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Control of an Omnidirectional Mobile Robot

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Abstract

Holonomic or nonholonomic omnidirectional mobile robots are known to be constructed by applying a specialized wheel or mobile mechanism. In particular, some representative control approaches are described for a holonomic and omnidirectional mobile robot with three lateral orthogonal-wheel assemblies. The uses of resolved acceleration control method, PID method, fuzzy model method, and stochastic fuzzy servo method are highlighted.

1. Introduction

Recently, a wheeled mechanism is now widely used as a mobile mechanism. Two independent driving wheels mechanism, or front-wheel steering and rear-wheel driving mechanism is well-known as a representative one, whose mechanism is often used for a conventional mobile robot. Note, however, that such a mechanism has a nonholonomic constraint, so that it prevents the robot from realizing a high mobility. It is required that a suitable desired trajectory satisfying the above constraint must be designed to control a nonholonomic mobile mechanism. In addition, there exists a problem that a feedback controller can not easily designed to control the nonholonomic robot accurately.

On the other hand, an industrial mobile robot has been required to have some abilities that it can freely travels in a confused factory and achieve an accurate positioning in a work station. For this purpose, an active research is now focused on the study of omnidirectional mobile robots. The omnidirectional mobile robot has been studied by using a variety of mechanisms. In particular, a holonomic vehicle has a full omnidirectionality with simultaneous and independently controlled rotational and translational motion capabilities.

In this paper, we first review several specialized wheel or mobile mechanisms for constructing an omnidirectional mobile robot. Then, we develop a dynamic model for an omnidirectional mobile robot, in which it is assumed that the platform is based on three lateral orthogonal-wheel assemblies. Then, in order to develop a control system with high performance for such a mobile robot, we discuss on resolved acceleration control method, PID method, fuzzy model method, and stochastic fuzzy servo method.

2. Review of Specialized Wheel or Mobile Mechanisms

The holonomic or nonholonomic omnidirectional mobile robots has been studied by using a variety of mechanisms. In other wards, several omnidirectional platforms have been known to be realized by developing a specialized wheel or mobile mechanism. From this point of view, such specialized mechanisms suitable for constructing an omnidirectional mobile robot are reviewed here.

2.1 Steered wheel mechanism

Nakano *et al.* [1],[2] has already developed a noholonomic-type omnidirectional mobile robot by using a driving wheel with steering, which needs a preparatory motion for arranging the wheel directions in a translational or rotational motion. Nevertheless, this mobile robot has some merits that the mechanism is simple, a specialized tire is not used, and there is no coupling between the actuator for driving the wheel and the actuator for steering [2].

2.2 Universal wheel mechanism

In the development of holonomic and omnidirectional mobile robot, a universal wheel (or Swedish wheel) concept [3] is well known for us. The wheel mechanism, which involves a large wheel with many small rollers mounted on its rim, can easily realize a holonomic and omnidirectional motion, if such wheels more than three are mounted on a platform and allocated at different directions. It is known that this type of wheels suffers from the successive shocks caused when individual rollers make contact with the ground.

2.3 Omni-alpha mechanism

Omni-alpha mechanism was developed to improve the remedy of the universal wheel mechanism. The original omni-alpha wheel is a combination of two plates, one of which has three barreled free-rollers. The three free-rollers of each plate are allocated with 120 [deg] so that the external form is consistent with the curvature of the wheel. Combining of two plates with 60 [deg] shift, the outer envelope of six free rollers constructs the external form of the wheel. Thus, this wheel can make successive contact with the ground, keeping a constant distance from a contact point. Asama *et al.* [4] developed an omnidirectional mobile robot with three degree-of-freedom decoupling drive mechanism and with a modified omni-alpha wheel in which one plate of the wheel consists with four barreled free-rollers.

2.4 Ball wheel mechanism

A nonholonomic mobile robot with two ball wheels was developed by Machida *et al.* [5]. This nonholonomic vehicle has been shown to have an omnidirectionality with controlled rotational and translational motion capabilities by using a particular control system based on the concept of a chained form. However, note that this type of robots can not achieve any three degree-of-freedom velocities at any time, because of its nonholonomic property. On the other hand, West and Asada studied a holonomic omnidirectional mobile robot using three ball wheels to achieve a full mobility and improved dead reckoning performance [6].

2.5 Orthogonal wheel mechanism

Pin and Killough [7] developed a holonomic and omnidirectional mobile robot with an orthogonal wheel concept that provides normal traction in a given direction while being free-wheeling in the other perpendicular direction. Two possibly assembly configurations of the wheels, labeled longitudinal and lateral assemblies were discussed and a prototype platform was also constructed by using the former assembly, in which its corresponding control system was implemented in a teleoperated mode or in a user-provided trajectory mode. A holonomic and omnidirectional mobile robot was also constructed in Tang *et al.* [8] by applying three lateral assemblies; however its construction is slightly more complicated than the longitudinal one, because it needs a gear or transmission belt to synchronize two rounded-tire wheels.

2.6 Crawler mechanism

West and Asada [9] and Nishikawa *et al.* [10] studied a holonomic and omnidirectional mobile robot called the Omnitrack by utilizing two rows of full spheres as rolling units, in which the ball wheels or spherical tires are arranged in two conveyer belts which produce forward and rotational motion of the platform. Two controlled rods in each track contact the top of the balls and, by rotating around an axis parallel to the track, provide sideways motion of the platform. Although the mechanism provides holonomic and omnidirectional motion, the rotational degree of freedom of the platform is extremely difficult to control because, like all tracked or skid-steer vehicles, significant sideways slippage of part of the track must occur during turns. Note also that it may lead to significant odometer and position tracking problems, because some of the ball wheels can temporarily lose contact with the ground on uneven terrain.

In order to improve this shortcoming, Hirose and Amano [11] developed a special-type crawler called the Vuton crawler, whose crawler has some cylindrical free rollers. As a result, a holonomic and omnidirectional mobile robot with four such crawlers has no sideways slippage and can bear a relatively high payload. Hirano *et al.* [12] also developed a composite crawler with free rollers, so that a holonomic and omnidirectional mobile robot with four such crawlers can run on offload. However, note that this vehicle can not bear a high payload, because it has relatively narrower contact space with the ground, compared to the vehicle with Vuton crawler.

2.7 Offset steered wheel mechanism

As suggested in above, the steered wheel mechanism of Nakano *et al.* [1], [2] requires a preparatory motion for arranging the wheel directions in a translational or rotational motion, because the steering axis is taken to be sideways (or perpendicular) to the rolling direction of the wheel, keeping a constant offset. In this steered wheel with sideways offset, when rotating the steering axis, the steering axis itself rotates with a circular motion around the contact point of the wheel, due to the reaction of the driving torque. Then, the translational velocity generated around the steering axis is always parallel to the wheel, and therefore the moving velocity of the steering axis is consequently parallel to the wheel when the wheel axis and the steering axis are simultaneously driven. Thus, the steered wheel with sideways offset can achieve the only one degree-of-freedom motion.

