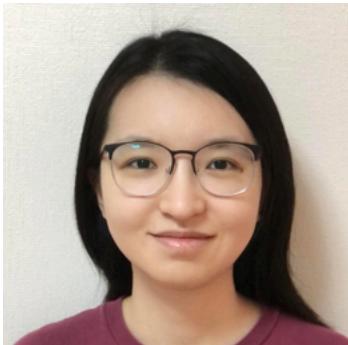


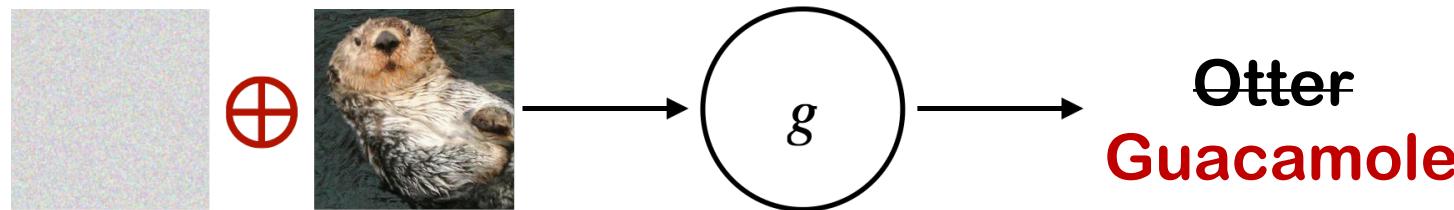
Adaptive Randomized Smoothing: Certified Adversarial Robustness for Multi-Step Defences

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Adversarial Examples

- Adversarial Examples (AE): test-time attacks to control model predictions with small crafted input perturbations.

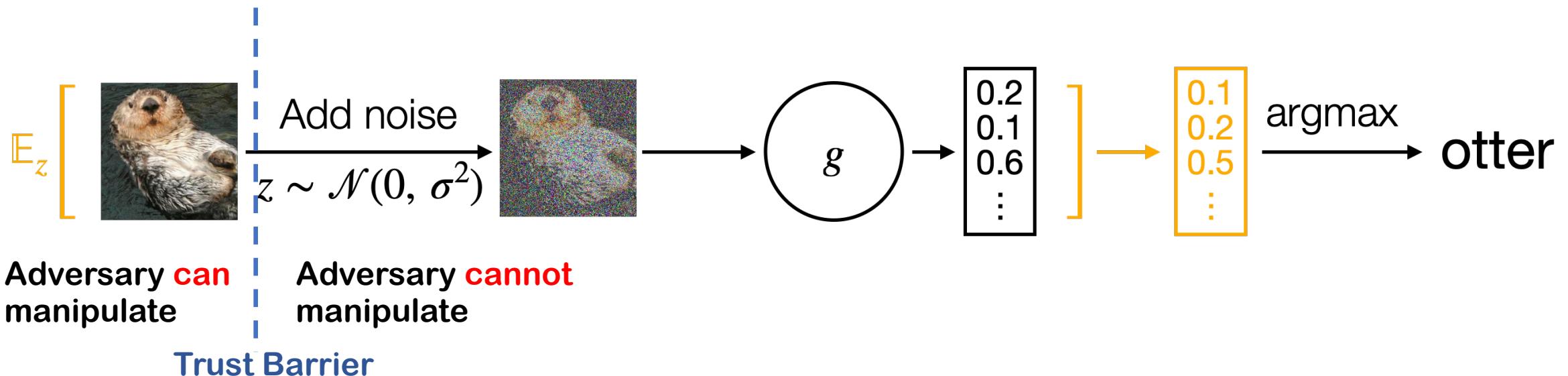


- The power of the adversary is determined by the maximum size of the attack:

$$\ell_2 \text{ attack: } \|\quad\|_2 \qquad \ell_\infty \text{ attack: } \|\quad\|_\infty$$

- Randomized Smoothing can provide provable defenses against adversarial examples!

