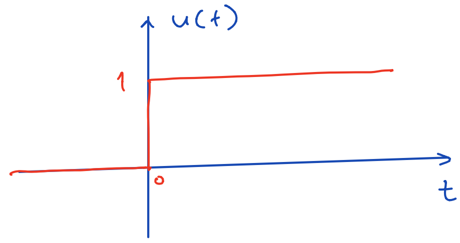
CHAPTER 5 : SIGNAL WAVEFORMS

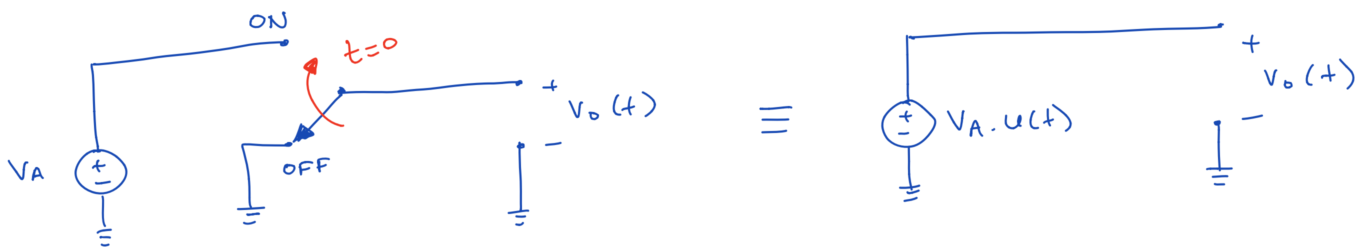
Waveform : an equation / graph defining the signal as a function of time.

$$v(t) = f(t) \quad , \quad i(t) = g(t)$$

Unit Step :

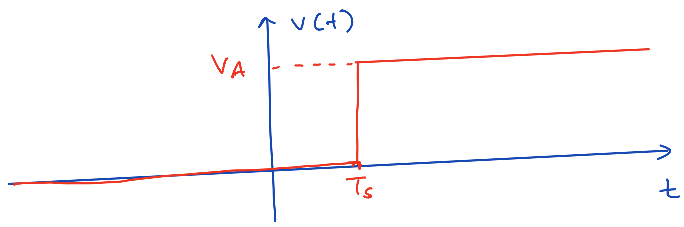
$$u(t) = \begin{cases} 0 & , \ t < 0 \\ 1 & , \ t \geq 0 \end{cases}$$



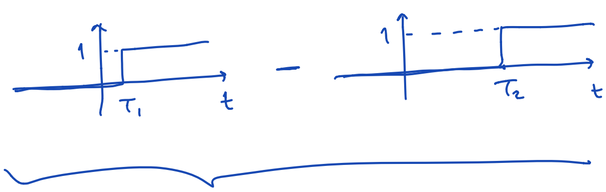


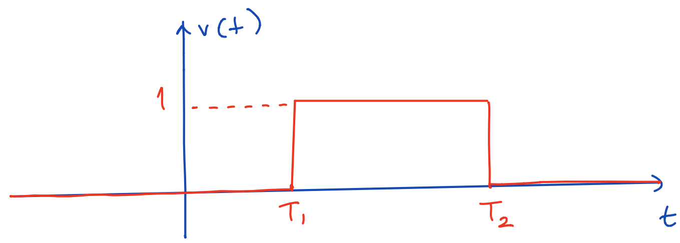
Time Delayed Step :

$$v(t) = V_A \cdot u(t - T_s) = \begin{cases} 0 & , \ \text{for} \ t < T_s \\ V_A & , \ \text{for} \ t \geq T_s \end{cases}$$

