Machine Learning Foundation Regression

Regularization and Gradient Descent

Introduction

In the field of machine learning, one of the fundamental challenges is finding the right balance between a model that captures underlying patterns in the data and one that avoids overfitting to noise. This project aims to explore that challenge through the lens of **polynomial regression**, with a focus on the powerful tools of **Regularization and Gradient Descent**.

We work with a sparse, noisy dataset generated from a known underlying function — the sine wave:

$y = sin(2\pi x)$

This function serves as our **ground truth**, and we compare it with a small sample of noisy observations to simulate real-world data scenarios where signals are often imperfect and incomplete.

As the complexity of a polynomial model increases, it tends to fit the training data more closely. While this can reduce training error, it often leads to poor generalization on unseen data — a phenomenon known as **overfitting**. Conversely, models with too little complexity may fail to capture important relationships — leading to **underfitting**. This project investigates how regularization techniques such as **Ridge Regression (L2)** and **Lasso Regression (L1)** help mitigate overfitting by constraining the model's coefficients.

In addition to regularization, we implement **Gradient Descent**, an optimization algorithm that minimizes the loss function by iteratively adjusting model parameters. We explore how gradient descent behaves under different regularization strengths and observe the impact of hyperparameters like the learning rate and regularization coefficient.

Through a combination of analytical reasoning, numerical simulation, and visual inspection, this project provides a hands-on understanding of the **bias-variance trade-off**, **model complexity**, and the **importance of regularization** in building robust predictive models.

By the end of this project, you will:

- Understand how polynomial regression can be prone to overfitting.
- See how Ridge and Lasso regression penalize model complexity in different ways.

- Gain intuition for how gradient descent works to minimize cost functions with and without regularization.
- Learn to visualize and evaluate model performance in the presence of noise and sparse data.

This foundation is crucial not only for academic understanding but also for real-world applications where interpretability, generalizability, and robustness are essential.

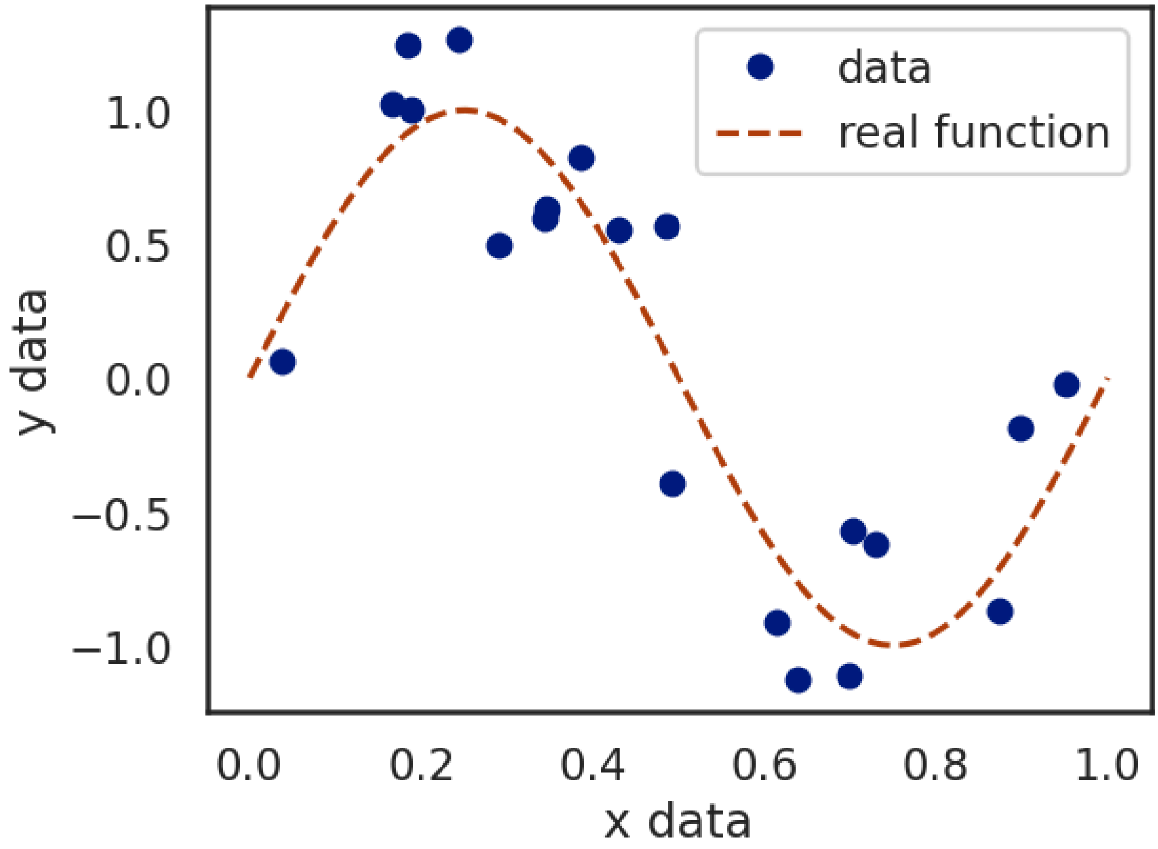
We will begin with a short tutorial on regression, polynomial features, and regularization based on a very simple, sparse data set that contains a column of `x` data and associated `y` noisy data. The data file is called `X_Y_Sinusoid_Data.csv`.

In [1]:
```
import os
data_path = ['data']
```

- Import the data.
- Also generate approximately 100 equally spaced x data points over the range of 0 to 1. Using these points, calculate the y-data which represents the "ground truth" (the real function) from the equation: $y = sin(2\pi x)$
- Plot the sparse data (`x` vs `y`) and the calculated ("real") data.

