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EEE 202 CIRCUIT THEORY
Final, Spring 2013-14

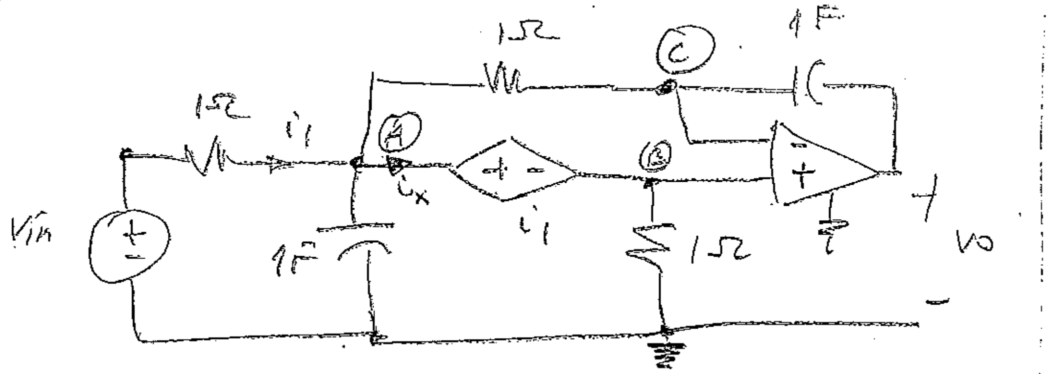
No credits will be given for unjustified answers. Good luck.

Prob. 1 : (25 pt.s)

Consider the following circuit. You may assume zero initial conditions for capacitor voltages. Assume that the op-amp is ideal and operates in the linear region.

i : Find the transfer function $H(s) = \frac{V_o(s)}{V_{in}(s)}$.

ii : Now assume that the circuit is in sinusoidal steady state and that $v_{in}(t) = \cos t$. Find $v_o(t)$.



i) 15/10

$V_A - V_B = i_1 = V_{in} - V_A \Rightarrow \boxed{2V_A - V_B = V_{in}}$ 02

$\boxed{i_x = V_B}$ 02 $\Rightarrow$ (A) $V_A - V_{in} + i_x + sV_A + V_A - V_C = 0$ 02

$\boxed{V_B = V_C}$ 01 $\Rightarrow (s+2)V_A - V_B + i_x = V_{in} \Rightarrow \boxed{V_A = \frac{V_{in}}{s+2}}$

02

$V_B = 2V_A - V_{in} = \left(\frac{2}{s+2} - 1\right)V_{in} = \frac{2-s-2}{s+2}V_{in} = \boxed{-\frac{s}{s+2}V_{in}}$

(C)

$V_C - V_A + \cancel{s}(V_C - V_o) = 0 \Rightarrow (s+1)V_B - V_A = sV_o$ 02

$\Rightarrow V_o = \frac{s+1}{s}V_B - \frac{1}{s}V_A = -\frac{s+1}{s+2}V_{in} - \frac{1}{s(s+2)}V_{in} = -\frac{s^2+s+1}{s(s+2)}V_{in}$

$\Rightarrow \boxed{H(s) = -\frac{s^2+s+1}{s(s+2)}}$ 04

ii) 10/10

$H(j\omega) = -\frac{1-\omega^2+j\omega}{j\omega(j\omega+2)}$ 02 $\omega = 1 \Rightarrow H(j1) = \frac{-j}{j(j+2)} = \frac{-1}{j+2}$ 02

$V_o = \frac{1}{\sqrt{5}} e^{j\pi} \cdot e^{-j\tan^{-1}\frac{1}{2}} = \frac{1}{\sqrt{5}} e^{j(\pi - \tan^{-1}\frac{1}{2})}$ 02

$\boxed{v_o(t) = \frac{1}{\sqrt{5}} \cos\left(t + \pi - \tan^{-1}\frac{1}{2}\right)}$ 04

Prob. 2 : (25 pt.s) Consider the circuit given in Figure 1. You may assume zero initial conditions for this problem. Here, the block given by $H(s)$ is a linear circuit and $H(s) = \frac{V_o(s)}{V_{in}(s)}$. (Note that unit step response is the solution of the circuit when input is unit step, and impulse response is the solution of the circuit when the input is an impulse.)

i : We want the unit step response of the circuit given in Figure 1 as $v_o(t) = 1 + e^{-t} - 2e^{-2t}$. How do you choose $H(s)$ to satisfy this requirement?

ii : Assume that the unit step response of the circuit in Figure 1 is as given above. Find the impulse response of such a circuit.

iii : A student suggests that the transfer function found in i could be realized with the circuit given in Figure 2. In you agree, find the relevant element values. (Op-amps are ideal and operate in the linear region. Note that some of the elements may not be necessary). If you disagree, design a circuit which realizes the transfer function found in i.

