EEE202 Circuit Theory

Lab 3 Report

Maximum Power Transfer

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Section 02

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Purpose

The purpose of this lab was to design at least two different passive linear circuits to transfer maximum power to 180ohm load resistor from a voltage source with output impedance 50Ω at a frequency between 5 and 10Mhz.

Methodology

The assignment is to design the circuit shown in figure 1, in 2 different ways.

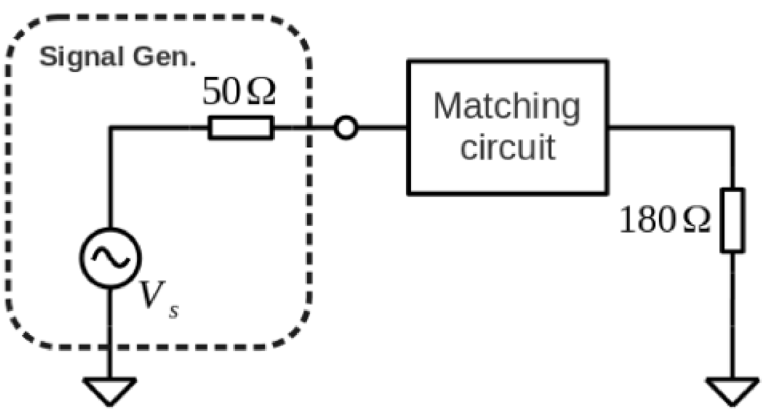


Fig1. Matching Circuit

To be able to transfer maximum power to the load, the load and source impedance should match, and the relationship between them can be expressed as:

$$Z_s = Z_L^* \quad (1)$$

$$I = \frac{V}{Z_L + Z_S}$$

$$Z_L = R_L + jX_L \quad (a)$$

$$Z_S = R_S + jX_S \quad (b)$$

$$P_L = Re\left\{\frac{1}{2} V_L I^*\right\}$$

$$P_L = Re\left\{\frac{1}{2} |I|^2 Z_L\right\}$$

$$P_L = Re\left\{\frac{1}{2} \frac{|V_S|^2}{|Z_S + Z_L|^2} (R_L + jX_L)\right\}$$

$$P_L = \frac{1}{2} \frac{|V_S|^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \quad (2)$$

We are looking for the max value of (2)

$$\left.\begin{array}{l} (X_S + X_L) = 0 \Rightarrow X_S = -X_L \\ R_S = R_L \end{array}\right\} \quad Z_S = Z_L^*$$

From here

$$P_{L,max} = \frac{|V_S|^2}{8R_S} = P_A$$ ← available power from source

Fig2: Max Power Calculation

By doing this we will be matching the load impedance with the load impedance. To achieve this we can use L-section, narrowband transformer, wideband transformer, π section and T section.

For this lab,

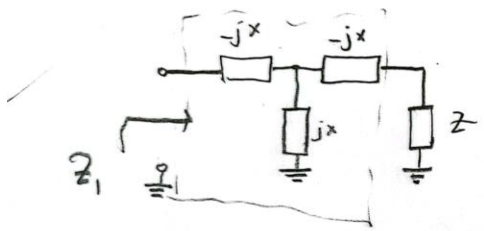
- π section and T section was used
- 7.5 Mhz
- 5V sinusoidal signal

From here we expect the max power to be for the chosen sinusoidal signal where R_L=R_S=50 Ohm,

$P_{available}$ = 62.5 mW

T Section and π Section Calculation

To be able to derive the values of capacitor and inductor the formula from analog electronics, which can also be derived from node equations.



$$Z_1 = -jx + \frac{1}{(1/jx) + 1/(-jx+Z)} = \frac{jx\cdot(-jx+Z)}{Z} - jx = \frac{x^2 + jxZ - jxZ}{Z} = \frac{x^2}{Z}$$

$$Z_1 = \frac{x^2}{Z}$$

Fig3: Calculation of node equations in T section

Though the node equation is different the result is same for π section.

