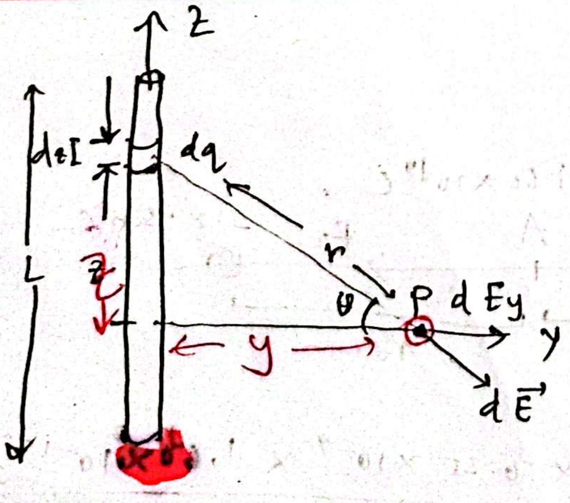
$$F_2 = \frac{-9\times10^9 \times 1.60\times10^{-19} \times 12.5\times10^{-9}}{(10\times10^{-2})^2}$$

$F_{net} = \sqrt{F_1 \,?\, F_2}$.

Electric field of continuous charge distributions

We divide charge distn into infinitesimal elements dq, expressing the charge element dq as λds, σdA or ρdv depending on whether the charge is distributed over a line (λ = linear charge density or charge per unit length), surface (σ = surface charge density or charge per unit area) or volume (ρ = volume charge density).

A uniform Line of ~~Charge~~ Charge



Consider a thin charged rod of line charge of length L having uniform positive linear charge density $\lambda = q/L$ where q = total charge carried by the rod.

The electric field at point P is due to total effect of all charge elements such as dq.

We have $d\vec{E}$ vectors from two charge vector components { dE_y and dE_z $dE_z = 0$ since equal + & −)

$q =$

$$dE_y = dE\cos\theta = \frac{1}{4\pi\epsilon_0} \frac{\lambda dz}{r^2} \times \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{\lambda dz}{y^2 + z^2} \times \frac{y}{\sqrt{y^2+z^2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda y \, dz}{(y^2+z^2)^{3/2}}$$

$$\therefore \quad E_y = \int dE_y = \int_{-L/2}^{+L/2} \frac{1}{4\pi\epsilon_0} \times \frac{\lambda y \, dz}{(y^2+z^2)^{3/2}}$$

$$= \frac{\lambda y}{4\pi\epsilon_0} \int_{-L/2}^{+L/2} \frac{dz}{(y^2+z^2)^{3/2}}$$

take,
$(z = y\tan u)$

$$= \frac{\lambda y}{4\pi\epsilon_0}$$

$$\frac{1}{4\pi\epsilon_0}\int \frac{\lambda y\, dz}{(y^2+z^2)^{3/2}}$$

$$\left[y^2(1+\tan^2 u)\right]^{3/2}$$

$$= y^3 \left(\cancel{1+\tan^2 u}\right)^{3/2} \quad \sec^2 u$$

$$= \frac{\lambda y}{4\pi\epsilon_0}\int \frac{dz}{(y^2+z^2)^{3/2}} \quad = \frac{\lambda \cancel{y}}{4\pi\epsilon_0} \times \frac{1}{\cancel{y^2}\, y} \frac{z}{\sqrt{z^2+y^2}}$$

$$= \frac{\lambda z}{4\pi\epsilon_0\, y\sqrt{z^2+y^2}}$$

$$\int \frac{dz}{(y^2+z^2)^{3/2}} =$$

$z = y\tan u$.

$dz = y\sec^2 u\, du$.

$\tan u = z/y$.

