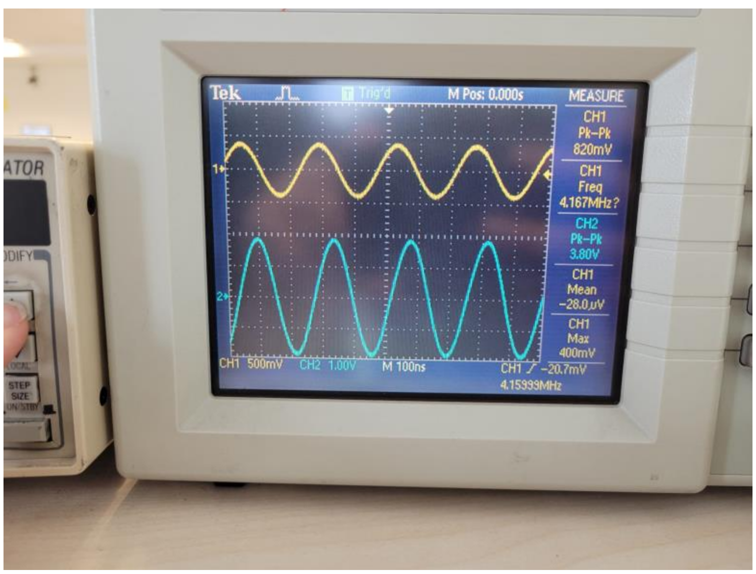

Fig 13: Response in 4.167MHz

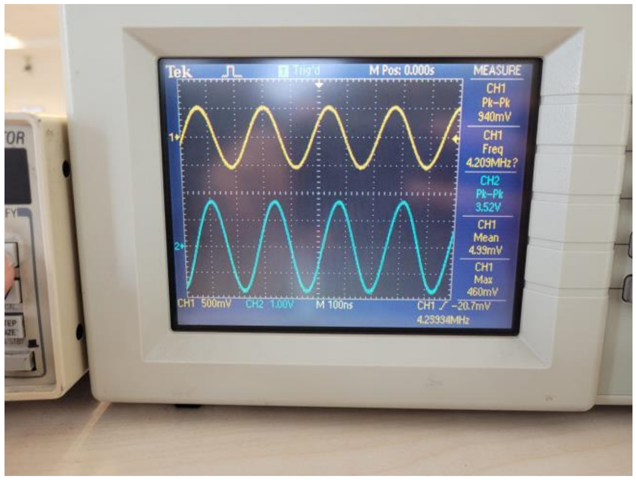

Fig 14: Response in 4.209MHz

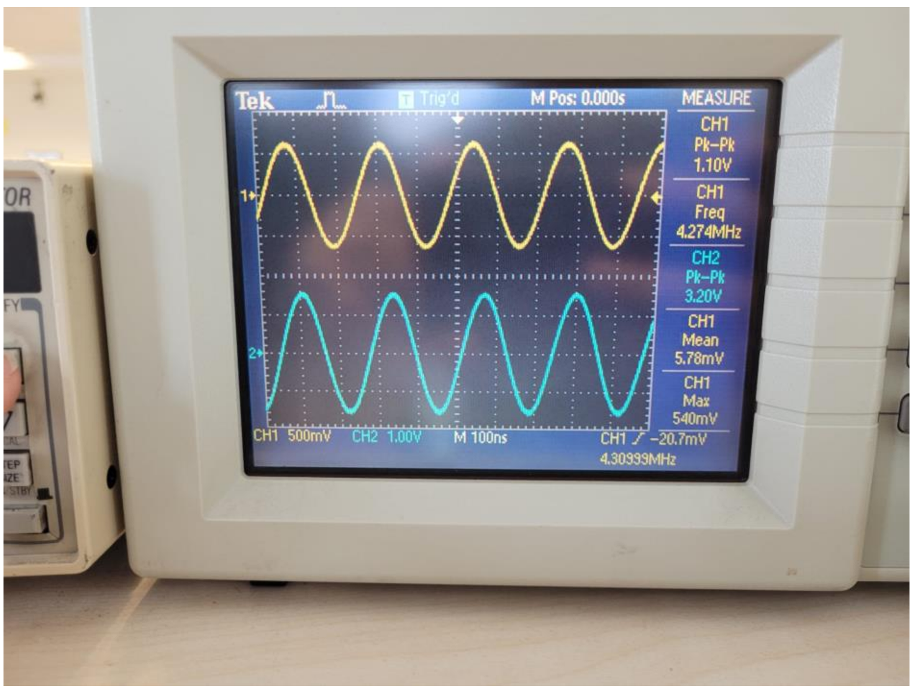

Fig 15: Response in 4.274MHz

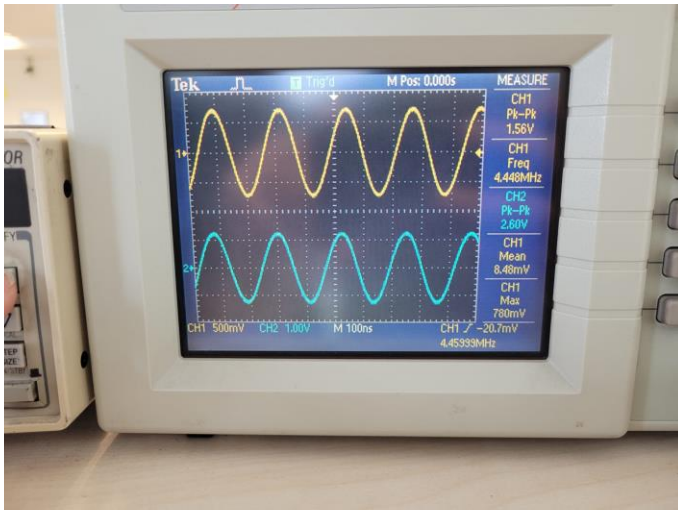

