Example calculation: loss-oriented hazard-consistent incremental dynamic analysis

Overview

Incremental dynamic analysis (IDA) (Vamvatsikos and Cornell 2002) is a nonlinear dynamic analysis procedure typically conducted by scaling the amplitude of a generic record set, thereby avoiding the arduous site-specific record selection process required for multiple stripe analysis (MSA) (Jalayer 2003). Unlike MSA, traditional IDA is, however, limited in its ability to produce hazard-consistent estimates of engineering demand parameters (HC-EDPs) (Chase et al. 2021; Zhong et al. 2022; Chandramohan 2016). This issue is addressed by Loss-oriented Hazard-Consistent Incremental Dynamic Analysis (LOHC-IDA) developed by Sistla et al. (2026), which post-processes IDA results to ensure hazard consistency.

LOHC-IDA comprises three main steps: *(i)* performing IDA using a generic set of ground motions; *(ii)* post-processing the IDA results to develop simplified surrogate models at different intensity levels to predict engineering demand parameters (EDPs) using secondary ground motion intensity measures and estimating the correlations among EDPs; and *(iii)* using the simplified EDP surrogate models and correlation matrices to adjust the original EDPs and simulate HC-EDPs that are consistent with the site- and intensity-specific ground motion characteristics (Bradley 2010).

This documentation serves as an example illustrating all calculation steps in the LOHC-IDA framework. A Python code to perform these calculations can be found at: github.com/Saiteja802.

All the equations used in this document and the Python code can be found in Sistla et al. (2026). For more details, please refer to and cite the following article if you use the Python script: Sistla S, Chandramohan R, Sullivan TJ. *Loss-oriented hazard-consistent incremental dynamic analysis. Structural Safety.* 2026 Jan 28:102692. Link: LOHC-IDA paper

LOHC-IDA application

The calculations for applying the LOHC-IDA framework are demonstrated here, considering only the EDPs in the X direction of a case-study two-storey steel BRBF building discussed in Sistla et al. (2026). The calculated results summarised in this document are identical to those obtained from the Python script hosted at github.com/Saiteja802.

The fundamental period (T_1) of this building is 0.5 s and further details can be found in Sistla et al. (2026). In this example, $S_a(T_1)$ is the primary intensity measure (IM^p), while S_aRatio and PGA are the secondary intensity measures (IM^s). The EDPs considered are the peak storey drift ratio at storey 1 ($PSDR_{1x}$), the peak storey drift ratio at storey 2 ($PSDR_{2x}$), the peak floor acceleration at floor 1 (PFA_{1x}), and the peak floor acceleration at floor 2 (PFA_{2x}).

The HC-EDP simulation process is demonstrated at a ground motion intensity level corresponding to 0.2 % probability of exceedance (PoE) in 50 years, with $S_a(0.5\,\text{s}) = 2.1\,\text{g}$. At this ground motion intensity, 5 out of 50 ground motions resulted in structural collapse. Therefore, the S_aRatio and PGA values for the remaining 45 ground motions, scaled to this intensity level, are summarised in Table 1a, while the corresponding EDP values are presented in Table 1b. The IM values in Table 1a correspond to the $\boldsymbol{IM^s}|_{IM^p}$ matrix in Equation 3, and the EDP values in Table 1b correspond to the $\boldsymbol{EDP}|_{IM^p}$ matrix in Equation 4. The EDP correlation matrix ($\boldsymbol{R}_{\mathbf{ln\,EDP}}|_{IM^p}$) is computed using Equation 6a, and its values are summarised in Table 2.

Linear regression models are fit to the EDPs using IM^s (Equation 5a). The coefficients of linear regression models for all EDPs are summarised in Table 3, where β_0 is the intercept, β_1 is the coefficient of lnS_aRatio, and β_2 is the coefficient of $lnPGA$. The variance of the error term ($\sigma^2_{e|IM^p}$) is calculated using Equation 5b.

