Robust Laguerre based model predictive control of nonholonomic mobile robots under slip conditions

Marzieh Jamalabadi
Department of Mechanical engineering
Amirkabir University of Technology
Tehran, Iran
marziejamalabadi@aut.ac.ir

Mahyar Naraghi
Department of Mechanical engineering
Amirkabir University of Technology
Tehran, Iran
naraghi@aut.ac.ir

Iman Sharifi
Department of Electrical engineering
Amirkabir University of Technology
Tehran, Iran
imansharifi@aut.ac.ir

Elnaz Firouzmand
Department of Electrical engineering
Amirkabir University of Technology
Tehran, Iran
efsh@aut.ac.ir

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***Abstract*—This paper studies the trajectory tracking of a nonholonomic mobile robot. A robust model predictive control based on Laguerre parametrization is proposed. The slip parameter is the source of uncertainty in the constrained mobile robot and is modeled with the bounded additive disturbance. By designing tubes as the positive disturbance invariant set around the nominal robot trajectory, the trajectory tracking is achieved robust to the introduced disturbances. Using Laguerre functions for input signal parametrization in the tube-based model predictive control structure, it is demonstrated that it does remain real-time with better tracking performance. Furthermore, stability is guaranteed with a sub-optimal procedure. Illustrative simulations are presented to show the applicability of the proposed method.**

***Index Terms*--Robust MPC, Tube Based MPC, Nonholonomic Mobile Robots, Laguerre functions, Trajectory Tracking**

I. INTRODUCTION

The problem of trajectory tracking for mobile robots covers an active research direction in the field of robotic systems [1]. Motion constraints of nonholonomic mobile robots are not integrable. These physical limitations impose hard constraints on the actuators of the mobile robots. Furthermore, mobile robots' control suffers from unknown wheel slip parameters in the model, which arises when the robot is navigating under off-road conditions [2]-[4].

Model Predictive Control (MPC) is a popular control strategy for controlling constrained systems while giving an optimal solution in the receding horizon. The constrained finite-time optimal control problem is solved at each step for a given horizon. Only the first member of the optimal controller sequence is implemented in the system [5]. Solving an open-loop optimal control at every time step is a restricting task in some practical scenarios due to the computation time. Considerable efforts have been made into the investigation in keeping the computations time of MPC algorithms as small as possible. In [6] authors use look-up tables to prevent solving optimization problems in real-time. Another approach in reducing computation time is by reducing the number of decision variables resulting from the parameterization of control actions with Laguerre functions [7]. Laguerre function for quadrotor is discussed in [8] and [9] demonstrate how tube-based MPC with Laguerre functions affect the constraints.

The traditional MPC has a degree of robustness to modeling uncertainties. However, when dealing with uncertain systems with external disturbances, it is critical to guarantee MPC's stability and feasibility. Using the min-max approach, the earlier approaches, in which the minimization of objective functions was considered for the worst possible uncertainty, imposed on the constrained systems [10]-[12]. However, these methods are computationally inefficient and only are applicable in small-sized systems or systems with slow dynamics. The other exciting idea in designing robust MPC is by tightening the constrained set of the uncertain system relative to the disturbance set's size. Hence, the MPC problem is solved for the nominal system while guaranteeing the real system trajectory does not violate the constraints in the presence of disturbances [13]-[15]. The authors of [16] address the problem of implementing stabilizing MPC in the virtue of limited computation time. They introduced mild conditions under which the sub-optimal MPC is stabilizable, even if optimality is lost. In [17], orthogonal functions are applied in MPC for LTV systems to reduce the decision variables, resulting in a lower computation time. They proposed using sub-optimality and relaxing the orthogonality of Laguerre functions. The system's performance is satisfactory, and the asymptotic stability and recursive feasibility of the MPC are guaranteed.

This paper aims to design a robust model predictive control with lower decision variables than traditional MPC for trajectory tracking of a nonholonomic mobile robot. Thus, applying Laguerre based MPC approach similar to [17], we extend it to design a robust tube-based model predictive control. Furthermore, we show that the proposed robust LMPC is stabilizable with the sub-optimal method, as presented in [17]. Simulation results for the suitable performance of the proposed control system are provided.

The remainder of the paper is organized as follows. The modeling of a nonholonomic mobile robot under slip conditions is presented in Section 2. The Laguerre based MPC (LMPC) is discussed in Section 3. Afterward, the idea of introducing tubes in LMPC is for achieving robust property to modeling uncertainty is proposed in Section 4. Simulations are presented in Section 5.

