Bottom-Up Parsing

- A **bottom-up parser** creates the parse tree of the given input starting from leaves towards the root.
- A bottom-up parser tries to find the right-most derivation of the given input in the reverse order.

  $S \Rightarrow \ldots \Rightarrow \omega$ (the right-most derivation of ω)

  $\leftarrow$ (the bottom-up parser finds the right-most derivation in the reverse order)
- Bottom-up parsing is also known as **shift-reduce parsing** because its two main actions are shift and reduce.
  - At each shift action, the current symbol in the input string is pushed to a stack.
  - At each reduction step, the symbols at the top of the stack (this symbol sequence is the right side of a production) will replaced by the non-terminal at the left side of that production.
  - There are also two more actions: accept and error.

Shift-Reduce Parsing

- A shift-reduce parser tries to reduce the given input string into the starting symbol.

  a string $\xrightarrow[\text{reduced to}]{}$ the starting symbol

- At each reduction step, a substring of the input matching to the right side of a production rule is replaced by the non-terminal at the left side of that production rule.
- If the substring is chosen correctly, the right most derivation of that string is created in the reverse order.

Rightmost Derivation: $S \overset{*}{\underset{rm}{\Rightarrow}} \omega$

Shift-Reduce Parser finds: $\omega \underset{rm}{\Leftarrow} \ldots \underset{rm}{\Leftarrow} S$

Shift-Reduce Parsing -- Example

$S \rightarrow aABb$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

input string: aaabb

aaAbb

aAbb ⇓ reduction

aABb

S

$$S \underset{rm}{\Rightarrow} aABb \underset{rm}{\Rightarrow} aAbb \underset{rm}{\Rightarrow} aaAbb \underset{rm}{\Rightarrow} aaabb$$

Right Sentential Forms

- How do we know which substring to be replaced at each reduction step?

Handle

- Informally, a **handle** of a string is a substring that matches the right side of a production rule.
  - But not every substring matches the right side of a production rule is handle

- A **handle** of a right sentential form γ ($\equiv \alpha\beta\omega$) is

  a production rule $A \rightarrow \beta$ and a position of γ

  where the string β may be found and replaced by A to produce

  the previous right-sentential form in a rightmost derivation of γ.

$$S \overset{*}{\underset{rm}{\Rightarrow}} \alpha A \omega \underset{rm}{\Rightarrow} \alpha\beta\omega$$

- If the grammar is unambiguous, then every right-sentential form of the grammar has exactly one handle.
- We will see that ω is a string of terminals.

Handle Pruning

- A right-most derivation in reverse can be obtained by **handle-pruning**.

$$S=\gamma_0 \underset{rm}{\Rightarrow} \gamma_1 \underset{rm}{\Rightarrow} \gamma_2 \underset{rm}{\Rightarrow} \cdots \underset{rm}{\Rightarrow} \gamma_{n-1} \underset{rm}{\Rightarrow} \gamma_n = \omega$$

input string

- Start from γ_n, find a handle $A_n \rightarrow \beta_n$ in γ_n, and replace β_n in by A_n to get γ_{n-1}.
- Then find a handle $A_{n-1} \rightarrow \beta_{n-1}$ in γ_{n-1}, and replace β_{n-1} in by A_{n-1} to get γ_{n-2}.
- Repeat this, until we reach S.

A Shift-Reduce Parser

$E \rightarrow E+T \mid T$
$T \rightarrow T*F \mid F$
$F \rightarrow (E) \mid id$

Right-Most Derivation of `id+id*id`

$E \Rightarrow E+T \Rightarrow E+T*F \Rightarrow E+T*id \Rightarrow E+F*id$
$\Rightarrow E+id*id \Rightarrow T+id*id \Rightarrow F+id*id \Rightarrow id+id*id$

Right-Most Sentential Form	Reducing Production
$\underline{id}+id*id$	$F \rightarrow id$
$\underline{F}+id*id$	$T \rightarrow F$
$\underline{T}+id*id$	$E \rightarrow T$
$E+\underline{id}*id$	$F \rightarrow id$
$E+\underline{F}*id$	$T \rightarrow F$
$E+T*\underline{id}$	$F \rightarrow id$
$E+\underline{T*F}$	$T \rightarrow T*F$
$\underline{E+T}$	$E \rightarrow E+T$
E	

<u>Handles</u> are red and underlined in the right-sentential forms.

