Queue-Based Strategy to Achieve Maximum Stable rate in Multi-user Network

Linear Programming Class Math-5593 Fall 2019

Sajjad Nassirpour

John McFarlane

Department of Electrical Engineering

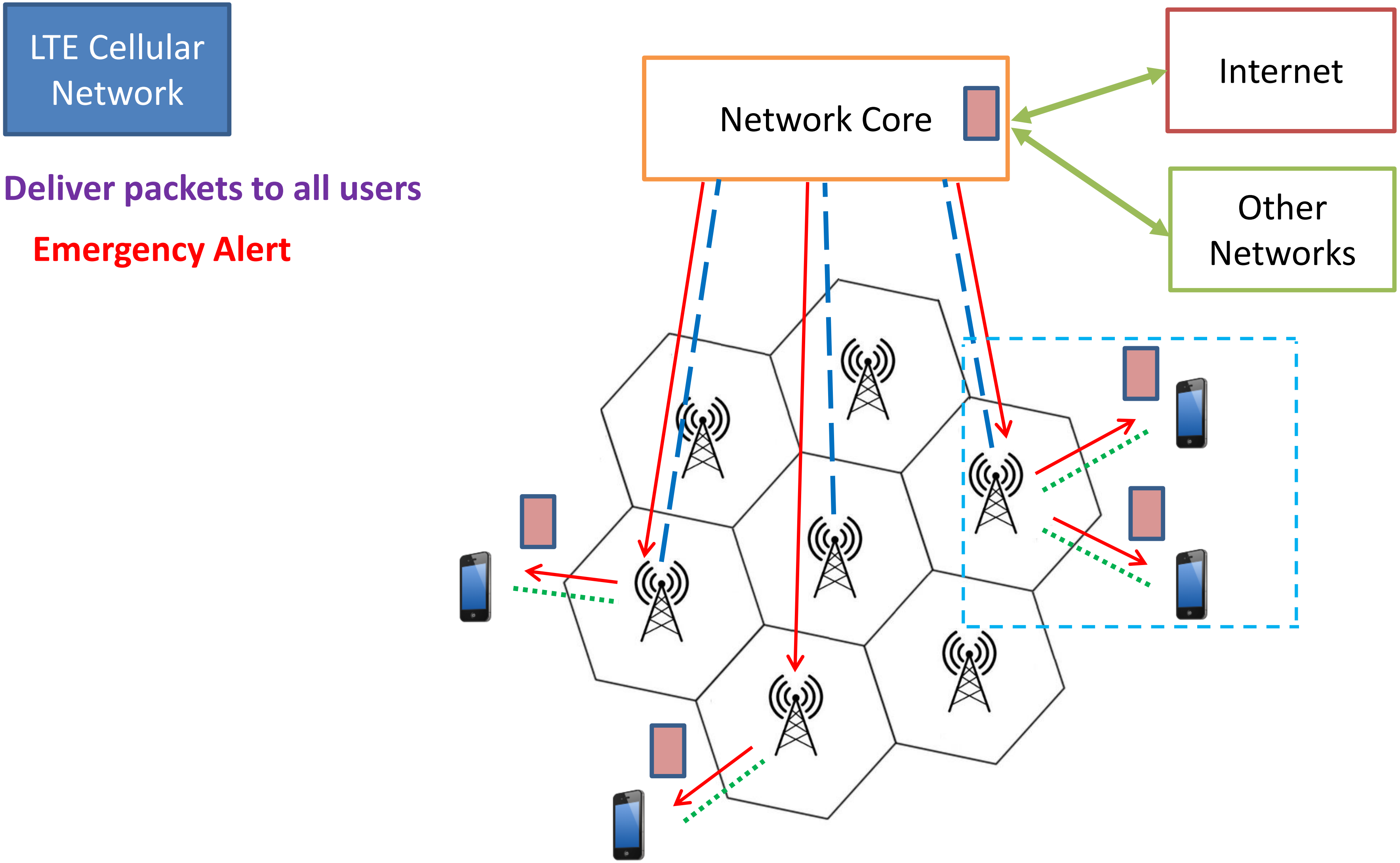
University of Colorado Denver

Email: sajjad.nassirpour@ucdenver.edu

Email: john.mcfarlane@ucdenver.edu

University of Colorado **Denver**

