1. Probability Addition Theorem

The addition theorem is used to calculate the probability of the occurrence of at least one of two events. It depends on whether the events are **mutually exclusive** (cannot occur together) or not mutually exclusive (can occur together).

- **Mutually Exclusive**: Events that cannot happen at the same time. E.g., getting a 2 and a 5 in one single die roll.
- **Not Mutually Exclusive**: Events that can happen together. E.g., drawing a red card and a king (since some red cards are kings too).

Formula:

- **Mutually exclusive**:

$$P(A \cup B) = P(A) + P(B)$$

- **Not mutually exclusive**:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Program:-

```
def addition_theorem(P_A, P_B, P_A_and_B=None):
    if P_A_and_B is None:
        return P_A + P_B
    else:
        return P_A + P_B - P_A_and_B
P_A = float(input("Enter P(A): "))
P_B = float(input("Enter P(B): "))
choice = input("Are the events mutually exclusive? (yes/no): ")
if choice.lower() == "yes":
    result = addition_theorem(P_A, P_B)
else:
    P_A_and_B = float(input("Enter P(A ∩ B): "))
    result = addition_theorem(P_A, P_B, P_A_and_B)
print(f"P(A ∪ B) = {result}")
```

Output:

```python
def addition_theorem(P_A, P_B, P_A_and_B=None):
    if P_A_and_B is None:
        return P_A + P_B
    else:
        return P_A + P_B - P_A_and_B
P_A = float(input("Enter P(A): "))
P_B = float(input("Enter P(B): "))
choice = input("Are the events mutually exclusive? (yes/no): ")
if choice.lower() == "yes":
    result = addition_theorem(P_A, P_B)
else:
    P_A_and_B = float(input("Enter P(A ∩ B): "))
    result = addition_theorem(P_A, P_B, P_A_and_B)
print(f"P(A ∪ B) = {result}")
```

```
Enter P(A): 0.6
Enter P(B): .5
Are the events mutually exclusive? (yes/no): n
Enter P(A ∩ B): 0.3
P(A ∪ B) = 0.8

=== Code Execution Successful ===
```

Q) In a class:

- 40% of students like Math → $P(A)=0.6$
- 50% like Science → $P(B)=0.5$
- 20% like both Math and Science → $P(A \cap B)=0.3$

What is the probability that a student likes Math or Science?

Solution:

$$P(A \cup B)=P(A)+P(B)-P(A \cap B)$$

$$P(A \cup B)=0.6+0.5-0.3=0.8$$

So, the probability that a student likes either Math or Science is **0.8**.

2. Probability Multiplication Theorem

Explanation:

The multiplication theorem helps us find the probability of both events A and B occurring. This depends on whether events are independent or dependent.

- **Independent**: One event doesn't affect the other (e.g., flipping a coin and rolling a die).
- **Dependent**: One event affects the outcome of the other (e.g., drawing cards without replacement).

Formula:

- **Independent Events**:

$$P(A \cap B)=P(A) \cdot P(B)$$

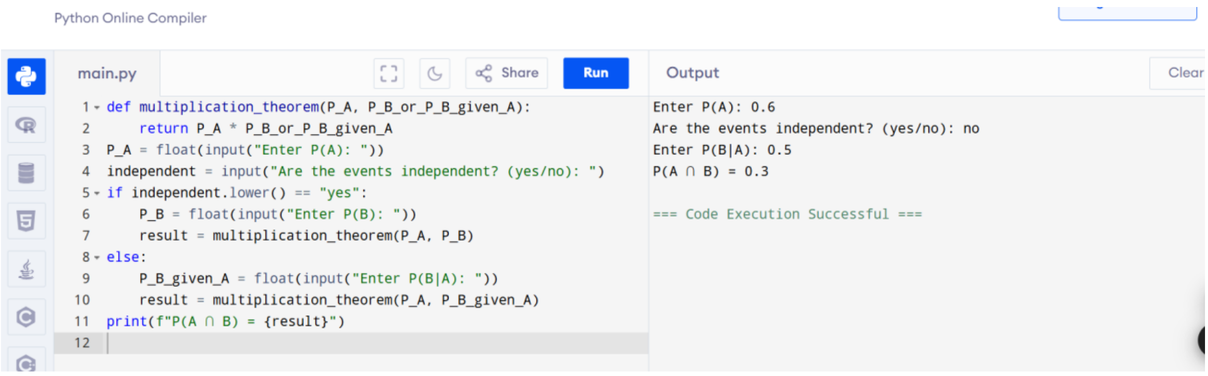
- **Dependent Events**:

$$P(A\cap B)=P(A)\cdot P(B|A)$$

Program:-

```
def multiplication_theorem(P_A, P_B_or_P_B_given_A):
    return P_A * P_B_or_P_B_given_A
P_A = float(input("Enter P(A): "))
independent = input("Are the events independent? (yes/no): ")
if independent.lower() == "yes":
    P_B = float(input("Enter P(B): "))
    result = multiplication_theorem(P_A, P_B)
else:
    P_B_given_A = float(input("Enter P(B|A): "))
    result = multiplication_theorem(P_A, P_B_given_A)
print(f"P(A ∩ B) = {result}")
```

Output:-



Q) A coin is tossed and a die is rolled:

• Probability of getting heads → P(A)=0.5P(A) = 0.5P(A)=0.5
• Probability of getting a 4 on the die → P(B)=16≈0.1667P(B) = \frac{1}{6} \approx 0.1667P(B)=61≈0.1667

What is the probability that both events occur — getting heads and getting a 4?

Solution: $P(A\cap B)=P(A)\cdot P(B)$

$P(A\cap B)=0.5\cdot 0.1667=0.0833$

So, the probability of getting heads and a 4 is **0.0833.**

3. Bayes’ Theorem

Explanation:

Bayes’ theorem is used to find the probability of an event A given that another event B has occurred, especially when we know the reverse conditional probability, P(B|A)P(B|A)P(B|A).

It’s very useful in medical testing, spam filtering, machine learning, and other areas where we revise beliefs based on new evidence.

Formula:

$$P(A|B)=P(A) \cdot P(B|A)/P(B)$$

Program:-

```
def bayes_theorem(P_A, P_B_given_A, P_B):
    return (P_B_given_A * P_A) / P_B
P_A = float(input("Enter P(A): "))
P_B_given_A = float(input("Enter P(B|A): "))
P_B = float(input("Enter P(B): "))
result = bayes_theorem(P_A, P_B_given_A, P_B)
print(f"P(A|B) = {result}")
```

Output:-

Q) A medical test is 95% accurate.

• 1% of people have the disease → P(A)=0.01P(A) = 0.01P(A)=0.01
• If a person has the disease, the test is positive 95% of the time → P(B|A)=0.95P(B|A) = 0.95P(B|A)=0.95
• The probability of testing positive in the entire population → P(B)=0.05P(B) = 0.05P(B)=0.05

What is the probability that a person actually has the disease, given that they tested positive?

Solution:

$P(A|B)=P(B)P(B|A)\cdot P(A)$

$P(A|B)=0.5 *0.95/0.01=0.050.0095=0.19$

So, the probability that a person actually has the disease given a positive test is 0.19 (or 19%).