Ejercicios-GAE

Saturday, 15 June 2024 7:12 p.m.

$$R_o = \frac{2P_o}{|I_A|^2} = \frac{1}{|I_A|^2}\frac{R_s}{2\pi a}\int_{-L.2}^{L/2} |I(z)|^2\,dz. \qquad \text{(2-173)}$$

3.2-4 Show that the ohmic resistance of a half-wave dipole from (2-173) is given by

$$R_o = \frac{R_s}{2\pi a}\frac{\lambda}{4}$$

3.2-5 Use the results of Prob. 3.2-4 to calculate the radiation efficiency of a half-wave dipole at 100 MHz if it is made of aluminum wire 6.35 mm (0.25 in.) in diameter. Assume the radiation resistance to be 70 Ω.

3.2-4. $\theta = \frac{2\pi}{\lambda}$

$l = \frac{\lambda}{2}$

$$E = \frac{jz_o I_o e^{-jkr}}{2\pi r}\left(\frac{\cos(\pi/2 \cos\theta)}{sen\,\theta}\right)$$

$$H = \frac{jI_o e^{-jkr}}{2\pi r}\left(\frac{\cos(\pi/2 \cos\theta)}{sen\,\theta}\right)$$

$$\langle s_r \rangle = \frac{z_o I_o^2}{8\pi^2 r^2}\left(\frac{\cos(\pi/2 \cos\theta)}{sen\,\theta}\right)^2 \hat{a}_r$$

$f = 100\ MHz$ $\qquad R_r = 70\Omega$

$r = \frac{6.35\times10^{-3}}{2}$

$$R_s = \sqrt{\frac{\omega\mu_o}{2\sigma}} = \sqrt{\frac{(2\pi)(1MHz)(4\pi\times10^{-7})}{2(3.5\times10^{7})}} = 335\ \mu\Omega$$

=

$$R_p = \frac{\Delta z\ R_s}{2\pi a} = \frac{\Delta z\ (335\mu\Omega)}{2\pi\left(\frac{6.35\times10^{-3}}{2}\right)} = \boxed{16.83\times10^{-3}\ \Delta z}$$

$$\eta = \frac{70}{70 + 16.83\times10^{-3}\Delta z} = 0.99 = 99\%$$

3.4-9 An AM broadcast receiver operating at 1200 kHz uses a ferrite rod antenna with 1000 turns of No. 30 copper wire wound evenly on a rod core of ferrite with $\mu_r = 50$ that is 30 mm long and 8 mm in diameter. Find the radiation resistance, the radiation efficiency neglecting ferrite core losses, and the reactance.

$f = 1200\ kHz$

$$R_r = 31200\left(\frac{AE}{\lambda^2}\right)^2 = 31200\left(\frac{175.93mm^2}{\frac{3\times10^8}{1200\times10^3}}\right)^2 = \boxed{4.428\times10^{-7}\Omega}$$

$$AE = N^2 A\left(4\pi\times10^{-7}(50)(1000)^2\frac{\pi(4\times10^{-3})^2}{0.03}\right)$$

$= 175.93\ mm^2$

$$X = 2\pi(1200\times10^3)\left(4\pi\times10^{-7}(50)(1000)^2\frac{\pi(4\times10^{-3})^2}{0.03}\right)$$

$\boxed{= 79.42\Omega}$

3.4-10 An AM broadcast receiver operating at 1 MHz uses a ferrite rod antenna with 500 turns of No. 30 copper wire wound on a core of ferrite with $\mu_r = 38$ that is 25 mm long and with a cross-section that is 1 cm by 3 mm. Find the radiation resistance and the radiation efficiency neglecting ferrite core losses.

$$L = (4\pi\times10^{-7})(38)(500)^2\left(\frac{3\times10^{-5}}{25\times10^{-3}}\right)$$

$= 180\ mH$

$$R_r = 31200\left(\frac{500^2(3\times10^{-5})\left(\frac{0.180}{0.025}\right)}{\frac{3\times10^8}{1\times10^6}}\right)$$

$\boxed{= 3.92\times10^{-7}\Omega}$

$$R_p = \frac{500(6.5\times10^{-3})(\pi)}{(5.8\times10^7)(5.07\times10^{-8})} = 34.6\ m\Omega$$

$$\eta = \frac{3.92\times10^{-7}}{3.92\times10^{-7} + 34.6\times10^{-3}} = \boxed{1.13\times10^{-5}}$$

3.4-11 A single-turn square loop antenna that is 0.5 m on each side operates at 30 MHz. The wire is aluminum with a diameter of 2 cm. Compute: (a) radiation resistance, (b) input impedance, and (c) radiation efficiency.

$f = 30MHz$

$$R_r = 31200\left(\frac{0.5^2}{\frac{3\times10^8}{30\times10^6}}\right)^2 = \boxed{195\ m\Omega}$$

$$R_p = \frac{l}{\sigma \cdot S} = \frac{2}{(3.5\times10^7)(\pi(0.01)^2)} = \boxed{1.82\times10^{-4}\Omega}$$

$$L = 4\pi\times10^{-7}\left(\frac{\pi}{\ln(100)-1}\right) = 1.09\times10^{-6}H$$

$$\eta = \frac{0.195}{0.195 + 1.82\times10^{-4}} = 0.9991$$

99%

$$X = 2\pi FL = 2\pi(30\times10^6)(1.09\times10^{-6})$$

$= 205.68\Omega$

$$Z_o = R_r + R_p + jX = 0.195 + 1.82\times10^{-4} + j(205.68)$$

b) $= 0.195 + j\,205.68$

4.4-5 Suppose a transmitting antenna produces a maximum far-zone electric field in a certain direction given by $E = 90\ I\frac{e^{-j\beta r}}{r}$ where I is the peak value of the terminal current. The input resistance of the lossless antenna is 50 Ω. Find the maximum effective aperture of the antenna, A_{em}. Your answer will be a number times wavelength squared.

$$E = 90 I \frac{e^{j\beta r}}{r}$$

$Z_o = 50\Omega$

$$A_{em} = 4\pi r^2\left(\frac{1}{2}\frac{|E|^2}{\eta}\right)$$

$$P_{rm} = \frac{|E|^2}{2\eta}(4\pi r^2) = \frac{I^2 Z_o}{2}$$

$$A_{em} = \frac{P_{rm}}{S_{rm}} = \frac{I^2 Z_o \lambda^2}{|E|^2} = \frac{\cancel{I^2}(50)\lambda^2}{90^2(\cancel{I^2})} =$$

$$A_{em} = 6.17\times10^{-3}\lambda^2$$

4.4-6 A parabolic reflector antenna with a circular aperture of 3.66-m diameter has a 6.30 m² effective aperture area. Compute the gain in dB at 11.7 GHz.

$f = 11.7\ GHz$

$$G = \frac{4\pi A_{ef}}{\lambda^2} = \frac{4\pi(6.3)}{\left(\frac{3\times10^8}{11.7\times10^6}\right)^2} = 120\ kW$$

$$10\log_{10}(6) = 50.8dB$$

effective aperture area. Compute the gain in dB at 11.7 GHz.

4.4-7 The effective aperture of a 1.22-m diameter parabolic reflector antenna is 55% of the physical aperture area. Compute the gain in dB at 20 GHz.

$$G = \frac{4\pi(0.55)\left(\frac{1.22}{2}\right)^2\pi}{\left(\frac{3\times10^8}{20\times10^6}\right)^2} = 35.9\times10^{-3}\ W$$

$$G_{dB} = 10\log_{10}(G) = \boxed{45.55dB}$$

4.4-8 Compute the gain in dB of 0.3-m circular diameter aperture antennas with 70% aperture efficiency at 5, 10, and 20 GHz. This problem approximates the performance of a small satellite earth terminal antenna over the range of commonly used frequencies and quantifies the frequency dependence of gain for a fixed aperture size.

$$A = \pi\left(\frac{0.3}{2}\right)^2 = 0.0707m^2$$

$$AE = 0.7(A) = 0.0495m^2$$

a) $$G_1 = \frac{4\pi(0.0495)}{\left(\frac{3\times10^8}{5\times10^9}\right)^2} = 173 \qquad G_{dB} = 22.38dB$$

b) $$G_2 = \frac{4\pi(0.0495)}{\left(\frac{3\times10^8}{10\times10^6}\right)^2} = 692 \qquad G_{2dB} = 28.4dB$$

c) $$G_3 = \frac{4\pi(0.0495)}{\left(\frac{3\times10^8}{20\times10^9}\right)^2} = 2770 \qquad G_{dB} = 34.43dB$$

4.4-11 An FM broadcast radio station has a 2-dB gain antenna system and 100 kW of transmit power. Calculate the effective isotropically radiated power in kW.

$$G_{lineal} = 10^{\frac{2}{10}} = 1.585$$

$$EIRP = Pt(G_{lineal})$$

$= 100\times10^3(1.585)$

$= 158.5\ kW$

4.4-14 A 150-MHz VHF transmitter delivers 20 W into an antenna with 10 dB gain. Compute the power in W available from a 3-dB gain receiving antenna 50 km away.

$f = 150\ MHz$

$P_t = 20W$

$G_T = 10dB$

$G_r = 3dB$

$r = 50\ km$

$$P_{rec} = \frac{P_t G_t G_r \lambda^2}{(4\pi r)^2} = \frac{(20W)(0.01W)(1.99mW)\left(\frac{3\times10^8}{150\times10^9}\right)}{(4\pi(50\times10^3))^2} =$$

$= 2.01\times10^{-15}\ W$

4.5-4 A pager receiver operating at 152.65 MHz uses a loop antenna constructed of a copper band with rectangular cross-section 3.65 mm × 0.70 mm in a single turn that is 41.6 mm long and 13 mm wide. (a) Compute the radiation efficiency. (b) Compute the effective aperture after first finding the gain. (c) Compute the power output from the antenna for an input electric field intensity of 13 μV/m.

$f = 152.65\ MHz$

$$R_r = 31200\left(\frac{N\,l^2}{\lambda^2}\right)^2\Omega$$

$$= 31200\left(\frac{41.6\times10^{-3}(13\times10^{-3})}{\left(\frac{3\times10^8}{152.65\times10^6}\right)^2}\right)^2$$

$$= 6.09\times10^{-4}\Omega$$

$$R_p = \frac{2(l_1 + l_2)}{\sigma A} = \frac{2(41.6\times10^{-3} + 13\times10^{-3})}{(5.8\times10^7)(3.65\times10^{-3})(0.7\times10^{-3})}$$

$$R_p = 7.36\times10^{-4}\Omega$$

a) $$\eta = \frac{6.09\times10^{-4}}{6.09\times10^{-4} + 7.36\times10^{-4}} = 0.4529\times100 = \boxed{45.29\%}$$

b) $$A_e = \frac{\lambda^2 G}{4\pi} = \frac{\left(\frac{3\times10^8}{152.65\times10^6}\right)^2(0.4529)(1.5)}{4\pi} = \boxed{0.208\ m^2}$$

c) $$P = \frac{E^2 A_e}{2R_p} = \frac{(13\times10^{-6})^2(0.208)}{2(7.36\times10^{-4})} = \boxed{23.88\ nW}$$