

## INDEX

1. If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses-

(a) 4-cell reuse.                      (b) 7-cell reuse.                      (c) 12-cell reuse.

If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

2. If a signal to interference ratio of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is-

(a)  $n = 4$ .                                      (b)  $n = 3$ .

Assume that there are 6 co-channels cells in the first tier and all of them are at the same distance from the mobile. Use suitable approximations.

3. How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

(a) 1,                      (b) 5,                      (c) 10,                      (d) 20,                      (e) 100.

Assume each user generates 0.1 Erlangs of traffic.

4. An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

5. A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute-

- a) The number of cells in the service area,
- b) The number of channels per cell,
- c) Traffic intensity of each cell,
- d) The maximum carried traffic,
- e) The total number of users that can be served for 2% GOS,
- f) The number of mobiles per channel, and
- g) The theoretical maximum number of users that could be served at one time by the system.

6. If a transmitter produces 50 watts of power, express the transmit power in units of  
 a) dBm, and b) dBW.  
 If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,
  - a) Find the received power in dBm at a free space distance of 100 m from the antenna,
  - b) What is P (10 km)?
 Assume unity gain for the receiver antenna.
7. Determine the path loss of a 900MHz cellular system in a large city from a base station with the height of 100m and mobile station installed in a vehicle with antenna height of 2m. The distance between mobile and base station is 4Km.
8. Determine the path loss between base station (BS) and mobile station (MS) of a 1.8GHz PCS system operating in a high-rise urban area. The MS is located in a perpendicular street to the location of the BS. The distances of the BS and MS to the corner of the street are 20 and 30 meters, respectively. The base station height is 20m.
9. A mobile is located 5 km away from a base station and uses a vertical  $\lambda/4$  monopole antenna with a gain of 2.55 dB to receive cellular 3 radio signals. The E-field at 1 km from the transmitter is measured to be V/m. The carrier frequency used for this system is 900 MHz.
  - a) Find the length and the gain of the receiving antenna.
  - b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5m above ground.
10. A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and  $\lambda =$  call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call-
  - a) How many users per square kilometer will this system support?
  - b) What is the probability that a delayed call will have to wait for more than 10s?
  - c) What is the probability that a call will be delayed for more than 10 seconds?

**Problem-01:** If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses-

- (a) 4-cell reuse.                      (b) 7-cell reuse.                      (c) 12-cell reuse.

If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

**Solution:**

Given that,

$$\begin{aligned}
 \text{Total bandwidth} &= 33 \text{ MHz,} \\
 &= 33,000 \text{ kHz} \\
 \text{Channel bandwidth} &= 25 \text{ kHz} \times 2 \text{ simplex channels} \\
 &= 50 \text{ kHz / duplex channel} \\
 \text{Total available channels} &= \text{Total Bandwidth / Channel Bandwidth} \\
 &= 33,000 / 50 \\
 &= 660 \text{ channels}
 \end{aligned}$$

*If 1 MHz of the allocated spectrum is dedicated to control channels,*

$$\text{i.e. Control channel bandwidth} = 1000 \text{ kHz}$$

$$\begin{aligned}
 \text{The number of available control channel} &= \text{Control channel bandwidth / Channel bandwidth} \\
 &= 1000 / 50 \\
 &= 20 \text{ channels}
 \end{aligned}$$

**(a)**

Given that,

$$\text{Cluster size, } N = 4$$

$$\begin{aligned}
 \text{Total number of channels available per cell} &= \text{Total available channels} / N \\
 &= 660 / 4 \\
 &\approx 165 \text{ channels.}
 \end{aligned}$$

**∴ Equitable distribution of,**

$$\begin{aligned}
 \text{Voice Channel} &= (\text{Total available channels} - \text{The number of available control channel}) / N \\
 &= (660 - 20) / 4 \\
 &\approx 160 \text{ channels}
 \end{aligned}$$

$$\begin{aligned}
 \text{Control Channel} &= \text{Total number of channels available per cell} - \text{Voice Channel} \\
 &= 165 - 160 \\
 &= 5 \text{ channels}
 \end{aligned}$$

(b)

Given that,

Cluster size,  $N = 7$

$$\begin{aligned}\text{Total number of channels available per cell} &= \text{Total available channels} / N \\ &= 660/7 \\ &\approx 94 \text{ channels.}\end{aligned}$$

$\therefore$  Equitable distribution of,

$$\begin{aligned}\text{Voice Channel} &= (\text{Total available channels} - \text{The number of available control channel}) / N \\ &= (660 - 20) / 7 \\ &\approx 91 \text{ channels}\end{aligned}$$

$$\begin{aligned}\text{Control Channel} &= \text{Total number of channels available per cell} - \text{Voice Channel} \\ &= 94 - 91 \\ &= 3 \text{ channels}\end{aligned}$$

(c)

Given that,

Cluster size,  $N = 12$

$$\begin{aligned}\text{Total number of channels available per cell} &= \text{Total available channels} / N \\ &= 660/12 \\ &\approx 55 \text{ channels.}\end{aligned}$$

$\therefore$  Equitable distribution of,

$$\begin{aligned}\text{Voice Channel} &= (\text{Total available channels} - \text{The number of available control channel}) / N \\ &= (660 - 20) / 12 \\ &\approx 53 \text{ channels}\end{aligned}$$

$$\begin{aligned}\text{Control Channel} &= \text{Total number of channels available per cell} - \text{Voice Channel} \\ &= 55 - 53 \\ &= 2 \text{ channels}\end{aligned}$$

### Source Code:

```
# Constants
total_bandwidth_mhz = 33 # Total allocated bandwidth in MHz
control_bandwidth_mhz = 1 # 1 MHz dedicated to control channels
channel_bandwidth_khz = 25 # Simplex channel bandwidth in kHz
duplex_channel_bandwidth_khz = 2 * channel_bandwidth_khz # Full duplex channel
bandwidth in kHz
reuse_factors = [4, 7, 12] # Reuse factors to be evaluated
```

```

# Convert bandwidth to kHz
total_bandwidth_khz = total_bandwidth_mhz * 1000 # Total bandwidth in kHz
control_bandwidth_khz = control_bandwidth_mhz * 1000 # Control bandwidth in kHz

# Bandwidth available for voice channels (in kHz)
voice_bandwidth_khz = total_bandwidth_khz - control_bandwidth_khz

# Total number of voice channels available
total_voice_channels = voice_bandwidth_khz // duplex_channel_bandwidth_khz

# Total number of control channels available
total_control_channels = control_bandwidth_khz // duplex_channel_bandwidth_khz

# Function to calculate channels per cell for different reuse factors
def calculate_channels_per_cell(reuse_factors):
    results = []
    for reuse_factor in reuse_factors:
        # Voice channels per cell
        voice_channels_per_cell = total_voice_channels // reuse_factor

        # Control channels per cell (rounded for equitable distribution)
        control_channels_per_cell = round(total_control_channels / reuse_factor)

        # Total number of channels per cell
        total_channels_per_cell = voice_channels_per_cell + control_channels_per_cell

        # Append results for this reuse factor
        results.append({
            'Reuse Factor': reuse_factor,
            'Total Channels per Cell': total_channels_per_cell,
            'Voice Channels per Cell': voice_channels_per_cell,
            'Control Channels per Cell': control_channels_per_cell
        })

    return results

# Call the function and display the results
results = calculate_channels_per_cell(reuse_factors)

# Display the results in the required format
for result in results:
    print(f"For Cluster size N = {result['Reuse Factor']}")
    print(f"Total number of channels available per cell : {result['Total Channels per Cell']} channels")
    print(f"Voice Channels : {result['Voice Channels per Cell']} channels")
    print(f"Control Channels : {result['Control Channels per Cell']} channels")
    print('-' * 40)

```

**Input:**

Enter Cluster Sizes: [4, 7, 12]

**Output:**

For Cluster size  $N = 4$

Total number of channels available per cell: 165 channels

Voice Channels: 160 channels

Control Channels: 5 channels

-----

For Cluster size  $N = 7$

Total number of channels available per cell: 94 channels

Voice Channels: 91 channels

Control Channels: 3 channels

-----

For Cluster size  $N = 12$

Total number of channels available per cell: 55 channels

Voice Channels: 53 channels

Control Channels: 2 channels

-----

**Problem-2:** If a signal to interference ratio of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is-

(a)  $n = 4$ .

(b)  $n = 3$ .

Assume that there are 6 co-channels cells in the first tier and all of them are at the same distance from the mobile. Use suitable approximations.

**Solution:**

Given that,

Minimum Required Signal-to-Noise interference ratio,  $S/I = 15$  dB,

The number of Co-channel interfering cells,  $i_0 = 6$

We Know,

Number of cell reuse,  $N = i^2 + i*j + j^2$  ..... (i)

First, let us consider a 7-cell reuse pattern,  $N = 7$  For  $i=1, j=2$

Also,

The Frequency Reuse Factor,  $Q = D/R$

$= \sqrt{(3N)}$  ..... (ii)  
 $= 4.583$ .

Where,

$D$  = Distance between centers of the nearest Co-channel cells.

$R$  = Radius of the cell.

**(a)**

Given that,

Path Loss exponent,  $n = 4$

**Frequency Reuse Factor,  $Q = 4.583$ .**

We know,

Signal-to-Noise interference ratio,  $S/R = 10 \log (Q^n / i_0)$   
 $= 10 \log ((4.583)^4 / 6)$   
 $= 18.66$  dB.

Since this is greater than the minimum required  $S/I$  ( $18.66 > 15$ ),  $N = 7$  can be used.

**(b)**

Given that,

Path Loss exponent,  $n = 3$

We know,

Signal-to-Noise interference ratio,  $S/R = 10 \log (Q^n / i_0)$  ..... (iii)  
 $= 10 \log ((4.583)^3 / 6)$   
 $= 12.05$  dB.

*Since this is less than the minimum required S/I ( $12.05 < 15$ ), we need to use a larger N.*

Using equation (i), the next possible value of  $N = 12$ ; For  $i = j = 2$ .

The corresponding co-channel ratio is given by equation (ii) as-

**Frequency Reuse Factor,  $Q = 6$ .**

Using equation (iii) the signal-to-interference ratio,  $S/I = 15.56$  dB.

**Since, this is greater than the minimum required S/I ( $15.56 > 15$ ),  $N = 12$  can be used.**

### Source Code:

```
import math
# Function to convert SIR from linear to dB
def linear_to_dB(SIR_linear):
    return 10 * math.log10(SIR_linear)
# Function to calculate SIR in dB, cluster size N, and frequency reuse factor Q
def calculate_sir_and_cluster_size(n):
    # Given SIR target
    SIR_target_dB = 15
    SIR_target_linear = 10 ** (SIR_target_dB / 10)

    # Number of co-channel cells in the first tier
    i0 = 6

    # Calculate the cluster size N
    numerator = (SIR_target_linear * i0) ** (2 / n)
    N = math.ceil(numerator / 3) # Round up to nearest integer for cluster size

    # Calculate SIR based on the computed N
    SIR_computed_linear = (3 * N) ** (n / 2) / i0
    SIR_computed_dB = linear_to_dB(SIR_computed_linear)

    # Calculate frequency reuse factor Q
    Q = math.sqrt(3 * N)

    return N, SIR_computed_dB, Q

# Input: Path Loss exponents
path_loss_exponents = [4, 3]

# Process each path loss exponent
for n in path_loss_exponents:
    N, SIR_computed_dB, Q = calculate_sir_and_cluster_size(n)

    # Display the results in the required format
    print(f'For Path Loss Exponent, n = {n}')
    print(f'Signal-to-Interference Ratio, S/I: {SIR_computed_dB:.3f} dB > 15 dB')
    print(f'Hence, Cluster size, N : {N}')
    print(f'Frequency Reuse Factor, Q : {Q:.3f}')
    print('-' * 40)
```



**Input:**

Enter Path Loss exponent: [4 3]

**Output:**

For Path Loss Exponent,  $n = 4$

Signal-to-Interference Ratio, S/I: 15.740 dB > 15 dB

Hence, Cluster size, N: 5

Frequency Reuse Factor, Q: 3.873

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For Path Loss Exponent,  $n = 3$

Signal-to-Interference Ratio, S/I: 15.563 dB > 15 dB

Hence, Cluster size, N: 12

Frequency Reuse Factor, Q: 6.000  
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**Problem-3:** How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

- (b) 1, (b) 5, (c) 10, (d) 20, (e) 100.

Assume each user generates 0.1 Erlangs of traffic.

**Solution:**

Given that,

Blocking Probability,  $P_B = 0.5\%$ ,

Traffic Intensity,  $A_u = 0.1$  Erlangs

We Know,

For Erlangs B, Grade of Service,  $GOS = P_B$

$$= 0.005$$

And, Total number of user,  $U = A / A_u$ ..... (i)

Where,

$A$  = Offered Traffic Intensity.

Also,

**Table 3.1:** Capacity of an Erlang B System.

Number of Channels $C$	Capacity (Erlangs) for GOS			
	= 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

From Table 3.1, we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels.

(a)

Given that,

Trunked channels,  $C = 1$

From table 3.1, For  $C = 1$  we obtain,  $A = 0.005$

From equation (i), we have-

$$\begin{aligned} \text{Total number of user, } U &= A / A_u \\ &= 0.05 \text{ users.} \end{aligned}$$

**But, actually one user could be supported on one channel. So,  $U = 1$ .**

**(b)**

Given that,

Trunked channels,  $C = 5$

From table 3.1, For  $C = 5$  we obtain,  $A = 1.13$  Erlang.

From equation (i), we have-

Total number of user,  $U = A / A_u$   
 $\approx 11$  users.

**(c)**

Given that,

Trunked channels,  $C = 10$

From table 3.1, For  $C = 10$  we obtain,  $A = 3.96$  Erlang

From equation (i), we have-

Total number of user,  $U = A / A_u$   
 $\approx 39$  users.

**(d)**

Have given,

Trunked channels,  $C = 20$

From table 3.1, For  $C = 20$  we obtain,  $A = 11.10$  Erlang

From equation (i), we have-

Total number of user,  $U = A / A_u$   
 $\approx 110$  users.

**(e)**

Have given,

Trunked channels,  $C = 100$

From table 3.1, For  $C = 100$  we obtain,  $A = 80.9$  Erlang.

From equation (i), we have-

**Total number of user,  $U = A / A_u$**   
 $\approx 809$  users.

### Source Code:

```
GOS = 0.5 / 100
Au = 0.1
# from table
A = [0.005, 1.13, 3.96, 11.1, 80.9]
c = [1, 5, 10, 20, 100]

# Display information
print('Blocking probability')
print(Gos)
print('Traffic intensity per user ')
print(Au)
print('Traffic intensity')
print(A)
print('Channel')
print(c)

# Calculate number of users
U = [a / Au for a in A]
u = [round(u_val) for u_val in U]

# Display number of users
print('Number of users')
print(u)
```

### Output:

```
Blocking probability: 0.005
Traffic intensity per user: 0.1
Traffic intensity:      [0.005, 1.13, 3.96, 11.1, 80.9]
Channel:                [1, 5, 10, 20, 100]
Number of users:        [0, 11, 40, 111, 809]
```

**Problem-4:** An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

**Solution:**

Given that,

Blocking Probability,  $P_B = 2\%$ ,  
 The average number of call requests per unit time  $\lambda = 2$ .  
 The average duration of a call,  $H = 3/60$  seconds  
 There are 2 million residents in the given urban area  $= 2000000$

We Know,

For Erlangs B, Grade of Service,  $GOS = P_B = 0.02$   
 And, Traffic Intensity,  $A_u = \lambda H = 0.1$  Erlangs

Also, Total number of user,  $U = A / A_u \dots\dots\dots(i)$

Where,

$A =$  Offered Traffic Intensity.

**Table 4.1:** Capacity of an Erlang B System.

Erlang B Traffic Table												
Maximum Offered Load Versus B and N												
N/B	B is in %											
	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20	30	40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
56	33.49	36.13	37.46	41.23	43.32	45.88	50.54	56.06	60.98	65.94	77.00	90.97
57	34.27	36.95	38.29	42.11	44.22	46.82	51.55	57.14	62.14	67.18	78.43	92.64
94	64.25	68.07	69.98	75.41	78.43	82.17	89.10	97.53	105.3	113.2	131.2	154.3
95	65.08	68.93	70.85	76.33	79.37	83.13	90.12	98.63	106.4	114.4	132.6	155.9
100	69.27	7~.25	75.24	80.91	84.06	87.97	95.24	104.1	112.3	120.6	139.7	164.3

N is the number of servers. The numerical column headings indicate blocking probability B in %. Table generated by Dan Dexter

Also, From Table 4.1, we can find the total capacity in Erlangs for the 2% GOS for different numbers of channels.

### For System-A

Given that,

Number of channels per cell used in the system,  $C = 19$

From table 4.1, For  $C = 19$  and  $GOS = 0.02$  we obtain,  $A = 12$  Erlangs

From equation (i), we have-

$$\begin{aligned}\text{Total number of user, } U &= A / A_u \\ &= 120 \text{ users.}\end{aligned}$$

*Since there are 394 cells, the total number of subscribers that can be supported by **System A** is equal to  $120 \times 394 = 47280$ .*

**Since, the percentage market penetration**  $= 47280/2000000 = 2.36\%$

### For System-B

Given that,

Number of channels per cell used in the system,  $C = 57$

From table 4.1, For  $C = 57$  and  $GOS = 0.02$  we obtain,  $A = 45$  Erlangs

From equation (i), we have-

$$\begin{aligned}\text{Total number of user, } U &= A / A_u \\ &= 450 \text{ users.}\end{aligned}$$

*Since there are 98 cells, the total number of subscribers that can be supported by **System B** is equal to  $450 \times 98 = 44,100$ .*

**Since, the percentage market penetration**  $= 44100/2000000 = 2.205\%$

### For System-C

Given that,

Number of channels per cell used in the system,  $C = 100$

From table 4.1, For  $C = 100$  and  $GOS = 0.02$  we obtain,  $A = 88$  Erlangs

From equation (i), we have-

$$\begin{aligned}\text{Total number of user, } U &= A / A_u \\ &= 880 \text{ users.}\end{aligned}$$

*Since there are 49 cells, the total number of subscribers that can be supported by **System C** is equal to  $880 \times 49 = 43,120$ .*

**Since, the percentage market penetration**  $= 43,120/2000000 = 2.156\%$

Therefore, total number of cellular subscribers that can be supported by these three systems are  $(47280 + 44100 + 43120) = 134500$  users.

***The market penetration of the three systems combined is equal to  $134500/2000000 = 6.725$***

## Source Code:

```
blocking_p = 2 / 100 # Blocking probability
lamda = 2 # Average call rate per user (calls per hour)
H = 3 / 60 # Average call duration (hours)
Au = lamda * H # Traffic per user in Erlangs
total_population = 2000000 # Total population of the area

# System A
print('For System A')
channel_a = 19
cell_A = 394
A_a = 12 # Traffic handled by each cell in System A
Ua = A_a / Au # Number of users supported per cell in System A
subscriber_A = Ua * cell_A # Total subscribers in System A
print('Number of users in System A per cell:', Ua)
print('Total number of subscribers in System A:', subscriber_A)
percentage_market_penetration_for_A = (subscriber_A / total_population) * 100
print('Percentage market penetration for System A:', percentage_market_penetration_for_A)

# System B
print('\nFor System B')
channel_b = 57
cell_B = 98
A_b = 45 # Traffic handled by each cell in System B
Ub = A_b / Au # Number of users supported per cell in System B
subscriber_B = Ub * cell_B # Total subscribers in System B
print('Number of users in System B per cell:', Ub)
print('Total number of subscribers in System B:', subscriber_B)
percentage_market_penetration_for_B = (subscriber_B / total_population) * 100
print('Percentage market penetration for System B:', percentage_market_penetration_for_B)

# System C
print('\nFor System C')
channel_c = 100
cell_C = 49
A_c = 88 # Traffic handled by each cell in System C
Uc = A_c / Au # Number of users supported per cell in System C
subscriber_C = Uc * cell_C # Total subscribers in System C
print('Number of users in System C per cell:', Uc)
print('Total number of subscribers in System C:', subscriber_C)
percentage_market_penetration_for_C = (subscriber_C / total_population) * 100
print('Percentage market penetration for System C:', percentage_market_penetration_for_C)

# Total subscribers and market penetration for all systems
total_subscribers = subscriber_A + subscriber_B + subscriber_C
total_market_penetration = (total_subscribers / total_population) * 100
print('\nTotal number of subscribers across all systems:', total_subscribers)
print('Total market penetration for all three systems:', total_market_penetration)
```

## Output:

For System A

Number of users in System A per cell: 120.0

Total number of subscribers in System A: 47280.0

Percentage market penetration for System A: 2.3640000000000003

For System B

Number of users in System B per cell: 450.0

Total number of subscribers in System B: 44100.0

Percentage market penetration for System B: 2.205

For System C

Number of users in System C per cell: 880.0

Total number of subscribers in System C: 43120.0

Percentage market penetration for System C: 2.156

Total number of subscribers across all systems: 134500.0

Total market penetration for all three systems: 6.7250000000000005



**Problem-5:** A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute-

- The number of cells in the service area,
- The number of channels per cell,
- Traffic intensity of each cell,
- The maximum carried traffic,
- The total number of users that can be served for 2% GOS,
- The number of mobiles per channel, and
- The theoretical maximum number of users that could be served at one time by the system.

**Solution:**

**(a)**

Given that,

Total coverage area = 1300 miles

Cell radius = 4 miles

We know,

The area of a cell (hexagon) can be shown to be  $2.5981R^2$

Thus each cell covers  $2.5981 \times (4)^2 = 41.57$  sq km.

**Hence, the total number of cells,  $N_c = 1300/41.57 = 31$  cells**

**(b)**

Given that,

Allocated spectrum = 40, 000,000 Hz

Channel width = 60,000 Hz

Frequency reuse factor, N = 7 cells

We know,

The total number of channels per cell,  $C = \text{Allocated spectrum} / (\text{Channel width} \times N)$

$= 40, 000,000 / (60,000 \times 7)$

$= 95$  channels/cell

**(c)**

Given that,

From (b) No,  $C = 95$

And, GOS = 0.02

From the table 4.1 (Erlang B chart) For  $C = 95$  and  $GOS = 0.02$ , we have-

Traffic intensity per cell,  $A = 84$  Erlangs/cell

**(d)**

Given that,

From (a), Number of cells = 31 cells

From (c), Traffic intensity per cell = 84 Erlangs/cell

We Know,

$$\begin{aligned}\text{Maximum carried traffic} &= \text{Number of cells} \times \text{Traffic intensity per cell} \\ &= 31 \times 84 \\ &= 2604 \text{ Erlangs.}\end{aligned}$$

**(e)**

Given that,

Traffic per user,  $A_u$  = 0.03 Erlangs

From (d), Total traffic,  $A$  = 2604 Erlangs.

We Know,

$$\begin{aligned}\text{Total number of users, } U &= A / A_u \\ &= 2604 / 0.03 \\ &= 86,800 \text{ users.}\end{aligned}$$

**(f)**

Given that,

Allocated spectrum = 40,000,000 Hz

Channel width = 60,000 Hz

From (e), Number of users,  $U$  = 86,800 users.

We Know,

$$\begin{aligned}\text{Number of channels} &= \text{Allocated Spectrum} / \text{Channel Width} \\ &= 40,000,000 / 60,000 \\ &\approx 666 \\ \text{Number of mobiles per channel} &= \text{Number of users} / \text{Number of channels} \\ &= 86,800 / 666 \\ &\approx 130 \text{ mobiles/channel}\end{aligned}$$

**(g)**

Given that,

From (b) No,  $C$  = 95 channels/cell

From (a), the total number of cells,  $N_c$  = 31 cells.

From (e) Total number of users,  $U$  = 86,800 users.

We Know,

$$\begin{aligned}\text{The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied)} \\ &= C \times N_c \\ &= 95 \times 31 = 2945 \text{ users,}\end{aligned}$$

Which is  $(2945/86,800) \times 100 = 3.4\%$  of the customer base.

### Source Code:

```
import math

# (a) Total number of cells
total_city_coverage_area = 1300 # in square miles
radius = 4 # radius of each cell in miles
cell_area = (2.591 * radius**2) # Each cell covers this area in square miles
Nc = round(total_city_coverage_area / cell_area) # Total number of cells, Nc
print(f"(a) Total number of cells, Nc: {Nc} cells\n")

# (b) Number of channels per cell
allocated_spectrum = 40000 # Allocated spectrum = 40 MHz = 40000 KHz
channel_width = 60 # Full duplex channel bandwidth = 60 KHz
N = 7 # 7-cell reuse pattern
C = round(allocated_spectrum / (channel_width * N)) # Total number of channels per cell
print(f"(b) The total number of channels per cell, C: {C} channels/cell\n")

# (c) Traffic intensity per cell (from Erlang B chart)
A = 84 # Traffic intensity per cell in Erlangs (from Erlang B table, GOS=0.02)
print(f"(c) Traffic intensity per cell, A: {A} Erlangs/cell\n")

# (d) Maximum carried traffic
max_c_t = math.floor(Nc * A) # Maximum carried traffic in Erlangs
print(f"(d) Maximum carried traffic: {max_c_t} Erlangs\n")

# (e) Total number of users
Au = 0.03 # Traffic per user in Erlangs
U = round(max_c_t / Au) # Total number of users
print(f"(e) Total number of users, U: {U} users\n")

# (f) Number of mobiles per channel
no_of_channel = math.floor(allocated_spectrum / channel_width) # Total number of channels
no_of_m_p_c = math.floor(U / no_of_channel) # Number of mobiles per channel
print(f"(f) Number of mobiles per channel: {no_of_m_p_c} mobiles/channel\n")

# (g) Theoretical maximum number of users that could be served
g = C * Nc # Theoretical maximum number of users that could be served
print(f"(g) Theoretical maximum number of users that could be served: {g} users\n")
```