II. Modeling

This section presents the mathematical modeling of a mobile robot under slip conditions for trajectory tracking. Consider a nonholonomic mobile robot depicted in Figure 1.

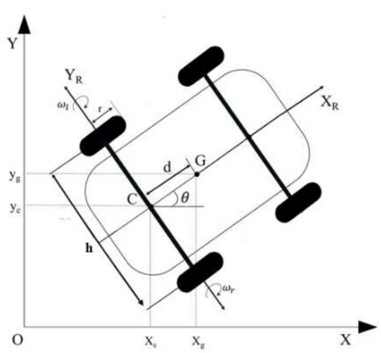

Figure 1. Mobile robot in configuration space

The kinematic modeling of a mobile robot is defined as follows:

$$\dot{x} = r(\frac{\omega_l + \omega_r}{2})\cos\theta; \dot{y} = r(\frac{\omega_l + \omega_r}{2})\sin\theta; \dot{\theta} = r(\frac{\omega_r - \omega_l}{h}) \quad (1)$$

where, ω_l and ω_r denote the angular velocity of the left and right wheels, and h is the distance between the center of the wheels as it showed Figure 1. According to the road condition, there is a difference between the actual and theoretical linear velocity of wheels. Therefore, the slip is formulated as the following

$$i = 1 - \frac{v}{r\omega} \quad (2)$$

Where $0 \le i < 1$ and v and r denote the wheel's actual linear velocity and the wheel radius, respectively. It is assumed while moving at relatively low speeds, the dynamic effects on a robot and lateral slip effects are neglectable [18]. Therefore, by incorporating longitudinal slip effect on mobile robots, it is modeled by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1-i_l}{2} & \frac{1-i_r}{2} \\ -\frac{1-i_l}{h} & \frac{1-i_r}{h} \end{bmatrix} \begin{bmatrix} r\omega_l \\ r\omega_r \end{bmatrix} \quad (3)$$

A reference trajectory in the global frame is adopted to control the mobile robot to track a predefined trajectory. Hence, a real mobile robot has to follow a reference trajectory with the same dynamic as (3) with the state vector defined by (x^{rf}, y^{rf}, $theta^{rf}$). Let us define the error between the reference and the robot as follows:

$$\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^{rf} - x \\ y^{rf} - y \\ \theta^{rf} - \theta \end{bmatrix} \quad (4)$$

By differentiating the error with respect to time and linearize the model around the equilibrium points (details in [18]), it leads to the following linear error dynamic defined on the frame fixed on the mobile robot:

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\theta \end{bmatrix} = \begin{bmatrix} 0 & \dot{\theta}^{rf} & 0 \\ -\dot{\theta}^{rf} & 0 & v^{rf}(t) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} + \begin{bmatrix} \frac{1-i_l}{2} & \frac{1-i_r}{2} \\ 0 & 0 \\ \frac{1-i_l}{h} & -\frac{1-i_l}{h} \end{bmatrix} \begin{bmatrix} r\omega_l^{fb} \\ r\omega_r^{fb} \end{bmatrix} \quad (5)$$

The discrete-time linear model of the error dynamic with sampling time T_s is obtained as follows:

$$\begin{bmatrix} e_x(k+1) \\ e_y(k+1) \\ e_\theta(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T_s\dot{\theta}^{rf}(k) & 0 \\ -T_s\dot{\theta}^{rf}(k) & 1 & T_s v^{rf}(k) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_x(k) \\ e_y(k) \\ e_\theta(k) \end{bmatrix} + \begin{bmatrix} T_s\frac{1-i_l}{2} & T_s\frac{1-i_r}{2} \\ 0 & 0 \\ -T_s\frac{1-i_l}{h} & T_s\frac{1-i_r}{h} \end{bmatrix} \begin{bmatrix} r\omega_l^{fb}(k) \\ r\omega_r^{fb}(k) \end{bmatrix} \quad (6)$$

As we do not know the exact value of slip factor, we divided that into two parts; nominal part $(\bar{i})$ and uncertain part (Δi) which is bounded to $0 \le \bar{i} + \Delta i < 1$.

$$i = \bar{i} + \Delta i \quad (7)$$

Now by substituting (7) in (6), the uncertainty that appeared in the equations of motion can be modeled with a bounded additive disturbance term as follows:

in which, the disturbance has abounded time-varying nature.

