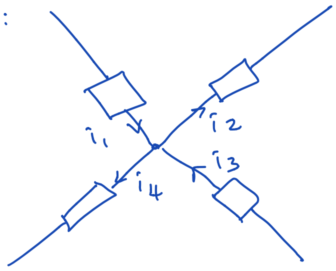
CHAPTER 10 – s-DOMAIN CIRCUIT ANALYSES

KCL :

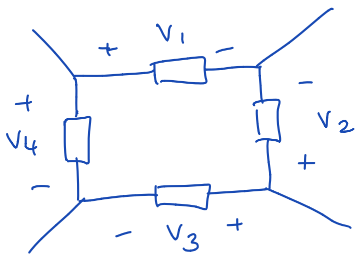


$$-i_1(t) + i_2(t) - i_3(t) + i_4(t) = 0$$

uniqueness & linearity

$$\Rightarrow -I_1(s) + I_2(s) - I_3(s) + I_4(s) = 0$$

KVL :



$$v_1(t) - v_2(t) + v_3(t) - v_4(t) = 0$$

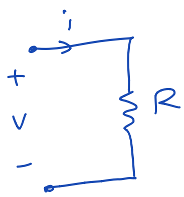
uniqueness & linearity

$$\Rightarrow V_1(s) - V_2(s) + V_3(s) - V_4(s) = 0$$

* KCL and KVL retain their form in s-domain.

Branch Relations :

Resistor :



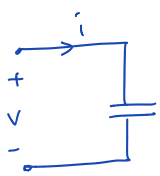
$$v(t) = R\, i(t)$$

$$V(s) = R \cdot I(s)$$

$\Rightarrow$ resistor retains its form

In general : any algebraic relation retaints its form in s-domain, due to linearity of Laplace transform.

Capacitor :



$$i(t) = C \frac{dv(t)}{dt}$$

$$I(s) = C \cdot \left[s V(s) - v(0) \right]$$

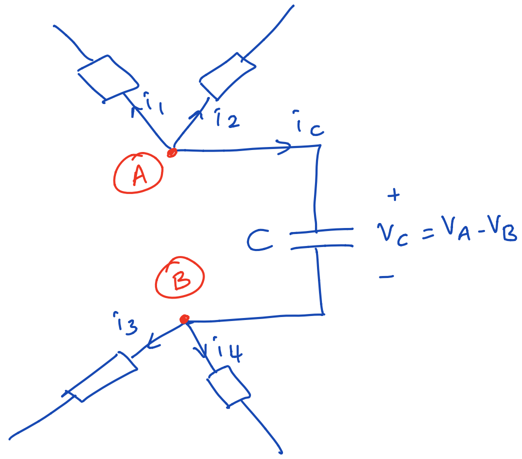
$$\boxed{I(s) = sC\, V(s) - C\, v(0)}$$

useful for node analysis

$$\boxed{V(s) = \frac{1}{sC} I(s) + \frac{1}{s} v(0)}$$

useful for mesh analysis

Example :

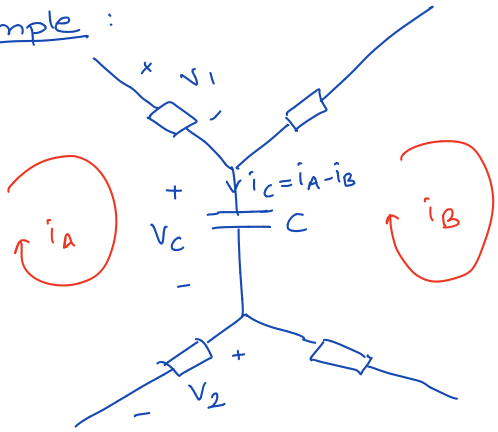


KCL at A :

$$I_1 + I_2 + I_c = 0$$

$$I_1 + I_2 + \left[sC(V_A - V_B) - C V_c(0) \right] = 0$$

Example :

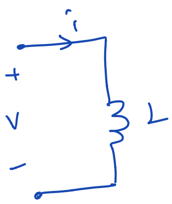


KVL at mesh A :

$$\ldots + V_1 + V_c + V_2 + \ldots = 0$$

$$\ldots + V_1 + \left[\frac{1}{sC}(I_A - I_B) + \frac{1}{s} v_c(0) \right] + V_2 + \ldots = 0$$

Inductor :



$$v(t) = L \frac{di(t)}{dt}$$

$$V(s) = L\left(sI(s) - i(0)\right)$$

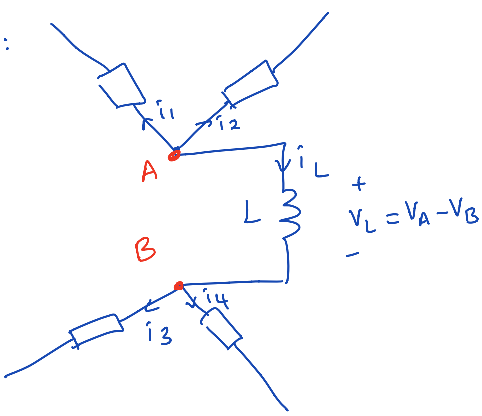
$$\boxed{V(s) = sL\, I(s) - L \cdot i(0)}$$

useful for mesh analysis

$$\boxed{I(s) = \frac{1}{sL} V(s) + \frac{1}{s} i(0)}$$

useful for node analysis

Example :

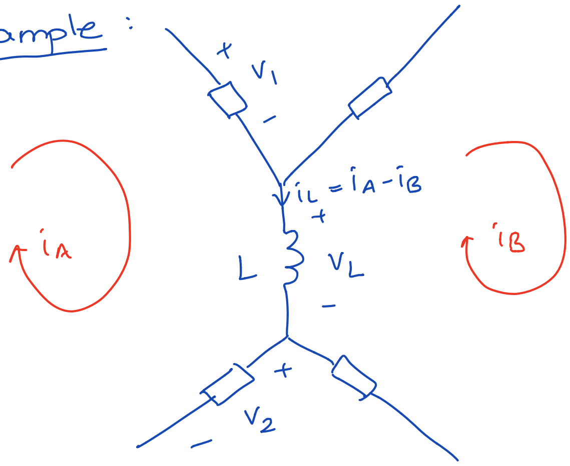


KCL at A :

$$I_1 + I_2 + I_L = 0$$

$$I_1 + I_2 + \left[\frac{1}{sL}(V_A - V_B) + \frac{1}{s} i(0) \right] = 0$$

Example :

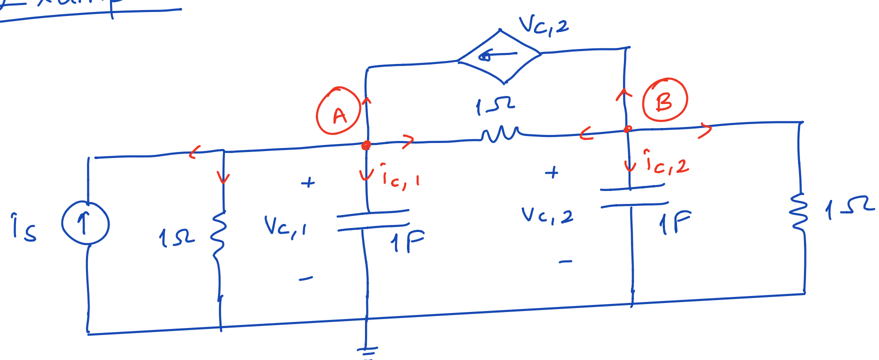


KVL at mesh A :

$$\cdots + V_1 + V_L + V_2 + \cdots = 0$$

$$\cdots + V_1 + \left[sL\,(I_A - I_B) - L\,i(0) \right] + V_2 + \cdots = 0$$

Example : Node analysis , General 2^{nd}-order circuit



$V_{c,1}(0) = 2\,V$

$V_{c,2}(0) = 1\,V$

$i_s(t) = u(t)$ A

$V_{c,2}(t) = ?$ for $t \geq 0$

$V_{c,2}(t) = V_B(t)$

$I_s(s) = \frac{1}{s}$, For capacitors: $I(s) = sC\,V(s) - C\,V(0)$

KCL @ A : $$-I_s + \frac{V_A}{1} + \underbrace{\left[sC_1 V_A - C_1 V_{c,1}(0) \right]}_{I_{c,1}} - \underbrace{V_{c,2}}_{V_B} + \frac{V_A - V_B}{1} = 0$$

$$-\frac{1}{s} + V_A + sV_A - 2 - V_B + V_A - V_B = 0$$

$$(s+2)V_A - 2V_B = 2 + \frac{1}{s} = \frac{2s+1}{s}$$

KCL @ B : $$\frac{V_B - V_A}{1} + \underbrace{V_{c,2}}_{V_B} + \underbrace{\left[sC_2 V_B - C_2 V_{c,2}(0) \right]}_{I_{c,2}} + \frac{V_B}{1} = 0$$

$$V_B - V_A + V_B + sV_B - 1 + V_B = 0$$

$$V_A = (s+3)V_B - 1$$

* Insert into first equation:

$$(s+2)\left[(s+3)V_B - 1\right] - 2V_B = \frac{2s+1}{s}$$

$$(s^2 + 5s + 6 - 2)V_B = (s+2) + \frac{2s+1}{s}$$

$$(s^2 + 5s + 4)V_B = \frac{s^2 + 4s + 1}{s}$$

$$V_B(s) = \frac{s^2+4s+1}{s(s^2+5s+4)} = \frac{s^2+4s+1}{s(s+1)(s+4)}$$

$$= \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+4}$$

$$k_1 = s\,V_B(s)\Big|_{s=0} = \frac{s^2+4s+1}{(s+1)(s+4)}\Big|_{s=0} = \frac{1}{4}$$

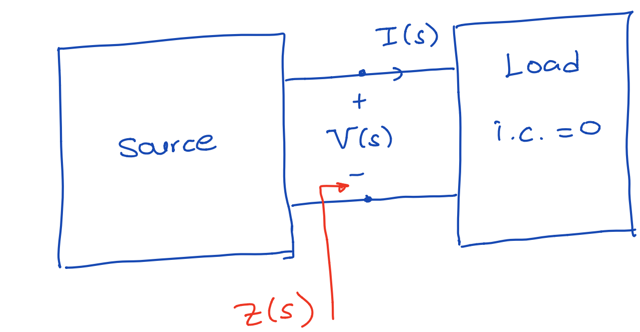
$$k_2 = (s+1)\,V_B(s)\Big|_{s=-1} = \frac{s^2+4s+1}{s(s+4)}\Big|_{s=-1} = \frac{1-4+1}{-1\,.\,3} = \frac{2}{3}$$

$$k_3 = (s+4)\,V_B(s)\Big|_{s=-4} = \frac{s^2+4s+1}{s(s+1)}\Big|_{s=-4} = \frac{16-16+1}{-4\,.\,(-3)} = \frac{1}{12}$$

Then,

$$V_{c,2}(t) = V_B(t) = \Big[\underbrace{\frac{1}{4}}_{\text{forced response}} + \underbrace{\frac{2}{3}e^{-t} + \frac{1}{12}e^{-4t}}_{\text{natural response}}\Big]u(t) \quad V$$

Impedance : For zero initial conditions

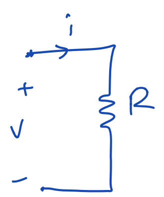


$$Z(s) = \frac{V(s)}{I(s)} \quad : \text{impedance in s-domain}$$

$$Y(s) = \frac{I(s)}{V(s)} \quad : \text{admittance in s-domain}$$

$$Z(s) = \frac{1}{Y(s)}$$

Resistor :

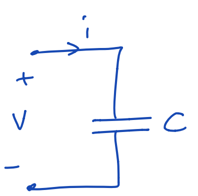


$$V(s) = R\,.\,I(s)$$

$$Z_R(s) = R$$

$$Y_R(s) = \frac{1}{R}$$

Capacitor :

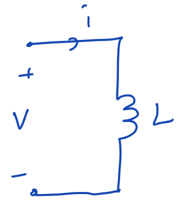


$$I(s) = sC\,V(s) - \cancel{C\,v(0)}$$ assume zero i.c.

$$Z_c(s) = \frac{1}{sC}$$

$$Y_c(s) = sC$$

Inductor:



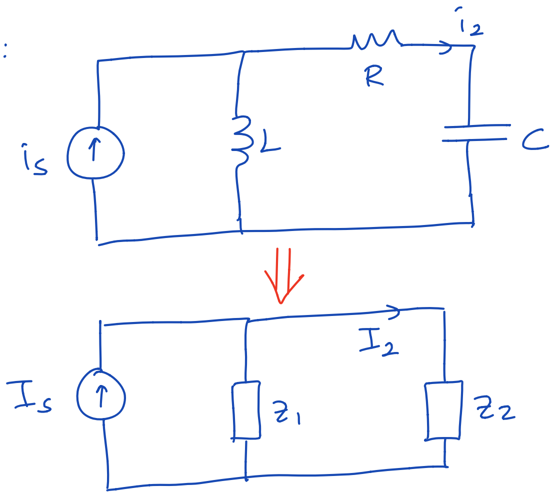
$$V(s) = sL\,I(s) \cancel{- L\,i(0)}$$ assume zero i.c.

$$Z_L(s) = sL$$

$$Y_L(s) = \frac{1}{sL}$$

* Series/parallel connection: works the same as in resistive circuits.
* voltage/current division: works the same as in resistive circuits.

Example:



Find $I_2(s)$ assuming zero initial conditions.

$$Z_2 = R + \frac{1}{sC}$$

$$Z_1 = sL$$

$$I_2(s) = \frac{Z_1}{Z_1+Z_2} \cdot I_s(s) \quad \text{(current divider)}$$

$$= \frac{sL}{\frac{RsC+1}{sC} + sL} \cdot I_s(s) = \frac{s^2LC}{s^2LC + sRC + 1} \cdot I_s(s)$$

Superposition in s-domain

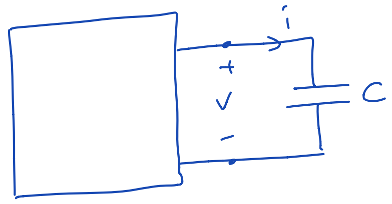
any solution = (zero-state solution) + (zero-input solution)

zero-state solution = initial conditions are zero, only the inputs are effective

zero-input solution = inputs are zero, only initial conditions are effective

How to deal with the initial conditions

Capacitor :

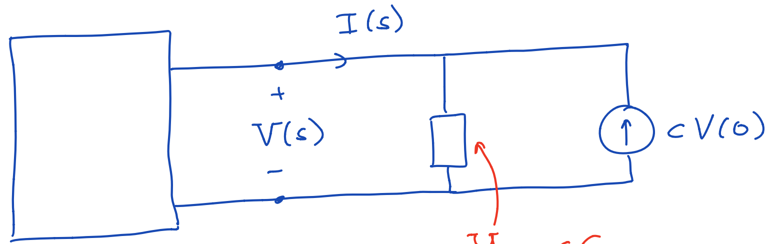


$$i(t) = C\frac{dv(t)}{dt}$$

$$I(s) = C\left[sV(s) - v(0)\right]$$

$$I(s) = \underbrace{sC}_{\text{admittance of capacitor}}\,V(s) - Cv(0)$$

Norton model



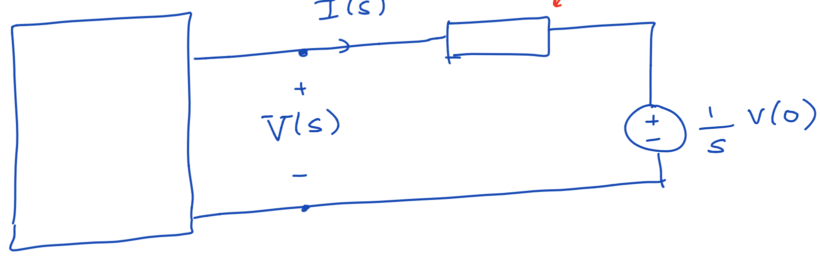
$Y_c = sC$
$Z_c = \frac{1}{sC}$ → capacitor with zero i.c.

* OR, equivalenty :

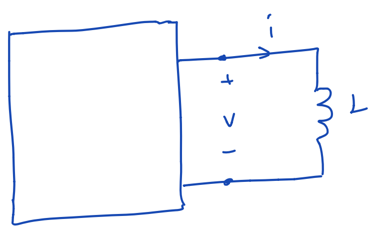
$$V(s) = \underbrace{\frac{1}{sC}}_{\text{impedance of capacitor}}\,I(s) + \frac{1}{s}v(0)$$

$Z_c = \frac{1}{sC}$

Thevenin model



Inductor :



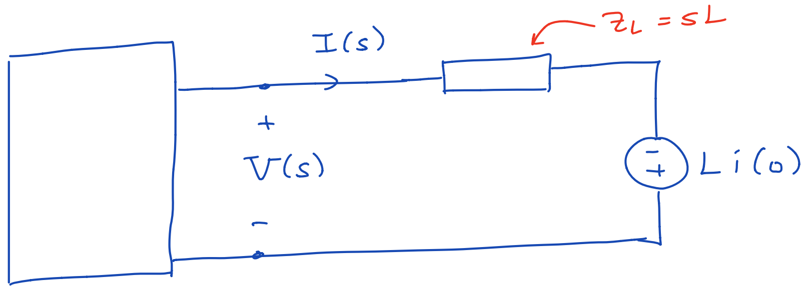
$$v(t) = L\frac{di(t)}{dt}$$

$$V(s) = L\left[sI(s) - i(0)\right]$$

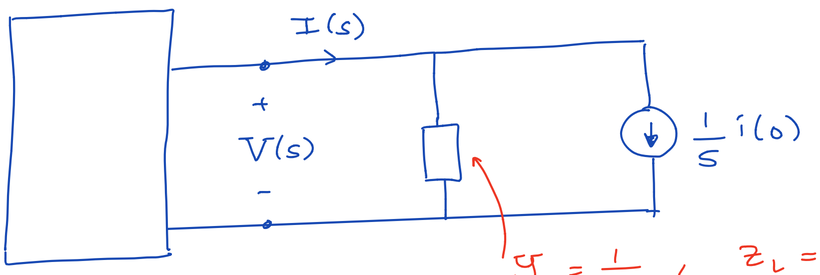
$$V(s) = \underbrace{sL}_{\text{impedance of inductor}}\,I(s) - Li(0)$$

$Z_L = sL$

Thevenin model



* OR, equivalently: $I(s) = \underbrace{\frac{1}{sL}}_{\text{admittance of inductor}} . V(s) + \frac{1}{s} i(0)$



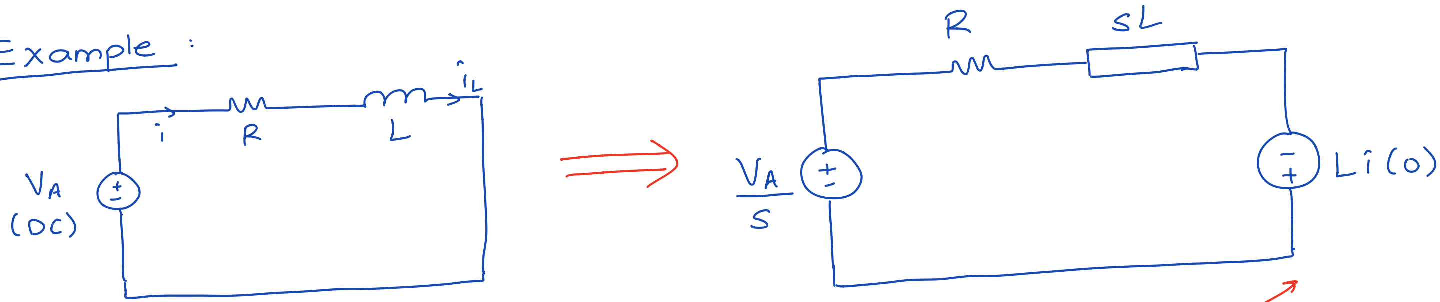
Norton model

$y_L = \frac{1}{sL}$, $Z_L = sL$

* These transformations are known as <u>initial condition transformations</u>.

* Application of superposition as follows:
  - Apply an appropriate initial condition transformation.
  - Treat initial conditions as equivalent <u>independent sources</u> in s-domain.
  - Apply superposition to this circuit in s-domain.
  - The contributions of equivalent sources due to initial conditions give the <u>zeo-input response</u>.
  - The contributions of actual independent sources give the <u>zero-state response</u>.

<u>Example</u>:

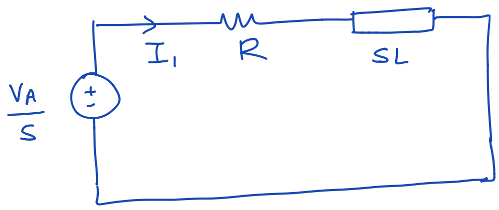


$i_L(0)$ is given.
Find $i(t)$, $t \geq 0$.

as if we have 2 independent sources

Apply superposition:

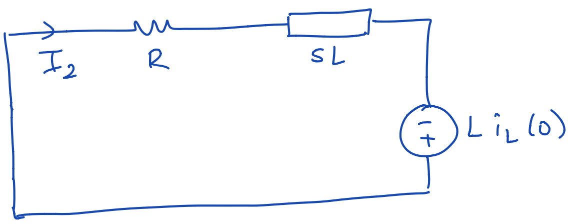
* Contribution of V_A:



$$I_1 = \frac{V_A/s}{R+sL} = \frac{V_A}{s(sL+R)}$$

$$= \frac{V_A/L}{s\left(s+\frac{R}{L}\right)}$$

* Contribution of $i_L(0)$:



$$I_2 = \frac{L i_L(0)}{R+sL}$$

$$= \frac{i_L(0)}{s+\frac{R}{L}}$$

* Now,

$$I(s) = I_1(s) + I_2(s) = \underbrace{\frac{V_A/L}{s(s+R/L)}}_{\text{zero-state}} + \underbrace{\frac{i_L(0)}{s+R/L}}_{\text{zero-input}}$$

$$I(s) = \frac{k_1}{s} + \frac{k_2}{s+R/L} + \frac{i_L(0)}{s+R/L}$$

$$k_1 = s\, I_1(s)\Big|_{s=0} = \frac{V_A/L}{s+R/L}\Big|_{s=0} = \frac{V_A}{R}$$

$$k_2 = (s+R/L)\cdot I_1(s)\Big|_{s=-R/L} = \frac{V_A/L}{s}\Big|_{s=-R/L} = -\frac{V_A}{R}$$

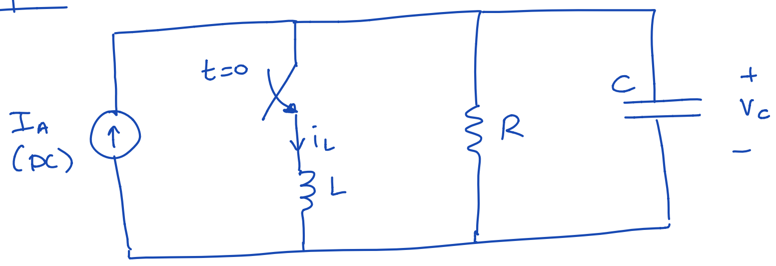
* Finally,

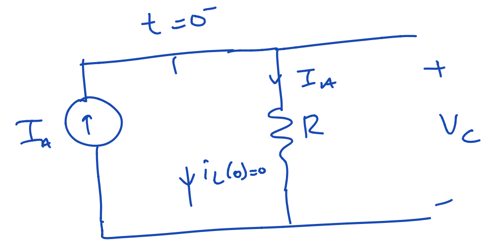
$$i(t) = \left[\underbrace{\overset{\text{forced}}{\frac{V_A}{R}} - \frac{V_A}{R}\cdot e^{-\frac{R}{L}t}}_{\text{zero-state response}} + \underbrace{i_L(0)\cdot e^{-\frac{R}{L}(t)}}_{\text{zero input response}}\right] u(t)$$

natural response

Example:

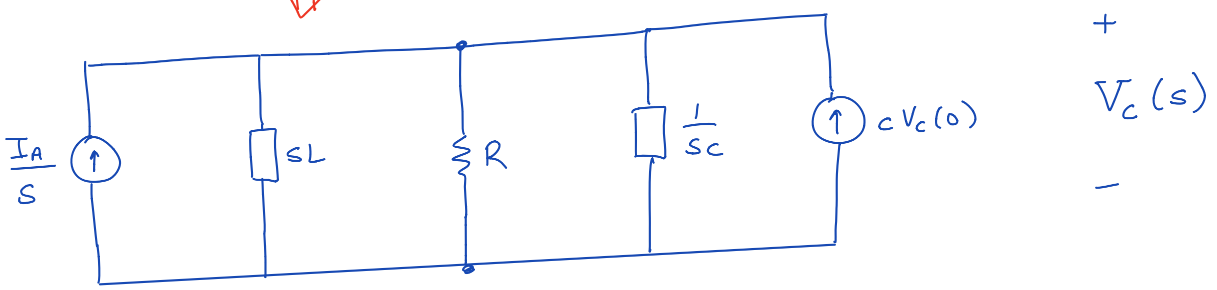
$V_c(t) = ?$ for $t \geq 0$.



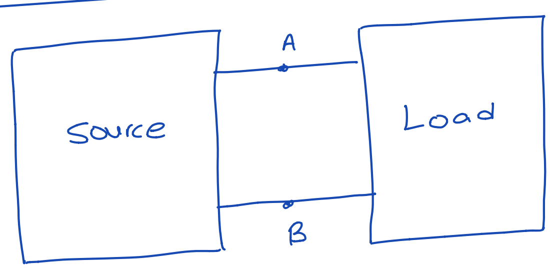


$i_L(0) = 0$, $V_c(0) = I_A \cdot R$

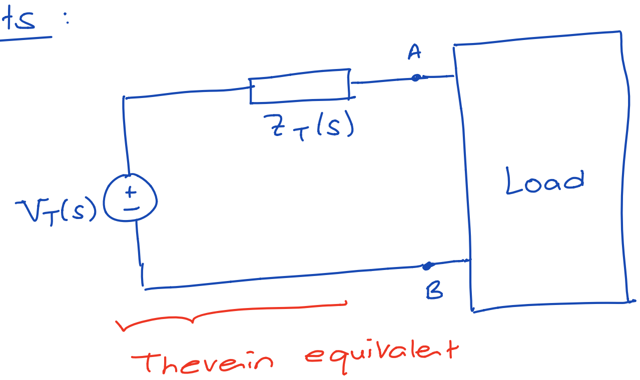
to s-domain

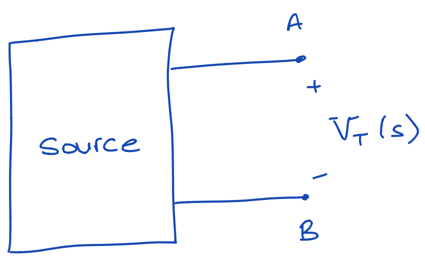


Thevenin-Norton Equivalent Circuits:

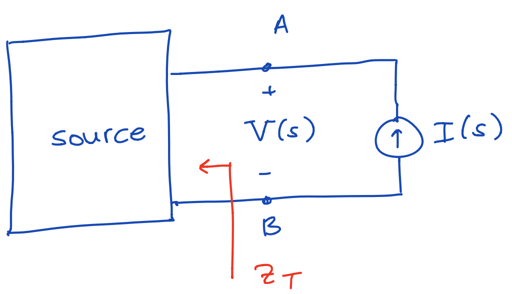


$\equiv$





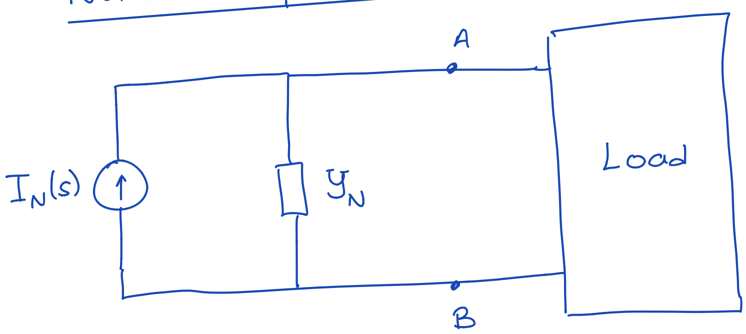
$V_T(s)$: open circuit voltage in s-domain.
Keep all independent sources inside the source part, including initial condition sources.



Kill all independent sources inside the source part, including initial condition sources.

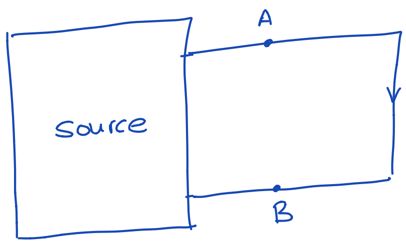
$$Z_T(s) = \frac{V(s)}{I(s)}$$

Norton equivalent :



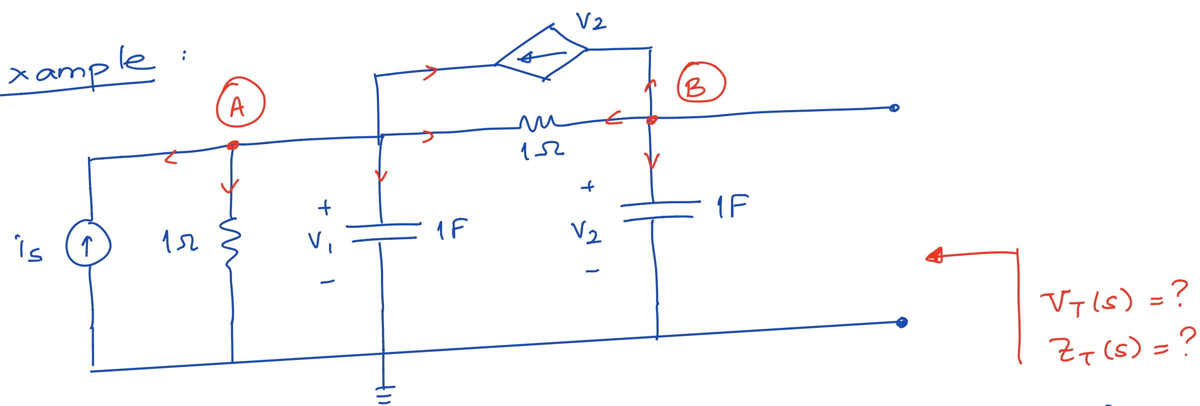
$$Z_T = \frac{1}{Y_N} = Z_N$$

$$Z_T = \frac{V_T(s)}{I_N(s)}$$



$I_N(s)$: short circuit current in s-domain. Keep all independent sources inside the source part, including the initial condition sources.

Example :



Assume zero-state (i.e., zero initial conditions)

$V_2 = V_B$

* $V_T(s)$:

$V_T = V_B$

KCL @ A : $-I_s + \frac{V_A}{1} + \frac{V_A}{\frac{1}{s.1}} + \frac{V_A - V_B}{1} - V_B = 0$

$(s+2)V_A - 2V_B = I_s$ .... (Eqn. 1)

KCL @ B : $V_B + \frac{V_B - V_A}{1} + \frac{V_B}{\frac{1}{s.1}} = 0$

$(s+2)V_B = V_A$

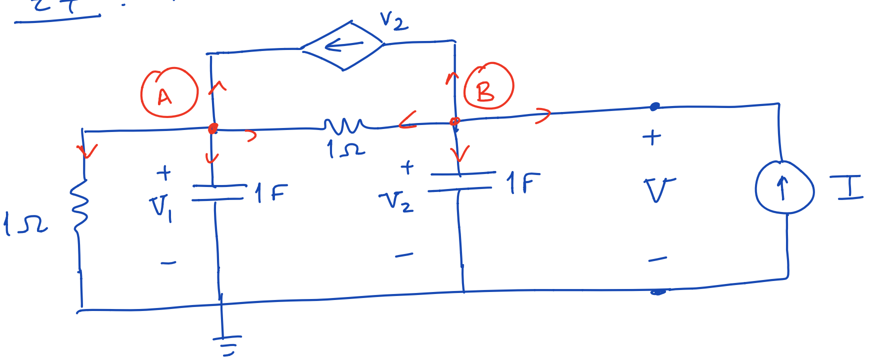
Insert into Eqn. 1:

$$(s+2)(s+2)V_B - 2V_B = I_s$$

$$(s^2+4s+2)V_B = I_s$$

$$\boxed{V_T(s) = V_B(s) = \frac{I_s(s)}{s^2+4s+2}}$$

* $\underline{Z_T}$: kill all independent sources



$$V_2 = V_B = V$$

$$Z_T(s) = \frac{V(s)}{I(s)}$$

KCL @ A: $\frac{V_A}{1} + \frac{V_A}{\frac{1}{s\cdot 1}} + \frac{V_A - V_B}{1} - V_B = 0$

$$(s+2)V_A = 2V_B = 2V \Rightarrow V_A = \frac{2V}{s+2}$$

KCL @ B: $\frac{V_B - V_A}{1} + V_B + \frac{V_B}{\frac{1}{s\cdot 1}} - I = 0$

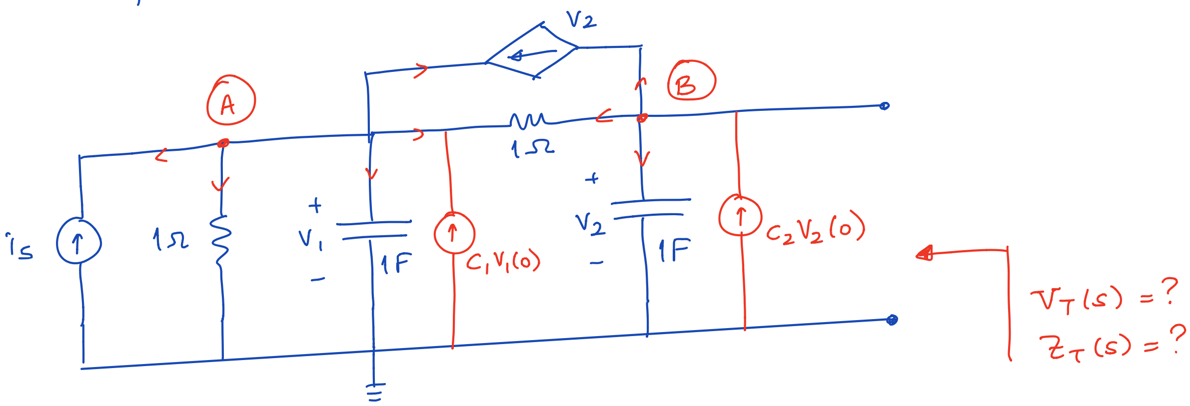
$$(s+2)V_B - V_A = I$$

$$(s+2)V - \frac{2V}{s+2} = I$$

$$\frac{(s^2+4s+2)}{s+2} V = I$$

$$\boxed{Z_T(s) = \frac{V}{I} = \frac{s+2}{s^2+4s+2}}$$

* If we had initial conditions :



* Now, $V_T(s)$ will depend on $V_1(0)$, $V_2(0)$, and $I_s(s)$.
* $Z_T(s)$ remains the same as in zero initial condition case.