$\cot u = \frac{y}{z}$

$1+\cot^2 u = \frac{z^2+y^2}{z^2}$

$\mathrm{cosec}^2 u = \frac{z^2+y^2}{z^2}$

$\Rightarrow \sin^2 u = \frac{z^2}{z^2+y^2}$

$\Rightarrow \sin u = \frac{z}{\sqrt{z^2+y^2}}$

$$= \int \frac{y\sec^2 u\, du}{(y^2+y^2\tan^2 u)^{3/2}}$$

$$= \int \frac{\cancel{y}\sec^2 u\, du}{\cancel{y^3}\, y^2 (1+\tan^2 u)^{3/2}}$$

$$= \int \frac{\sec^2 u\, du}{y^2(\sec^2 u)^{3/2}}$$

$$= \frac{1}{y^2}\int \frac{du}{\sec u}$$

$$= \frac{1}{y^2}\times \int \cos u\, du$$

$$= \frac{1}{y^2}\sin u = \frac{1}{y^2}\times \frac{z}{\sqrt{z^2+y^2}}$$

$$= \frac{\lambda \cancel{y} x}{4\pi\epsilon_0} \left[\frac{1}{y^2} \times \frac{z}{\sqrt{z^2+y^2}} \right]_{-L/2}^{+L/2}$$

$$= \frac{\lambda}{4\pi\epsilon_0 y} \left[\frac{+L/2}{\sqrt{y^2+L^2/4}} - \frac{(-L/2)}{\sqrt{y^2+L^2/4}} \right]$$

$$= \frac{\lambda}{4\pi\epsilon_0 y} \left[\frac{L/2 + L/2}{\sqrt{L^2/4 + y^2}} \right]$$

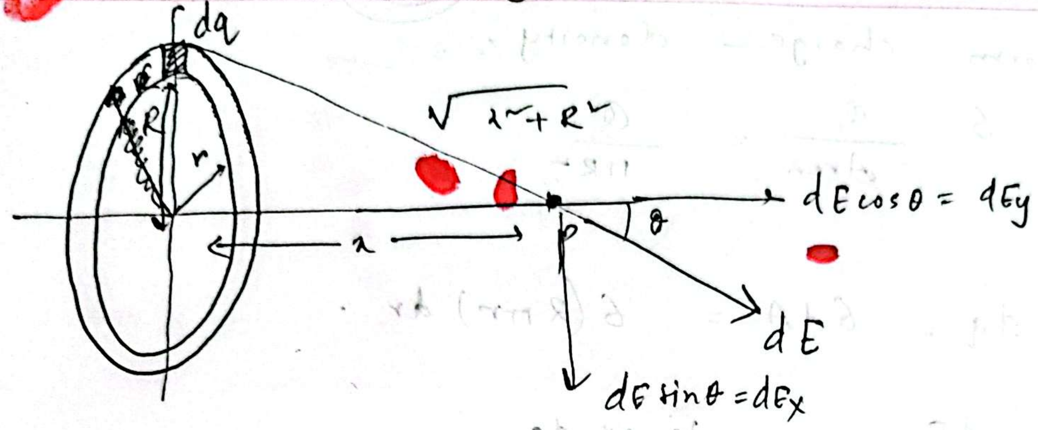
$$\boxed{E_y = \frac{\lambda L}{4\pi\epsilon_0 \, y\sqrt{y^2+L^2/4}}}$$

→ Electric field for a uniform line of charge.

A Uniform ring ~~of~~ or disk of charge

Consider, a ring of uniform positive charge having radius R. A differential element of charge ~~accp~~ occupies a length ds.

(1) Uniformly charged ring - This element sets up an electric field $d\vec{E}$ at point p.



$$dE = \frac{k\,dq}{x^2+R^2}$$

$$dE_x = dE\cos\theta$$

$$\int E = \int \frac{k\,dq}{x^2+R^2} \times \int \frac{x}{\sqrt{x^2+R^2}}$$

$$= \frac{kx}{(x^2+R^2)^{3/2}} \int dq$$

$$\boxed{E = \frac{kxQ}{(x^2+R^2)^{3/2}}}$$

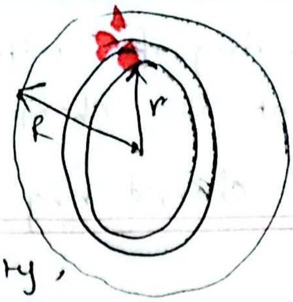
Cases

(I) At the centre of the ring :

$x = 0 \qquad E \approx 0$.

(II) Very far away : $x \gg R$. $E = \frac{kxQ}{\cancel{x^3}\,x^2} = \frac{kQ}{x^2}$

(2) from disk.