On the contrary, a steered wheel mechanism with forward offset was proposed by Wada *et al.* [13], in which the steering axis is taken to be forward to the rolling direction of the wheel, keeping a constant offset. In this mechanism, the translational velocity generated around the steering axis is always perpendicular to the wheel, and the translational velocity due to the wheel driving directs the direction of the wheel movement, so that they always give the orthogonal velocities to the steering axis. Thus, by controlling their velocities, the moving velocity of the steering axis can be generated at any direction and with any largeness.

3. Omnidirectional Mobile Robot with Orthogonal Wheels

In this section, it is assumed that the omnidirectional mobile robot consists of the orthogonal-wheel assembly mechanism proposed by Pin and Killough [7].

The basic lateral orthogonal-wheel assembly is illustrated in Fig. 1. The major components are two spheres of equal diameter which have been sliced to resemble wide, i.e., rounded-tire wheels. The axle of each wheel is perpendicular to the sliced surfaces and is mounted using ball bearing so that the wheel is freewheeling around its axle. Through a bracket which is holding the extremities of the wheel axle, each wheel can be driven to roll on its portion of spherical surface, rotating around an axis Z, perpendicular to the wheel axle. When these axes Z_1 and Z_2 are maintained parallel and at a constant distance from each other, and when the wheel rotations around these axes are synchronized, contact with the ground can be assured by at least one wheel, while allowing enough space for the brackets holding the wheel axles to clear the ground. These assemblies can be constructed by using more than two wheels, in which each wheel axle should be allocated with less than 90 [deg] offset.

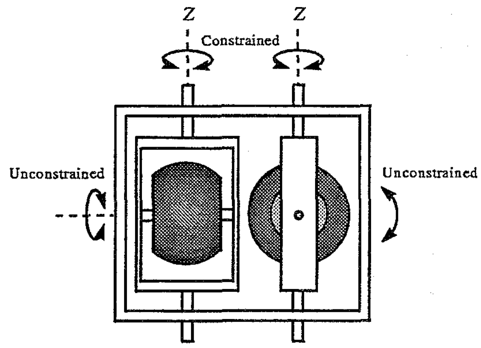

Figure 1: The lateral orthogonal-wheel assembly

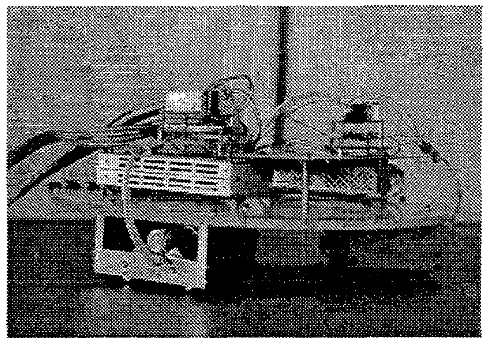

Figure 2: The appearance of an omnidirectional mobile robot

In the following, it was assumed that a platform is based on three assemblies allocated at an equal distance from the center of gravity (c.g.) for the robot, in which one assembly consists of two wheels. The actual experimental mobile robot is appeared in Fig. 2.

4. Derivation of Dynamic Model for the Omnidirectional Mobile Robot

Let the mobile robot be rigid moving on the work space. It is assumed that the absolute coordinate system $O_w - X_w Y_w$ is fixed on the plane and the moving coordinate system $O_m - X_m Y_m$ is fixed on the c.g. for the mobile robot as shown in Fig. 3.

When defining the position vector of the c.g. for the mobile robot such as $\boldsymbol{S}_w = [x_w \; y_w]^T$, we have

$$\boldsymbol{M}\ddot{\boldsymbol{S}}_w = \boldsymbol{F}_w \tag{1}$$

where $\boldsymbol{F}_w = [F_x \; F_y]^T$ is the force vector in the absolute coordinate system applied to the center of gravity for the mobile robot and $\boldsymbol{M}$ is a symmetric positive-definite matrix as $\boldsymbol{M} = \mathrm{diag}(M, M)$ with the mass M.

Let ϕ denote the angle between X_w- and X_m-

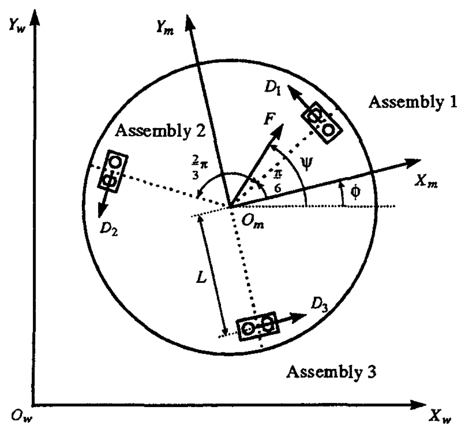

Figure 3: Model of an omnidirectional mobile robot

coordinates, i.e., the rotational angle of the moving coordinate system with respect to the absolute coordinate system. When introducing the coordinate transformation matrix from the absolute coordinate system to the moving coordinate system such as

$$ {}^{w}R_m = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \tag{2}$$

it follows that

$$\dot{\boldsymbol{S}}_w = {}^{w}R_m \dot{\boldsymbol{s}}_m \tag{3}$$
$$\boldsymbol{F}_w = {}^{w}R_m \boldsymbol{f}_m \tag{4}$$

where $\boldsymbol{s}_m = [x_m \ y_m]^T$ and $\boldsymbol{f}_m = [f_x \ f_y]^T$ are the position vector of the c.g. and the force vector applied to the c.g. in the moving coordinate system, respectively.

Therefore, transforming equation (1) to the moving coordinate system gives

$$\boldsymbol{M}({}^{w}R_m^{T}\,{}^{w}\dot{R}_m \dot{\boldsymbol{s}}_m + \ddot{\boldsymbol{s}}_m) = \boldsymbol{f}_m \tag{5}$$

Then, the dynamic properties for the mobile robot can be described as [14]

$$M(\ddot{x}_m - \dot{y}_m\dot{\phi}) = f_x \tag{6}$$
$$M(\ddot{y}_m + \dot{x}_m\dot{\phi}) = f_y \tag{7}$$
$$I_v\ddot{\phi} = M_I \tag{8}$$

where I_v is the moment of inertia for the robot, M_I is the moment around the c.g. for the robot, and f_x, f_y, M_I are given by

$$f_x = -\frac{1}{2}D_1 - \frac{1}{2}D_2 + D_3 \tag{9}$$
$$f_y = \frac{\sqrt{3}}{2}D_1 - \frac{\sqrt{3}}{2}D_2 \tag{10}$$
$$M_I = (D_1 + D_2 + D_3)L \tag{11}$$

In addition, the driving system property [15] for each assembly is assumed to be given by

$$I_w\dot{\omega}_i + c\omega_i = ku_i - rD_i \quad (i = 1, 2, 3) \tag{12}$$

where L is the distance between any assembly and the c.g. of the robot; c is the viscous friction factor for the wheel; D_i is the driving force for each assembly; r is the radius of the wheel; I_w is the moment of inertia of the wheel around the driving shaft; ω_i is the rotational rate of the wheel; k is the driving gain factor; and u_i is the driving input torque.