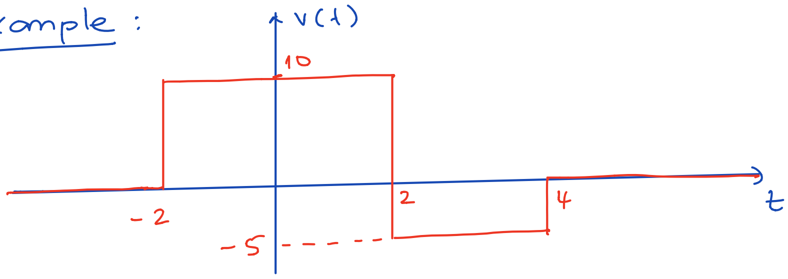


Pulse : $v(t) = u(t - T_1) - u(t - T_2)$



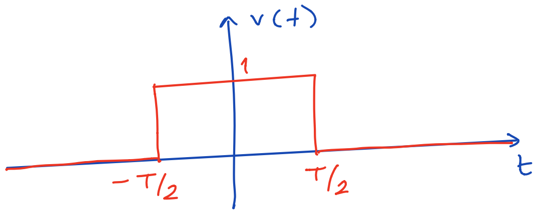


Example :



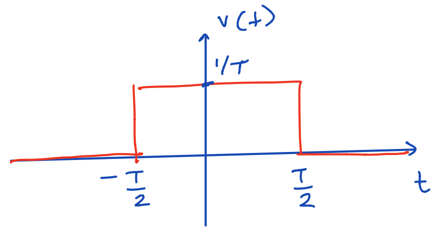
$$v(t) = 10 \cdot u(t+2) - 15 \cdot u(t-2) + 5\, u(t-4)$$

* When $T_1 = -T_2$ and $T_2 = T/2$ for a pulse, we have



* A special pulse :

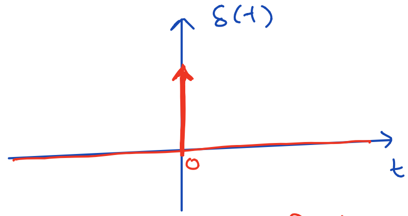
$$v(t) = \frac{1}{T}\left[u(t+T/2) - u(t-T/2)\right]$$



$$\int_{-\infty}^{\infty} v(t)\,dt = 1$$, independent of T.

Impulse :

$$\delta(t) = \lim_{T \to 0} \frac{1}{T}\left[u(t+T/2) - u(t-T/2)\right]$$



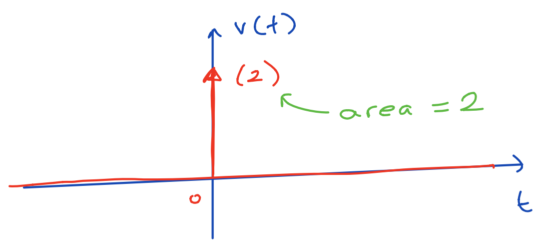
very special function

properties : $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

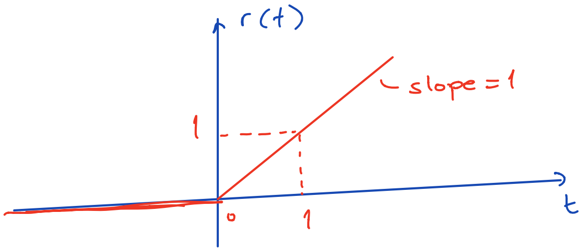
$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = \left\{ \begin{array}{ll} 0, & t < 0 \\ 1, & t > 0 \end{array} \right\} = u(t)$$

OR, $\delta(t) = \dfrac{d\,u(t)}{dt}$

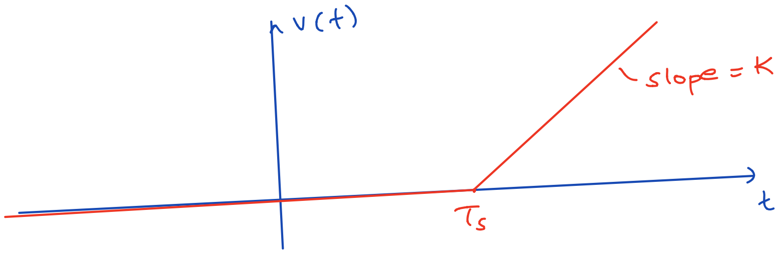
Example : $v(t) = 2\,\delta(t)$



Unit ramp function: $r(t) = \begin{cases} 0 , & t < 0 \\ t , & t \geq 0 \end{cases}$

