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Point estimation in sign-restricted SVARs based on independence criteria with an application to rational bubbles☆

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ABSTRACT

The median and median target estimates in sign-restricted SVARs are driven by a highly informative prior for the set-identified structural parameters. This paper proposes an approach for point elicitation by minimizing the evidence against the null hypothesis of independence with respect to the orthogonalized residuals implied by the identified set. Finite sample properties of the estimator are studied in a Monte Carlo experiment. As an empirical illustration, we analyze monetary policy effects within the rational bubble model of equity valuation (Galí, 2014). The detected monetary policy shocks lead to distinct response profiles of the fundamental and bubble components of asset prices.

1. Introduction

Following the pioneering work of Canova and De Nicolo (2002), Faust (1998) and Uhlig (2005), sign restrictions have become a relevant approach to identification in structural vector autoregressions (SVARs, VARs). Compared with the use of conventional zero restrictions (Blanchard and Quah, 1989; Sims, 1980), identification by means of sign restrictions relies on much weaker assumptions, which often align with a broad class of theoretical models.[1] However, imposing sign restrictions produces a set of structural models that equally align with both identifying assumptions and observations, which complicates the inferential analysis of structural objects of interest. Given the point-identified reduced form parameter, all

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[1] Minimalist restrictions such as those employed in Uhlig (2005) often deliver an identified set that could be too wide to draw meaningful economic conclusions or policy advice. As a means to sharpen the identified set, Antolín-Díaz and Rubio-Ramírez (2018) propose using narrative information that may offer further restrictions on either the sign or size of some particular structural shocks or on the historical decompositions of observables. With the same purpose, Arias et al. (2018) suggest combining sign restrictions with traditional exclusion restrictions and provide a general framework for this approach.

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structural models implied by the identified set have the same likelihood and thus are indistinguishable from a frequentist perspective. Owing to this, conditional Bayesian priors for the structural parameter remain unrevised in the view of new data and will continue to influence the posterior even in the asymptotic case. Therefore, results from (standard) Bayesian posterior inference about the structural parameter must be interpreted with caution.

Alternative approaches to estimation and inference in sign-restricted SVAR models have been recently developed in the literature (e.g., Gafarov et al., 2018; Giacomini and Kitagawa, 2021; Granziera et al., 2018).[2] While the provision of confidence sets is of key importance in statistical inference, a natural complement of representative intervals is the elicitation of a particular point estimator of structural model parameters (incl. IRFs). Because many sign restrictions have only weak identification power, the resulting credible intervals are often too wide to deliver meaningful economic implications. Therefore, in many empirical applications, researchers typically report certain point estimates with the purpose of communicating the core implications of the identified set conveniently. However, problems related to commonly-reported posterior median estimates have been powerfully underlined in the recent literature (see e.g., Baumeister and Hamilton, 2015).

In this paper, we show that the identification power of sign restrictions crucially depends on the correlation of the reduced form residuals, whereas consistent point estimation is especially essential when the employed restrictions are conditionally weak for the structural objects of interest. However, because the conditional priors for set-identified parameters are not updated through the likelihood function (Moon and Schorfheide, 2012; Poirier, 1998), both the median and the median target estimators may be driven by uncontrolled prior information. Thus, conclusions based upon these point estimates could be misleading and should be treated with caution.[3] As an alternative, we propose a non-parametric way to elicit point estimators of structural model parameters by exploiting additional statistical information of the data. Specifically, we take advantage of a developing body of literature (e.g., Gouriéroux et al., 2017; Herwartz, 2019; Keweloh, 2021; Lanne and Luoto, 2021; Moneta et al., 2013) that suggests tools of independent component analysis (ICA) as a means of identification in structural models. Explicitly, by assuming statistical independence of the structural shocks, a 'most plausible' choice of the structural parameter is obtained from maximizing the *p*-value of a non-parametric independence test based on the distance covariance (dCov, Székely et al., 2007) as suggested by Matteson and Tsay (2017). Accordingly, we refer to this eliciting approach as sign-restricted Hodges–Lehman (SRHL) estimation (Dufour, 1990; Hodges and Lehmann, 1963). From an economic perspective and depending on the matter of interest, the assumption of mutual independence could appear quite strong. In this regard, the SRHL approach may serve as a complementary tool, which allows for testing the stronger assumptions (i.e., mutual independence) conditional on sign restrictions that are often consensual and, hence, weak. Such a practice could provide valuable information on the statistical properties of the shocks – which might reassure the selected economic framework – or on the higher-order dependence patterns of orthogonal shocks.

We also develop a less restrictive framework for achieving (partial) identification that only relies on group-wise independence (instead of mutual independence). Since shocks within the same group are allowed to be dependent, this can be considered as a weaker form of independence, as called for by Montiel Olea et al. (2022). Unlike purely data-driven identification methods, SRHL does not suffer from local identification under mild conditions and delivers outcomes that are truly structural by construction in terms of economic interpretations as given by the employed sign restrictions. In addition to the non-parametric method of Matteson and Tsay (2017), the proposed framework for point elicitation can be easily modified to incorporate alternative independence criteria. We explicitly address two of them: The pseudo maximum likelihood estimation (PML) (Gouriéroux et al., 2017) and the generalized method of moments (GMM) approach of Lanne and Luoto (2021). Owing to its robustness in various circumstances as shown in previous simulation studies (Herwartz and Maxand, 2020; Herwartz et al., 2022b; Matteson and Tsay, 2017), we consider the non-parametric approach based on dCov statistics as a flexible framework for testing independence, when the distribution of the structural innovations is unknown. We provide simulation-based evidence on the performance of the suggested SRHL estimator in both small and large samples and highlight its consistency and robustness under various distributional scenarios featuring super-Gaussian, nearly Gaussian and sub-Gaussian shocks. Combining statistical independence with set-identifying sign restrictions has also been advocated by Drautzburg and Wright (2021). Unlike the approach developed in this paper that aims for a point elicitation, Drautzburg and Wright (2021) focus on the refinement of the identified set. While the interval estimation suggested by these authors is conducted by taking the intersection between the confidence interval obtained by sign restrictions and the region, for which the null hypothesis of independence cannot be rejected at a certain significance level, the point estimator developed in this study is obtained by minimizing the evidence against the null hypothesis conditional on the sign restrictions.

We employ SRHL estimation to reconsider the empirical assessment of monetary policy (MP) effects within the rational bubble model of equity valuation (Galí, 2014). The adoption of (weak and consensual) sign restrictions with further narrative resolution holds particular merit for this purpose, since the recursive identification scheme employed in Galí and Gambetti (2015) has been considered as crucial for detecting rational bubbles (see e.g., Paul, 2020). Without imposing timing restrictions, the proposed coupling of set-identification and point estimation allows the data to speak in favor of results

[2] Gafarov et al. (2018) suggest a delta-method confidence interval for scalar structural objects of interest using directional derivatives of endpoints of the identified set. Granziera et al. (2018) analyze set-identified impulse response functions (IRFs) in a moment-inequality-based minimum distance framework. Giacomini and Kitagawa (2021) adopt a multiple-prior robust Bayesian approach and construct credible sets that are valid in a frequentist sense.

[3] The common approach of sampling orthogonal matrices from the Haar distribution (Rubio-Ramírez et al., 2010) is shown to be highly informative for the structural models in the identified set (Baumeister and Hamilton, 2015; 2018; Watson, 2019). Adding to the literature, we detect further complexity entering the prior through the imposition of sign restrictions in various forms.

that are similar to those in Galí and Gambetti (2015). In specific, an unexpected monetary tightening invokes a significant decline in dividends such that the fundamental component of the stock price exhibits a negative short-run response that tapers off. However, the response of the bubble component is positive and tends to dominate the long-term responses of asset prices. The core findings remain largely robust when we relax the independence assumption by allowing for dependence among non-MP shocks and by employing a GMM test that accounts for the possible prevalence of stochastic volatility. Going beyond this benchmark study, we underline the informational content of the detected MP shocks by highlighting their strong linear correlations with MP measures that have been retrieved in a theoretically and conceptually distinct context. Among others, the strongest correlation is detected with reference to the innovations in the MP reaction function in the DSGE model estimated by Smets and Wouters (2007).

The remainder of this paper is organized as follows. Section 2 introduces the sign-restricted SVAR models and discusses in detail the proposed approach for point elicitation. Monte Carlo evidence in Section 3 sheds light on the finite sample properties of the estimator. We employ the suggested approach in Section 4 to the work of Galí and Gambetti (2015) on the link between asset valuation and monetary policy in the US. Section 5 concludes. The online Appendix provides additional theoretical analysis and simulation evidence on point elicitation and implicit priors in sign-restricted SVARs (Appendices A, B and D) and further empirical results for US monetary policy and rational bubbles (Appendix C).

2. Methodology

In this section, we first sketch the identification problem in SVARs and sign restrictions as a theory-guided and mildly restrictive approach to set identification and subsequently introduce the SRHL procedure.

2.1. Sign-restricted structural VAR models

2.1.1. Model setup and specifications

The dynamic model of interest takes the structural form

$$Wy_t = \Pi \mathrm{x}_{t-1} + \xi_t, \ \ t = 1, \dots, T, \tag{1}$$

where y_t is a $K \times 1$ vector of observable variables, whose contemporaneous structural relations are characterized by a $K \times K$ invertible matrix W. The $K \times (Kp+1)$ structural autoregressive coefficient matrix Π links Wy_t to an intercept and past realizations of y_t up to lag order p, which are collected by $\mathrm{x}_{t-1} = (1, y'_{t-1}, y'_{t-2}, \dots, y'_{t-p})'$. The $K \times 1$ vector ξ_t comprises structural shocks that are uncorrelated with a zero mean and identity covariance matrix, i.e., $\xi_t \sim (0, I_K)$. The reduced form representation of model (1) is

$$y_t = A\mathrm{x}_{t-1} + u_t, \ \ t = 1, \dots, T, \tag{2}$$

where $A = B\Pi = (A_0, A_1, \dots, A_p)$ and $B = W^{-1}$, through which the structural shocks are mapped to the reduced form system:

$$u_t = B\xi_t \quad \text{and} \quad E\left[u_t u'_t\right] = \Sigma = BB'. \tag{3}$$

Under the weak stationarity condition, $\det A(z) = \det\left(I_K - \sum_{j=1}^{p} A_j z^j\right) \neq 0$ for $|z| \leq 1$, the process has a Wold moving average (MA) representation, which holds particular importance for tracing the dynamic effects of structural shocks on the system variables characterized by structural IRFs

$$y_t = \mu + \sum_{i=0}^{\infty} \Phi_i u_{t-i} \tag{4}$$

$$= \mu + \sum_{i=0}^{\infty} \Phi_i B \xi_{t-i} \ = \mu + \sum_{i=0}^{\infty} \Theta_i \xi_{t-i}, \tag{5}$$

where $\Theta_i = \Phi_i B = f_i(A, B)$ with $f_0(A, B) = f_0(B) = B$.[4] While reduced form parameters in $r = \left(\mathrm{vec}(A)', \mathrm{vech}(\Sigma)'\right)'$ can be consistently estimated from the observations, the structural parameter matrix B remains unidentified without further theoretical or statistical assumptions. The rank condition for solving K^2 distinct unknowns in B is not fulfilled, since the moment condition in (3) provides only $K(K+1)/2$ equations. More specifically, any orthogonal rotation of B – i.e., BQ, $Q \in \mathcal{O}(K)$, $\mathcal{O}(K) = \{Q = (q_1, \dots, q_K) : q_i \in \mathbb{R}^K, q'_i q_j = \delta_{ij}, \forall i, j = 1, \dots, K\}$ (with δ_{ij} being the Kronecker delta) – is observationally equivalent if shocks are multivariate Gaussian. One popular way to achieve (point) identification is to restrict the matrix B as the lower-triangular Cholesky factor (C) of the covariance matrix Σ imposing a recursive causal structure among the model variables (Sims, 1980). This approach has a tradition in the identification of MP shocks. However, the Cholesky scheme has been

[4] In specific, coefficients in (4) can be sequentially determined from the reduced form parameters in A, i.e., $f_i(A, B) = \Phi_i B = \sum_{j=1}^{i} A_j \Phi_{i-j} B$ with $A_j = 0$ for $j > p$, $\Phi_0 = I_K$ and $\mu = A(1)^{-1} A_0$.

criticized for being inconsistent with a rich variety of DSGE models, which typically feature contemporaneous responses of macroeconomic variables to the policy shock (see, e.g., Smets and Wouters, 2007).