On the other hand, the geometrical relationships among variables $\dot{\phi}$, $\dot{x}_m$, $\dot{y}_m$, and ω_i, i.e., the inverse kinematics are given by

$$r\omega_1 = -\frac{1}{2}\dot{x}_m + \frac{\sqrt{3}}{2}\dot{y}_m + L\dot{\phi} \tag{13}$$
$$r\omega_2 = -\frac{1}{2}\dot{x}_m - \frac{\sqrt{3}}{2}\dot{y}_m + L\dot{\phi} \tag{14}$$
$$r\omega_3 = \dot{x}_m + L\dot{\phi} \tag{15}$$

Therefore, using equations (6)~(15) gives

$$\ddot{x}_m = a_1\dot{x}_m + a_2'\dot{y}_m\dot{\phi} - b_1(u_1 + u_2 - 2u_3) \tag{16}$$
$$\ddot{y}_m = a_1\dot{y}_m - a_2'\dot{x}_m\dot{\phi} + \sqrt{3}b_1(u_1 - u_2) \tag{17}$$
$$\ddot{\phi} = a_3\dot{\phi} + b_2(u_1 + u_2 + u_3) \tag{18}$$

where

$$a_1 = -3c/(3I_w + 2Mr^2)$$
$$a_2' = 2Mr^2/(3I_w + 2Mr^2)$$
$$a_3 = -3cL^2/(3I_wL^2 + I_vr^2)$$
$$b_1 = kr/(3I_w + 2Mr^2)$$
$$b_2 = krL/(3I_wL^2 + I_vr^2)$$

It is easy to find that combining equations (2), (3), (16) and (17) yields the appropriate dynamic equations in the absolute coordinate system for the robot. Thus, defining the state variable for the robot as $\boldsymbol{x} = [x_w \ y_w \ \phi \ \dot{x}_w \ \dot{y}_w \ \dot{\phi}]^T$, the manipulated variable as $\boldsymbol{u} = [u_1 \ u_2 \ u_3]^T$, and the output variable as $\boldsymbol{y} = [\dot{x}_w \ \dot{y}_w \ \dot{\phi}]^T$ yields the following state equation [16]:

$$\dot{\boldsymbol{x}} = A(\boldsymbol{x})\boldsymbol{x} + B(\boldsymbol{x})\boldsymbol{u} \tag{19}$$
$$\boldsymbol{y} = C\boldsymbol{x} \tag{20}$$

where

$$A(\boldsymbol{x}) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_1 & -a_2\dot{\phi} & 0 \\ 0 & 0 & 0 & a_2\dot{\phi} & a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_3 \end{bmatrix}$$

$$B(\boldsymbol{x}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ b_1\beta_1 & b_1\beta_2 & 2b_1\cos\phi \\ b_1\beta_3 & b_1\beta_4 & 2b_1\sin\phi \\ b_2 & b_2 & b_2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$a_2 = 1 - a_2' = 3I_w/(3I_w + 2Mr^2)$
$\beta_1 = -\sqrt{3}\sin\phi - \cos\phi, \quad \beta_2 = \sqrt{3}\sin\phi - \cos\phi$
$\beta_3 = \sqrt{3}\cos\phi - \sin\phi, \quad \beta_4 = -\sqrt{3}\cos\phi - \sin\phi$

Let the translational velocity of the robot in the absolute coordinate system be $V = (\dot{x}_w^2 + \dot{y}_w^2)^{\frac{1}{2}}$ and the azimuth of the robot in the absolute coordinate system be $\psi = \theta + \phi$. Here, θ denotes the angle between X_m-coordinate and $\boldsymbol{f}_m$, i.e., the azimuth of the robot in the moving coordinate system. Then, it is found that

$$\dot{x}_w = V\cos\psi \tag{21}$$

$$\dot{y}_w = V\sin\psi \tag{22}$$

$$\psi = \arctan\frac{\dot{y}_w}{\dot{x}_w} \tag{23}$$

where note that a counterclockwise rotation denotes the positive direction for the rotational motion of the robot.

It should be noted that X_w- and Y_w-directional motions in (19) are coupled, because the equation of motions is derived in the absolute coordinate system. However, since the rotational angle of the robot can be assured as $\phi = \psi - \theta$ even if the azimuth of the robot is changed arbitrarily, the mobile robot can realize a translational motion without changing the pose.

5. Resolved Acceleration Control

Since the dynamic equation is finally considered in the absolute coordinate system, solving equation (19) with respect to $u_i, i = 1,2,3$ gives the following resolved acceleration control:

$$u_1 = \frac{\beta_1(\ddot{x}_w^* - a_1\dot{x}_w + a_2\dot{\phi}\dot{y}_w)}{6b_1} + \frac{\beta_3(\ddot{y}_w^* - a_1\dot{y}_w - a_2\dot{\phi}\dot{x}_w)}{6b_1} + \frac{\ddot{\phi}^* - a_3\dot{\phi}}{3b_2} \tag{24}$$

$$u_2 = \frac{\beta_2(\ddot{x}_w^* - a_1\dot{x}_w + a_2\dot{\phi}\dot{y}_w)}{6b_1} + \frac{\beta_4(\ddot{y}_w^* - a_1\dot{y}_w - a_2\dot{\phi}\dot{x}_w)}{6b_1} + \frac{\ddot{\phi}^* - a_3\dot{\phi}}{3b_2} \tag{25}$$

$$u_3 = \frac{\cos\phi(\ddot{x}_w^* - a_1\dot{x}_w + a_2\dot{\phi}\dot{y}_w)}{3b_1} + \frac{\sin\phi(\ddot{y}_w^* - a_1\dot{y}_w - a_2\dot{\phi}\dot{x}_w)}{3b_1} + \frac{\ddot{\phi}^* - a_3\dot{\phi}}{3b_2} \tag{26}$$

where $\ddot{x}_w^*, \ddot{y}_w^*$ and $\ddot{\phi}^*$ are given by adding PI-servo or PD-servo to $\ddot{x}_{wd}, \ddot{y}_{wd}$ and $\ddot{\phi}_d$ such that

$$\ddot{x}_w^* = \ddot{x}_{wd} + K_{\dot{x}p}e_{\dot{x}} + K_{\dot{x}i}\int_0^t e_{\dot{x}}dt \tag{27}$$

$$\ddot{y}_w^* = \ddot{y}_{wd} + K_{\dot{y}p}e_{\dot{y}} + K_{\dot{y}i}\int_0^t e_{\dot{y}}dt \tag{28}$$

$$\ddot{\phi}^* = \ddot{\phi}_d + K_{\phi v}\dot{e}_\phi + K_{\phi p}e_\phi \tag{29}$$

