CHAPTER 3 : CIRCUIT ANALYSES TECHNIQUES

NODE ANALYSIS : Variables are node voltages

Basic Steps :

Step 1 : Choose a reference node (Datum = Ground). Specify reference directions for element currents and hence voltages. Now, we can write each element voltage in terms of node voltages.

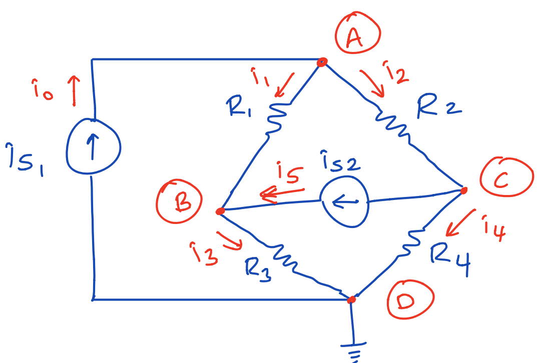
Step 2 : Write KCL equations at all nodes except for the reference node.

Step 3 : Use element relations (e.g., $i = GV$) to express currents in terms of node voltages.

Step 4 : Substitute these in KCL equations.

Result : We will have $(n-1)$ equations in terms of $(n-1)$ node voltages.

Example :



Step 1 : Choose D as the ground node.

$V_1 = V_A - V_B$ $\qquad V_3 = V_B$

$V_2 = V_A - V_C$ $\qquad V_4 = V_C$

Step 2 : KCL at A : $-i_0 + i_1 + i_2 = 0$

KCL at B : $-i_1 + i_3 - i_5 = 0$

KCL at C : $-i_2 + i_4 + i_5 = 0$

Step 3 : $i_0 = i_{s_1}$ $\qquad i_3 = G_3 \cdot V_B$

$i_1 = G_1 (V_A - V_B)$ $\qquad i_4 = G_4 \cdot V_C$

$i_2 = G_2 (V_A - V_C)$ $\qquad i_5 = i_{s2}$

Step 4 : KCL at A : $-i_{s1} + G_1(V_A - V_B) + G_2(V_A - V_C) = 0$

$$(G_1 + G_2)V_A - G_1 V_B - G_2 V_C = i_{s1}$$

KCL at B : $-G_1(V_A - V_B) + G_3 \cdot V_B - i_{s2} = 0$

$$-G_1 V_A + (G_1 + G_3)V_B = i_{s2}$$

KCL at C : $-G_2(V_A - V_C) + G_4 \cdot V_C + i_{s2} = 0$

$$-G_2 V_A + (G_2 + G_4)V_C = -i_{s2}$$

* we can write these equations in matrix form as:

$$G\,\underline{V} = \underline{u}$$

$\underline{u}$ — current source vector. $(n-1)\times 1$

$\underline{V}$ — vector of node voltages $(n-1)\times 1$

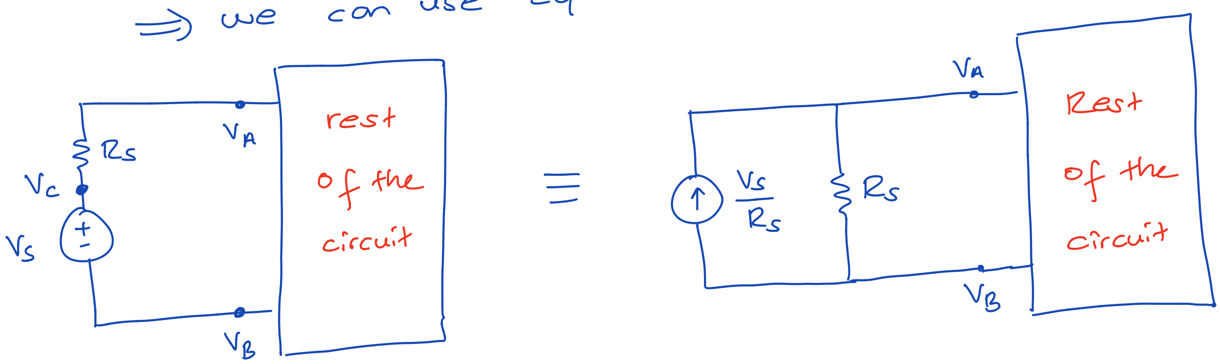
G — Conductance (admittance) matrix : $(n-1)\times(n-1)$

* For the previous example:

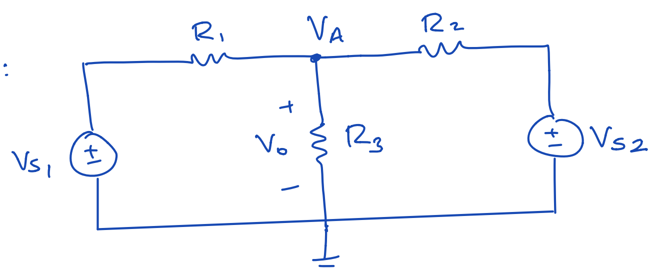
$$\begin{bmatrix} (G_1+G_2) & -G_1 & -G_2 \\ -G_1 & (G_1+G_3) & 0 \\ -G_2 & 0 & (G_2+G_4) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s2} \\ -i_{s2} \end{bmatrix}$$

* What happens when we have voltage sources as well?
$\Rightarrow$ modify the same idea

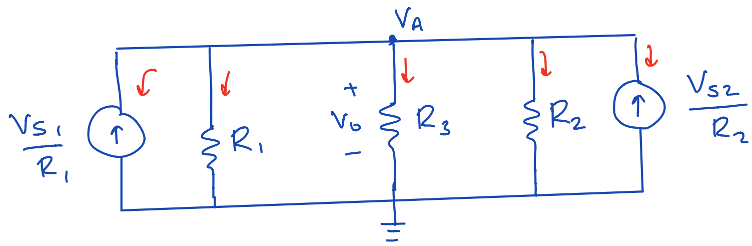
Case 1 : If a voltage source is in series with a resistor
$\Rightarrow$ we can use equivalence (i.e., source transformation)



Example :



$V_o = ?$

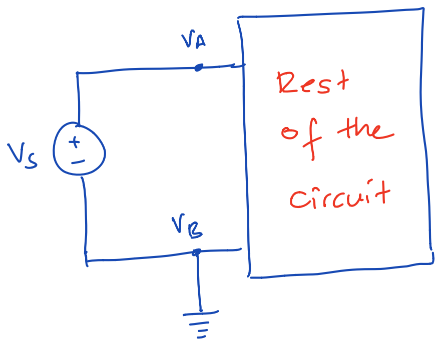


KCL at A : $-\frac{V_{S1}}{R_1} + \frac{V_A}{R_1} + \frac{V_A}{R_3} + \frac{V_A}{R_2} - \frac{V_{S2}}{R_2} = 0$

$$(G_1 + G_2 + G_3) V_A = G_1 . V_{S1} + G_2 . V_{S2}$$

$$V_o = V_A = \frac{G_1 . V_{S1} + G_2 . V_{S2}}{G_1 + G_2 + G_3}$$

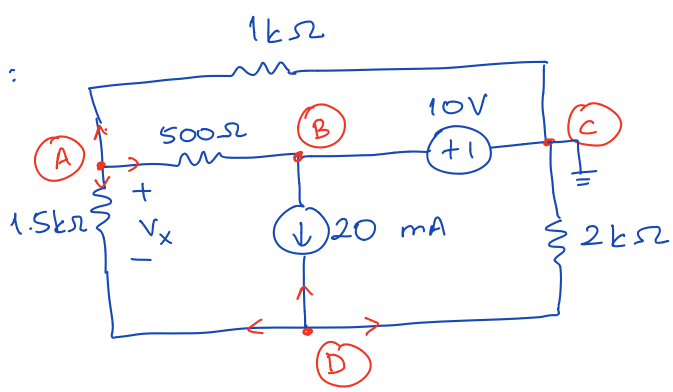
Case 2 : If a voltage source is not in series with a resistor of the voltage source
$\Rightarrow$ we may choose one end of the voltage source as the reference node (ground).



* Do not write node equation at node A.
* There remains $(n-2)$ nodes, write node equations for these as usual.
* we get $(n-2)$ equations $+ \; V_A = V_S$

$\underbrace{\qquad\qquad}_{(n-1) \text{ equations}}$. we can find all the node voltages.

