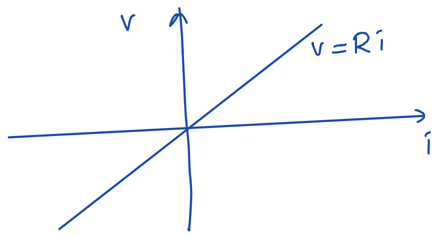
CHAPTER 4: ACTIVE CIRCUITS

* A circuit element is called passive if for all possible cases we have $p = v.i \geq 0$. Otherwise, it is called an active element.

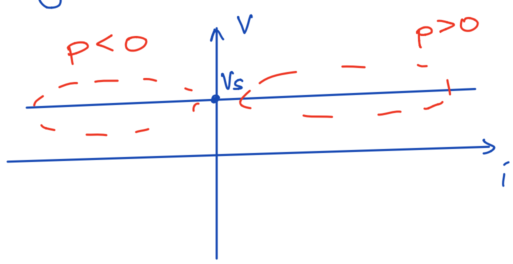
* Hence, an active element satisfies $p < 0$ for some cases (not necessarily for all possible cases.

* Typical passive element: Linear resistor with $R \geq 0$



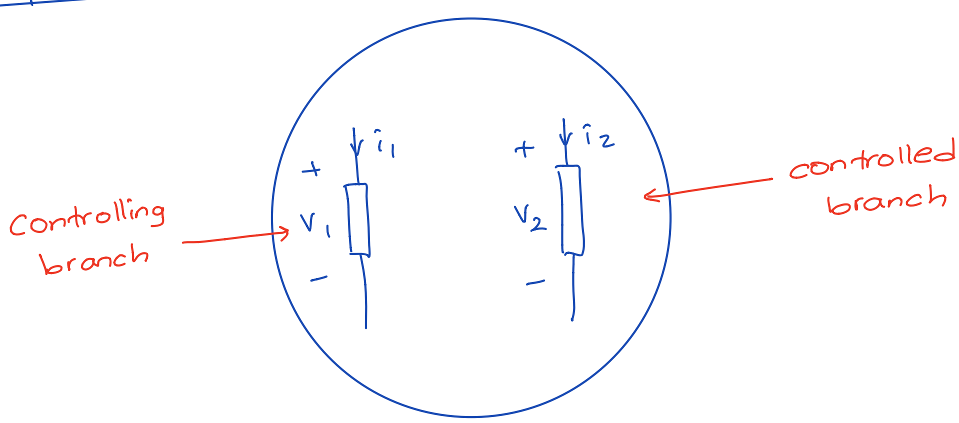
$p = v.i = Ri^2 \geq 0$

* Typical active element: independent sources



⇐ voltage source example

Dependent Sources:



* A branch voltage/current (v_2, i_2) depends on another branch voltage/current (v_1, i_1).

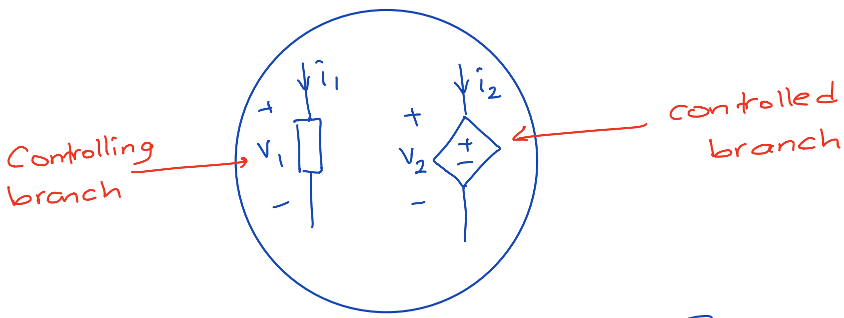
* mathematically, this can be expressed in matrix form as:

$$\begin{bmatrix} V_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ i_1 \end{bmatrix}$$

* In this class, we will represent dependent sources using a diamond symbol.

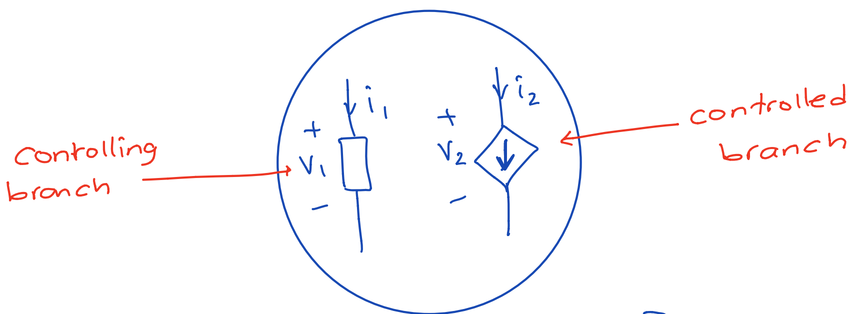
There are 4 possibilites:

1) Current controlled voltage source (CCVS)



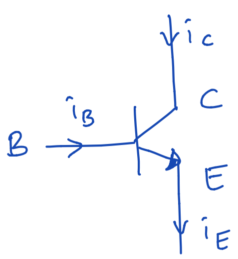
Controlling variable : i_1
Controlled variable : V_2 $\Big\}$ $V_2 = r . i_1$

2) Current Controlled Current Source (CCCS)



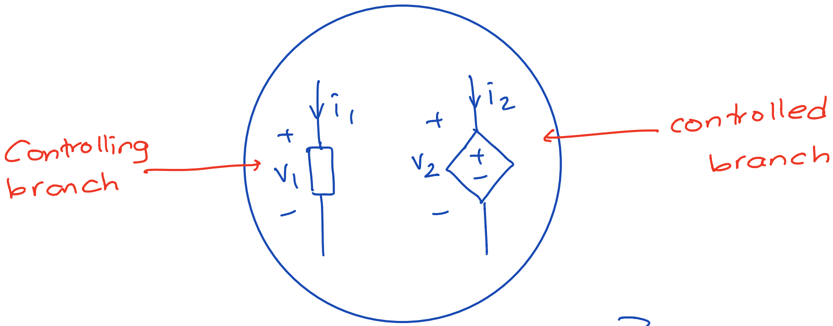
Controlling variable : i_1
Controlled variable : i_2 $\Big\}$ $i_2 = \beta . i_1$

Example : BJT transistors



$i_C = \beta . i_B$

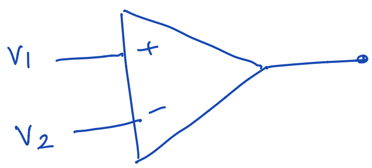
3) Voltage Controlled Voltage Source (VCVS)



Controlling variable : v_1
Controlled variable : v_2

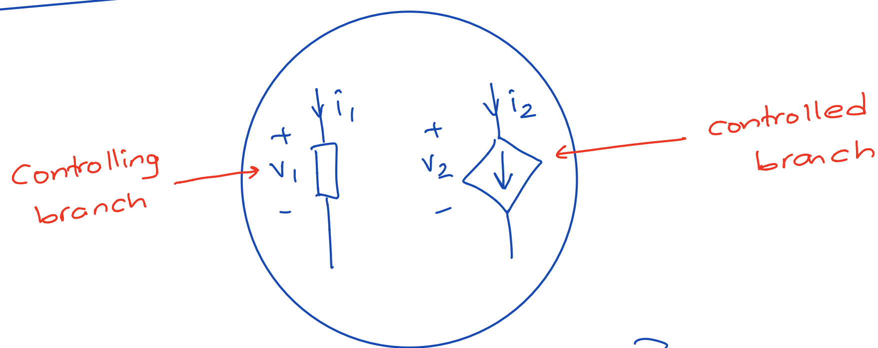
$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \quad v_2 = \mu \cdot v_1$$