Here, $K_{\cdot p}$ is the proportional gain, $K_{\cdot i}$ is the integral gain, $K_{\cdot v}$ is the derivative gain, and each error is defined by

$$e_{\dot{x}} = \dot{x}_{wd} - \dot{x}_w \tag{30}$$

$$e_{\dot{y}} = \dot{y}_{wd} - \dot{y}_w \tag{31}$$

$$e_\phi = \phi_d - \phi \tag{32}$$

where $\dot{x}_{wd}, \dot{y}_{wd}$ and ϕ_d denote the references for $\dot{x}_w, \dot{y}_w$ and ϕ, respectively. It should be noted that the solutions $u_i, i = 1,2,3$ exit for all t, because the input distribution factors b_1 and b_2 are always non-zero constants.

For a control result using this resolved acceleration control system, see [17].

6. PID Control Method

The state and output equations (19) and (20) are described in the absolute coordinate system. It should be noted, however, that the control input $\boldsymbol{u}$ is the quantity in the moving coordinate system. Therefore, the control input in the absolute coordinate system must be transformed into the control input for each assembly, i.e., signal expressed in the moving coordinate system, if the control input is designed in the absolute coordinate system. Here, defining the control input in the moving coordinate as ${}^m\boldsymbol{u}$ and one in the absolute coordinate as ${}^w\boldsymbol{u}$, then the transformation for each input can be derived from the kinematics as follows

$${}^m\boldsymbol{u} = {}^wJ^T\,{}^w\boldsymbol{u} \tag{33}$$

where ${}^wJ$ is the Jacobian matrix defined by

$${}^wJ^T = \begin{bmatrix} \frac{\beta_1}{3} & \frac{\beta_3}{3} & \frac{1}{3L} \\ \frac{\beta_2}{3} & \frac{\beta_4}{3} & \frac{1}{3L} \\ \frac{2\cos\phi}{3} & \frac{2\sin\phi}{3} & \frac{1}{3L} \end{bmatrix}$$

In the following, it is enough to only consider the design of the control system in the absolute coordinate.

6.1 An adaptive PI controller

In this section, we show an adaptive PI controller. Let $u_{\dot{x}}, u_{\dot{y}}$ and u_ϕ be the control inputs using ${}^w\dot{x}, {}^w\dot{y}$ and ϕ, respectively and denote ${}^w\boldsymbol{u} = [u_{\dot{x}}, u_{\dot{y}}, u_\phi]$.

In the conventional research, it is known [18] that PD control is effective to control the rotational angle, i.e.,

$$u_\phi = K_{\phi p} e_\phi + K_{\phi v} \dot{e}_\phi \tag{34}$$

and P control is effective for controlling each velocity,

$$u_{\dot{x}} = K_{\dot{x}p} e_{\dot{x}}, \quad u_{\dot{y}} = K_{\dot{y}p} e_{\dot{y}}$$

But the concrete method does not exist for the selection of the 'P' gains, $K_{\dot{x}p}$ and $K_{\dot{y}p}$. Thus, a trial and error procedure is required until a desired performance will be obtained. Moreover, if the system parameters are changed due to mounting a weight on the robot or changing of the physical parameters, then the design parameters must be selected again.

On the other hands, a simple output feedback strategy

$$u(t) = K(t)e(t), \quad \dot{K}(t) = e^2(t)$$

is known [19],where u is the scalar input, e is the scalar error, and K is a scalar time-varying gain. This control strategy is a kind of P type controller with an adaptive gain tuning, where P gain is adjusted automatically according to the error signal. This control strategy is very attractive, but a steady state error may remain, if we only use the P-type controller. In a constant output feedback problem, this can be easily overcome by introducing an integral term to the control law.

In the paper of [20], the following adaptive PI controller for $\dot{x}_w$ was adopted,

$$u_{\dot{x}}(t) = K_{\dot{x}c}\left[K_{\dot{x}1}(t)e_{\dot{x}}(t) + \int_0^t K_{\dot{x}2}(\tau)e_{\dot{x}}(\tau)d\tau\right] \tag{35}$$

with

$$\begin{aligned} K_{\dot{x}1}(t) &= K_{\dot{x}p}(t) + \alpha_{\dot{x}1}K_{\dot{x}i}(t) \\ K_{\dot{x}2}(t) &= \alpha_{\dot{x}2}K_{\dot{x}i}(t) \\ \dot{K}_{\dot{x}p}(t) = e_{\dot{x}}^2(t), &\quad \dot{K}_{\dot{x}i}(t) = e_{\dot{x}}^2(t) \end{aligned}$$

where $K_{\dot{x}c}, \alpha_{\dot{x}1}$, and $\alpha_{\dot{x}2}$ are some positive constants. Similarly,

$$u_{\dot{y}}(t) = K_{\dot{y}c}\left[K_{\dot{y}1}(t)e_{\dot{y}}(t) + \int_0^t K_{\dot{y}2}(\tau)e_{\dot{y}}(\tau)d\tau\right] \tag{36}$$

with

$$\begin{aligned} K_{\dot{y}1}(t) &= K_{\dot{y}p}(t) + \alpha_{\dot{y}1}K_{\dot{y}i}(t) \\ K_{\dot{y}2}(t) &= \alpha_{\dot{y}2}K_{\dot{y}i}(t) \\ \dot{K}_{\dot{y}p}(t) = e_{\dot{y}}^2(t), &\quad \dot{K}_{\dot{y}i}(t) = e_{\dot{y}}^2(t) \end{aligned}$$

for $\dot{y}_w$, where $K_{\dot{y}c}, \alpha_{\dot{y}1}$ and $\alpha_{\dot{y}2}$ are some positive constants. This control strategy is a kind of PI type controller with adaptive gain tuning, where P and I gains are adjusted automatically according to the error signals. By using this control strategy, when the error signal becomes large because of the plant parameter changing, the PI gains are tuned adaptively and look forward to converging the error signal. The output response can be improved by setting the greater $K_{\dot{x}c}$ and $K_{\dot{y}c}$, but it requires a large initial input. On the other hands, large $\alpha_{\dot{x}1}$, $\alpha_{\dot{y}1}$ in the P gain and $\alpha_{\dot{x}2}$, $\alpha_{\dot{y}2}$ in the I gain may contribute to the improvement of rise time and offset removing without causing a large initial input.