Example :



Find node voltages.
Find V_x

$V_B = 10 \, V$

KCL at A : $\underset{(3)}{\frac{V_A}{1k\Omega}} + \underset{(2)}{\frac{V_A - V_D}{1.5k\Omega}} + \underset{(6)}{\frac{V_A - 10}{0.5k\Omega}} = 0$

$$11 V_A - 2 V_D = 60$$

KCL at D : $\frac{V_D - V_A}{1.5k\Omega} - 20mA + \frac{V_D}{2k\Omega} = 0$

(4) (6k) (3)

$$-4V_A + 7V_D = 120$$

* 2 equations, 2 unknowns. Solve together:

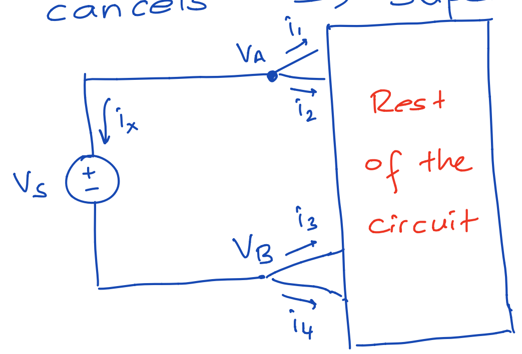
$V_A \approx 9.6V$

$V_D \approx 22.6V$

* Finally, $V_x = V_A - V_D \approx -13V$

Case 3 : If a voltage source is not in series with a resistor and we cannot (or do not want to) choose one end of the voltage source as reference node :

Sum the KCL equations for both nodes of the voltage source $\Rightarrow$ Current of the voltage source cancels $\Rightarrow$ Supernode



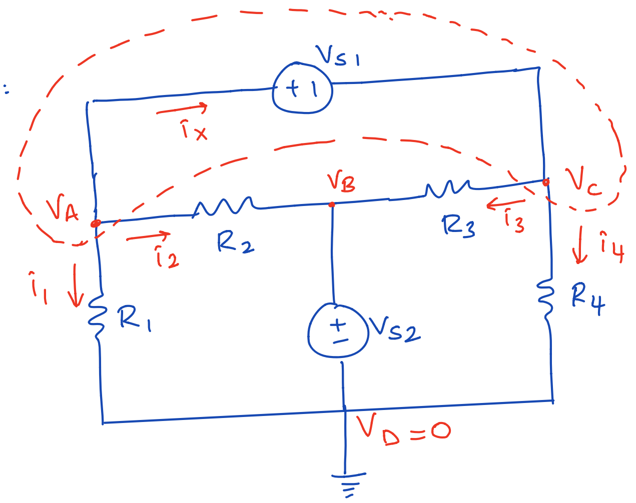
KCL at A : $i_1 + i_2 + \cancel{i_x} = 0$

KCL at B : $-\cancel{i_x} + i_3 + i_4 = 0$

$$+ \quad i_1 + i_2 + i_3 + i_4 = 0$$

Also, $V_A - V_B = V_s$

Example :



$V_B = V_{S2}$

KCL at A : $i_1 + i_2 + i_x = 0$

KCL at C : $-i_x + i_3 + i_4 = 0$

KCL at supernode : $i_1 + i_2 + i_3 + i_4 = 0$ $\Rightarrow$ can be obtained by using the Gaussian volume

$$G_1 . V_A + G_2 (V_A - V_B) + G_3 . (V_C - V_B) + G_4 V_C = 0$$

Using $V_B = V_{S2}$,

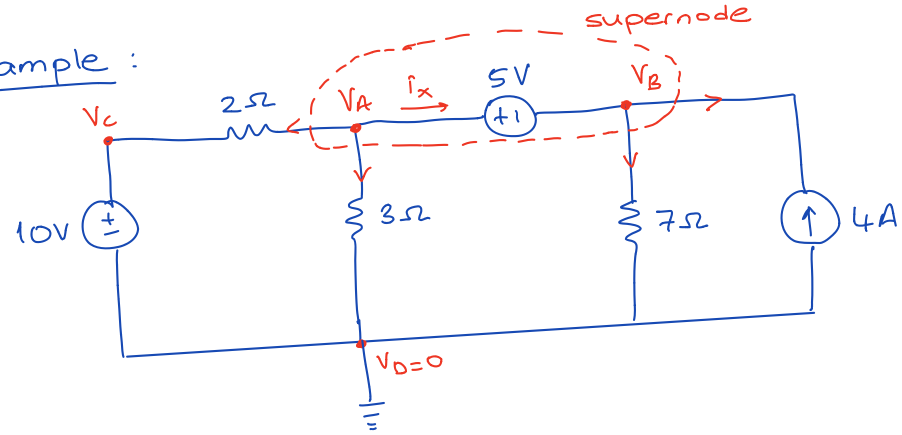
$$(G_1 + G_2) V_A + (G_3 + G_4) V_C = (G_2 + G_3) V_{S2}$$

* Need one more equation :

KVL at A and C : $V_A - V_C = V_{S1}$

* Now, we have two equations and two unknowns (V_A , V_C)

Example :



$i_x = ?$

$V_C = 10\text{ V}$

KCL at supernode (A+B) : $\dfrac{V_A - 10}{2} + \dfrac{V_A}{3} + \dfrac{V_B}{7} - 4 = 0$

(21) (14) (6) (42)

$$35 V_A + 6 V_B = 210 + 168 = 378$$

Also : $V_A - V_B = 5\text{ V}$ (KVL at supernode)

$V_B = V_A - 5$

Insert back : $35 V_A + 6 (V_A - 5) = 378$

$$V_A = \frac{408}{41} \approx 9.95\text{ V}$$

$$V_B = V_A - 5 \approx 4.95\text{ V}$$

* Now, to find i_x, write KCL at A :

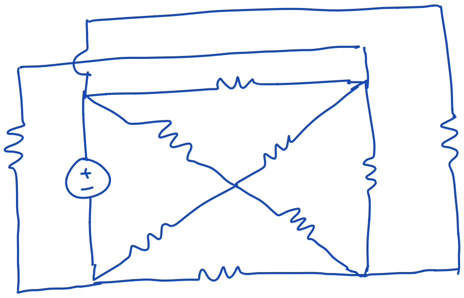
$$\frac{V_A - 10}{2} + \frac{V_A}{3} + i_x = 0$$

$$i_x = -\frac{V_A}{3} + \frac{10 - V_A}{2} = -3.29\text{ A}$$

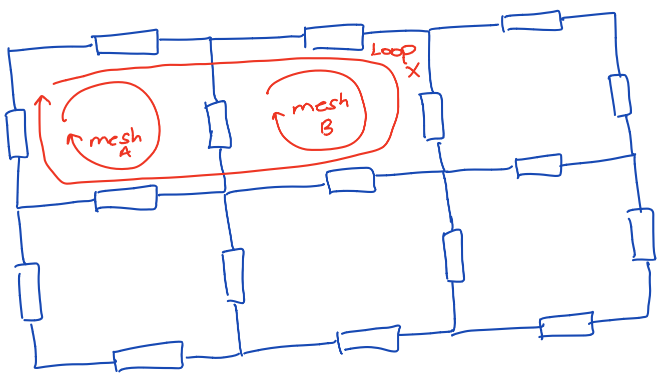
MESH ANALYSES : Variables are mesh currents

Planar Circuit : A circuit that can be drawn on a plane without crossovers.

Counter example :

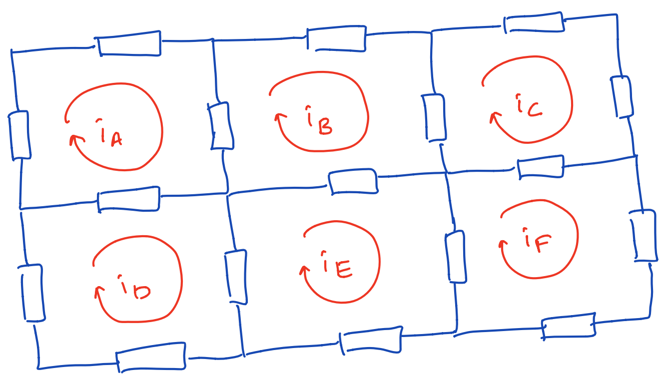


Mesh : Given a planar circuit, a mesh is a loop which does not contain any elements inside.



Fact : Given a planar circuit that has b two-terminal elements and n nodes, there are exactly $(b-n+1)$ meshes $\Rightarrow$ same as the number of independent KVL equations

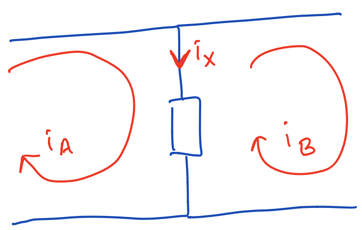
Mesh current : In each mesh, define a circulating current called a mesh current.
Necessary for writing KVL equations.



** Mesh currents are conceptually the duals of node voltages. However, they are not as natural. Node voltages can be measured easily, whereas mesh current is an abstract variable that simplifies analysis. They may be impossible to measure directly.

* Convention : Clockwise circulation direction for mesh currents.

* If a branch current i_x is common between mesh A and mesh B :



$$i_x = i_A - i_B$$

* Hence, any element current can be written in terms of mesh currents.

Basic Steps :

Step 1 : Choose meshes. Specify reference directions for element currents and hence voltages. Write each element current in terms of mesh currents.

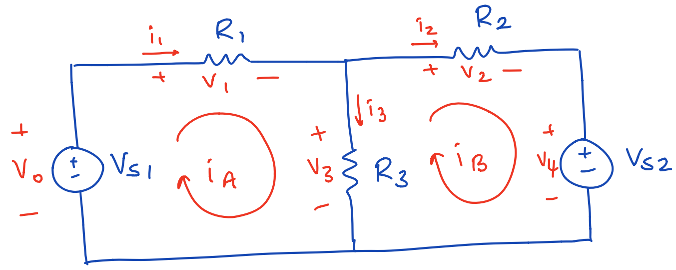
Step 2 : Write KVL equations at all meshes.

