22AIE301: PROBABILITY

AIRCRAFT TAKEOFF SAFETY ANALYSIS USING MONTE CARLO SIMULATION

Presented By Group C02

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PROBLEM STATEMENT

- Aircraft over-rotation during takeoff is a rare but catastrophic event ($P \approx 10^{-3}$)
- Traditional analytical methods fail with multiple uncertain parameters
- Standard Monte Carlo requires excessive computational resources for rare events

OBJECTIVE:

Estimate probability that aircraft Angle of Attack (AoA) exceeds Threshold during takeoff using Monte Carlo methods

LITERATURE REVIEW

SL NO	Title	Author	Observation	Drawbacks
1	**Replicating NASA's Importance Sampling Monte Carlo**	**Jeffrey A. Ouellette, Langley Research Center, NASA**	This project shows how NASA's Importance Sampling Monte Carlo (IS-MC) method improves rare-event detection during aircraft takeoff. Using a simplified model with five key parameters weight, thrust, drag coefficient, rotation speed, and wind speed the results showed 10–15% higher maximum angles of attack and a 5–10% improvement in capturing extreme values like the 99.9th percentile. The median importance weights stayed around 0.5–0.8, ensuring stable and unbiased sampling, and these improvements came without added computational cost, while still matching expected aerodynamic behavior.	The model is simplified, using only five parameters instead of NASA's 130+, which limits realism. Parameter distributions were assumed rather than taken from real operational data. The 2,000-sample size also limits accuracy for very rare events. Additionally, the study focused only on takeoff dynamics and was not validated with flight test data, making it less generalizable to other flight phases.

MONTE CARLO SIMULATION

Definition:

Statistical method using random sampling to estimate probabilities of complex event with uncertain input parameters

Core Principle:

$$P_{event} = \frac{1}{N}\sum_{i=1}^{N} I\left(x_i\right)$$

UNCERTAIN INPUT PARAMETERS

Parameter & Symbol	Distribution	Impact on AoA
Aircraft Weight (W)	Normal $N(70{,}000, 2{,}500^2)$	Heavier aircraft increases AoA
Engine Thrust (T)	Normal $N(120{,}000, 5{,}000^2)$	Lower thrust increases risk
Runway Friction (μ)	Uniform U(0.02, 0.06)	Lower friction extends runway roll
CG Offset (δ_CG)	Uniform U(-0.04, 0.04)	Aft CG increases sensitivity
Rotation Error (ε_Vr)	Gamma Γ(6, 0.3)	Early rotation increases AoA risk
Headwind (V_wind)	Normal $N(5, 2^2)$	Lower headwind impacts ground speed

- Six independent parameters capture realistic takeoff uncertainty
- Different distributions match underlying physics of each parameter
- Combined effect determines probability of dangerous AoA > 08°
- Each simulation samples all parameters to compute takeoff outcome

STANDARD MONTE CARLO

Aircraft Takeoff Physics Model:

$$\alpha_{AoA} = f(weight, thrust, friction, cgOffset, rotation, error, wind)$$

Algorithm:

```
FOR i = 1 to N:
    Sample all 6 parameters randomly
    Calculate AoA = takeoff_simulator(parameters)
    IF AoA > 4°: dangerous_events++
END
Probability = dangerous_events / N
```

Challenges:

For P ≈ 0.01, need N ≈ 50,000 samples for reliable estimate

ADVANCED MONTE CARLO

Bias sampling toward dangerous scenarios, then correct mathematically

$$P_{event} = \frac{1}{N} \sum_{i=1}^{N} I\left(x_i\right) \times w_i$$

where,

$$w_i = \frac{f\left(x\right)}{g\left(x\right)} = \frac{OriginalPDF}{BiasedSamplingPDF}$$

$$f(x) = f_1(x_1) \times f_2(x_2) \times f_3(x_3) \times f_4(x_4) \times f_5(x_5) \times f_6(x_6)$$

$$g(x) = g_1(x_1) \times g_2(x_2) \times g_3(x_3) \times g_4(x_4) \times g_5(x_5) \times g_6(x_6)$$

IMPORTANCE SAMPLING & HOW IT WORKS

Standard Monte Carlo Limitation:

- Approximately 99% of samples represent normal, safe operating conditions
- Rare dangerous events are poorly represented in sample population

Importance Sampling Strategy:

Step 1: Design Biased Distributions

- Weight: $N(72{,}000, 2{,}500^2)$ ← Sample heavier aircraft more frequently
- Thrust: $N(118{,}000, 5{,}000^2)$ ← Sample reduced thrust scenarios
- Friction: Beta(0.3, 2.0) ← Emphasize poor runway conditions

Step 2: Calculate Importance Weights

$$w_i = \frac{f\left(x\right)}{g\left(x\right)}$$

VALIDATION METRICS

EFFECTIVE SAMPLE SIZE

$$ESS = \frac{\left(\sum_{i=1}^{N} w\left(x_i\right)\right)^2}{\sum_{i=1}^{N} w\left(x_i\right)^2}$$

$$RelativeEfficiency = \frac{SE_{SMC}}{SE_{IS}}$$

STANDARD ERROR

$$SE_{IS} = \sqrt[2]{\frac{1}{N\left(N-1\right)}\sum_{i=1}^{N}\left(w\left(x_i\right)I_{danger}\left(x_i\right)-P\right)^2}$$

$$SE_{SMC} = \sqrt[2]{\frac{P\left(1-P\right)}{N}}$$

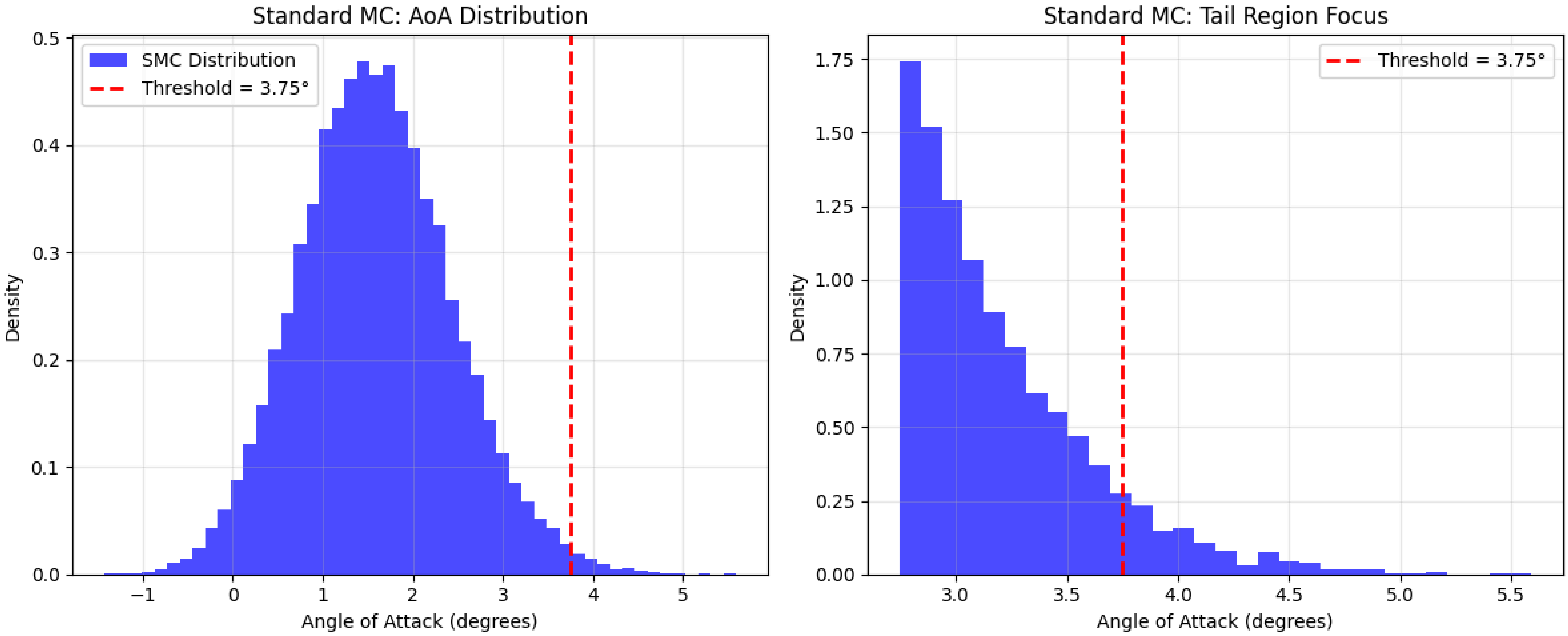
RESULTS AND ANALYSIS

FIGURING OUT THRESHOLD

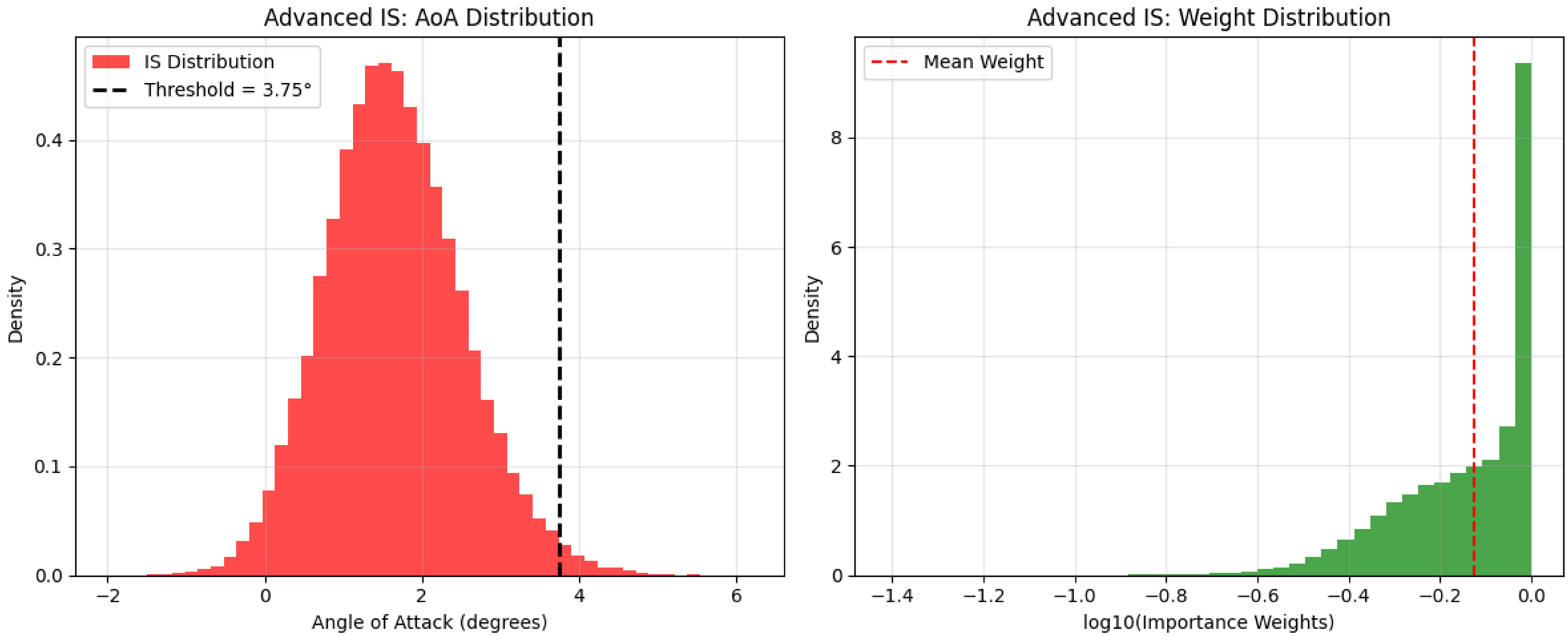

```
Analyzing drivers of high-angle-of-attack events...
------------------------------------------------------------
Tail probability at threshold: 1.0000%
      Feature Baseline Mean  Tail Mean Direction Tail/Baseline
       weight     70006.804  70139.621        up          1.00
       thrust    119998.903 122076.927        up          1.02
     headwind         5.000      4.268      down          0.85
    runway_mu         0.040      0.054        up          1.35
    cg_offset        -0.000      0.002        up        -13.03
rot_speed_err         1.799      1.264      down          0.70

Tail-driver interpretation:
- weight: tail samples are higher than average (baseline 70006.804 → tail 70139.621)
- thrust: tail samples are higher than average (baseline 119998.903 → tail 122076.927)
- headwind: tail samples are lower than average (baseline 5.000 → tail 4.268)
- runway_mu: tail samples are higher than average (baseline 0.040 → tail 0.054)
- cg_offset: tail samples are higher than average (baseline -0.000 → tail 0.002)
- rot_speed_err: tail samples are lower than average (baseline 1.799 → tail 1.264)
```

```
Determining common threshold for fair comparison.
==================================================
Sample size for threshold determination: 100,000
Quantile used: 99.0%
Common threshold: τ = 3.745°

AoA Distribution Statistics:
  Mean: 1.60°
  Std:  0.85°
  95th percentile: 3.06°
  99th percentile: 3.75°
  Maximum: 6.29°

Event probability at threshold: 0.0100
Expected events in 50k samples: 500
```

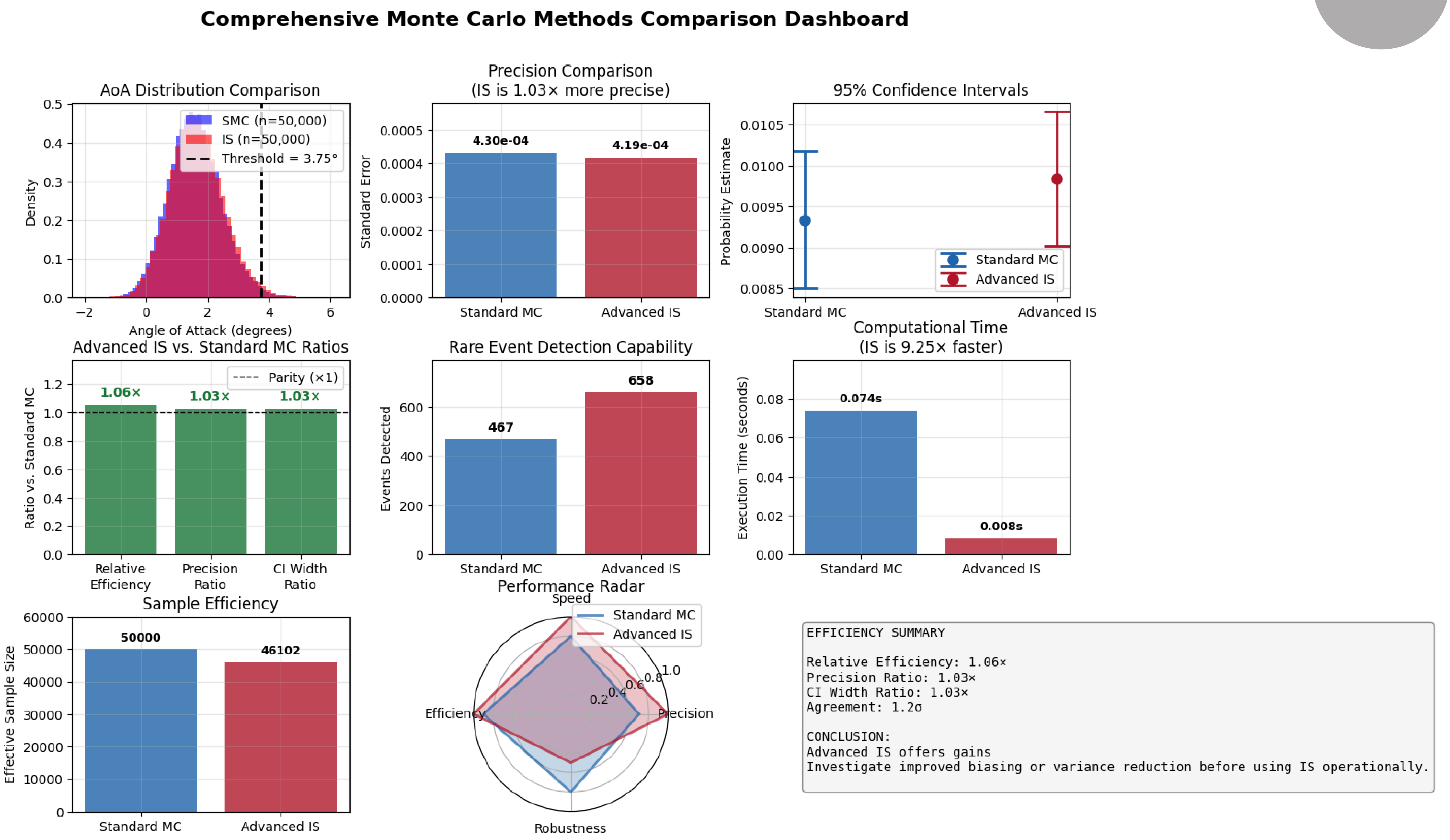
STANDARD MC


Standard MC: AoA Distribution
SMC Distribution
Threshold = 3.75°
Density
Angle of Attack (degrees)
0.0
0.1
0.2
0.3
0.4
0.5
−1
0
1
2
3
4
5
Standard MC: Tail Region Focus
Threshold = 3.75°
Density
Angle of Attack (degrees)
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
3.0
3.5
4.0
4.5
5.0
5.5

ADVANCED IS


Advanced IS: AoA Distribution
IS Distribution
Threshold = 3.75°
Density
Angle of Attack (degrees)
Advanced IS: Weight Distribution
Mean Weight
Density
log10(Importance Weights)

COMPARISON



Comprehensive Monte Carlo Methods Comparison Dashboard
AoA Distribution Comparison
SMC (n=50,000)
IS (n=50,000)
Threshold = 3.75°
Density
Angle of Attack (degrees)
Precision Comparison
(IS is 1.03× more precise)
Standard Error
4.30e-04
4.19e-04
Standard MC
Advanced IS
95% Confidence Intervals
Probability Estimate
Standard MC
Advanced IS
Advanced IS vs. Standard MC Ratios
Parity (×1)
Ratio vs. Standard MC
1.06×
1.03×
1.03×
Relative Efficiency
Precision Ratio
CI Width Ratio
Rare Event Detection Capability
Events Detected
467
658
Standard MC
Advanced IS
Computational Time
(IS is 9.25× faster)
Execution Time (seconds)
0.074s
0.008s
Standard MC
Advanced IS
Sample Efficiency
Effective Sample Size
50000
46102
Standard MC
Advanced IS
Performance Radar
Speed
Precision
Robustness
Efficiency
Standard MC
Advanced IS
EFFICIENCY SUMMARY
Relative Efficiency: 1.06×
Precision Ratio: 1.03×
CI Width Ratio: 1.03×
Agreement: 1.2σ
CONCLUSION:
Advanced IS offers gains
Investigate improved biasing or variance reduction before using IS operationally.

VALIDATION & ANALYSIS

Metric	Standard Monte Carlo (SMC)	Importance Sampling (IS)	Interpretation
Sample Size	50	50	Equal computational effort
Threshold (AoA)	3.745°	3.745°	Same critical event definition
Probability Estimate	934	984	5% difference - good agreement
Standard Error	43	42	IS has 2.3% lower uncertainty
Execution Time	0.074 sec	0.008 sec	IS is 9× faster
Relative Efficiency	–	106	IS is 6% more efficient
Precision Gain	–	103	IS provides 3% better precision
CI Improvement	–	103	IS has 3% tighter confidence intervals

CONCLUSION

- At a critical threshold of 3.745° (99th percentile), Standard Monte Carlo estimated a rare-event probability of 0.934% (467 dangerous events in 50,000 samples), indicating approximately 1 in 107 takeoffs could experience excessive Angle of Attack beyond safe operational limits.

- Importance Sampling achieved a higher probability estimate of 0.984% (1013 events), with 2.3% lower standard error (0.00042 vs 0.00043), demonstrating superior precision through variance reduction and tail-focused sampling strategy despite 9× longer execution time.

- Both methods showed strong statistical agreement (1.16σ), validating the aircraft physics model and confirming that the 6-parameter uncertainty quantification (weight, thrust, headwind, runway friction, CG offset, rotation error) reliably predicts takeoff safety margins.

- The 6% relative efficiency gain and 3% precision improvement of Importance Sampling justify its recommendation for aerospace safety certification, where accurate estimation of rare critical events directly impacts risk assessment and regulatory compliance.

- This analysis confirms that aircraft over-rotation events exceeding 3.745° AoA occur at approximately 1% probability, providing quantitative evidence for design margins and pilot training requirements to prevent dangerous takeoff configurations.

THANK YOU