For the rotational angle control, we also use the conventional PD control as shown in (34).

7. Fuzzy Model Method

For the fuzzy servo, a rough construction procedure of fuzzy models is as follows:

1. Substitute a representative angle ϕ into Eq.(19) and construct a linear model.
2. Discretize the linear model.
3. Construct the augmented system.
4. Design an optimal regulator for the augmented system and determine the feedback gain matrix by solving the associated Riccati equation using any numerical approach.

Here, we prepare four kind of models within the range of $-2\pi \le \phi \le 2\pi$. The value of ϕ at each model rule is given by

$$\begin{aligned} &\text{Model rule 1} : \phi = 0 \text{ or } 2\pi \text{ or } -2\pi \\ &\text{Model rule 2} : \phi = \pi/2 \text{ or } -3\pi/2 \\ &\text{Model rule 3} : \phi = \pi \text{ or } -\pi \\ &\text{Model rule 4} : \phi = 3\pi/2 \text{ or } -\pi/2 \end{aligned}$$

7.1 Linear fuzzy models

Since the representative angle is assumed to be constant in each model rule, the system matrix of linear model is same for all of model rules such as

$$A_c = \begin{bmatrix} 0_{3\times3} & & I_{3\times3} & \\ & a_1 & 0 & 0 \\ 0_{3\times3} & 0 & a_1 & 0 \\ & 0 & 0 & a_3 \end{bmatrix}$$

The distribution matrix B_{ci} is provided by

$$B_{c1} = \begin{bmatrix} & 0_{3\times3} & \\ -b_1 & -b_1 & 2b_1 \\ \sqrt{3}b_1 & -\sqrt{3}b_1 & 0 \\ b_2 & b_2 & b_2 \end{bmatrix}$$

$$B_{c2} = \begin{bmatrix} & 0_{3\times3} & \\ -\sqrt{3}b_1 & \sqrt{3}b_1 & 0 \\ -b_1 & -b_1 & 2b_1 \\ b_2 & b_2 & b_2 \end{bmatrix}$$

$$B_{c3} = \begin{bmatrix} & 0_{3\times3} & \\ b_1 & b_1 & -2b_1 \\ -\sqrt{3}b_1 & \sqrt{3}b_1 & 0 \\ b_2 & b_2 & b_2 \end{bmatrix}$$

$$B_{c4} = \begin{bmatrix} & 0_{3\times3} & \\ \sqrt{3}b_1 & -\sqrt{3}b_1 & 0 \\ b_1 & b_1 & -2b_1 \\ b_2 & b_2 & b_2 \end{bmatrix}$$

where the subscript i of B_{ci} denotes the number of model rule.

7.2 Discrete-time models

The linear model in the discrete-time is given by

$$x(k+1) = \Phi x(k) + \Gamma_i u_i(k) \qquad (37)$$

$$y(k) = Cx(k) \qquad (38)$$

where it was assumed that Φ and Γ_i were calculated by using the physical parameter conditions given by

$$I_v = 11.25 \text{ [kgm}^2], \; M = 9.4 \text{ [kg]}$$
$$L = 0.178 \text{ [m]}, \; k = 0.448, \; c = 0.1889 \text{ [kgm}^2/\text{s]}$$
$$I_w = 0.02108 \text{ [kgm}^2], \; r = 0.0245 \text{ [m]}$$

and using the sampling width of 50 [ms]. The results are as follows:

$$\Phi = \begin{bmatrix} & 0.0416 & 0 & 0 \\ I_{3\times3} & 0 & 0.0416 & 0 \\ & 0 & 0 & 0.0475 \\ & 0.684 & 0 & 0 \\ 0_{3\times3} & 0 & 0.684 & 0 \\ & 0 & 0 & 0.903 \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} -\gamma_1 & -\gamma_1 & \gamma_2 \\ \gamma_3 & -\gamma_3 & 0 \\ \gamma_4 & \gamma_4 & \gamma_4 \\ -\gamma_5 & -\gamma_5 & \gamma_6 \\ \gamma_7 & -\gamma_7 & 0 \\ \gamma_8 & \gamma_8 & \gamma_8 \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} -\gamma_3 & \gamma_3 & 0 \\ -\gamma_1 & -\gamma_1 & \gamma_2 \\ \gamma_4 & \gamma_4 & \gamma_4 \\ -\gamma_7 & \gamma_7 & 0 \\ -\gamma_5 & -\gamma_5 & \gamma_6 \\ \gamma_8 & \gamma_8 & \gamma_8 \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} \gamma_1 & \gamma_1 & \gamma_2 \\ -\gamma_3 & \gamma_3 & 0 \\ \gamma_4 & \gamma_4 & \gamma_4 \\ \gamma_5 & \gamma_5 & -\gamma_6 \\ -\gamma_7 & \gamma_7 & 0 \\ \gamma_8 & \gamma_8 & \gamma_8 \end{bmatrix}$$

$$\Gamma_4 = \begin{bmatrix} \gamma_3 & -\gamma_3 & 0 \\ \gamma_1 & \gamma_1 & -\gamma_2 \\ \gamma_4 & \gamma_4 & \gamma_4 \\ \gamma_7 & -\gamma_7 & 0 \\ \gamma_5 & \gamma_5 & -\gamma_6 \\ \gamma_8 & \gamma_8 & \gamma_8 \end{bmatrix}$$

with

$$\gamma_1 = 1.628 \times 10^{-4} \quad \gamma_2 = 3.257 \times 10^{-4}$$
$$\gamma_3 = 2.820 \times 10^{-4} \quad \gamma_4 = 2.696 \times 10^{-4}$$
$$\gamma_5 = 6.126 \times 10^{-3} \quad \gamma_6 = 1.225 \times 10^{-2}$$
$$\gamma_7 = 1.061 \times 10^{-2} \quad \gamma_8 = 1.060 \times 10^{-2}$$