$$\begin{bmatrix} e_x(k+1) \\ e_y(k+1) \\ e_\theta(k+1) \end{bmatrix} =$$

$$\begin{bmatrix} 1 & T_s\dot{\theta}^{rf}(k) & 0 \\ -T_s\dot{\theta}^{rf}(k) & 1 & T_s v^{rf}(k) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_x(k) \\ e_y(k) \\ e_\theta(k) \end{bmatrix}$$

$$+ \begin{bmatrix} T_s\frac{1-\bar{i}_l}{2} & T_s\frac{1-\bar{i}_r}{2} \\ 0 & 0 \\ -T_s\frac{1-\bar{i}_l}{h} & T_s\frac{1-\bar{i}_r}{h} \end{bmatrix} \begin{bmatrix} r\omega_l^{fb}(k) \\ r\omega_r^{fb}(k) \end{bmatrix}$$

$$\underbrace{+ \begin{bmatrix} -T_s\frac{\Delta i_l}{2} & -T_s\frac{\Delta i_r}{2} \\ 0 & 0 \\ T_s\frac{\Delta i_l}{b} & -T_s\frac{\Delta i_r}{b} \end{bmatrix} \begin{bmatrix} r\omega_l^{fb}(k) \\ r\omega_r^{fb}(k) \end{bmatrix}}_{d(k)} \Rightarrow$$

$$e(k+1) = A(k)e(k) + Bu(k) + d(k), \tag{8}$$

$$d(k) = -rT_s \begin{bmatrix} \frac{\omega_l^{fb}}{2} & \frac{\omega_r^{fb}}{2} \\ 0 & 0 \\ -\frac{\omega_l^{fb}}{b} & \frac{\omega_r^{fb}}{b} \end{bmatrix} \begin{bmatrix} \Delta i_l \\ \Delta i_r \end{bmatrix}$$

III. LAGUERRE BASED MODEL PREDICTIVE CONTROL

Laguerre functions are obtained by computing the inverse z-transform of Laguerre networks and can be formulated with the following dynamic model:

$$\Gamma(k+1) = A_l\Gamma(k) \tag{9}$$

where the initial Laguerre vector ($\Gamma(0) \in R^n$, n is the degree of the Laguerre network) and the state matrix is defined by:

$$A_l = \begin{bmatrix} a & 0 & 0 & \cdots & 0 \\ \beta & a & 0 & \cdots & 0 \\ -a\beta & \beta & a & \cdots & 0 \\ a^2\beta & -a\beta & \beta & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (-1)^{N-2}a^{N-2}b & (-1)^{N-3}a^{N-3}b & \cdots & \beta & a \end{bmatrix}; \tag{10}$$

$$\Gamma(0) = \sqrt{\beta} \begin{bmatrix} 1 \\ -a \\ a^2 \\ -a^3 \\ \vdots \\ (-1)^{N-1}a^{N-1} \end{bmatrix}$$

in which, a is the pole of the Laguerre network and $\beta = 1 - a^2$. In order to decrease the computation time in MPC, the sequence of control signals (v) over a prediction horizon is estimated by Laguerre functions, which results are in the following transformation:

$$v = \Gamma^T\eta \tag{11}$$

where η is the Laguerre coefficients, which comprise the new decision vector in MPC. According to the Laguerre network's degree, the number of decision variables can be decreased compared to the traditional MPC. The final LMPC is formulated as follows at step k_i:

$$J(e_{k_i}, \eta_{k_i}; k_i) = \|e(k_i + N_p)\|_P^2 + \sum_{k=k_i}^{k_i+N_p-1} \|e(k)\|_Q^2 + \|u(k)\|_R^2$$

$$J^*(e_{k_i}; i) = \min_{\eta_{k_i}} J(e_{k_i}; k_i)$$

$$\eta_{k_i}^* = \arg\min_{\eta_{k_i}} J(e_{k_i}, \eta_{k_i}; k_i)$$

subject to (8)

$$u(N_P; k_i) = \Gamma\eta_{k_i}$$

$$u(k) \in U(k), k = k_i, k_i+1, \ldots, k_i+N_P-1$$

$$e(k) \in E(k), k = k_i, k_i+1, \ldots, k_i+N_P-1 \tag{12}$$

where J is the cost function for a state e_{k_i} which initiates at step k_i, N_p is the prediction horizon; P and Q are the PSD (positive semi-definite) weight matrices used as a terminal cost and stage cost [13], respectively, and R is the PD weight matrix for control input.