Randomized Smoothing (RS)



- Randomized Smoothing gives provable robustness by averaging over noisy predictions:
- **Theorem (Cohen et al. 2019):** with $\mathbb{P}(f(X + z) = y_+) \geq \underline{p}_+ \geq \bar{p}_- \geq \max_{y_- \neq y_+} \mathbb{P}(f(X + z) = y_-)$ we have:

$$\text{No } \ell_2 \text{ attack with } \| \quad \|_2 \leq r_X = \frac{\sigma}{2} (\Phi^{-1}(\underline{p}_+) - \Phi^{-1}(\bar{p}_-))$$

certificate

Limitations of RS

- Noise degrades accuracy.
- Difficulty scaling to high dimensional inputs for ℓ_∞ threat models:

$$r_X \leq \|\quad\|_2 \leq \sqrt{d} \|\quad\|_\infty$$

- Does not support test-time adaptivity to adapt the accuracy/robustness trade-off to the input.

We use **Gaussian differential privacy** to address these shortcomings!

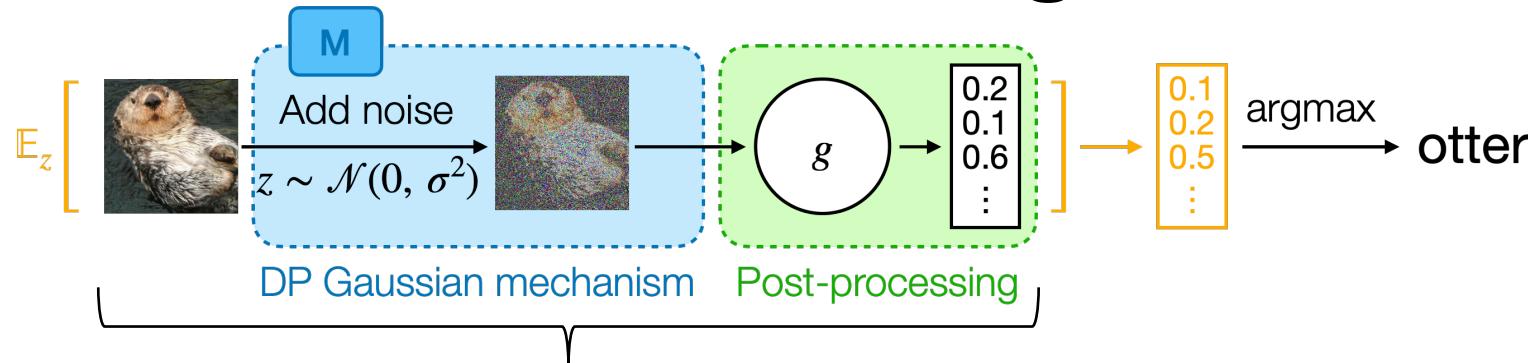
Gaussian Differential Privacy

- We can frame privacy as a hypothesis test between $\mathcal{H}_0: D$ and $\mathcal{H}_1: D'$ (i.e. does $x \in D$?). This enables a hypothesis test definition of DP.
- A tradeoff function f bounds the power of any statistical test of \mathcal{H}_0 v.s. \mathcal{H}_1 .

(Theorem 2.7 Dong et al. 2019) For a Gaussian mechanism $\mathcal{M}(D) = \theta(D) + z, z \sim \mathcal{N}\left(0, \frac{r^2}{\mu^2}\right)$, such that for any neighboring D, D' , $\theta(D) - \theta(D') \in B_2(r)$ (i.e., the ℓ_2 sensitivity of θ is r), we have that \mathcal{M} is G_μ -DP with function $f = G_\mu$ defined by :
$$G_\mu(\alpha) = \Phi(\Phi^{-1}(1 - \alpha) - \mu), \text{ for all } \alpha \in [0, 1]$$

- **Composition:** the composition of an G_{μ_1} -DP Gaussian mechanism and an G_{μ_2} -DP Gaussian mechanism is G_μ -DP Gaussian mechanism with $\mu = \sqrt{\mu_1^2 + \mu_2^2}$.