7.3 Construction of augmented system

If the type-2 servo system is considered here, using the polynomial equation with the time transition operator z described by

$$f(z^{-1}) = (1 - z^{-1})^2 = 1 - 2z^{-1} + z^{-2} \qquad (39)$$

yields

$$f(z^{-1}) y_d(k) = 0 \qquad (40)$$

for the reference output vector $y_d(k)$. Here, z^{-1} denotes the one-step delay operator, and the reference error vector $e(k)$ is defined by

$$e(k) = y_d(k) - y(k) \qquad (41)$$

Operating $f(z^{-1})$ to the discrete-time model (37) and the above reference error vector, and defining $v(k) = [f(z^{-1})x(k) \; e(k-1) \; e(k-2)]^T$ as the state variable vector for the augmented system, we have

$$v(k+1) = \Phi_e v(k) + \Gamma_{ei} f(z^{-1}) u_i(k) \qquad (42)$$

where

$$\Phi_e = \begin{bmatrix} \Phi & 0_{6\times3} & 0_{6\times3} \\ -C & 2I_{3\times3} & -I_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} \end{bmatrix}$$

$$\Gamma_{ei} = \begin{bmatrix} \Gamma_i \\ 0_{3\times3} \\ 0_{3\times3} \end{bmatrix}$$

7.4 Design of optimal regulator

After selecting the suitable weighting matrices Q_i and R_i for each model rule, solve the positive definite matrix P_i to the associated algebraic Riccati equation given by

$$P_i = Q_i + \Phi_e^T P_i \Phi_e - \Phi_e^T P_i \Gamma_{ei} (R_i + \Gamma_{ei}^T P_i \Gamma_e)^{-1} \Gamma_{ei}^T P_i \Phi_e \qquad (43)$$

It should be noted, however, that the augmented system does not assure the controllability. That is, it is difficult to obtain a unique stabilizable solution $P_i > 0$. But, since there exist uncontrollable modes on the unit circle [21], it is possible to obtain $P_i \geq 0$ as a strong solution [22] by applying an iterative approach. The strong solution P_i is obtained as a steady-state solution of the following iterative relation:

$$P_i[j+1] = Q_i + \Phi_e^T P_i[j]\Phi_e - \Phi_e^T P_i[j]\Gamma_{ei}(R_i + \Gamma_{ei}^T P_i[j]\Gamma_e)^{-1}\Gamma_{ei}^T P_i[j]\Phi_e \quad (44)$$

with initial condition of $P_i[0] = 0$. Anyway, if P_i is calculated, then the optimal feedback law can be realized as

$$f(z^{-1})\boldsymbol{u}_i(k) = -G_i\boldsymbol{v}(k) \quad (45)$$

Here,

$$G_i = \left[R_i + \Gamma_{ei}^T P_i \Gamma_{ei}\right]^{-1} \Gamma_{ei}^T P_i \Phi_e = [F_i \; K_i \; L_i] \quad (46)$$

7.5 Construction of fuzzy servo system

Since the actual manipulated input to the robot is $\boldsymbol{u}_i(k)$, it is necessary to arrange Eq.(45). Therefore, introducing an intermediate variable $\boldsymbol{\xi}(k)$ that satisfies

$$f(z^{-1})\boldsymbol{\xi}(k) = \boldsymbol{e}(k) \quad (47)$$

gives the following state equation:

$$\begin{bmatrix} \boldsymbol{\xi}(k) \\ \boldsymbol{\xi}(k-1) \end{bmatrix} = \begin{bmatrix} 2I_{3\times3} & -I_{3\times3} \\ I_{3\times3} & 0_{3\times3} \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}(k-1) \\ \boldsymbol{\xi}(k-2) \end{bmatrix} + \begin{bmatrix} I_{3\times3} \\ 0_{3\times3} \end{bmatrix} \boldsymbol{e}(k) \quad (48)$$

Dividing the both sides of Eq.(45) by $f(z^{-1})$ and using Eq.(47) in it, we have

$$\boldsymbol{u}_i(k) = -F_i\boldsymbol{x}(k) - K_i\boldsymbol{\xi}(k-1) - L_i\boldsymbol{\xi}(k-2) \quad (49)$$

where it is assumed that $\boldsymbol{\xi}(-2) = \boldsymbol{\xi}(-1) = 0$.

The membership function is of a Gaussian type with the center value w_c, the reciprocal value of standard deviation w_d, and the input variable x, whose confidence μ is given by

$$\mu = \exp\left\{\ln(0.5)(x - w_c)^2 w_d^2\right\} \quad (50)$$

Since there exist two or three multiple labels in one model rule, the confidence of i-th model rule in the antecedent part is assumed to be given by

$$\mu_i = \max_j \{\mu_{ij}\} \quad (51)$$

where j denotes the number of fuzzy labels in the i-th model rule. When defining the normalized confidence given by

$$p_i = \mu_i / \sum_{j=1}^{4} \mu_j \quad (52)$$

the manipulated quantity can be consequently expressed [23] by

$$\begin{aligned} \boldsymbol{u}(k) &= \sum_{i=1}^{4} \boldsymbol{u}_i(k) \\ &= -\left[\sum_{i=1}^{4} F_i p_i\right] \boldsymbol{x}(k) - \left[\sum_{i=1}^{4} K_i p_i\right] \boldsymbol{\xi}(k-1) \\ &\quad - \left[\sum_{i=1}^{4} L_i p_i\right] \boldsymbol{\xi}(k-2) \end{aligned} \quad (53)$$

The corresponding simulation results are shown in [24].

8. Stochastic Fuzzy Servo Method

It is assumed that the controlled object is completely known, linear, and dynamic system described by the following model:

$$\boldsymbol{x}(k+1) = \Phi\boldsymbol{x}(k) + \Gamma\boldsymbol{u}(k) + \boldsymbol{w}(k) \quad (54)$$

$$\boldsymbol{y}(k) = C\boldsymbol{x}(k) + \boldsymbol{v}(k) \quad (55)$$

where $\boldsymbol{x}(k) \in \mathcal{R}^n$ is the state variable vector, $\boldsymbol{u}(k) \in \mathcal{R}^p$ is the control input vector, $\boldsymbol{w}(k) \in \mathcal{R}^n$ is the system noise vector, $\boldsymbol{v}(k) \in \mathcal{R}^m$ is the measurement noise vector, and $\boldsymbol{y}(k) \in \mathcal{R}^m$ is the measurement vector. Here, it is assumed that $\{\boldsymbol{w}(k), \boldsymbol{v}(k)\}$ are independent each other for $k = 0, 1, \ldots$ and zero-mean white Gaussian noises with covariance matrices Q and R. The initial state vector $\boldsymbol{x}(0)$ is assumed to be subject to the normal distribution with its mean $\bar{\boldsymbol{x}}_0$ and covariance P_0, i.e., $\boldsymbol{x}(0) \sim N(\bar{\boldsymbol{x}}_0, P_0)$, and be independent of $\{\boldsymbol{w}(k), \boldsymbol{v}(k)\}$. In addition, let the data set of $\boldsymbol{y}(k)$ be described by $\mathcal{Y}_k \triangleq \{\boldsymbol{y}(0), \ldots, \boldsymbol{y}(k)\}$ and similarly the data set of $\boldsymbol{u}(k)$ be denoted by $\mathcal{U}_k \triangleq \{\boldsymbol{u}(0), \ldots, \boldsymbol{u}(k)\}$.