Example : OPAMP



$V_{out} = A(v_1 - v_2)$

4) Voltage Controlled Current Source (VCCS)



Controlling variable : v_1
Controlled variable : i_2

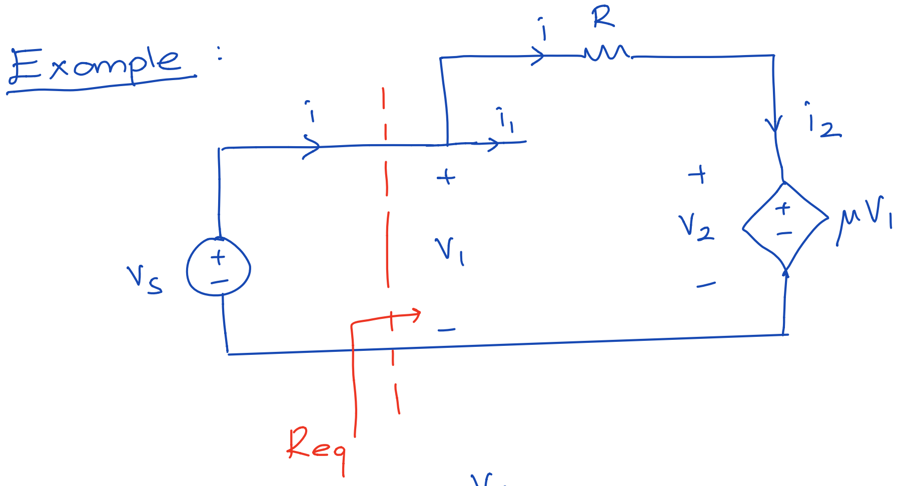
$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \quad i_2 = g \cdot v_1$$

* KCL an KVL can be written the same way.
  For element relations : $Mv + Ni = u$, only the entries of M and/or N will change.

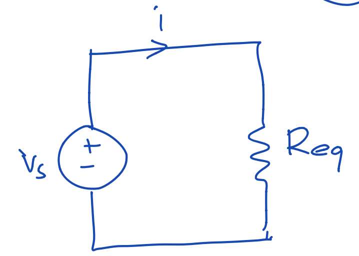
  $\Longrightarrow$ Linearity is preserved.

Consequence : Superposition, Thévenin / Norton equivalence still apply.

Example :



$\equiv$



$V_s = R_{eq} \cdot i$

* Find $R_{eq} = \frac{V_s}{i}$

KVL : $V_1 = R \cdot i + \mu V_1 \Rightarrow (1-\mu) V_1 = R \cdot i$

Also : $V_1 = V_s$

so, $(1-\mu) V_s = R \cdot i$

$$R_{eq} = \frac{V_s}{i} = \frac{R}{1-\mu}$$

Note : if $\mu > 1 \Rightarrow R_{eq} < 0 \Rightarrow$ negative resistance !!

* $P_2 = V_2 \cdot i_2 = \mu V_s \cdot i = \mu \cdot V_s \cdot \frac{(1-\mu) \cdot V_s}{R}$

$$= \frac{\mu (1-\mu) V_s^2}{R}$$

$P_2 < 0$ when $\mu > 1$

(Note that $P_2 > 0$ when $0 < \mu < 1$)

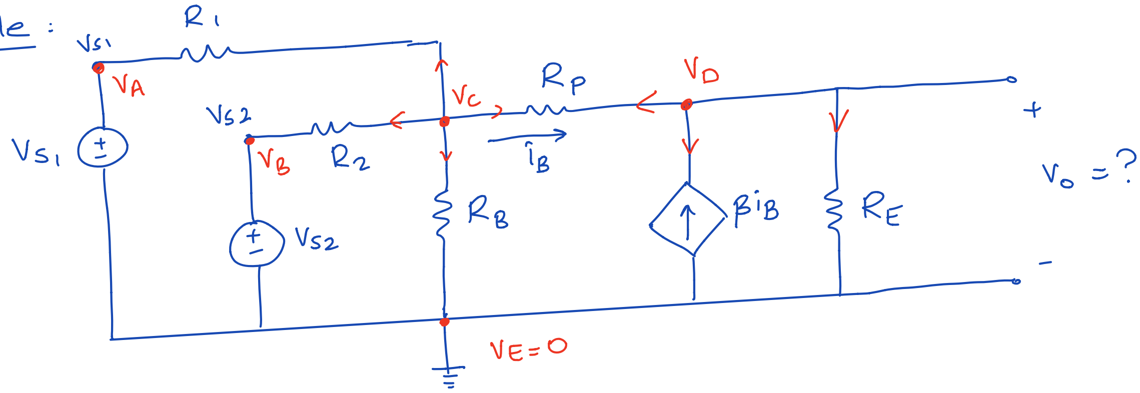
Node Analysis with Dependent Sources

Basic steps are the same as before.

Modification : Add element relations for dependent sources to the list of KCL equations.

Result : We will have $(n-1+x)$ equations in terms of $(n-1)$ node voltages + x new equations for dependent sources.

Example :



$V_A = V_{S1}$

$V_B = V_{S2}$

KCL at C : $G_1(V_C - V_A) + G_2(V_C - V_B) + G_B \cdot V_C + G_P \cdot (V_C - V_D) = 0$

$$(G_1 + G_2 + G_B + G_P)V_C - G_P V_D = G_1 \cdot V_{S1} + G_2 \cdot V_{S2}$$

KCL at D : $G_P(V_D - V_C) - \beta \cdot i_B + G_E \cdot V_D = 0$

Extra equation : $i_B = G_P \cdot (V_C - V_D)$

Insert into second equation :

$$G_P(V_D - V_C) - \beta \cdot G_P (V_C - V_D) + G_E \cdot V_D = 0$$

$$-G_P(1+\beta)V_C + \left[G_P(1+\beta) + G_E\right] \cdot V_D = 0$$

write in matrix form :

$$\begin{bmatrix} (G_1+G_2+G_B+G_P) & -G_P \\ -G_P(1+\beta) & G_P(1+\beta)+G_E \end{bmatrix} \begin{bmatrix} V_C \\ V_D \end{bmatrix} = \begin{bmatrix} G_1 \cdot V_{S1} + G_2 \cdot V_{S2} \\ 0 \end{bmatrix}$$

* Note that the symmetry of the conductance matrix is broken when there are dependent sources.