GDP and Randomized Smoothing

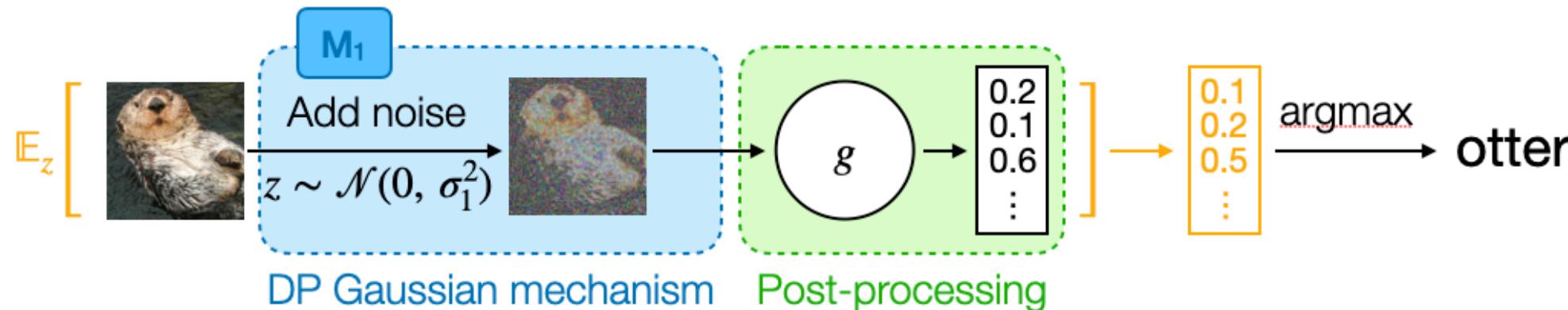


GDP under neighbouring definition $D' = D + \delta, \|\delta\|_p \leq r$.

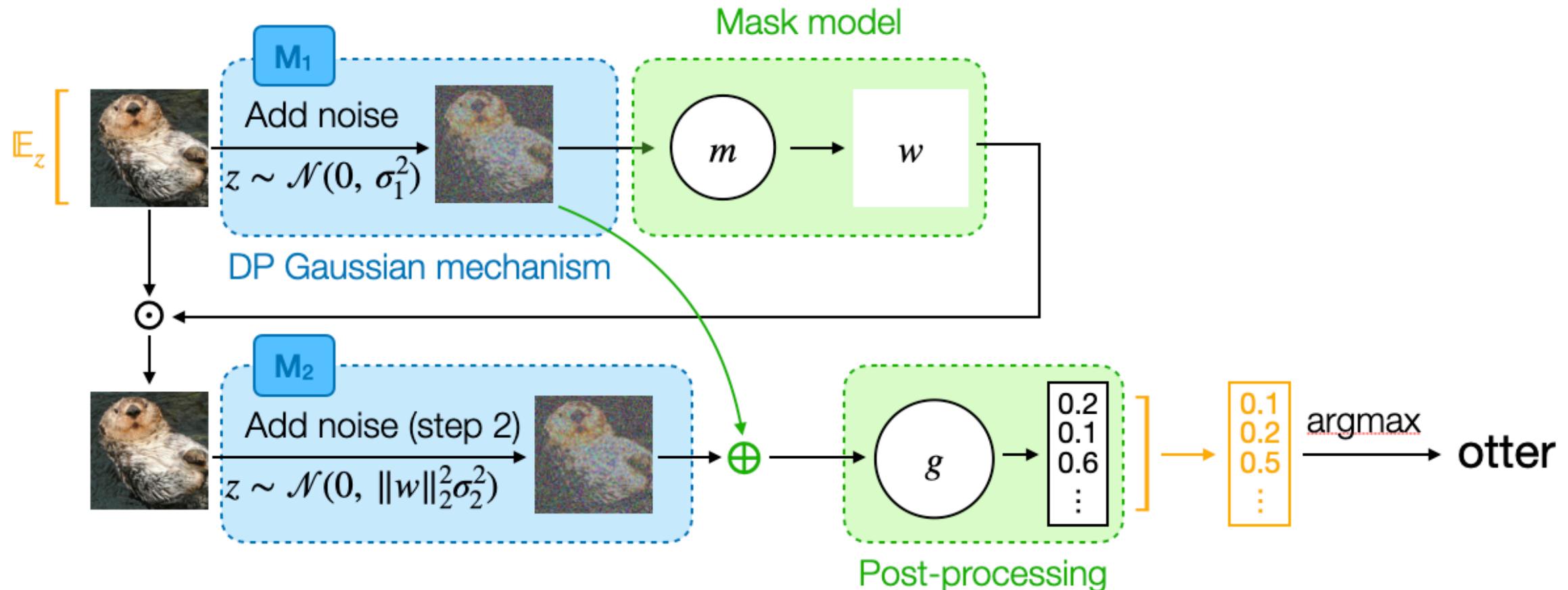
- We prove that our GDP randomized smoothing mechanism satisfies
 $f(1 - \underline{p}_+) \geq 1 - f(\overline{p}_-) \Rightarrow \forall \|\delta\|_p \leq r, M_S(D + \delta) = y_+$
- Using this result and GDP, we prove that:

No ℓ_2 attack is possible with $\|\Delta\|_2 \leq r_X = \frac{\sigma}{2} (\Phi^{-1}(\underline{p}_+) - \Phi^{-1}(\overline{p}_-))$

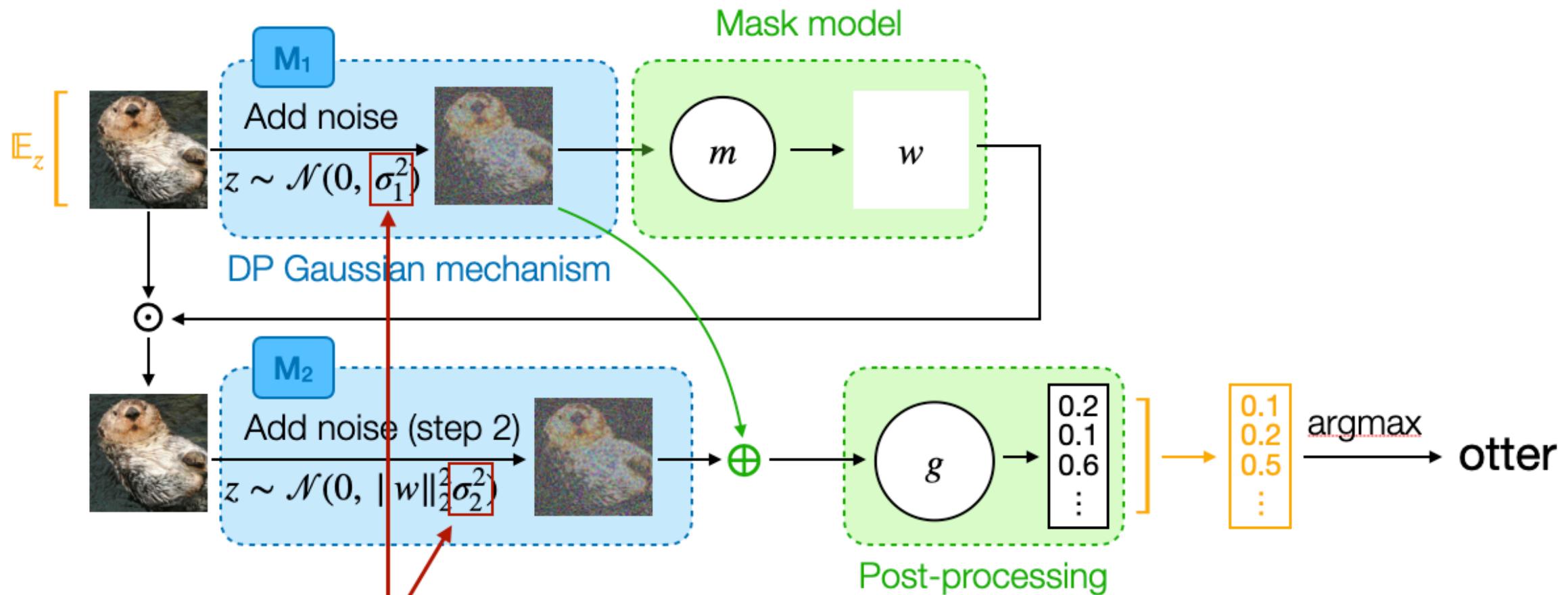
Adaptive Randomized Smoothing for ℓ_∞



Adaptive Randomized Smoothing for ℓ_∞

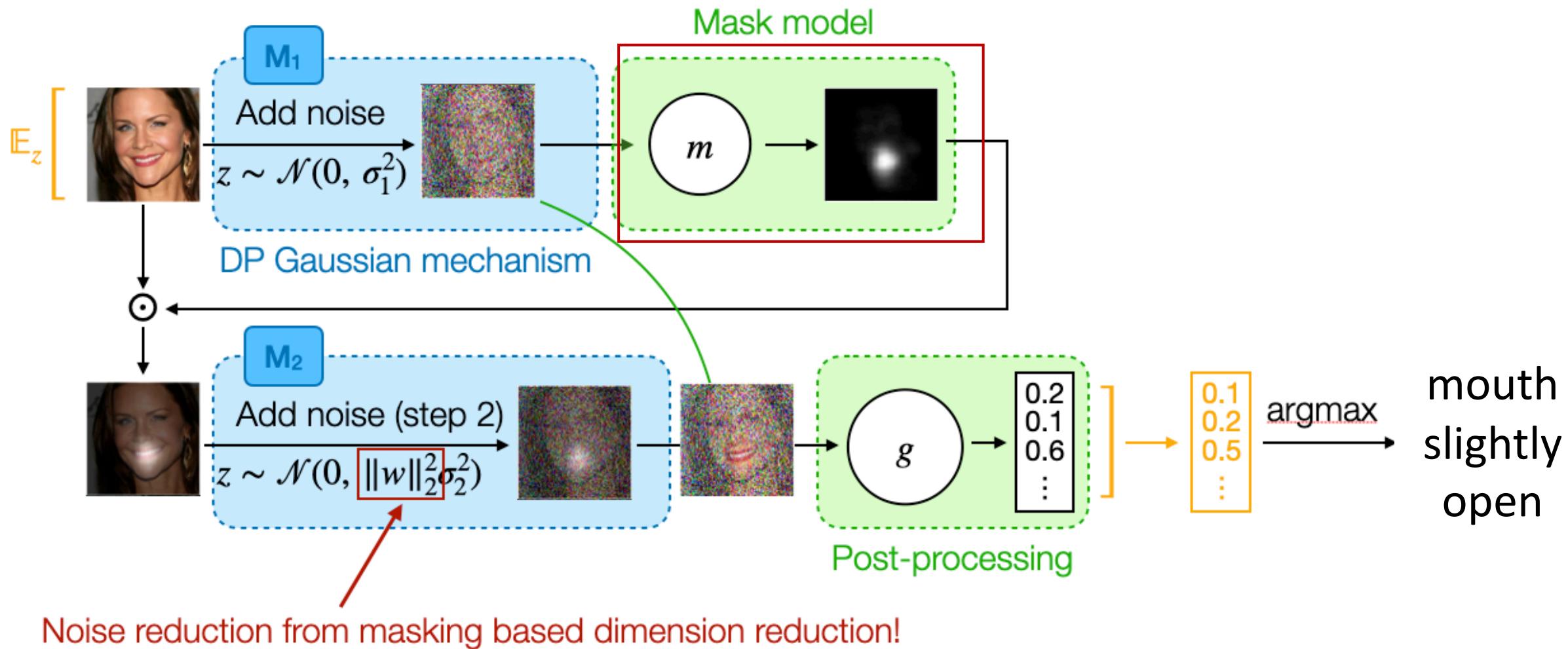