2.1.2. Set identification with sign restrictions

Compared with exclusion restrictions as implied by the Cholesky scheme, sign restrictions are often considered to be relatively weak and more consensual. In general, the imposition of sign restrictions on IRFs up to horizon $h \geq 0$ can be formulated as[5]

$$F_h(A, B) = J\phi_h \geq \mathbf{0}, \tag{6}$$

where $\phi_h = \left(\text{vec}(f_0(A, B))', \ldots, \text{vec}(f_h(A, B))'\right)'$, with J being a selection matrix, which consists of either 1, -1 or zero and has the same number of rows as the number of imposed sign restrictions. At the implementation side, the imposition of sign restrictions proceeds by sampling from a structured space of reduced form covariance decompositions. A proposal for the structural parameter B is obtained by uniformly performing orthogonal rotations of a benchmark decomposition of the covariance matrix Σ, such as a Cholesky factor (C)

$$B = CQ, \ \ Q \in \mathcal{O}(K) \text{ and } CC' = \Sigma. \tag{7}$$

A draw of B is accepted if the imposed sign restriction is satisfied, i.e., $F_h(A, B) \geq \mathbf{0}$; otherwise, it will be discarded. Therefore, given the reduced form – characterized by the parameter vector r –, identification of the structural form essentially proceeds by imposing restrictions on the orthogonal matrix Q. Unlike the Cholesky scheme, which restricts Q to be a single point in $\mathcal{O}(K)$ (i.e., $Q = I_K$), the outcome from sign restrictions is a set of points in $\mathcal{O}(K)$, which leads to a set of structural parameters, to which we refer as the 'identified set' .

Conditional on the reduced form parameters r, the identified set – denoted by $IS_r(B)$ – refers to the set that contains all non-singular matrices B which diagonalize the covariance matrix and fulfill the imposed sign restriction, i.e.,$IS_r(B) = \{B \in \mathbb{R}^{K\times K} : BB' = \Sigma, \ F_h(A, B) \geq \mathbf{0}\}$. Given the sampling scheme in (7), there exists also a set of matrices Q associated with $IS_r(B)$, i.e., $IS_r(Q) = \{Q \in \mathcal{O}(K) : F_h(A, CQ) \geq \mathbf{0}\}$ and thus $IS_r(B) = \{CQ : Q \in IS_r(Q)\}$. In order to summarize core implications of an identified set, researchers often choose to report certain 'point estimates' – such as the posterior median, mean or median target as suggested by Fry and Pagan (2011) – of the structural object of interest. However, these estimators are based on posterior distributions and may reflect uncontrolled prior information without the prospect of convergence to the 'true' parameters from a frequentist perspective.[6] It has been widely recognized that the conditional prior beliefs for set-identified parameters are not updated through the likelihood function, such that the asymptotic equivalence between the Bayesian and the frequentist inference disappears (see Moon and Schorfheide, 2012; Poirier, 1998). Hence, a sound communication of point estimates requires that one must be crystal clear regarding the prior information that has been exploited for eliciting a specific 'most plausible model' from the identified set (see online Appendix A for a stylized example).

Given the consequential role played by the prior, the Haar distribution (i.e., uniform drawing Q from $\mathcal{O}(K)$, see Rubio-Ramírez et al. 2010) that underlies the common approach of sampling orthogonal matrices has been subjected to a critical assessment in the recent literature (Baumeister and Hamilton, 2015; 2018; Watson, 2019). In line with these studies, we show in online Appendix B that the implicit prior induced by such a sampling scheme is highly informative for the structural quantities of interest.[7]

2.1.3. Conditionally weak vs. strong restrictions

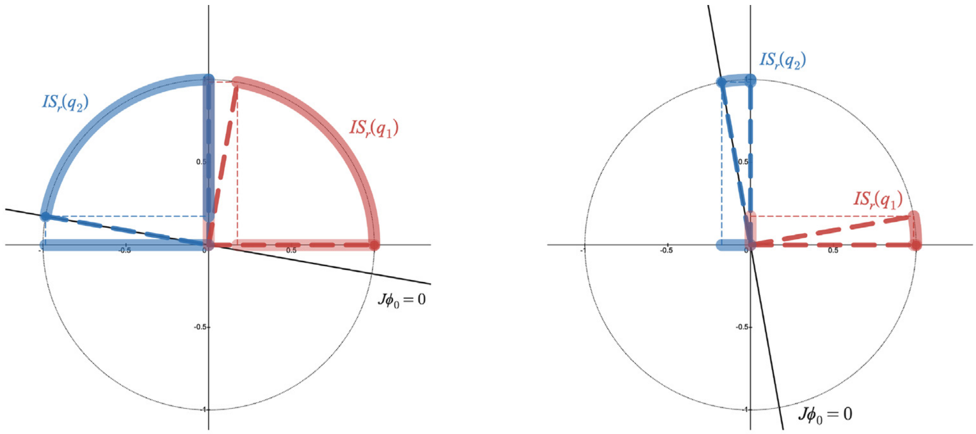
Before introducing our methods for point estimation, there is a further point worth addressing. Although the data remain silent about any particular point within the identified set, they are informative about the boundaries of the set (see Figs. 1 and 2 in Watson 2019). In practice, the boundaries of the identified set for a certain (scalar) structural quantity of interest can be sufficiently close to each other, such that the distance between any two points within the set is less than or equal to a small number $\epsilon > 0$. In this case, the critical assessment of the problems outlined above (and those in Baumeister and Hamilton 2015; 2018) may lose practical essence. This concern was also raised by Inoue and Kilian (2020). In this regard, we distinguish between conditionally weak and strong sign restrictions.

As an illustration, consider a bivariate VAR model consisting of the quantity and price of certain commodity driven by a demand and a supply shock. Since a demand shock contemporaneously moves both variables in the same direction, while a (negative) supply shock invokes an increase in price but a reduction of quantity, one can formulate the following sign

[5] One can also impose sign restrictions on other functions of the structural parameters, such as historical decompositions. For expository purposes, we consider in this section only restrictions imposed on IRFs.

[6] The posterior median (mean) estimate of a structural parameter is obtained by minimizing the expectation of the absolute loss (quadratic loss) evaluated by means of the marginal posterior distribution of that parameter.

[7] Adding to the literature, we detect further complexity entering the prior through the imposition of sign restrictions in various forms. In particular, imposing a sign restriction on a single parameter will not only truncate the prior of that parameter, but will generally affect the location and shape of the prior density of other parameters as well. Similarly, once higher-order IRFs are subject to restrictions, the prior also involves reduced form autoregressive parameters. In fact, it seems particularly difficult (if not impossible) to control such implicit priors (see the discussion in online Appendix B for details).



(a) restriction (8) is conditionally weak (b) restriction (8) is conditionally strong

Fig. 1. Identified sets of q_1 (red) and q_2 (blue) as implied by the sign restriction (8) given that $\sigma_{12} > 0$. Thick dashed lines indicate boundaries of identified sets for q_1 (red) and q_2 (blue), respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

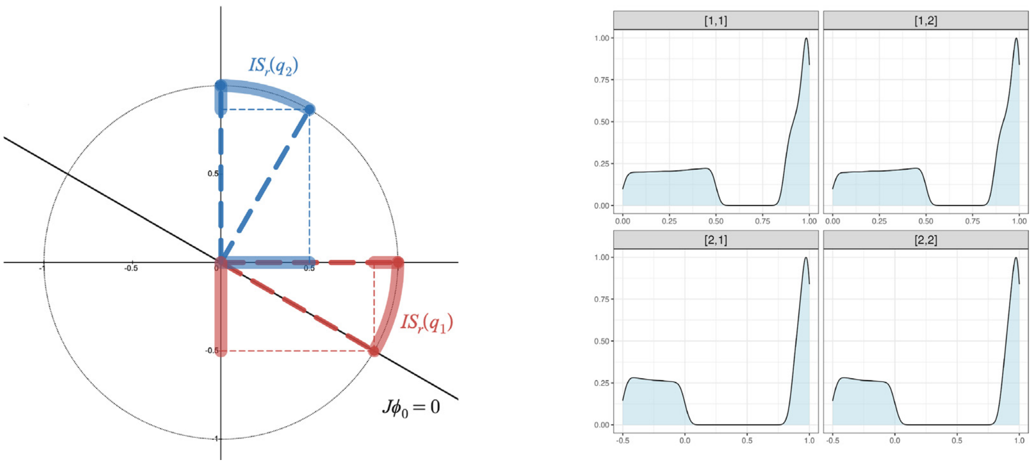


Fig. 2. Sign restriction is uninformative for a permutation of q_1 (red) and q_2 (blue). The right-hand side panel shows the marginal distribution of elements in Q. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

restrictions on the structural impact multiplier:

$$J\phi_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathrm{vec}(CQ) = \begin{bmatrix} \sqrt{\sigma_{11}} q_{11} \\ \frac{\sigma_{12}}{\sqrt{\sigma_{11}}} q_{11} + \sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}} q_{21} \\ -\sqrt{\sigma_{11}} q_{12} \\ \frac{\sigma_{12}}{\sqrt{\sigma_{11}}} q_{12} + \sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}} q_{22} \end{bmatrix} \geq \mathbf{0}, \tag{8}$$

where σ_{ij} is the $[i, j]$ element in Σ. Define $c \equiv \sigma_{12}/\sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} = \rho_{12}/\sqrt{1 - \rho_{12}^2}$, where ρ_{12} is the correlation of the reduced form residuals.[8] The restrictions in (8) imply

$$q_{11} \geq 0, \;\; q_{21} \geq -cq_{11} \quad \text{and} \quad q_{12} \leq 0, \;\; q_{22} \geq -cq_{12}.$$

[8] Notice that c is a monotonically increasing function of ρ_{12} and $c \geq 0$ ($c < 0$) holds, when $\rho_{12} \geq 0$ ($\rho_{12} < 0$) where the equality holds if and only if the correlation is zero.