A Stack Implementation of A Shift-Reduce Parser

- There are four possible actions of a shift-parser action:

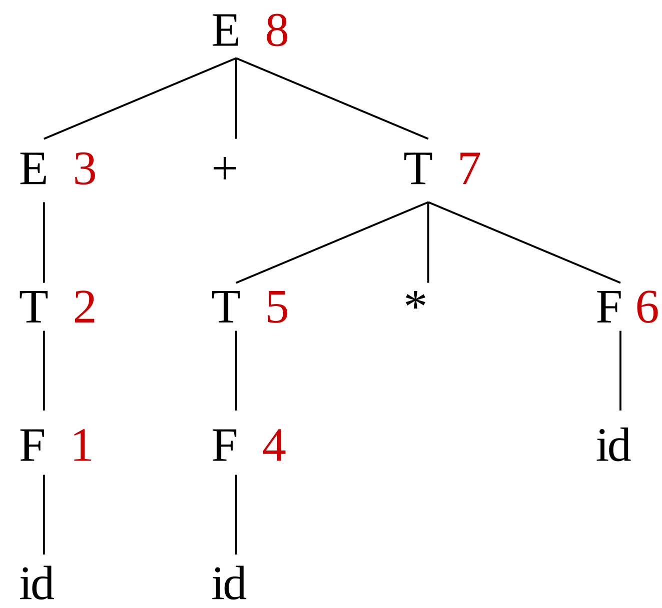
  1. **Shift** : The next input symbol is shifted onto the top of the stack.
  2. **Reduce**: Replace the handle on the top of the stack by the non-terminal.
  3. **Accept**: Successful completion of parsing.
  4. **Error**: Parser discovers a syntax error, and calls an error recovery routine.

- Initial stack just contains only the end-marker $.
- The end of the input string is marked by the end-marker $.

A Stack Implementation of A Shift-Reduce Parser

Stack	**Input**	**Action**
$	id+id*id$ shift	
$id	+id*id$	reduce by F → id
$F	+id*id$	reduce by T → F
$T	+id*id$	reduce by E → T
$E	+id*id$	shift
$E+	id*id$	shift
$E+id	*id$	reduce by F → id
$E+F	*id$	reduce by T → F
$E+T	*id$	shift
$E+T*	id$	shift
$E+T*id	$	reduce by F → id
$E+T*F	$	reduce by T → T*F
$E+T	$	reduce by E → E+T
$E	$	accept

Parse Tree



Conflicts During Shift-Reduce Parsing

- There are context-free grammars for which shift-reduce parsers cannot be used.
- Stack contents and the next input symbol may not decide action:
  - **shift/reduce conflict**: Whether make a shift operation or a reduction.
  - **reduce/reduce conflict**: The parser cannot decide which of several reductions to make.
- If a shift-reduce parser cannot be used for a grammar, that grammar is called as non-LR(k) grammar.

left to right scanning

right-most derivation

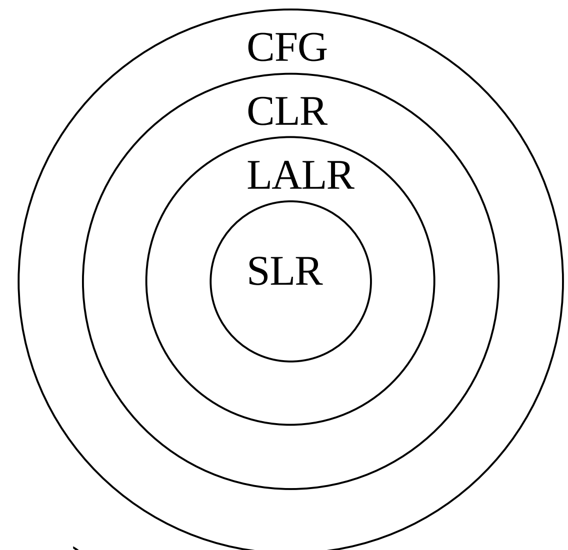
k lookhead

- An ambiguous grammar can never be a LR grammar.