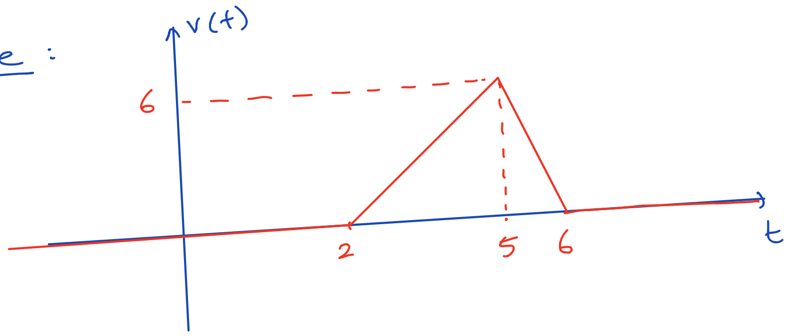


Delayed Ramp Function: $v(t) = K \cdot r(t - T_s) = \begin{cases} K \cdot (t - T_s) , & t \geq T_s \\ 0 , & t < T_s \end{cases}$



Triangular waveform: can be obtained from ramps

Example:



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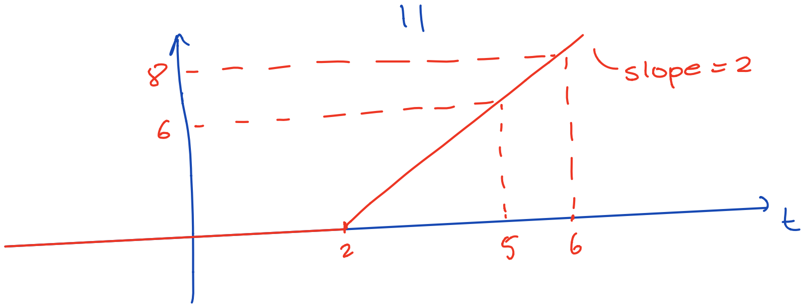
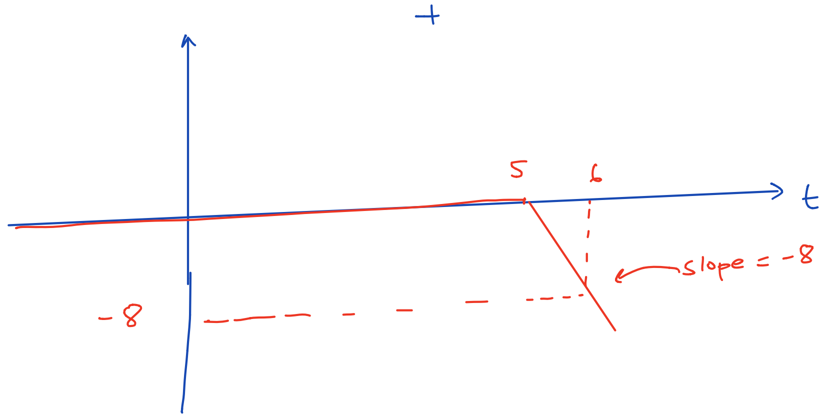


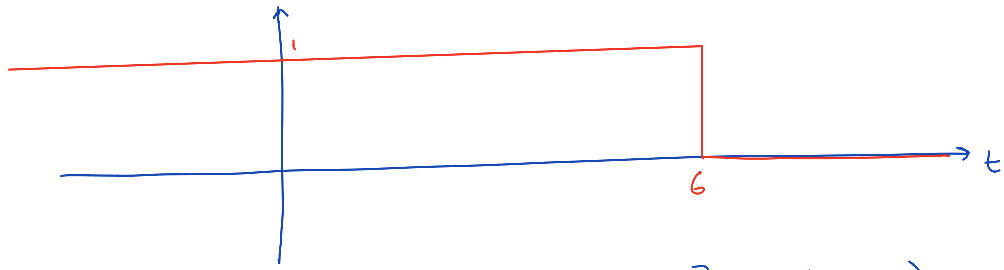
$r_1(t) = 2r(t-2)$

+



$r_2(t) = -8r(t-5)$

Then, multiply with a reversed unit step:



$u(6-t)$

So, $v(t) = \left[2r(t-2) - 8r(t-5)\right] \cdot u(6-t)$

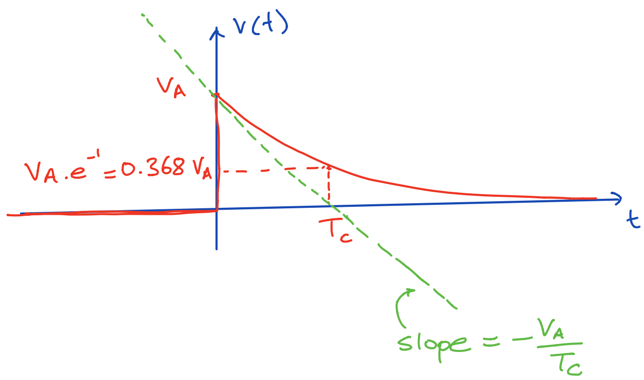
OR, $v(t) = \left[2r(t-2) - 8r(t-5)\right] \cdot \left[u(t-2) - u(t-6)\right]$

OR $v(t) = 2r(t-2) - 8r(t-5) + 6r(t-6)$

Impulse		Unit step		Ramp
$\delta(t)$	$\longrightarrow$	$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$	$\longrightarrow$	$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$
$\delta(t) = \frac{du(t)}{dt}$	$\longleftarrow$	$u(t) = \frac{dr(t)}{dt}$	$\longleftarrow$	$r(t)$

Exponential waveforms:

$$v(t) = V_A \cdot e^{-t/T_c} \cdot u(t)$$



V_A: amplitude
T_c: time constant

$$\frac{dv(t)}{dt} = -\frac{1}{T_c}\, v(t)$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = -\frac{1}{T_c} \cdot V_A$$

$v(0) = V_A$
$v(T_c) = V_A \cdot e^{-1} \approx 0.368\, V_A$
$v(5T_c) = V_A \cdot e^{-5} \approx 0.0067\, V_A$

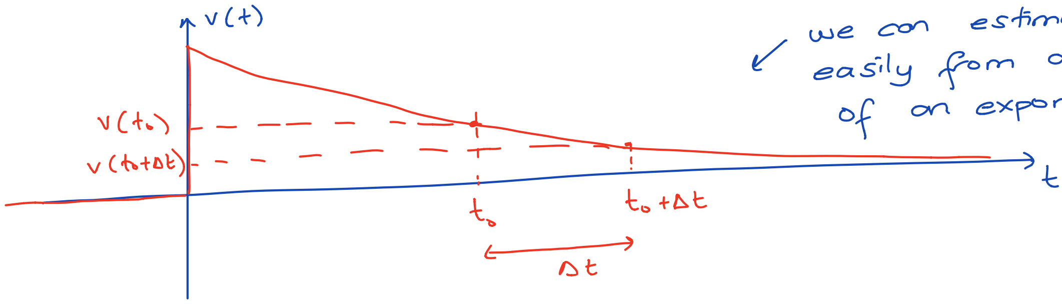
$v(t) \leq 1\,\%$ of $v(0)$, $t \geq 5T_c$
$v(t) \leq 2\,\%$ of $v(0)$, $t \geq 4T_c$

Rule of Thumb:
We assume convergence in ~ 5 time constants

$$\frac{v(t+\Delta t)}{v(t)} = \frac{V_A \cdot e^{-(t+\Delta t)/T_c}}{V_A \cdot e^{-t/T_c}} = e^{-\Delta t/T_c}$$