Table 1: a) $S_a Ratio$ and PGA of the ground motions scaled to a $S_a(0.5\,\mathrm{s})$ value of 2.1 g (0.2 % PoE in 50 years intensity level) and b) EDPs of the case-study 2-storey steel BRBF building at the same intensity level.

a)

	$S_a Ratio$	PGA (g)
record no. 1	1.42	0.515
record no. 2	1.9	0.796
record no. 3	1.95	1.03
record no. 4	1.06	0.718
record no. 5	1.89	0.756
record no. 6	1.28	1.16
record no. 7	3.67	0.706
record no. 8	1.47	0.81
record no. 9	1.58	0.945
record no. 10	1.44	0.968
record no. 11	1.37	2.33
record no. 12	1.16	1.03
record no. 13	2.13	0.639
record no. 14	1.4	1.05
record no. 15	0.999	1.31
record no. 16	2.41	1.33
record no. 17	1.52	0.85
record no. 18	1.29	1.37
record no. 19	1.23	1.23
record no. 20	2.3	1.15
record no. 21	1.33	0.81
record no. 22	1.34	0.676
record no. 23	2.89	0.907
record no. 24	1.82	0.857
record no. 25	2.17	0.774
record no. 26	0.974	1.44
record no. 27	2.16	0.836
record no. 28	1.29	0.788
record no. 29	2.81	0.717
record no. 30	0.963	4.66
record no. 31	1.66	1.03
record no. 32	1.03	1.02
record no. 33	1.17	1.21
record no. 34	1.34	0.901
record no. 35	1.78	1.04
record no. 36	1.23	0.902
record no. 37	1.12	2.12
record no. 38	1.38	2.27
record no. 39	1.12	0.778
record no. 40	2.07	0.546
record no. 41	1.2	0.871
record no. 42	4.51	0.633
record no. 43	3.0	0.902
record no. 44	1.18	2.88
record no. 45	1.84	1.1

b)

	$PSDR_{1x}$ (rad)	$PSDR_{2x}$ (rad)	PFA_{1x} (g)	PFA_{2x} (g)
record no. 1	0.011	0.017	0.577	0.643
record no. 2	0.0201	0.0377	0.816	0.767
record no. 3	0.0146	0.0239	1.13	0.695
record no. 4	0.0116	0.0194	0.611	0.626
record no. 5	0.0166	0.0271	0.739	0.715
record no. 6	0.012	0.0168	1.43	0.718
record no. 7	0.00924	0.0172	0.603	0.678
record no. 8	0.0193	0.0381	0.777	0.888
record no. 9	0.0132	0.0188	0.805	0.693
record no. 10	0.0169	0.0316	1.22	0.87
record no. 11	0.0222	0.0411	1.08	0.848
record no. 12	0.0533	0.0862	1.68	0.955
record no. 13	0.0119	0.0171	0.694	0.686
record no. 14	0.0225	0.0348	0.942	0.909
record no. 15	0.023	0.0376	1.27	0.862
record no. 16	0.0103	0.0179	1.02	0.703
record no. 17	0.0389	0.0521	0.876	0.998
record no. 18	0.0138	0.0176	1.37	0.719
record no. 19	0.0389	0.0669	1.08	1.02
record no. 20	0.0163	0.0298	0.864	0.755
record no. 21	0.0144	0.0232	0.758	0.74
record no. 22	0.0181	0.0283	0.657	0.703
record no. 23	0.0138	0.0208	1.03	0.665
record no. 24	0.0145	0.0291	0.738	0.783
record no. 25	0.0228	0.0398	0.832	0.94
record no. 26	0.031	0.0572	1.28	1.02
record no. 27	0.0074	0.0112	0.684	0.59
record no. 28	0.0215	0.0328	0.992	0.788
record no. 29	0.00952	0.0169	0.595	0.719
record no. 30	0.0412	0.0488	1.69	0.861
record no. 31	0.0192	0.0282	1.16	0.756
record no. 32	0.0537	0.057	0.888	0.901
record no. 33	0.0273	0.0466	0.924	0.843
record no. 34	0.0177	0.0258	0.678	0.706
record no. 35	0.0187	0.0287	1.04	0.796
record no. 36	0.0224	0.0339	0.774	0.767
record no. 37	0.0323	0.0461	2.19	1.04
record no. 38	0.0191	0.0278	1.92	0.888
record no. 39	0.0286	0.0419	0.823	0.851
record no. 40	0.012	0.0157	0.53	0.653
record no. 41	0.0198	0.0278	0.798	0.821
record no. 42	0.00951	0.0149	0.535	0.652
record no. 43	0.0156	0.026	0.907	0.783
record no. 44	0.0304	0.043	1.26	0.818
record no. 45	0.00936	0.0208	0.845	0.81