Shift-Reduce Parsers

- There are two main categories of shift-reduce parsers

1. **Operator-Precedence Parser**
   – simple, but only a small class of grammars.



2. **LR-Parsers**
   – covers wide range of grammars.
     - SLR – simple LR parser
     - Canonical LR – most general LR parser
     - LALR – intermediate LR parser (lookhead LR parser)

   – SLR, CLR and LALR work same, only their parsing tables are different.

LR Parsers

- The most powerful shift-reduce parsing (yet efficient) is:

LR(k) parsing.

left to right scanning | right-most derivation | k lookhead (k is omitted ➔ it is 1)

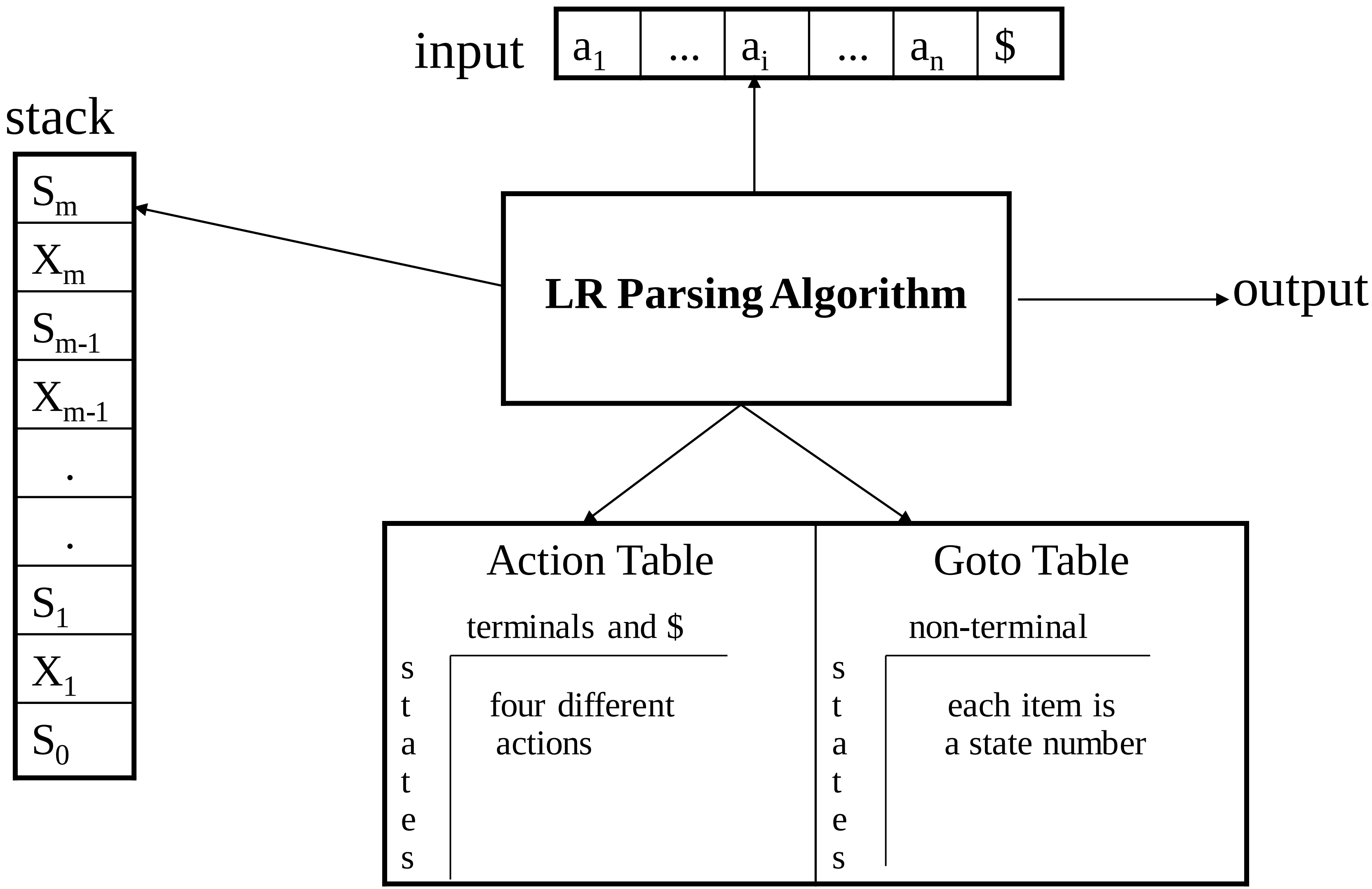
- LR parsing is attractive because:
  - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient.
  - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers.

