EEE202 HW4 solution

Q1) 9-19(a)

$$f(t) = [500 + 100e^{-500t}\, t\cos(1000t)]\,u(t)$$

$$\cos(\beta t) \rightarrow \frac{s}{s^2+\beta^2}$$

$\alpha = 500$
$\beta = 1000$

$$e^{-\alpha t}\cos(\beta t) \longrightarrow \frac{s+\alpha}{(s+\alpha)^2+\beta^2}$$

$$t e^{-\alpha t}\cos(\beta t) \rightarrow ?$$

If we multiply a function $f(t)$ by t
then $\mathcal{L}\{t f(t)\} = -\frac{d}{ds}\mathcal{L}\{F(s)\}$

proof: $F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$

$$-\frac{d}{ds}F(s) = \int_{0^-}^{\infty} -f(t)\left(\frac{d}{ds}e^{-st}\right)dt = \int_{0^-}^{\infty} t f(t)e^{-st}\,dt$$

Q.E.D.

$$\therefore\ t e^{-\alpha t}\cos(\beta t) \rightarrow -\frac{d}{ds}\left[\frac{s+\alpha}{(s+\alpha)^2+\beta^2}\right]$$

$$= -\left[\frac{(s+\alpha)^2+\beta^2-(s+\alpha)[2(s+\alpha)]}{[(s+\alpha)^2+\beta^2]^2}\right]$$

$$= -\frac{(s+\alpha)^2+\beta^2-2(s+\alpha)^2}{[(s+\alpha)^2+\beta^2]^2} = \frac{(s+\alpha)^2-\beta^2}{((s+\alpha)^2+\beta^2)^2}$$

$$\therefore F(s) = \frac{500}{s} + 100 \times \frac{(s+500)^2 - 1000^2}{\left[(s+500)^2 + 1000^2\right]^2} = \frac{500}{s} + 100 \frac{(s+500)^2 - 10^6}{\left[(s+500)^2 + 10^6\right]^2}$$

$$= \frac{500\left[(s+500)^2+10^6\right]^2 + \left[(s+500)^2 - 10^6\right] \times 100s}{s\left[(s+500)^2+10^6\right]^2}$$ define $s+500=y$

$$= \frac{500\left[y^2+10^6\right]^2 + 100(y-500)\left[y^2 - 10^6\right]}{s\left[(s+500)^2+10^6\right]^2}$$

$$= \frac{500\left[y^4 + 2\times10^6 y^2 + 10^{12}\right] + 100y^3 - 10^8 y - 5\times10^4 y^2 + 5\times10^{10}}{s\left[(s+500)^2+10^6\right]^2}$$

$$= \frac{500y^4 + 100y^3 + y^2(10^9 - 5\times10^4) - 10^8 y + 5\times10^{14} + 5\times10^{10}}{s\left[(s+500)^2+10^6\right]^2}$$ found from $s = y - 500$

zeroes are $-509 \pm 1005.6j$, $y = -491.1 \pm j994.4$

poles are $p_1 = 0$, $p_{2,3} = -500 \pm 1000j$, $p_{4,5} = -500 \pm 1000j$

Q2) 9-32 a)

$$F_1(s) = \frac{300(s+50)}{s^2(s^2+40s+300)} = \frac{300(s+50)}{s^2(s+10)(s+30)} = \frac{K_1}{s} + \frac{K_2}{s^2} + \frac{K_3}{s+10} + \frac{K_4}{s+30}$$

$$K_2 = s^2 \times F_1(s)\Big|_{s=0} = \frac{300(s+50)}{(s+10)(s+30)}\Big|_{s=0} = \frac{300\times50}{10\times30} = 50$$

$$K_1 = \frac{d}{ds} s^2 F_1(s)\Big|_{s=0} = \frac{300(s+10)(s+30) - 300(s+50)(2s+40)}{(s+10)^2(s+30)^2}\Big|_{s=0}$$

$$= \frac{300\times10\times30 - 300\times50\times40}{10^2\times30^2} = \frac{-300\times1700}{300\times300} = \frac{-1700}{300} = \frac{-17}{3}$$