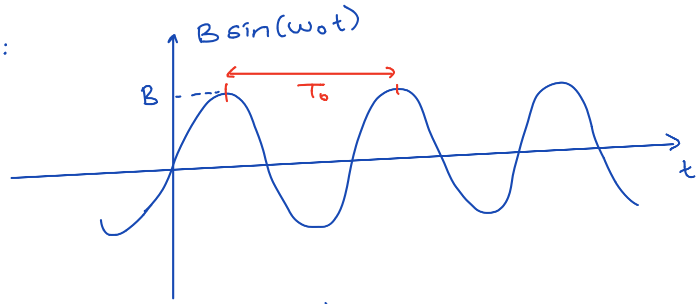
$\Rightarrow$ independent of amplitude V_A and starting time point t

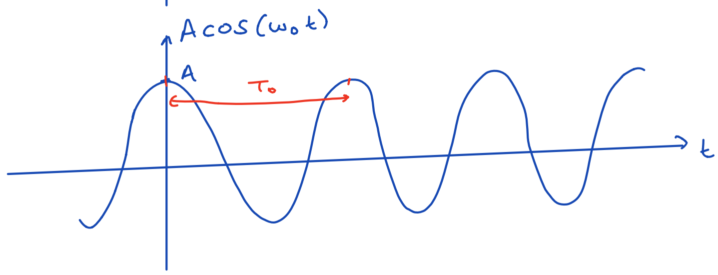
$$T_c = \frac{\Delta t}{\ln\left(\frac{v(t)}{v(t+\Delta t)}\right)} \Longrightarrow \text{estimate } T_c$$



we can estimate T_c easily from a graph of an exponential waveform

Sinusoidal signal:

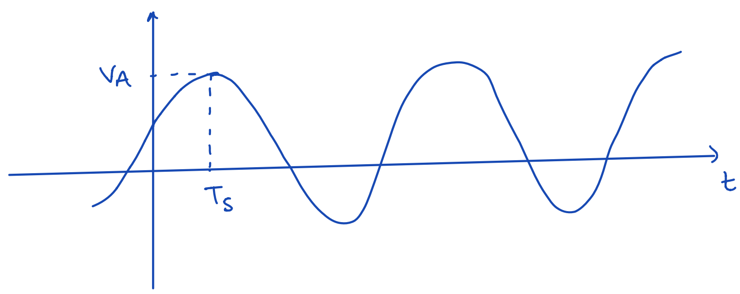




$\omega_0 = \frac{2\pi}{T_0}$ angular frequency, in rps

$f_0 = \frac{1}{T_0} = \frac{\omega_0}{2\pi}$ (cyclic) frequency, in $\frac{1}{\text{sec}} = \text{Hz}$

shifted sinusoid:



$$v(t) = V_A \cdot \cos\left(\frac{2\pi}{T_0}(t - T_s)\right)$$

$$= V_A \cdot \cos\left(\underbrace{\frac{2\pi}{T_0}}_{\omega_0} t - \frac{2\pi T_s}{T_0}\right) \quad \phi$$

$$= V_A \cdot \cos(\omega_0 t + \phi)$$, where ϕ : phase shift

Sum of Sinusoids:

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$v(t) = V_A \cos(\omega_0 t + \phi) = V_A \cdot \cos(\omega_0 t)\cos(\phi) - V_A \cdot \sin(\omega_0 t) \cdot \sin(\phi)$$

$$= \underbrace{V_A \cos(\phi)}_{a} \cdot \cos(\omega_0 t) \underbrace{- V_A \sin(\phi)}_{b} \cdot \sin(\omega_0 t)$$

$$v(t) = a \cdot \cos(\omega_0 t) + b \sin(\omega_0 t)$$

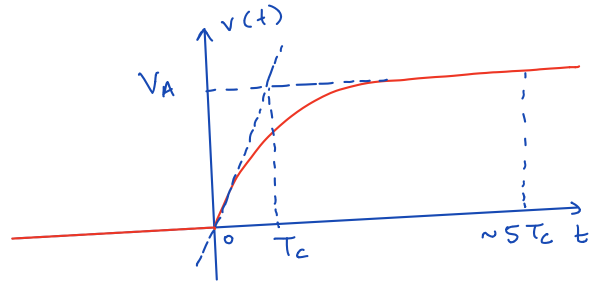
Here, $a = V_A \cos(\phi)$, $b = -V_A \cdot \sin(\phi)$ $\Bigg\}$ $V_A = \sqrt{a^2 + b^2}$, $\tan(\phi) = \dfrac{-b}{a}$ $\Bigg\}$ useful in phasor analysis

