22AIE301: PROBABILITY

AIRCRAFT TAKEOFF SAFETY ANALYSIS USING MONTE CARLO SIMULATION

Presented By Group C02

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PROBLEM STATEMENT

- Aircraft over-rotation during takeoff is a rare but catastrophic event ($P \approx 10^{-3}$)
- Traditional analytical methods fail with multiple uncertain parameters
- Standard Monte Carlo requires excessive computational resources for rare events

OBJECTIVE:

Estimate probability that aircraft Angle of Attack (AoA) exceeds 08° during takeoff using Monte Carlo methods

LITERATURE REVIEW

SL NO	Title	Author	Observation	Drawbacks
1	**Replicating NASA's Importance Sampling Monte Carlo**	**Jeffrey A. Ouellette, Langley Research Center, NASA**	This project shows how NASA's Importance Sampling Monte Carlo (IS-MC) method improves rare-event detection during aircraft takeoff. Using a simplified model with five key parameters weight, thrust, drag coefficient, rotation speed, and wind speed the results showed 10–15% higher maximum angles of attack and a 5–10% improvement in capturing extreme values like the 99.9th percentile. The median importance weights stayed around 0.5–0.8, ensuring stable and unbiased sampling, and these improvements came without added computational cost, while still matching expected aerodynamic behavior.	The model is simplified, using only five parameters instead of NASA's 130+, which limits realism. Parameter distributions were assumed rather than taken from real operational data. The 2,000-sample size also limits accuracy for very rare events. Additionally, the study focused only on takeoff dynamics and was not validated with flight test data, making it less generalizable to other flight phases.

MONTE CARLO SIMULATION

Definition:

Statistical method using random sampling to estimate probabilities of complex event with uncertain input parameters

Core Principle:

$$P_{event} = \frac{1}{N}\sum_{i=1}^{N} I\left(x_i\right)$$

UNCERTAIN INPUT PARAMETERS

Parameter & Symbol	Distribution	Impact on AoA
Aircraft Weight (W)	Normal N(70,000, 2,500²)	Heavier aircraft increases AoA
Engine Thrust (T)	Normal N(120,000, 5,000²)	Lower thrust increases risk
Runway Friction (μ)	Uniform U(0.02, 0.06)	Lower friction extends runway roll
CG Offset (δ_CG)	Uniform U(-0.04, 0.04)	Aft CG increases sensitivity
Rotation Error (ε_Vr)	Gamma Γ(6, 0.3)	Early rotation increases AoA risk
Headwind (V_wind)	Normal N(5, 2²)	Lower headwind impacts ground speed

- Six independent parameters capture realistic takeoff uncertainty
- Different distributions match underlying physics of each parameter
- Combined effect determines probability of dangerous AoA > 08°
- Each simulation samples all parameters to compute takeoff outcome

STANDARD MONTE CARLO

Aircraft Takeoff Physics Model:

$$\alpha_{AoA} = f\,(weight, thrust, friction, cgOffset, rotation, error, wind)$$

Algorithm:

```
FOR i = 1 to N:
    Sample all 6 parameters randomly
    Calculate AoA = takeoff_simulator(parameters)
    IF AoA > 8°: dangerous_events++
END
Probability = dangerous_events / N
```

Challenges:

For P ≈ 0.01, need N ≈ 100,000 samples for reliable estimate

ADVANCED MONTE CARLO

Bias sampling toward dangerous scenarios, then correct mathematically

$$P_{event} = \frac{1}{N} \sum_{i=1}^{N} I\left(x_i\right) \times w_i$$

where,

$$w_i = \frac{f\left(x\right)}{g\left(x\right)} = \frac{Original PDF}{Biased Sampling PDF}$$

$f(x) = f_1(x_1) \times f_2(x_2) \times f_3(x_3) \times f_4(x_4) \times f_5(x_5) \times f_6(x_6)$

$g(x) = g_1(x_1) \times g_2(x_2) \times g_3(x_3) \times g_4(x_4) \times g_5(x_5) \times g_6(x_6)$

IMPORTANCE SAMPLING & HOW IT WORKS

Standard Monte Carlo Limitation:

- Approximately 99% of samples represent normal, safe operating conditions
- Rare dangerous events are poorly represented in sample population

Importance Sampling Strategy:

Step 1: Design Biased Distributions

- Weight: N(72,000, 2,500²) ← Sample heavier aircraft more frequently
- Thrust: N(118,000, 5,000²) ← Sample reduced thrust scenarios
- Friction: Beta(0.3, 2.0) ← Emphasize poor runway conditions

Step 2: Calculate Importance Weights

$$w_i = \frac{f(x)}{g(x)}$$

EXPECTED OUTCOMES

Primary Event Analysis:

- Critical Event Probability: Likelihood that aircraft Angle of Attack (AoA) exceeds 08° during takeoff
- Risk Quantification: Statistical estimate of over-rotation event frequency

Statistical Measures:

- Confidence Intervals: Uncertainty bounds around probability estimates
- Effective Sample Size (ESS): Quality measure of importance sampling performance
- Relative Efficiency (RE): Precision improvement factor over standard methods

Validation & Impact:

- Cross-validation: Verification against large-scale benchmark simulations
- Aerospace Applications: Statistical evidence for flight test planning and safety certification

PRELIMINARY RESULTS

THANK YOU