$$K_3 = \frac{300(s+50)}{s^2(s+30)}\Big|_{s=-10} = \frac{300\times40}{10^2\times20} = 6$$

$$K_4 = \frac{300(s+50)}{s^2(s+10)}\Big|_{s=-30} = \frac{300\times20}{30^2\times(-20)} = -\frac{300}{900} = -\frac{1}{3}$$

$$f_1(t) = -\frac{17}{3} + 50t + 6e^{-10t} - \frac{1}{3}e^{-30t} \quad t \geq 0$$

b) $F_2(s) = \dfrac{1000s^2}{(s+5)(s^2+4s+8)} = \dfrac{1000s^2}{(s+5)(s+2-2j)(s+2+2j)}$

poles are: -5, $-2+2j$, $-2-2j$

$$= \frac{K_1}{s+5} + \frac{K_2}{s+2-2j} + \frac{K_2^*}{s+2+2j}$$

$$K_1 = \frac{1000s^2}{s^2+4s+8}\Bigg|_{s=-5} = \frac{1000\times 25}{25-20+8} = \frac{1000\times 25}{13} = \frac{25000}{13}$$

$$K_2 = \frac{1000s^2}{(s+5)(s+2+2j)}\Bigg|_{s=-2+2j} = \frac{1000\times(-2+2j)^2}{(-2+2j+5)(-2+2j+2+2j)}$$

$$= \frac{1000\,(4-4-8j)}{(3+2j)\times 4j} = \frac{-2000}{3+2j} = \frac{-2000(3-2j)}{13}$$

$$K_2^* = \frac{-2000(3+2j)}{13}$$

$\therefore$ $f_2(t) = \dfrac{25000}{13}e^{-5t} - \dfrac{2000(3-2j)}{13}e^{-2t}e^{j2t} - \dfrac{2000}{13}(3+2j)e^{-2t}e^{-j2t}$

$$= \frac{25000}{13}e^{-5t} - \frac{2000}{13}e^{-2t}\times 3\times\left(e^{j2t}+e^{-j2t}\right)$$
$$+\frac{2000}{13}\times(+2j)\,e^{-2t}\left(e^{j2t}-e^{-j2t}\right)\times\frac{2j}{2j}$$

$$= \frac{25000}{13}e^{-5t} - \frac{12000}{13}e^{-2t}\cos 2t - \frac{8000}{13}e^{-2t}\sin 2t$$

$$= \frac{25000}{13}e^{-5t} - \frac{12000}{13}e^{-2t}\left(\cos 2t + \frac{2}{3}\sin 2t\right) \quad t \geq 0$$

$$= \frac{25000}{13}e^{-5t} - \frac{12000}{13}\times\frac{\sqrt{13}}{3}\,e^{-2t}\cos(2t-0.588)$$

$\leftarrow 33.69^\circ$

$$= 1923e^{-5t} - 1109.4\cos(2t-0.588)$$

$$= 1923e^{-5t} + 1109.4\cos(2t+2.554)$$

$\leftarrow +146.31^\circ$

Q3) 9+51:

$$\frac{d^2v}{dt^2} + 20\frac{dv}{dt} + 1000v = 0 \qquad v(0^-) = 20 \quad \frac{dv}{dt}(0^-) = 0$$

$$\left(s^2V(s) - s\times 20 - 0\right) + 20\left(sV(s) - 20\right) + 1000V(s) = 0$$

$$\left(s^2 + 20s + 1000\right)V(s) = 20s + 400$$

40

$$V(s) = \frac{20s+400}{s^2+20s+1000} = \frac{20s+400}{(s+10-30j)(s+10+30j)}$$

$$= \frac{K}{s+10-30j} + \frac{K^*}{s+10+30j}$$

$$K = \left.\frac{20s+400}{s+10+30j}\right|_{s=-10+30j} = \frac{20(-10+30j)+400}{-10+30j+10+30j}$$

$$= \frac{200+600j}{60j} = 10 - \frac{10}{3}j \qquad K^* = 10 + \frac{10}{3}j$$

$$|K| = \sqrt{100 + \frac{100}{9}} = \sqrt{\frac{1000}{9}} = \frac{\sqrt{1000}}{3}$$

$$\angle K = \mathrm{atan2}\left(-\frac{10}{3}, 10\right) = -0.3218 \text{ radians}$$
$$= -18.435^\circ$$