Figure 1 shows the identified sets for the two unit (column) vectors in Q and their entries in the case of $\sigma_{12} > 0$.[9] It can be verified that as the correlation of the reduced form residuals ρ_{12} approaches one (minus one) – or equivalently $c \to \infty$ ($c \to -\infty$) –, the sign restriction in (8) becomes increasingly stronger. In this case, especially for parameters q_{22} and q_{11} (see Fig. 1(b)), any two points selected from the set do not materially differ from each other. The uncertainty related to these parameters predominantly stems from the estimation of the reduced form parameters, which determines the boundaries of the set.[10] In the limiting case where the reduced form residuals exhibit a complete correlation with $\rho_{12} = 1$ (-1) and Σ is singular, the identified set for Q is a singleton – i.e., $IS_r(Q) = \{I_2\}$ ($IS_r(Q) = \left\{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right\}$) – and 'point elicitation' becomes redundant. However, if the reduced form residuals are moderately or weakly correlated, the same restriction leads to an identified set that implies a much wider range for the structural parameters of interest (see Fig. 1(a)). In the borderline scenario where $\rho_{12} = 0$, the identified sets for q_1 and q_2 would cover the entire $\pi/4$ arc in the first and second quadrants, respectively. These are the cases for which point elicitation becomes more interesting because the identified set includes a broad spectrum of structural models each having a distinct economic implication.[11]

Motivated by these considerations, we let point estimation be guided by additional statistical assumptions and provide a data-based method to extract a point estimator from the identified set. Our estimation strategy is fully compatible with the widespread convention for the implementation of sign restrictions. The resulting point estimates summarize useful information from the data on the one hand. On the other hand, they have the interpretation of 'the most plausible' model under testable assumptions. We next describe the approach.

2.2. Least dependent shocks

2.2.1. Independent component estimation under the HL principle

If structural shocks of interest are mutually independent and non-Gaussian distributed, the structural parameter matrix B can be identified up to sign and column permutation. While the latter condition is rarely critical in most realistic scenarios, mutual independence requires that an economic 'shock' reflects an exogenous piece of information that originates in one (economic) sector without any (linear or non-linear) relation to ongoings in other segments of the economy. The uniqueness of independent components in SVARs follows from early results on the properties of linear combinations of non-Gaussian random variables. Accordingly, the linear weighting scheme $u_t = B\xi_t$ allows a recovery of the unique independent shocks in ξ_t from u_t, if and only if at most one element of ξ_t exhibits a Gaussian distribution (Comon, 1994).

The supposition of independent non-Gaussian shocks has led to various identification methods. While the approach followed by Moneta et al. (2013) relies on an a-priori recursive model structure, the performance of parametric methods proposed by Lanne et al. (2017) and generalized by Gouriéroux et al. (2017) depends on particular distributional assumptions for the shocks in ξ_t. Instead of mutual independence, approaches that rely on a weaker form of independence up to third and fourth order moments (co-skewness and co-kurtosis) have been recently developed and discussed by Hafner et al. (2022), Lanne and Luoto (2021) and Keweloh (2021). Representing a group of fully non-parametric identification schemes, the identification technique of Matteson and Tsay (2017) has been successfully applied recently (e.g., Herwartz, 2019; Herwartz and Plödt, 2016). Simulation evidence provided by Herwartz et al. (2022b) suggests that such a non-parametric approach promises consistent and robust assessments of IRFs under various distributional settings and (co)variance shifts.

Unlike purely statistical identifications that pursue an unrestricted optimization, the approach proposed in this study consists of a minimization of remaining dependence for structural models implied by the identified set $IS_r(B)$.[12] Using formal tests of the null hypothesis of mutual independence, the principle of so-called Hodges–Lehmann (HL) estimation allows for obtaining a point estimate of B from solving the minimization problem[13]

$$\min\{\text{Mutual dependence of components in } \xi_t(B)\}, \tag{9}$$

[9] The identified sets for q_1 (red) and q_2 (blue) are highlighted by the arcs of the unit circle. The sets for parameters in the first (q_{11} and q_{12}) and second row of Q (q_{21} and q_{22}) are given by the projection of the arcs on the abscissa and ordinate, respectively. Without loss of generality, q_2 is defined to be orthogonal to q_1 in counterclockwise direction and thus the matrix Q is interpreted as a rotation. Alternatively, one could also express Q as a reflection matrix by choosing the opposite orientation.

[10] In practice, the moment-inequality-based inference proposed by Granziera et al. (2018) can provide an indication of whether the imposed sign restriction is weak or strong conditional on a given reduced form model. However, because the result crucially depends on the reduced form specification (hence '*conditionally* weak or strong'), it should not be taken as direct empirical evidence in favor of or against the economic theory that motivates the imposed sign restrictions.

[11] Consider – for example – $b_{12} = \sqrt{\sigma_{11}}q_{12}$ and assume that $\sigma_{11} = 2$ is estimated without uncertainty. The identified set for b_{12} implied by this particular case in Fig. 1(a) is $[-1.39, 0]$ (since $IS_r(q_{12}) = [-0.98, 0]$). If b_{12} represents the impact effect of a certain policy action or the price elasticity of demand for a certain commodity, it would make a big difference whether the econometrician chooses to communicate -1.3 or -0.3 as a representative point estimate.

[12] An important shortcoming of shocks that are identified by purely statistical methods is that they do not necessarily allow for an economically meaningful interpretation. By contrast, the issue of so-called 'shock labeling' (see discussions in Herwartz and Lütkepohl, 2014; Lanne and Luoto, 2020) hardly applies to our approach. By construction, shocks in $IS_r(B)$ are truly structural in terms of implied economic properties as given by the employed sign restrictions.

[13] If model parameters of interest (ψ, say) can be subjected to testing by means of a nuisance-free test statistic, the HL estimator is the particular parameter value ψ_0 that produces the largest p-value of testing the null hypothesis $H_0 : \psi = \psi_0$ (Dufour, 1990; Hodges and Lehmann, 1963).

with $\xi_t(B) = B^{-1}(y_t - Ax_{t-1})$ s.t. $B \in IS_r(B)$. The parameter matrix B that minimizes the conditional loss function (9) is our point estimator, to which we refer henceforth as the sign-restricted-HL (SRHL) estimator. In fact, the performance of the SRHL estimator critically depends on the power features of the independence diagnostic employed. Herwartz and Maxand (2020) provide a simulation-based comparison of alternative tests of mutual independence and confirm a particular result of Matteson and Tsay (2017) who find that as a non-parametric independence test the dCov statistic of Székely et al. (2007) has favorable power features against a wide range of violations of the null hypothesis. Since the independence test based on dCov statistics has power against any form of dependence, and in light of its favorable power features, HL estimation of structural parameters as applied in this work builds upon this statistic.[14] We briefly outline the dCov statistic and the independence test.

2.2.2. Distance covariance and the SRHL estimator

The dCov statistic $\mathcal{V}^2$ between samples from two (groups of) random variables amounts to the difference between the characteristic function of the joint distribution function and the product of the characteristic functions of the respective marginal distributions. For random variables $X \in \mathbb{R}^{d_1}$ and $Y \in \mathbb{R}^{d_2}$ with $E|X|_{d_1} < \infty$ and $E|Y|_{d_2} < \infty$, Székely et al. (2007) introduce the test for independence between X and Y based on the distance covariance

$$\mathcal{V}^2(X,Y) = ||\varphi_{X,Y}(m,n) - \varphi_X(m)\varphi_Y(n)||^2_w, \tag{10}$$

where φ_X, φ_Y and $\varphi_{X,Y}$ denote the marginal and joint characteristic functions, respectively.[15] The distance covariance is a nonnegative number and is zero if and only if the two considered sets X and Y are independent. For two random samples $\mathbf{X} = (X_1, \dots, X_T)'$ and $\mathbf{Y} = (Y_1, \dots, Y_T)'$, where $X_i = (x_1, \dots, x_{d_1})'$ and $Y_i = (y_1, \dots, y_{d_2})'$, $i = 1, \dots, T$, the empirical version of the distance covariance is

$$\mathcal{V}^2_T(\mathbf{X},\mathbf{Y}) = ||\varphi^T_{\mathbf{X},\mathbf{Y}}(m,n) - \varphi^T_{\mathbf{X}}(m)\varphi^T_{\mathbf{Y}}(n)||^2_w = S_1 + S_2 - 2S_3, \tag{11}$$

where $\varphi^T_{\mathbf{X}}, \varphi^T_{\mathbf{Y}}$ are marginal empirical characteristic functions of the samples $\mathbf{X}$ and $\mathbf{Y}$ and $\varphi^T_{\mathbf{X},\mathbf{Y}}$ is the empirical characteristic function of the sample $\{(X_1, Y_1), \dots, (X_T, Y_T)\}$. Let $|\cdot|_d$ denote the Euclidean distance in $\mathbb{R}^d$. Then, the statistics in (11) are

$$S_1 = \frac{1}{T^2}\sum_{i=1}^{T}\sum_{j=1}^{T}|X_i - X_j|_{d_1}|Y_i - Y_j|_{d_2}$$

$$S_2 = \left(\frac{1}{T^2}\sum_{i=1}^{T}\sum_{j=1}^{T}|X_i - X_j|_{d_1}\right)\left(\frac{1}{T^2}\sum_{i=1}^{T}\sum_{j=1}^{T}|Y_i - Y_j|_{d_2}\right)$$

$$\text{and } S_3 = \frac{1}{T^3}\sum_{m=1}^{T}\sum_{i=1}^{T}\sum_{j=1}^{T}|X_m - X_i|_{d_1}|Y_m - Y_j|_{d_2}$$

Székely et al. (2007) showed that the test statistic $\mathcal{V}^2_T(\mathbf{X},\mathbf{Y})$ has an asymptotic distribution under the null hypothesis of independence

$$T\mathcal{V}^2_T(\mathbf{X},\mathbf{Y})/S_2 \xrightarrow{d} \mathcal{Q}, \tag{12}$$

where $\mathcal{Q}$ is a quadratic form of centered Gaussian random variables with $E[\mathcal{Q}] = 1$. For X and Y with finite first-order moments, the test is consistent against any form of dependence, since

$$\mathcal{V}^2_T(\mathbf{X},\mathbf{Y}) \xrightarrow{a.s.} \mathcal{V}^2(X,Y)$$

(see Theorem 2 in Székely et al. (2007) for a proof).

Matteson and Tsay (2017) suggested a composite approach for testing mutual independence based on distance covariances $\mathcal{V}^2$. Adapting the test to a K-dimensional random vector $\xi(B)$ with $K \ll T$, let $\boldsymbol{\Xi}(B) = (\xi_1(B), \dots, \xi_T(B))'$ denote the sample of structural shocks implied by B. To keep the notation uncluttered, we temporally omit the matrix B that signifies the dependence of ξ on B. Let $\xi^{(1)}, \dots, \xi^{(K)}$ denote a partition of the elements in $\boldsymbol{\Xi}(B)$ into samples of K marginal components. We apply the test to these components based on the fact that ξ consists of mutually-independent components if and

[14] More powerful tests of joint independence are worth considering in case an analyst has additional information on a specific (i.e., parametric) source of joint dependence of the shocks in ξ_t. However, for the general case addressed in this work, we believe that the availability of such information is likely context specific.