**Output:**

- (a) Total number of cells,  $N_c$ : 31 cells
- (b) The total number of channels per cell,  $C$ : 95 channels/cell
- (c) Traffic intensity per cell,  $A$ : 84 Erlangs/cell
- (d) Maximum carried traffic: 2604 Erlangs
- (e) Total number of users,  $U$ : 86800 users
- (f) Number of mobiles per channel: 130 mobiles/channel
- (g) Theoretical maximum number of users that could be served: 2945 users

**Problem-6:** If a transmitter produces 50 watts of power, express the transmit power in units of

a) dBm, and b) dBW.

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

c) Find the received power in dBm at a free space distance of 100 m from the antenna,

d) What is P (10 km)?

Assume unity gain for the receiver antenna.

**Solution:**

Given that,

Transmitter power,  $P_t = 50 \text{ W}$

Carrier frequency,  $f_c = 900 \text{ MHz}$

**(a)**

We know,

$$\begin{aligned} \text{Transmitter power, } P_t(\text{dBm}) &= 10 \log[P_t(\text{mW})/(1\text{mW})] \\ &= 10 \log [50 \times 10^3] \\ &= 47.0 \text{ dBm} \end{aligned}$$

**(b)**

We know,

$$\begin{aligned} \text{Transmitter power, } P_t(\text{dBW}) &= 10 \log[P_t(\text{W})/(1\text{W})] \\ &= 10 \log [50] \\ &= 17.0 \text{ dBW} \end{aligned}$$

**(c)**

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

Given that,

$$\begin{aligned} \text{Transmitter Gain, } G_t &= 1 \\ \text{Receiver Gain, } G_r &= 1 \\ \text{Wave length } \lambda &= c / f = 1/3 \text{ m} \\ \text{The T-R separation distance, } d &= 100\text{m} \\ \text{The system loss factor, } L &= 1 \end{aligned}$$

We know,

$$\begin{aligned} \text{The received power, } P_r &= (P_t \times G_t \times G_r \times \lambda^2) / (4\pi^2 \times d^2 \times L) \\ &= (50 \times 1 \times 1 \times (1/3)^2) / ((4\pi)^2 \times 100^2 \times 1) \\ &= 3.5 \times 10^{-3} \text{ mW} \end{aligned}$$

$$\begin{aligned} \text{Received power, } P_r(\text{dBm}) &= 10 \log[P_r(\text{mW})] \\ &= 10 \log[P_r(3.5 \times 10^{-3})] \\ &= -24.5 \text{ dBm} \end{aligned}$$

(d)

Given that,

$$d_o = 10 \text{ km} = 10000 \text{ m}$$

We Know,

The received power at 10 km can be expressed in terms of dBm, we have

$$\begin{aligned}\therefore P_r(10 \text{ km}) &= P_r(100) + 20 \log[d / d_o] \\ &= P_r(100) + 20 \log[100 / 10000] \\ &= -24.5 - 40 \\ &= -64.5 \text{ dBm}\end{aligned}$$

### Source Code:

```
# given
import math

Pt = 50 # Transmitter Power
fc = 900 # Carrier Frequency

# a
PtdBm = 10 * math.log10(Pt * 1e3)
print('Transmitted Power: %.1f dBm' % PtdBm)

# b
PtBW = 10 * math.log10(Pt)
print('Transmitted Power: %.1f dBW' % PtBW)

# received power
Gt, Gr, lam, d, L = 1, 1, (1 / 3), 100, 1
Pr = (Pt * Gt * Gr * (lam ** 2)) / (((4 * math.pi) ** 2) * (d ** 2) * L)
PrdBm = 10 * math.log10(Pr * 1e3)
print('Received Power: %.1f dBm' % PrdBm)

# Pr(10Km)
Pr10Km = PrdBm + 20 * math.log10(100 / 10000)
print('Received Power: %.1f dBm' % Pr10Km)
```

### Output:

```
Transmitted Power: 47.0 dBm
Transmitted Power: 17.0 dBW
Received Power: -24.5 dBm
Received Power: -64.5 dBm
```

**Problem-7:** Determine the path loss of a 900MHz cellular system in a large city from a base station with the height of 100m and mobile station installed in a vehicle with antenna height of 2m. The distance between mobile and base station is 4 km.