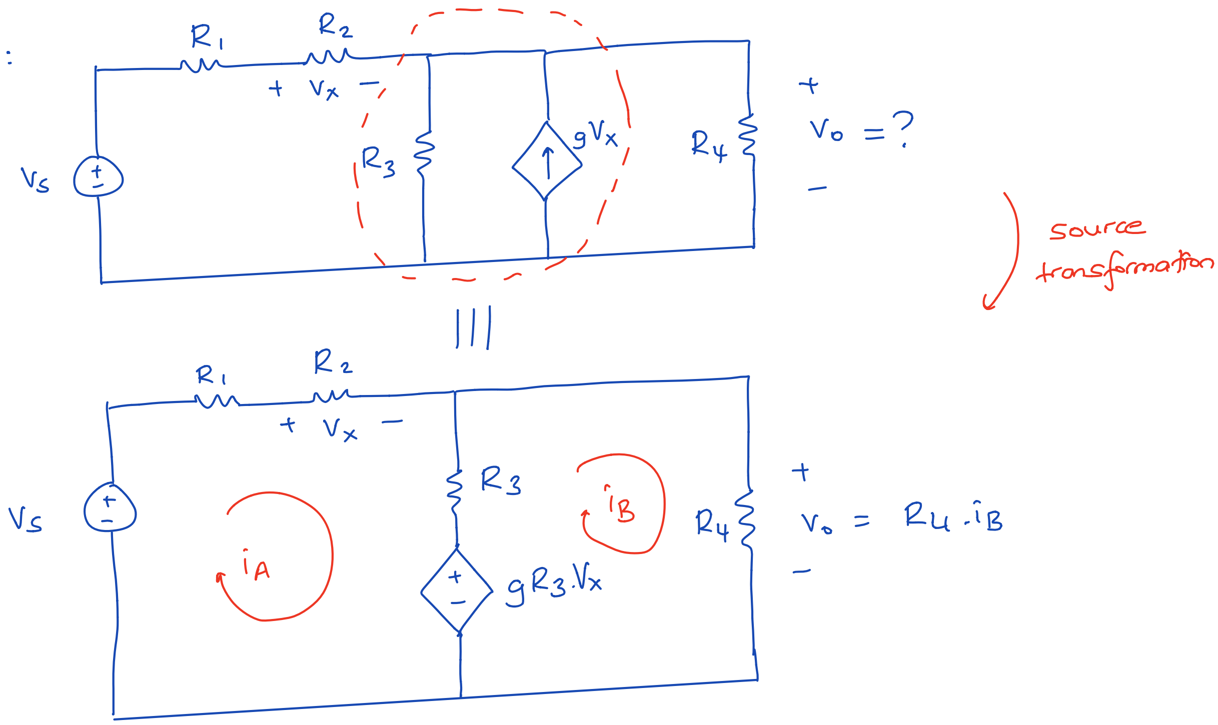
Mesh Analysis with Dependent Sources

Basic steps are the same as before

Modification : Add the element relations for the dependent sources to the list of KVL equations.

Result : We will have $(b-n+1+y)$ equations in terms of $(b-n+1)$ mesh currents + y new equations for dependent sources.

Example :



After reduction :

Mesh A : $R_1 \cdot i_A + R_2 \cdot i_A + R_3(i_A - i_B) + gR_3 V_x - V_s = 0$

$$(R_1 + R_2 + R_3) i_A - R_3 i_B + g R_3 V_x = V_s$$

Mesh B : $R_4 \cdot i_B - g R_3 V_x + R_3 (i_B - i_A) = 0$

$$-R_3 \cdot i_A + (R_3 + R_4) i_B - g R_3 V_x = 0$$

Extra equation : $V_x = R_2 \cdot i_A$

Re-write : Mesh A : $(R_1 + R_2 + R_3 + gR_2R_3) i_A - R_3 i_B = V_s$

Mesh B : $-(R_3 + gR_2R_3) i_A + (R_3 + R_4) i_B = 0$

Write in matrix form:

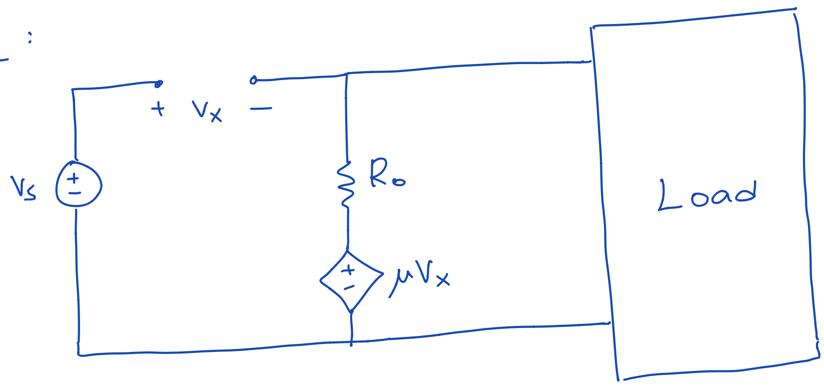
$$\begin{bmatrix} R_1+R_2+R_3+gR_2R_3 & -R_3 \\ -(R_3+gR_2R_3) & (R_3+R_4) \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix}$$

* Note that the symmetry of the resistance matrix is broken when there are dependent sources.

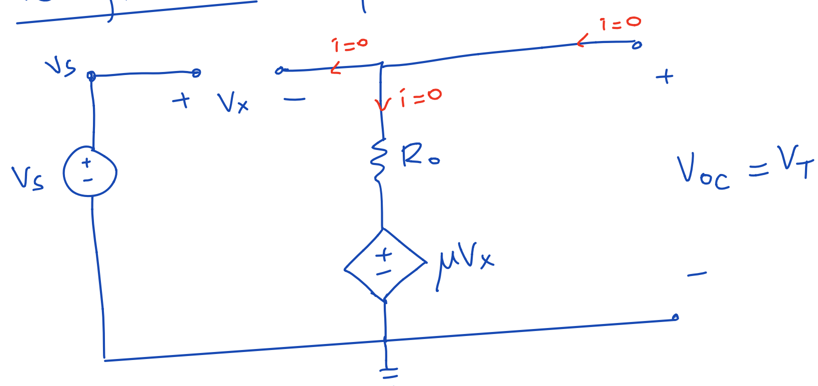
Superposition, Thévenin, Norton with Dependent Sources

* Same as before, but keep in mind that dependent sources should be treated as ordinary circuit elements (e.g., like resistors), not as independent sources.

Example:



* To find V_T: open circuit voltage



From KVL:

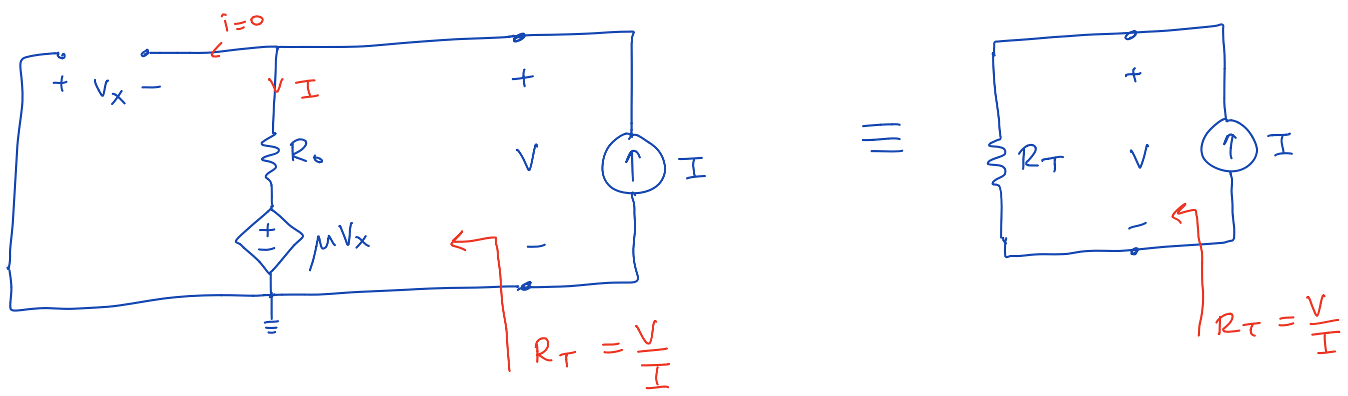
$$\left.\begin{aligned} V_T &= \mu . V_x \\ V_x &= V_s - V_T \end{aligned}\right\}$$

$$V_T = \mu\,(V_s - V_T)$$

$$(1+\mu)\,V_T = \mu . V_s$$

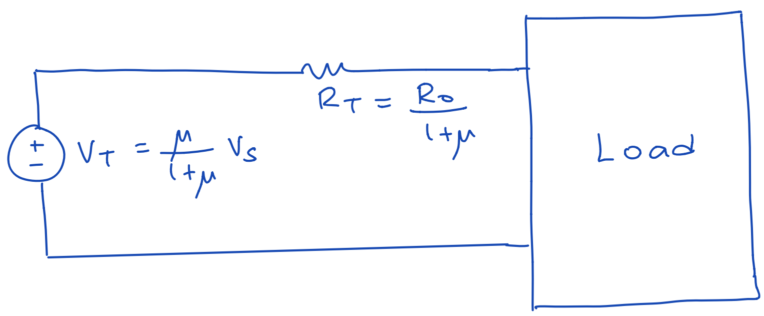
$$V_T = \frac{\mu}{1+\mu} . V_s$$

* To find R_T : Kill all independent sources, but keep dependent source as it is.