Uniform charge density,

$$\sigma = \frac{Q}{\text{Area}} = \frac{Q}{\pi R^2}$$

$$dq = \sigma\, dA = \sigma (2\pi r)\, dr.$$

$$dE_{ring} = \frac{k\, x\, dq}{[redacted]\,(x^2+r^2)^{3/2}}$$

$$= \frac{k\, \sigma \times x\, 2\pi r\, dr}{[redacted]\,(x^2+r^2)^{3/2}}$$

$$E_{disk} = \int dE_{ring} = \int_0^R \frac{k \times \sigma x x(2\pi r\, dr)}{(x^2+r^2)^{3/2}}$$

$$= k\,\sigma \times 2\pi x \int_0^R \frac{r\, dr}{(x^2+r^2)^{3/2}}.$$

let, $x^2+r^2 = u$

$2r\, dr = du$

$r \to 0$

$u \to x^2$

$r = R$

$u = x^2+R^2$

$$= k \times \sigma \times 2\pi x \int_{x^2}^{x^2+R^2} \frac{du/2}{u^{3/2}}$$

$$= \frac{\cancel{2}\pi \sigma\, k x}{\cancel{2}} \int_{x^2}^{x^2+R^2} \frac{\cancel{du}}{\cancel{Q}}\; u^{-3/2}\, du$$

$$= \pi \sigma k x \left[\frac{u^{-3/2+1}}{-3/2+1} \right]_{x^2}^{x^2+R^2}$$

$$= \pi \sigma k x \left[\frac{u^{-1/2}}{-1/2} \right]_{x^2}^{x^2+R^2}$$

$$= -2\pi \sigma k \times x \left[\frac{1}{\sqrt{(x^2+R^2)}} - \frac{1}{x} \right]$$

$$= -2\pi \sigma k \cancel{x} \frac{x - \sqrt{x^2+R^2}}{x\sqrt{x^2+R^2}}$$

$$= 2\pi \sigma k x \frac{\sqrt{x^2+R^2} - x}{x\sqrt{x^2+R^2}}$$

$$\boxed{E \approx 2\pi \sigma k x \left| \frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}} \right|}$$

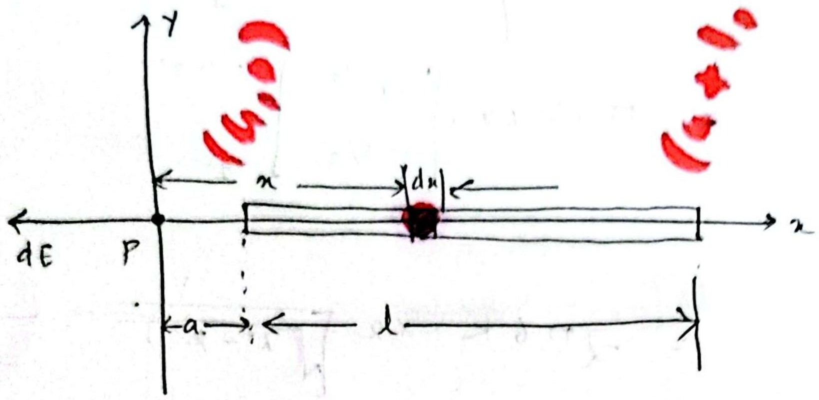
$x \to 0$

$$E_{disk} \approx \cancel{\alpha} \cdot \frac{2\cancel{\pi}\sigma}{\frac{4\cancel{\pi}\epsilon_0}{2}} \times \left[\frac{x}{|x|} - \frac{x}{\sqrt{x^2+R^2}} \right]$$

$$= \frac{\sigma}{2\epsilon_0} [1 - 0]$$

$$\boxed{E_{disk} = \frac{\sigma}{2\epsilon_0}} \rightarrow \text{Gauss's law.}$$

Electric field due to a charged rod :



The electric field at P due to a uniformly charged rod lying along the x axis. The magnitude of the field at P due to the segment of charge dq is $k_e\, dq/x^2$. The total field at p is the vector sum over all segments of the rod.