IV. ROBUST LMPC

After linearization and discretization, including the additive uncertainty term described in (8), the tube-based model predictive controller is designed as follows.

The problem that we introduced in (8) is a linear and discrete-time system formulated in the general form:

$$e^+ = Ae + Bu + d, \qquad e \in E, u \in U \tag{13}$$

where $e \in R^3$ and $u \in R^2$, denote the current error and control, respectively. $E \subset R^3$ and $U \subset R^2$ constraints are polytopes, and each set contains the origin in its interior. e^+ is the successor error; d is bounded with $d \in D$ limitation where $D \subset R^3$ is a polytope which is containing the origin, as well. Based on Tube-based theory [13], the control signal is formulated as follows:

$$u = \bar{u} + K(e - \bar{e}) \tag{14}$$

in which K is chosen to be stabilizing (make A_K=A+BK stable) and, $u = Kx$ is the optimal controller for the unconstrained (A, B, Q, R) problem. Furthermore, $\bar{e}$ is the nominal error state and $\bar{u}$ is obtained by solving the LMPC problem for a nominal system (disturbance-free) as described by:

$$\bar{e}^+ = A\bar{e} + B\bar{u}, \quad \bar{e} \in E \ominus Z, \bar{u} \in U \ominus KZ \tag{15}$$

where $\ominus$ denotes set subtraction and Z is a disturbance invariant set [19] (note that $U \ominus KZ$ is not an empty set), for the controlled uncertain system (16):

$$y^{+} = A_k y + d \tag{17}$$

with the definition of:

$$y = e - \bar{e} \tag{18}$$

Proposition [13]: if $e \in \bar{e} + Z$, then $e^{+} = Ae + B(\bar{u} + k(e - \bar{e})) + d$ and $\bar{e}^{+} = A\bar{e} + B\bar{u}$ satisfy $e^{+} \in \bar{e}^{+} \oplus Z$ for all $u \in U$, all $d \in D$.

Lemma : Reference [17] has proven the stability of the MPC method for linear systems which parameterized by orthonormal functions (like Laguerre functions); according to that, for the nominal system, find the $\bar{e}$ and $\bar{u}$ then if $e \in \bar{e} + Z$, and $e \in E_f$ then $e^{+} = Ae + B(\bar{u} + k(e - \bar{e})) + d$ and $\bar{e}^{+} = A\bar{e} + B\bar{u}$ satisfy $e^{+} \in \bar{e}^{+} + Z$ for all $u \in U$, all $d \in D$.

Proof: if the initial state is in the feasible set (E_f), there exists $\bar{u}$ obtained by suboptimal MPC (similar to MPC based Laguerre Functions), that guarantees asymptotic stability (feasibility implies stability) [16], then there exists a tube with the center of nominal states ($\bar{e}$) that computed by suboptimal MPC controller input ($\bar{u}$: first element of $\bar{u}$ sequence in the prediction horizon at every step). Because of the disturbance, the real state (e) has deviation (y) from its center, and this deviation belonges to the disturbance invariant set; so the system remains robust.

V. Simulations

To demonstrate the applicability of the proposed method, we have provided a scenario for simulation. In this scenario, we consider a trajectory with a circle shape.

Consider a nonholonomic mobile robot formulated with Equation (8). It is worth noting that under circular trajectories, the mobile robot model is considered an LTI system. The angular velocity of mobile robot wheels is bounded to ($-20 \leq \omega_r, \omega_l \leq 20$ both in *rad/s*); i.e. modelling the practical limitations of actuators. Error constraints are ($-0.5m \leq e_x, e_y \leq 0.5m$, $-0.5rad \leq e_\theta \leq 0.5rad$). We consider a sampling time T_s = 0.2s for discretizing the model and the prediction horizon of Np=15 for MPC. The state and input penalty matrices are considered as follows:

$$Q = 100\begin{bmatrix} 30 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, R = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

The degree of N=[2,2] and the pole a=0.9 are chosen to define a Laguerre network. The additive time-varying disturbance appeared in (8) is considered as $-0.015 \leq d_1, d_3 \leq 0.015$; $d_2 = 0$, which has been chosen randomly; that should be noted, imposing disturbance in which state, affect the other states because the system is coupled. The mobile robot's parameters are h=0.15m, r=0.05m, $\bar{\iota}_l, \bar{\iota}_r = 0.15$.

The trajectory is defined by:

$$x_{rf} = 3\cos(\frac{\pi}{16}t),\ y_{rf} = -3\sin(\frac{\pi}{16}t)$$

The initial error is [-10cm, -20cm, -0.2rad].