**Solution:**

Have given,

The frequency, $f_c$	= 900 MHz (150 MHz to 1500MHz)
The effective transmitter (base station) antenna height, $h_{te}$	= 100m
The effective transmitter (mobile) antenna height, $h_{re}$	= 2m
T-R separation distance, $d$	= 4 km

Now, The correction factor for effective mobile antenna height,

$$a(h_{re}) = 3.2 (\log 11.75 h_{re})^2 - 4.97 \text{ dB for } f_c \geq 300 \text{ MHz}$$

From Okumura-Hata Model we know,

The path loss in urban areas is given by

$$\begin{aligned}
 L_{50}(\text{urban})(\text{dB}) &= 69.55 + 26.16 \log f_c - 13.82 \log h_{te} - a(h_{re}) \\
 &\quad + (44.9 - 6.55 \log h_{te}) \log d \\
 &= 69.55 + 26.16 \times 2.954 - 13.82 \times 2 - 1.045 + (44.9 - 13.1) \times 0.6 \\
 &= 137.3 \text{ dB}
 \end{aligned}$$

**Source Code:**

```
import math

# Given values
hre = 2 # Height of the receiving antenna (meters)
hte = 100 # Height of the transmitting antenna (meters)
fc = 900 # Frequency (MHz)
d = 4 # Distance between antennas (kilometers)

# Calculate a_hre
a_hre = 3.2 * (math.log10(11.75 * hre)) ** 2 - 4.97

# Calculate path loss
Lp = 69.55 + 26.16 * math.log10(fc) - 13.82 * math.log10(hte) - a_hre + (44.9 - 6.55 *
math.log10(hte)) * math.log10(d)

print('Path loss: %.2f % Lp)
```

**Output:**

Path loss: 137.29

**Problem-8:** Determine the path loss between base station (BS) and mobile station (MS) of a 1.8GHz PCS system operating in a high-rise urban area. The MS is located in a perpendicular street to the location of the BS. The distances of the BS and MS to the corner of the street are 20 and 30 meters, respectively. The base station height is 20m.

### Solution:

Given that,

$$\begin{aligned} \text{The frequency, } f_c &= 1.8 \text{ GHz (0.9 to 2 GHz)} \\ \text{The effective transmitter (base station) antenna height, } h_b &= 20\text{m} \\ \text{T-R separation distance, } d = \sqrt{(20^2 + 30^2)} &= 0.036 \text{ km} \end{aligned}$$

From Okumura-Hata Model we know,

The path loss in a high-rise urban areas with Perpendicular Street to the location of the Base Station is given by-

$$\begin{aligned} L_p &= 135.41 + 12.49 \log f_c - 4.99 \log h_b + [46.84 - 2.34 \log h_b] \log d \\ &= 135.41 + 12.49 \times \log (1.8) - 4.99 \times \log 20 + [46.84 - 2.34 \log 20] \times \log 0.036 \\ &= 68.91 \text{ dB} \end{aligned}$$

### Source Code:

```
import math

# Given values
fc = 1.8 # Frequency (GHz)
hb = 20 # Height of the base station antenna (meters)
d = math.sqrt(20 ** 2 + 30 ** 2) / 1000 # Distance between the base station and mobile station (kilometers)

print('Distance: %.4f Km' % d)

# Calculate path loss
PathLoss = 135.41 + (12.49 * math.log10(fc)) - (4.99 * math.log10(hb)) + ((46.82 - 2.34 * math.log10(hb)) * math.log10(d))

print('PathLoss: %.4f dB' % PathLoss)
```

### Output:

Distance: 0.0361 Km  
PathLoss: 68.9368 dB



**Problem-9:** A mobile is located 5 km away from a base station and uses a vertical  $\lambda/4$  monopole antenna with a gain of 2.55 dB to receive cellular 3 radio signals. The E-field at 1 km from the transmitter is measured to be V/m. The carrier frequency used for this system is 900 MHz.

- Find the length and the gain of the receiving antenna.
- Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5m above ground.

**Solution:**

Given that,

Frequency of operation, $f$	= 900 MHz
Gain of antenna, $G$	= 1.8 = 2.55 dB

(a)

We Know,

Wave length,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = 0.333 \text{ m.}$$

$$\text{Length of the antenna, } L = \lambda/4 = 0.333/4 = 0.0833 \text{ m} = 8.33 \text{ cm.}$$

And, Gain of antenna,  $G = 2.55 \text{ dB}$ .

(b)

Given that,

T-R separation distance, $d$	= 5 km
E-field at a distance of 1 km, $E_0$	= $10^{-3} \text{ V/m}$
Transmitter distance $d_0$	= 1km
Transmitting antenna height, $h_t$	= 50m
Receiving antenna height, $h_r$	= 1.5m
Wave length, $\lambda$	= 0.333

We Know,

Since  $d \gg \sqrt{h_t h_r}$ , the electric field is given by

$$\begin{aligned} E_R(d) &\approx \frac{2E_0 d_0}{d} \frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{ V/m} \\ &= \frac{2 \times 10^{-3} \times 1 \times 10^3}{5 \times 10^3} \left[ \frac{2\pi (50) (1.5)}{0.333 (5 \times 10^3)} \right] \\ &= 113.1 \times 10^{-6} \text{ V/m.} \end{aligned}$$

Here, Effective Aperture

$$\begin{aligned} A_e &= \frac{\lambda^2}{4\pi} G \\ &= 0.016 \text{ m}^2 \end{aligned}$$

Now, the received power at a distance d can be obtained using

$$\begin{aligned} P_r(d) &= P_d A_e = \frac{|E|^2}{120\pi} A_e \\ &= ((113.1 \times 10^{-6})^2 \times 0.016) / 337 \\ &= 5.4 \times 10^{-13} \text{ W} \\ &= -122.68 \text{ dBW} \\ &= -92.68 \text{ dBm} \end{aligned}$$

### Source Code:

```
import math

# Given values
T_R_distance = 5 # Distance between transmitter and receiver in km
E_field = 1e-3 # Electric field in V/m
f = 900 # Frequency in MHz
d0 = 1000 # Distance for electric field measurement in meters
c = 3e8 # Speed of light in m/s

# Converting frequency to Hz
f *= 1e6

# Calculating wavelength
lamda = c / f

# Height of transmitting and receiving antennas
ht = 50 # in meters
hr = 1.5 # in meters

# Distance from transmitter to receiver
d = T_R_distance * 1000 # in meters

# a
length_of_antenna = lamda / 4
gain = 10 ** (2.55 / 10)
gain_rcv = 10 * math.log10(gain)
print('Gain of Receiving Gain: %.2f dB' % gain_rcv)
effective_aperture = (gain * (lamda ** 2)) / (4 * 3.1416)
# print('Effective aperture is', effective_aperture)
```

```

# b
Er_d = (2 * E_field * d0 * 2 * 3.1416 * ht * hr) / (lamda * d ** 2)
print('Electric Field {:e} V/m'.format(Er_d))

pr_d = ((Er_d ** 2) / (120 * 3.1416)) * effective_aperture
received_power_at_5km_distance = 10 * math.log10(pr_d)
print(f'Received power at distance in dBW
{received_power_at_5km_distance:.2f}')

received_power2 = 10 * math.log10(pr_d * 1000)
print(f'Received power at distance in dBm {received_power2:.2f}')

```

### Output:

Gain of Receiving Gain: 2.55 dB  
 Electric Field 1.130976e-04 V/m  
 Received power at distance in dBW -122.68  
 Received power at distance in dBm -92.68

**Problem-10:** A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and  $\lambda$  = call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call-

- How many users per square kilometer will this system support?
- What is the probability that a delayed call will have to wait for more than 10s?
- What is the probability that a call will be delayed for more than 10 seconds?

**Solution:**

Given that,

Cell radius, R	= 1.387 km
Area covered per cell is $2.598 \times (1.387)^2$	$\approx 5$ sq km
Number of cells per cluster, n	= 4
Total number of channels, N	= 60
Therefore, number of channels per cell = $60 / 4$	= 15 channels.
From Erlang C chart, for 5% probability of delay with C = 15,	
Traffic intensity, A	= 9.0 Erlangs.

**(a)**

Given that,

Traffic per user,  $A_u = 0.029$  Erlangs.

We know,

$$\begin{aligned} \text{The number of users, } U &= A / A_u \\ &= 9.0 / 0.029 \\ &= 310 \text{ users} \end{aligned}$$

$$\begin{aligned} \text{The number of users per square km} &= 310 \text{ users} / 5 \text{ sq km} \\ &= 62 \text{ users /sq km} \end{aligned}$$

**(b)**

Given that,

$$\begin{aligned} \text{Wave length, } \lambda &= 1 \text{ call/hour} \\ \text{Holding time, } H &= A_u / \lambda \\ &= 0.029 \text{ hour} \\ &= 104.4 \text{ seconds.} \\ \text{Time, } t &= 10\text{s} \end{aligned}$$

We know,

The conditional probability that a delayed call will have to wait for more than t seconds is  $P_r[\text{delay} > t \mid \text{delay}] = \exp(-(C-A)t/H)$   
 $= \exp(-(15-9)10/104.4)$   
 $= 56.29 \%$

(c)

Given that,

The probability of delayed call,  $P_r[\text{delay} > 0] = 5 \% = 0.05$

We know,

Probability that a call is delayed more than 10 seconds,

$$\begin{aligned} P_r[\text{delay} > 10] &= P_r[\text{delay} > 0] \times P_r[\text{delay} > t \mid \text{delay}] \\ &= 0.05 \times 0.5629 \\ &= 2.81 \% \end{aligned}$$

### Source Code:

```
import math

# Given values
radius = 1.387
cluster = 4
total_channel = 60
channel_per_cell = total_channel / cluster
each_cell_covers = math.ceil(2.5981 * (radius ** 2))
traffic_per_user = 0.029
t = 10
blocking_probability = 5 / 100

# extra
print('Number of channel per cell: %d' % channel_per_cell)
print('Area Covered per cell is: %d sq km' % each_cell_covers)

traffic_intensity = 9
no_of_user = math.floor(traffic_intensity / (traffic_per_user *
each_cell_covers))
print(f'(a) Number of users: {no_of_user}')

print('(b)')
lambda = 1 # Au = lambda/H_holding time
holding_time = (traffic_per_user / lambda) * 60 * 60
print('Holding time: %.2f seconds' % holding_time)
probability_to_wait = math.exp(-(channel_per_cell - traffic_intensity) * t /
holding_time) * 100
```

```
print(f'Probability to wait: %.2f' % probability_to_wait, '%')

probability_of_delay = blocking_probability * probability_to_wait
print(f'(c) Probability of delay %.2f ' % probability_of_delay, '%')
```

**Output:**

Number of channel per cell: 15  
Area Covered per cell is: 5 sq km  
(a) Number of users: 62  
(b) Holding time: 104.40 seconds  
Probability to wait: 56.29 %  
  
(c) Probability of delay 2.81 %