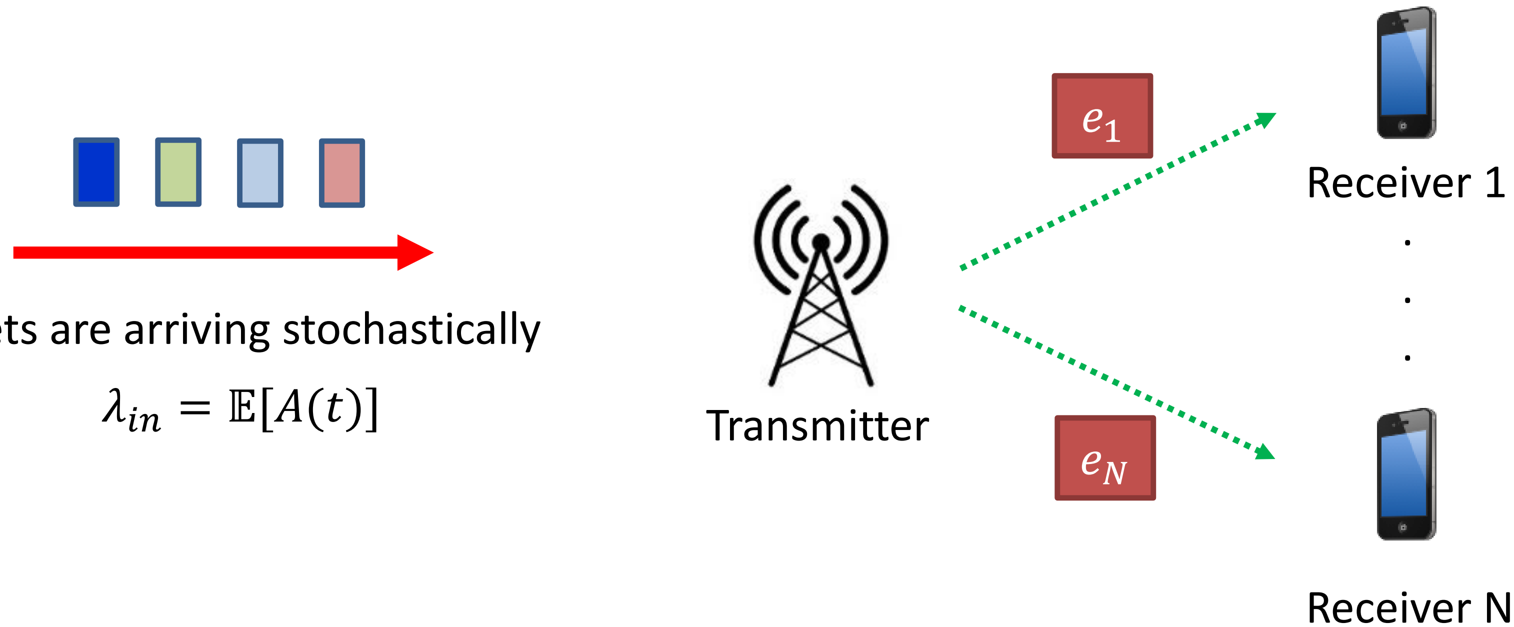
What is Multicast Transmission Model



Deliver packets to all users

- **Emergency Alert**

Multicast Transmission Model



$$\lambda_{in} = \mathbb{E}[A(t)]$$

$e_1 = 0.1$
\# of packets $= 1000$

→ User 1 receives 900 packets successfully

Our Goal
Deliver packets to all receivers without loss $\lambda_{out} = \lambda_{in} = \lambda$

Solution → Queue-based packet management

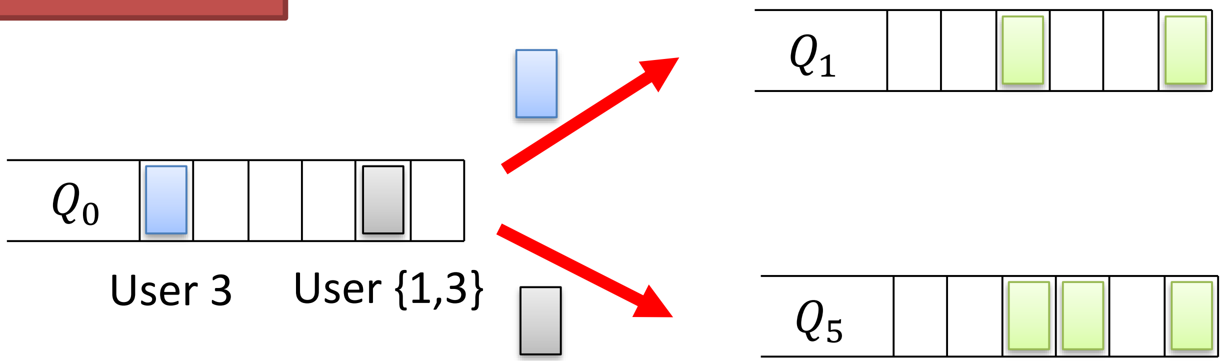
Details Of Queue-based strategy

$N = 3$

Queue	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Index set	{1,2,3}	{1,2}	{2,3}	{1,3}	{1}	{2}	{3}

Deliver packets by re-transmission

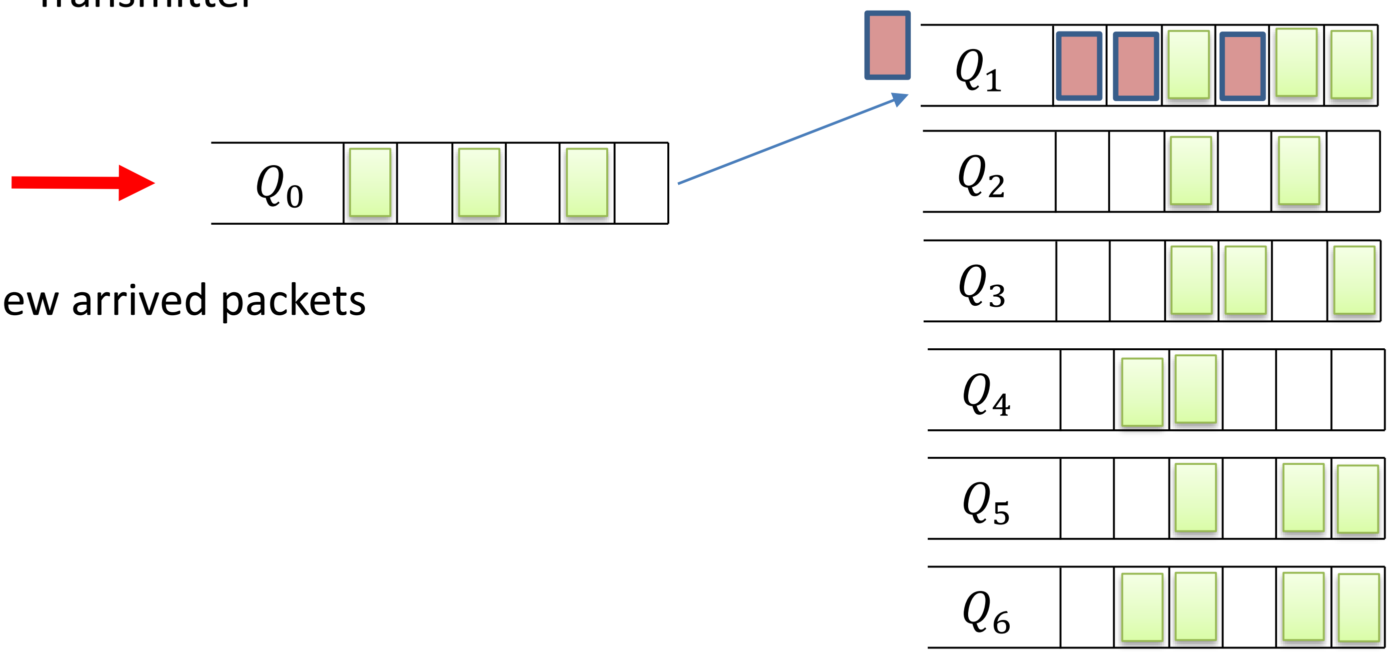
Packet Movement Rules



Problem of Queue-based strategy

Transmitter

Queue	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Index set	{1,2,3}	{1,2}	{2,3}	{1,3}	{1}	{2}	{3}



Problem → Backlogged packets → Need to stabilize the system

Queue Stability Analysis

$$U_i(t+1) = \max[U_i(t) - \mu_i(t), 0] + A_i(t)]$$

# of packets inside the Q_i at time t+1

# of packets arrived at Q_i

# of packets left the Q_i

Network Stability

A network is strongly stable if all individual queues of the network are strongly stable

Stability Constraints

Network Stability

A network is strongly stable if all individual queues of the network are strongly stable

Lemma

If a queue is strongly stable we should have

$\mathbb{E}[A_i(t)] \leq A_{max}$ or $\mathbb{E}[\mu_i(t) - A_i(t)] \leq D_{max}$ and then,

$$\lim_{t \to \infty} \frac{\mathbb{E}[U_i(t)]}{t} = 0$$

Solution $\rightarrow$ Incoming packets $\leq$ outcoming packets

Assign Network Coding to Queue-based strategy

Queue	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Index set	{1,2,3}	{1,2}	{2,3}	{1,3}	{1}	{2}	{3}

Random Coding	Probability Of Use	Responsible Queues
Network Coding # 0	P0	Q_0
Network Coding # 1	P1	Q_4, Q_5, Q_6
Network Coding # 2	P2	Q_1, Q_6
Network Coding # 3	P3	Q_2, Q_4
Network Coding # 4	P4	Q_3, Q_5

Assign Network Coding to Queue-based strategy

Number of NC schemes

Bell Number

Random Coding	Probability Of Use	Responsible Queues
Network Coding # 0	P0	Q_0
Network Coding # 1	P1	Q_4, Q_5, Q_6
Network Coding # 2	P2	Q_1, Q_6
Network Coding # 3	P3	Q_2, Q_4
Network Coding # 4	P4	Q_3, Q_5

$$B(n) = \sum_{k=0}^{n} S(n,k)$$

Stirling numbers of the second kind

$$S(n,k) = \frac{1}{k!} T(n,k)$$

$$T(n,k) = k^n - C(k,1)(k-1)^n + C(k,2)(k-2)^n - \cdots + (-1)^{(k-1)} C(k,k-1)1^n$$

$$B(3) = \sum_{k=0}^{3} S(3,k) = 0 + 1 + 3 + 1 = 5$$

single-line notation for combination

Linear Programming

maximize λ

Subject to: stability constraints in each queue

$$\sum_i P_i = 1$$

$$P_i \geq 0$$

Stability Constraints

Queue	Q_1
Index set	{1,2}

A packet leaving Q_1 has 4 possible outcomes:

Outcome 1: the packet is received by R_1 and R_2, so the packet is received by all three receivers and leaves the queuing system with transition probability $P_2(1-e_1)\,(1-e_2)$.

Outcome 2: the packet is received by R_1 but not R_2, so the packet moves to $\boldsymbol{Q_5}$ with transition probability $P_2(1-e_1)\,e_2$

Outcome 3: the packet is received by R_2 but not R_1, so the packet moves to $\boldsymbol{Q_4}$ with transition probability $P_2 e_1(1-e_2)$.

Outcome 4: The packet is received by none of the receivers and it stays in $\boldsymbol{Q_1}$ with transition probability $P_2 e_1 e_2$.

Random Coding	Probability Of Use	Responsible Queues
Network Coding # 2	**P2**	$\boldsymbol{Q_1, Q_6}$

$$Q_1: P_0 e_1 e_2(1-e_3) \leq P_2[(1-e_1)\,(1-e_2) + (1-e_1)\,e_2 + e_1(1-e_2)]$$

Linear Programming

maximize λ

Subject to:

$$Q_0{:}\ \ \lambda \le P_0[e_1\, e_2(1-e_3)+(1-e_3)\,(1-e_2)e_1 + (1-e_1)\,(1-e_2)\,(1-e_3)+ (1-e_3)\,(1-e_1)e_2+(1-e_1)\,e_2e_3+(1-e_1)\,(1-e_2)e_3+e_3\,(1-e_2)e_1]$$

$$Q_1{:}\ \ P_0e_1e_2(1-e_3)\le P_2[(1-e_1)\,(1-e_2)+(1-e_1)\,e_2+e_1(1-e_2)]$$

$$Q_2{:}\ \ P_0e_2e_3(1-e_1)\le P_3[(1-e_3)\,(1-e_2)+(1-e_3)\,e_2+e_3(1-e_2)]$$

$$Q_3{:}\ \ P_0e_1e_3(1-e_2)\le P_4[(1-e_3)\,(1-e_1)+e_3\,e_1+e_3(1-e_1)]$$

$$Q_4{:}\ \ P_2e_1(1-e_2)+P_0e_1(1-e_2)\,(1-e_3)+P_4e_1(1-e_3)\le (P_1+P_3)\,(1-e_1)$$

$$Q_5{:}\ \ P_2e_2(1-e_1)+P_0e_2(1-e_1)\,(1-e_3)+P_3e_2(1-e_3)\le (P_1+P_4)\,(1-e_2)$$

$$Q_6{:}\ \ P_3e_3(1-e_2)+P_0e_3(1-e_2)\,(1-e_1)+P_4e_3(1-e_1)\le (P_1+P_2)\,(1-e_3)$$

$$\sum_i P_i = 1 \qquad\qquad P_i \ge 0$$

Simulation Results

Random Coding	Probability Of Use	Responsible Queues
Network Coding # 0	P0	Q_0
Network Coding # 1	P1	Q_4, Q_5, Q_6
Network Coding # 2	P2	Q_1, Q_6
Network Coding # 3	P3	Q_2, Q_4
Network Coding # 4	P4	Q_3, Q_5

Queue	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Index set	{1,2,3}	{1,2}	{2,3}	{1,3}	{1}	{2}	{3}

e_1	e_2	e_3	λ_{max}	P_0	P_1	P_2	P_3	P_4
0.1	0.1	0.1	0.8993	0.9002	0.0745	0.0082	0.0082	0.0089
0.2	0.2	0.2	0.79633	0.8028	0.113	0.0268	0.0268	0.0306
0.1	0.1	0.2	0.79906	0.8007	0.1625	0.0064	0.0148	0.0156
0.1	0.2	0.3	0.699274	0.7035	0.2279	0.0100	0.0404	0.0182

Max error probability

```
ampl: reset;model Project.mod; data Project.dat;solve; display
P0,P1,P2,P3,P4,lambda;
MINOS 5.51: optimal solution found.
3 iterations, objective 0.8992877634
P0 = 0.900188
P1 = 0.074542
P2 = 0.00818353
P3 = 0.00818353
P4 = 0.00890296
lambda = 0.899288
```

Backlog description for $\lambda > \lambda_{max}$

$e_1 = 0.1 \quad e_2 = 0.2 \quad e_3 = 0.3$

Constraint	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Dual Price	1	0	0.998963	1.04193	0	0	0.998963

Backlog

Queue	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Index set	{1,2,3}	{1,2}	{2,3}	{1,3}	{1}	{2}	{3}

Complexity for N>3

Number of NC schemes

$$B(n) = \sum_{k=0}^{n} S(n,k)$$

For a multicast network with N users, the number of network coding schemes is $S_N = B(N)$

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1(2^N-1)} \\ \vdots & \ddots & \vdots \\ x_{S_N 1} & \cdots & x_{S_N(2^N-1)} \end{bmatrix}_{S_N \times (2^N - 1)} \qquad x_{ij} = \{0,1\}$$

$$O(S_N)$$

Complexity for N>3

Queues' inter-connection

- $N = 3$
- Number of queue: $2^3 - 1 = 7$

- $N = \mathbf{10}$
- Number of queue: $2^{10} - 1 = 1023$

$O(exponential)$

Thanks for your attention