$*$ $V_A \cdot \sin(\omega t) = V_A \cdot \cos(\omega t - \frac{\pi}{2})$

$-V_A \cdot \cos(\omega t) = V_A \cdot \cos(\omega t - \pi)$ $\Big\}$ useful in phasor analysis

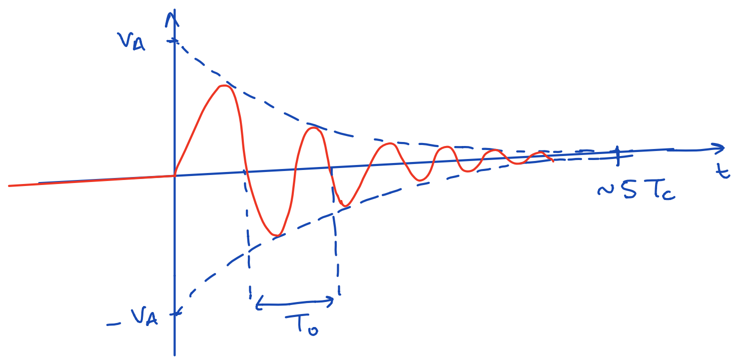
Composite waveforms:

$*$ $v(t) = V_A (1 - e^{-t/T_c}) \cdot u(t)$

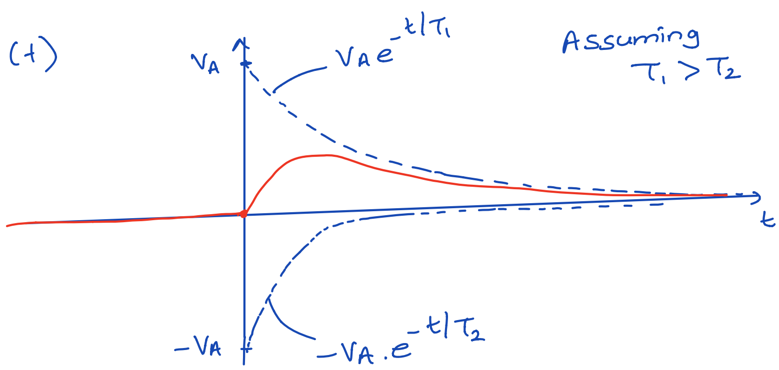


$*$ $v(t) = V_A \cdot e^{-t/T_c} \cdot \sin(\omega_0 t)\, u(t)$

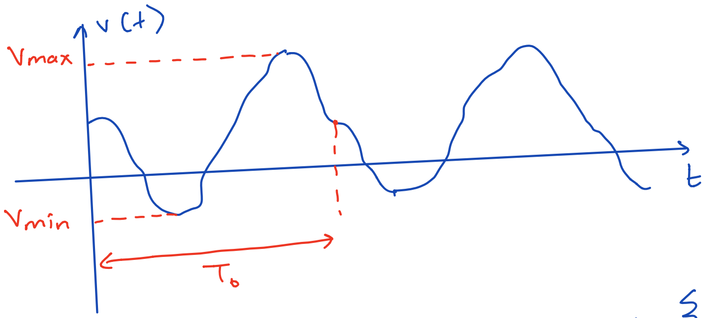
decaying sinusoid
OR
damped sinusoid



$*$ $v(t) = V_A \left[e^{-t/T_1} - e^{-t/T_2} \right] u(t)$



Periodic Signal Properties



$$v(t+T_0) = v(t)$$

peak value $= V_p = \max\{|V_{max}|, |V_{min}|\}$

max. of absolute value

peak-to-peak value $= V_{pp} = V_{max} - V_{min}$

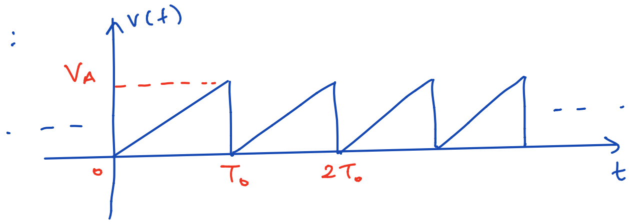
average value $= V_{avg} = \dfrac{1}{T_0} \displaystyle\int_t^{t+T_0} v(x)\,dx$, t : arbitrary (can be zero)

root-mean-square value $= V_{rms} = \sqrt{\dfrac{1}{T_0} \displaystyle\int_t^{t+T_0} [v(x)]^2\,dx}$

Example: $v(t) = V_A \cdot \sin(\omega_0 t + \phi)$

$$V_{rms} = \frac{V_A}{\sqrt{2}} \quad \text{(exercise)}$$

Example:



$$v(t) = V_A \cdot \frac{t}{T_0}, \quad 0 \le t \le T_0$$