Table 2: EDP correlation matrix ($\boldsymbol{R}_{\ln \mathbf{EDP}}|_{IM^p}$) at 0.2 % PoE in 50 years intensity level.

	$PSDR_{1x}$	$PSDR_{2x}$	PGA_x	PFA_{1x}	PFA_{2x}
$PSDR_{1x}$	1.0	0.954	0.464	0.539	0.82
$PSDR_{2x}$	0.954	1.0	0.433	0.508	0.878
PGA_x	0.464	0.433	1.0	0.794	0.472
PFA_{1x}	0.82	0.878	0.472	0.6	1.0
PFA_{2x}	0.539	0.508	0.794	1.0	0.6

Note: PGA is included when estimating the correlation matrix. These correlations will be used to simulate HC-EDPs.

Table 3: Coefficients of EDP regression models.

Coefficient	$\boldsymbol{PSDR_{1x}}$	$\boldsymbol{PSDR_{2x}}$	$\boldsymbol{PFA_{1x}}$	$\boldsymbol{PFA_{2x}}$
β_0	-3.668	-3.261	-0.019	-0.178
β_1	-0.728	-0.617	-0.151	-0.144
β_2	0.246	0.227	0.563	0.102

The parameters of the conditional bivariate lognormal probability density function (Equation 7a and Equation 7b) of the target IM^s at the site, at 0.2 % PoE in 50 years intensity level, are shown in Table 4. The means of lnS_aRatio and $lnPGA$ are represented by $\mu_{lnIM^s_{1,T}|IM^p}$ and $\mu_{lnIM_{2,T}|IM^p}$, respectively. The variances of lnS_aRatio and $lnPGA$ are represented by $\Sigma_{11,T|_{IM^p}}$ and $\Sigma_{22,T|_{IM^p}}$, respectively. The covariance of lnS_aRatio and $lnPGA$ is denoted as $\Sigma_{21,T|_{IM^p}}$.

Table 4: Parameters of the conditional bivariate lognormal probability density function of the target IM^s at the site, at 0.2 % PoE in 50 years intensity level.

Parameter	$\mu_{lnIM^s_{1,T}\|IM^p}$	$\mu_{lnIM^s_{2,T}\|IM^p}$	$\Sigma_{11,T\|_{IM^p}}$	$\Sigma_{22,T\|_{IM^p}}$	$\Sigma_{21,T\|_{IM^p}}$
Value	0.677	-0.452	0.084	0.0916	-0.0836

The hazard-consistent mean ($\mu_{\ln \text{HC-EDP}_k\,|_{IM^p}}$) and variance ($\sigma^2_{\ln \text{HC-EDP}_k\,|_{IM^p}}$) of the EDPs are computed using Equations 8a and 8b, respectively. These results are summarised in Table 5.

As an example, the regression model fit to predict PFA_{1x} is illustrated in

Table 5: Hazard-consistent mean ($\mu_{\ln \text{HC-EDP}_k|IM^p}$) and variance ($\sigma^2_{\ln \text{HC-EDP}_k|IM^p}$) of the EDPs.