Step 3 : Use element relations (e.g., $v = Ri$) to express voltages in terms of mesh currents.

Step 4 : Substitute these in KVL equations.

Result : We will have $(b-n+1)$ equations in terms of $(b-n+1)$ mesh currents.

Example :



Step 1 :

$$i_1 = i_A$$
$$i_2 = i_B$$
$$i_3 = i_A - i_B$$

Step 2 : KVL at mesh A : $V_1 + V_3 - V_0 = 0$

KVL at mesh B : $V_2 + V_4 - V_3 = 0$

step 3 : Element relations

$$V_0 = V_{S1}$$
$$V_1 = R_1 . i_1 = R_1 . i_A$$
$$V_2 = R_2 . i_2 = R_2 . i_B$$
$$V_3 = R_3 . i_3 = R_3 (i_A - i_B)$$
$$V_4 = V_{S2}$$

Step 4 : KVL at mesh A : $R_1 i_A + R_3 (i_A - i_B) - V_{S1} = 0$

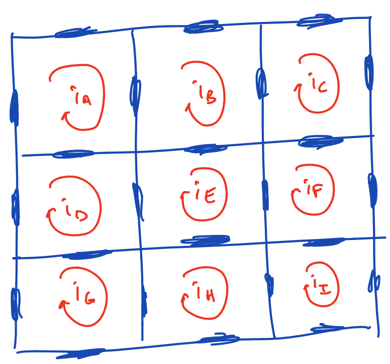
$$(R_1 + R_3) i_A - R_3 i_B = V_{S1}$$

KVL at mesh B : $R_2 . i_B + V_{S2} - R_3 (i_A - i_B) = 0$

$$-R_3 i_A + (R_2 + R_3) i_B = -V_{S2}$$

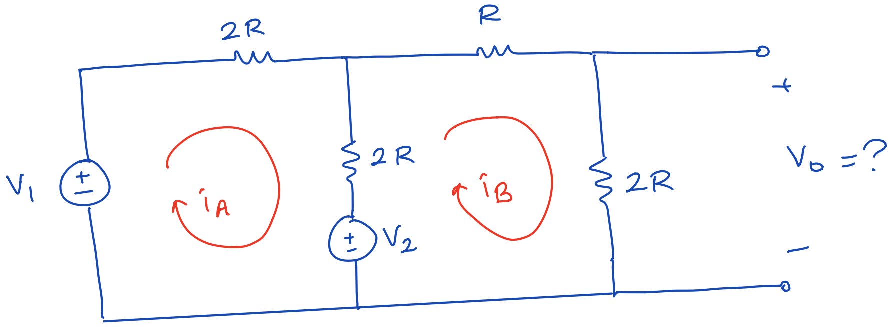
* 2 equations , 2 unknowns (i_A, i_B)

* Side note :



Cannot measure i_E directly. It is an abstract variable.

Example :



KVL at mesh A : $2R \cdot i_A + 2R\,(i_A - i_B) + V_2 - V_1 = 0$

$$4R \cdot i_A - 2R\, i_B = V_1 - V_2$$

KVL at mesh B : $R \cdot i_B + 2R \cdot i_B - V_2 + 2R\,(i_B - i_A) = 0$

$$-2R\, i_A + 5R \cdot i_B = V_2$$

×2

$$-4R\, i_A + 10R \cdot i_B = 2V_2$$

$$+ \quad 4R \cdot i_A - 2R \cdot i_B = V_1 - V_2$$

$$8R\, i_B = V_1 + V_2$$

$$i_B = \frac{V_1 + V_2}{8R}$$

Then, $V_o = 2R \cdot i_B = \dfrac{V_1 + V_2}{4}$

* We can write these equations in matrix form as:

$$R\, i = u$$

voltage source vector : $(b-n+1) \times 1$

vector of mesh currents : $(b-n+1) \times 1$

Resistance (impedance) matrix : $(b-n+1) \times (b-n+1)$

* For the previous example:

$$\begin{bmatrix} 4R & -2R \\ -2R & 5R \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} V_1 - V_2 \\ V_2 \end{bmatrix}$$

* Simplest case of mesh equations : circuits with independent voltage sources and resistors $\Rightarrow$ easy to write KVL.

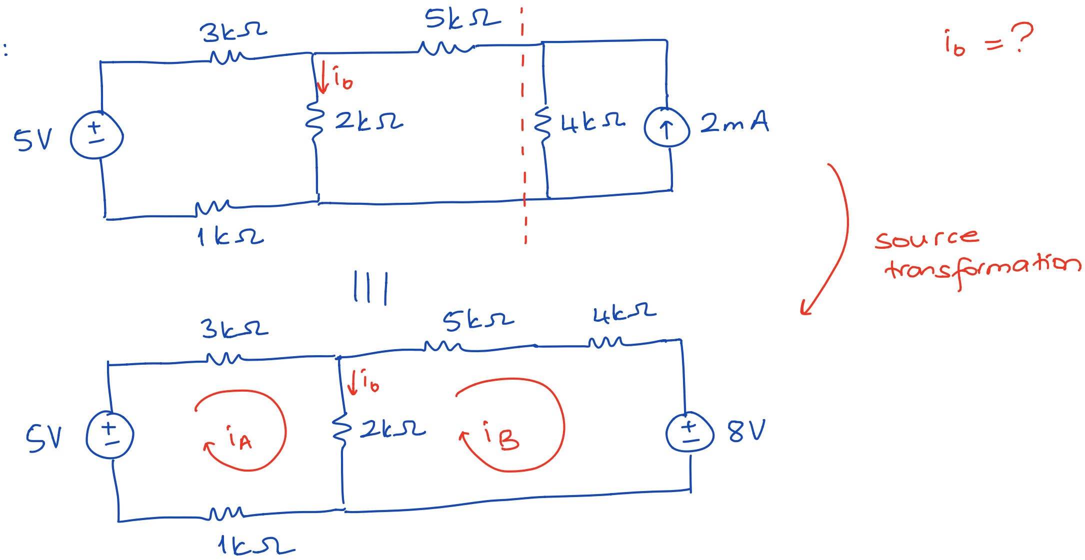
Mesh Equations with Current Sources:

We have 3 different cases:

* **Case 1**: If a current source is in parallel with a resistor $\Rightarrow$ apply source transformation, i.e., replace with a voltage source in series with a resistor

Example:

$i_o = ?$



KVL at mesh A: $i_A \cdot 3k + (i_A - i_B) \cdot 2k + i_A \cdot 1k - 5V = 0$

$$6 i_A - 2 i_B = 5 \qquad (i_A, i_B \text{ in mA})$$

KVL at mesh B: $i_B \cdot 5k + i_B \cdot 4k + 8V + (i_B - i_A) \cdot 2k = 0$

$$-2 i_A + 11 i_B = -8$$

$\times 3$

$$-6 i_A + 33 i_B = -24$$
$$+\ \ 6 i_A - 2 i_B = 5$$

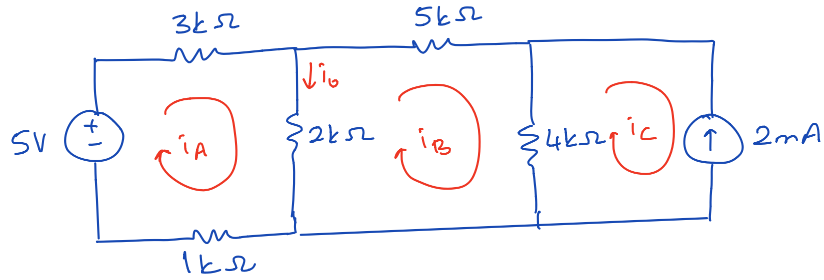
$$31 i_B = -19 \Rightarrow i_B \approx -0.61\,mA$$

$$i_A = \frac{5 + 2 i_B}{6} \approx 0.63\,mA$$

* Then, $i_o = i_A - i_B \approx 1.24\,mA$

* **Case 2**: If a current source is in only one mesh known.
$\Rightarrow$ that mesh current is already known.
Hence, write the remaining mesh equations.

Example :



$i_0 = ?$

$i_C = -2mA$

KVL at mesh A : (same as before)

$6i_A - 2i_B = 5$

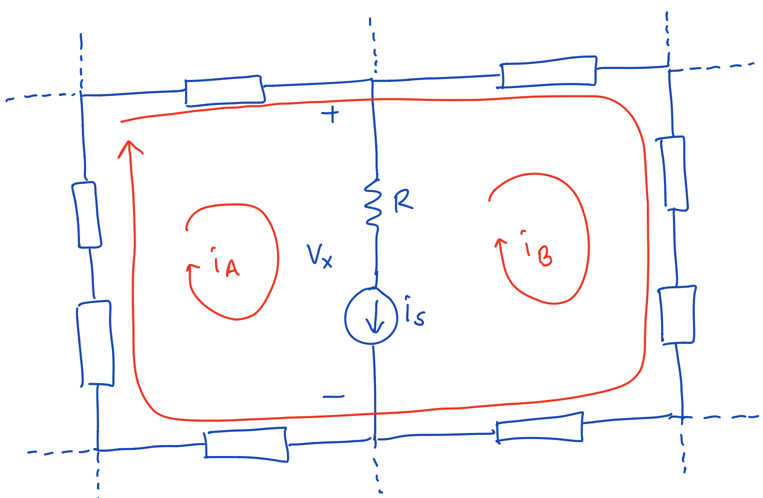
KVL at mesh B : $i_B \cdot 5k + (i_B - \overset{-2mA}{i_C}) \cdot 4k + (i_B - i_A) \cdot 2k = 0$

$-2i_A + 11\, i_B = 4i_C = -8 \Rightarrow$ same as before.

* Answer will be the same as before.

Case 3 : If a current source is between two meshes

$\Rightarrow$ we can combine these two mesh equations into a single mesh equation $\Rightarrow$ supermesh



mesh A : $\dots + \cancel{V_x} + \dots = 0$

mesh B : $\dots - \cancel{V_x} + \dots = 0$

+

mesh A + mesh B : V_x is eliminated

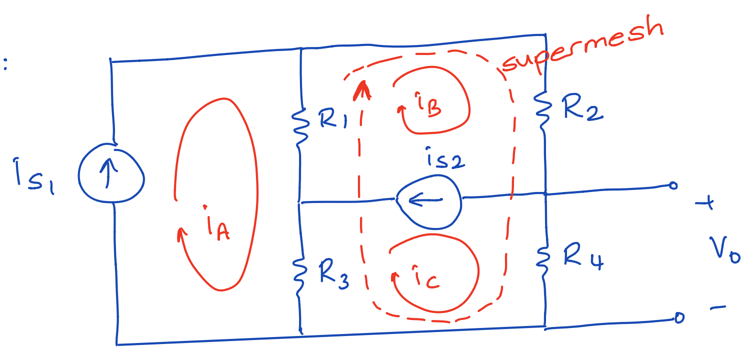
$\Rightarrow$ supermesh equation

* we need one more equation:

$i_s = i_A - i_B$

* we again get $(b-n+1)$ equations for $(b-n+1)$ unknowns.

Example :



$V_o = ?$

* $i_A = i_{s1}$
* Supermesh (B+C) : $R_2 . i_B + R_4 . i_C + R_3(i_C - i_A) + R_1(i_B - i_A) = 0$

$$-(R_1+R_3)i_A + (R_1+R_2)i_B + (R_3+R_4)i_C = 0$$

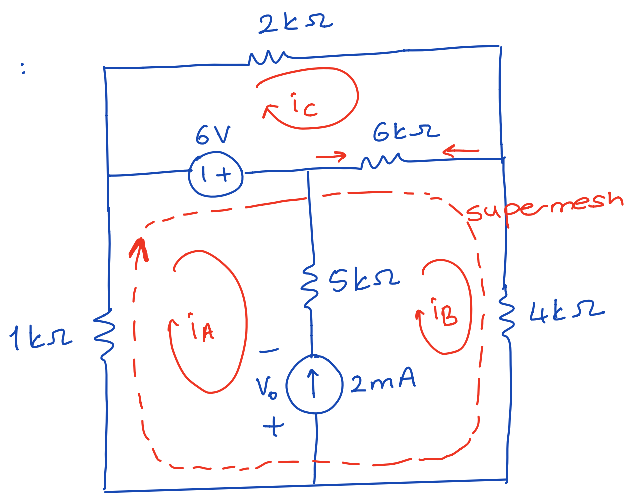
$$(R_1+R_2)i_B + (R_3+R_4)i_C = (R_1+R_3).i_{s1}$$

* Also, from supermesh : $i_B - i_C = i_{s2}$

2 equations
2 unknowns

* Then, $V_o = R_4 . i_C$

Example :



$V_o = ?$

$i_B - i_A = 2mA \Rightarrow i_A = i_B - 2$

Supermesh (A+B) : $-6 + 6(i_B - i_C) + 4 i_B + 1 . i_A = 0$ (currents in mA)

$$i_A + 10 i_B - 6 i_C = 6$$

$$(i_B - 2) + 10 i_B - 6 i_C = 6$$

$$11 i_B - 6 i_C = 8 \quad .... \quad \text{(Eqn. 1)}$$

mesh C : $2 . i_C + 6(i_C - i_B) + 6 = 0$

$$-6 i_B + 8 i_C = -6 \quad .... \quad \text{(Eqn. 2)}$$

4 × Eqn. 1 : $44 i_B - 24 i_C = 32$

3 × Eqn. 2 : $-18 i_B + 24 i_C = -18$

$+$

$$26 i_B = 14 \Rightarrow i_B = \frac{14}{26} \approx 0.54 \text{ mA}$$

$$i_A = i_B - 2 \approx -1.46 \text{ mA}$$

$$i_C = \frac{6 i_B - 6}{8} \cong -0.35 \text{ mA}$$

Note : 5kΩ resistor has no effect on currents.

* Now, to find V_o :

KVL at mesh B : $6(i_B - i_C) + 4 \cdot i_B + V_o + 5\underbrace{(i_B - i_A)}_{2mA} = 0$

$$V_o = -10 - 10 i_B + 6 i_C$$
$$\approx -17.5\ V$$

5kΩ resistor effects V_o.

LINEAR CIRCUITS :

Implicants : Superposition, Thevenin / Norton equivalent circuits

* A circuit is called linear if it contains linear elements + independent sources.

* Circuit Equations : Combained constraints

KCL : $Ai = 0$

KVL : $Bv = 0$

Element : $Mv + Ni = u$

$$\underbrace{\begin{bmatrix} 0 & A \\ B & 0 \\ M & N \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} v \\ i \end{bmatrix}}_{w} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix}}_{u_s}$$

* Resulting equations are linear : $Tw = u_s$

* b : number of two-terminal elements

* T : $2b \times 2b$ matrix that depends on circuit elements, but not on independent sources.

w : $2b \times 1$ vector of unknowns

u_s : $2b \times 1$ vector that depends only on the independent sources.

Consequences of Linearity :

Homogeneity : $f(\alpha x) = \alpha f(x)$, $\alpha \in \mathbb{R}$

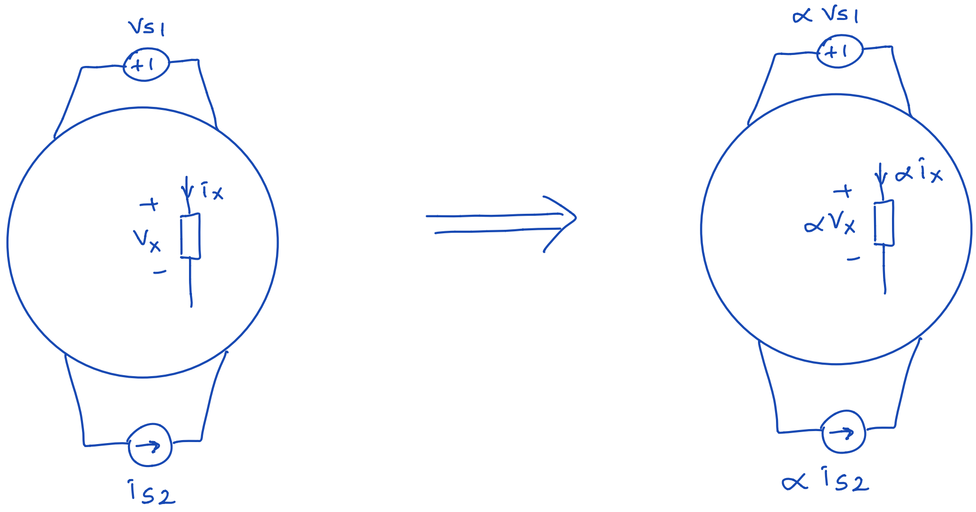
In our case : input sources | output element voltages, currents

$$u_s \longrightarrow w$$
$$\Rightarrow \alpha u_s \longrightarrow \alpha w$$

Proof : $T\omega = u_s$

$$\alpha u_s = \alpha T\omega = T(\alpha\omega)$$

Circuit Interpretation :



* Note that all independent sources must be multiplied by the same constant α.

Additivity : $f(x_1 + x_2) = f(x_1) + f(x_2)$

In our case :

input		output
u_{s1}	$\longrightarrow$	ω_1
u_{s2}	$\longrightarrow$	ω_2

$$\Longrightarrow \quad u_{s1} + u_{s2} \longrightarrow \omega_1 + \omega_2$$

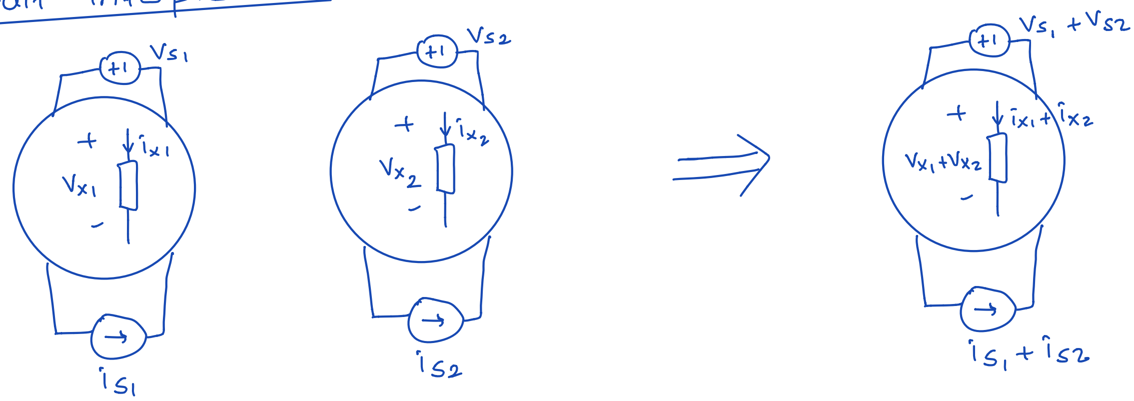
Proof : $T\omega_1 = u_{s1}$, $T\omega_2 = u_{s2}$

$$T\omega_1 + T\omega_2 = u_{s1} + u_{s2}$$

$$T(\omega_1 + \omega_2) = u_{s1} + u_{s2}$$

So, $u_{s1} + u_{s2} \longrightarrow \omega_1 + \omega_2$

Circuit Interpretation :



Superposition:

* A consequence of linearity. Assume that the source vector u_s is given as:

$$U_s = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ V_{s_1} \\ i_{s_1} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ V_{s_1} \\ 0 \end{bmatrix}}_{U_{s_1}} + \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ i_{s_1} \end{bmatrix}}_{U_{s_2}} = U_{s_1} + U_{s_2}$$

we have only two sources (one voltage source, one current source)

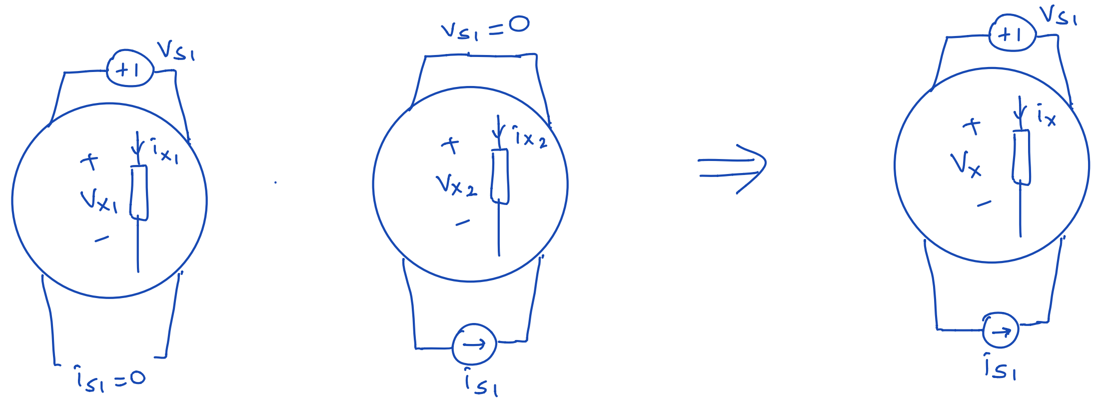
* Note that for U_{s_1}, only V_{s_1} is operational.
  $i_{s_1} = 0 \Rightarrow i_{s_1}$ current source set to zero
  $\Rightarrow$ set to OPEN CIRCUIT

* Similarly, for U_{s_2}, only i_{s_1} is operational.
  $V_{s_1} = 0 \Rightarrow V_{s_1}$ voltage source set to zero
  $\Rightarrow$ set to SHORT CIRCUIT.

* $U_{s_1} \rightarrow w_1$, $U_{s_2} \rightarrow w_2$

From linearity: $U_s = U_{s_1} + U_{s_2} \Rightarrow w = w_1 + w_2$

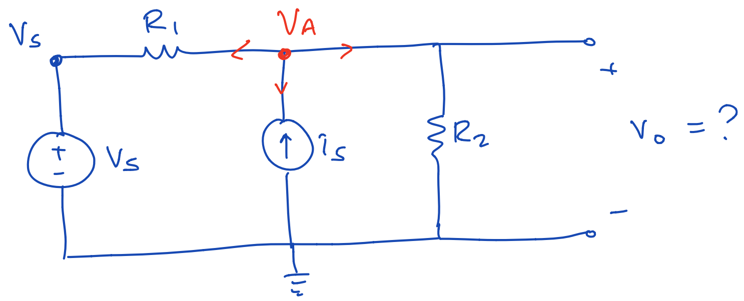
Circuit interpretation:



$i_x = i_{x_1} + i_{x_2}$

$V_x = V_{x_1} + V_{x_2}$

Example :



Direct way : $V_o = V_A$

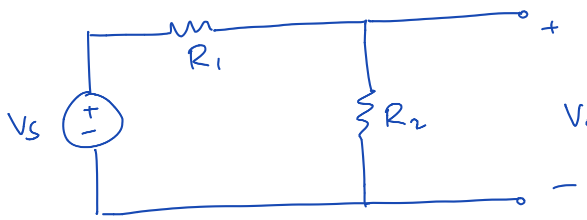
KCL at A : $\dfrac{V_A - V_s}{R_1} - i_s + \dfrac{V_A}{R_2} = 0$

$$(R_1 + R_2)\, V_A = R_2 . V_s + R_1 R_2\, i_s$$

$$V_A = V_o = \frac{R_2}{R_1 + R_2} V_s + \frac{R_1 R_2}{R_1 + R_2} i_s$$

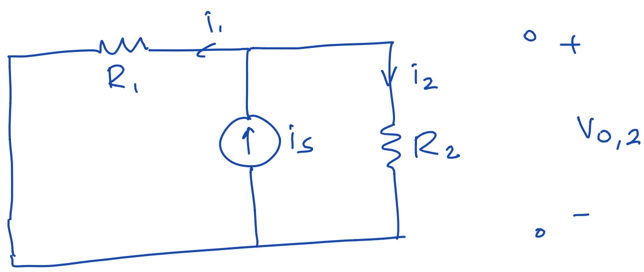
Superposition :

Due to V_s : Set $i_s = 0$ $\Longrightarrow$ OPEN CIRCUIT



$V_{o,1} = \dfrac{R_2}{R_1 + R_2} . V_s$ (voltage divider)

Due to i_s : Set $V_s = 0$ $\Longrightarrow$ SHORT CIRCUIT



$i_2 = \dfrac{R_1}{R_1 + R_2} . i_s$ (current divider)

$V_{o,2} = R_2 . i_2 = \dfrac{R_1 R_2}{R_1 + R_2} . i_s$

Add the results : $V_o = V_{o,1} + V_{o,2}$

$$= \underbrace{\frac{R_2}{R_1 + R_2}}_{K_1} V_s + \underbrace{\frac{R_1 R_2}{R_1 + R_2}}_{K_2} i_s = K_1 . V_s + K_2 . i_s$$

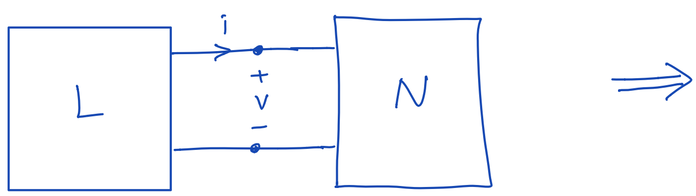
In general : If we have a linear circuit with n independent voltage sources $v_{s1}, \dots, v_{sn}$, and m independent current sources $i_{s1}, \dots, i_{sm}$, and y denotes an arbitrary branch voltage or current :

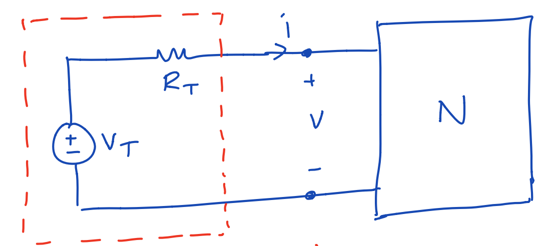
$$y = K_1 . v_{s1} + \dots + K_n . v_{sn} + H_1 . i_{s1} + \dots + H_m . i_{sm}$$

* Here, K_i and H_j are constants that depend only on the circuit elements but not on the independent sources themselves.

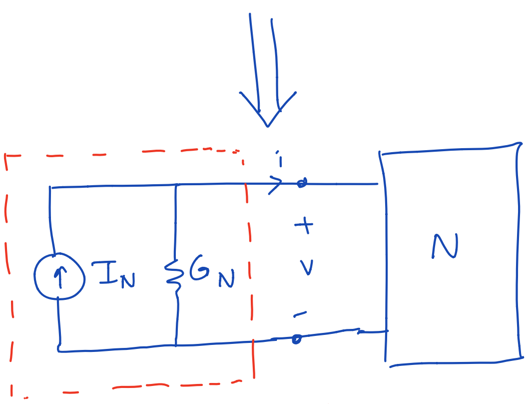
Thévenin and Norton Equivalent Circuits :

* N is any circuit (which may be nonlinear), and L is any linear circuit. The total circuit is assumed to have a unique solution.





Thévenin equivalent



Norton equivalent

V_T : Thevenin eq. voltage source
R_T : " " resistance
I_N : Norton eq. current source
G_N : " " conductance

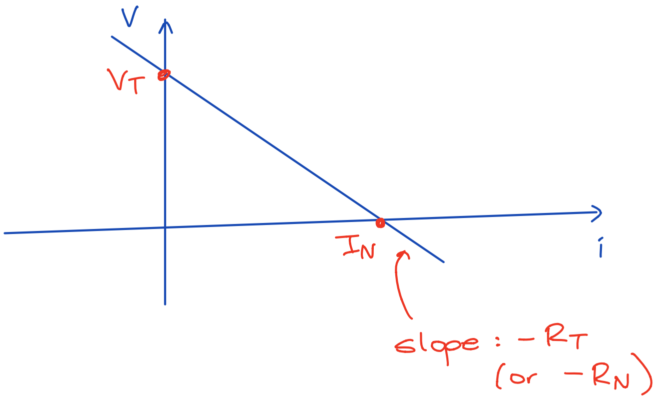
For Thévenin eq. circuit : $V_T = R_T . i + v$
For Norton eq. circuit : $I_N = \dfrac{v}{R_N} + i$

$\Big\}$ compare

$$\left.\begin{aligned} i &= \frac{V_T}{R_T} - \frac{V}{R_T} \\ i &= I_N - \frac{V}{R_N} \end{aligned}\right\} \text{compare}$$

* Hence, Thévenin and Norton equivalent circuits are equal to each other if:

$$R_N = R_T \quad , \quad I_N = \frac{V_T}{R_T}$$

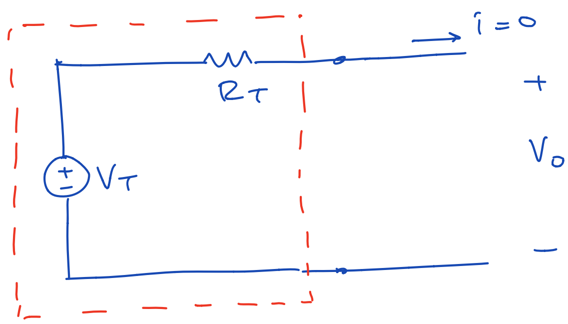


$V = \underbrace{V_T}_{\text{y-intercept}} - i . \overbrace{R_T}^{\text{slope}}$ (Thévenin)

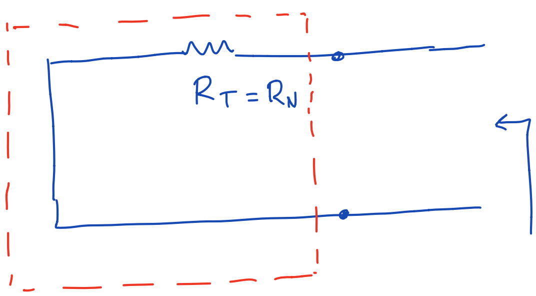
OR

$i = \underbrace{I_N}_{\text{x-intercept}} - \frac{V}{\underbrace{R_N}_{\text{slope}}}$ (Norton)

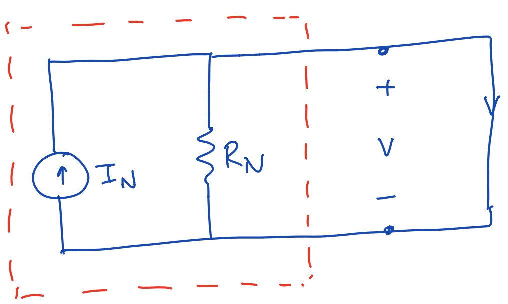
*



$V_{oc} = V_T$ (open circuit voltage is V_T)



Kill all sources inside, find $R_T = R_N$

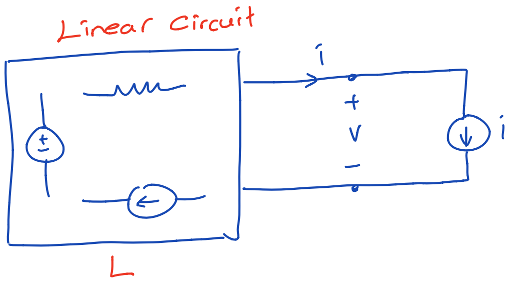


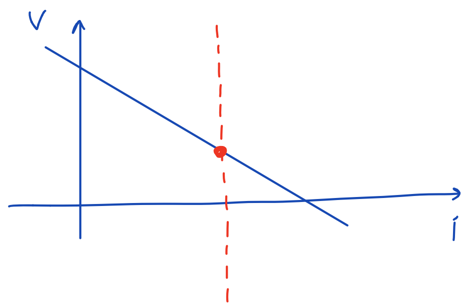
$i_{sc} = I_N$ (short circuit current is I_N)

So,

$$V_T = V_{oc}$$
$$I_N = i_{sc}$$
$$R_N = R_T = \frac{V_{oc}}{i_{sc}} = \frac{V_T}{I_N}$$

Proof of Thévenin equivalent circuit :





* Replace N by a current source (this is called substitution).
  This circuit is uniquely solvable.
  Apply superposition :

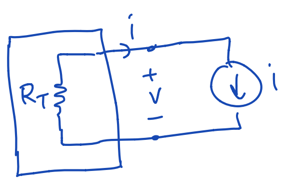
$$v = H_0 . i + \underbrace{K_1 v_{S1} + \dots + K_n . v_{Sn} + H_1 . i_{S1} + \dots + H_m . i_{Sm}}_{\text{contribution of sources inside } L}$$

Compare with : $v = -R_T . i + V_T$ (Thévenin eq. circuit)

* $H_0 = -R_T$, V_T = contribution of sources inside L

* How to find R_T : Set $V_T = 0 \Rightarrow$ all sources inside L set to zero.

Then, $v = H_0 . i \Rightarrow H_0 = -R_T = \frac{v}{i}$

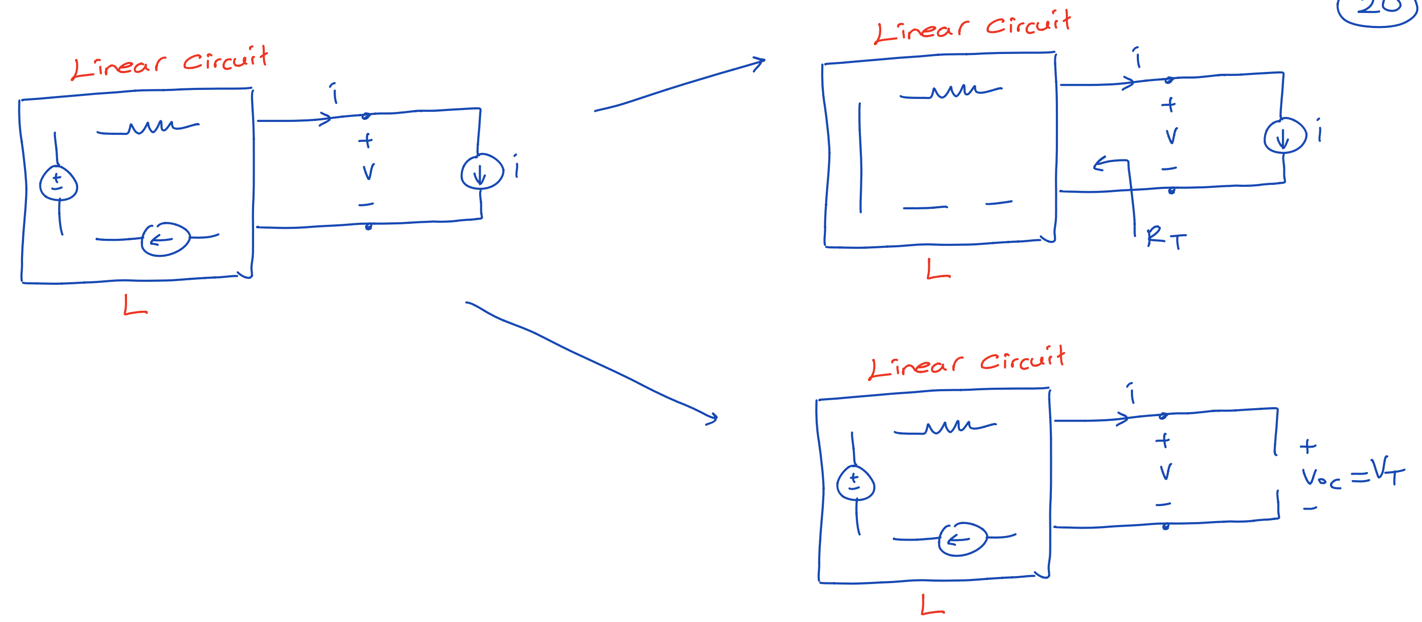


$$\left.\begin{array}{l} v = H_0 . i \\ v = -R_T . i \end{array}\right\} \quad H_0 = -R_T$$

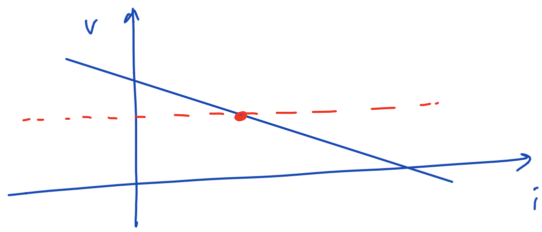
* How to find V_T : Set $i = 0 \Rightarrow$ set right hand side to OPEN CIRCUT.