$$X^2 = R_S * R_L \quad (3)$$

$$X = 94.87 \quad (4)$$

For jx elements inductor, for -jx elements capacitor needs to be used as it allows us to escape from complex numbers.

$$jX = jwL, \quad -jx = \frac{1}{jwC} \qquad (5)$$

$$L = \frac{X}{2\pi f}, \; C = \frac{1}{X 2\pi f} \quad (6)$$

R_s is source resistance and R_L is load resistance. As capacitor, are easier to handle in the lab two -jX and one jX element is used for T and π section.

From here we get capacitance and inductance as

C = 223 pF

L = 2.01 uH

For the values to fit into standard values in the lab 220pF and 2uH was used.

Software Part and Result

Without the matching circuit,

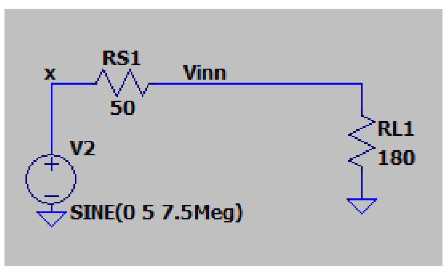


Fig 4: 180ohm Without the matching circuit

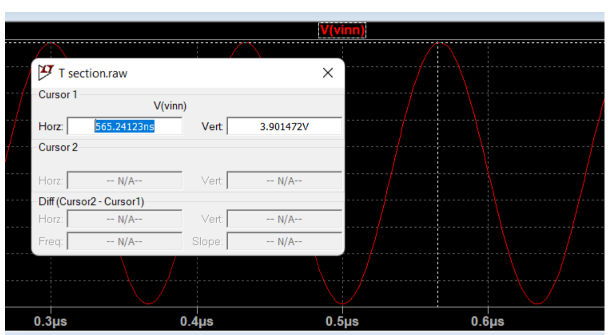


Fig 5: Result of the circuit

The average power delivered to R_L in such case is,

- P_L = 42mW

Our goal is to increase this power to about 62.5 mW using impedance matching

1. T section

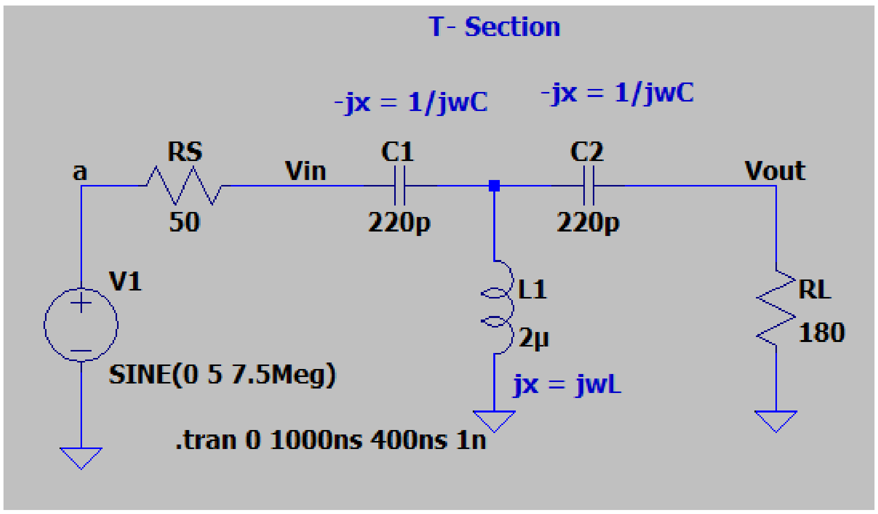


Fig 6: T section

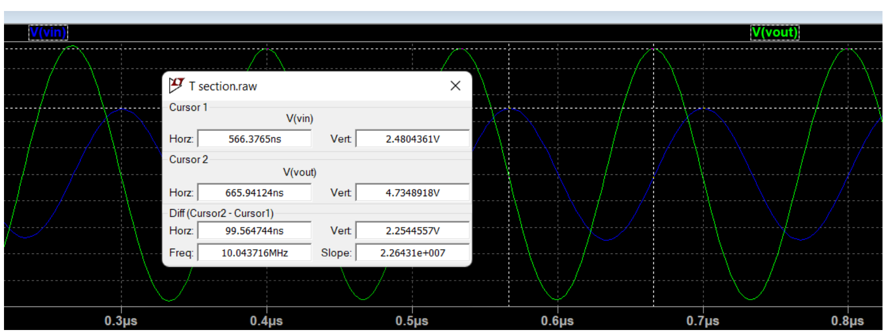


Fig 7: Vin and Vout of T-section

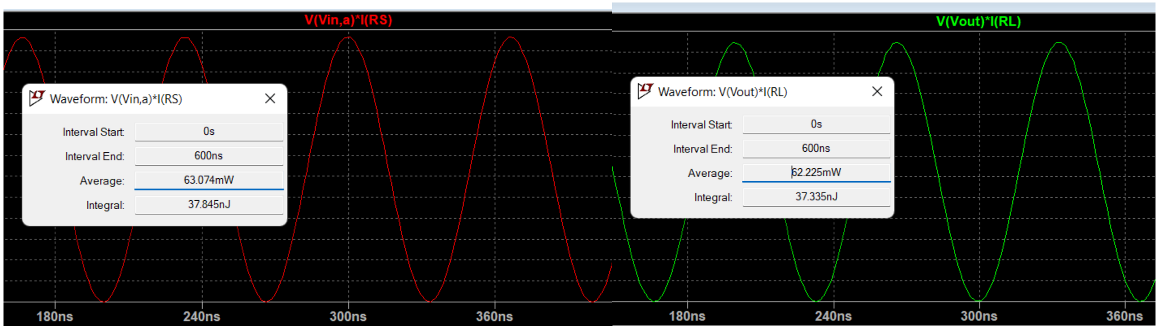


Fig 8 & 9: Power on source resistance and Delivered power