    LL(1)-Grammars $\subset$ LR(1)-Grammars
  - An LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input.

LR Parsers

- **LR-Parsers**
  - covers wide range of grammars.
  - SLR – simple LR parser
  - CLR – most general LR parser
  - LALR – intermediate LR parser (look-head LR parser)
  - SLR, CLR and LALR work same (they used the same algorithm), only their parsing tables are different.

LR Parsing Algorithm



A Configuration of LR Parsing Algorithm

- A configuration of a LR parsing is:

$$(\underbrace{S_o\ X_1\ S_1 \ldots X_m\ S_m}_{\text{Stack}},\ \underbrace{a_i\ a_{i+1} \ldots a_n\ \$}_{\text{Rest of Input}})$$

- S_m and a_i decides the parser action by consulting the parsing action table. (*Initial Stack* contains just S_o)

- A configuration of a LR parsing represents the right sentential form:

$$X_1 \ldots X_m\ a_i\ a_{i+1} \ldots a_n\ \$$$

Actions of A LR-Parser

1. **shift s** -- shifts the next input symbol and the state **s** onto the stack

   $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ ) \Rightarrow (S_o\ X_1\ S_1 \ldots X_m\ S_m\ a_i\ s,\ a_{i+1} \ldots a_n\ \$\ )$

2. **reduce A→β** (or **`rn`** where n is a production number)
   - pop 2|**β**| (=r) items from the stack;
   - then push **A** and **s** where **s=goto[s_{m-r},A]**

   $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ ) \Rightarrow (S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ A\ s,\ a_i \ldots a_n\ \$\ )$

   - Output is the reducing production reduce A→β

3. **Accept** – Parsing successfully completed

4. **Error** -- Parser detected an error (an empty entry in the action table)

Reduce Action

- pop 2|**β**| (=r) items from the stack; let us assume that **β** = $Y_1 Y_2 \ldots Y_r$
- then push **A** and **s** where **s=goto[s_{m-r},A]**

$$(S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ Y_1\ S_{m-r} \ldots Y_r\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ )$$

$$\rightarrow (S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ A\ s,\ a_i \ldots a_n\ \$\ )$$

- In fact, $Y_1 Y_2 \ldots Y_r$ is a handle.

$$X_1 \ldots X_{m-r}\ A\ a_i \ldots a_n\ \$ \Rightarrow X_1 \ldots X_m\ Y_1 \ldots Y_r\ a_i\ a_{i+1} \ldots a_n\ \$$$

(SLR) Parsing Tables for Expression Grammar

1) $E \rightarrow E+T$
2) $E \rightarrow T$
3) $T \rightarrow T*F$
4) $T \rightarrow F$
5) $F \rightarrow (E)$
6) $F \rightarrow id$