$$dq = \lambda\, dx$$

$$\therefore dE = \frac{k_e \lambda\, dx}{x^2}$$

$$\therefore E = k_e \lambda \int_a^{a+l} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{l+a}$$

$$= k_e \lambda \left[-\frac{1}{l+a} + \frac{1}{a} \right]$$

$$= k_e \lambda \left[\frac{-\cancel{a} + l + \cancel{a}}{a(l+a)} \right]$$

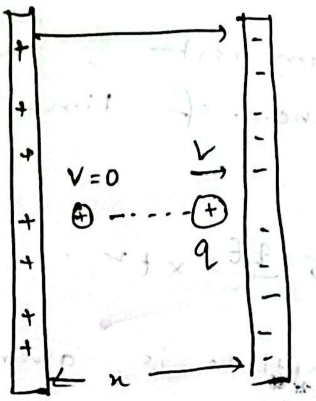
$$\lambda = \frac{q}{l}$$

$$= k_e \lambda \times \frac{l}{a(l+a)}$$

$$= k_e \frac{q}{\cancel{l}} \times \frac{\cancel{l}}{a(l+a)}$$

$$= \frac{k_e q}{a(l+a)}.$$

Motion of an accelerating positive charge in an electric field



$2 \times 1.69 \times 10^{-7}$
$\times 1.5 \times 2$
$\cancel{2.94 \times 10^{-7}}$
3.339×10^{-7}

A positive point charge q of mass m is released from rest in a uniform electric field E directed along the x axis. The acceleration is const and is given by →

$$a = \frac{qE}{m}.$$

The motion is simple linear motion along the

x axis.

Therefore, we can apply the equations of kinematics in one dimension.

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i).$$

Choosing ~~as~~ the initial position of the charge as $x_i = 0$ and assigning $v_i = 0$ because the particle starts from rest, the position of the particle as a func. of time is,

$$x_f = \frac{1}{2} a t^2 = \frac{1}{2} \frac{qE}{m} \times t^2.$$

the speed of the particle is given by →

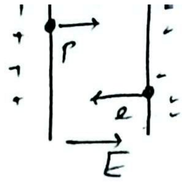
$$v_f = at = \frac{qE}{m} \times t.$$

~~Therefore~~ The third kinematic eqn gives →

$$(v_f)^2 = 2a_x x_f = \left(\frac{2qE}{m}\right) \times x_f.$$

from which we can find the kinetic energy of the charge after it has moved a distance

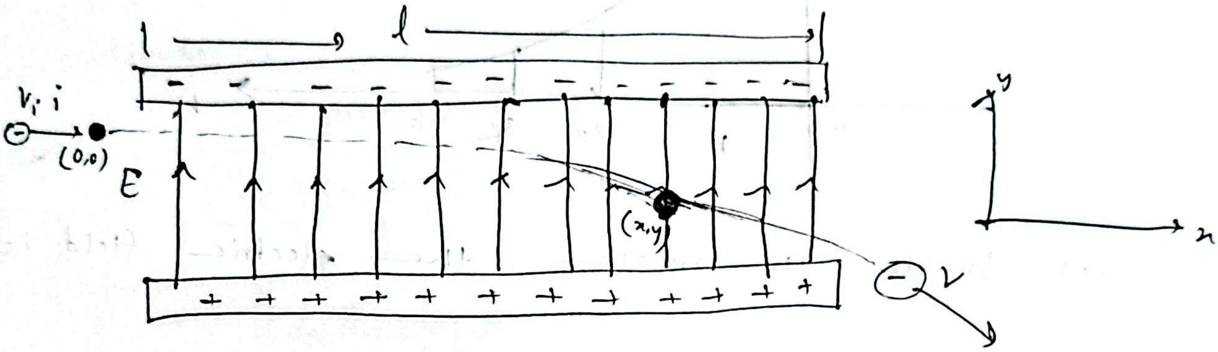
$$\Delta x = x_f - x_i$$



$$\therefore K = \frac{1}{2} m v_f^2 = \frac{1}{2} m \left(\frac{2qE}{m}\right) \Delta x = qE \Delta x .$$

same as work done $= F \Delta x$
$= qE \Delta x$

Motion of an accelerating electron in an electric field



if E is uniform (const), the acceleration is constant. If ~~negative~~ charge negative, the acceleration will be in the direction opposite the electric field.

Acceleration, $a = -\frac{eE}{m}$

$$v_x = v_i = \text{const} .$$

$$v_y = a_y t = \frac{-eE}{m} \times t .$$

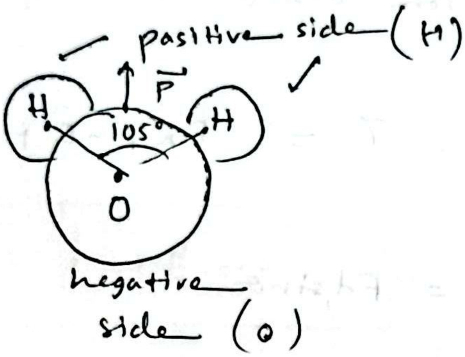
$$x = v_i t \; ; \quad y = \frac{1}{2} a_y t^2 = -\frac{1}{2} \frac{eE}{m} t^2 .$$

A dipole in an Electric field

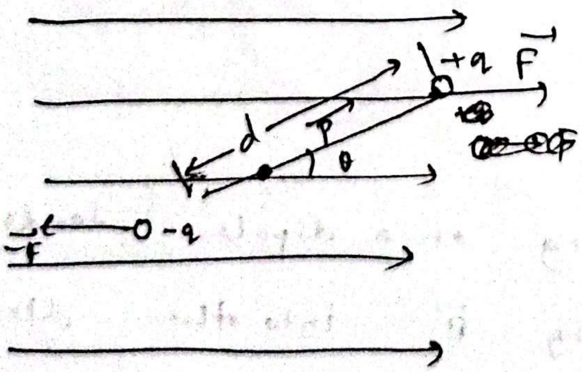
The electric dipole moment $\vec{p}$ of an electric dipole is a vector that points from the negative to the positive end of the dipole.

Example:

(1) A molecule of water (H_2O) is an electric dipole.



Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions and with the same magnitude $F = qE$.



from the electric field
Because $\vec{E}$ is uniform and $F_{net} = 0$ on the dipole, the center of mass of the dipole does not move.

However, the forces on the charged ends produce net torque $\vec{\tau}$ on the dipole about the C. of M.
The C.M, lies on the line connecting the charged ends, at some distance x from one end and a distance $d-x$ from other end.

$$\therefore \text{Net torque, } \tau = Fx\sin\theta + F(d-x)\sin\theta$$

$$= Fd\sin\theta$$

$p = qd$.

$F = qE$.

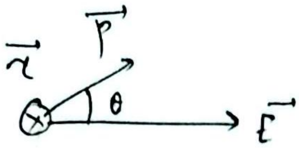
$\therefore \tau = qE \times d\sin\theta$.

$\tau = pE\sin\theta$

$\therefore \vec{\tau} = \vec{p} \times \vec{E}$

The torque acting on a dipole tends to reduce θ, by rotating $\vec{p}$ into the direction of $\vec{E}$.

for, clockwise rotation, $\vec{\tau} = -\vec{p} \times \vec{E}$



Potential energy of electric dipole :

$$\Delta U = -W$$

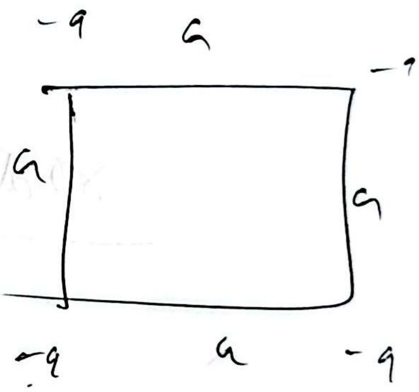
$$= -\int \tau \, d\theta$$

$$= -\int_{90^\circ}^{\theta} pE \sin\theta \, d\theta .$$

$$= \cancel{b} pE \cancel{\cos} \cdot \left[-\cos\theta\right]_{90^\circ}^{\theta}$$

$$= pE \left[-\cos\theta - (-\cos 90^\circ)\right] .$$

$$\boxed{U = -pE\cos\theta .}$$



$$\therefore \boxed{U = -\vec{p} \cdot \vec{E} .}$$

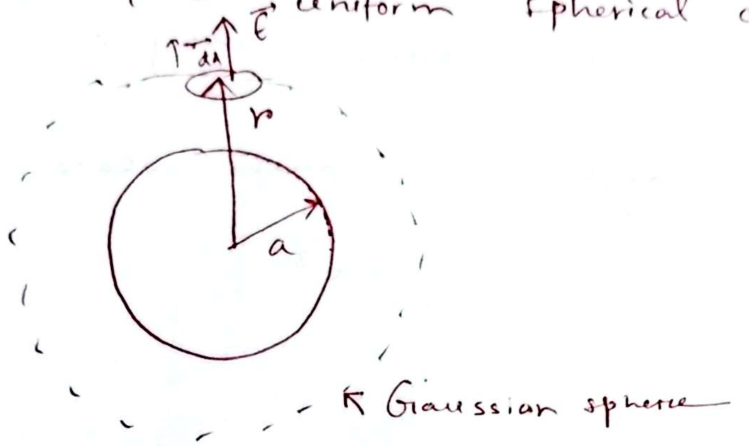
when, $\theta = 0$.

$$U = -pE$$

$\theta = 90^\circ$, $U = 0$.

$\theta = 180^\circ$ $U = pE$ (max).

Electric field for a uniform spherical charge:



Consider a uniform spherical distn of charge. This must be charge held in place in an insulator. Charge on a conductor would be free to move and would end up on the surface. This charge density is uniform along the sphere.

Suppose, charge Q is uniformly distributed throughout a sphere of radius a. Find E at a radius r.

If $r > a$, then electric field points outside the spher

Just as before (for point charge) we start with Gauss's law,

$$\Phi = E \times A = E \times 4\pi r^2$$

$$\Phi = \frac{Q}{\varepsilon}$$

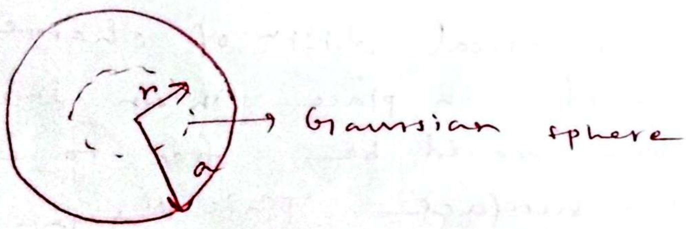
$$\therefore \frac{Q}{\epsilon_0} = E \times 4\pi r^2$$

$$\therefore E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

That is, E outside the sphere is exactly the same as if there were only a point charge Q.

Now, more inside the sphere where $r < a$, The

~~Now,~~ volumetric charge density $\rho = \frac{Q}{V_a} = \frac{Q}{\frac{4}{3}\pi a^3}$



The charge contained within a sphere of radius r is,

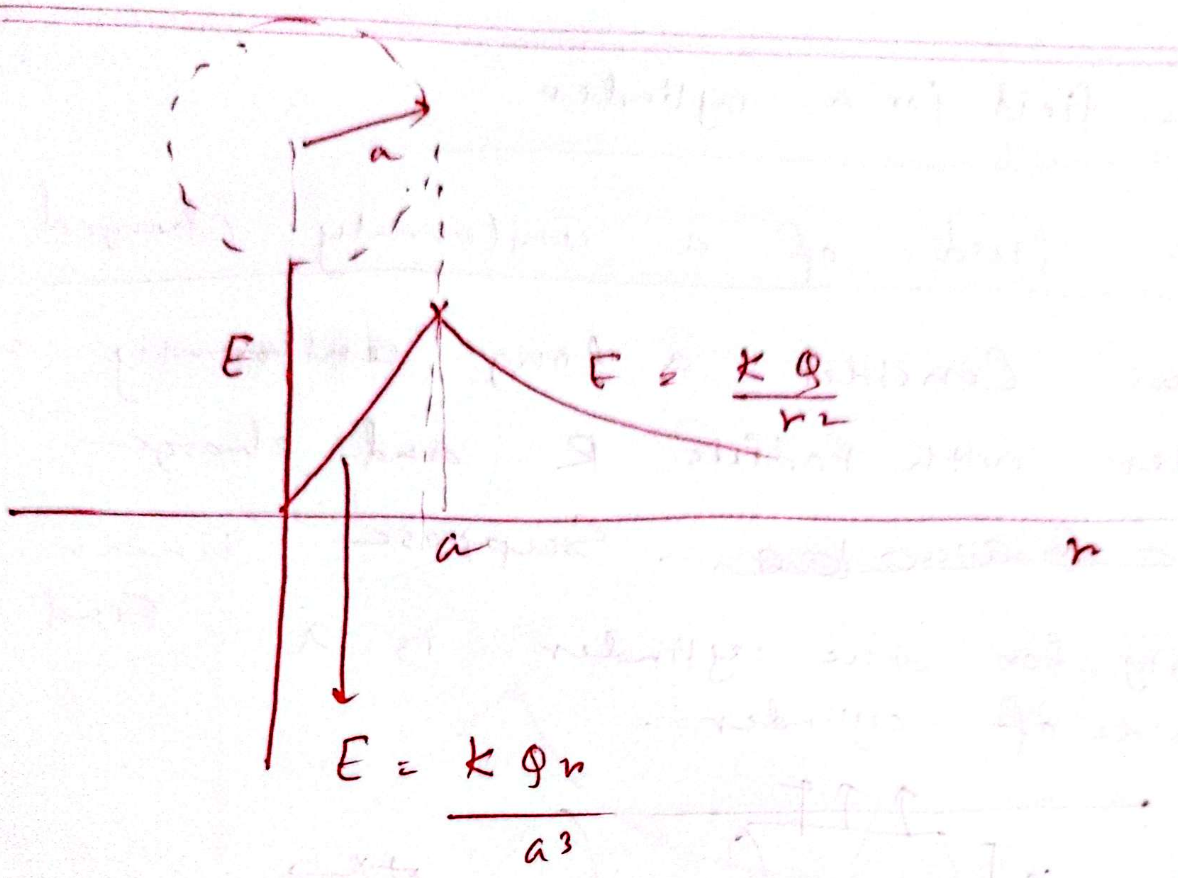
$$Q'(r) = Q' = V\rho = \frac{4}{3}\pi r^3 \times \frac{Q}{\frac{4}{3}\pi a^3}$$

$$\therefore Q' = \frac{r^3}{a^3} \times Q.$$

$$\therefore E \times 4\pi \cancel{r^2} = \frac{\cancel{r^3}\, r}{a^3} \times \frac{Q}{\epsilon}$$

$$\Rightarrow \boxed{E = k\frac{rQ}{a^3}}$$

That is electric field inside the sphere of uniform charge is Z at $r = 0$, and change linearly with radius r.

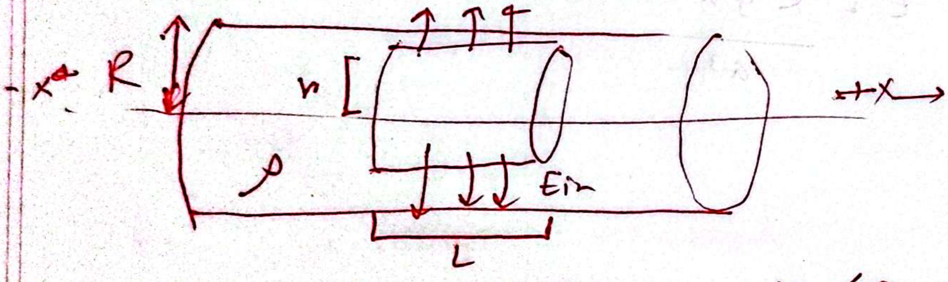


a
E
E = kQ/r2
a
r
E = kQr/a3

Electric field for a cylinder

Electric field of a uniformly charged cylinder

~~Suppose~~ Consider a long uniformly charged cylinder with radius R and charge density ρ. ~~Using Gauss's Law~~. Suppose linear charge density for the cylinder is λ. Find E inside & outside of cylinder.



Inside of cylinder : $r < R$

According to Gauss's Law, $\oint \vec{E} \cdot \vec{dA} = \dfrac{Q_{enc}}{\epsilon_0}$.

$$\Rightarrow E \int dA = \frac{Q_{enc}}{\epsilon_0}.$$

$$\Rightarrow E \times 2\pi \cancel{r} \cancel{L} = \frac{\rho \times \pi r^{\cancel{2}} \cancel{L}}{\epsilon_0}.$$

$$\Rightarrow E = \frac{\rho r}{2\pi \epsilon_0}.$$

for,

E_{in} if $r = 0$.

$E_{in} = 0$.

$$E_{in}(R) = \frac{\lambda}{2\pi\epsilon_0}$$

$$E_{out}(R) = \frac{\lambda}{2\pi\epsilon_0 R}$$