As seen in figures 8 and 9 , input voltage of the signal generator is 2.45V(Vin in figure 8), whereas output voltage is 4.74V (Vout in figure 8). For the chosen sinusoidal signal, average power from Vin is 63.074mW and average power transferred is 62.22mW. This shows, T-section is a viable method for impedance matching.

	Calculated Available Power (mW)	**Power on source resistance (mW)**	**Delivered Power (mW)**	**Error**
T-section	62.5mW	63.074mW	-	0.9%
	62.5 mW	-	62.22mW	0.4%

Table 1: Power table of T section

- Delivered Power to expected Calculated Power ratio is 99.6%, indicating the matching circuit is working efficiently.
- We have managed to raise 42mW to 62.22mW

	Input Voltage (Vin)	**Output Voltage (Vout)**
T-section	2.48V	4.74V

Table 2: Voltages on T section

2. π Section

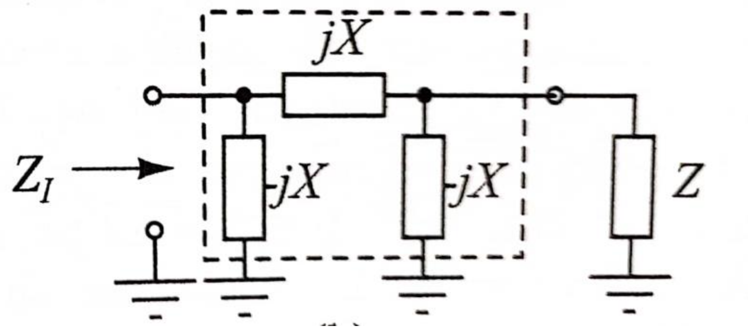


Fig 10: π section

The calculation are same in terms of their result however the node equation this time is,

$$Z_1 = \frac{1}{-jx} + \frac{1}{jx + \left(\frac{1}{-jx} + \frac{1}{Z}\right)^{-1}} = \frac{x^2}{Z}$$

Fig 11: Calculation for π Section

As calculated before the capacitor values are 220pF and inductor value is 2uH.