Action Table / Goto Table

state	id	+	*	(	)	$		E	T	F
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

Actions of A (S)LR-Parser -- Example

stack	input	action	output
0	id*id+id$	shift 5	
0id5	*id+id$	reduce by F→id	F→id
0F3	*id+id$	reduce by T→F	T→F
0T2	*id+id$	shift 7	
0T2*7	id+id$	shift 5	
0T2*7id5	+id$	reduce by F→id	F→id
0T2*7F10	+id$	reduce by T→T*F	T→T*F
0T2	+id$	reduce by E→T	E→T
0E1	+id$	shift 6	
0E1+6	id$	shift 5	
0E1+6id5	$	reduce by F→id	F→id
0E1+6F3	$	reduce by T→F	T→F
0E1+6T9	$	reduce by E→E+T	E→E+T
0E1	$	accept	

Constructing SLR Parsing Tables – LR(0) Item

- An **LR(0) item** of a grammar G is a production of G a dot at the some position of the right side.
- Ex: $A \rightarrow aBb$ Possible LR(0) Items: (four different possibility)

$$A \rightarrow \bullet aBb$$
$$A \rightarrow a \bullet Bb$$
$$A \rightarrow aB \bullet b$$
$$A \rightarrow aBb \bullet$$

- Sets of LR(0) items will be the states of action and goto table of the SLR parser.
- A collection of sets of LR(0) items (**the canonical LR(0) collection**) is the basis for constructing SLR parsers.
- *Augmented Grammar*:

  G' is G with a new production rule $S' \rightarrow S$ where S' is the new starting symbol.

The Closure Operation

- If ***I*** is a set of LR(0) items for a grammar G, then ***closure(I)*** is the set of LR(0) items constructed from I by the two rules:

1. Initially, every LR(0) item in I is added to closure(I).
2. If $A \rightarrow \alpha \bullet B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow \bullet\gamma$ will be in the closure(I).
   We will apply this rule until no more new LR(0) items can be added to closure(I).

The Closure Operation -- Example

$E' \to E$
$E \to E+T$
$E \to T$
$T \to T*F$
$T \to F$
$F \to (E)$
$F \to id$

$closure(\{E' \to \bullet E\}) =$

$\{ \; E' \to \bullet E$ ← kernel items
$E \to \bullet E+T$
$E \to \bullet T$
$T \to \bullet T*F$
$T \to \bullet F$
$F \to \bullet (E)$
$F \to \bullet id \; \}$

Goto Operation

- If I is a set of LR(0) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:
  - If $A \rightarrow \alpha \bullet X\beta$ in I
    then every item in **closure({$A \rightarrow \alpha X \bullet \beta$})** will be in goto(I,X).

Example:

$I = \{\ E' \rightarrow \bullet E,\ \ E \rightarrow \bullet E+T,\ \ E \rightarrow \bullet T,$

$T \rightarrow \bullet T*F,\ \ T \rightarrow \bullet F,$

$F \rightarrow \bullet (E),\ \ F \rightarrow \bullet id\ \}$

$goto(I,E) = \{ E' \rightarrow E \bullet, E \rightarrow E \bullet +T \}$

$goto(I,T) = \{ E \rightarrow T \bullet, T \rightarrow T \bullet *F \}$

$goto(I,F) = \{ T \rightarrow F \bullet \}$

$goto(I,() = \{ F \rightarrow (\bullet E), E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F, T \rightarrow \bullet F,$

$F \rightarrow \bullet (E), F \rightarrow \bullet id \}$

$goto(I,id) = \{ F \rightarrow id \bullet \}$

Construction of The Canonical LR(0) Collection

- To create the SLR parsing tables for a grammar G, we will create the canonical LR(0) collection of the grammar G'.