8.1 Multiple-model optimal servo system

Under the assumption that in equations (54) and (55) the distribution matrix Γ and the initial state $\boldsymbol{x}(0)$ are uncertain, consider a multiple model adaptive control which treats with the candidate models as the hypotheses. Let the hypothesis H_i be described by

$$H_i : \boldsymbol{x}(k+1) = \Phi\boldsymbol{x}(k) + \Gamma_i\boldsymbol{u}(k) + \boldsymbol{w}(k) \quad (56)$$

$$\boldsymbol{y}(k) = C\boldsymbol{x}(k) + \boldsymbol{v}(k), i = 1, \ldots, M \quad (57)$$

where M is the number of candidate models and the initial state is assumed to be subject to $\boldsymbol{x}(0) \sim N(\bar{\boldsymbol{x}}_0^i, P_0^i)$.

The multiple model adaptive control in the steady-state solution satisfying the above hypotheses can be obtained [22] by

$$\boldsymbol{u}(k) = -\sum_{i=1}^{M} F_i\hat{\boldsymbol{x}}_i(k)p(H_i|k)$$

$$-\sum_{i=1}^{M} H_i z_i(k-1)p(H_i|k) \quad (58)$$

From the Kalman filter theory, the elemental state estimate $\hat{x}_i(k) \triangleq E[x(k)|H_i, \mathcal{Y}_{k-1}, \mathcal{U}_{k-1}]$ can be obtained as follows:

$$\hat{x}_i(k+1) = \Phi\hat{x}_i(k) + K\nu_i(k) + \Gamma_i u(k)$$
$$\hat{x}_i(0) = \bar{x}_0^i \quad (59)$$

$$\nu_i(k) = y(k) - \hat{y}_i(k) \quad (60)$$

where $\hat{y}_i(k) \triangleq C\hat{x}_i(k)$ and K is given as

$$K = \Phi PC^T[CPC^T + R]^{-1} \quad (61)$$

$$P = \Phi P\Phi^T - \Phi PC^T[CPC^T + R]^{-1}CP\Phi^T + Q \quad (62)$$

Note also that $z_i(k)$ is given by a recursive relation:

$$z_i(k) = z_i(k-1) + e_i(k), \quad z_i(-1) = y_d \quad (63)$$

$$e_i(k) \triangleq y_d - \hat{y}_i(k) \quad (64)$$

The elemental feedback gain $G_i \triangleq [F_i,\ H_i]$ is obtained by

$$G_i = [R_{ci} + \Gamma_{ai}^T P_{ci}\Gamma_{ai}]^{-1}\Gamma_{ai}^T P_{ci}\Phi_a \quad (65)$$

$$P_{ci} - \Phi_a^T P_{ci}\Phi_a + \Phi_a^T P_{ci}\Gamma_{ai}[R_{ci} + \Gamma_{ai}^T P_{ci}\Gamma_{ai}]^{-1}\Gamma_{ai}^T P_{ci}\Phi_a - Q_{ci} = 0 \quad (66)$$

where

$$\Phi_a = \begin{bmatrix} \Phi & 0 \\ -C & I \end{bmatrix}, \quad \Gamma_{ai} = \begin{bmatrix} \Gamma_i \\ 0 \end{bmatrix}$$

Moreover, the a posteriori probability for the hypothesis H_i, $p(H_i|k)$, is calculated by

$$p(H_i|k) = \frac{p_i(y(k))p(H_i|k-1)}{\sum_{j=1}^{M} p_j(y(k))p(H_j|k-1)} \quad (67)$$

$$p_i(y(k)) = (2\pi)^{-m/2}|S_i|^{-1}\exp[-\frac{1}{2}\|y(k) - \hat{y}_i(k)\|^2_{S_i^{-1}}] \quad (68)$$

in which $p_i(y(k))$ denotes the probability density function of the data $y(k)$ under the hypothesis H_i, and

$$S_i = CPC^T + R \quad (69)$$

8.2 Stochastic fuzzy servo system

For the multiple model optimal servo system using some linear dynamic models, let the hypothesis of $H_i, i = 1, ..., M$ be regarded as the fuzzy label $A_i, i = 1, ..., M$ for one vector data $y(k)$. Assume that the distributions of $y(k)$ under the hypothesis H_i are all equal forcedly (in fact, as can be seen from equation (69), S_i are all equal for $i = 1, ..., M$), as in the conventional stochastic fuzzy control [25]. Then, the multiple model adaptive control in steady-state is reduced to

$$u(k) = -\sum_{i=1}^{M} F_i\hat{x}_i(k)p(i|k) - \sum_{i=1}^{M} H_i z_i(k)p(i|k) \quad (70)$$

where the a posteriori probability $p(i|k)$ for the ith membership function (or ith control rule) can be written by

$$p(i|k) = \frac{\mu_{A_i}(y(k))p(i|k-1)}{\sum_{j=1}^{M}\mu_{A_j}(y(k))p(j|k-1)} \quad (71)$$

Here, $p(i|k-1)$ denotes the a priori probability for the ith membership function, and the confidence of data $y(k)$ with respect to the label A_i is given by

$$\mu_{A_i}(y(k)) = \exp[-\frac{1}{2}\|y(k) - \hat{y}_i(k)\|^2_{S_i^{-1}}] \quad (72)$$

In this case, note that the membership function moves on the support set with a fixed width, because $\{\hat{y}_i(k)\}$ can change in time. In addition, if the a priori probability is not assigned to the membership function (or control rule) as in the case of a conventional fuzzy control, then the a posteriori probability can be expressed by

$$p(i|k) = \frac{\mu_{A_i}(y(k))}{\sum_{j=1}^{M}\mu_{A_j}(y(k))} \quad (73)$$

Several simulation examples are given in [26].

9. Conclusions

In this paper, we have reviewed several specialized wheel or mobile mechanisms for constructing mainly a holonomic and omnidirectional mobile robot. Then, we concentrated our attention to a particular omnidirectional mobile robot, whose platform was composed of the three lateral orthogonal-wheel assemblies. After introducing the dynamic model for such a mobile robot, we discussed on the four control approaches to develop a feedback control system.

References

[1] Nakano, E. and Koyachi, N., "An Advanced Mechanism of the Omni-Directional Vehicle (ODV) and Its Application to the Working Wheelchair for the Disabled," *Proc. of '83 Int. Conf. Advanced Robotics*, 1983, pp. 277–284.