Parameter	$\boldsymbol{PSDR_{1x}}$	$\boldsymbol{PSDR_{2x}}$	$\boldsymbol{PGA_x}$	$\boldsymbol{PFA_{1x}}$	$\boldsymbol{PFA_{2x}}$
$\mu_{\ln \text{HC-EDP}_k|IM^p}$	-4.273	-3.782	-0.452	-0.376	-0.321
$\sigma^2_{\ln \text{HC-EDP}_k|IM^p}$	0.2	0.189	0.0916	0.097	0.019

Note: The mean and variance of PGA_x are included here because they are used to simulate HC-EDPs. The values of $\mu_{\ln \text{HC-EDP}_k|IM^p}$ and $\sigma^2_{\ln \text{HC-EDP}_k|IM^p}$ for PGA in this Table are identical to $\mu_{lnIM^s_{2,T}|IM^p}$ and $\Sigma_{22,T|IM^p}$, respectively, as reported in Table 4.

Figure 1. This figure also includes the estimation of the hazard-consistent mean of PFA_{1x} .

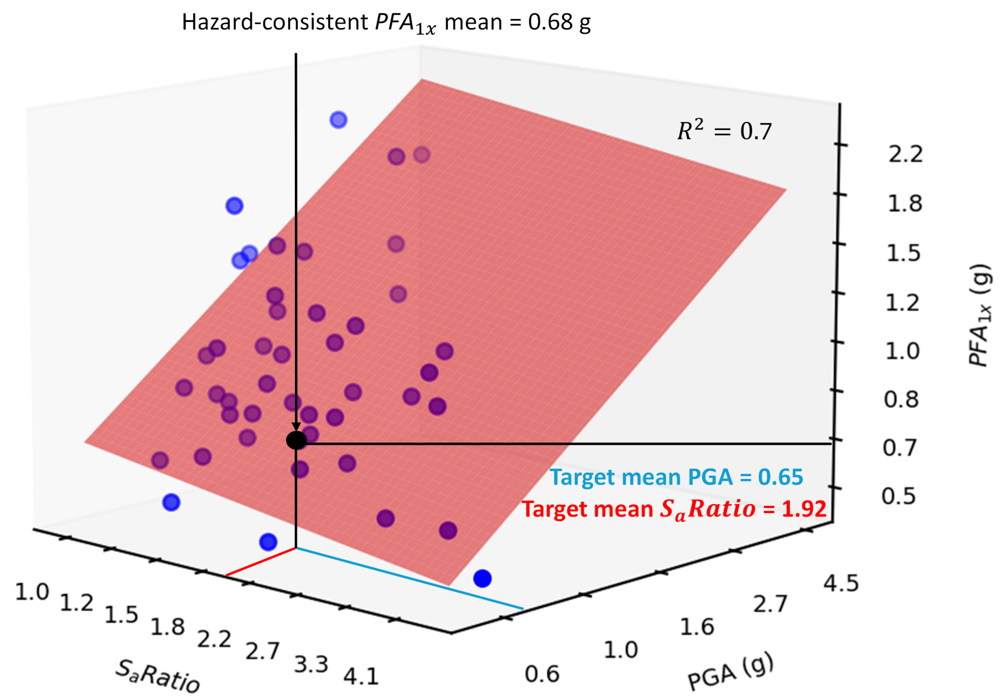

Figure 1: Hazard-consistent PFA_{1x} mean estimation at the 0.2 % PoE in 50 years intensity level.

Using the hazard-consistent mean ($\mu_{\ln \text{HC-EDP}_k|IM^p}$) and variance ($\sigma^2_{\ln \text{HC-EDP}_k|IM^p}$) of the EDPs (Table 5), along with the EDP correlation structure from IDA (Table 2), 100 sets of HC-EDPs are simulated assuming a lognormal multivariate distribution. The simulated EDPs are summarised in Table 6. The hazard-consistent standard deviation has been increased by 0.3 to account for modelling uncertainty.

Table 6: Hazard-consistent EDPs simulated using LOHC-IDA at the 0.2 % PoE in 50 years intensity level.

	Realization-1	Realization-2	Realization-3	......	Realization-100
$PSDR_{1x}$	0.011	0.011	0.01	......	0.014
$PSDR_{2x}$	0.015	0.03	0.012	......	0.029
PGA_{x}	0.335	0.726	0.368	......	0.358
PFA_{1x}	0.435	0.828	0.504	......	0.681
PFA_{2x}	0.61	0.902	0.455	......	0.925

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