Fig 16: Response in 4.448Mhz

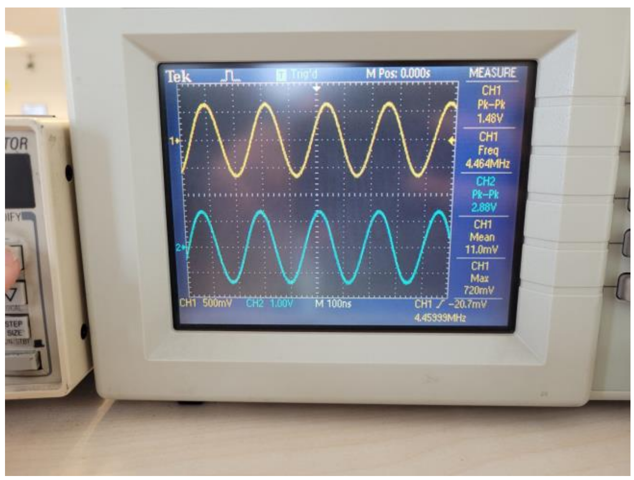

Fig 17: Response in 4.464MHz

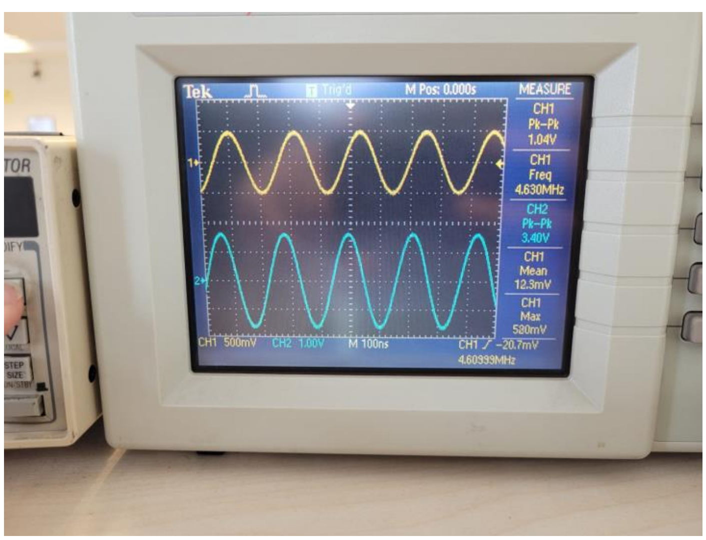

Fig 18: Response in 4.63 MHz

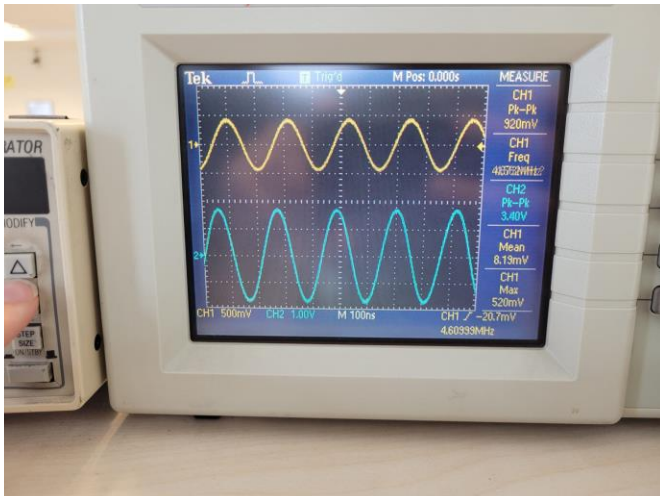

Fig 19: Response in 4.72 MHz