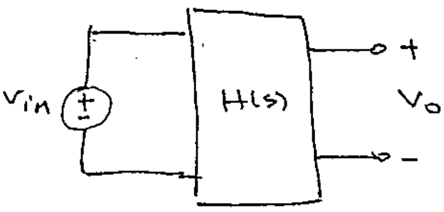


Fig. 1

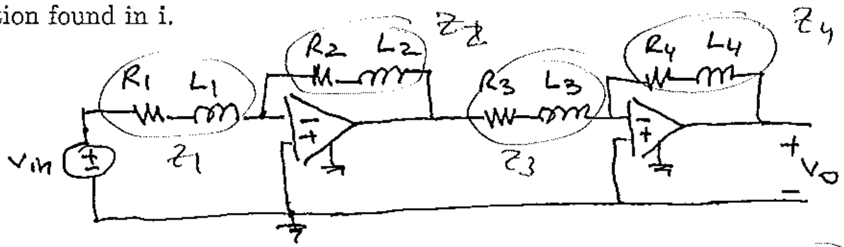


Fig. 2

10/10

i) $V_o(s) = \frac{1}{s} + \frac{1}{s+1} - \frac{2}{s+2} = \frac{s^2+3s+2+s^2+2s-2s^2-2s}{s(s+1)(s+2)} = \frac{3s+2}{s(s+1)(s+2)}$ 01 when $V_{in} = \frac{1}{s}$ 02

$(s+1)(s+2) \quad s(s+2) \quad s(s+1)$

$\Rightarrow \boxed{H(s) = \frac{V_o}{V_{in}} = \frac{3s+2}{(s+1)(s+2)}}$ 04

ii) impulse response $= \frac{d}{dt}(\text{step response}) = -e^{-t} + 4e^{-2t}$ 05/05

iii) $H(s) = -\frac{Z_2}{Z_1} \cdot \left(-\frac{Z_4}{Z_3}\right) = \frac{Z_2 \cdot Z_4}{Z_1 \cdot Z_3}$ 02

$Z_1 = R_1 + sL_1$
$Z_2 = R_2 + sL_2$
$Z_3 = R_3 + sL_3$
$Z_4 = R_4 + sL_4$

10/10

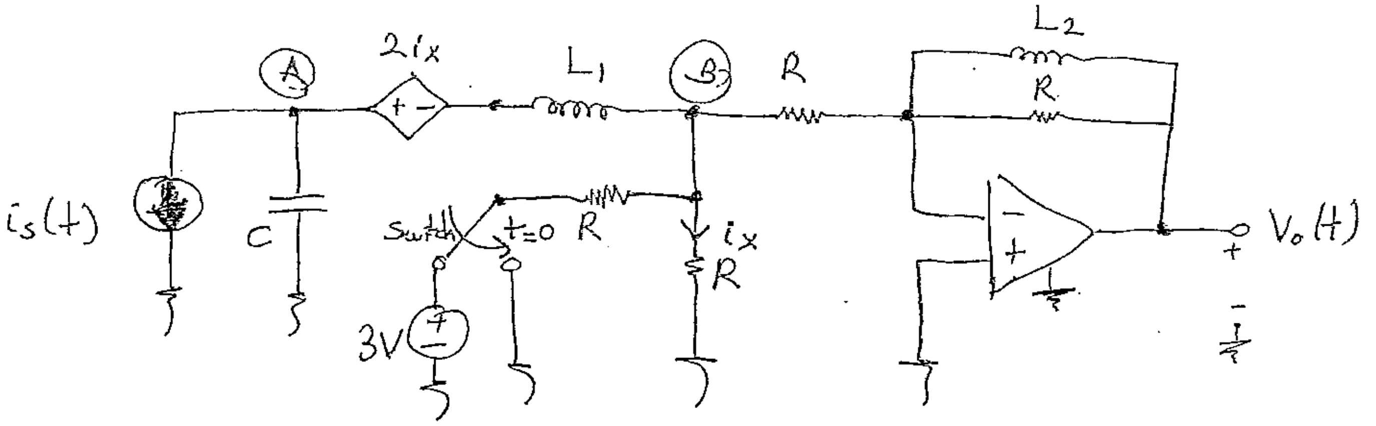
choose $Z_2 = 3s+2 = R_2 + sL_2 \Rightarrow \boxed{R_2 = 2\Omega,\ L_2 = 3H}$ 02

$Z_1 = s+1 = R_1 + sL_1 \Rightarrow \boxed{R_1 = 1\Omega,\ L_1 = 1H}$ 02

$Z_4 = 1 = R_4 + sL_4 \Rightarrow \boxed{L_4 = 0H,\ R_4 = 1\Omega}$ 02

$Z_3 = s+2 = R_3 + sL_3 \Rightarrow \boxed{R_3 = 2\Omega,\ L_3 = 1H}$ 02

Prob. 3 : (28 pt.s) Consider the following circuit. Assume that the op-amp is linear and operates in the linear region. Here $R = 3\ \Omega$, $C = 6/5$ F., $L_1 = 5/6$ H., $L_2 = 3$ H., $i_s(t) = 2e^{-2t}\ u(t)$, where $u(t)$ is the unit step function. For $-\infty < t < 0$, the switch is connected to the 3 V. DC source, and for $t \geq 0$ the switch is connected to the ground. Find $v_o(t)$ for $t \geq 0$.



Prob. 4 : (22 pt.s) Consider the circuit given below. You may assume zero initial conditions for this problem. Assume that $RC = 1$ sec.

i : Find the network function $H(s) = \frac{V_o(s)}{V_{in}(s)}$.

ii : Determine the type of this filter (e.g. low-pass, high-pass, etc.)

iii : Assume that $v_{in}(t) = \cos t$. By using Laplace transform analysis, find $v_o(t)$.