In [40]:
```
import os
import pandas as pd
import numpy as np

file_path = "/content/X_Y_Sinusoid_Data.csv"
data = pd.read_csv(file_path)

X_real = np.linspace(0, 1.0, 100)
Y_real = np.sin(2 * np.pi * X_real)
```

In [41]:
```
import matplotlib.pyplot as plt
import seaborn as sns
%matplotlib inline

sns.set_style('white')
sns.set_context('talk')
sns.set_palette('dark')

# Plot of the noisy (sparse)
ax = data.set_index('x')['y'].plot(ls='', marker='o', label='data')
ax.plot(X_real, Y_real, ls='--', marker='', label='real function')

ax.legend()
ax.set(xlabel='x data', ylabel='y data');
```

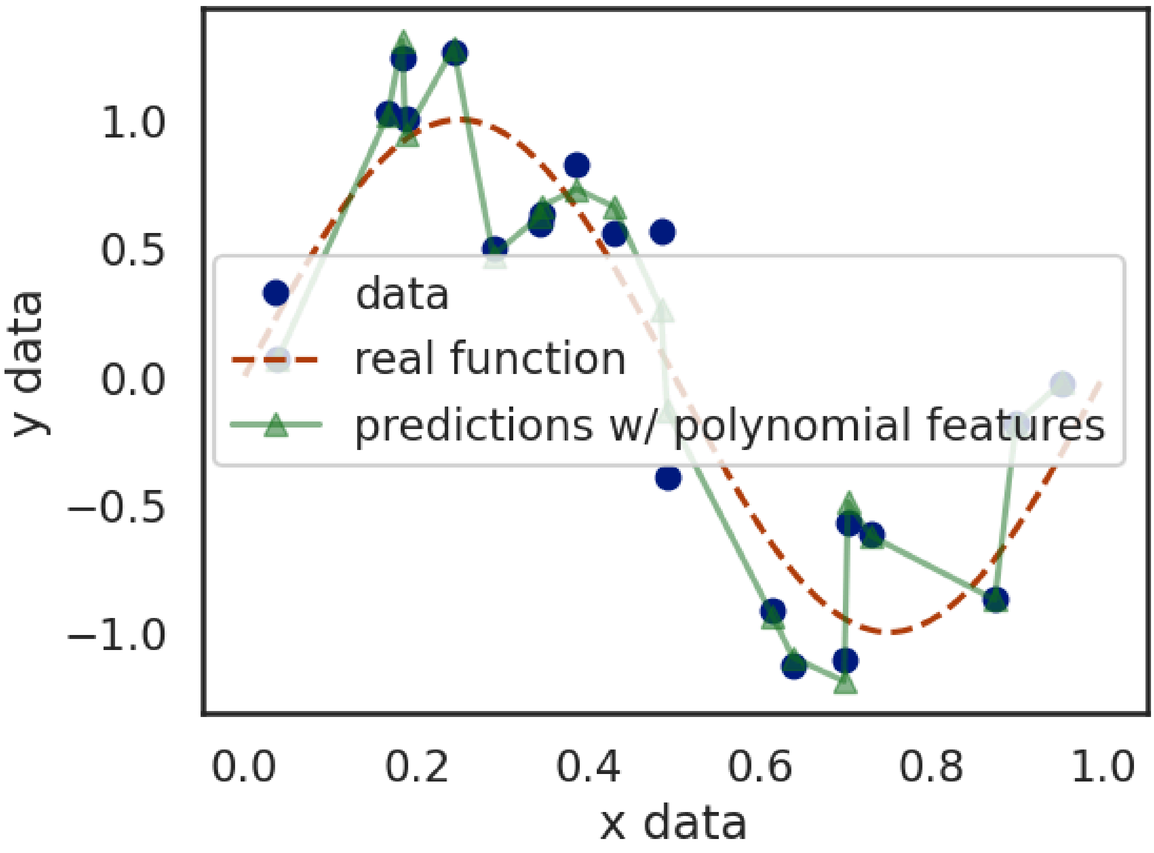


- Using the `PolynomialFeatures` class from Scikit-learn's preprocessing library, create 20th order polynomial features.
- Fit this data using linear regression.
- Plot the resulting predicted value compared to the calculated data.

Note that `PolynomialFeatures` requires either a dataframe (with one column, not a Series) or a 2D array of dimension (`X` , 1), where `X` is the length.

In [7]:
```
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression

# Setup the polynomial features
degree = 20
pf = PolynomialFeatures(degree)
lr = LinearRegression()

# Extract the X- and Y- data from the dataframe
X_data = data[['x']]
Y_data = data['y']

# Create the features and fit the model
X_poly = pf.fit_transform(X_data)
lr = lr.fit(X_poly, Y_data)
Y_pred = lr.predict(X_poly)

# Plot the result
plt.plot(X_data, Y_data, marker='o', ls='', label='data', alpha=1)
plt.plot(X_real, Y_real, ls='--', label='real function')
plt.plot(X_data, Y_pred, marker='^', alpha=.5, label='predictions w/ polynomial
plt.legend()
```

```
ax = plt.gca()
ax.set(xlabel='x data', ylabel='y data');
```