- ***Algorithm***:

  C is { closure({S'→ • S}) }

  repeat the followings until no more set of LR(0) items can be added to ***C***.

  for each I in ***C*** and each grammar symbol X

  if goto(I,X) is not empty and not in ***C***

  add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

The Canonical LR(0) Collection -- Example

I_0: $E' \rightarrow .E$
$E \rightarrow .E+T$
$E \rightarrow .T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_1: $E' \rightarrow E.$
$E \rightarrow E.+T$

I_2: $E \rightarrow T.$
$T \rightarrow T.*F$

I_3: $T \rightarrow F.$

I_4: $F \rightarrow (.E)$
$E \rightarrow .E+T$
$E \rightarrow .T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_5: $F \rightarrow id.$

I_6: $E \rightarrow E+.T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_7: $T \rightarrow T*.F$
$F \rightarrow .(E)$
$F \rightarrow .id$

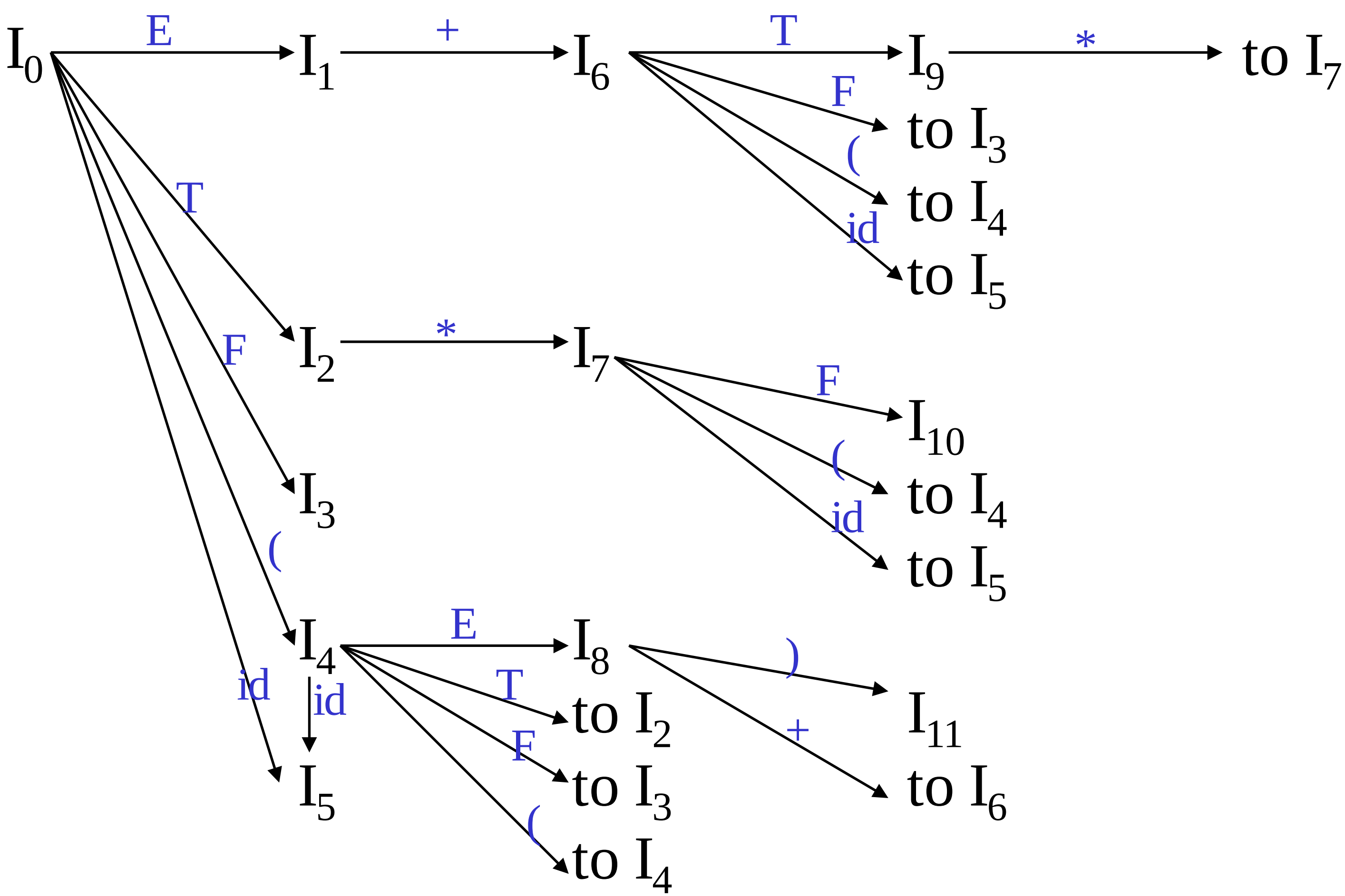
I_8: $F \rightarrow (E.)$
$E \rightarrow E.+T$