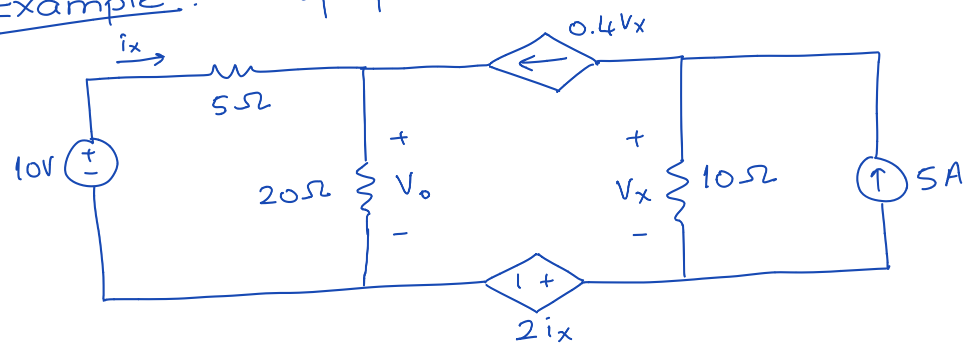


KVL : $V = R_o . I + \mu V_x$
$V_x = -V$

$V = R_o . I - \mu . V$
$(1+\mu) V = R_o . I$
$R_T = \frac{V}{I} = \frac{R_o}{1+\mu}$

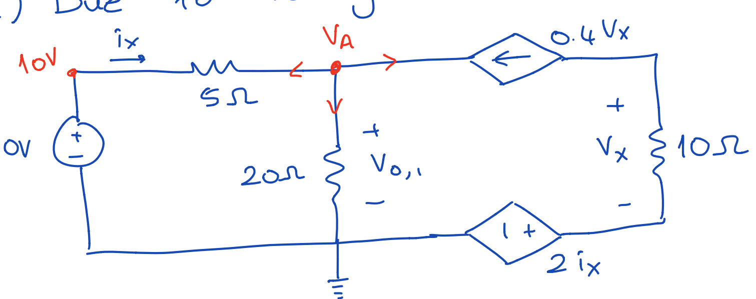


Example : Supeposition with dependent sources



$V_o = ?$

1) Due to voltage source :



KCL at A : $\frac{V_A - 10}{5} + \frac{V_A}{20} - 0.4V_x = 0$

(4) (20)

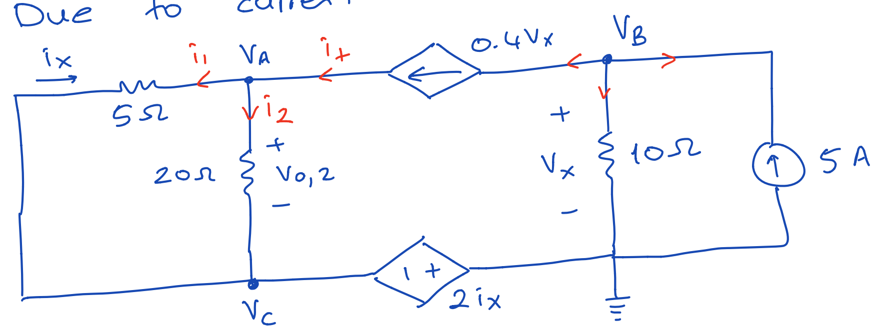
$5V_A = 40 + 8V_x$

Additional eq. : $V_x = 10 \cdot (-0.4V_x) = -4V_x$

$5V_x = 0 \Rightarrow V_x = 0$

Then, $5V_A = 40 \Rightarrow \boxed{V_A = V_{o,1} = 8V}$

2) Due to current source :



KCL at B : $0.4V_x + \frac{V_B}{10} - 5 = 0$

(10) (10)

$V_B = 50 - 4V_x$

Additional Eq. : $V_x = V_B$

$V_B = 50 - 4V_B$

$5V_B = 50$

$V_B = 10V$

* $i_t = 0.4V_x = 0.4V_B = 4A$

$i_2 = \frac{5}{5+20} \cdot 4A = 0.8A$ (current divider)

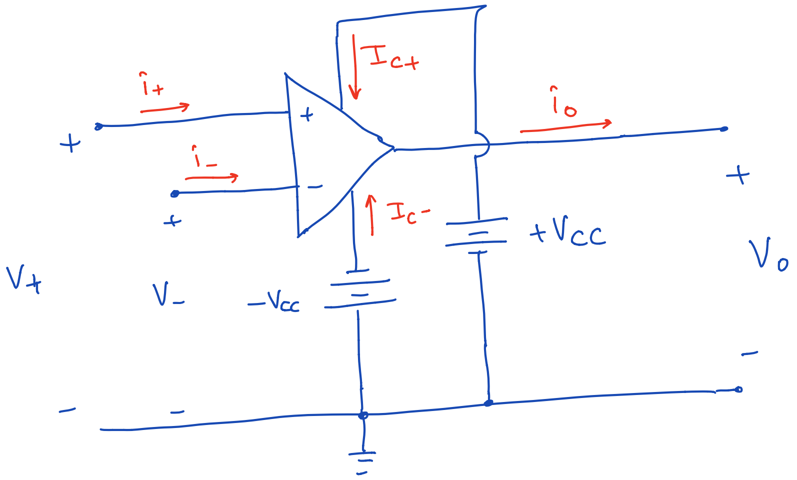
Then, $\boxed{V_{o,2} = 20 \cdot i_2 = 16V}$

* Finally, $V_o = V_{o,1} + V_{o,2} = 8 + 16 = \boxed{24V}$

OPERATIONAL AMPLIFIERS (OPAMPs)

* OPAMP is an integrated circuit, which is an important building block in many electronic circuits.

* To operate the OPAMP, we need to connect +Vcc and −Vcc DC sources to appropriate terminals. Including these connections, an OPAMP has 5 terminals.

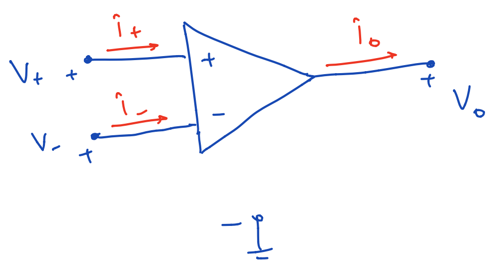


KCL for OPAMP:

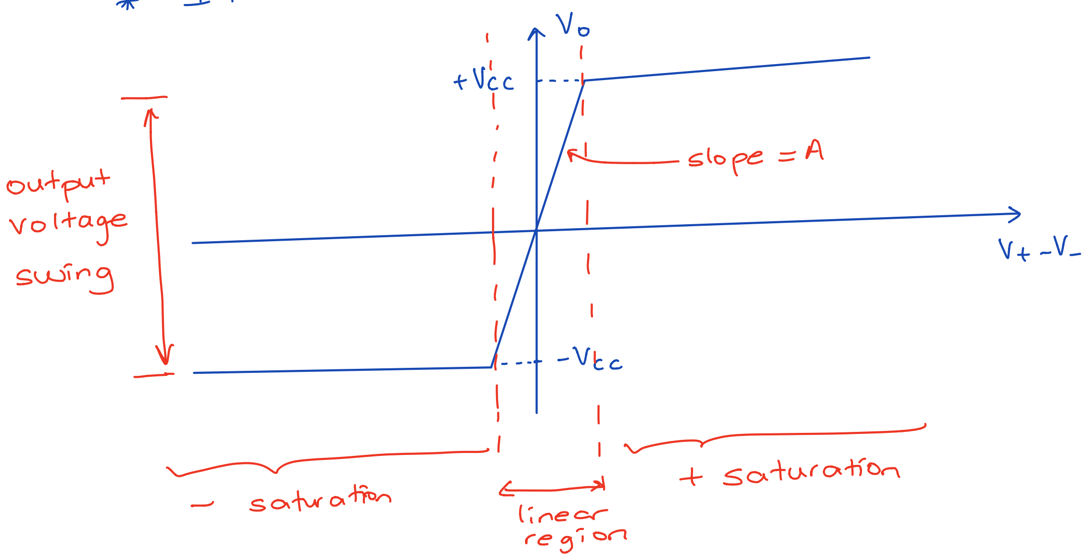
$$i_o = I_{C+} + I_{C-} + \underbrace{i_+}_{\approx 0} + \underbrace{i_-}_{\approx 0}$$

So, output current comes from the supply voltages.

* OPAMP is usually drawn as a 3-terminal device, omitting supplies.



* It turns out, v_o depends on $V_d = V_+ - V_-$.



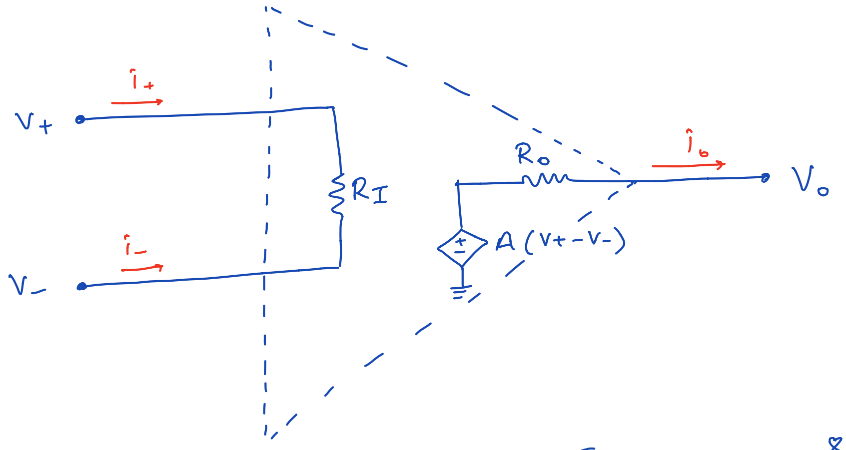
There are 3 operating modes for an OPAMP:

* $+$ Saturation : $V_o = +V_{cc}$, when $A(V_+ - V_-) > V_{cc}$
* Linear Region : $V_o = A(V_+ - V_-)$, when $-V_{cc} < V_o < V_{cc}$
* $-$ Saturation : $V_o = -V_{cc}$, when $A(V_+ - V_-) < -V_{cc}$

For each mode, we can represent/model the OPAMP by using the elements we have seen so far.

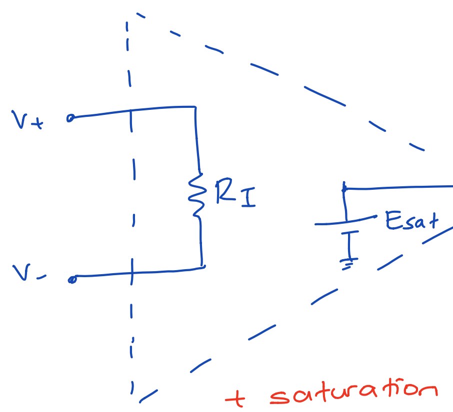
Finite Gain model :

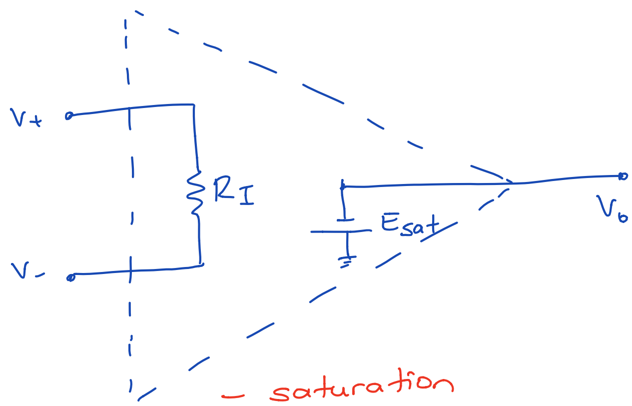
* In Linear Region :



* A : finite gain $\Rightarrow$ $10^5 < A < 10^8$ (very large)
* R_I : input resistance $\Rightarrow$ $10^6 < R_I < 10^{12}\ \Omega$ (very large)
* R_o : output resistance $\Rightarrow$ $10 < R_o < 100\ \Omega$ (small)

* In $\pm$ Saturation : Output behaves like a $\pm E_{sat}$ DC source.





Ideal OPAMP model : a.k.a. infinite gain model

Can be obtained from the finite gain model by taking the limits :

$R_I \longrightarrow \infty$, $R_o \longrightarrow 0$, $A \longrightarrow \infty$

Logical conclusions :

* $R_I \longrightarrow \infty \implies i_+ = i_- = 0$ (R_I is open circuit)

Then, there are 3 operating modes :

* + Saturation : $V_o = +V_{CC}$, when $(V_+ - V_-) > \frac{V_{CC}}{A} \longrightarrow 0$

i.e., when $(V_+ - V_-) > 0$

* Linear Region : $V_o = A(V_+ - V_-) \implies V_+ - V_- = \frac{V_o}{A} \longrightarrow 0$

$\boxed{V_+ - V_- = 0}$

when $-V_{CC} < V_o < V_{CC}$

* − Saturation : $V_o = -V_{CC}$, when $(V_+ - V_-) < \frac{-V_{CC}}{A} \longrightarrow 0$

i.e., when $(V_+ - V_-) < 0$

To summarize , in ideal OPAMP model :

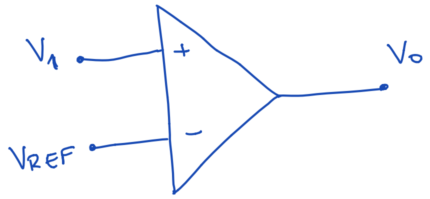
$i_+ = i_- = 0$ (in all regions)

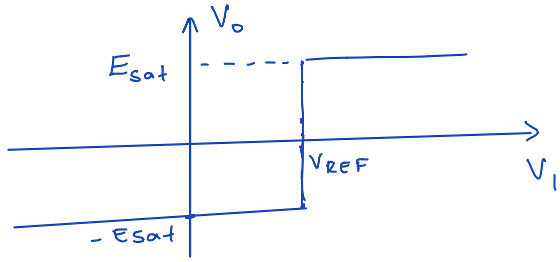
* Linear region : $V_+ = V_-$, when $-V_{CC} < V_o < V_{CC}$
* + Saturation : $V_o = +V_{CC}$, when $(V_+ - V_-) > 0$
* − Saturation : $V_o : -V_{CC}$, when $(V_+ - V_-) < 0$

* In most applications, OP AMP is used in the linear region.

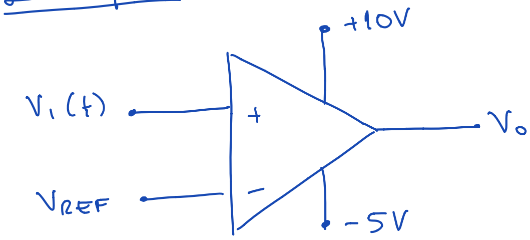
* We will use $\pm$ Esat for saturation voltage instead of V_{CC}.
Typically Esat $< V_{CC}$ (slightly).

Example : OPAMP in saturation mode : comparator



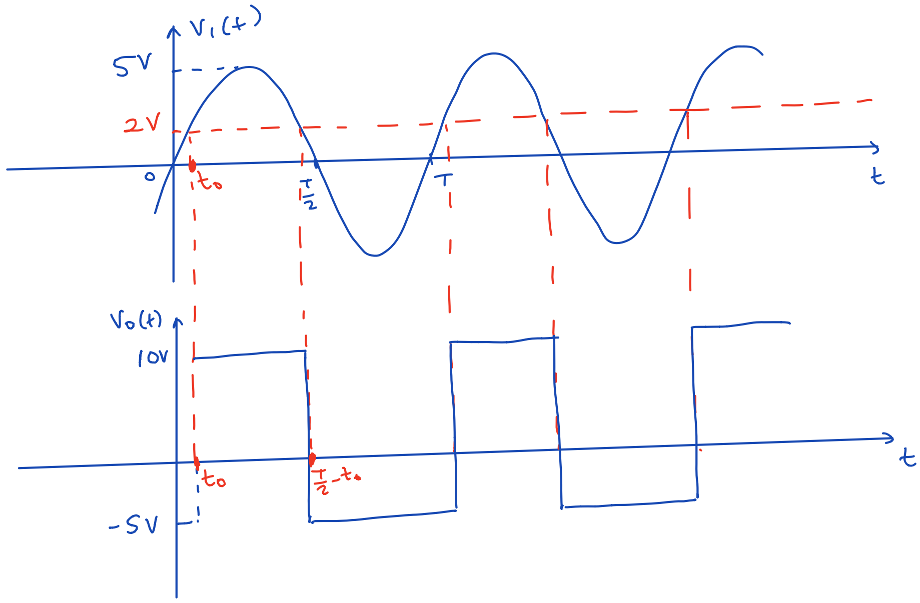


Example :



$V_1(t) = 5 \cdot \sin(2\pi f t)$ V

$V_{REF} = 2$ V

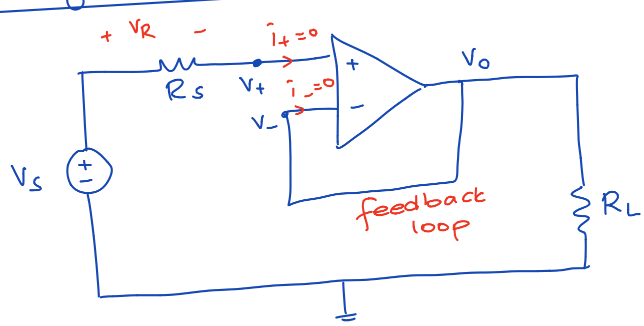


$5 \sin(2\pi f_o t_o) = 2$

$2\pi f_o t_o = \text{asin}(2/5)$

$t_o = \dfrac{1}{2\pi f_o} \text{asin}(2/5)$

Voltage Follower : Buffer



$V_R = 0$

$V_+ = V_s$ (KVL)

$V_- = V_o$

In linear region : $V_+ = V_-$ $\Rightarrow$ $\boxed{V_o = V_s}$, if $-E_{sat} < V_o < E_{sat}$

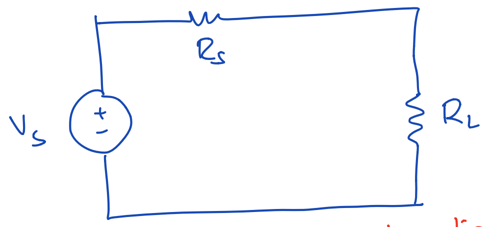
$\Rightarrow$ $-E_{sat} < V_s < E_{sat}$

to be in linear region

$\Rightarrow$ Prevents loading the source. We use the voltage source V_s without drawing any power from it.

Application of Buffer:

Compare with :

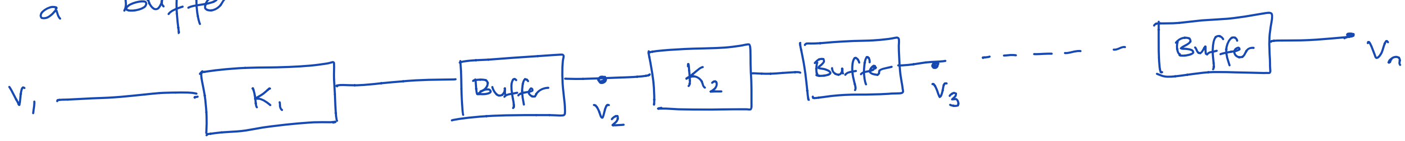


$$V_o = \frac{R_L}{R_L + R_s} \cdot V_s$$

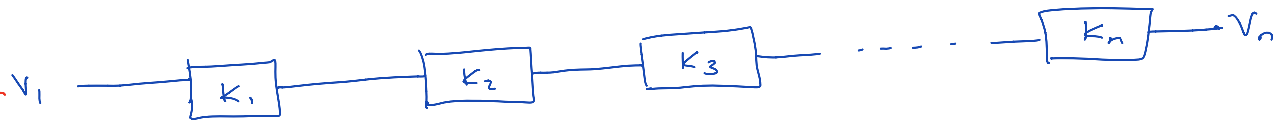
there is loading effect here.

Altenative interpretation: in the linear region, independent of anything else, we can use this circuit as a VCVS with unit gain.

* we can isolate building blocks from each other by putting a buffer in between.



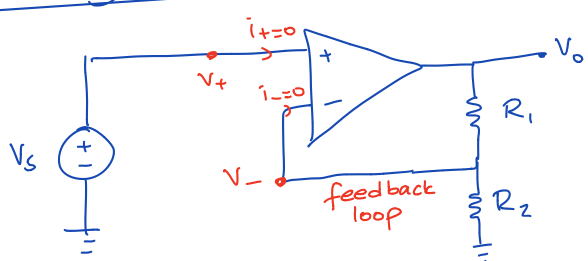
V.S.



→ we have $V_n = K_n K_{n-1} \dots K_1 . V_1$

→ we have $V_n \neq K_n K_{n-1} \dots K_1 . V_1$

Noninverting amplifier:



$V_+ = V_s$

$V_- = \frac{R_2}{R_1 + R_2} \cdot V_o$ (voltage divide)

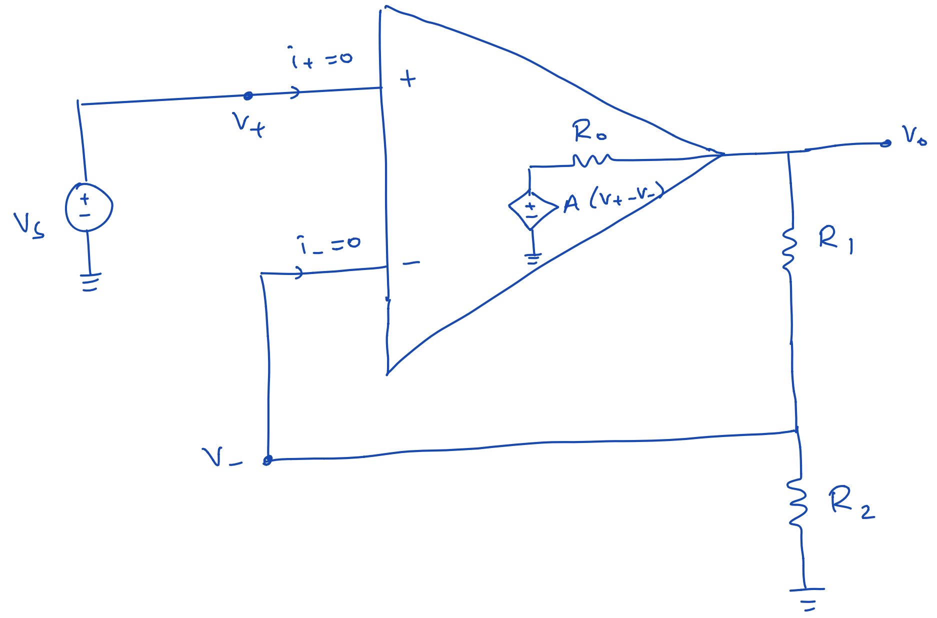
In linear region: $V_+ = V_-$

$$V_s = \frac{R_2}{R_1 + R_2} \cdot V_o \Rightarrow \boxed{V_o = \underbrace{\left(\frac{R_1 + R_2}{R_2}\right)}_{\text{"closed-loop" gain}} V_s}$$

if $-E_{sat} < V_o < E_{sat}$

$$-\frac{R_2}{R_1 + R_2} \cdot E_{sat} < V_s < \frac{R_2}{R_1 + R_2} \cdot E_{sat}$$

Effects of Finite OPAMP Gain (finite A):



$R_I \sim 10^6\,\Omega - 10^{12}\,\Omega \Rightarrow$ ignore it (OPEN CIRCUIT)

Then,

$$V_+ = V_s$$

$$V_- = \frac{R_2}{R_1+R_2} \cdot V_o$$

$$V_o = \frac{R_1+R_2}{R_o+R_1+R_2} \cdot A\,(V_+ - V_-)$$

So,

$$V_o = \frac{R_1+R_2}{R_o+R_1+R_2} \cdot A\left(V_s - \frac{R_2}{R_1+R_2} \cdot V_o\right)$$

$$V_o = \frac{(R_1+R_2) \cdot A\, V_s}{R_o+R_1+R_2} - \frac{R_2 \cdot A}{R_o+R_1+R_2} \cdot V_o$$

$$(R_o+R_1+R_2+R_2 A)\,V_o = (R_1+R_2)\,A \cdot V_s$$

$$V_o = \frac{A\,(R_1+R_2)}{R_o+R_1+R_2\,(1+A)} \cdot V_s$$

* In the limit as $A \longrightarrow \infty$

$$V_o = \frac{R_1+R_2}{R_2} \cdot V_s = K \cdot V_s$$

closed-loop gain

* Re-write V_o for finite gain case:

$$V_o = \frac{A \cdot \frac{R_1+R_2}{R_2}}{\frac{R_o+R_1+R_2}{R_2} + A} \cdot V_s \approx \frac{A \cdot \frac{R_1+R_2}{R_2}}{\frac{R_1+R_2}{R_2} + A} \cdot V_s$$

(assuming R_o is sufficiently small)

$$V_o = \frac{A \cdot K}{K + A} \cdot V_s$$

$$\boxed{V_o = \frac{K}{1 + K/A} \cdot V_s}$$

* One practical rule of thumb is to limit K (closed-loop gain) to less than 1% of the OPAMP open loop gain (A).

i.e., $K < \frac{A}{100}$.

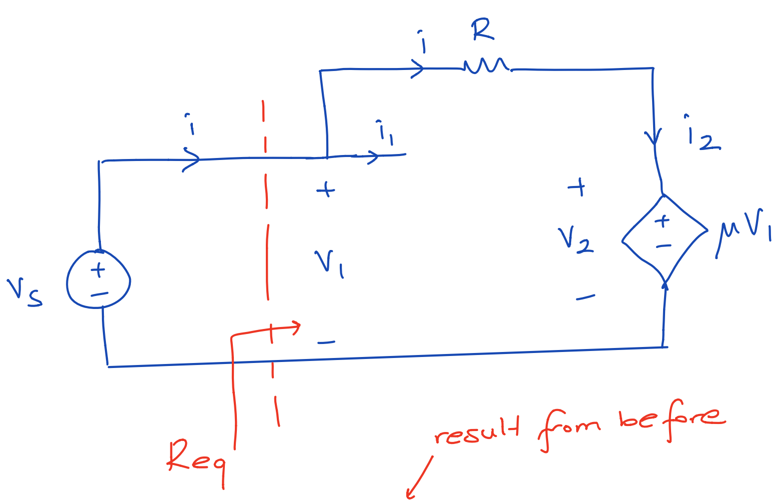
* Despite large variations in A, closed-loop gain remains constant as long as $K \ll A$. $\Longrightarrow$ importance of feedback loop !!

<u>Applications</u>:

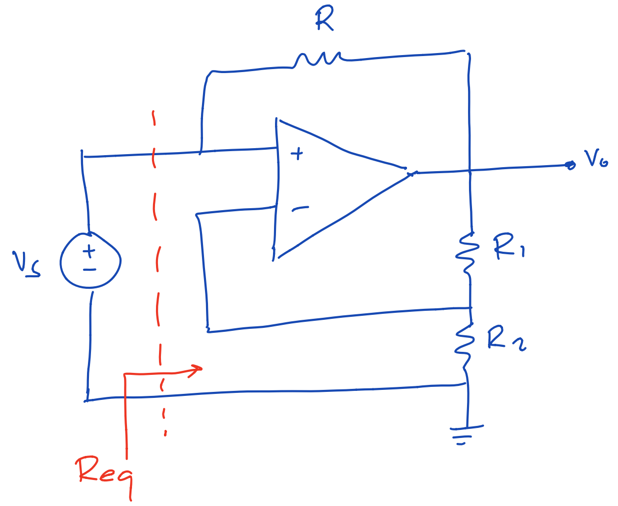
1) An amplifier with gain $\frac{R_1+R_2}{R_2}$

2) A VCVS with $\mu = \frac{R_1+R_2}{R_2} > 1$

3) Realization of a negative resistance.