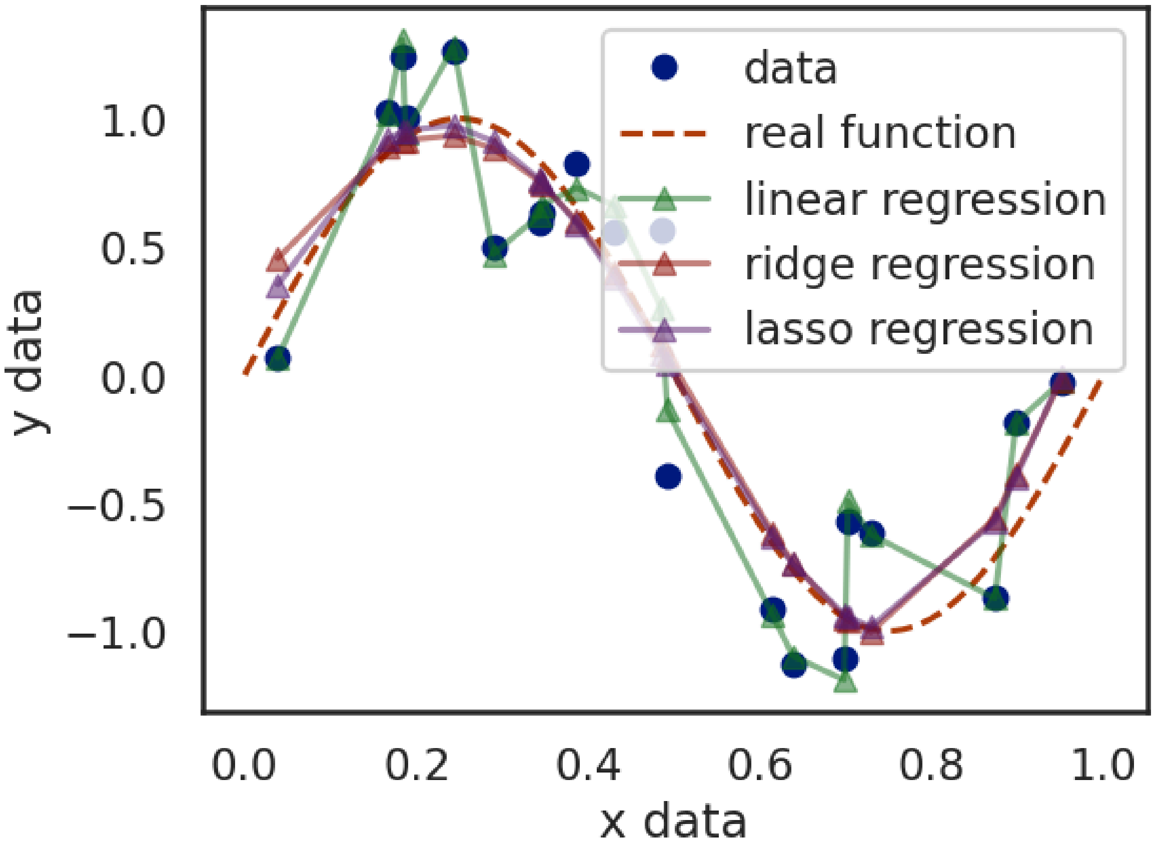
- Perform the regression on using the data with polynomial features using ridge regression (α=0.001) and lasso regression (α=0.0001).
- Plot the results, as was done in Question 1.
- Also plot the magnitude of the coefficients obtained from these regressions, and compare them to those obtained from linear regression in the previous question. The linear regression coefficients will likely need a separate plot (or their own y-axis) due to their large magnitude.

What does the comparatively large magnitude of the data tell us about the role of regularization?

In [8]:

```
# Mute the sklearn warning about regularization
import warnings
warnings.filterwarnings('ignore', module='sklearn')

from sklearn.linear_model import Ridge, Lasso

# The ridge regression model
rr = Ridge(alpha=0.001)
rr = rr.fit(X_poly, Y_data)
Y_pred_rr = rr.predict(X_poly)

# The lasso regression model
lassor = Lasso(alpha=0.0001)
lassor = lassor.fit(X_poly, Y_data)
Y_pred_lr = lassor.predict(X_poly)
```

```
# The plot of the predicted values
plt.plot(X_data, Y_data, marker='o', ls='', label='data')
plt.plot(X_real, Y_real, ls='--', label='real function')
plt.plot(X_data, Y_pred, label='linear regression', marker='^', alpha=.5)
plt.plot(X_data, Y_pred_rr, label='ridge regression', marker='^', alpha=.5)
plt.plot(X_data, Y_pred_lr, label='lasso regression', marker='^', alpha=.5)

plt.legend()

ax = plt.gca()
ax.set(xlabel='x data', ylabel='y data');
```



In [9]:
```
# let's look at the absolute value of coefficients for each model

coefficients = pd.DataFrame()
coefficients['linear regression'] = lr.coef_.ravel()
coefficients['ridge regression'] = rr.coef_.ravel()
coefficients['lasso regression'] = lassor.coef_.ravel()
coefficients = coefficients.applymap(abs)

coefficients.describe()  # Huge difference in scale between non-regularized vs r
```

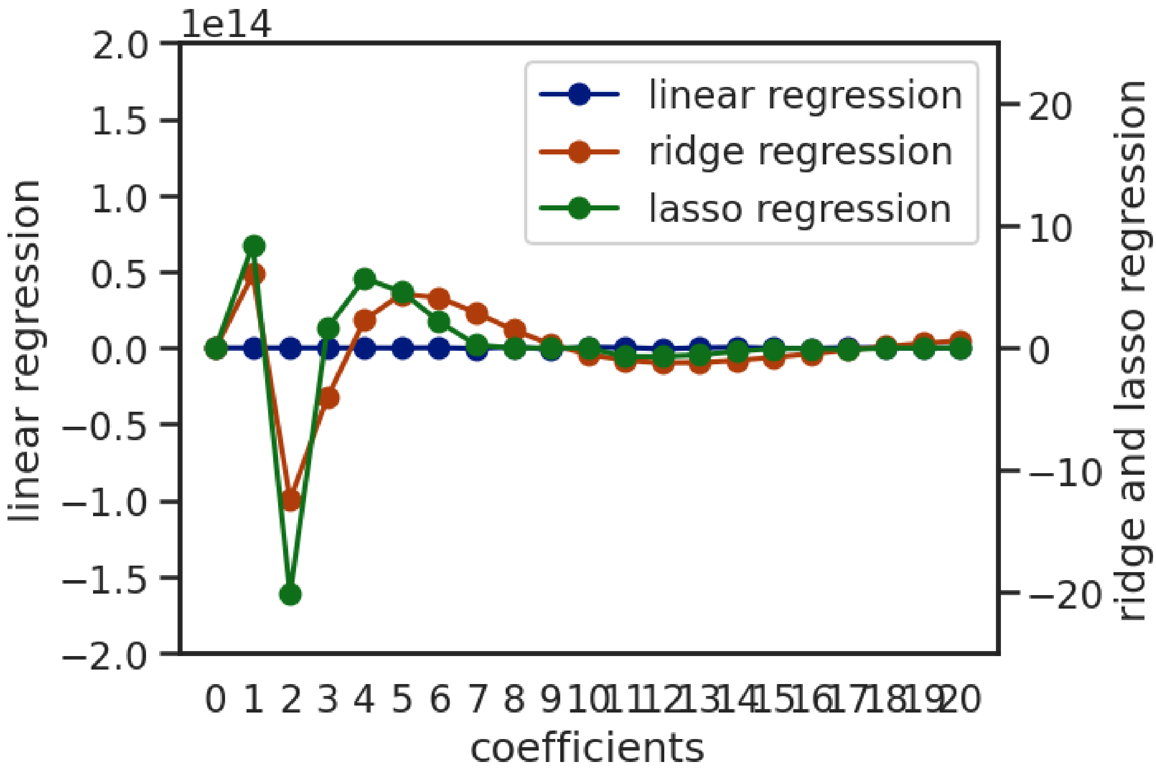
```
<ipython-input-9-10cb8b65aa37>:7: FutureWarning: DataFrame.applymap has been depr
ecated. Use DataFrame.map instead.
  coefficients = coefficients.applymap(abs)
```

Out[9]:

	linear regression	ridge regression	lasso regression
count	2.100000e+01	21.000000	21.000000
mean	2.427786e+11	2.169397	2.167284
std	2.709998e+11	2.900278	4.706731
min	1.866875e+06	0.000000	0.000000
25%	3.076136e+10	0.467578	0.000000
50%	1.258321e+11	1.017272	0.252181
75%	3.631114e+11	2.883507	1.641353
max	9.201049e+11	12.429635	20.176708

In [10]:

```
colors = sns.color_palette()

# Setup the dual y-axes
ax1 = plt.axes()
ax2 = ax1.twinx()

# Plot the linear regression data
ax1.plot(lr.coef_.ravel(),
         color=colors[0], marker='o', label='linear regression')

# Plot the regularization data sets
ax2.plot(rr.coef_.ravel(),
         color=colors[1], marker='o', label='ridge regression')

ax2.plot(lassor.coef_.ravel(),
         color=colors[2], marker='o', label='lasso regression')

# Customize axes scales
ax1.set_ylim(-2e14, 2e14)
ax2.set_ylim(-25, 25)

# Combine the legends
h1, l1 = ax1.get_legend_handles_labels()
h2, l2 = ax2.get_legend_handles_labels()
ax1.legend(h1+h2, l1+l2)

ax1.set(xlabel='coefficients',ylabel='linear regression')
ax2.set(ylabel='ridge and lasso regression')

ax1.set_xticks(range(len(lr.coef_)));
```



For the remaining questions, we will be working with the data set from last lesson, which is based on housing prices in Ames, Iowa. There are an extensive number of features--see the exercises from week three for a discussion of these features.

To begin:

- Import the data with Pandas, remove any null values, and one hot encode categoricals. Either Scikit-learn's feature encoders or Pandas `get_dummies` method can be used.
- Split the data into train and test sets.
- Log transform skewed features.
- Scaling can be attempted, although it can be interesting to see how well regularization works without scaling features.

In [13]:
```
filepath = 'Ames_Housing_Sales.csv'
data = pd.read_csv(filepath, sep=',')
```

Create a list of categorial data and one-hot encode. Pandas one-hot encoder (`get_dummies`) works well with data that is defined as a categorical.

In [15]:
```
# Get a Pd.Series consisting of all the string categoricals
# Use object instead of np.object to avoid the AttributeError
one_hot_encode_cols = data.dtypes[data.dtypes == object]  # filtering by string
one_hot_encode_cols = one_hot_encode_cols.index.tolist()  # list of categorical

# Here we see another way of one-hot-encoding:
# Encode these columns as categoricals so one hot encoding works on split data (
for col in one_hot_encode_cols:
    data[col] = pd.Categorical(data[col])
```

```
# Do the one hot encoding
data = pd.get_dummies(data, columns=one_hot_encode_cols)
```

Next, split the data in train and test data sets.

In [16]:
```
from sklearn.model_selection import train_test_split

train, test = train_test_split(data, test_size=0.3, random_state=42)
```

There are a number of columns that have skewed features--a log transformation can be applied to them. Note that this includes the `SalePrice`, our predictor. However, let's keep that one as is.

In [18]:
```
# Create a list of float colums to check for skewing
mask = data.dtypes == float  # Changed np.float to float
float_cols = data.columns[mask]
```

In [19]:
```
skew_limit = 0.75
skew_vals = train[float_cols].skew()

skew_cols = (skew_vals
             .sort_values(ascending=False)
             .to_frame()
             .rename(columns={0:'Skew'})
             .query('abs(Skew) > {0}'.format(skew_limit)))

skew_cols
```

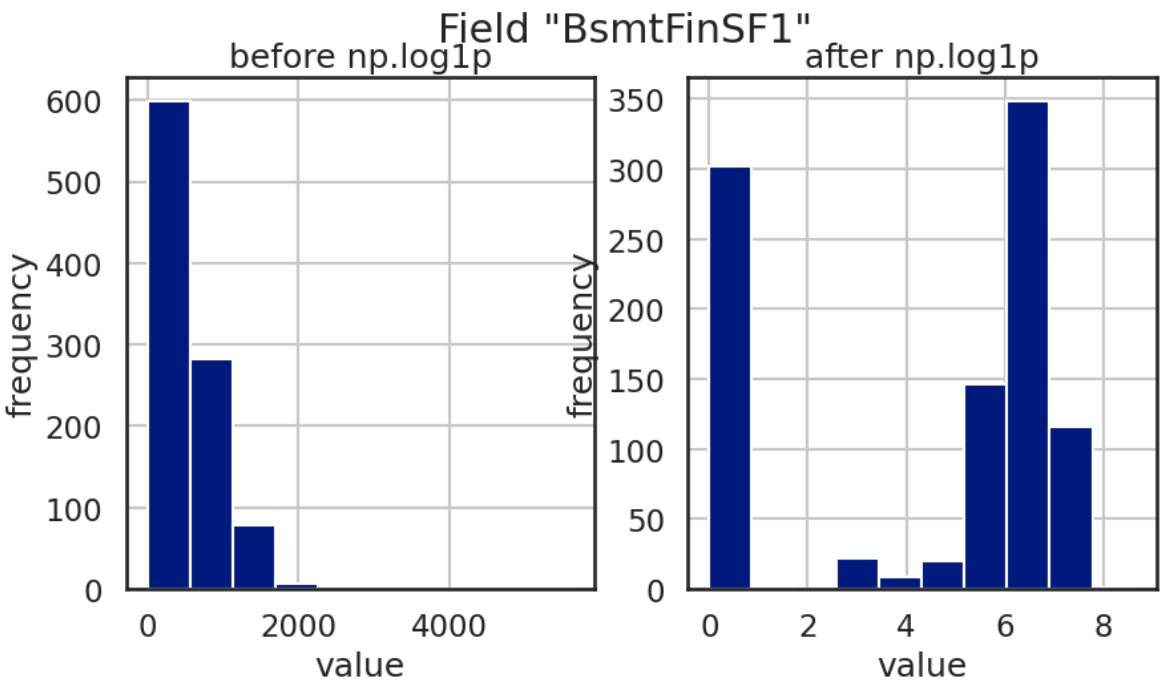
Out[19]:

	Skew
MiscVal	26.915364
PoolArea	15.777668
LotArea	11.501694
LowQualFinSF	11.210638
3SsnPorch	10.150612
ScreenPorch	4.599803
BsmtFinSF2	4.466378
EnclosedPorch	3.218303
LotFrontage	3.138032
MasVnrArea	2.492814
OpenPorchSF	2.295489
SalePrice	2.106910
BsmtFinSF1	2.010766
TotalBsmtSF	1.979164
1stFlrSF	1.539692
GrLivArea	1.455564
WoodDeckSF	1.334388
BsmtUnfSF	0.900308
GarageArea	0.838422
2ndFlrSF	0.773655

Transform all the columns where the skew is greater than 0.75, excluding "SalePrice".

In [20]:

```
#Let's look at what happens to one of these features, when we apply np.log1p vis

field = "BsmtFinSF1"
fig, (ax_before, ax_after) = plt.subplots(1, 2, figsize=(10, 5))
train[field].hist(ax=ax_before)
train[field].apply(np.log1p).hist(ax=ax_after)
ax_before.set(title='before np.log1p', ylabel='frequency', xlabel='value')
ax_after.set(title='after np.log1p', ylabel='frequency', xlabel='value')
fig.suptitle('Field "{}"'.format(field));
# a little bit better
```



In [21]:
```
# Mute the setting wtih a copy warnings
pd.options.mode.chained_assignment = None

for col in skew_cols.index.tolist():
    if col == "SalePrice":
        continue
    train[col] = np.log1p(train[col])
    test[col]  = test[col].apply(np.log1p)  # same thing
```

Separate features from predictor.

In [22]:
```
feature_cols = [x for x in train.columns if x != 'SalePrice']
X_train = train[feature_cols]
y_train = train['SalePrice']

X_test  = test[feature_cols]
y_test  = test['SalePrice']
```

- Write a function `rmse` that takes in truth and prediction values and returns the root-mean-squared error. Use sklearn's `mean_squared_error`.

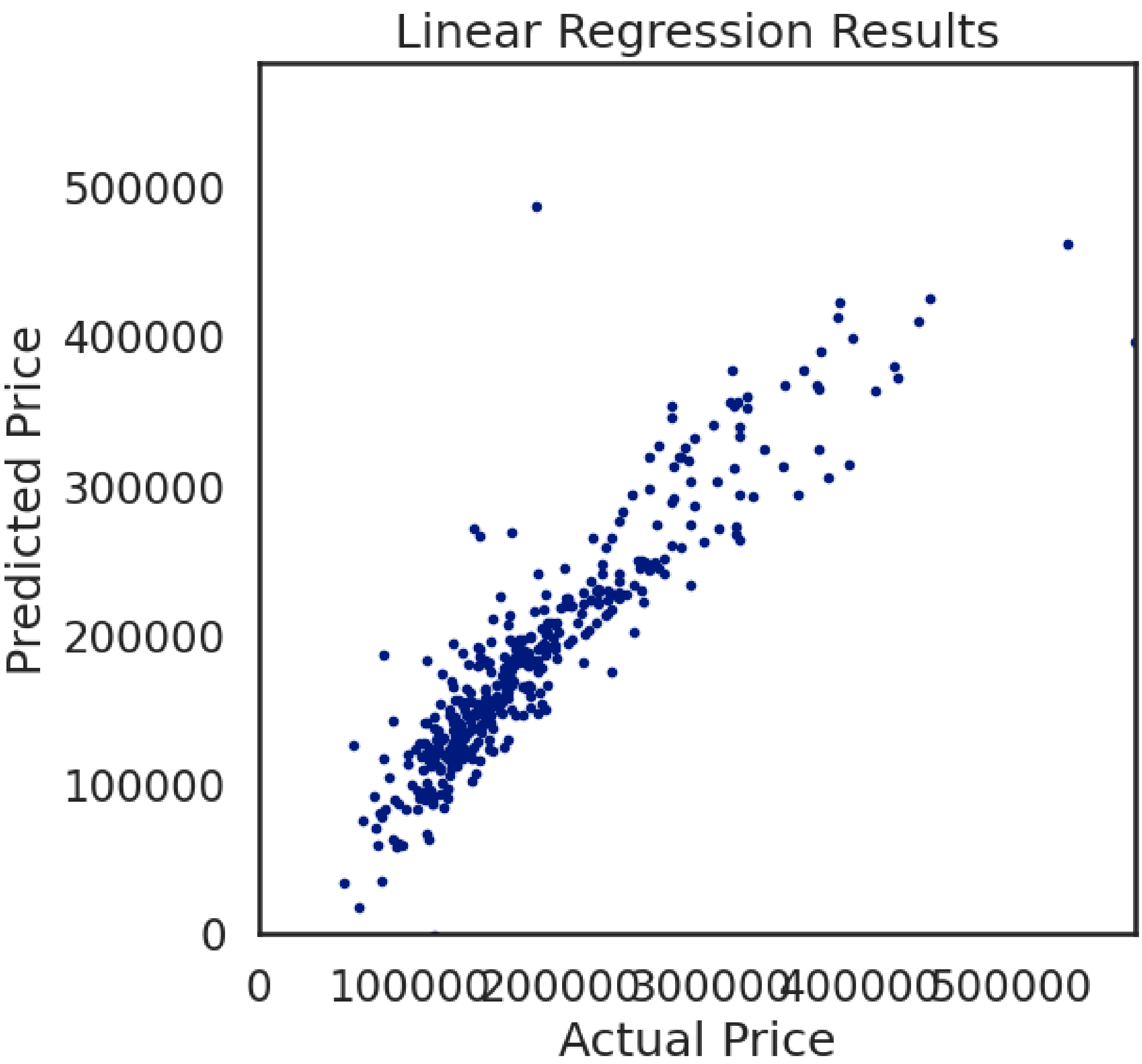
In [23]:
```
from sklearn.metrics import mean_squared_error
def rmse(ytrue, ypredicted):
    return np.sqrt(mean_squared_error(ytrue, ypredicted))
```

- Fit a basic linear regression model
- print the root-mean-squared error for this model
- plot the predicted vs actual sale price based on the model.

In [24]:
```
from sklearn.linear_model import LinearRegression
linearRegression = LinearRegression().fit(X_train, y_train)
linearRegression_rmse = rmse(y_test, linearRegression.predict(X_test))
print(linearRegression_rmse)
```

```
947309.7044202151
```

In [25]:
```
f = plt.figure(figsize=(6,6))
ax = plt.axes()
ax.plot(y_test, linearRegression.predict(X_test),
         marker='o', ls='', ms=3.0)
lim = (0, y_test.max())
ax.set(xlabel='Actual Price',
       ylabel='Predicted Price',
       xlim=lim,
       ylim=lim,
       title='Linear Regression Results');
```



Ridge regression uses L2 normalization to reduce the magnitude of the coefficients. This can be helpful in situations where there is high variance. The regularization functions in Scikit-learn each contain versions that have cross-validation built in.

- Fit a regular (non-cross validated) Ridge model to a range of α values and plot the RMSE using the cross validated error function I created above.
- Use

$$[0.005, 0.05, 0.1, 0.3, 1, 3, 5, 10, 15, 30, 80]$$

as the range of alphas.

- Then repeat the fitting of the Ridge models using the range of α values from the prior section. Compare the results.

Now for the `RidgeCV` method. It's not possible to get the alpha values for the models that weren't selected, unfortunately. The resulting error values and α values are very similar to those obtained above.

In [26]:
```
from sklearn.linear_model import RidgeCV
alphas = [0.005, 0.05, 0.1, 0.3, 1, 3, 5, 10, 15, 30, 80]
ridgeCV = RidgeCV(alphas=alphas,
                  cv=4).fit(X_train, y_train)
ridgeCV_rmse = rmse(y_test, ridgeCV.predict(X_test))
print(ridgeCV.alpha_, ridgeCV_rmse)
```

```
15.0 32195.778260172978
```

Much like the `RidgeCV` function, there is also a `LassoCV` function that uses an L1 regularization function and cross-validation. L1 regularization will selectively shrink some coefficients, effectively performing feature elimination.

The `LassoCV` function does not allow the scoring function to be set. However, the custom error function (`rmse`) created above can be used to evaluate the error on the final model.

Similarly, there is also an elastic net function with cross validation, `ElasticNetCV`, which is a combination of L2 and L1 regularization.

- Fit a Lasso model using cross validation and determine the optimum value for α and the RMSE using the function created above. Note that the magnitude of α may be different from the Ridge model.
- Repeat this with the Elastic net model.
- Compare the results via table and/or plot.

Use the following alphas:
`[1e-5, 5e-5, 0.0001, 0.0005]`

In [28]:
```
from sklearn.linear_model import LassoCV
alphas2 = np.array([1e-5, 5e-5, 0.0001, 0.0005])
lassoCV = LassoCV(alphas=alphas2,
                  max_iter=50000,  # use int, not float
                  cv=3).fit(X_train, y_train)
lassoCV_rmse = rmse(y_test, lassoCV.predict(X_test))
print(lassoCV.alpha_, lassoCV_rmse)
```

```
0.0005 37753.025305153475
```

We can determine how many of these features remain non-zero.

In [29]:
```
print('Of {} coefficients, {} are non-zero with Lasso.'.format(len(lassoCV.coef_
                                                                len(lassoCV.coef_
```

```
Of 283 coefficients, 264 are non-zero with Lasso.
```

Now try the elastic net, with the same alphas as in Lasso, and l1_ratios between 0.1 and 0.9

In [31]:
```
from sklearn.linear_model import ElasticNetCV
l1_ratios = np.linspace(0.1, 0.9, 9)
# Changed max_iter to an integer
elasticNetCV = ElasticNetCV(alphas=alphas2,
                            l1_ratio=l1_ratios,
                            max_iter=10000).fit(X_train, y_train)
elasticNetCV_rmse = rmse(y_test, elasticNetCV.predict(X_test))
print(elasticNetCV.alpha_, elasticNetCV.l1_ratio_, elasticNetCV_rmse)
```

```
0.0005 0.1 35009.076126258275
```

Comparing the RMSE calculation from all models is easiest in a table.

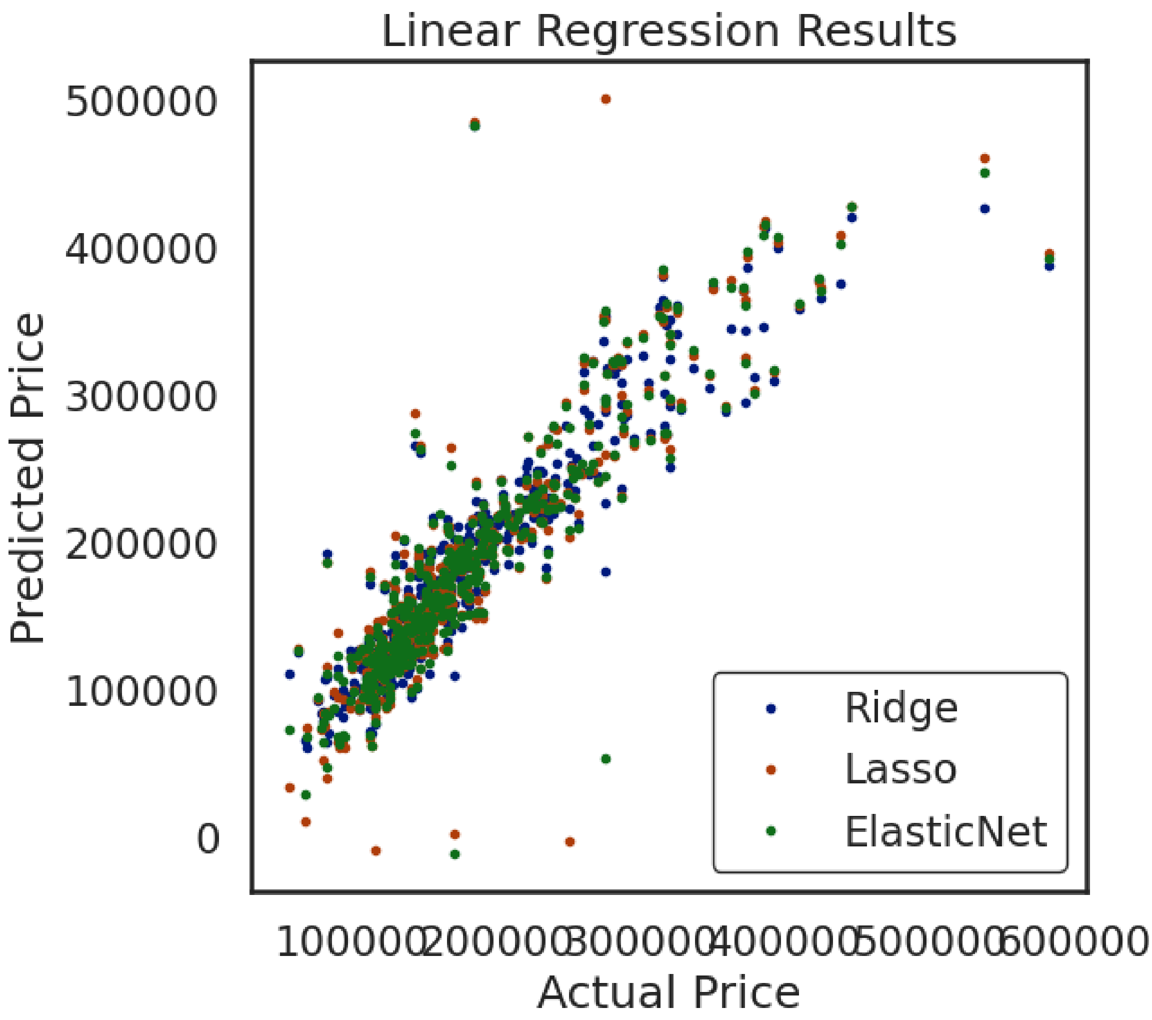
In [32]:
```
rmse_vals = [linearRegression_rmse, ridgeCV_rmse, lassoCV_rmse, elasticNetCV_rms
labels = ['Linear', 'Ridge', 'Lasso', 'ElasticNet']
rmse_df = pd.Series(rmse_vals, index=labels).to_frame()
rmse_df.rename(columns={0: 'RMSE'}, inplace=1)
rmse_df
```

Out[32]:

	RMSE
Linear	947309.704420
Ridge	32195.778260
Lasso	37753.025305
ElasticNet	35009.076126

We can also make a plot of actual vs predicted housing prices as before.

In [33]:
```
f = plt.figure(figsize=(6,6))
ax = plt.axes()
labels = ['Ridge', 'Lasso', 'ElasticNet']
models = [ridgeCV, lassoCV, elasticNetCV]
for mod, lab in zip(models, labels):
    ax.plot(y_test, mod.predict(X_test),
             marker='o', ls='', ms=3.0, label=lab)
leg = plt.legend(frameon=True)
leg.get_frame().set_edgecolor('black')
leg.get_frame().set_linewidth(1.0)

ax.set(xlabel='Actual Price',
       ylabel='Predicted Price',
       title='Linear Regression Results');
```



Let's explore Stochastic gradient descent in this exercise.
Recall that Linear models in general are sensitive to scaling. However, SGD is *very* sensitive to scaling.
Moreover, a high value of learning rate can cause the algorithm to diverge, whereas a too low value may take too long to converge.

- Fit a stochastic gradient descent model without a regularization penalty (the relevant parameter is `penalty`).
- Now fit stochastic gradient descent models with each of the three penalties (L2, L1, Elastic Net) using the parameter values determined by cross validation above.
- Do not scale the data before fitting the model.
- Compare the results to those obtained without using stochastic gradient descent.

In [35]:
```
# Import SGDRegressor and prepare the parameters

from sklearn.linear_model import SGDRegressor

model_parameters_dict = {
    'Linear': {'penalty': None},  # Change 'none' to None
    'Lasso': {'penalty': 'l2',
           'alpha': lassoCV.alpha_},
    'Ridge': {'penalty': 'l1',
           'alpha': ridgeCV_rmse},
    'ElasticNet': {'penalty': 'elasticnet',
                   'alpha': elasticNetCV.alpha_,
```

```
                    'l1_ratio': elasticNetCV.l1_ratio_}
}
new_rmses = {}
for modellabel, parameters in model_parameters_dict.items():
    # following notation passes the dict items as arguments
    SGD = SGDRegressor(**parameters)
    SGD.fit(X_train, y_train)
    new_rmses[modellabel] = rmse(y_test, SGD.predict(X_test))

rmse_df['RMSE-SGD'] = pd.Series(new_rmses)
rmse_df
```

Out[35]:

	RMSE	RMSE-SGD
Linear	947309.704420	1.112710e+16
Ridge	32195.778260	2.163545e+15
Lasso	37753.025305	2.526807e+14
ElasticNet	35009.076126	4.068309e+15

Notice how high the error values are! The algorithm is diverging. This can be due to scaling and/or learning rate being too high. Let's adjust the learning rate and see what happens.

- Pass in `eta0=1e-7` when creating the instance of `SGDClassifier`.
- Re-compute the errors for all the penalties and compare.

In [37]:

```
# Import SGDRegressor and prepare the parameters

from sklearn.linear_model import SGDRegressor

model_parameters_dict = {
    'Linear': {'penalty': None},  # Changed 'none' to None
    'Lasso': {'penalty': 'l2',
           'alpha': lassoCV.alpha_},
    'Ridge': {'penalty': 'l1',
           'alpha': ridgeCV_rmse},
    'ElasticNet': {'penalty': 'elasticnet',
                    'alpha': elasticNetCV.alpha_,
                    'l1_ratio': elasticNetCV.l1_ratio_}
}
new_rmses = {}
for modellabel, parameters in model_parameters_dict.items():
    # following notation passes the dict items as arguments
    SGD = SGDRegressor(eta0=1e-7, **parameters)
    SGD.fit(X_train, y_train)
    new_rmses[modellabel] = rmse(y_test, SGD.predict(X_test))

rmse_df['RMSE-SGD-learningrate'] = pd.Series(new_rmses)
rmse_df
```

Out[37]:

	RMSE	RMSE-SGD	RMSE-SGD-learningrate
Linear	947309.704420	1.112710e+16	74224.212568
Ridge	32195.778260	2.163545e+15	74380.103932
Lasso	37753.025305	2.526807e+14	76540.201863
ElasticNet	35009.076126	4.068309e+15	74020.328742

Now let's scale our training data and try again.

- Fit a `MinMaxScaler` to `X_train` create a variable `X_train_scaled`.
- Using the scaler, transform `X_test` and create a variable `X_test_scaled`.
- Apply the same versions of SGD to them and compare the results. Don't pass in a eta0 this time.

In [38]:

```
from sklearn.preprocessing import MinMaxScaler

scaler = MinMaxScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

new_rmses = {}
for modellabel, parameters in model_parameters_dict.items():
    # following notation passes the dict items as arguments
    SGD = SGDRegressor(**parameters)
    SGD.fit(X_train_scaled, y_train)
    new_rmses[modellabel] = rmse(y_test, SGD.predict(X_test_scaled))

rmse_df['RMSE-SGD-scaled'] = pd.Series(new_rmses)
rmse_df
```

Out[38]:

	RMSE	RMSE-SGD	RMSE-SGD-learningrate	RMSE-SGD-scaled
Linear	947309.704420	1.112710e+16	74224.212568	32803.820000
Ridge	32195.778260	2.163545e+15	74380.103932	77751.873859
Lasso	37753.025305	2.526807e+14	76540.201863	32753.637109
ElasticNet	35009.076126	4.068309e+15	74020.328742	32870.108340

In [39]:

```
from sklearn.preprocessing import MinMaxScaler

scaler = MinMaxScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

new_rmses = {}
for modellabel, parameters in model_parameters_dict.items():
    # following notation passes the dict items as arguments
    SGD = SGDRegressor(**parameters)
    SGD.fit(X_train_scaled, y_train)
    new_rmses[modellabel] = rmse(y_test, SGD.predict(X_test_scaled))
```

```python
rmse_df['RMSE-SGD-scaled'] = pd.Series(new_rmses)
rmse_df
```

Out[39]:

	RMSE	RMSE-SGD	RMSE-SGD-learningrate	RMSE-SGD-scaled
Linear	947309.704420	1.112710e+16	74224.212568	32910.584959
Ridge	32195.778260	2.163545e+15	74380.103932	77725.694135
Lasso	37753.025305	2.526807e+14	76540.201863	32996.928305
ElasticNet	35009.076126	4.068309e+15	74020.328742	32759.016296

Conclusion

This project has provided a hands-on exploration of the interplay between model complexity, regularization, and optimization in the context of polynomial regression. Using a simple yet powerful example — a noisy sinusoidal dataset — we observed how different polynomial degrees influence the model's ability to fit the data, and how easily overfitting can occur in high-capacity models.

To mitigate this, we introduced regularization techniques:

- Ridge Regression (L2), which penalizes large coefficients and stabilizes the model,
- Lasso Regression (L1), which not only reduces overfitting but can also eliminate irrelevant features through coefficient shrinkage.

These techniques proved effective in improving the model's generalization performance, especially in cases where the number of features is high relative to the data size — a common situation in real-world scenarios.

Additionally, we implemented **Gradient Descent** to optimize both regularized and unregularized cost functions. This iterative algorithm gave us insight into how learning rate, convergence, and regularization strength affect model training dynamics. We observed that:

- Regularization modifies the gradient updates to constrain parameter growth,
- The choice of learning rate is critical to ensuring convergence without oscillation or divergence,
- An optimal regularization parameter (λ) can be identified through validation performance, striking the right balance between underfitting and overfitting.

In essence, this project reinforced the following core machine learning principles:

- **More complex models are not always better** — simplicity guided by regularization often leads to better generalization.
- **Regularization is essential** in high-dimensional spaces or when data is noisy and limited.

- **Gradient descent is a versatile and powerful optimization tool**, foundational to training most modern machine learning models.

The insights and techniques applied here are directly transferable to broader machine learning problems — from linear models to deep neural networks — where the same concepts of regularization and optimization continue to play a central role.