Then, $V_T = V_{oc}$

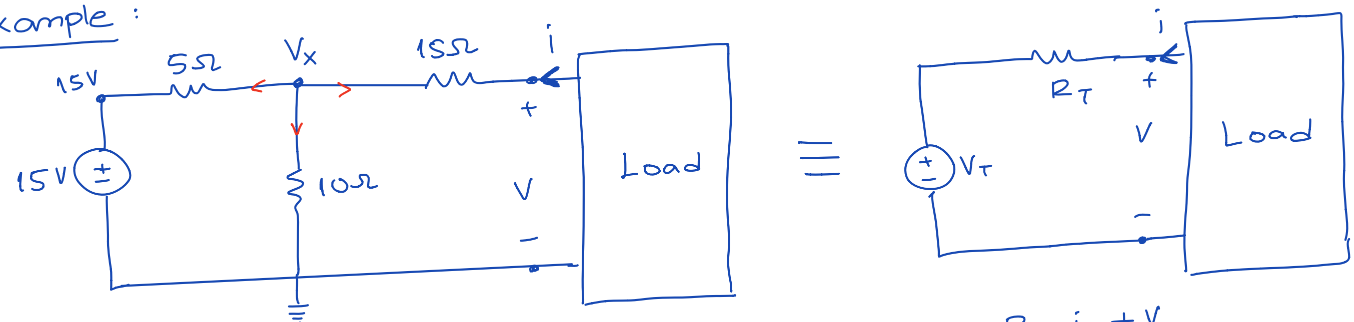
* Hence, Thévenin equivalent circuit is a direct consequence of superposition and unique solvability.



* It can be shown similarly that Norton eq. circuit is also a direct consequence of superposition.
the right-hand side, (connect a voltage source V to the right-hand side, in place of N).



Example :



$V = R_T . i + V_T$

Direct way :

KCL at X :
$$\underset{(6)}{\frac{V_x - 15}{5}} + \underset{(2)}{\frac{V_x - V}{15}} + \underset{(3)}{\frac{V_x}{10}} = 0$$

$$11 V_x = 90 + 2V \quad \Rightarrow \quad V_x = \frac{2V + 90}{11}$$

* Also : $V = 15 . i + V_x$

$$V = 15.i + \frac{2V + 90}{11}$$

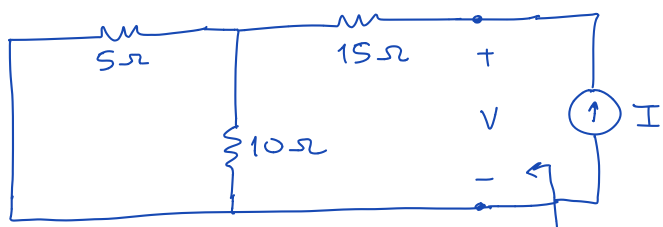
$$9V = 165.i + 90$$

$$\boxed{V = 18.3.i + 10} \Rightarrow \text{compare with } V = R_T.i + V_T$$

$$\Rightarrow \boxed{R_T = 18.3\,\Omega \quad , \quad V_T = 10\text{ V}}$$

Indirect way :

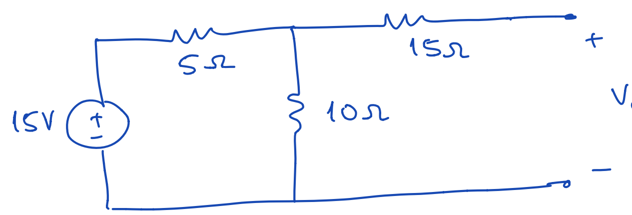
* To find R_T : Kill all sources,



$R_T = \frac{V}{I}$

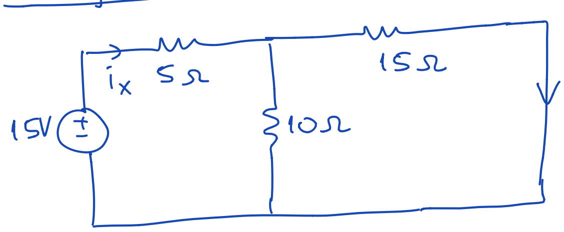
$$R_T = (5\,\Omega // 10\,\Omega) + 15\,\Omega$$
$$= 18.3\,\Omega$$

* To find V_T : measure open circuit voltage



$$V_{oc} = V_T = \frac{10}{5+10} . 15V = 10\text{ V}$$

* To find I_N : measure short circuit current

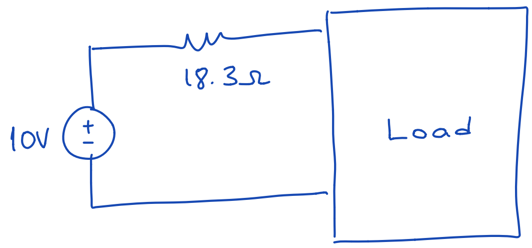


$i_{sc} = I_N$

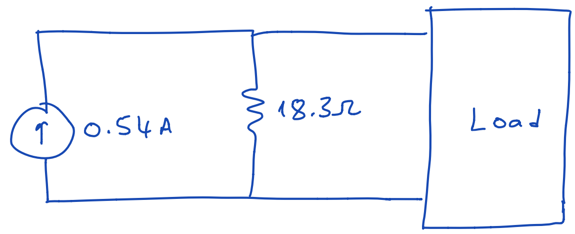
$$i_x = \frac{15\text{ V}}{5 + (10//15)} = \frac{15}{11}\text{ A} \approx 1.36\text{ A}$$

$$i_{sc} = I_N = \frac{10}{10+15} . i_x = \frac{10}{25} . \frac{15}{11} \approx 0.54\text{ A}$$

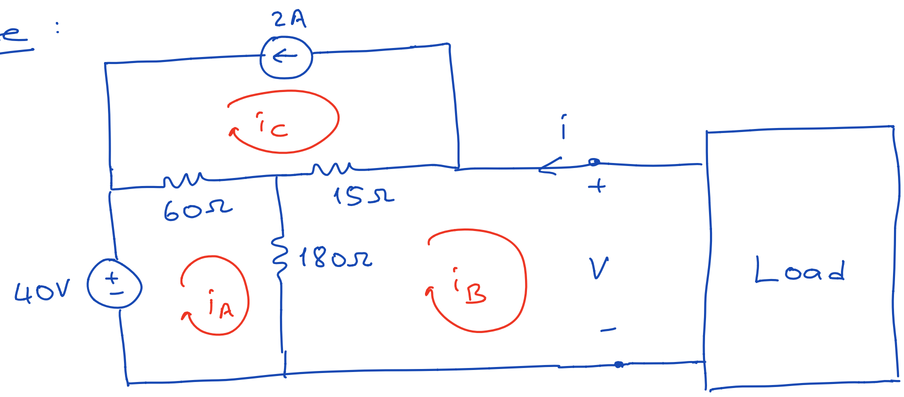
Doudle check : $R_T = \frac{V_T}{I_N} = \frac{10V}{0.54A} \approx 18.3\,\Omega$



OR



Example :



Direct Way :

$i_c = -2\,A$

$i = -i_B$

mesh A : $60(i_A - i_c) + 180(i_A - i_B) - 40 = 0$

$240\, i_A - 180\, i_B = 60\, i_c + 40 = -80 \quad \ldots \text{(Eqn.1)}$

mesh B : $15\,(i_B - i_c) + V + 180(i_B - i_A) = 0$

$-180\, i_A + 195\, i_B = 15\, i_c - V = -30 - V \quad \ldots \text{(Eqn.2)}$

$3 \times$ Eqn.1 : $720\, i_A - 540\, i_B = -240$

$4 \times$ Eqn 2 : $-720\, i_A + 780\, i_B = -120 - 4V$

+

$240\, i_B = -360 - 4V$

$i = -i_B \implies -240\, i = -360 - 4V$

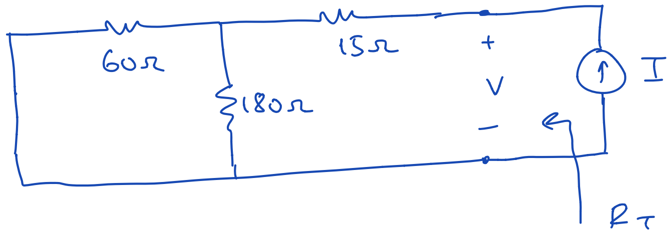
$$\boxed{V = 60\, i - 90}$$

Compare with : $V = R_T . i + V_T$

$$\boxed{R_T = 60\Omega \,,\quad V_T = -90V}$$

Indirect way:

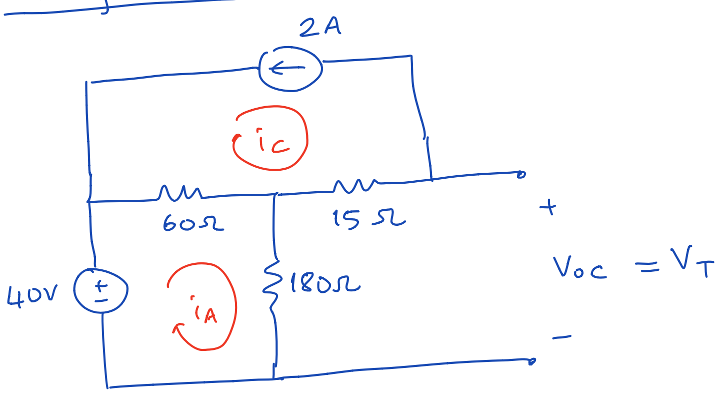
* To find R_T : kill all sources



$R_T = \frac{V}{I}$

$$R_T = (60 // 180) + 15 = 45 + 15 = 60\,\Omega$$

* To find V_T : open circuit voltage



$$i_c = -2\,A$$

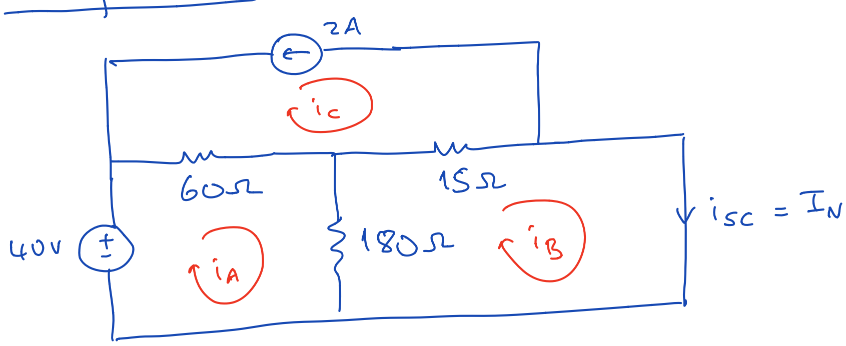
mesh A : $60(i_A - i_c) + 180 \cdot i_A - 40 = 0$

$$240\, i_A = 60\, i_c + 40 = -80$$

$$i_A = -\frac{1}{3}\,A$$

Then, $V_T = 15 \cdot i_c + 180 \cdot i_A = -30 - 60 = -90\,V$

* To find I_N : short circuit current



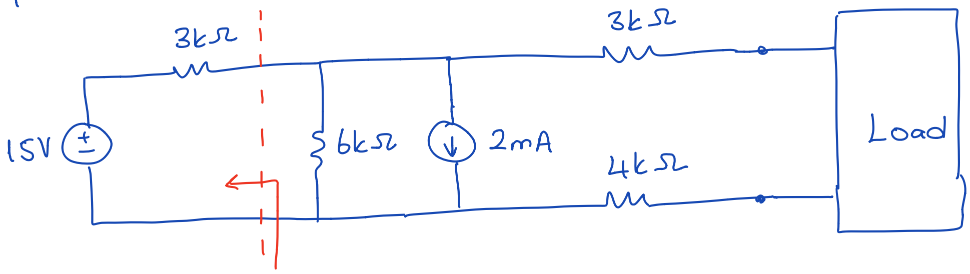
$I_N = I_B$

Do mesh analysis. Then, $I_N = I_B$.

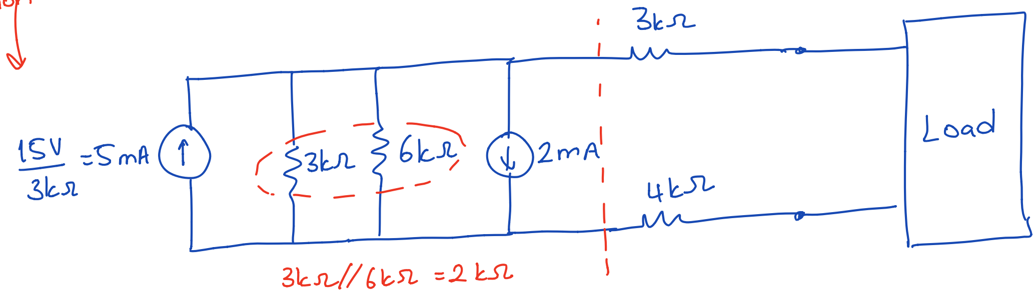
(exercise).

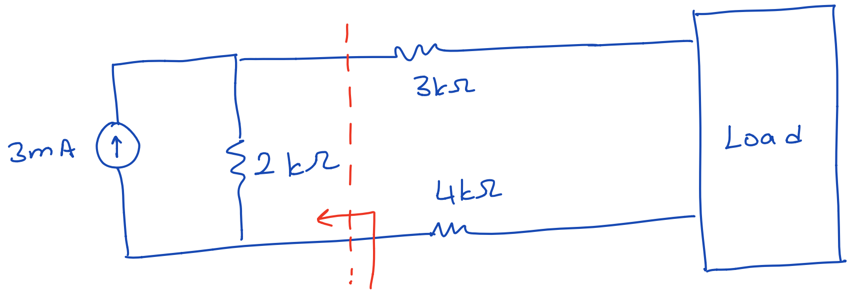
$$I_N = \frac{V_T}{R_T} = \frac{-90}{60} = -1.5\,A$$

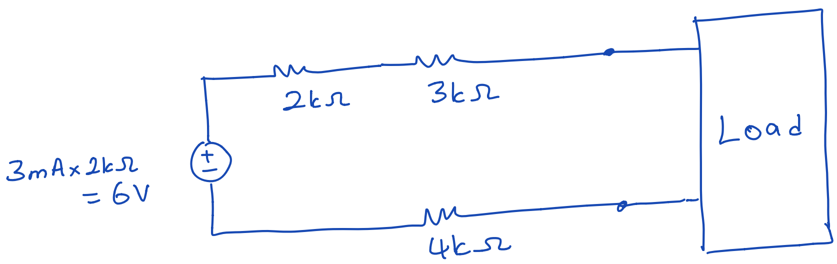
Example : By reduction

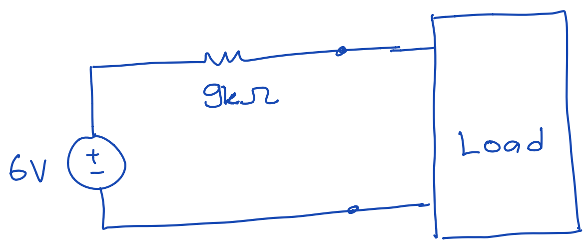


source transformation





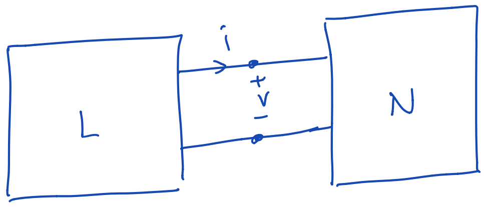


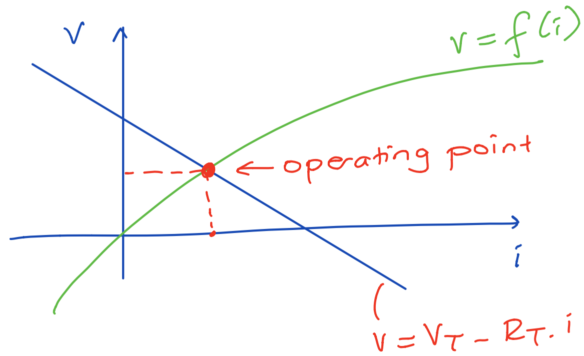


$V_T = 6V$, $R_T = 9k\Omega$, $I_N = \frac{6V}{9k\Omega} \simeq 0.67\ mA$

Typical Applications :

1) when N is a nonlinear device (e.g., a diode)





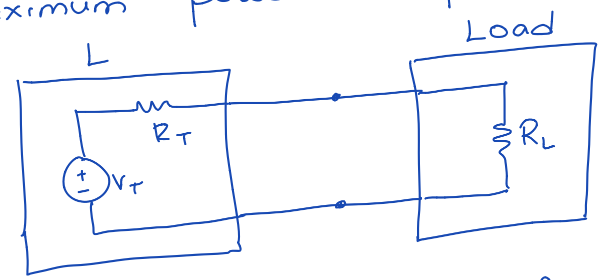
L : $v = V_T - R_T \cdot i$

N : $v = f(i)$

nonlinear equation : finding the solution may be difficult.

Thévenin / Norton equivalent circuits make it easie to find this solution.

2) maximum power transfer



maximum power is transfered to the load when $R_L = R_T$.

$\Longrightarrow$ called impedance matching