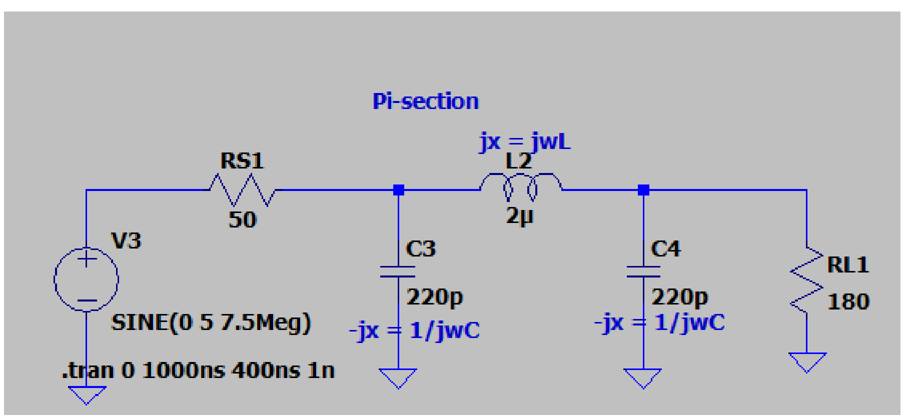


Fig 12: Resulting π section

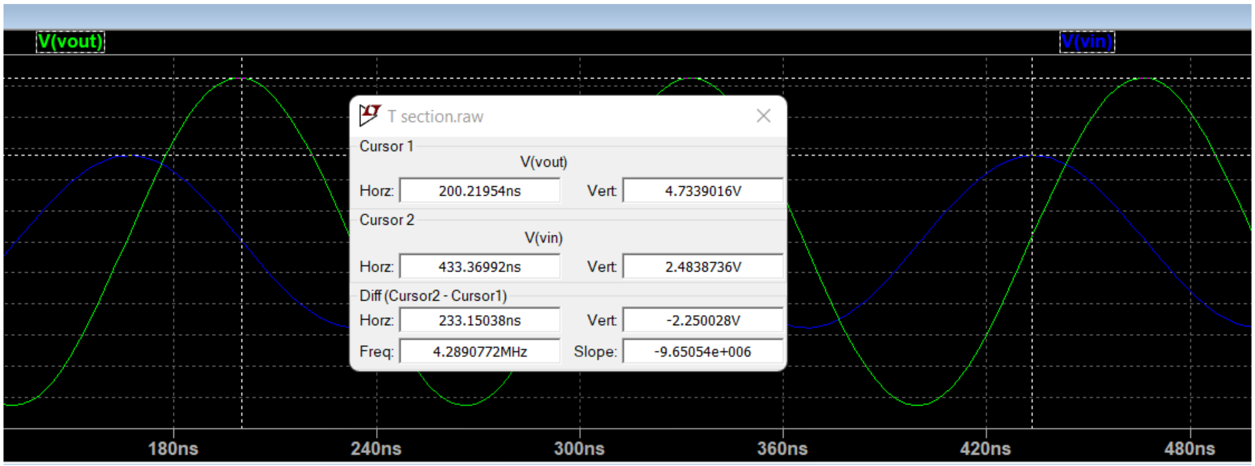


Fig 13: Vin and Vout

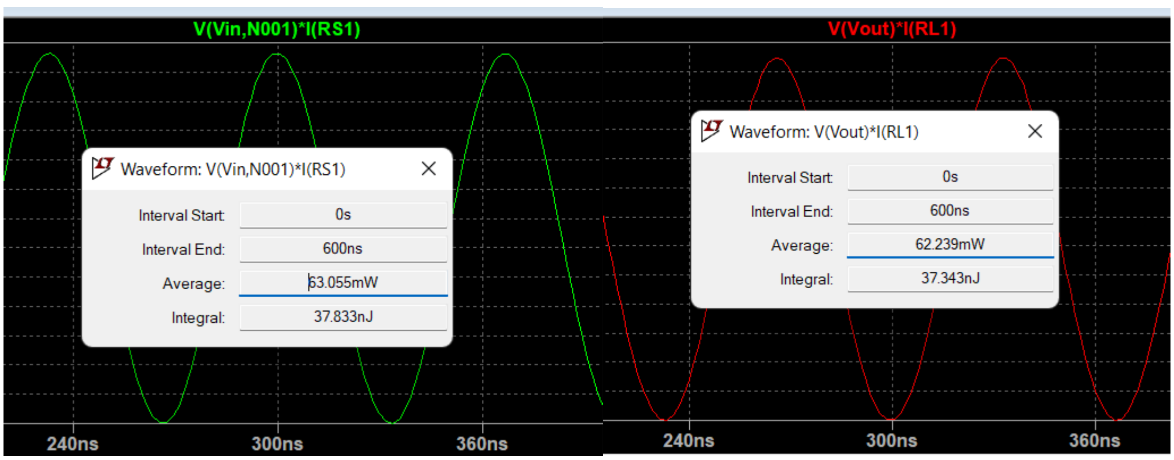


Fig 14 & 15: Average power on resistor and load

As seen in figures 14 and 15, input voltage of the signal generator is 2.48V (Vin in figure 14), whereas output voltage is 4.73V (Vout in figure 14). Average power on Vin is 63.055mW and average power transferred is 62.239mW. This shows, π-section is a good method for impedance matching.

	Calculated Available Power (mW)	Power from source resistor (mW)	Delivered Power (mW)	Error
π-section	62.5mW	63.055mW	-	0.88%
	62.5 mW	-	62.239mW	0.41%

Table 3: Power on π section

- Delivered power ratio to expected value of calculated available **power is 99.6%,** indicating we have a working matching circuit.
- We have managed to raise 42mW to 62.24mW .

	Input Voltage (Vin)	Output Voltage (Vout)
π -section	2.48V	4.73V

Table 4: Voltage on π section

Hardware Implementation

1. 47 Ohm part

The idea is to use voltage divider and then calculate power transferred to it allowing us to compare it to the calculated power in the methodology. Rout is 47 ohm and Rin is 50 ohm source resistance