[15] In (10), $||\cdot||_w$ corresponds to the norm in the (weighted) L_2-space of functions on $\mathbb{R}^{d_1+d_2}$ with a positive weight function $w(m,n) = (c_{d_1}c_{d_2}|m|^{d_1+1}_{d_1}|n|^{d_2+1}_{d_2})^{-1}$, where the constant is $c_d = \pi^{(1+d)/2}/\Gamma((1+d)/2)$ (see Lemma 1 in Székely et al., 2007). Hence, the dependence measure in (10) can be written as

$$\mathcal{V}^2(X,Y) = \frac{1}{c_{d_1}c_{d_2}}\int_{\mathbb{R}^{d_1+d_2}}\frac{|\varphi_{X,Y}(m,n) - \varphi_X(m)\varphi_Y(n)|^2}{|m|^{d_1+1}_{d_1}|n|^{d_2+1}_{d_2}}dmdn.$$

only if $\varphi_{\xi^{(k)},\xi^{(k_-)}} = \varphi_{\xi^{(k)}}\varphi_{\xi^{(k_-)}}$, for all $k = 1, \ldots, K-1$ and $k_- = \{1, \ldots, K\} \setminus \{k\}$. The null hypothesis of the test states that

$$H_0 : \varphi_{\xi^{(k)},\xi^{(k_-)}} = \varphi_{\xi^{(k)}}\varphi_{\xi^{(k_-)}}, \quad \text{for all } k = 1, \ldots, K \text{ and } k_- = \{1, \ldots, K\} \setminus \{k\}.$$

The test statistic reads as

$$\mathcal{U}_T(\Xi(B)) = T\sum_{k=1}^{K} \mathcal{V}_T^2(\xi^{(k)}, \xi^{(k_-)}). \tag{13}$$

As $T \to \infty$, the test statistic $\mathcal{U}_T(\Xi(B)) \to \infty$, if the components in any subset of $\Xi(B)$ are dependent. Let $p^{\mathcal{U}_T}(B)$ denote the p-value from testing mutual independence of components in $\Xi(B)$ based on the test statistic in (13). The SRHL estimator for B in this study is defined as the particular one from the set, for which the weakest evidence is obtained against the null hypothesis (i.e., the highest p-value),

$$\max p^{\mathcal{U}_T}(B), \ \text{s.t. } B \in IS_r(B). \tag{14}$$

Under the assumption of non-Gaussian independent components, the criterion in (14) leads to a unique estimate. However, the SRHL approach still leaves room for the data to object against the independence assumption. For instance, finding that p-values obtained for all $B \in IS_r(B)$ are below a certain threshold (say 5%) would prompt the econometrician to reflect upon the assumption that underlying shocks are mutually independent. On the contrary, a 'landscape' of insignificant p-values adds to the interpretation of shocks as purely exogenous surprises, and hence, offers reassuring evidence in favor of the identified set if an analyst considers 'mutual independence' as an intuitive feature of the economic shocks under scrutiny Moreover, insignificant p-values support the reliability of structural IRFs that are usually constructed under the conditioning scenario of $E[\xi_{jt}|\xi_{it} = 1] = 0$ which applies - strictly speaking - only in a joint Gaussian model or under mutual independence of the shocks. Therefore, we consider the Hodges–Lehman principle (here the reliance on a formal test of independence) as a natural complement of set identification.

2.2.3. Alternative independence criteria

An important alternative to the non-parametric SRHL procedure is to pursue ICA by means of (pseudo) ML estimation (Gouriéroux et al., 2017). Denoting the (non-Gaussian) density function of the kth independent shock as $g_k(x)$, $k = 1, \ldots, K$, the log-likelihood of B reads as

$$l(B) = \sum_{t=1}^{T}\sum_{k=1}^{K} \log g_k\left[w_k(B)'(y_t - Ax_{t-1})\right] - T\log|\det B| \tag{15}$$

with $w_i(B)'$ being the ith row of $W = B^{-1}$. Accordingly, one can elicit the point estimator from $IS_r(B)$ by maximizing the (pseudo) log-likelihood

$$\max \sum_{t=1}^{T}\sum_{k=1}^{K} \log g_k\left[w_k(B)'(y_t - Ax_{t-1})\right], \ \text{s.t. } B \in IS_r(B).$$

The last term disappears because $\det B = \sqrt{\det \Sigma}$, $\forall B \in IS_r(B)$. The ML approach is attractive due to its computational efficiency and because it allows for likelihood-based inference straightforwardly. For instance, conventional exclusion restrictions can be tested based on likelihood ratios (Lanne et al., 2017). However, an important drawback of this approach is that the likelihood function depends not only on the structural parameter of interest, but also on density functions $g_k(x)$ (and possibly parameters that are used to parameterize the density functions). The misspecification of (pseudo) densities proves to have an effect on the asymptotic accuracy and could lead to inconsistent estimation in certain cases. We show in Section 3 and in online Appendix D by Monte Carlo experiments that the performance of the PML estimator could crucially depend on the choices of g_k and the distribution, from which the true structural innovations are generated. By contrast, the non-parametric procedure based on the dCov statistic yields point estimates with great accuracy in a large variety of distributional scenarios without making any explicit assumptions on the probability density functions of elements in ξ_t (see Herwartz and Maxand 2020; Herwartz et al. 2022b; Matteson and Tsay 2017 for more simulation-based evidence on dCov-ICA).

It is noteworthy that the GMM approach (Keweloh, 2021; Lanne and Luoto, 2021) relying on a number of co-skewness and co-kurtosis restrictions has been recently applied by Drautzburg and Wright (2021) in a similar fashion as the one proposed in this study.[16] These authors achieve a refinement of the identified set using alternative criteria that are necessary (not sufficient) for independence. In specific, their criterion is based on a set of third- and fourth-order moment conditions that takes the form

$$E[\xi_{kt}^2\xi_{jt}] = 0, \quad \text{and} \quad E[\xi_{kt}^3\xi_{jt}] = 0, \ E[\xi_{kt}^2\xi_{jt}^2] - 1 = 0, \tag{16}$$

[16] Drautzburg and Wright (2021) also consider a similar non-parametric test of independence based on Hoeffding's $\mathcal{D}_{X,Y}$ (Hoeffding, 1948), i.e., $\mathcal{D}_{X,Y} = F_{X,Y} - F_X F_Y$ with F_X, F_Y and $F_{X,Y}$ denoting the marginal and joint distribution of random variables X and Y, respectively. A similar independence criterion was also considered by Herwartz and Maxand (2020) and Herwartz et al. (2022b), where $\mathcal{D}_{X,Y}$ is measured in a Cramér-von Mises sense (Genest et al., 2007). While a comparative analysis of alternative independence criteria could be of interest for future research, it goes beyond the scope of this study.

for $j \neq k$ and $j, k \in \{1, \ldots, K\}$, respectively. Given a set of L moment conditions $m(B)$, one can formulate a $(L \times 1)$ vector consisting of their sample counterparts $\bar{m}(B) = T^{-1} \sum_{t=1}^{T} m_t(B) = T^{-1} \sum_{t=1}^{T} (\xi_{kt}^2 \xi_{jt}, \xi_{kt}^3 \xi_{jt}, \xi_{kt}^2 \xi_{jt}^2 - 1)'$ based on $\Xi(B)$. A GMM test can be carried out in a usual sense with a test statistic

$$\mathcal{J}_T = T\bar{m}(B)'\Omega_m \bar{m}(B), \tag{17}$$

where Ω_m is a $L \times L$ positive semi-definite weighting matrix that may also depend on B. The p-value from the test – denoted by $p^{\mathcal{J}_T}(B)$ – can be determined either based on the asymptotic χ^2-distribution under the null hypothesis or by means of simulation as suggested by Drautzburg and Wright (2021). While Drautzburg and Wright (2021) focus on the refinement of the identified set by testing the null hypothesis $H_0 : m(B) = 0$ at a certain significance level and construct the corresponding confidence region based on models, for which H_0 cannot be rejected, the SRHL approach aims for a point elicitation.[17] Accordingly, the point estimator for B is obtained as

$$\max p^{\mathcal{J}_T}(B), \text{ s.t. } B \in IS_r(B).$$

Owing to its weaker assumptions, the moment-based method is quite appealing, since it does not impose mutual independence but only some necessary conditions on selected higher-order moments. Furthermore, the GMM approach might cope with stochastic volatility, if it builds upon odd cross moments. With this being said, however, it does require that the moments under consideration exist. Furthermore, estimation based on higher-order moments or cumulants is sensitive against outliers in the data and could be particularly vulnerable, if the distribution of structural shocks exhibits heavy tails. We illustrate this by means of Monte Carlo experiments in Section 3.

2.2.4. Extensions and discussion

Partial independence In many applications, the assumption that the components in $\Xi(B)$ are mutually independent appears restrictive.[18] One particular merit of the SRHL approach is that it allows for dependence among components in the subset of $\Xi(B)$. If the interest of the econometrician lies in a particular (say, the k^*th) shock and the shock is uniquely characterized by sign restrictions, one can (partially) identify the k^*th column of matrix B by maximizing the p-value of a single dCov test (Székely et al., 2007), i.e.,

$$T\mathcal{V}_T^2(\xi^{(k^*)}, \xi^{(k^*_-)}).$$

Notice that the components in $\xi^{(k^*_-)}$ with $k^*_- = \{1, \ldots, K\} \setminus \{k^*\}$ are allowed to be dependent. The resulting SRHL estimator exploits merely the partial independence (sometimes also called 'group-wise independence') between $\xi^{(k^*)}$ and the remaining shocks, which can be considered as a weaker form of an independence assumption, as called for by Montiel Olea et al. (2022).

Local identification A well-known issue related to the identification by means of ICA is that the identification is only local. Any flip of the sign and/or permutation of the columns of Q will equally satisfy the independence criterion.[19] Sign-restricted models are by construction unique for sign patterns, but can also be non-informative for column permutations (see Bacchiocchi and Kitagawa, 2020 for conditions for global identification). For instance, if the third restriction (on b_{12}) in (8) is changed so that all elements in B have to be positive, the resulting sign pattern is not informative for distinguishing between the first and second column of Q or B. This is illustrated in Fig. 2. As a result, the marginal distribution of elements in Q is not concave (in this case, bi-modal) on the parameter space and q_{11} (q_{21}) has an identical marginal distribution as q_{21} (q_{22}). Notice that the median estimate for these parameters would lie outside the identified set. In this regard, sign restrictions have to be carefully designed. For instance, restrictions like $\begin{bmatrix} + & + \\ + & + \end{bmatrix}$ or $\begin{bmatrix} - & - \\ - & - \end{bmatrix}$ are prone to local identification and should be avoided. Agnostic or partial restrictions could also fall into this category. With this being said, if the imposed sign restriction is informative for column permutations, the proposed SRHL estimation does not suffer from such issues. As an example, one can make sure that the shock of interest has a unique sign pattern by checking that no other columns in B exhibit the same sign pattern. This will be illustrated in Section 4.

Caveats Since the SRHL procedure searches for an optimal rotation in the subset of $\mathcal{O}(K)$, i.e., $IS_r(Q)$, it is more efficient than purely statistical identification approaches. However, in the unlikely case that sign restrictions are false, conditioning on these restrictions might be counterproductive. Consider – for instance – the stylized bivariate model in (8). If all parameters in B are restricted to be positive while the reduced form residuals are negatively correlated ($\rho_{12} < 0$), the resulting $IS_r(B)$

[17] It is worth noting that when eliciting a particular point from the identified set, no test decision (i.e., whether to reject H_0) is made and a SRHL point estimator is always obtainable. In case of an interval estimation à la Drautzburg and Wright (2021), the resulting confidence region depends on a particular test decision (i.e., not rejecting H_0). Depending on the test level, the confidence region implied by the models, for which H_0 cannot be rejected conditional on the sign restrictions, can be narrow or wide, indicating whether the additional independence assumption is binding.

[18] For instance, the volatility of certain macroeconomic and financial shocks may co-move, especially during periods of recession and financial stress. As another example, a shock to speculative (inventory) demand of a certain commodity may also depend on an eventual shock to the supply of such a commodity, because a surge in the former one becomes more likely in scenarios, where there could be potential disruption or bottleneck in the supply.