[2] Nakano, E., Mori, Y., and Takahashi, T., "Study of the Mechanism and Control of Omni-Directional Vehicle," *Procs. of the 12th Annual Conference of RSJ*, Fukuoka, Japan, Nov. 1994, vol. 1, pp. 369–379 (in Japanese).

[3] Muir, P.F. and Neuman, C.P., "Kinematic Modeling of Wheeled Mobile Robots," *J. Robotic Systems*, vol. 4, pp. 281–340, 1987.

[4] Asama, H., Sato, M., Bongoni, L., Kaetsu, H., Matsumoto, A., and Endo, I., "Development of an Omni-Directional Mobile Robot with 3 DOF Decoupling Drive Mechanism," *Procs. of 1995 IEEE Int. Conf. on Robotics and Automation*, Nagoya, Japan, May 1995, vol. 2, pp. 1925–1930.

[5] Machida, K., Luo, Z.-H., and Funaki, M., "Prototyping and Control Experiments of a Cartesian Nonholonomic Robot," *J. of the Robotics Society of Japan*, vol. 15, no. 2, pp. 243–248, 1997 (in Japanese).

[6] West, M. and Asada, H., "Design and Control of Ball Wheel Omnidirectional Vehicles," *Procs. of 1995 IEEE Int. Conf. on Robotics and Automation*, Nagoya, Japan, May 1995, vol. 2, pp. 1931–1938.

[7] Pin, F.G. and Killough, S.M., "A New Family of Omnidirectional and Holonomic Wheeled Platforms for Mobile Robots," *IEEE Trans. on Robotics and Automation*, vol. 10, no. 4, pp. 480–489, 1994.

[8] Tnag, J., Watanabe, K., and Shiraishi, Y., "Design of Traveling Experiment of an Omnidirectional Holonomic Mobile Robot," *Procs. of 1996 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS96)*, Osaka, Japan, Nov. 1996, vol. 1, pp. 66–73.

[9] West, M. and Asada, H., "Design of a Holonomic Omnidirectional Vehicle," *Procs. of 1992 IEEE Int. Conf. on Robotics and Automation*, Nice, France, May 1992, vol. 1, pp. 97–103.

[10] Nishikawa, A., West, M., and Asada, H., "Development of a Holonomic Omnidirectional Vehicle and an Accurate Guidance Method of the Vehicles," *J. of the Robotics Society of Japan*, vol. 13, no. 2, pp. 249–256, 1995 (in Japanese).

[11] Hirose, S. and Amano, S., "The VUTON: High Payload, High Efficiency Holonomic Omni-Directional Vehicle," *Proc. of JSME Annual Conference on Robotics and Mechatronics*, Sapporo, Japan, July 1993, pp. 350–355 (in Japanese).

[12] Hirano, T., Chen, P., and Toyota, T., "A Fundamental Study on Omni-Directional-Vehicle (1) – Omni-Directional Running Mechanism for Offroad –," *Proc. of the 14th SICE Kyushu Branch Annual Conference*, Kitakyushu, Japan, Nov. 1995, pp. 273–274 (in Japanese).

[13] Wada, M. and Mori, S., "Development of a Holonomic and Omnidirectional Mobile Robot," *J. of the Robotics Society of Japan*, vol. 15, no. 8, pp. 1139–1146, 1997 (in Japanese).

[14] Iwatsuki, M. Nakano, K., and Ohuchi, T., "Target Point Tracking Control of Robot Vehicle by Fuzzy Reasoning," *Trans. of the Society of Instrument and Control Engineers (SICE)*, vol. 27, no. 1, pp. 70–76, 1991 (in Japanese).

[15] Saito, M. and Tsumura, T., "Collision Avoidance Among Multiple Mobile Robots – A Local Approach Based on Non-linear Programming –," *Trans. of the Institute of Systems, Control and Information Engineers*, vol. 3, no. 8, pp. 252–260, 1990 (in Japanese).

[16] Watanabe, K., Shiraishi, Y., Tang, J., Fukuda, T., and Tzafestas, S.G., "Construction of the Dynamic Model for an Omnidirectional Mobile Robot," *Procs. of CESA'96 IMACS Multiconference, Symposium on Robotics and Cybernetics*, Lille, France, July 1996, pp. 643–647.

[17] Watanabe, K., Shiraishi, Y., and Tang, J., "A velocity type of computed torque control system for an omnidirectional mobile robot," *Proc. of the 15th SICE Kyushu Branch Annual Conference*, Saga, Japan, Nov. 1996, pp. 185–188 (in Japanese).

[18] Watanabe, K. and Shiraishi, Y., "A PID control system for an omnidirectional mobile robot," *Procs. of the 35th SICE Annual Conference Domestic Session Papers*, Tottori, Japan, July 1996, pp. 167–177 (in Japanese).

[19] Ilchmann, A., *Non-identifier-based high-gain adaptive control*, Lecture Notes in Control and Information Science, no. 189. Berlin, Springer-Verlag, 1993.

[20] Sato, K. and Watanabe, K., "Robust PI Control System for an Omnidirectional Mobile Robot with Three Orthogonal-Wheel Assemblies," *Proc. of JSME Kyushu Branch Memorial Conference*, Fukuoka, Japan, Oct. 1997, pp. 132–134 (in Japanese).

[21] Watanabe, K., Tang, J., and Han, F., "A Rotational Control for an Omnidirectional Mobile Robot Using a Fuzzy Modeling Approach," *Procs. of the 14th Annual Conference of RSJ*, Niigata, Japan, Nov. 1996, no. 1, pp. 53–54 (in Japanese).

[22] Watanabe, K., *Adaptive Estimation and Control*, Hertfordshire, UK, Prentice-Hall, 1992.

[23] K. Tanaka, *Advanced Fuzzy Control*, Tokyo, Kyoritsu-syuppan, pp. 95–192, 1994 (in Japanese).

[24] Watanabe, K., Izumi, K., Tang, J., and Han, F., "Trajectory Control of an Omnidirectional Mobile Robot Using a Fuzzy Servo System," *Proc. of the 2nd Int. Workshop on Advanced Mechatronics (IWAM'97)*, Unzenn, Japan, Dec. 1997, pp. 74–79.

[25] Watanabe, K., "Stochastic Fuzzy Control (Theoretical Derivation)," *JSME Int. Journal*, vol. 40, no. 2(C), June, pp. 224–230, 1997.

[26] Watanabe, K., Izumi, K., and Han F., "Stochastic Fuzzy Servo Control Using Multiple Linear Dynamic Models, *Procs. of 2nd Int. Conf. on Knowledge-based Intelligent Electronic Systems*, Adelade, Australia, April 1998, to be published.