$$Vout = Vin\frac{Rout}{Rin + Rout}$$

We can calculate average power from

$$P = V^2/2R$$

Fig 16: 47 Ohm

The expected voltage is 2.42V and power is 58.56 mW. The found result is,

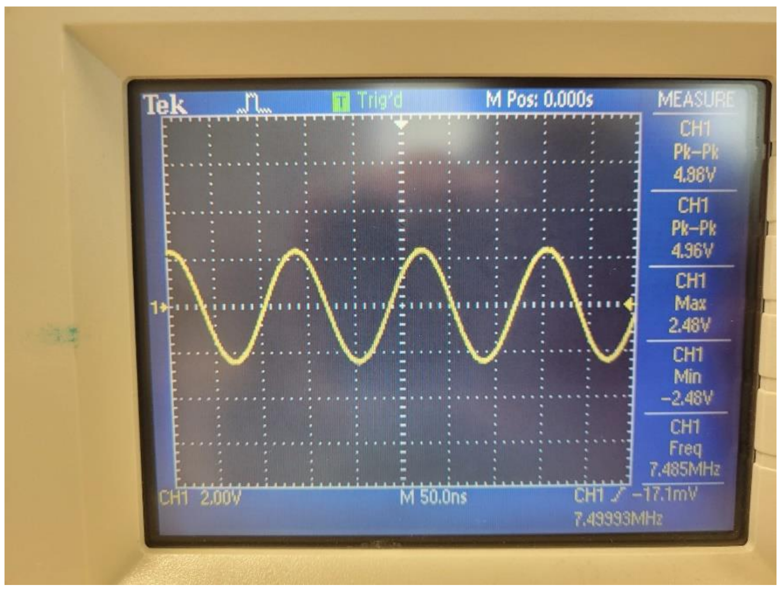


Fig 17: 47ohm voltage divider result

2.48volts, corresponding to 61.5 mW. Indicating that the voltage supply is supplying more than expected value. It

- The error for voltage is 2.41%
- Error for power is %5.02

2. T section

As there were 1uH inductors I connected two of them serially to achieve 2uH value. For capacitor as said before 220 pF was used.

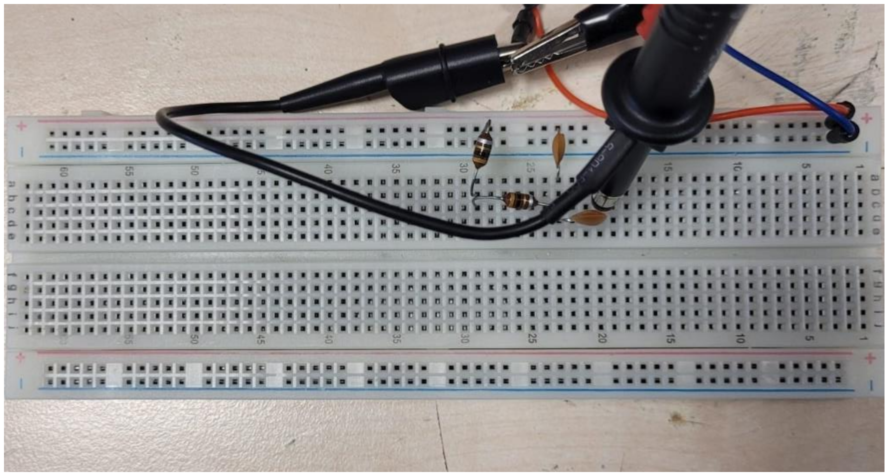


Fig 18: T section circuit

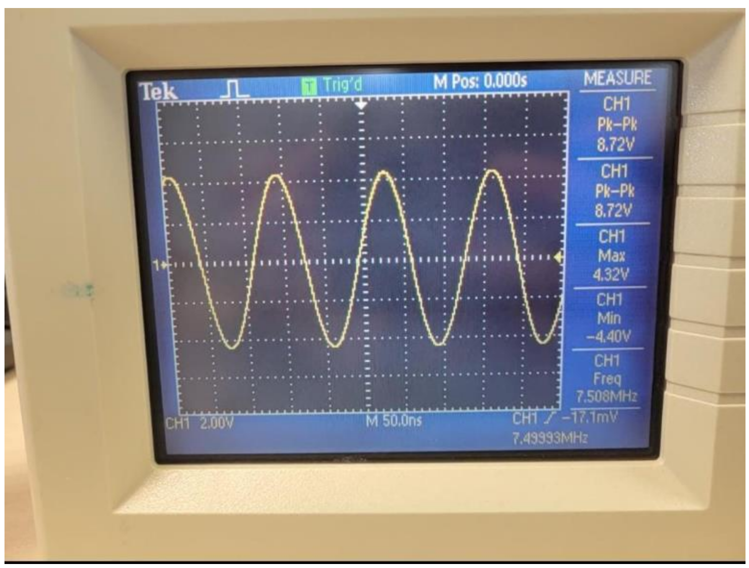


Fig 19: 8.72V Peak to Peak Voltage on T section

We get 4.36 volt from peak to peak. Calculating the average power and putting the voltage values in we get,

	Calculated Available Power	Delivered Power in Software	Delivered Power in Hardware	Error rate (Between Hardware and Calculated Power)
	62.5mW	62.22mW	52.80mW	15.52%

Table 5: Power for T section

Considering the max power transfer is

$$\frac{Delivered\ Power}{Calculated\ Available\ Power}$$

We transfer **84.48% of the power** to the load. I used calculated available power as 62.5 mW is the value we expect in ideal conditions and eliminates the error caused by 47ohm.