Adaptive Randomized Smoothing for ℓ_∞



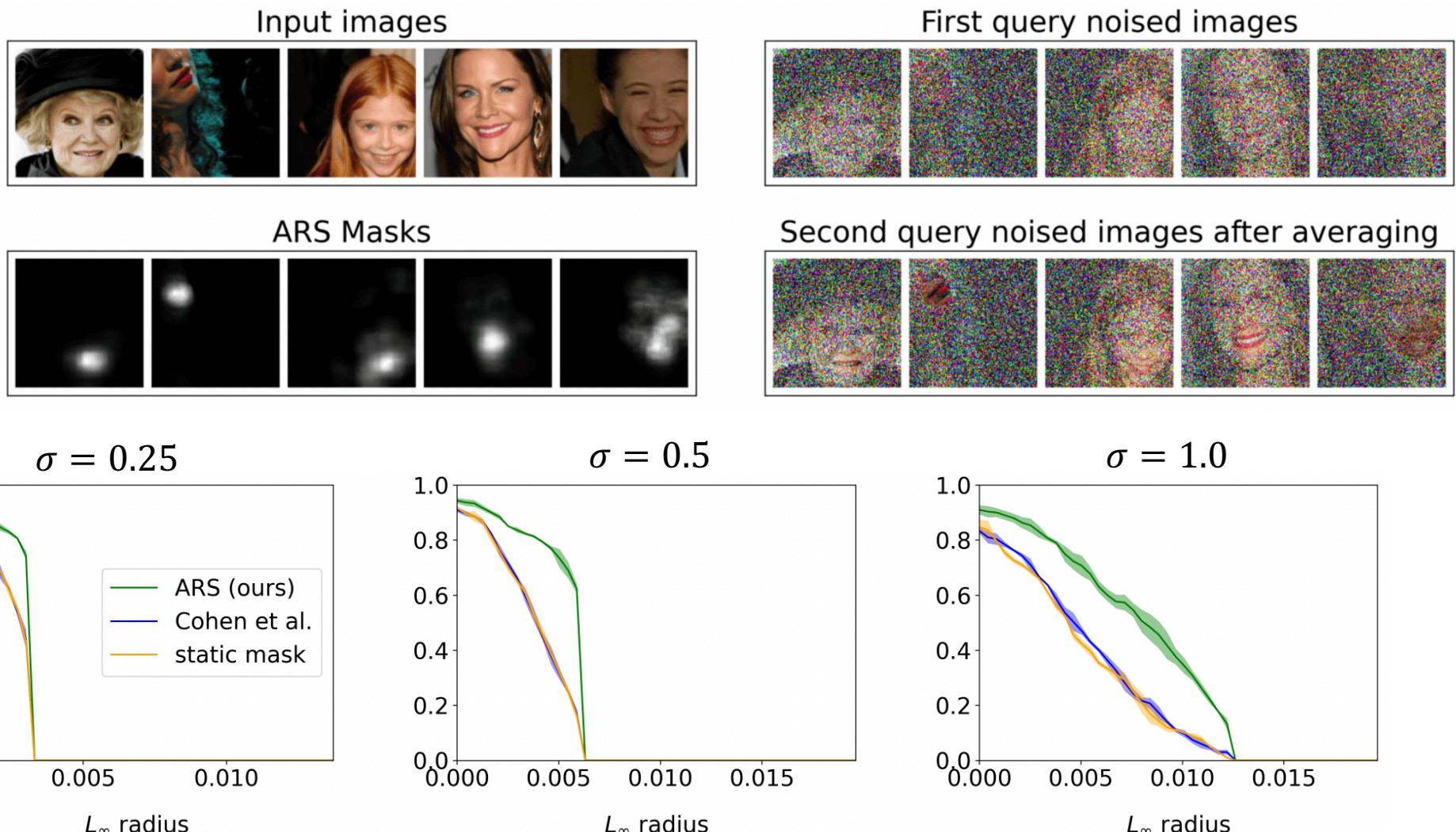
$$\text{From } f\text{-DP composition: } \frac{1}{\sigma^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

Why does ARS help?



Evaluations

- CelebA



Conclusion

- Adaptive Randomized Smoothing (ARS) uses DP composition post-processing properties to certify **adaptive multi-step models**.
- ARS learns to **adjust the scale of noise** based on the test input.
- ARS provides **higher accuracy** at a given level of provable robustness.

Link to our code