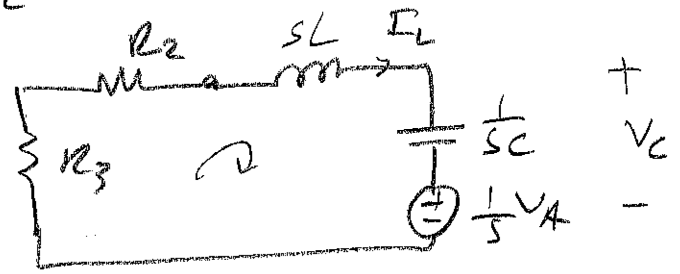
$$v(t) = e^{-10t} \times \frac{2\sqrt{1000}}{3}\cos(30t - 0.3218) \quad t \geq 0.$$

$$= 10^{-10t}\left[20\cos 30t + \frac{20}{3}\sin 30t\right] \quad t \geq 0$$

Q4) 10-32 (a&c)

a) $i_L(0) = 0$ $v_c(0) = V_A$

for $t \geq 0$



$$I_L(s) = -\frac{\frac{1}{s}V_A}{R_2+R_3+sL+\frac{1}{sC}} = -\frac{\frac{1}{s}V_A sC}{sC(R_2+R_3)+s^2LC+1}$$

$$= \frac{-V_A C}{LC\left(s^2+\frac{sC(R_2+R_3)}{LC}+\frac{1}{LC}\right)} = \frac{-V_A}{L}\,\frac{1}{s^2+\frac{R_2+R_3}{L}s+\frac{1}{LC}}$$

c) $R_1 = R_2 = 500$ $R_3 = 1000$ $L = 500mH = 0.5H$ $C = 0.15\mu F$ $V_A = 5$

$$I_L(s) = \frac{-5}{0.5} \times \frac{1}{s^2+\frac{1500}{0.5}s+\frac{10^9}{75}} = \frac{-10}{s^2+3000s+\frac{10^9}{75}}$$

$\frac{1}{LC} = \frac{1}{0.5 \times 0.15 \times 10^{-6}} = \frac{10^9}{75}$

poles are $-1500 \pm j3329.16$

$$I_L(s) = \frac{K}{s+1500-j3329.16} + \frac{K^*}{s+1500+j3329.16}$$

$$K = \left.\frac{-10}{s+1500+j3329.16}\right|_{s=-1500+j3329.16} = 0.0015j$$

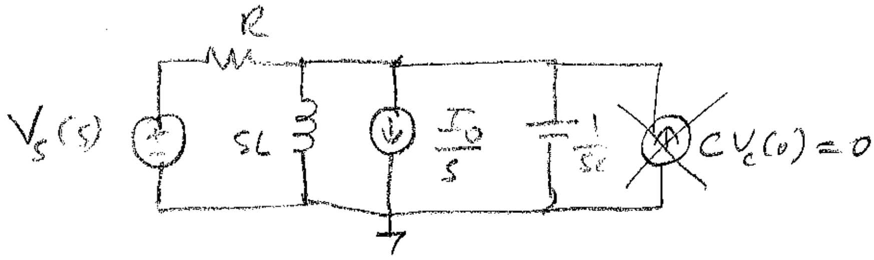
$\therefore$ $i_L(t) = 0.0015j\, e^{(-1500+3329.16j)t} - 0.0015j\, e^{(-1500-3329.16j)t}$

$$= -0.003\, e^{-1500t}\left(\frac{e^{3329.16jt} - e^{-3329.16jt}}{2j}\right)$$

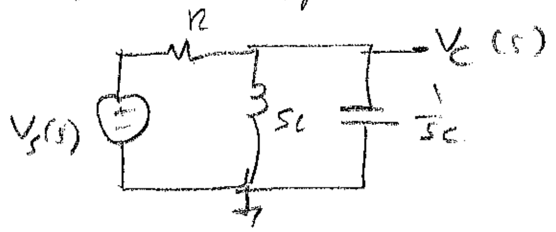
$-1500 \pm j33$

$$= -0.003\, e^{-1500t} \sin(3329.16t)$$

Q5) 10-36



Zero-state response:



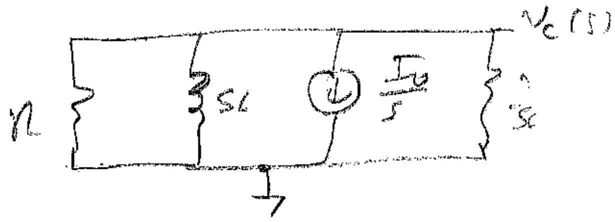
$$V_C(s) = V_s(s)\,\frac{sL // \frac{1}{sC}}{sL // \frac{1}{sC} + R}$$

$$sL // \frac{1}{sC} = \frac{\frac{sL}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1+s^2LC}$$

$$V_C(s) = V_s(s)\,\frac{\frac{sL}{1+s^2LC}}{\frac{sL}{s^2LC+1} + R} = \frac{sL}{sL + R + s^2LCR}\,V_s(s) = \frac{sL \times V_s(s)}{LCR\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)}$$

$$= \frac{V_s(s)}{CR}\,\frac{s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Zero-input response:



$$V_C(s) = -\frac{I_0}{s}\, R // sL // \frac{1}{sC}$$

$$= -\frac{I_0}{s}\, R // \frac{sL}{1+s^2LC}$$

$$= -\frac{I_0}{s}\,\frac{\frac{R \times sL}{1+s^2LC}}{R + \frac{sL}{1+s^2LC}}$$

$$= -\frac{I_0}{s}\,\frac{RLs}{R + s^2LCR + sL}$$

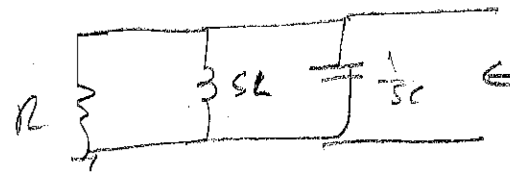
$$= -\frac{I_0}{s}\,\frac{RLs}{LCR\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)}$$

$$= -\frac{I_0}{C}\,\frac{1}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Q6) 10-38

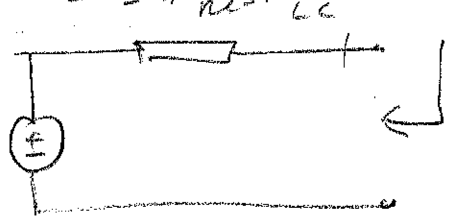
To find V_T: Since the circuit is at zero-state, from the previous problem, the open circuit voltage $= V_T = \frac{V_S(s)}{CR}\,\frac{s}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$

To find R_T:



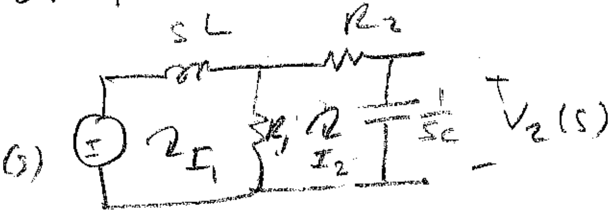
$Z_T = R // sL // \frac{1}{sC}$

$= \frac{1}{C}\,\frac{s}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$



Q7) 10-51 (a, b & d)

a)



$-V_1(s) + I_1(s)sL + (I_1(s) - I_2(s))R_1 = 0$

$(sL+R_1)I_1 - R_1 I_2 = V_1$ eqn 1

$R_2 I_2 + \frac{1}{sC} I_2 + (I_2 - I_1)R_1 = 0 \Rightarrow \left(R_1 + R_2 + \frac{1}{sC}\right)I_2 = R_1 I_1$ eqn 2.

b) $(sL+R_1)\,\frac{R_1+R_2+\frac{1}{sC}}{R_1}\,I_2 - R_1 I_2 = V_1$

$\frac{(sL+R_1)(sC(R_1+R_2)+1)}{sCR_1} I_2 - \frac{sCR_1^2}{sCR_1} I_2 = V_1$

$I_2 = V_1\,\frac{sCR_1}{s^2LC(R_1+R_2) + sL + sCR_1(R_1+R_2) + R_1 - sCR_1^2}$

$= \frac{V_1 s R_1 C}{s^2LC(R_1+R_2) + s(L + CR_1R_2) + R_1}$

poles are: $-666.67 \pm j942.81$

d) $$I_2(s) = \frac{\frac{20}{s}\times 5\times 0.5\times10^{-6}\times 2\times10^{7}}{s^2\times0.5\times10^{-6}\times3\times10^{3}+s(1+0.5\times10^{-6}\times2\times10^{6})+2\times10^{3}}$$

$$= \frac{2\times10^{-2}}{1.5\times10^{-3}s^2+2s+2\times10^{3}} = \frac{20}{1.5s^2+2000s+2000000}$$

$$= \frac{20}{1.5}\,\frac{1}{s^2+\frac{2000}{1.5}s+\frac{2000000}{1.5}} = \frac{20}{1.5}\,\frac{1}{(s+666.67-j942.81)\times(s+666.67+j942.81)}$$

$$= \frac{K}{s+666.67-j942.81} + \frac{K^*}{s+666.67+j942.81}$$

$$K = \frac{20}{1.5}\,\frac{1}{-666.67+j942.81+666.67+j942.81} = -j\frac{20}{1.5\times2\times942.81}$$

$$= -0.141j \Rightarrow K^* = 0.141j$$

$$|K| = 0.141 \quad \measuredangle K = -\pi+\frac{\pi}{2} = -\frac{\pi}{2}$$

$$\therefore\ i_2(t) = 0.282\,e^{-666.67t}\cos\left(942.81t-\frac{\pi}{2}\right)$$

$$= 0.282\,e^{-666.6t}\sin(942.81t)$$

Q8) 11-7

a) Driving point impedance = ∞

$$V_2(s) = \frac{V_1(s)}{R} \times \left(R /\!/ \frac{1}{sC}\right) + V_1(s)$$

$$R /\!/ \frac{1}{sC} = \frac{R/sC}{R+\frac{1}{sC}} = \frac{R}{1+sRC}$$

$$\frac{V_2(s)}{V_1(s)} = 1 + \frac{1}{R}\,\frac{R}{1+sRC} = 1 + \frac{1}{1+sRC}$$

$$= \frac{1+sRC+1}{1+sRC} = \frac{2+sRC}{1+sRC} = \frac{s+\frac{2}{RC}}{s+\frac{1}{RC}}$$

b) pole $= s = -\frac{1}{RC} = -2000$ rad/sec. $\Rightarrow RC = \frac{1}{2000}$

Take $R = 1\,k\Omega$. $\Rightarrow C = \frac{1}{1\times10^3 \times 2000} = 0.5\,\mu F$

zero $= s = -\frac{2}{RC} = -4000$ rad/sec.

Q9) 11-18

Impulse response $V_2(s) = \frac{50}{50+50+s\times10^{-2}} \times 1$ $\quad V_1(s) = 1$

$$V_2(s) = \frac{50}{100+\frac{s}{100}} = \frac{5000}{10000+s}$$

$V_2(t) = 5000\,e^{-10000t}$ impulse response. $t \geq 0$

Step response: $V_1(s) = \frac{1}{s}$

$$V_2(s) = \frac{1}{s}\,\frac{5000}{s+10000} = \frac{A}{s} + \frac{B}{s+10000}$$

$$A = \frac{5000}{10000} = \frac{1}{2} \qquad B = \left.\frac{5000}{s}\right|_{s=-10000} = -\frac{1}{2}$$

$$V_2(s) = \frac{1}{2}\,\frac{1}{s} - \frac{1}{2}\,\frac{1}{s+10000}$$

$t \geq 0$ $\quad V_2(t) = \frac{1}{2} - \frac{1}{2}\,e^{-10000t}$ $\quad$ check $\frac{dV_2}{dt} = 5000\,e^{-10000t}$ ✓

Q10) 11-27

$$\text{stepresponse} = g(t) = 50\left(e^{-25000t} - e^{-50000t}\right)u(t)$$

$$\text{impulse response} = h(t) = -50\times 25000\, e^{-25000t} + 50\times 50000\, e^{-50000t}$$

$$h(t) = 50\times 25000\left(-e^{-25000t} + 2e^{-50000t}\right)$$

$$H(s) = 50\times 25000\left(\frac{-1}{s+25000} + \frac{2}{s+50000}\right)$$

$$= 50\times 25000\left(\frac{-s-50000+2s+50000}{(s+25000)(s+50000)}\right)$$

$$= \frac{50\times 25000\times s}{(s+25000)(s+50000)}$$

$$G(s) = \frac{50\times 25000}{(s+25000)(s+50000)}$$

$$T(s) = H(s) = \frac{50\times 25000\times s}{(s+25000)(s+50000)}$$

Q.11) 11-39

$$\frac{I_2(s)}{I_1(s)} = -\frac{R_1}{R_2+sL} = -\frac{100}{400+s\times 100\times 10^{-3}} = \frac{-100}{400+0.1s} = \frac{-1000}{s+4000}$$

$w=50000$; $$\frac{I_2(jw)}{I_1(jw)} = \frac{-1000}{4000+jw} = \frac{-1000}{4000+j50000}$$ $I_1(jw)=10$ $\quad I_2(jw) = \dfrac{-10000}{4000+j50000}$

$$I_2(jw) = \frac{-10}{4+50j} = \frac{-10}{\sqrt{2516}\, e^{+1.49j}} \Rightarrow i_{2ss}(t) = \frac{-10}{\sqrt{2516}}\cos(50000t-1.49)$$

$$= -0.1994\cos(50000t-1.49)$$

$$= 0.1994\cos(50000t+1.65)$$

If $w=5000$, then $$\frac{I_2(jw)}{I_1(jw)} = \frac{-1000}{4000+j5000} = \frac{-1}{4+j5}$$

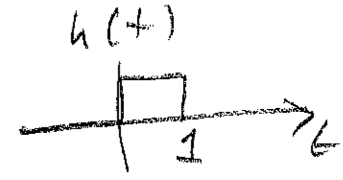
$$I_2(jw) = \frac{-10}{4+j5} = \frac{-10}{\sqrt{41}\, e^{j0.896}} \Rightarrow i_{2ss}(t) = \frac{-10}{\sqrt{41}}\cos(5000t-0.896)$$

$$= -1.56\cos(5000t-0.896)$$

$$= 1.56\cos(5000t+2.246)$$

pole is located at -4000.

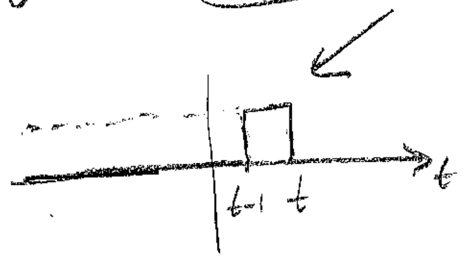
Q12) 11-57

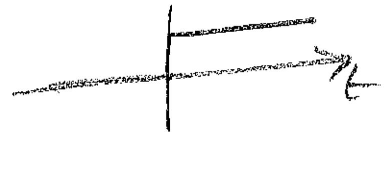


a) $h(t) = 10\,[u(t) - u(t-1)]$

$x(t) = e^{-t}u(t)$

$$y(t) = \int_0^t 10\,\underbrace{[u(t-\tau) - u(t-\tau-1)]}\,e^{-\tau}u(\tau)\,d\tau$$