[19] The test statistic in (13) is a modified version of the one presented in Matteson and Tsay (2017). Instead of joint estimation as in (13), they carry out the test based on sequential estimation with test statistics $T\sum_{k=1}^{K-1} \mathcal{V}_T^2(\xi^{(k)}, \xi^{(k_+)})$ and the null $H_0 : \varphi_{\xi^{(k)},\xi^{(k_+)}} = \varphi_{\xi^{(k)}}\varphi_{\xi^{(k_+)}}$, for all $k = 1, \ldots, K$ and $k_+ = \{k+1, \ldots, K\}$. It can be shown that in finite samples the test result based on sequential estimation depends on the column ordering of Q.

Table 1
Average mean squared errors (and their standard deviations in parentheses) for alternative independence criteria and sample sizes. The independent shocks are generated from the six distributions depicted in the left-hand-side panel of Fig. 3 with their kurtosis documented in the second row of the table.

		Student's $t(5)$		Student's $t(10)$		Student's $t(20)$		Gauss. mix (a)		Gauss. mix (b)		Gauss. mix (c)	
	Kurt.	8.243		4.003		3.374		2.500		2.274		1.514	
$T = 50$	dCov	0.117	(0.164)	0.091	(0.125)	0.091	(0.125)	0.077	(0.105)	0.071	(0.104)	0.053	(0.069)
	PML(+)	0.087	(0.143)	0.129	(0.171)	0.149	(0.175)	0.255	(0.164)	0.287	(0.135)	0.297	(0.039)
	PML(−)	0.280	(0.151)	0.239	(0.176)	0.228	(0.186)	0.111	(0.168)	0.057	(0.124)	0.005	(0.004)
	GMM	0.258	(0.170)	0.223	(0.180)	0.200	(0.181)	0.134	(0.175)	0.101	(0.162)	0.019	(0.062)
$T = 500$	dCov	0.039	(0.105)	0.043	(0.077)	0.049	(0.074)	0.043	(0.084)	0.035	(0.082)	0.036	(0.049)
	PML(+)	0.005	(0.015)	0.036	(0.106)	0.103	(0.169)	0.295	(0.081)	0.294	(0.045)	0.293	(0.012)
	PML(−)	0.298	(0.057)	0.291	(0.113)	0.250	(0.161)	0.011	(0.052)	0.002	(0.003)	0.000	(0.000)
	GMM	0.180	(0.163)	0.176	(0.172)	0.192	(0.177)	0.045	(0.112)	0.028	(0.086)	0.028	(0.086)
$T = 1,000$	dCov	0.034	(0.112)	0.039	(0.085)	0.038	(0.047)	0.037	(0.088)	0.024	(0.071)	0.029	(0.046)
	PML(+)	0.003	(0.032)	0.008	(0.036)	0.059	(0.131)	0.295	(0.053)	0.293	(0.030)	0.293	(0.008)
	PML(−)	0.294	(0.060)	0.294	(0.080)	0.279	(0.139)	0.003	(0.003)	0.001	(0.001)	0.000	(0.000)
	GMM	0.152	(0.171)	0.128	(0.159)	0.157	(0.174)	0.029	(0.084)	0.034	(0.095)	0.034	(0.093)
$T = 5,000$	dCov	0.021	(0.094)	0.022	(0.083)	0.029	(0.049)	0.020	(0.088)	0.003	(0.007)	0.019	(0.040)
	PML(+)	0.000	(0.000)	0.001	(0.002)	0.007	(0.034)	0.293	(0.022)	0.293	(0.014)	0.293	(0.004)
	PML(−)	0.286	(0.048)	0.294	(0.035)	0.293	(0.074)	0.001	(0.001)	0.000	(0.000)	0.000	(0.000)
	GMM	0.114	(0.160)	0.065	(0.127)	0.065	(0.124)	0.035	(0.095)	0.048	(0.109)	0.033	(0.092)

would be an empty set.[20] Therefore, sign restrictions imposed should be conditionally weak and reasonably consensual (to avoid emptiness), but also sufficiently strong to be informative with regard to alternative column permutations. An example of such a restriction is given by the identifying assumption 1 in our empirical application.

3. Monte Carlo analysis

In this section, we investigate the simulation performance of the proposed SRHL estimator. Several papers have studied the non-parametric identification of independent components and provided rich simulation-based evidence for its finite sample performance in various circumstances. Matteson and Tsay (2017) compare the performance of dCov-ICA with some prominent ICA procedures, including the FastICA that uses the negentropy as a contrast function. Within a selection of 18 source distributions, they find that dCov-ICA outperforms the most competing methods. Adapting dCov-ICA for the purpose of identification in SVARs, Herwartz and Maxand (2020) and Herwartz et al. (2022b) find that the non-parametric approach based on dCov statistics yields point estimates for structural IRFs with great accuracy in a data-generating process that features realistic stochastic properties such as volatility breaks. With this being said, estimators discussed in these studies are purely data-driven and thereby suffer from local identification (i.e., their estimates are only unique up to permutations and signs of the columns of the structural mixing matrix). In this section, we compare the performance of the proposed estimator conditional on a sign pattern that leads to global identification. The simulation exercise is conducted based on a stylized bivariate model, in which the linearly transformed shocks $u_t = B\xi_t$ are directly observed, with a much sharper focus on a comparative assessment of the alternative independence criteria for point elicitation discussed in the previous section. In practice, the residual vector u_t is estimated from the data and its estimation uncertainty has to be properly accounted for.[21] In the online Appendix D, we provide further evidence on the performance of the SRHL estimator conditional on the typical agnostic sign pattern employed for monetary policy identification. In this context, the SRHL estimator provides sufficiently accurate assessments of the structural parameters with data generated from a calibrated three-equation DSGE model even in small samples.

The structural shocks are independently drawn from one of the six distributions displayed on the left-hand-side panel of Fig. 3, including Student's t with various degrees of freedom and different symmetric Gaussian mixtures. The shocks are adjusted by their population variance to have unit variances. The kurtoses of the standardized distributions are documented in the second row of Table 1. It goes from 8.4 (super-Gaussian) to 3.3 (nearly Gaussian) and towards 1.5 (sub-Gaussian). The mixing matrix B is formulated as

$$B = CQ = \begin{bmatrix} 1 & 0 \\ \sigma_{12} & \sqrt{1-\sigma_{12}^2} \end{bmatrix} \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix}, \tag{18}$$

where we choose a moderate correlation of the observables $\sigma_{12} = 0.4$ and rotate the Cholesky factor by an angle $\vartheta = \pi/3$. Accordingly, the true structural parameter matrix reads as $B = \begin{bmatrix} 0.500 & -0.866 \\ 0.994 & 0.112 \end{bmatrix}$. In each experiment, the econometrician

[20] The identified set for Q produced by such restrictions is illustrated in the left panel of Fig. 2. If $\rho_{12} < 0$, the slope of the straight line in that Figure $(-c)$ will be positive. Because all elements in matrix Q have to be positive, there exists no real solution for $q_1' q_2 = q_{11}q_{12} + q_{21}q_{22} = 0$.

[21] While studying the coverage properties of the respective confidence intervals is interesting, it lies beyond the scope of this paper.

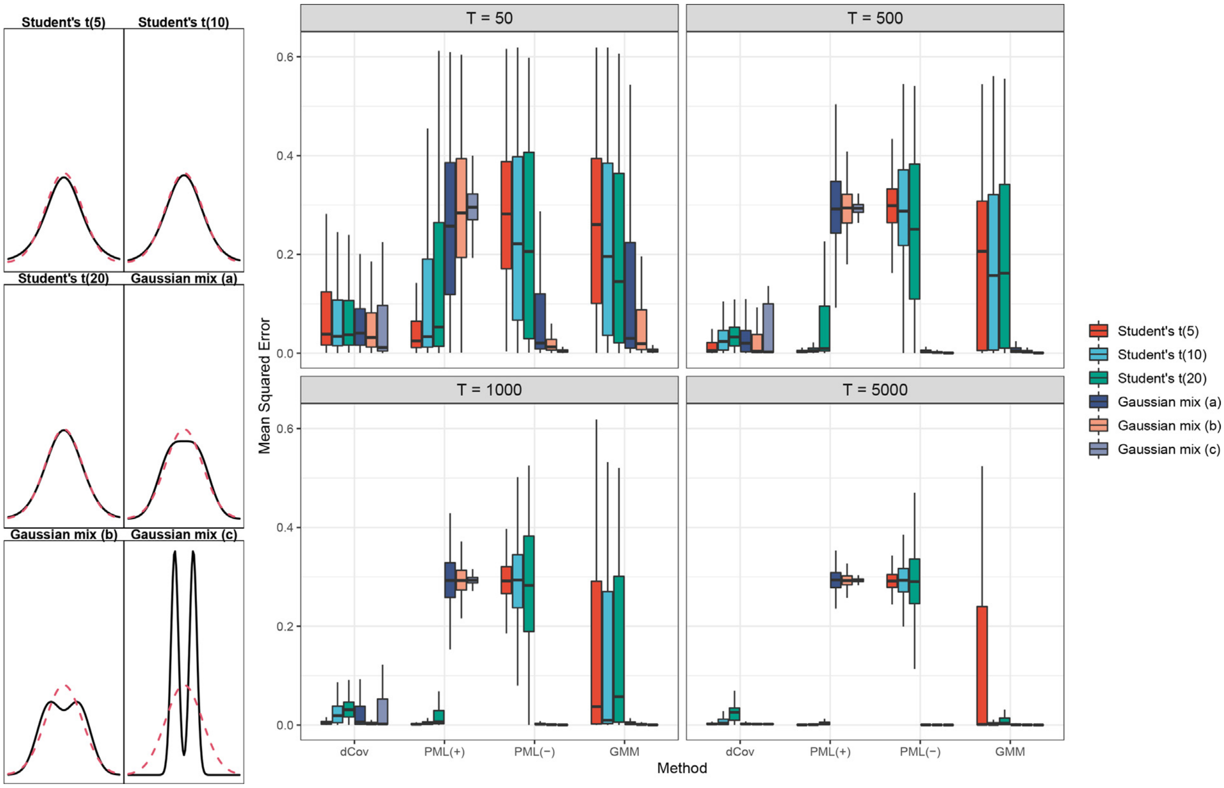


Fig. 3. The left-hand-side panel shows six data-generating-distributions, including Student's t and mixtures of Gaussian distributions. The Gaussian density is indicated by the red dashed curve. The right-hand-side panel shows boxplots of the mean squared errors for alternative independence criteria (with horizontal lines marking the medians). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

observes the sequence $\{u_t\}_{t=1}^T$, estimates the covariance σ_{12} and thus the Cholesky factor C. Based on this, she imposes the correct sign pattern and approximates the identified set with 1000 draws using the sampling procedure of Rubio-Ramírez et al. (2010).[22] Then she elicits four point estimates for B based on the proposed non-parametric dCov test, PML based on a Student's $t(5)$ and a Gaussian mixture density, and finally on the GMM test using the high-order cross-moments in (16). Owing to the correctly imposed sign restriction, there is no ambiguity about the ordering and sign of the column of B, so that we can directly assess the performance of these estimates in a conventional mean-squared-error (MSE) sense. The MSE of an estimate $\hat{B}$ is computed by taking the product $\text{vec}(\hat{B}-B)'\text{vec}(\hat{B}-B)$ and dividing it by 4 (i.e., number of unknown structural parameters). The experiment is performed with 1000 replications.