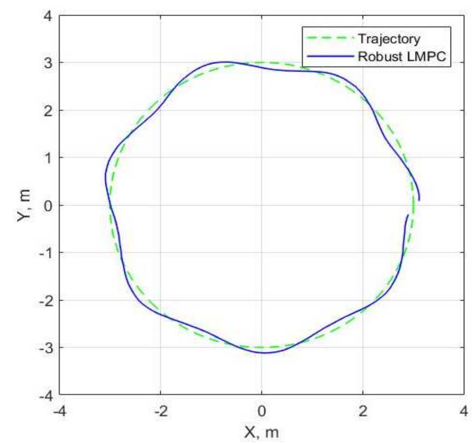

Figure 2. Trajectory tracking of mobile robot

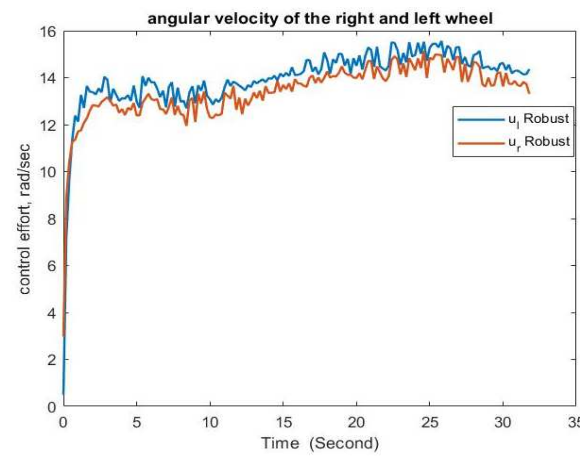

Figure 3. Control of robust LMPC for the right and left wheel

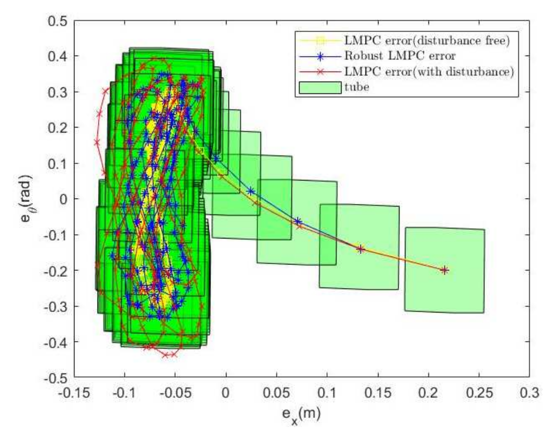

Figure 4. error in x and θ direction

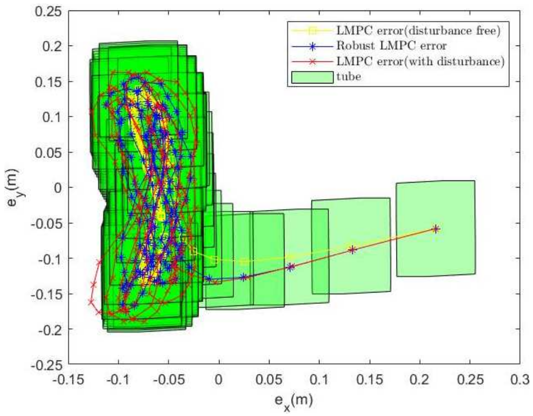

Figure 5. error in x and y direction

TABLE 1. ERROR AND EXECUTION TIME FOR CIRCLE TRAJECTORY

Trajectory	**Simulation results**			
	Execution time(average) per step (sec)	***Max Execution Time (sec)***	***Min Execution Time (sec)***	***Final error [e_x, e_y, e_θ] (In cm & deg)***
LMPC case	0.0288	0.047	0.023	[13.3;9.5;5.9]
Robust LMPC case	0.052	0.084	0.045	[11.3;8.2;-1.5]

As shown from Figure 2 robot with robust LMPC, followed the reference trajectory with satisfactory performance. The control that comes from robust LMPC is shown in Figure 3 with handling the constraints. The robustness of the proposed controller has demonstrated in Figure 4 and Figure 5 which shows robust LMPC remains in the tube along the trajectory against LMPC results. It is shown in the LMPC case, multiple times the trajectory of error came out from the tube, especially after rejecting the initial error and when the disturbance free case, reached the steady state error.