$t<0 \quad y(t) = 0$

$t<1 \quad y(t) = \int_0^t 10\,e^{-\tau}\,d\tau = -10\,e^{-\tau}\Big|_0^t$

$= -10(e^{-t} - 1)$

$= 10(1 - e^{-t})$

$t>1 \quad y(t) = \int_{t-1}^t 10\,e^{-\tau}\,d\tau = -10\,e^{-\tau}\Big|_{t-1}^t = -10(e^{-t} - e^{-t+1})$

$= -10\,e^{-t}(1-e)$

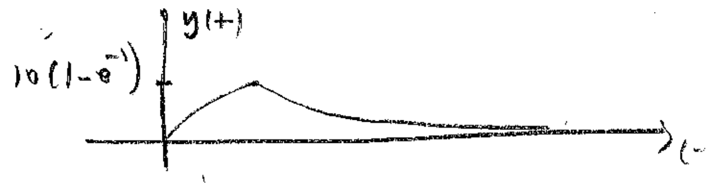
$= 10(e-1)\,e^{-t}$

$\therefore\ y(t) = 10(1-e^{-t})u(t) + [10(e-1)e^{-t} - 10(1-e^{-t})]u(t-1)$

$= 10(1-e^{-t})u(t) + [10(e-1)e^{-t} - 10 + 10e^{-t}]u(t-1)$

$= 10(1-e^{-t})u(t) + [10e\,e^{-t} - 10]u(t-1)$

$= 10(1-e^{-t})u(t) + 10(e^{1-t} - 1)u(t-1)$



11/12

5) $H(s) = 10\left(\frac{1}{s} - \frac{e^{-s}}{s}\right)$

$X(s) = \frac{1}{s+1}$

$\Rightarrow Y(s) = H(s)X(s) = \frac{10}{s+1}\left(\frac{1}{s} - \frac{e^{-s}}{s}\right)$

$= \frac{10}{(s+1)s} - \frac{10e^{-s}}{(s+1)s}$

$f(s) = \frac{10}{(s+1)s} = \frac{A}{s+1} + \frac{B}{s}$ $A = \left.\frac{10}{s}\right|_{s=-1} = -10$

$= \frac{-10}{s+1} + \frac{10}{s}$ $B = \left.\frac{10}{s+1}\right|_{s=0} = 10$

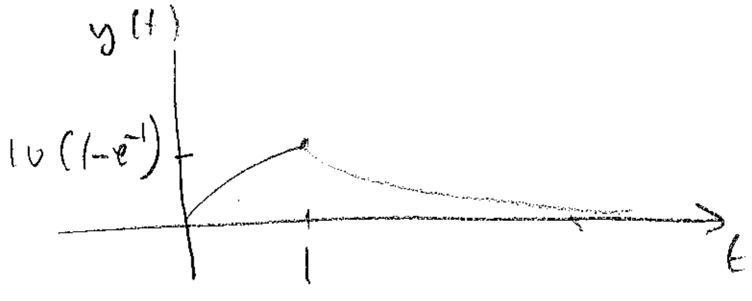
$f(t) = (10 - 10e^{-t})u(t)$

$g(t) = -\frac{10}{(s+1)s}e^{-s} \Rightarrow g(t) = -f(t-1)u(t-1)$

$= -(10 - 10^{-(t-1)})u(t-1)$

$= (10e^{1-t} - 10)u(t-1)$

$\therefore y(t) = (10 - 10e^{-t})u(t) + (10e^{1-t} - 10)u(t-1)$



$t>1$ $(\cancel{10} - 10e^{-t} + 10e^{1-t} - \cancel{10})$

$= -10e^{-t} + 10e \cdot e^{-t}$

$= 10e^{-t}(e-1)$

$t=1$ $10e^{-1}(e-1) = 10(1-e^{-1})$