3. Π Section

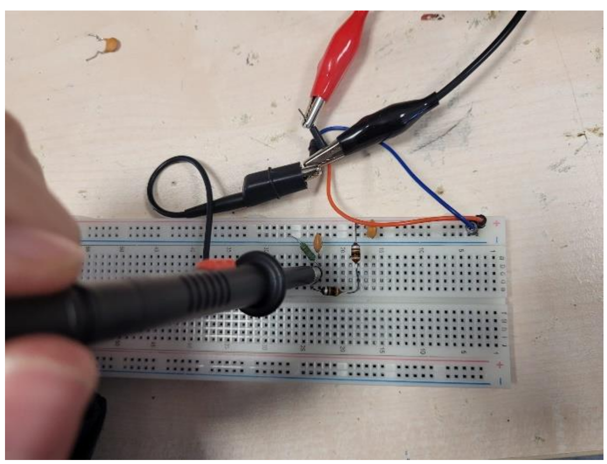


Fig 20: π Section Circuit

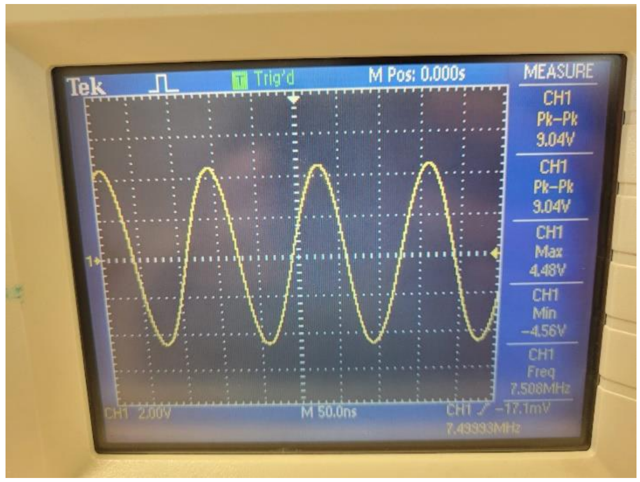


Fig 21: 9.04 Peak to Peak Voltage on π Section

We have 4.52 volts from 9.04 Volt peak to peak voltage. Calculating power from $P= V^2/2R$ and putting into the table we have

	Calculated Available Power	Delivered Power in Software	Delivered Power in Hardware	Error rate (Between Hardware and Calculated Power)
	62.50mW	62.24mW	56.75mW	9.2%

Table 6: Power on π Section

From Max Power Transfer we get,

90.8%,

of the power transferred to the load. In both T and P section the error for max power transfer is below 20%

Conclusion

It is possible that the available power is different than the ideal value we have found as we found different value while doing the voltage divider. However, whether this error would dissipate or not if I had used a 50 ohm resistor is unknown as the error may be caused by the signal generator itself.

Moreover, Pi section result were higher than the result of T section which could be caused by the breadboards or the components itself I have used. In both circuits the error is less than 20% and the power delivered is always higher than 42mW,which was the value without the matching circuit, indicating the power is transferred appropriate.

This lab was the most confusing one in terms of understanding what is meant by max power transfer. I have decided to use 62.5 mW for the max power transfer ratio even though if I had used the measured value from 47ohm, it would give me less error. This was done to protect the accuracy of the result. In an ideal circuit the expected value was 62.5mW meaning we should calculate error and possible max power transfer according to it as well.