I_9: $E \rightarrow E+T.$
$T \rightarrow T.*F$

I_{10}: $T \rightarrow T*F.$

I_{11}: $F \rightarrow (E).$

Transition Diagram (DFA) of Goto Function



Constructing SLR Parsing Table

(of an augumented grammar G')

1. Construct the canonical collection of sets of LR(0) items for G'.
   $C \leftarrow \{I_0,...,I_n\}$

2. Create the parsing action table as follows
   - If a is a terminal, $A \rightarrow \alpha.a\beta$ in I_i and $goto(I_i,a)=I_j$ then action[i,a] is ***shift j.***
   - If $A \rightarrow \alpha.$ is in I_i , then action[i,a] is ***reduce*** $\boldsymbol{A \rightarrow \alpha}$ for all a in FOLLOW(A) where $A \neq S'$.
   - If $S' \rightarrow S.$ is in I_i , then action[i,$] is ***accept***.
   - If any conflicting actions generated by these rules, the grammar is not SLR(1).

3. Create the parsing goto table
   - for all non-terminals A, if $goto(I_i,A)=I_j$ then goto[i,A]=j

4. All entries not defined by (2) and (3) are errors.

5. Initial state of the parser contains $S' \rightarrow .S$

Parsing Tables of Expression Grammar

Action Table Goto Table

state	id	+	*	(	)	$		E	T	F
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

SLR(1) Grammar

- An LR parser using SLR(1) parsing tables for a grammar G is called as the SLR(1) parser for G.
- If a grammar G has an SLR(1) parsing table, it is called SLR(1) grammar (or SLR grammar in short).
- Every SLR grammar is unambiguous, but every unambiguous grammar is not a SLR grammar.

shift/reduce and reduce/reduce conflicts

- If a state does not know whether it will make a shift operation or reduction for a terminal, we say that there is a **shift/reduce conflict**.

- If a state does not know whether it will make a reduction operation using the production rule `i` or `j` for a terminal, we say that there is a **reduce/reduce conflict**.

- If the SLR parsing table of a grammar G has a conflict, we say that that grammar is not SLR grammar.

Conflict Example

$S \rightarrow L{=}R$
$S \rightarrow R$
$L \rightarrow {*}R$
$L \rightarrow id$
$R \rightarrow L$

I_0: $S' \rightarrow .S$
$S \rightarrow .L{=}R$
$S \rightarrow .R$
$L \rightarrow .{*}R$
$L \rightarrow .id$
$R \rightarrow .L$

I_1: $S' \rightarrow S.$

I_2: $S \rightarrow L.{=}R$
$R \rightarrow L.$

I_3: $S \rightarrow R.$

I_4: $L \rightarrow {*}.R$
$R \rightarrow .L$
$L \rightarrow .{*}R$
$L \rightarrow .id$

I_5: $L \rightarrow id.$

I_6: $S \rightarrow L{=}.R$
$R \rightarrow .L$
$L \rightarrow .{*}R$
$L \rightarrow .id$

I_7: $L \rightarrow {*}R.$

I_8: $R \rightarrow L.$

I_9: $S \rightarrow L{=}R.$

Problem

FOLLOW(R)={=,$}

= → shift 6
→ reduce by $R \rightarrow L$

shift/reduce conflict

Conflict Example2

$S \rightarrow AaAb$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$

$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S$

$S \rightarrow .AaAb$

$S \rightarrow .BbBa$

$A \rightarrow .$

$B \rightarrow .$

Problem

FOLLOW(A)={a,b}

FOLLOW(B)={a,b}

a → reduce by $A \rightarrow \varepsilon$

a → reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

b → reduce by $A \rightarrow \varepsilon$

b → reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict