



# Multiobjective design of Work-In-Process buffer for scheduling repetitive building projects

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## ABSTRACT

Variability in production is one of the largest factors that negatively impacts construction project performance. A common construction practice to protect production systems from variability is the use of buffers (Bf). Construction practitioners and researchers have proposed buffering approaches for different production situations, but these approaches have faced practical limitations in their application. A multiobjective analytic model (MAM) is proposed to develop a graphical solution for the design of Work-In-Process (WIP) Bf in order to overcome these practical limitations to Bf application, being demonstrated through the scheduling of repetitive building projects. Multiobjective analytic modeling is based on Simulation–Optimization (SO) modeling and Pareto Fronts concepts. Simulation–Optimization framework uses Evolutionary Strategies (ES) as the optimization search approach, which allows for the design of optimum WIP Bf sizes by optimizing different project objectives (e.g., project cost, time and productivity). The framework is tested and validated on two repetitive building projects. The SO framework is then generalized through Pareto Front concepts, allowing for the development of the MAM as nomographs for practical use. The application advantages of the MAM are shown through a project scheduling example. Results demonstrate project performance improvements and a more efficient and practical design of WIP Bf. Additionally, production strategies based on WIP Bf and lean production principles in construction are discussed.

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## 1. Introduction

Variability in production is one of the largest factors that negatively impacts construction project performance. It can induce dynamic and unexpected conditions, unsteady project objectives and obscuring the means to achieve them. To understand the effect of variability on production processes, Hopp and Spearman [1] distinguished two kinds of variability in manufacturing systems: 1) the time process of a task and 2) the arrival of jobs or workflow at a workstation. Koskela [2] proposes a similar classification to variability in construction systems, where the processes duration and the flow of preconditions for executing construction processes (e.g., space, equipment, workers, component and materials, among others) are understood as variable production phenomena. From a practical standpoint, construction practitioners everyday observe this behavior in the project environment through varying production rates, labor productivity, schedule control, cost control, etc.

Several researchers have shown that variability is a well-known problem in construction projects, which leads to a general deterioration of project performance on dimensions such as: cycle time [3–7], labor productivity [8,9], project cost [10], planning efficiency [11,12],

among others. A way to deal with variability impacts in production systems is through the use of buffers (Bf). By using a Bf, a production process can be isolated from the environment as well as the processes depending on it [2]. Buffers can circumvent the loss of throughput, wasted capacity, inflated cycle times, larger inventory levels, long lead times, and poor customer service by shielding a production system against variability [1]. Hopp and Spearman [1] define three generic types of Bf for manufacturing, which can be applied in construction as:

1. Inventory: In-excess stock of raw materials, Work in Process (WIP) and finished goods, categorized according their position and purposes in the supply chain [13].
2. Capacity: Allocation of labor, plants and equipment capacity in excess so that they can absorb actual production demand problems [14].
3. Time: Reserves in schedules as contingencies used to compensate for adverse effects of variability. Float in a schedule is analogous to a Bf for time since it protects critical path from time variation in non-critical activities.

Theoretically, the analysis of Bf in this paper is based on lean production principles. Lean production is a management philosophy focused on adding value from raw materials to finished product. It allows avoiding, eliminating and/or decreasing waste from this so-called value stream. Among this waste, production variability decreasing is a central point within the lean philosophy from a system standpoint [15]. Lean

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production, as applied in construction, focuses mainly on: i) decreasing non-value-adding activities or waste (e.g. wait times); ii) increasing value-adding activities efficiency (e.g. process duration); iii) decreasing variability [2]; and iv) optimizing the production system performance as a whole [15].

In construction, current buffering practices generally follow an intuitive and/or informal pattern, leading to poor variability control [3,5,7,14,16–24]. Recently, several researchers and practitioners have proposed new Bf approaches to manage variability in construction, which have allowed industry to partially avoid informal and intuitive methods of designing and managing Bf in construction [3,7,14,18,25–27]. However, these methods have been either too theoretical in design or too difficult to apply in practice. In fact, there is limited evidence showing any use of practical buffering design approaches in construction practice [28].

This paper presents a buffering approach that is applicable for Work-In-Process (WIP) in repetitive building projects. In construction, WIP can be defined as the difference between cumulative progress of two consecutive and dependent processes, which characterizes work units ahead of a crew that will perform work (e.g., work units that have not been processed yet, but that will be). This definition of WIP is clearer in repetitive projects where processes are repeated continuously (e.g. highways, railways, pipelines, sewers, etc.) or in discrete repeated units (e.g. high-rise buildings, multistorey building, and repetitive residential projects, etc.) [29]. Existing research explores, the use of WIP Bf in repetitive projects, both implicit and explicitly, and demonstrates the limitations of its application [3,5,23,26–28,30–34]. This body of research suggests opportunities to improve the use of WIP Bf and to overcome practical limitations in current buffering approaches.

However, WIP Bf application in a production system is neither an apparent nor a direct task. The use of WIP Bf is controversial from a lean production perspective since the lean ideal suggests that zero inventories, or non-buffered production systems, are desirable [15]. Nevertheless, a production system without WIP implies a production system without throughput. Hopp and Spearman [1] recognize this issue and state that pull mechanisms in a production system do not avoid the use of buffers. However, the use of large WIP Bf to ensure throughput in production systems will inherently increase cycle times and costs. Therefore, it appears that a 'balance problem' exists between the use of WIP Bf to reduce variability impacts and overall production system performance based on lean principles.

Simulation–Optimization (SO) modeling can address this balance problem. Simulation–Optimization modeling can help to design appropriate WIP Bf sizes by addressing the trade-off between decreasing variability through larger WIP Bf sizes and increasing production system performance by lowering WIP Bf sizes to the theoretical limit of zero. In designing optimal WIP Bf sizes, SO modeling must account for different project objectives (project cost, time and/or productivity). Computer simulation is being actively applied as a research tool to investigate how buffering strategies affect construction production systems [3,14,23,30,31,35,36]. To date, research has only addressed specific cases of buffering strategies and it has not effectively addressed the balance problem. The first application of SO to model Bf in construction was proposed by [5], and a similar SO approach to model Bf in a construction scheduling context was also developed by [33]. Though both explicitly addressed the balance problem in theory, the research was not applied to an actual WIP Bf design in construction.

## 2. Research objective

The main goal of this research is to propose and validate a simple graphical approach to design WIP Bf in repetitive building projects. Accomplishment of this goal requires the development of a multiobjective analytic model (MAM) based on SO modeling which uses Evolutionary Strategies (ES) as the optimization search approach and Pareto Front concepts. To be practically applicable, this MAM should result in nomographs to facilitate its use in the process of WIP Bf design. The

paper addresses the development, testing and validation of SO approach and resultant MAM and the proposed graphical approach to design WIP Bf.

## 3. Research methodology

The research methodology consists on three stages: 1) definition of the SO framework to design WIP Bf; 2) testing and validation of the SO frame; and 3) development and application of the MAM to design WIP Bf. A discrete event simulation modeling architecture is employed as a basis for developing the SO framework. The SO framework is then applied to two multifamily residential building projects for testing and validation. The application includes the construction of discrete event simulation models for repetitive processes, SO modeling to design optimum WIP Bf sizes, and the development and implementation of buffered construction schedules. Finally, using the SO framework and Pareto Front concepts, this research develops the MAM for practical application of the concepts, thereby achieving its goal for a simple and practical tool to design WIP Bf in repetitive building projects. Multiobjective model development involves: i) the definition of multiobjective nomographs to address the design WIP Bf sizes with various project objectives; ii) sensitivity analysis and selection of WIP Bf sizes according to project preferences; iii) development of buffered construction schedules; and iv) application on a construction project example.

## 4. Describing WIP Bf in repetitive construction processes

In repetitive projects, WIP Bf can be characterized by a Linear Scheduling Diagram. Fig. 1 shows the diagram for  $n$  processes in a

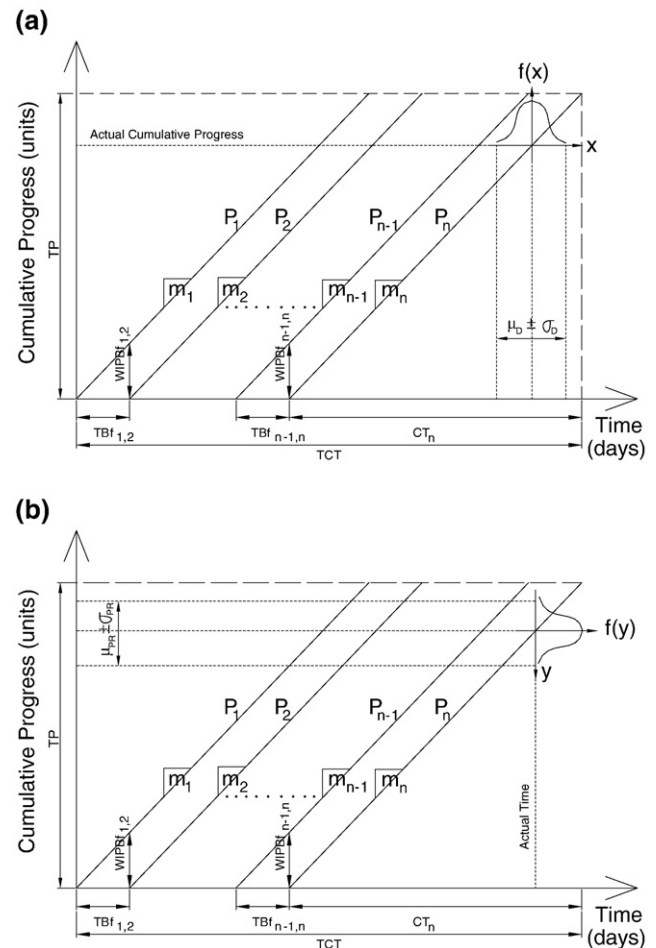


Fig. 1. Graphical representation of model for WIP Bf characterizing  $n$  processes: (a) unitary duration PDF, and (b) daily production rate PDF.

repetitive project with their different production parameters. Let repetitive and sequential processes  $P_1, P_2, \dots, P_{n-1}, P_n$  with average production rates and standard deviation called  $m_1, m_2, \dots, m_{n-1}, m_n$  (units/day) and  $SD_1, SD_2, \dots, SD_{n-1}, SD_n$  (units/day), respectively. Production rates ( $m_i$ ) for each process are an average value with a certain variation ( $SD_i$ ). This variable behavior can be mathematically captured by means of probability density functions (PDF) of duration by production unit or daily production rate (see Fig. 1a and b). Fig. 1a shows the duration PDF ( $f(x)$ ), with an expected duration by production unit ( $\mu_D$ ) and a certain standard deviation ( $\sigma_D$ ) for actual cumulative progress. Fig. 1b shows production rate PDF ( $f(y)$ ), with an expected progress or production rate by day ( $\mu_{PR}$ ) and a certain standard deviation ( $\sigma_{PR}$ ) for actual time.

Variability of a process, represented by a PDF for duration or production rate in this case, impacts the succeeding processes. For instance,  $P_1$  variability impacts  $P_2$ ,  $P_2$  variability impacts  $P_3$ , and so on. The production variability has a cumulative effect from upstream processes to downstream processes in repetitive production systems (i.e., a ripple effect) [1,7]. WIP Bf decreases this effect, isolating and protecting downstream processes from upstream processes variability [3,7,23]. The location and size of WIP Bf for repetitive project can be seen in Fig. 1. Let  $WIP Bf_{1,2}, WIP Bf_{2,3}, \dots, WIP Bf_{n-1,n}$  which have the corresponding Time Bf called  $T Bf_{1,2}, T Bf_{2,3}, \dots, T Bf_{n-1,n}$ , respectively. The main assumption relating to the location and size of WIP Bf within production processes is that these are restrictions applied only at the beginning of processes, which could change during the progression of work between processes.

Modeling requires definitions for the various states and boundary conditions relating to WIP Bf sizes. Minimum WIP Bf (MWIP Bf) is the minimum amount of work units ahead of a crew, from which the crew can perform its work and avoid any technical problem relating to buffering (e.g., the Bf to avoid crew congestion). This is a boundary condition for modeling and it has a related Time Bf that is defined as Minimum Time Bf (MT Bf). The Initial WIP Bf (IWIP Bf) is the amount of work units allocated ahead of a crew at the beginning of the downstream processes to protect them from the process duration or production rate variability of the upstream processes (e.g., the Bf to avoid idle or waiting time for lack of production units to perform work). It also has a related Time Bf that is defined as Initial Time Bf (IT Bf).

## 5. WIP Bf design approach using Simulation–Optimization

### 5.1. Simulation architecture and modeling assumptions

In this research, a discrete event simulation approach is used to design WIP Bf. Discrete event simulation describes systems evolving over time where state variables change instantaneously at separate points in time [37]. A discrete event simulation is an appropriate simulation approach to represent construction processes in repetitive building projects. A discrete event simulation software, Extend™, was selected to perform simulation modeling given its powerful features to visualize and to handle highly dynamic and complex systems [38]. Fig. 2 illustrates the simulation modeling architecture for two linear sequential processes, which is made up by two kinds of hierarchical blocks: processes and WIP Bf. These blocks emulate the ‘Parade of Trades’ that has been modeled previously in repetitive projects [7]. Inside these blocks, there are individual blocks, logical decision processes and stochastic inputs (e.g. process duration or production rate).



Fig. 2. Simulation modeling architecture showing two linear sequential processes and the corresponding WIP Bf.

Extend™ is based on a simulation strategy called Process Interaction where entities flow as integer units throughout the system [39]. For the simulation modeling architecture in this research, work units as houses or floors for building projects are the entities flowing through the system from “INPUT” to “OUTPUT” states (Fig. 2). At the beginning, work units flow from “INPUT” to “PROCESS 1” blocks, where the work units are either accumulated according to some defined production rate PDF for an unitary time (e.g., one day) or processed over a duration basis according to some defined process duration PDF in the process block. Before making the simulation experiments, project decision makers must choose the type of PDF. In the model, the production rate or duration PDF is estimated from historical data or expert judgement. Finally, the selection of production rate or process duration PDF for each process block depends on the preferences of the project decision makers. In practice, they will choose the production parameter more familiar with their planning and control procedures of site processes. Once work units have been processed, they accumulate in the “WIP BUFFER” block until the specified amount of work units is reached, i.e. MWIP Bf or IWIP Bf as described in Section 4. The “WIP BUFFER” releases units, the “PROCESS 2” processes units and then releases them as system “OUTPUT”. The production cycle is complete when all work units have been processed by all hierarchical process blocks. Notice that the simulation model has an algorithm by which processes cannot crossover each other. It always defines at least a minimum amount of work units (i.e. MWIP Bf) between processes to avoid conflicts.

In this research, the main modeling assumptions consider the following: 1) dependent linear sequence between processes for both simulation models and real projects; 2) only one crew is used for each process; 3) multiple crews are not allowed to expedite the process; 4) all crews follow the same sequence while no work units can be skipped; 5) no penalty is applied for discontinuous resource utilization, which may cause come-back delays; and 6) no technical lead-time is required between processes. These assumptions not only guide the simulation models, but also guide the on-site tests.

### 5.2. General Simulation–Optimization approach to design WIP Bf

The notion of optimization WIP Bf design for construction processes was initially explored by [3]. This initial work applied sensitivity analyses to find WIP Bf sizes relating to minimum cost. González et al. [5] have improved on this notion by explicitly proposing the use of SO models to design WIP Bf for minimum cycle time. A similar SO approach was studied by [33], which allows for the design of WIP Bf sizes for maximum project profits and continuous resources utilization (mainly labor). In practice, a SO model allows for the optimization (to maximize or minimize) of a key output performance measure, finding the better combination of input variables [37]. Thus, a simulator can represent a function  $\phi(x_1, \dots, x_n)$  for some input parameter vector  $x = (x_1, \dots, x_n)$ . The optimization goal is to find  $\min_{x \in W} E[\phi(x)]$  or  $\max_{x \in W} E[\phi(x)]$ , where the response  $E[\phi(x)]$  is the expectation of  $\phi(x)$  and  $W$  is a feasible range for the parameters [40]. The software Extend™ provides an Evolutionary Optimizer Module which the authors employ to optimize WIP Bf sizes on the basis of additional project parameters (e.g., project cost, time and productivity). This module is based on Evolutionary Strategies (ES) belonging to the family of Evolutionary Algorithms (EA). The ES are algorithms similar to genetic algorithms that mimic the principles of natural evolution as a method to solve parameter optimization problems [41]. A key feature of ES is that they do not require restrictive assumptions or prior knowledge about the problem being solved [42].

### 5.3. Evolutionary Strategies in optimization problems

ES usually use mutation, recombination, and selection applied to a population of individuals containing candidate solutions in order to evolve iteratively better and better solutions. The canonical versions of

the ES are denoted by  $(\mu/\rho, \lambda)$ -ES (comma-selection) or  $(\mu/\rho+\lambda)$ -ES (plus-selection) respectively. Here  $\mu$  refers to the number of parents,  $\rho \leq \mu$  the mixing number (e.g., the number of parents involved in the procreation of an offspring), and  $\lambda$  the number of offspring. In the comma-selection, the parents are deterministically selected (e.g., deterministic survivor selection) from the set of offspring. In contrast, the plus-selection deterministically chooses them from the population of parents and offspring. For the both cases  $\mu \leq \lambda$  must hold. In the case of combinatorial optimization problems (with discrete finite size search space), plus-selection is the more effective ES [43], which is used in Extend<sup>TM</sup>.

In the plus-selection,  $\mu$  parents (candidate solutions) produce  $\lambda$  offspring (new solutions) by mutation. One the most promising features of ES is the use of adaptive step sizes for mutation [44]. When a parent is mutated to produce an offspring, each object variable is mutated independently using self-adaptive mutation rates [45]. Basically, mutation creates new points by adding random normal distributed quantities with mean zero and variance  $\sigma_i^2$ . It is important to note that, for each decision variable, an individual standard deviation  $\sigma_i$  is used controlling the step-size (also called mutation strength).

During the search, the step sizes for mutation are adapted and several self-adaptation schemes are possible [44,46]. In general, self-adaptation to the optimal mutation strength requires a definition of the selection pressure  $\lambda/\mu$  which guides the constant size of parents and offsprings during a search [43]. Afterwards, plus-selection members are sorted according to their objective functions values (individual fitness). Thus, the best  $\mu$  of all the plus-selection members are selected to become parents in next generation according to their highest fitness (one generation embraces the cycle recombination–mutation–selection) [44, 47]. To guarantee that only the  $\mu$  best individuals from the selection pool in each generation are transferred, a truncation rate is used [43]. The recombination operator is, on before mutation, to recombine randomly a set of chosen parents to find a new solution, in which  $0 \leq \rho \leq 1$  [44]. However, in this paper there is no recombination since  $\rho = 1$ . This is due to the fact that the derivation of design rules for recombining operators in combinatorial optimization problems is still a challenge [43]. The termination criteria for the ES process used in this paper was based on the maximum number of generations and convergence of fitness value (mean value for objective functions) [43]. A brief algorithm for a typical plus-selection ES is shown below (adapted from Beyer and Schwefel [43]):

- Step 1 Create the initial  $\mu$  parent population randomly, and evaluate the fitness of each individual in the population;
- Step 2 Use evolutionary operators, i.e. recombination (if it applies) and mutation, to generate  $\lambda$  offspring and count them to the initial population;
- Step 3 Evaluate the fitness of each member in the new population ( $\mu+\lambda$ );
- Step 4 Select new  $\mu$  parent based on their fitness values; and
- Step 5 Go to Step 2 if termination criteria are not met.

In the past, ES have proved to be reliable tools to perform single objective optimizer. However, most problems in engineering include several objectives competing simultaneously across a high-dimensional problem space [44]. The solution set of a multiobjective optimization problem consists of all those vectors in which all of their components cannot be simultaneously improved. This is known as the Pareto concept of optimality. The solution set is a so-called the Pareto-optimal set, where solutions are defined as non-dominated solutions [48]. The Pareto Front concept allow comparing solutions in multiobjective optimization that have no unified criterion with respect to optima, helping to find good compromises or 'tradeoffs' rather than a single solution [49]. In EA, multiobjective problems are solved by means of the scalarization of the objective vectors in order to provide scalar fitness information to EA to work on. In several problems, objectives are often

artificially combined or aggregated, into scalar function given a certain level of knowledge about the problem, and then, implemented in EA [48]. This approach provides guidelines to ES multiobjective search.

Fonseca and Fleming [48] have characterized four types of evolutionary approaches. 1) Aggregated methods, where objectives are combined into a higher scalar function which is used for fitness calculation. It produces one single solution and requires a profound domain knowledge which is not always available. 2) Population-based non-Pareto approaches, in which multiple non-dominated solutions evolve in parallel, where population is controlled by non-dominated solutions. 3) Pareto-based approaches, where EA compare solutions given the dominance relation in order to determine the reproduction probability of each individual. 4) Niche induction techniques mostly implement fitness sharing, which is based on that individuals in a particular niche tend to share the resources available, mimicking to nature. Therefore, the fitness value of a certain individual is degraded if more individuals are located in its neighbourhood (defined geometrically as a distance measure called niche radius). Several multiobjective ES have also been developed, mainly the Multiobjective Elitist Evolution Strategy and the Memetic Pareto Archived Evolution Strategy, which are Pareto-based approaches. The Multiobjective Elitist Evolution Strategy incorporates the use of a secondary population that acts as an elite archive of solutions. The Memetic Pareto Archived Evolution Strategy employs multiples instances of the (1+1)-Pareto Archived Evolution Strategy algorithm to update individual solutions, coupled with mechanisms for handling a global and many local archives of solutions [50].

Optimizing a combination of objectives has the advantage of producing a single compromise solution. However, several problems can emerge to accept a reliable solution. For instance, the function used excludes unknown aspects of the problem prior to optimization, or an inappropriate setting of the coefficients of the combining function is selected [48]. Extend<sup>TM</sup> uses a single optimization ES approach which avoids this issue, since its focus is one objective at time. The next sections explain the approach developed in this research to develop multi-objective models to design WIP Bf.

#### 5.4. WIP Bf optimization using Evolutionary Strategies in simulation approach

Balancing the use of WIP Bf and project performance can be solved by means of the search process of WIP Bf sizes in order to optimize various project objectives. Then, the SO approach explores the IWIP Bf sizes as the production decision variables to optimize project performance. From the production parameters analyzed in Fig. 1, the objective functions are defined as:

1. Minimize total cycle time (Min TCT): decrease the project total time.  
where:

TCT      total cycle time for processes package, (days).

2. Minimize total cost (Min TC): decrease the project total cost.  
then:

$$TC = \text{Direct Cost} + \text{Indirect Cost} \quad (1)$$

where:

TC      total cost for processes package, (\$).

$$\text{Direct Cost} = DC = TP \times \sum_{i=1}^n MUC_i + \sum_{i=1}^n CT_i \times (LDC_i + EqDC_i) \quad (2)$$

and:

$$CT_i = TP/m_i \quad (3)$$



where:

$CT_i$	cycle time for process $i$ , (days), $i=1\dots n$ .
$TP$	total production, (units).
$MUC_i$	material unit cost for process $i$ , (\$/unit), $i=1\dots n$ .
$LDC_i$	labor daily cost for process $i$ , (\$/day), $i=1\dots n$ .
$EqDC_i$	equipment daily cost for process $i$ , (\$/day), $i=1\dots n$ .

$$\text{Indirect Cost} = IC = TCT \times DOC + FC \quad (4)$$

where:

$DOC$	daily overhead cost for processes package, (\$/day).
$FC$	fix cost, (\$).

In this paper,  $FC$  for cost analysis in simulated and real cases was neglected since  $FC$  is assumed as a constant value for every case, cancelling its effect over total cost variations.

3. Maximize average total production rate (Max ATm): increase the average project production rate of  $n$  processes.  
then:

$$ATm = \frac{\sum_{i=1}^n m_i}{n} \quad (5)$$

where:

$ATm$	average production rate for processes package, (units/day).
$n$	number of processes.

At the beginning (first generation), the ES guide the evolution of the IWIP Bf sizes in the SO process creating a random parent population (candidate IWIP Bf), estimating the fitness values (production responses in cost, time, or production rate for each IWIP Bf depending on what type of project objective is optimized during the search). By using the mutation operator, an offspring population (new IWIP Bf) from parent population is produced. The new IWIP Bf is added to the initial parent population (saving their fitness values). The fitness of each individual (all IWIP Bf) is evaluated by selecting the IWIP Bf with the best fitness values, which are incorporated in a new parent population of IWIP Bf for the next generation. The process is terminated if the maximum number of generations and/or the maximum convergence level with respect to the mean fitness values is reached. In Extend™, the convergence level is estimated in relation to the mean production responses of the all actual IWIP Bf sizes (population). The solution space where optimum IWIP Bf size can be searched is given by the following restrictions:

$$MWIPBf_{1,2} \leq IWIPBf_{1,2} \leq TP, \dots, MWIPBf_{n-1,n} \leq IWIPBf_{n-1,n} \leq TP.$$

A suitable selection of the tuning parameters in ES can result in an efficient search process (at local and global level). Nonetheless, there is no clearly defined procedure to define these parameters, being used only heuristics coming from empirical analysis [43]. According to guidelines propose by several researchers [42,43,46,51], the following levels for ES tuning parameters were set as follows:  $\mu=10$ ,  $\lambda=20$ , mutation rate=0.25, selection pressure=2 (with tournament selection), truncation rate=0.2. Similarly, the levels of the termination rules parameters were set as follows: convergence level=99.5%, maximum number of generations=1000, maximum sampling by generation

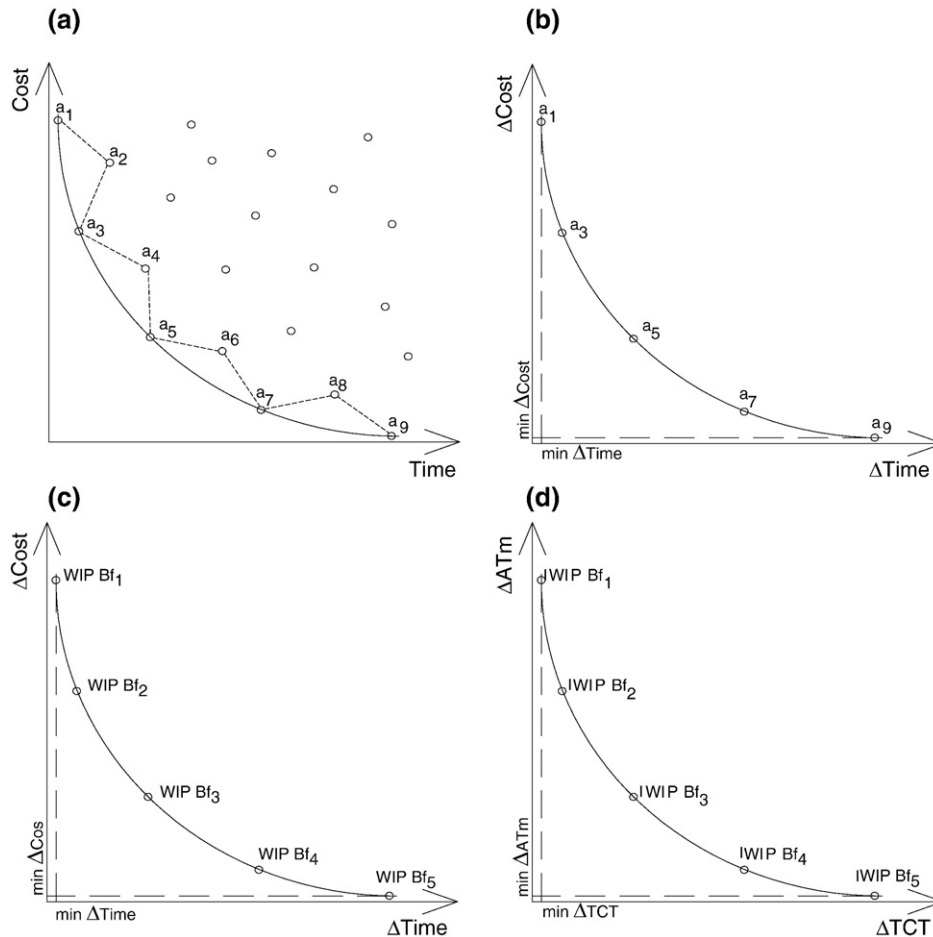


Fig. 3. Pareto Front curves: (a) Cost–Time trade-off, (b)  $\Delta\text{Cost}$ – $\Delta\text{Time}$  trade-off, (c)  $\Delta\text{Cost}$ – $\Delta\text{Time}$  trade-off for WIP Bf, and (d)  $\Delta\text{ATm}$ – $\Delta\text{TCT}$  trade-off for IWIP Bf.

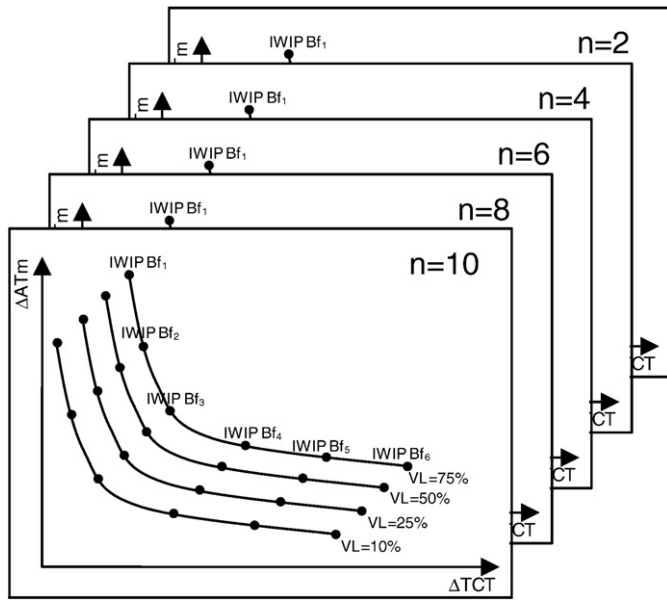


Fig. 4. WIP Bf design nomographs for different number of processes ( $n$ ) and variability levels ( $VL$ ).

(simulation runs)=100. The latter values have the purpose of producing enough long simulation runs, which guarantees reliable results avoiding local convergence and sub-optimal solutions.

Simulation–Optimization modeling allows for the design of optimum WIP Bf sizes for each objective function. The process suggests that there will be a maximum or minimum value for WIP Bf sizes. Knowing the extreme values (e.g. maximum and minimum) allows for a more general approach to designing WIP Bf based on multiple project objectives. The next section addresses the framework for a multiobjective model to design WIP Bf sizes using SO modeling as a basis.

## 6. Multiobjective model to design WIP Bf

The design of WIP Bf can be viewed as a multiobjective decision process. Simulation–Optimization models are particularly useful in

understanding the decision tradeoffs in design. However, simulation techniques are not a simple and common practice among construction practitioners [52]. In this paper, a practical method to design WIP Bf through MAM, resulting in a simple set of nomographs is proposed. While the use of simulation is not common practice at this time, the use of nomographs are common in engineering disciplines (e.g., hydrologic engineering) and can be easily applied by construction practitioners.

To get a rational solution for the proposed nomographs, Pareto Front concepts are introduced. Fig. 3a shows the common Cost–Time trade-off problem in construction faced through Pareto Front concepts [49,53,54]. Points  $a_1, \dots, a_n$  represent a resource mix for a given project (crew sizes, equipment, work methods, technologies, etc.). Points  $a_1, a_3, a_5, a_7$ , and  $a_9$  are along Pareto Front line and they represent non-dominated solutions, e.g. point  $a_3$  is a non-dominated solution, being partially better than points  $a_2$  or  $a_4$ . Points  $a_1$  and  $a_9$  are optimum points for minimum Time and Cost respectively, and they bound the whole Pareto Front Line [49]. Thus, solutions will depend on project decision makers preferences. For example in Fig. 3a, if a project decision maker chooses to save time, he will use more productive equipment and/or hiring more workers, but will be aware of the related increase in cost [53].

Fig. 3b shows another approach, where  $\Delta\text{Cost}$  and  $\Delta\text{Time}$  are the difference between actual and expected (or planned) budget and schedule, respectively. Obviously, a higher  $\Delta\text{Cost}$  and  $\Delta\text{Time}$  values means a higher actual budget and schedule for construction processes with respect to expected estimations. Fig. 3c describes a similar  $\Delta\text{Cost}$ – $\Delta\text{Time}$  trade-off but for different WIP Bf sizes (WIP Bf<sub>1</sub>, ..., WIP Bf<sub>5</sub>), keeping a constant resources mix. Due to the fact that the cost variable is a function of specific project characteristics, changing from project to project, it is a rather difficult task to generalize nomographs for repetitive projects, particularly when one of its variables is cost-based. To avoid this limitation, Fig. 3d describes a complementary approach replacing  $\Delta\text{Cost}$  by  $\Delta\text{ATm}$ , being defined as the difference between expected and actual average production rates for construction processes. A higher  $\Delta\text{ATm}$  value means a lower actual average production rates for construction processes in relation to expected estimations. In general, production rates are a more flexible project objective commonly found in construction production analysis and design. Fig. 3d regards IWIP Bf as points along Pareto Front line since Bf design is the focus of this paper. In addition,  $\Delta\text{Time}$  is replaced by  $\Delta\text{TCT}$  (using the nomenclature of this paper) and extreme points IWIP Bf<sub>1</sub> and IWIP Bf<sub>5</sub> represent the minimum  $\Delta\text{TCT}$  and  $\Delta\text{ATm}$  respectively.

Table 1  
Production characteristics of the case studies

Project	Kind of production units	Measure of production units	Type of Production Units <sup>a</sup>	Number of production units	Average area by production unit (m <sup>2</sup> )	Number of analyzed processes	Precedence relationship	Planned total cycle time (days) <sup>b</sup>	Planned total cost (\$) <sup>c</sup>
A	Houses	Units	1	32	180	5	Finish–Start	58	56,883.8
B			2	32	85	4		35	31,173.3

<sup>a</sup> Given for simulation effects.

<sup>b</sup> Execution Time for analyzed processes package.

<sup>c</sup> Approximated Budget in U.S. dollars for analyzed processes package.

Table 2  
Planned production and cost parameters for Project A

Planned production parameters and direct costs								Planned indirect costs		
Process	Type of process	MUC <sub>i</sub> (\$/unit)	TP (units)	m <sub>i</sub> (units/day)	CT <sub>i</sub> (days)	LDC <sub>i</sub> (\$/day)	DC (\$)	DOC (\$/day)	TCT (days)	Overhead (\$)
P <sub>1</sub>	Drywall ceiling	\$199.4	32	0.6	54.0	\$101.5	\$11,794.7	\$100.0	58	\$5800.0
P <sub>2</sub>	Partition	\$245.8	32	0.6	54.0	\$71.6	\$11,682.8	Total (2)	\$5800.0	
P <sub>3</sub>	Doors installation	\$177.7	32	0.6	54.0	\$57.9	\$8774.6			
P <sub>4</sub>	Water-proofing	\$136.6	32	0.6	54.0	\$67.7	\$7982.2			
P <sub>5</sub>	Kitchen floor (ceramic)	\$179.5	32	0.6	54.0	\$95.7	\$10,849.6			
						Total (1)	\$51,083.8	Total (1)+(2)		\$56,883.8

**Table 3**  
Production responses for simulated base case – Project A (non-buffered)

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
TP (units)	32.0				
CT (days)	54.7	60.9	63.4	70.1	69.5
MWIP Bf (units)	Bf no.	Bf <sub>12</sub>	Bf <sub>23</sub>	Bf <sub>34</sub>	Bf <sub>45</sub>
	Size	0.6			
$m_i$ (units/day)	0.60	0.53	0.51	0.46	0.46
Standard deviation of $m_i$ (units/day)	0.59	0.48	0.64	0.25	0.58
COV of $m_i$ (%)	99.1%	90.5%	125.8%	54.6%	125.4%
Average Worker Days (wd)	2.77	3.73	3.50	1.12	2.91
Total Average Worker Days (wd)	2.81				
Average Labor Productivity (units/wd)	0.215	0.142	0.145	0.411	0.159
ATm (units/day)	0.51				
Average COV of $m_i$ (%)	99.1%				
Total Average Labor Productivity (units/wd)	0.214				
TCT for 32 units (days)	80.6				
TC (\$)	63.086,4				

Using this approach, nomographs are constructed by first bounding the extreme points for minimum  $\Delta ATm$  and  $\Delta TCT$  through the SO process (see points IWIP Bf<sub>1</sub> and IWIP Bf<sub>5</sub> in Fig. 3d).  $\Delta ATm$  and  $\Delta TCT$  for intermediate IWIP Bf are obtained by simple simulation runs for each IWIP Bf size between extremes point (see  $\Delta ATm$  and  $\Delta TCT$  responses for points IWIP Bf<sub>2</sub>, IWIP Bf<sub>3</sub>, and IWIP Bf<sub>4</sub> in Fig. 3d). Fig. 4 illustrates examples of different nomographs, where repetitive projects could hold from  $n=2$  to  $n=10$  processes and face different levels of processes variability (VL), being a constant value for the  $n$  processes. In this research, the Coefficient of Variation (COV) is used as a measure of VL, where COV is the ratio between  $\sigma_D$  and  $\mu_D$  (see Fig. 1). The use of SO requires a selection of PDF functions. A Beta function was selected for the duration PDFs due to its general flexibility and adaptability to the processes duration in construction [55,56]. Ultimately, decision makers can use the nomographs from this research, both graphic and analytically (e.g. by using multivariate regression models), designing constant IWIP Bf sizes between processes according to their preferences on  $\Delta ATm$ ,  $\Delta TCT$  given  $n$  and VL.

In practice, a decision maker may need to choose an IWIP Bf size not only by its impact on time and production rate, but also in project cost. In this research, it is assumed that the IWIP Bf size that minimizes  $\Delta Cost$  is between extreme points for minimum  $\Delta TCT$  and  $\Delta ATm$ . The next sections of this paper demonstrate the validity of this assumption. It is necessary to note that the variable for project cost in this paper is called TC. A decision maker can use the nomograph in Fig. 3d to develop a sensitivity analysis for  $\Delta TCT$ ,  $\Delta ATm$  and  $\Delta TC$ . The optimum IWIP Bf size will be selected according to the preferred objective function.

To evaluate the impacts of IWIP Bf size using nomographs, the expressions for ATm, TCT and TC are as follows:

$$\text{Actual ATm} = \text{Planned ATm} \times (1 - \Delta ATm) \quad (6)$$

(ATm is computed over processes of critical path in a construction schedule).

$$\text{Actual TCT} = \text{Planned TCT} \times (1 + \Delta TCT) \quad (7)$$

(TCT is computed over processes of critical path in a construction schedule).

$$\text{Actual TC} = TP \times \sum_{i=1}^n MUC_i + \sum_{i=1}^n CT_i \times (LDC_i + EqDC_i) + TCT \times DOC \quad (8)$$

Replacing Eq. (3) in Eq. (8):

$$\text{Actual TC} = TP \times \sum_{i=1}^n MUC_i + \sum_{i=1}^n \frac{TP}{\text{Actual } m_i} \times (LDC_i + EqDC_i) + \text{Actual TCT} \times DOC \quad (9)$$

Regarding actual production rate by process  $i$  as follows:

$$\text{Actual } m_i = \text{Planned } m_i \times (1 - \Delta ATm) \quad (10)$$

And replacing Eqs. (7) and (10) in Eq. (9), Actual TC is as:

$$\text{Actual TC} = TP \times \sum_{i=1}^n MUC_i + \sum_{i=1}^n \frac{TP}{\text{Planned } m_i \times (1 - \Delta ATm)} \times (LDC_i + EqDC_i) + \text{Planned TCT} \times (1 + \Delta TCT) \times DOC \quad (11)$$

(TC is computed with respect to all processes of a project).

## 7. Testing and validation of the Simulation–Optimization approach

### 7.1. Project description

To test and validate the proposed SO approach on site, two multifamily residential building projects located in Santiago-Chile were used as case studies. The test involved the implementation of buffered construction schedules including optimum IWIP Bf sizes, and the collection of historical data or expert opinion from projects. This research was performed between June and December 2006 as part of an ongoing research to explore production strategies based on WIP Bf [5]. Table 1 shows the general production characteristics of both case studies.

### 7.2. Project A

Table 2 shows the planned production parameters and total costs of selected processes package. Equipment costs are not regarded. The

**Table 4**  
WIP Bf sizes and production responses of the Case A for each project objective after SO search and 1000 simulation runs (given a 95% confidence interval)

Simulation experiment	WIP Bf strategy	WIPBf size (units)				Average total cycle time (days) <sup>a</sup>	Average total cost (\$) <sup>a</sup>	Average production rate (units/day) <sup>a</sup>
		WIP Bf <sub>12</sub>	WIP Bf <sub>23</sub>	WIP Bf <sub>34</sub>	WIP Bf <sub>45</sub>			
Base case	MWIP Bf	0.6	0.6	0.6	0.6	80.56±0.29	63,086.4±132.3	0.51±0.002
Min TCT	IWIP Bf	<b>1</b>	<b>1</b>	<b>1</b>	<b>6</b>	<b>80.71±0.29</b>	<b>60,790.6±145.8</b>	<b>0.57±0.003</b>
Min TC		3	5	4	11	89.83±0.32	60,455.7±95.2	0.59±0.002
Max ATm		13	13	11	14	133.78±0.54	64,078.9±110.0	0.62±0.002
Differences with base case								
Simulation experiment	WIP Bf strategy	Average WIPBf Size (units)				Average total cycle time <sup>b</sup>	Average total cost <sup>b</sup>	Average production rate <sup>b</sup>
Min TCT	IWIP Bf	<b>2.3</b>				<b>0.18%</b>	<b>-3.64%</b>	<b>10.75%</b>
Min TC		5.8				11.50%	-4.17%	16.20%
Max ATm		12.8				66.07%	1.57%	22.05%

<sup>a</sup> 95% Confidence Interval.

<sup>b</sup> Difference calculated with respect to the mean production responses.

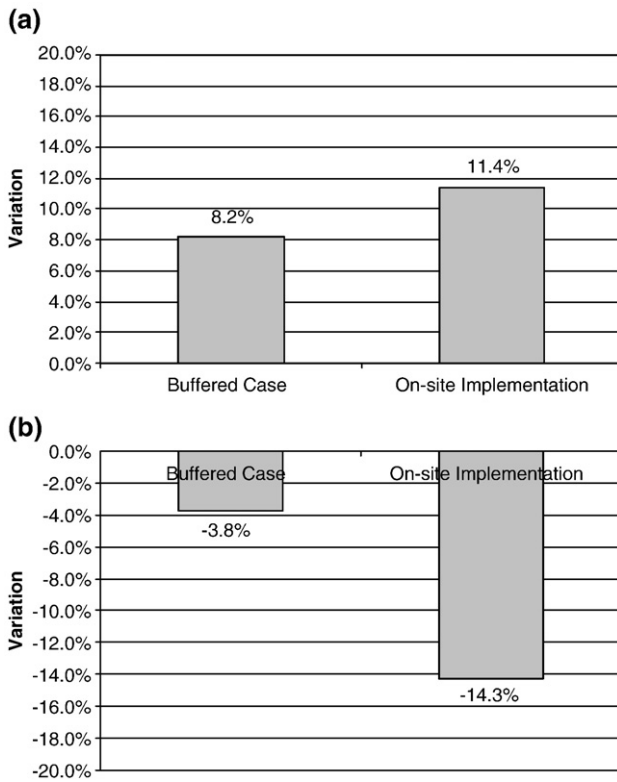


Fig. 5. Project performance impacts after IWIP Bf implementation over Project A: (a) Average Labor Productivity variation in relation to base case, and (b) TC variation regarding in relation to base case.

MWIP Bf size for all processes was 0.6 units. In this project, site personnel were more familiar with production rates than total process durations. Therefore, from historical data production rates PDFs for each process were fitted. Historical data was not used to represent real conditions of base case, since it was gathered from different time horizons and number of production units for each process. Furthermore, the analysis of their IWIP Bf sizes did not provide clear information. A simulation model of process packages with MWIP Bf was used to set a base case. To validate the base case simulations, intermediate and final outputs were examined by site personnel. The base case simulation model was found to provide a reliable description of the base production system.

Table 3 shows the main Base Case A production results using 1000 simulation runs. Average Worker Days were collected from historical data to estimate labor productivity. The 'Total Average Worker Days' and 'Total Average Labor Productivity' rows represent the average value for worker days and labor productivity respectively. Next, SO experiments for various project objectives were developed. Table 4 provides the main results for the optimum solutions (IWIP Bf size combination) in the SO model. To avoid sub-optimum solutions, two

steps are followed: 1) The termination rules defined for a SO search (Section 5.4); and 2) at least, two SO are performed for each project objective. Additionally, to improve accuracy in production responses for optimum solutions, 1000 simulations runs with a 95% confidence interval are performed for each solution. The optimal solutions for each project objective are selected based on its production results. For instance, if the project objective is Min TC, it will be searched with the IWIP Bf size combination that has the least cost. Table 4 adopts the same denominations defined in Fig. 1 for WIP Bf. It is necessary to note that the manner of estimating the 'difference for Average Production Rate' and the  $\Delta ATm$  are similar in nature, since they characterize the difference between 'high' and 'low' levels in production rates. However, there is a slight difference in the procedure of estimate the first. In the Average Production Rate, the difference is computed by comparing the actual optimized value (replacing the expected value) with the base case value. However, both are focused on maximizing the average production rate.

In Project A, decision makers choose the IWIP Bf strategy that will minimize the TCT difference between base and buffered case (marked in bold letters in Table 4). If pressure exists to finish project on time, decision makers will choose this solution. The confidence interval provides a better notion of risk in stochastic scenarios where values may have a wider range of variation than the only mean. In this case, confidence intervals for the base case and Min TCT do not show significant differences in cycle times. However, the Min TCT case provides a better option in relation to base case to define the IWIP Bf from the perspective of lower cost and higher production rate given the confidence intervals shown in Table 4.

It is interesting to address the effect of WIP Bf sizes on project performance. For example, higher WIP Bf in buffered cases reduce variability in labor productivity assuring a continuous resource utilization of crews (avoiding idle times by lack of WIP Bf). It causes a reduction in individual process cycles times and ultimately in total cost. How the direct cost is the most important component of total cost (based on real conditions of projects studied) which is productivity driven-cost, the improvements in total cost are reasonable. This can be seen in Table 4 for Min TCT and Min TC cases. However, Table 4 shows that, at the limit, the highest WIP Bf size is found in the Max ATM case which produces not only the highest productivity, but also the highest total cost. In spite of the high productivity, the selected WIP Bf sizes produce the maximum total cycle time level which leads to an increment of the indirect cost component, cancelling the positive effects of the better productivity in direct cost. Although the use of WIP Bf has pros and cons, the ultimate management objective is to optimize cost and time through them (which is the purpose of this paper).

To test on site the SO approach, a buffered construction schedule was implemented with the support of project personnel. This buffered construction schedule used IWIP Bf sizes of IWIP Bf<sub>1,2</sub> = 1.8 units, IWIP Bf<sub>2,3</sub> = 1.0 units, IWIP Bf<sub>3,4</sub> = 1.4 units, and IWIP Bf<sub>4,5</sub> = 5.5 units. During a 4 months period daily performance measurements for each process were gathered. Inputs for simulation models were gathered during the summer period and implementation of simulation outputs (IWIPBf)

Table 5  
Planned production and cost parameters for Project B

Planned production parameters and direct costs								Planned indirect costs		
Process	Type of Process	MUC <sub>i</sub> (\$/unit)	TP (units)	<i>m<sub>i</sub></i> (units/day)	CT <sub>i</sub> (days)	LDC <sub>i</sub> (\$/day)	DC (\$)	DOC (\$/day)	TCT (days)	Overhead (US\$)
<i>P</i> <sub>1</sub>	Dry Wall Ceiling	\$94.4	20	1	20	\$85.0	\$3588.8	\$58.0	35	\$2030.0
		\$87.6	12	1	12	\$78.8	\$1997.4	Total (2)	\$2030.0	
<i>P</i> <sub>2</sub>	Partition—1st layer	\$123.3	20	1	20	\$111.0	\$4685.7			
		\$106.3	12	1	12	\$95.7	\$2424.7			
<i>P</i> <sub>3</sub>	Plumbing and electrical installation	\$170.4	20	1	20	\$153.4	\$6476.8			
		\$156.6	12	1	12	\$140.9	\$3570.6			
<i>P</i> <sub>4</sub>	Partition—2nd layer	\$111.0	20	1	20	\$99.9	\$4217.2			
		\$95.7	12	1	12	\$86.1	\$2182.2			
						Total (1)	\$29,143.3	Total (1)+(2)		\$31,173.3



**Table 6**

Production responses for simulated base case—Project B (non-buffered)

	$P_1$	$P_2$	$P_3$	$P_4$
TP (units)	32.0	32.0	32.0	32.0
CT (days)	25.5	25.4	26.3	26.2
MWIP Bf (units)	Bf no. Size	Bf <sub>12</sub> 1.0	Bf <sub>23</sub>	Bf <sub>34</sub>
$m_i$ (units/day)	1.26	1.26	1.22	1.23
Standard deviation of $m_i$ (units/day)	0.67	0.69	0.65	0.67
COV of $m_i$ (%)	53.2%	54.9%	53.5%	54.9%
Average Worker Days (wd)	2.00	6.00	4.00	6.00
Total Average Worker Days (wd)	4.50			
Average Labor Productivity (units/wd)	0.628	0.210	0.305	0.204
ATm (units/day)	1.24			
Average COV of $m_i$ (%)	54.1%			
Total Average Labor Productivity (units/wd)	0.337			
TCT for 32 units (days)	27.6			
TC (\$)	27,894.9			

were developed during the winter period. To avoid analysis limitations for weather effect and to perform reliable comparisons between the simulation estimations and real application, the on-site implementation eliminated days with bad weather (i.e. non-working days). Production rates and cycle times were not analyzed since the number of Total Average Worker Days to simulation models (estimated from historical data) suffered an increment with respect to on-site implementation of 10% (from 2.81 wd to 3.09 wd, respectively). This variation in the number of workers could bias the improvement analysis of WIP Bf implementation. To avoid this limitation, data analysis focused on the impacts of the WIP Bf strategy over the total Average Labor Productivity, TC, and production rate variability (COV of  $m_i$ ). These performance measures are less sensitive to variation in the number of worker days than the other ones, allowing for a major accuracy in the analysis of WIP Bf impacts.

Fig. 5 summarizes improvements of the Total Average Labor Productivity and TC. Analysis regarding the planned estimation of labor productivity and cost were not included since, in this project, the estimated values were determined to be overoptimistic. Therefore, the base case was used as a baseline for measurement. Fig. 5a shows that the buffered case increases the Average Labor Productivity by 8.2% in relation to the base case. However, on-site implementation shows an even higher improvement for Average Labor Productivity with an average of 11.4%. The labor productivity differences between the buffered case and the on-site results can be explained as followed: The buffered case had an incremental improvement on Average Labor Productivity of 5.5% for processes  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_4$  in relation to the base case, which had an IWIP Bf size of 1.0 unit higher than MWIP Bf equal to

0.6 units. On the other hand, these processes improved Average Labor Productivity by 8% in relation to the base case for on-site implementation with an average IWIP Bf size equal to 1.6 units. Also, the process Total Average Worker Days increased by 12% in relation to the buffered case. Even though there was a high number of workers and continuous resource utilization was not totally effective due to the high interdependence between processes (i.e. low WIP Bf size). The on-site implementation for these processes showed productivity improvements. As long as, process  $P_5$  for the buffered case had an improvement of 23.4% on labor productivity in relation to base case and an IWIP Bf size of 6.0 units. The on-site implementation showed an improvement of 30.7% on labor productivity in relation to base case and an IWIP Bf size of 5.5 units. Although the number of workers was higher than the buffered case by only 3.8% for Average Worker Days, Process  $P_5$  is the improvement driver in the analyzed processes.

On-site observations and project personnel interviews provided evidence that there is a natural disposition of crews to slow down their progress pace to perform work when they observe that the WIP Bf size decreases. In contrast, when WIP Bf size increase to a level that allows continuous resource utilization, crews tend to increase the speed of progress pace to perform the available WIP Bf, improving their productivity. Then, on-site implementation of the IWIP Bf sizes in Case A shows this situation among processes with a higher WIP Bf level, providing a more efficient use of labor and reaching even higher productivity levels than the buffered case.

Fig. 5b shows TC improvements for the buffered case and on-site implementation of 3.8% and 14.3% respectively. These differences can be explained by the reasons given above for the improvements of labor productivity. Note that one of the most important instances is in the labor cost for the process  $P_5$  (with a 24.2% improvement). For the on-site implementation, indirect Costs were computed regarding of total days that processes were executed (71 days), and direct costs (specifically labor costs) were computed regarding to the days effectively worked for each process (without weather effect), providing more reliable comparisons of production scenarios.

Variability levels using average COV of  $m_i$  for the buffered case show a reduction of –10.66% in the base case, supporting the expected lean impacts of buffers in relation to production variability. It should be noted however, that variability levels in the on-site implementation increased 3.95% in relation to base case when weather conditions were taken into account.

### 7.3. Project B

Table 5 shows the planned production parameters and total costs for selected process packages, excluding also equipment costs. The MWIPBf size is equal to 1.0 units. In this case, site personnel were

**Table 7**

WIP Bf sizes and production responses of the Case B for each project objective after SO search and 1000 simulation runs (given a 95% confidence interval)

Simulation experiment	WIP Bf strategy	WIPBf size (units)			Average total cycle time (days) <sup>a</sup>	Average total cost (\$) <sup>a</sup>	Average production rate (units/day) <sup>a</sup>
		WIP Bf <sub>12</sub>	WIP Bf <sub>23</sub>	WIP Bf <sub>34</sub>			
Base case	MWIP Bf	1	1	1	27.57±0.08	27,894.9±35.85	1.24±0.004
Min TCT	IWIP Bf	<b>1</b>	<b>1</b>	<b>13</b>	<b>27.50±0.08</b>	<b>27,127.8±35.64</b>	<b>1.37±0.004</b>
Min TC		24	1	23	41.25±0.08	25,288.8±16.8	2.20±0.004
Max ATm		28	21	28	55.85±0.09	26,105.2±17.1	2.22±0.003
Differences with base case							
Simulation experiment	WIP Bf strategy	Average WIPBf size (units)			Average total cycle time <sup>b</sup>	Average total cost <sup>b</sup>	Average production rate <sup>b</sup>
		WIP Bf <sub>12</sub>	WIP Bf <sub>23</sub>	WIP Bf <sub>34</sub>			
Min TCT	IWIP Bf	<b>5.0</b>			<b>-0.25%</b>	<b>-2.75%</b>	<b>10.21%</b>
Min TC		16.0			49.65%	-9.34%	77.23%
Max ATm		25.7			102.61%	-6.42%	78.93%

<sup>a</sup> 95% Confidence Interval.<sup>b</sup> Difference calculated with respect to the mean production responses.

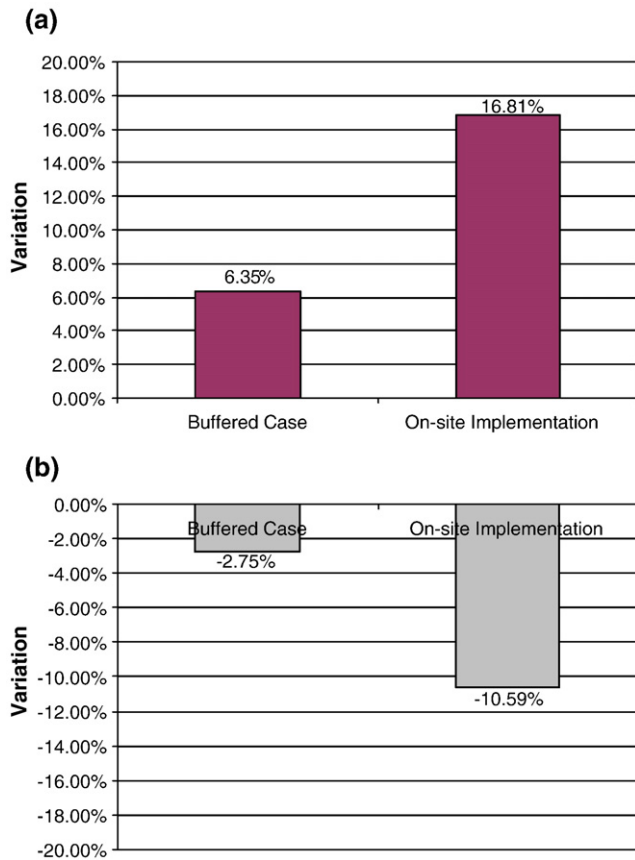


Fig. 6. Project performance impacts after IWIP Bf implementation over Project B: (a) Average Labor Productivity variation in relation to base case, and (b) TC variation regarding in relation to base case.

familiar with process unit duration. Given the lack of historical data, the process durations PDF were captured from expert judgement using the Visual Interactive Beta Estimation (VIBES) algorithm proposed by [55]. It is necessary to note that there were two kinds of processes duration used according to site personnel opinion: units type 1 and units type 2. The first 20 units were regarded as type 1 and the last 12 units as type 2.

Due to both the lack of historical data and similarities with Project A, a simulation model of process packages with MWIP Bf was used as a non-buffered base case. As well, site personnel were used to validate intermediate and final simulation outputs of the base case. Table 6 shows the main production results for Base Case B. Average Worker Days were estimated from expert judgement.

Simulation–Optimization experiments following the same procedure of Project A were developed. Table 7 shows the optimal solutions for the IWIP Bf size combinations. Similarly to the reasons stated for Case A, decision makers may chose an IWIP Bf strategy that minimizes TCT difference between the base and buffered cases (marked in bold letters). Also, the performance response followed a similar behavior pattern for the different WIP Bf sizes that the explained in the Case A.

As in the Project A, an on-site implementation of the buffered construction schedule was developed. This buffered construction schedule used IWIP Bf sizes of: IWIP Bf<sub>1,2</sub>=1.7 units, IWIP Bf<sub>2,3</sub>=2.4 units, and IWIP Bf<sub>3,4</sub>=8.8 units. Simulation models were developed and implemented for the summer period, making comparisons for different production scenarios without any filters. The TCT for the buffered case was slightly better than the base case with a difference of -0.3%. The on-site implementation of the buffered construction schedule also showed a slight difference in relation to the base case of 1.6%, making it a good description of production system. However, impacts on project performance were focused on labor productivity, cost and variability. Cycle times and production rates were not the focus due to the fact that the Total Average Worker Days between simulation assumptions and on-site implementation in the experiment had an increase of 13.0% (from 4.5 wd to 5.09 wd). Furthermore, the project personnel were not confident in the estimated values for the planned production rates (e.g. see Tables 1 and 5). Therefore, these values were increased during simulation and on-site experimentations. Given these reasons, the base case was used to make comparisons.

Fig. 6 summarizes improvements on Total Average Labor Productivity and TC against the base case. Fig. 6a shows that the improvement in labor productivity for the buffered case is 6.35% and the improvement from the on-site implementation is 16.81%. These differences between both cases can be explained, first, by the tendency to underestimate Total Average Worker Days by 12% for the simulated scenario when comparing the buffered case to the actual on-site results, according to site personnel opinion; and second, by the on-site improvements causing for the WIP Bf (i.e. similar to the reasons given to Case A). As a result, the on-site performance of labor

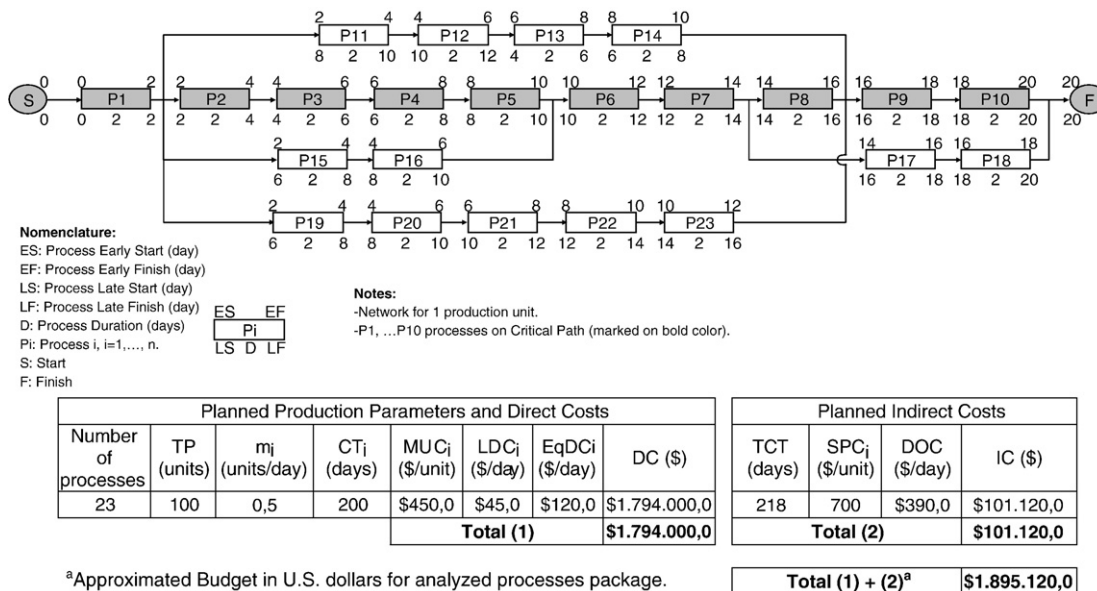


Fig. 7. Construction scheduling network for the application example of MAM, including planned budget and production responses.

productivity is better than simulated estimation. Processes  $P_1$ ,  $P_2$  and  $P_3$  continued to be very dependent on Bf size given their small IWIP Bf size (IWIP Bf<sub>1,2</sub>=1.7 units and IWIP Bf<sub>2,3</sub>=2.4 units). This fact limit the effect of continuous resource utilization though the Total Average Worker Days for on-site implementation as it increased by 21.75% in relation to buffered case. On the other hand, process  $P_5$  experienced an improvement of labor productivity for buffered case and on-site implementation of 40.7% and 70.3% respectively. The latter productivity value was reached with a 4% less Average Worker Days in relation to the buffered case. Also,  $P_5$  had a smaller on-site IWIP Bf size than the buffered case (8.8 units against 13 units). However,  $P_5$  performed the job more efficiently than other processes with less workers (better continuous resource utilization), making it the driver for labor productivity improvements for on-site implementation.

Fig. 6b shows that improvements on TC for the buffered case and on-site implementation were -2.75% and -10.59% respectively, compared to the base case. Cost differences in the buffered case and on-site implementations can be fundamentally explained by improvements in labor productivity. Particularly, the underestimation of labor productivity for the buffered case led to better cost performance in the on-site implementation. Nevertheless, there are improvements that can be attributed to the WIP Bf. On the other hand, the average COV of  $m_i$  is increased for the buffered case and on-site implementation up to 12.34% and 44.34% respectively, in relation to the base case. The main reason is the high increment of variability in process  $P_4$ . In contrast, the remaining processes tend to maintain their variability levels.  $P_4$  had the higher  $m_i$  due to the IWIP Bf size impact (8.80 units) induced by higher levels of production rates for type 1 units and lower levels of production rates for type 2 units. This range of production rates between type 1 and 2 units produces high levels of buffered and on-site COV for  $m_i$ .

#### 7.4. Discussion of SO testing and validation

Site personnel for both projects agreed with the project improvements after WIP Bf implementation. They found increasing efficiency of crews and reduction in production system variability. Also, they perceived that the SO approach was a reliable tool to design WIP Bf, requiring a minimum effort of implementation, control and measurement supported by scheduling construction process.

The SO testing and validation showed that the IWIP Bf size for Min TC is between the IWIP Bf sizes for Max ATm and Min TCT respectively. Table 4 shows that the average IWIP Bf sizes in Project A for Min TCT, Min TC and Max ATm are 2.3 units, 5.8 and 12.8 units respectively, where the location of the average IWIP Bf size for Min TC is between Min TCT and Max ATm. Furthermore, the average IWIP Bf size for Min TC has time and production rate responses located between the same ones for Min TCT and Max ATm. Table 7 shows a similar behavior for Project B, where average IWIP Bf sizes for Min TC (16.0 units) is between the average IWIP Bf sizes for Min TCT (5.0 units) and Max ATm (25.7 units), being the location for its time and production responses similar to Project A. Evidence showed that the type of project objectives for nomographs (Figs. 3d and 4) and location of IWIP Bf sizes for Min TC were appropriate, demonstrating that the assumptions for the MAM were correct.

#### 8. MAM application

To illustrate the development and application of the MAM, an example of project schedule is used. In doing so, a repetitive building project can be tested. Fig. 7 shows the scheduling network, planned production parameters and costs. The expected duration for each process is 2 days for each production unit and the MWIP Bf is 1 unit.

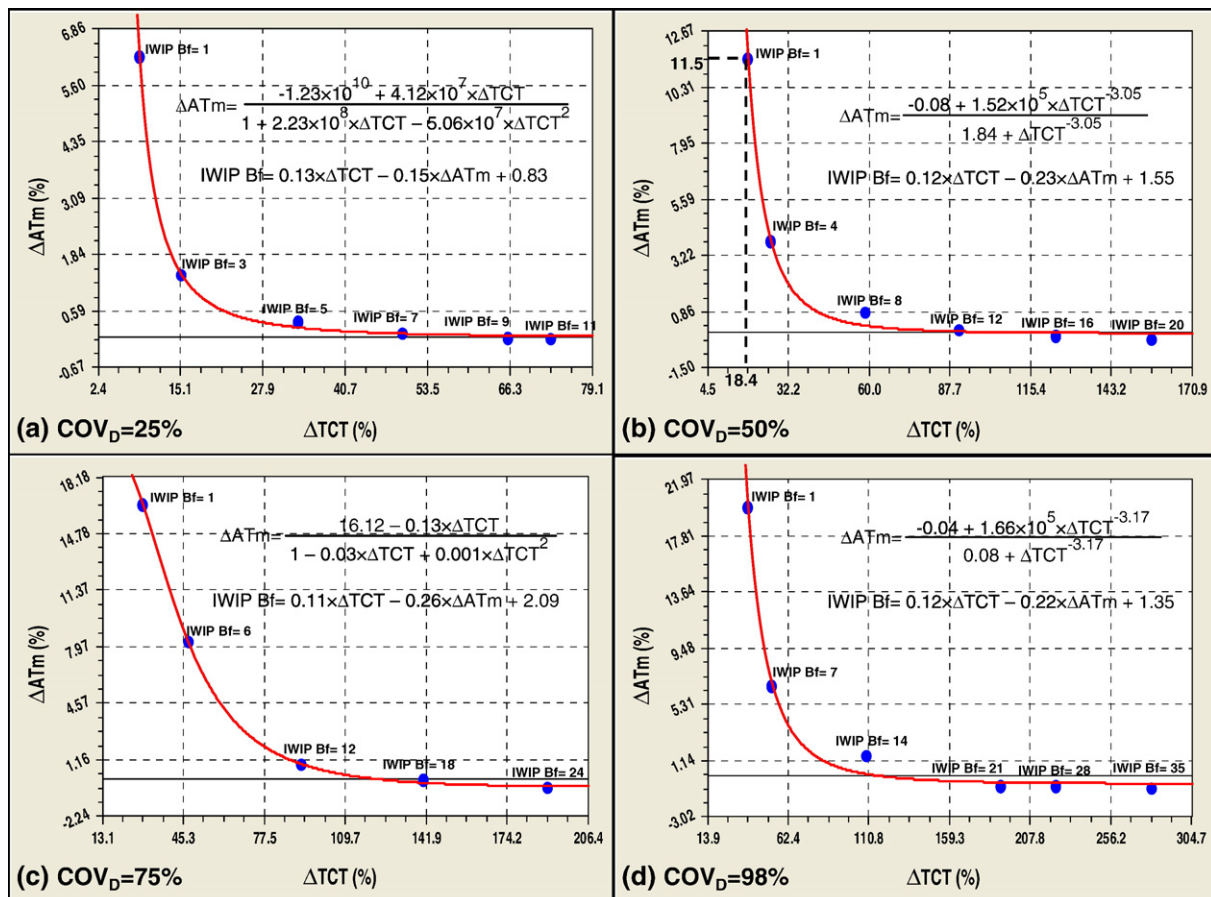


Fig. 8. WIP Bf design nomographs for 10 repetitive processes, with an expected duration by unit of 2 days: (a)  $COV_D=25\%$ , (b)  $COV_D=50\%$ , (c)  $COV_D=75\%$ , and (d)  $COV_D=98\%$ . IWIP Bf sizes are production units.



**Table 8**  
Sensitivity analysis to choose the optimum IWIP Bf size

WIP Bf (units)	Actual ATm (units/day)	Actual TCT (days)	Actual TC (\$)	$\Delta$ ATm	$\Delta$ TCT	$\Delta$ TC <sup>a</sup>
1	0.44	258	\$2,009,216	11.49%	18.36%	6.02%
4	0.48	275	\$1,947,261	3.81%	25.98%	2.75%
8	0.50	346	\$1,951,267	0.81%	58.71%	2.96%
12	0.50	416	\$1,972,811	0.06%	90.85%	4.10%
16	0.50	489	\$1,999,392	-0.16%	124.09%	5.50%
20	0.50	560	\$2,026,231	-0.32%	157.03%	6.92%

<sup>a</sup> Estimated as the difference between actual and planned TC.

To get nomographs for this project example, SO modeling for 10 processes was developed (equal to the number of processes over Critical Path for example shown in Fig. 7). Using Beta PDFs for the duration of these processes, nomographs for variability levels ( $COV_D$ ) of 25%, 50%, 75% and 98% were developed. Note that the Beta PDFs limited the level of variability to 98% due to the nature of the function. Expected duration by unit of 2 days for each process and  $COV_D$  value was estimated, evaluating the Beta PDF. In practice, nomographs for different number of processes and variability levels could be available. A decision maker will need to choose the most appropriate nomographs according to project characteristics and/or the decision maker's preferences.

Initially to develop the nomograph, IWIP Bf sizes for Min ATm and Min TCT (nomographs extreme points) for each  $COV_D$  were computed through SO processes in the same procedure used to obtain optimum IWIP Bf in the case studies. Afterwards, intermediate IWIP Bf sizes were determined for each  $COV_D$  by simple inspection (i.e. inspection of integer values for IWIP Bf sizes contained between extreme points). Final responses over  $\Delta$ ATm and  $\Delta$ TCT after 1000 simulation runs for each case were estimated, stating the Pareto Front lines in Fig. 8.

By using multivariate linear regression, two kinds of analytical expressions for each nomograph were developed (Fig. 8). The first one provides a relationship between  $\Delta$ ATm and  $\Delta$ TCT. A decision maker can then estimate both graphic and analytically the expected results for  $\Delta$ ATm and  $\Delta$ TCT. For example, Fig. 8b shows graphically that for  $\Delta$ TCT of 18.4%, there will be a  $\Delta$ ATm of 11.5% (i.e. there will be an increment of TCT and a reduction of average production rates for construction processes, respectively, compared with initial estimations). Additionally, a decision maker can compute with the analytical expression shown in Fig. 8b the value of  $\Delta$ ATm given  $\Delta$ TCT. The second expression provides the relationship of IWIP Bf size to both  $\Delta$ ATm and  $\Delta$ TCT. Similarly, the decision maker can estimate the IWIP Bf size both graphic and analytically. For example in Fig. 8b, the IWIP Bf is 1 unit. All analytic expressions in Fig. 8 had a coefficient of determination ( $R^2$ ) higher or equal to 0.98 and a  $P$ -value (at  $\alpha$  level=0.05) lesser or equal to 0.002, demonstrating their statistical significance.

For this example, the nomograph from Fig. 8b applies because it is estimated that processes could reach variability levels of 50% during the execution phase. The Actual ATm, TCT and TC can be computed using the IWIP Bf sizes from the nomograph and Eqs. (6), (7) and (11) respectively. The sensitivity analysis is shown in Table 8. In this case, it has been assumed that a decision maker could be interested in minimizing project cost. Therefore, the optimum IWIP Bf size is 4 units (see Table 8).

To analyze the impacts of the WIP Bf strategy design, a network schedule IWIP Bf of 4 units was constructed for a hypothetical repetitive building project. Two scenarios were taken into account: a

**Table 10**  
Project performance comparisons between WIP Bf scenarios at strategic level

Production scenario	WIP Bf size (units)	$\Delta$ ATm	$\Delta$ TCT	$\Delta$ TC
Real case without Bf	1	23.31%	26.40%	14.54%
Buffered case	4	17.40%	27.58%	10.99%

base case (with MWIP Bf size equal to 1 unit) and a buffered case. The processes duration PDFs are shown in Table 9. The IWIP Bf on the repetitive building project is shown in Table 10 with 1000 simulation runs for each scenario, which indicates improvements in cost using IWIP Bf size equal to 4 units. This approach decreases the impacts of variability on labor productivity and stimulates continuous resource utilization. It is interesting to comment that variability levels and process durations for stochastic production situations (without and with buffer) were higher than the nomograph of Fig. 8b. However, the conservative results of the nomograph do not negate the beneficial impacts of WIP Bf on the variable production scenarios.

In summary, nomographs allow project decision makers to design IWIP Bf sizes for a construction schedule following steps: 1) Select a variability level for project through  $COV_D$ , 2) Determine production responses on cost ( $\Delta$ TCT) and production rates ( $\Delta$ ATm) for each IWIP Bf size defined along Pareto Front lines, 3) Develop sensitivity analysis over actual cost, time and production rates using production responses of abacus (i.e.  $\Delta$ TCT and  $\Delta$ ATm) and Eqs.(6), (7) and (11), and (4) Select optimum IWIP Bf size according to decision makers' preferences on production objectives, i.e. to minimize cost or time and to maximize production rates.

## 9. Conclusions

This research has demonstrated the feasibility of designing WIP Bf strategies for construction projects to decrease the negative impacts of variability in production processes and to increase project performance. By doing so, a MAM to design WIP Bf based on SO modeling and Pareto Front concepts was proposed. A SO approach was tested and validated by means of two case studies, allowing for different levels of performance improvement after the application of WIP Bf strategies. However, the magnitude of the improvements depends on the context of the application (e.g., seasonality, execution complexity, types of processes, variability levels, modeling assumptions, etc.), the project decision makers and site personnel willingness to apply buffering strategies, and the level of supply chain control.

The MAM was developed as nomographs using only two production variables: time and production rates. This framework allowed for a simple and practical method of designing WIP Bf for scheduling repetitive building projects with independence of cost. The framework is supported by evidence from the SO case studies. This statement was demonstrated through cost improvements obtained in the project examples after application of the MAM. It was apparent that the use of MAM reduced the interdependencies between processes for a given level of variability. This paper provides the first application of the MAM approach to generalize the application of WIP Bf in construction through simple and practical means. It is hoped that this approach will facilitate the use of WIP Bf in the construction industry and contribute to reduce the gap between theory and practice in the body of knowledge for the buffer management. Because there is variability in construction, more rational use of buffers is necessary. In addition, further research is necessary in order to produce more nomographs to design WIP Bf for other production situations and contexts, stimulating its generalization and facilitating its industry adoption as a practical tool.

This paper also documents a two-level methodology, both strategic and tactical, to design WIP Bf. It is demonstrated in the scheduling process for repetitive building projects. The MAM approach can be applied at the strategic level, while the SO approach can be applied at the tactical level. This methodology can reduce the management cost

**Table 9**  
Duration PDFs for processes of project example

Processes	Average duration by unit (days)	$COV_D$ (%)	PDF type	PDF parameters
$P_1, \dots, P_{10}$	2.34	57.39	Beta	$\alpha=0.7$ $b=1.01$ $L=0.5$ $U=5.0$
$P_{11}, \dots, P_{18}$	2.15	57.74	Uniform	$\alpha=0$ $b=4.3$
$P_{19}, \dots, P_{23}$	2.20	56.36	Gamma	$\alpha=3.17$ $\beta=0.694$



and supervision effort of labor, due to the fact that labor permanency on site is decreased while its efficiency is increased. Alternatively, the increment of labor efficiency can be related to more profits for subcontractors given the reduction of labor permanency in projects. As a result, labor can be assigned to other projects. This methodology can also reduce on-site waste, decreasing waiting times and stimulating continuous resource utilization. The methodology can also contribute to reduction of rework by assuring the quality of WIP for downstream crews (stimulation of value-adding activities).

This paper is part of an ongoing research to generalize the design and management of WIP Bf in repetitive projects based on lean production principles. The next step in this research is to develop WIP Bf management process at the operational level (not only scheduling, but also planning and controlling). Currently, this is being approached through the development and investigation of decision-making models to forecast and control more rationally on-site production commitments in construction, including the management of WIP Bf designed at lower production levels. Future articles will address this topic.

## 10. Notation

The following symbols are used in this paper:

$\Delta AT_m$	— difference between expected and actual $AT_m$
$\Delta TC$	— difference between actual and expected TCT (budget)
$\Delta TCT$	— difference between actual and expected TCT (schedule)
$AT_m$	— average production rate for processes package
Bf	— buffer
COV	— coefficient of variation
$CT_i$	— cycle time
DC	— direct cost
DOC	— daily overhead cost for processes package
$E[\cdot]$	— response expectation of $\phi$ for optimization search
$EqDC_i$	— equipment daily cost
EA	— evolutionary algorithms
ES	— Evolutionary Strategies
FC	— fix cost
IC	— indirect cost
IT Bf	— initial time buffer
IWIP Bf	— initial work in place buffer
$LDC_i$	— labor daily cost
MAM	— multiobjective analytic model
Max $AT_m$	— maximize average total production rate
$m_i$	— average production rates
Min TC	— minimize total cost
Min TCT	— minimize total cycle time
MT Bf	— minimum time buffer
$MUC_i$	— material unit cost
MWIP Bf	— minimum work in place buffer
$n$	— number of processes
PDF	— probability density function
$P_i$	— repetitive and sequential process
$SD_i$	— standard deviation of $m_i$
SO	— Simulation–Optimization
TC	— total cost
TCT	— total cycle time
TP	— total production
VL	— processes variability
$W$	— feasible range of optimization search for $x_i$
WIP	— work in process
$\sigma_D$	— certain standard deviation for actual cumulative progress
$\sigma_{PR}$	— certain standard deviation for actual time
$\sigma_i$	— standard deviation of mutation process or mutation strength controlling the step-size.
$\sigma_i^2$	— variance of mutation process

$x_i$	— decision variables for $\phi$
$\lambda$	— offsprings population for a ES search
$\mu$	— parents population for a ES search
$\mu_D$	— expected duration by production unit
$\mu_{PR}$	— expected progress or production rate by day
$\rho$	— mixing number for a ES search
$\phi$	— general expression for objective function

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