$\Longleftrightarrow$



result from before

$$Req = \frac{R}{1-\mu}$$, for non-inverting OPAMP part, $\mu = \frac{R_1+R_2}{R_2}$

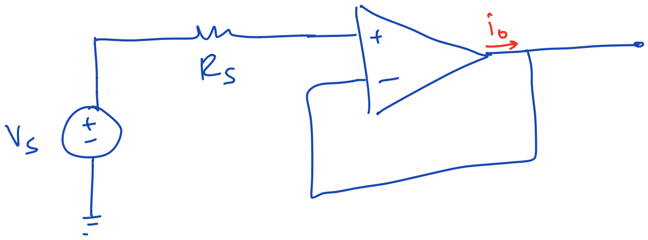
$$\Longrightarrow 1-\mu = -\frac{R_1}{R_2}$$

$$Req = -\frac{R \cdot R_2}{R_1} < 0$$

* used in oscillators and active filters

About the negative feedback loop :

* For stability, feedback loop should enter the negative input of the OPAMP



$$V_o = A\,(V_+ - V_-)$$ √ stable

non-inverting input

inverting input

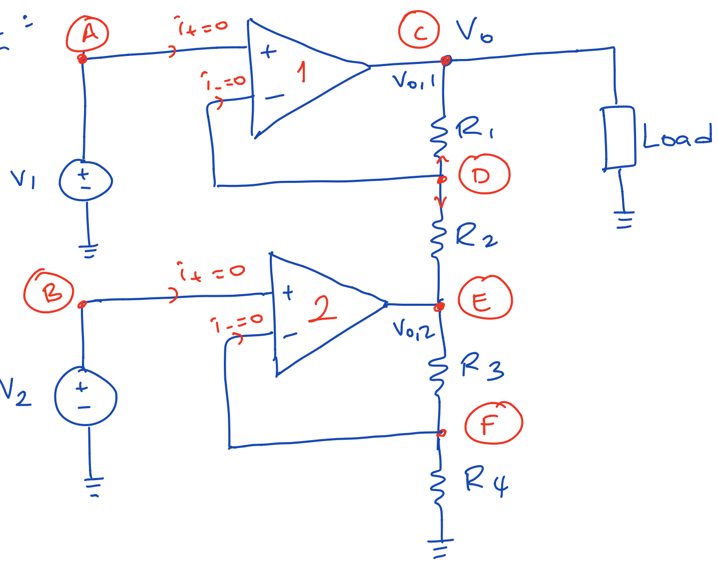
* If we use positive feedback : $V_o = A\,(V_+ - V_-)$ X not wanted

Using Node Equations for OP-AMP Circuits

*** write node equations at all non-reference nodes except for the OPAMP output node.
(Note that $i_o \neq 0$ in general)

* By using OPAMP equations, solve the node equations.

Example :



$V_A = V_1$, $V_B = V_2$

Do not write node equations at nodes C and E.

KCL at D : $\dfrac{V_D - V_o}{R_1} + \dfrac{V_D - V_E}{R_2} = 0$

$$(R_1 + R_2)\,V_D - R_2 V_o - R_1 V_E = 0$$

Assuming OPAMP 1 is in linear region : $V_A = V_D = V_1$

$$(R_1+R_2)V_1 - R_2 V_o - R_1 V_E = 0$$

KCL at F: $V_F = \dfrac{R_4}{R_3+R_4} V_E$

Assuming OPAMP 2 is in linear region: $V_B = V_F = V_2$

$$V_2 = \frac{R_4}{R_3+R_4} V_E \Rightarrow V_E = \frac{R_3+R_4}{R_4} V_2 = V_{o,2}$$

* Insert into first equation:

$$(R_1+R_2)V_1 - R_2 V_o - R_1 . \frac{(R_3+R_4)}{R_4} . V_2 = 0$$

$$\boxed{V_o = \frac{R_1+R_2}{R_2} . V_1 - \frac{R_1 . (R_3+R_4)}{R_2 . R_4} . V_2}$$

if both OPAMPs are in linear region.

* if we choose $R_1 = R_4 = R_A$
$R_2 = R_3 = R_B$

Then, $\boxed{V_o = \dfrac{R_A+R_B}{R_B} (V_1 - V_2)}$

This relation is valid when:

$-E_{sat} < V_{o,1} < E_{sat}$, $-E_{sat} < V_{o,2} < E_{sat}$

$$-\frac{R_B}{R_A+R_B} . E_{sat} < V_1 - V_2 < \frac{R_B}{R_A+R_B} . E_{sat}$$, for OPAMP 1 to be in linear region

$$-\frac{R_A}{R_A+R_B} E_{sat} < V_2 < \frac{R_A}{R_A+R_B} . E_{sat}$$, for OPAMP 2 to be in linear region

* Let's say $E_{sat} = 10V$, $-E_{sat} = -10V$ } for both OPAMPs

$R_1 = R_4 = R_A = 1\,k\Omega$
$R_2 = R_3 = R_B = 2\,k\Omega$

In linear region: $V_E = V_{o,2} = \dfrac{R_A+R_B}{R_A} . V_2 = 3 V_2$

$$V_o = V_{o,1} = \frac{R_A+R_B}{R_B} (V_1 - V_2) = \frac{3}{2} (V_1 - V_2)$$

Case 1 : if $V_1 = 2V$, $V_2 = 3V$

$V_E = V_{o,2} = 9V$ ✓ in linear region

$V_o = V_{o,1} = \frac{3}{2}(V_1 - V_2) = -\frac{3}{2} V$ ✓ in linear region

Case 2 : if $V_1 = 2V$, $V_2 = 4V$

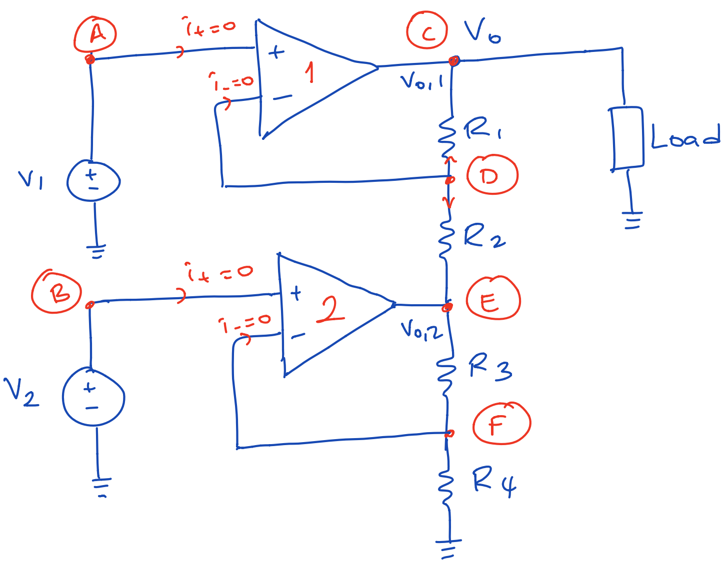
$V_E = V_{o,2} = 12 V$ ✗ cannot happen! OPAMP 2 is saturated

$$\boxed{V_E = V_{o,2} = 10 V}$$

Then, $V_o = V_{o,1} = \frac{3}{2}(V_1 - V_2)$ is no longer valid!!

Side note : $V_{+,2} = V_2 = 4V$
$V_{-,2} = V_F = \frac{R_4}{R_3+R_4} \cdot V_E = \frac{1}{3} \cdot 10 \approx 3.33 V$ } $V_{+2} > V_{-2}$ + saturation

Re-write KCL at D : Assuming that OPAMP 1 is not saturated : $V_o = V_A = V_1$



$$\frac{V_1 - V_o}{R_1} + \frac{V_1 - V_E}{R_2} = 0$$

$$R_2 V_o = (R_1 + R_2) V_1 - R_1 V_E$$

$$V_o = \left(\frac{R_1+R_2}{R_2}\right) V_1 - \frac{R_1}{R_2} \cdot V_E$$

$$= \frac{3}{2} V_1 - \frac{1}{2} V_E$$

$$= 3 - 5 = \boxed{-2V}$$

✓ in linear region

* In general :

- Assume a region of operation for each OPAMP.
- Solve the circuit.
- Check the validating inequalities for the solution obtained.

If they are satisfied, the solution is correct. Otherwise, choose another operating region for the OPAMP(s) and solve the circuit again.