Data of the execution time came into Table 1; which is shown that according to the time step (T_s is 0.2s) and by imposing a constraint on max iteration in optimization problem [20], it does remain real-time.

The simulations have been performed on a system with a Core i7 2.4 GHz CPU and 8 GB of RAM. The problem formulation and modeling are done in Matlab and with Multi-Parametric Toolbox (MPT) [21]. It is an open-source toolbox that is available for computational geometry and parametric optimization.

VI. CONCLUSIONS

In this paper, the case study is trajectory tracking of a nonholonomic mobile robot under slip conditions. The wheel slip is the source of uncertainty in this study. A circular trajectory has been examined with satisfactory performance. Using Laguerre functions, decision variables decreased compared to traditional MPC, causing a decrease in computation cost. Designing tubes provided constraint fulfillment together with suitable robustness and better performance.

VII. REFERENCES

1. Scaglia, et al., *A linear-interpolation-based controller design for trajectory tracking of mobile robots.* Control Engineering Practice 2010. **18**(3): p. 318-329.
2. Gonzalez, et al., *Adaptive control for a mobile robot under slip conditions using an LMI-based approach.* European Journal of Control 2010. **16**: p. 144-155.
3. Iossaqui, Camino, and Zampieri, *Adaptive tracking control of tracked mobile robots with unknown slip parameter*, in *XVIII Congresso de Automatica* 2010: Brasileiro.
4. Wong, *Theory of ground vehicles*. 2008: John Wiley & Sons.
5. Borrelli, Bemporad, and Morari, *Predictive control for linear and hybrid systems*. 2017: Cambridge University Press.
6. Bemporad, et al., *The explicit linear quadratic regulator for constrained systems.* Automatica, 2002. **38**(1): p. 3-20.
7. Wang, *Discrete model predictive controller design using Laguerre functions.* Journal of Process Control. **14**(2): p. 131-142.
8. Villarreal, Rossiter, and Shin, *Laguerre-based Adaptive MPC for Attitude Stabilization of Quad-rotor*, in *2018 UKACC 12th International Conference on Control (CONTROL)*. 2018, IEEE: Sheffield, UK.
9. Spasic, et al., *Tube Model Predictive Control based on Laguerre functions with an Auxiliary Sliding Mode Controller*, in *IEEE 15th International Symposium on Intelligent Systems and Informatics*. 2017: Subotica, Serbia.
10. Bemporad, Borrelli, and Morari, *Min-max control of constrained uncertain discrete-time linear systems.* IEEE Transactions on automatic control 2003. **48**: p. 1600-1606.
11. Lazar, et al., *On input-to-state stability of min–max nonlinear model predictive control.* Systems & Control Letters, 2008. **57**(1): p. 39-48.
12. Scokaert and Mayne, *Min-max feedback model predictive control for constrained linear systems.* IEEE Transactions on Automatic Control, 1998. **43**: p. 1136-1142.
13. Mayne, Seron, and Raković, *Robust model predictive control of constrained linear systems with bounded disturbances.* Automatica, 2005. **41**(2): p. 219-224.
14. Nikou and Dimarogonas, *Decentralized tube-based model predictive control of uncertain nonlinear multiagent systems.* International Journal of Robust and Nonlinear Control 2019. **29**(10): p. 2799-2818.
15. Rakovic, et al., *Simple robust control invariant tubes for some classes of nonlinear discrete time systems*, in *Proceedings of the 45th IEEE Conference on Decision and Control*. 2006, IEEE.
16. Scokaert, Mayne, and Rawlings, *Suboptimal model predictive control (feasibility implies stability).* IEEE Transactions on Automatic Control 1999. **44**(3): p. 648-654.
17. Hemmasian Ettefagh, et al., *Orthonormal function parametrisation of model predictive control for linear time-varying systems.* International Journal of Systems Science, 2018.
18. González, Rodríguez, and Luis Guzmán, *Autonomous Tracked Robots in Planar Off-Road Conditions*. Modelling, Localization and Motion Control. 2014: Springer.
19. Kolmanovsky and Gilbert, *Theory and computation of disturbance invariant sets for discrete-time linear systems.* Mathematical Problems in Engineering, 1998. **4**: p. 317-367.
20. K, M, and A, *Suboptimal model predictive control of a laboratory crane*, in *8th IFAC Symposium on Nonlinear Control Systems*. 2010: University of Bologna, Italy.
21. Herceg, et al., *Multi- parametric toolbox 3.0*, in *European control conference (ECC)*. 2013, IEEE. p. 502-510.