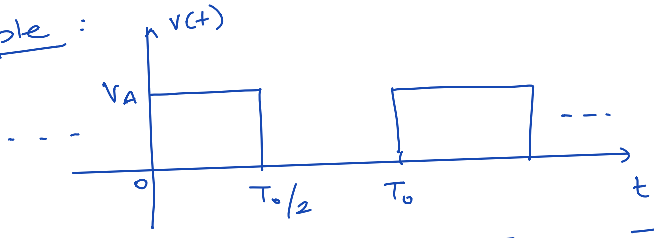
$$V_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} v^2(t)\,dt} = \sqrt{\frac{1}{T_0}\int_0^{T_0} V_A^2 \frac{t^2}{T_0^2}\,dt}$$

$$= \sqrt{\frac{V_A^2}{T_0^3} \left.\frac{t^3}{3}\right|_0^{T_0}} = \sqrt{\frac{V_A^2}{\cancel{T_0^3}} \cdot \frac{\cancel{T_0^3}}{3}} = \frac{V_A}{\sqrt{3}}$$

$$V_{avg} = \frac{1}{T_0}\int_0^{T_0} v(t)\,dt = \frac{1}{T_0}\int_0^{T_0} V_A \cdot \frac{t}{T_0}\,dt$$

$$= \frac{V_A}{T_0^2} \left.\frac{t^2}{2}\right|_0^{T_0} = \frac{V_A}{2}$$

Example :

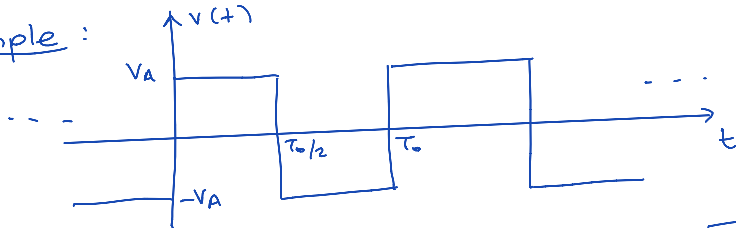


$$V_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} v^2(t)\,dt} = \sqrt{\frac{1}{T_0}\int_0^{T_0/2} V_A^2\,dt}$$

$$= \sqrt{\frac{1}{T_0} V_A^2 . t \Big|_0^{T_0/2}} = \sqrt{\frac{V_A^2}{\cancel{T_0}} . \frac{\cancel{T_0}}{2}} = \frac{V_A}{\sqrt{2}}$$

$$V_{avg} = \frac{V_A}{2}$$ (exercise)

Example :



$$V_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} v^2(t)\,dt} = \sqrt{\frac{1}{T_0}\int_0^{T_0} V_A^2\,dt} = \sqrt{\frac{1}{\cancel{T_0}} V_A^2 . \cancel{T_0}}$$

$$= V_A$$

$$V_{avg} = 0$$ (exercise)

Homework : Read chapter 6.