Results from the Monte Carlo exercise are displayed in Fig. 3 and Table 1. Four alternative sample sizes are considered $T = 50$, 500, 1000 and 5000, which are indicative for both the small-sample performance and asymptotic properties of the estimators. The performance of the proposed SRHL estimator is markedly robust against the underlying shock distributions. Pointing to the consistency of the estimator, the MSE converges towards zero as the sample size T increases. The performance of the PML estimator crucially depends on the underlying data-generating distribution. It performs very well, if the underlying source distribution coincides with the pseudo density employed. However, it severely suffers from missspecification and inconsistency, once the source distribution is super-Gaussian but the pseudo density is sub-Gaussian, or vice versa. GMM estimation based on higher-order moments performs quite well – also in small samples – if the data-generating distribution exhibits negative excessive kurtosis. However and as mentioned previously, in cases when the underlying shocks are super-Gaussian with heavy tails (e.g., Student's $t(5)$), GMM estimation requires considerably more observations to reach a similar performance as the one based on the dCov test.[23] Results documented for Student's $t(20)$ shocks (see the corresponding column in Table 1) demonstrate that for nearly Gaussian shocks, the suggested SRHL estimator outperforms the competing independence criteria in small to medium-sized samples. Despite the correctly specified (except for the de-

[22] Notice that the sign pattern is informative for column permutations, such that the resulting estimates do not suffer from local identification.

[23] It is worth mentioning that GMM estimation allows for potential heteroskedasticity and co-heteroskedasticity in the data, while the distributional result in (12) only holds if observations in the sample are identically distributed. In this regard, if there is clear evidence for heteroskedasticity in the data, the less restrictive moment-based methods (Drautzburg and Wright, 2021; Lanne and Luoto, 2021) should be considered. For this reason, we employ the GMM method as a robustness check in the empirical application in Section 4.

grees of freedom parameter) pseudo density, PML(+) manages to capture the slightly positive excessive kurtosis (of about 0.37) and outperforms dCov only in the very large sample with $T = 5,000$. In summary, the SRHL estimator based on the non-parametric dCov test can be considered as a flexible framework for detecting structural shocks, when their underlying distributions are unknown.

4. What are the effects of monetary policy on the fundamental and bubble components of equity prices?

Echoing a variety of potential amplifiers of MP transmission (e.g., channeling through wealth Goodhart and Hofmann, 2008, investment, i.e., Tobin's q, and credit Bernanke et al., 1999), policy-makers and academics alike have developed considerable interest in uncovering causal linkages among asset valuation and MP. Meanwhile, SVARs have become a prominent approach to assessing the relationship between MP and asset prices empirically. While early studies (e.g., Patelis, 1997) rely on recursive model structures for identification, Bjørnland and Leitemo (2009) have initiated a debate of potential bidirectional relations and suggested a combination of short- and long-run restrictions for identification. Interestingly, their restrictions have been rejected in Lütkepohl and Netšunajev (2017) who gain overidentifying information from a heteroskedasticity-based approach to identification. Results from independence-based identification of a six-dimensional VAR in Herwartz et al. (2022a) indicate that MP shocks have a negative and sluggish effect on US house prices, while the effect on equity is rather short-lived and less pronounced. After experiencing how persistent deviations between asset prices and fundamentals could become an important source of real economic turmoil, the so-called 'lean-against-the-wind' debate has gained momentum to unravel the scope of MP to actively steer asset prices towards their fundamental values. In this debate, the notion of rational bubbles (Galí, 2014) has challenged the conventional view that asset prices respond negatively to surprise hikes in interest rates. On the empirical side, Galí and Gambetti (2015) have provided strong evidence in favor of an ultimately indeterminate response of asset prices to contractionary MP. More specifically, based on a systematic distinction of fundamental and bubble components of asset prices, the ultimate (or net) response of asset prices to a contractionary increase in interest rates is given by the sum of an expected decline in the fundamental and an expected increase in the bubble component.

Regarding the identification of their structural model, Galí and Gambetti (2015) derived impact effects of structural shocks within a six-dimensional model employing a Cholesky decomposition. As a result of their variable ordering, the adopted exclusion restrictions imply that monetary authorities could respond contemporaneously to surprise information originating in dividends but not to such information entering through asset prices (i.e., the FFR is ordered between (real) dividend growth and stock returns). However, the literature on the MP asset valuation link has shed considerable doubt on recursive model structures. Paul (2020) makes a strong point against the recursive model of Galí and Gambetti (2015), noticing that their findings in favor of the rational bubbles model 'are driven by the Cholesky identification of shocks that is subject to the mentioned concerns' . Moreover, given that the recursive model is a point-identified model specification, it is interesting to systematically compare the structural implications of the SRHL estimator and point estimates obtained from a recursive structural model.[24] Next, we describe our (weak) identifying assumptions that allow us to formulate a set of sign restrictions, and subsequently turn to elicitation of particular candidates out of set-identified admissible structural models (i.e., the median and median target and the SRHL estimator), and aim to unravel how US MP has affected fundamental and bubble components of equity values separately and after aggregation.

The empirical model of interest is a structural VAR comprising six observable variables ($K = 6$), which are collected as $y_t = (\Delta x_t, \Delta d_t, \Delta p_t, \Delta p_t^c, i_t, \Delta q_t)'$, where $x_t,\ p_t,\ p_t^c,\ q_t,\ d_t$ denote (in log) real GDP, the GDP deflator, the World Bank commodity price index, the S&P 500 Composite Index and the corresponding dividend series (both deflated by the GDP deflator), respectively, and i_t is the FFR.[25] The data are measured at a quarterly frequency and cover the period from 1960Q1 until 2011Q3. We use the same reduced form specification as Galí and Gambetti (2015), which includes an intercept term and lags up to order 4. The reduced form is estimated by means of ML.

4.1. Identification

Unlike using a lower-triangular covariance decomposition for identification as in Galí and Gambetti (2015), the MP shock is identified by means of sign restrictions in this illustration. We keep our identifying restrictions rather minimalist by focusing on fairly uncontroversial sign patterns and remain agnostic about, e.g., signs of impact responses of output and asset prices. To avoid issues of local identification, the imposed sign restrictions have to be informative for column permutations of B and arguably unique in characterizing the structural shock of interest.

[24] Galí and Gambetti (2015) largely base their empirical conclusions on a corresponding model with time-varying coefficients. Recognizing that it is debatable whether the reduced form is in fact subject to time variation (Sims and Zha, 2006), we mainly focus on the constant coefficient model for two reasons. On the one hand, it allows for a straightforward comparison of alternative identification schemes. On the other hand, the constant coefficient model as such provides an interesting 'average' perspective on the effects of MP on (stylized components) of US equity prices.

[25] Following Galí and Gambetti (2015), we interpret an exogenous MP shock as a shock in the policy rate (FFR). The replication files for Galí and Gambetti (2015) are downloaded from openICPSR. In particular, we use the `MakeData.m` file provided by these authors to transform the data collected in `DataQ.xlsx`.

To capture the unique counter-directional co-movement of inflation and interest rates induced by a MP shock (Uhlig, 2005), we first assume that an exogenous monetary tightening uniquely induces instantaneous positive responses of interest rates i_t and negative responses of both inflation ($\Delta p_t < 0$) and dividends ($\Delta d_t < 0$).

Identifying assumption 1. A contractionary monetary policy shock is the only shock in the system that causes a positive contemporaneous response of the FFR and negative contemporaneous responses of inflation and dividend growth.

While the agnostic restrictions of Assumption 1 encompass a wide range of empirical and theoretical models (including, e.g., Galí, 2014), they have also been criticized for being too weak to allow reliable inference (Antolín-Díaz and Rubio-Ramírez, 2018; Arias et al., 2019). To further sharpen the implications of admissible models, we follow Arias et al. (2019) – who hint at the informational content of the Taylor rule for the identification of MP shocks – and impose restrictions on the systematical components of the MP reaction function. Furthermore, we add a narrative restriction in a similar fashion as Antolín-Díaz and Rubio-Ramírez (2018) by focusing on the extraordinary disinflationary measures taken by the Fed Chair Paul Volcker in late 1979. This results in the following identifying assumptions:

Identifying assumption 2. The contemporaneous response of the FFR to output and prices is positive.

Identifying assumption 3. The monetary policy shock was the most important driver of the increase in the FFR observed in 1979Q4. In other words, the monetary policy shock had the largest positive contribution to the FFR.[26]

Unlike a recursive identification scheme, our identification strategy is not sensitive to the ordering of the variables in y_t. Nevertheless, we try to stay as closely related to Galí and Gambetti (2015) as possible and assume – without loss of generality – that the MP shock is the fifth shock in the system, i.e., $\xi_t^{mp} = \xi_{5t}$. The identifying assumption 1 imposes three sign restrictions on the IRFs at horizon $i = 0$ and can be formally expressed as

$$F_0(B) = J_1\phi_0 = J_1 \mathrm{vec}(B) = \begin{bmatrix} -b_{2k} \\ -b_{3k} \\ b_{5k} \end{bmatrix} > \mathbf{0},\ \exists! k \in \{1, \ldots, 6\} \tag{19}$$

where $J_1 = (-e_{6k-4}, -e_{6k-3}, e_{6k-1})'$ with e_j being the jth column of an identity matrix of dimension 36×36. Furthermore, notice that the fifth equation in the structural model (1) is the reaction function of the central bank and reads as

$$i_t = \tau_x \Delta x_t + \tau_d \Delta d_t + \tau_p \Delta p_t + \tau_{p^c} \Delta p_t^c + \tau_q \Delta q_t + \boldsymbol{\pi}_5' \mathrm{x}_{t-1} + \xi_{5t}, \tag{20}$$

where $\boldsymbol{\pi}_5'$ is the fifth row in Π and $\tau_x = -w_{55}^{-1} w_{51}$, $\tau_d = -w_{55}^{-1} w_{52}$, $\tau_p = -w_{55}^{-1} w_{53}$, $\tau_{p^c} = -w_{55}^{-1} w_{54}$, $\tau_q = -w_{55}^{-1} w_{56}$ with w_{ij} being the $[i, j]$ element in $W = B^{-1}$, respectively. The two sign restrictions given by the identifying assumption 2 are $\tau_x > 0$ and $\tau_p > 0$, while other structural parameters in the reaction function are unconstrained. In analogy to the identifying assumption 1, these sign restrictions can be written as

$$F_{TR}(B) = \left[I_2 \otimes (J_{2,1} \mathrm{vec}(B^{-1}))\right] J_{2,2} \mathrm{vec}(B^{-1}) = \begin{bmatrix} -w_{55} w_{51} \\ -w_{55} w_{53} \end{bmatrix} > \mathbf{0}, \tag{21}$$

where $J_{2,1} = -e_{29}'$ and $J_{2,2} = (e_5, e_{17})'$. Finally, the narrative restriction 3 is imposed on the historical decomposition of the variable i_t. The time series is approximated by the sum of the cumulative effect of each of the six shocks, which is characterized by the structural MA representation (5), i.e.,$i_t \approx \tilde{i}_t = \sum_{i=0}^{t-1} e_5' \Theta_i \xi_{t-i} = \sum_{k=1}^{6} \tilde{i}_t^{(k)}$, where $\tilde{i}_t^{(k)} = \sum_{i=0}^{t-1} \theta_{5k} \xi_{k,t-i}$ quantifies the cumulative effect of the kth shock on variable i_t and e_5 is the fifth column of the identity matrix I_6. Let $\Delta\tilde{i}_{T_2-T_1}^{(k)}$ denote the approximated cumulative change in i_t between two dates T_1 and T_2 (with $T_1 < T_2$) contributed by the kth shock, i.e., $\Delta\tilde{i}_{T_2-T_1}^{(k)} = \tilde{i}_{T_2}^{(k)} - \tilde{i}_{T_1}^{(k)}$. The identifying assumption 3 can be expressed as

$$F_{NA|\mathbf{y}_{T_1-p:T_2}}(A, B) = \begin{bmatrix} \Delta\tilde{i}_{T_2-T_1}^{(5)} \\ \Delta\tilde{i}_{T_2-T_1}^{(5)} - \Delta\tilde{i}_{T_2-T_1}^{(1)} \\ \Delta\tilde{i}_{T_2-T_1}^{(5)} - \Delta\tilde{i}_{T_2-T_1}^{(2)} \\ \Delta\tilde{i}_{T_2-T_1}^{(5)} - \Delta\tilde{i}_{T_2-T_1}^{(3)} \\ \Delta\tilde{i}_{T_2-T_1}^{(5)} - \Delta\tilde{i}_{T_2-T_1}^{(4)} \\ \Delta\tilde{i}_{T_2-T_1}^{(5)} - \Delta\tilde{i}_{T_2-T_1}^{(6)} \end{bmatrix} > \mathbf{0}, \tag{22}$$

[26] While the narrative restriction is inspired by the type-I restriction proposed in Antolín-Díaz and Rubio-Ramírez (2018), which specifies a particular shock as the most important driver of the *unexpected change* of certain variable, our restriction is imposed in a slightly different way. In fact, the restriction is twofold. First, the contribution of the MP shock to the FFR must be positive. Second, this contribution is larger than those of any other shock. Note that the sign of contributions of other shocks is left unrestricted and a negatively large (in absolute value) contribution from non-MP shocks is allowed. Instead of focusing on the sign of the shock (e.g., the restriction 4 employed in Antolín-Díaz and Rubio-Ramírez, 2018), we impose a restriction on the sign of the historical decomposition, which proves to be fruitful for analysing the Volcker's period as recently shown in Herwartz et al. (2022b).

Table 2
Element-wise Jarque-Bera tests and bootstrap tests of the number of Gaussian components based on fourth-order blind identification (FOBI) for $\hat{u}_t$.

Jarque–Bera tests of components in the residual vector						
Component	1	2	3	4	5	6
Test statistic	36.02	6.18	1.54	390.80	1197.2	106.25
p-value	0.00	0.05	0.46	0.00	0.0	0.00
FOBI test for the number of Gaussian components ($K-k_0$)						
$K-k_0$	5	4	3	2		
Test statistic	40570.29	5076.52	1696.67	36.33		
p-value	0.00	0.00	0.00	0.91		

Table 3
Correlation between SRHL-implied MP shocks and alternative measures from other studies.

	Alternative monetary policy measures						
•	SW	SZ	RR	GSS	RR*	SZ*	HRW
Cor($\xi_t^{mp,SRHL}, \xi_t^{mp,\bullet}$)	0.763	0.712	0.556	0.356	0.412	0.601	0.601
p-value	0.000	0.000	0.000	0.001	0.000	0.000	0.000
Sample size	175	168	154	88	154	149	171

where $\mathbf{y}'_{T_1-p:T_2} = (y'_{T_1-p}, \ldots, y'_{T_2})'$. Notably, besides the reduced-form and structural parameters, the narrative sign restriction also depends on the residual vector u_t between T_1 and T_2, which in turn depends on the data vector and its relevant lags.[27]

Remark 1. Following assumptions 1–3, the identified set is $IS_r(B) = \{B \in \mathbb{R}^{K\times K} : BB' = \Sigma,\ F_0(B) > \mathbf{0},\ F_{TR}(B) > \mathbf{0},\ F_{NA|\mathbf{y}_{T_1-p:T_2}}(A,B) > \mathbf{0}\}$, where $F_0(B)$, $F_{TR}(B)$ and $F_{NA|\mathbf{y}_{T_1-p:T_2}}(A,B)$ are defined in (19), (21) and (22), respectively.

Identifying assumption 4. Structural shocks in the system are independent components, and at most one shock exhibits a Gaussian distribution.

For a respective diagnostic analysis, we apply fourth-order blind identification to components in the VAR residual, as suggested by Nordhausen et al. (2017). Results documented in Table 2 indicate strong deviations from Gaussianity. The null hypothesis stating that the last (first) $K-k_0$ (k_0) components are Gaussian (non-Gaussian) can be rejected for $k_0 = 3$, 2 and 1 with 1% significance.[28]

4.2. Monetary policy shocks

Since US MP shocks obtained from the SRHL approach can be considered economically meaningful by virtue of the identifying assumptions 1 to 3, it is tempting to compare these point-identified shocks with alternative policy shock measures provided in related studies. In specific, we compare our benchmark estimates with i) innovations to the estimated MP reaction function in the medium-scale DSGE model of Smets and Wouters (2007) (SW: 1959Q1–2004Q4); ii) the best-fitting SVAR shock from Sims and Zha (2006) featuring volatility changes (SZ: 1960Q4–2003Q1); iii) narrative measures obtained by Romer and Romer (2004) based on Federal Open Market Committee (FOMC) minutes (RR: 1969Q3–2007Q4); iv) policy measures retrieved from high-frequency changes in the three-month fed funds futures within a 30-minutes-window around FOMC announcements (see e.g., Gürkaynak et al., 2005) (GSS: 1990Q1–2016Q4); v) MP shocks obtained from a three-dimensional SVAR identified by means of (co)variance changes in Herwartz et al. (2022b) (HRW: 1966Q1–2008Q3) and vi) - vii) RR and SZ series adjusted by Herwartz et al. (2022b) conditional on their (co)variance change model (RR* and SZ*). Correlations between SRHL-estimates with these shocks (joint with effective time series length and p-values from testing $H_0: \rho = 0$) are documented in Table 3.

The benchmark MP shocks obtained in this paper are highly correlated with these alternative measures at a significance level of 1%. Although strong and significant correlations between identified shocks do not necessarily imply that the shocks are identified correctly, it is worth recalling that these MP shocks are obtained from quite distinct econometric frameworks (e.g., in terms of the employed model specification, data and identification assumptions). In this regard, two observations

[27] Since the (conditional) prior related to such narrative restriction depends not only on the relevant parameters, but also on the set of observations, one has to resort to the concept of 'conditional identified set' for Bayesian inference (see Giacomini et al., 2021).

[28] Specifically, we employ the function `FOBIboot` from the `R` package `ICtest` developed by Nordhausen et al. (2017) with 2000 bootstrap replications. We also apply Jarque–Bera (JB) tests to each VAR residual series and have arrived at equivalent test decisions. Furthermore, we perform JB tests for MP shocks implied by the 1000 draws conditional on the identified set defined in Remark 1. The frequencies of obtaining p-values less than 0.1, 0.05 and 0.01 are, 76%, 73% and 67%, respectively. This provides arguably strong evidence for deviations of US MP shocks from a Gaussian distribution.

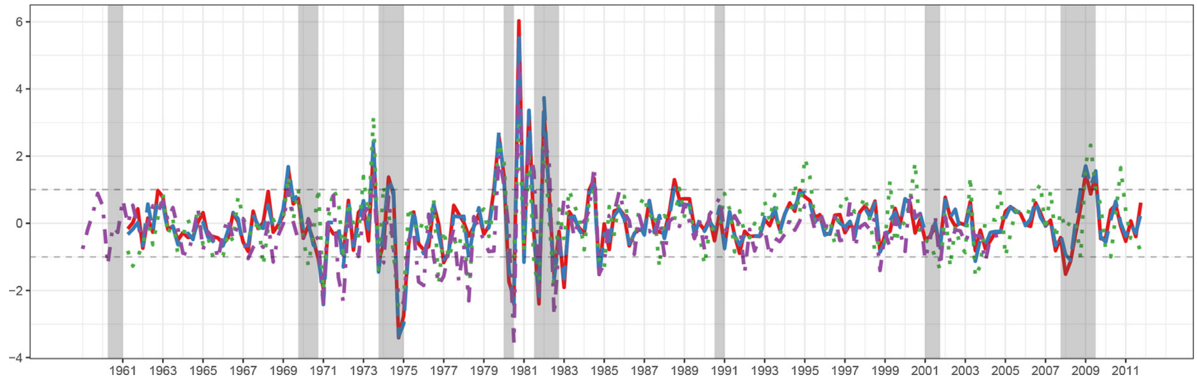


Fig. 4. The solid red, dashed blue, dotted green and purple dot-dashed lines indicate estimates for MP shocks obtained in a recursive SVAR, by means of SRHL estimation and the median target approach in a sign-restricted SVAR, and as innovations to the policy reaction function in a medium-scale DSGE model of Smets and Wouters (2007), respectively. Shaded areas represent episodes of economic recession as defined by the National Bureau of Economic Research. Dashed horizontal lines indicate locations of ±1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

are worth remarking. First, we display in Fig. 4 the DSGE shock of Smets and Wouters (2007) alongside MP measures obtained from the sign-restricted SVAR model. It becomes evident that the informational content of shocks to the MP reaction function in Smets and Wouters (2007) is strikingly similar to benchmark estimates in this study, showing a correlation coefficient of 0.763 when compared with the SRHL estimates. Second, related studies (Braun and Brüggemann, 2022; Nguyen, 2018) have recently cast doubt on the instrumental validity of the narrative measures provided by Romer and Romer (2004). Interestingly, the shock measures – both unadjusted (RR) and adjusted (RR*) – of Romer and Romer (2004) show the weakest correlation with the SRHL estimates among the alternative measures with a similar sample size.

4.3. Impulse responses

According to Galí (2014), the price of an infinite-lived asset Q_t is the sum of a fundamental component Q_t^F and a bubble component Q_t^B, i.e., $Q_t = Q_t^F + Q_t^B$. The fundamental component is the expected present value of future payoffs D_{t+i}

$$Q_t^F \equiv E_t\left\{\sum_{i=1}^{\infty}\left(\prod_{j=0}^{i-1}\frac{1}{\mathrm{R}_{t+j}}\right)D_{t+i}\right\}, \tag{23}$$

where R_t is the risk-free interest rate. Building on this definition of the fundamental component, Galí and Gambetti (2015) derived its response (in log) to an exogenous MP shock ξ_t^{mp}, which is linked to the responses of dividends and the real rate at future horizons

$$\frac{\partial q_{t+i}^F}{\partial \xi_t^{mp}} = \sum_{j=0}^{\infty}\Lambda^j\left[(1-\Lambda)\frac{\partial d_{t+j+1+i}}{\partial \xi_t^{mp}} - \frac{\partial \mathrm{r}_{t+j+i}}{\partial \xi_t^{mp}}\right]. \tag{24}$$

In (24), Λ is defined as the ratio between the gross rates of dividend growth and interest on a balanced growth path.[29] We follow Galí and Gambetti (2015) and calibrate $\Lambda = 0.99$ in our work. Equation (24) suggests that an exogenous surprise in MP influences the fundamental part of asset prices directly through the real interest (discount) rate channel on the one hand, and on the other hand indirectly through its effects on consumption and investment decisions of agents and thereby expected dividends.

To jointly account for both estimation and identification uncertainty (i.e., the uncertainty related to the independence test employed), we consider bootstrap variants of the two alternative point estimators of both the recursive model and the outcome of SRHL estimation. We resort to the residual-based moving block bootstrap developed by Brüggemann et al. (2016), which provides valid inference for IRFs given that the structural parameters are point-identified under our assumption. Moreover, the block bootstrap procedure is robust under conditional heteroskedasticity and capable of mimicking the relevant fourth-order moment structure of the residuals.[30] Conditional on each bootstrap sample and the respective ML estimates of the reduced form parameters, the matrix Q is uniformly sampled from $\mathcal{O}(6)$ à la Rubio-Ramírez et al. (2010) and

[29] This can be easily seen from the log-linearized form of (23):

$$q_t^F \propto \sum_{j=0}^{\infty}\Lambda^j\big[(1-\Lambda)E_t(d_{t+j+1}) - E_t(\mathrm{r}_{t+j})\big].$$

[30] For simulation-based evidence on the properties of the inferential methods for IRFs, we refer the reader to Brüggemann et al. (2016). We employ a fixed design and use in each bootstrap replication the (lagged) original observations rather than recursively generating them with the model residuals,

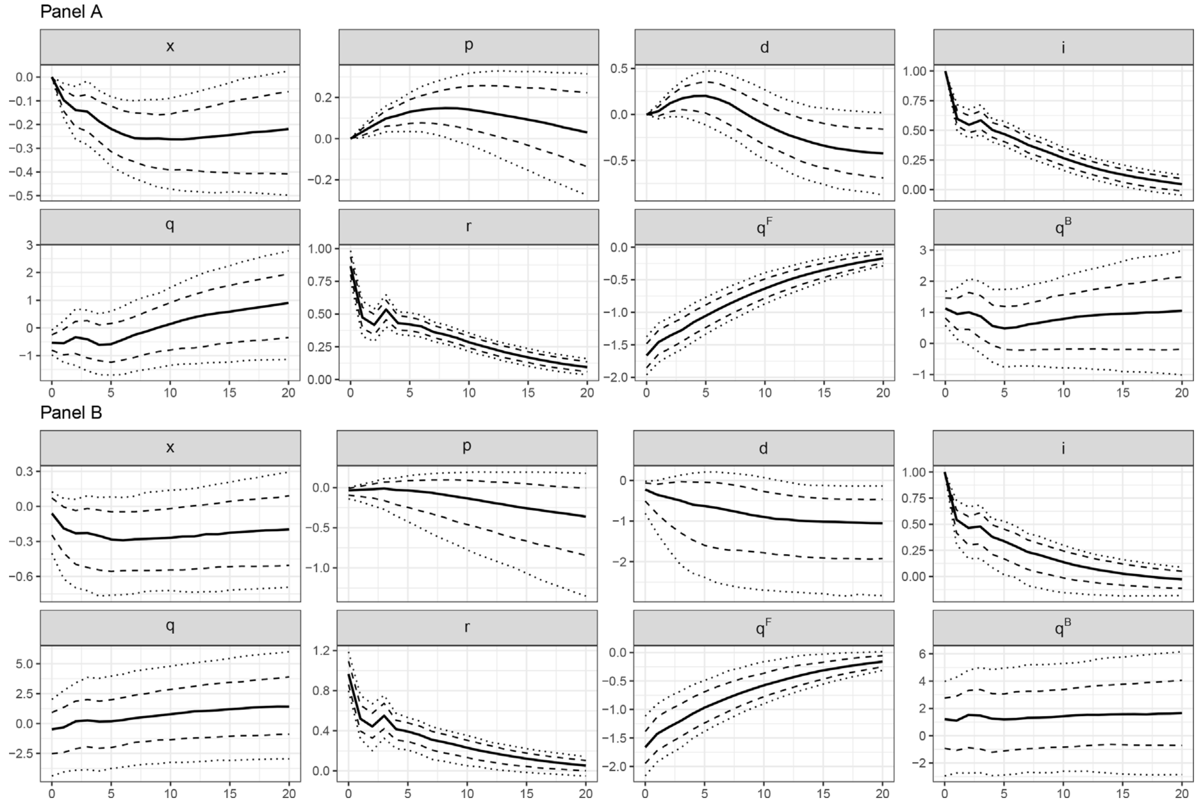


Fig. 5. Responses to a contractionary monetary policy shock identified by a Cholesky scheme (Panel A) and the SRHL approach (Panel B). Solid, dashed and dotted lines indicate the median and confidence intervals with a coverage of 68% and 90% based on the moving block bootstrap with a block length of 15 and 499 replications, respectively.

is accepted if the implied structural model satisfies the restrictions 1–3. In each bootstrap replication, the identified set $IS_r(B) = \{CQ : Q \in IS_r(Q)\}$ described in Remark 1 is approximated based on 1000 draws. For the matrices $B \in IS_r(B)$, we apply the dCov test to the implied orthogonalized shocks $\Xi(B)$ and obtain a point estimate by maximizing $p^{\mathcal{U}_T}(B)$.[31] Providing strong indications that the underlying shocks satisfy the independence assumption 4, the minimum p-value obtained from 499 bootstrap replications is 0.32 with an average value of 0.94. To facilitate the comparison of IRFs derived from alternative structural models, the MP shocks are normalized such that the shock has a unit on-impact effect on the FFR, i.e., the IRFs to a standard-deviation shock are divided by the impact response of the FFR in each bootstrap replication. For purposes of comparison, we also display IRFs obtained from a Cholesky covariance decomposition (as in Galí and Gambetti, 2015).

Bootstrap IRFs displayed in panel A of Fig. 5 are broadly in line with the benchmark point estimates of Galí and Gambetti (2015). However, the short-run responses of dividends to a contractionary MP shock are (significantly) positive if a lower-triangular covariance factor is used for identification.[32] Apart from the indicated 'dividend puzzle' the identification by means of Cholesky factors is also prone to being associated with pronounced price puzzles. At the same time, the results from identification by means of sign restrictions (SRHL point estimation, panel B of Fig. 5) show that an unexpected monetary tightening invokes a hike of real rates and a decline in dividends such that the fundamental component of equity valuation exhibits a negative impact response that tapers off sluggishly. Despite the lack of significance, the response of the bubble component is positive and tends to dominate the long-term response of asset prices. The empirical evidence for both puzzles is markedly weaker for IRFs determined by means of SRHL estimation. While this result comes with no surprise regarding impact effects (due to identifying assumption 1), it is worth noticing that the dynamic effects have been left unrestricted. Summarizing this evidence, we can conclude that the confirmation of rational bubbles in asset valuation is not necessarily restricted to using a recursive identification scheme as in Galí and Gambetti (2015).

so that the historical period of the Volcker's disinflation is preserved for the conditioning of bootstrap data and the implementation of the narrative restriction. As a robustness check, we also conduct a heteroskedasticity-robust wild bootstrap, in which the residuals are resampled based on a Rademacher-distributed random variable. The resulting IRFs are found to be very close to their counterparts obtained from the moving block bootstrap. We provide the corresponding baseline results in the online Appendix C.2.4.

[31] We also consider other less restrictive tests (partial independence and GMM) as a robustness check in Section 4.4.

[32] Bootstrap estimates of dividend IRFs markedly differ from those in Galí and Gambetti (2015), which can be attributed to their use of an iid bootstrap that lacks validity under heteroskedastic residual distributions.

While structural IRFs highlight structural relations in an unconditional manner, historical decompositions are informative on the time varying relevance of MP shocks for the fundamental and bubble components of asset prices. In this regard, the online Appendix C.3 provides insights into the cumulative effects of MP shocks for a set of eight selected periods of stock market booms. As it turns out, the role of rational bubbles has been limited in earlier decades (i.e., during the 1960s and 1970s). Concerning the so-called dot-com bubble in the late 1990s, two cycles of policy tightening have resulted in a sizeable contribution to rational bubbles.

4.4. Robustness

The identification of US MP is among the most active fields in contemporary macroeconometrics. As such, this literature provides several potential directions for robustness analysis that we will undertake in this section. First, to analyze the extent to which quantitative easing policies deserve a qualification of the benchmark results, we replace the FFR in the VAR with the shadow rate of Wu and Xia (2016).[33] Moreover, as introduced in Section 2.2, the proposed SRHL procedure allows for partial identification that relies on a weaker form of independence assumption, i.e., group-wise dependence as opposed to mutual independence in assumption 4. Thus, as the second direction of robustness analysis, we replace assumption 4 with a less restrictive counterpart

Identifying assumption 4a. The monetary policy shock exhibits a non-Gaussian distribution and it is independent of all other orthogonalized model residuals.

Finally, we employ the GMM method as a weaker and more flexible framework for testing independence under the possible existence of stochastic volatility. In doing so, we formulate four types of moment conditions that allow for stochastic volatility on the one hand, and dependence among non-policy shocks on the other. Again, denoting the MP shock with ξ_{5t}, we impose the following moment conditions ($L = 20$)

$$E[\xi_{kt}^i \xi_{5t}] = 0, \;\; E[\xi_{5t}^i \xi_{kt}] = 0, \;\; i = 2, 3 \text{ and } k \in \{1, 2, 3, 4, 6\}. \tag{25}$$

Following Drautzburg and Wright (2021), the weighting matrix in the GMM test statistic (17) is $\Omega_m(B) = T^{-1} \sum_{t=1}^{T} m_t(B) m_t(B)'$. Assumption 4 is replaced by

Identifying assumption 4b. The monetary policy shock exhibits a non-Gaussian distribution and it is independent of all other orthogonalized model residuals up to the moment conditions in (25). Normalized structural IRFs obtained from the three alternative directions of robustness analysis are reported in online Appendix C.2 and reveal that the benchmark outcomes are fairly robust throughout. Hence, we can conclude that the SRHL-based support of the rational bubbles model of Galí and Gambetti (2015) does not result from neglecting quantitative easing effects. In addition, SRHL estimation is fairly robust when focusing on either a complete system of independent shocks or on the notion of a marginally independent MP shock.

5. Conclusion

The introduction of weak (and often consensual) sign restrictions for identification in structural VARs has boosted many fields of macroeconometric research, in particular MP analysis. Because structural parameters lack point identification, given the reduced form parameters, any prior beliefs on set-identified parameters influence the posterior even in the asymptotic case. The common approach to implementing sign restrictions introduces an implicit prior, which is not only informative as pointed out in the literature, but also highly complicated due to employed restrictions and seems hardly controllable in practice. Building upon the literature integrating tools of ICA in SVAR identification (Drautzburg and Wright, 2021; Gouriéroux et al., 2017; Herwartz, 2019; Keweloh, 2021; Lanne and Luoto, 2021; Moneta et al., 2013), the SRHL approach as suggested in this work elicits point estimates from solving an optimization problem tailoring at implied shocks with minimum dependence in terms of distance covariance statistics (Matteson and Tsay, 2017; Székely et al., 2007). Owing to the consistency of the independence test, the SRHL estimator benefits from convergence towards the true model with sample information approaching infinity. Simulation exercises show that the SRHL approach is effective in highlighting a most plausible model with sufficient accuracy. For empirical practice and depending on the matter of interest, the SRHL approach benefits from complementing conceptually weak sign restrictions with valuable information on the absence or presence of higher-order dependencies among economic shocks under consideration.

As an empirical illustration, we reconsider the assessment of MP effects in Galí and Gambetti (2015) and clarify whether their recursive identification scheme can be considered as crucial for the detection of rational bubbles. Combining set identification by means of established sign restrictions (Antolín-Díaz and Rubio-Ramírez, 2018; Arias et al., 2019; Uhlig, 2005) and the suggested approach to point estimation allows the data to support the core findings of the benchmark study. Monetary policy shocks detected by the SRHL approach lead to distinct response profiles of the fundamental and bubble components of asset prices. In addition, we confirm that the suggested estimator results in MP shocks that significantly correlate with

[33] Before 2008, the FFR and the shadow rate are identical. The shadow rate can be downloaded from the web https://www.frbatlanta.org/cqer/research/wu-xia-shadow-federal-funds-rate.

prominent suggestions from the related literature. While the current study focuses on point estimation, developing inferential approaches for valid interval estimation and testing procedures within the SRHL framework is an important direction of future work.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jedc.2023.104630.

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