Queue-Based Strategy to Achieve Maximum Stable rate in Multi-user Network

Linear Programming Class Math-5593 Fall 2019

Sajjad Nassirpour
John McFarlane
Department of Electrical Engineering
University of Colorado Denver
Email: sajjad.nassirpour@ucdenver.edu
Email: john.mcfarlane@ucdenver.edu

University of Colorado **Denver**

Presentation Outline

- **Motivation**
  - Multicast Transmission Model
- **Queue-based strategy**
- **System Stabilizing**
  - Problem of Queue-based strategy
  - Queue Stability Analysis

Presentation Outline

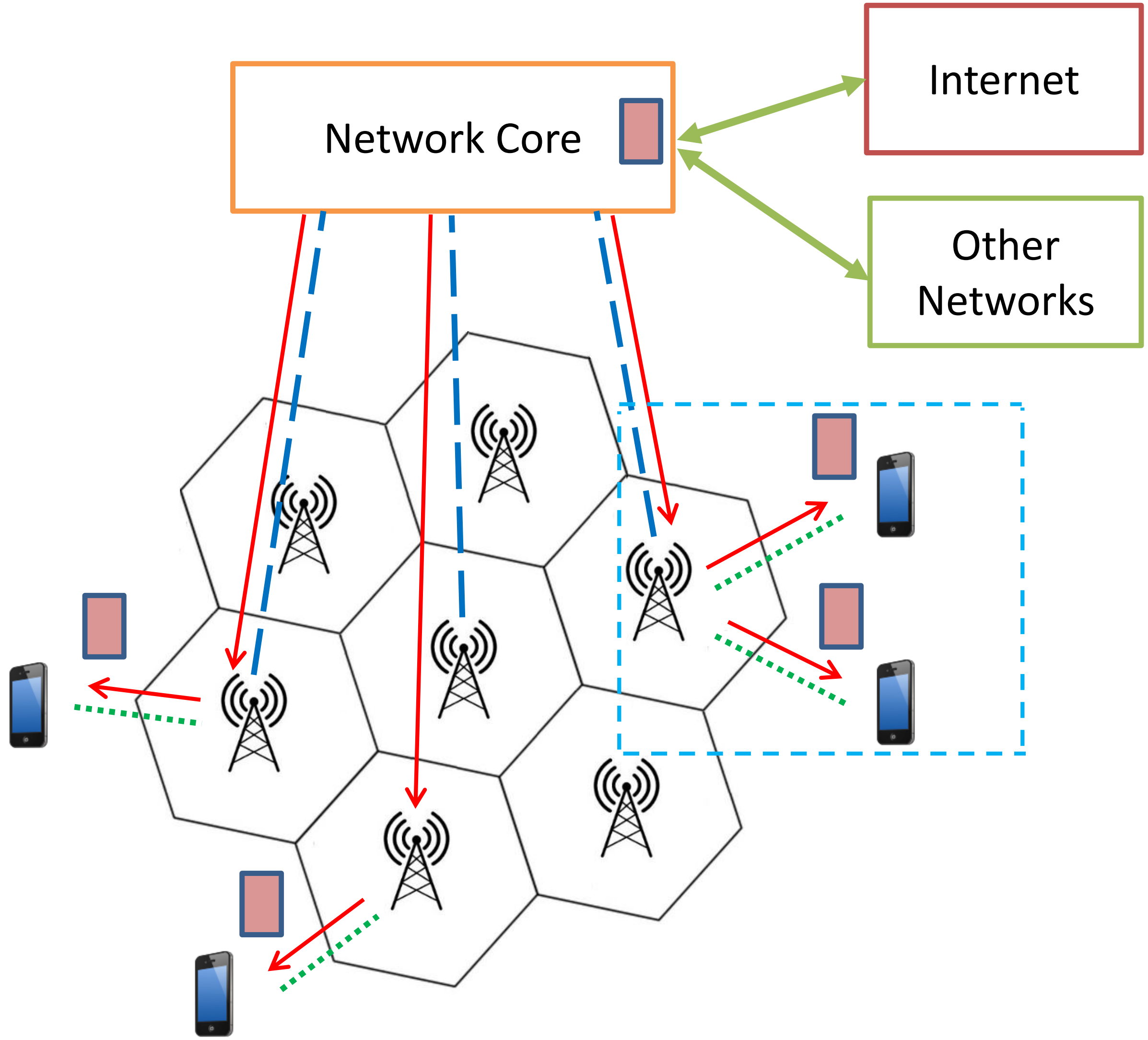
- **Linear Programming**
  - Stability constraints
  - Simulation Results
- **Backlog description**
- **Complexity for N>3**
  - Number of Network Coding
  - Inter connection between queues

What is Multicast Transmission Model

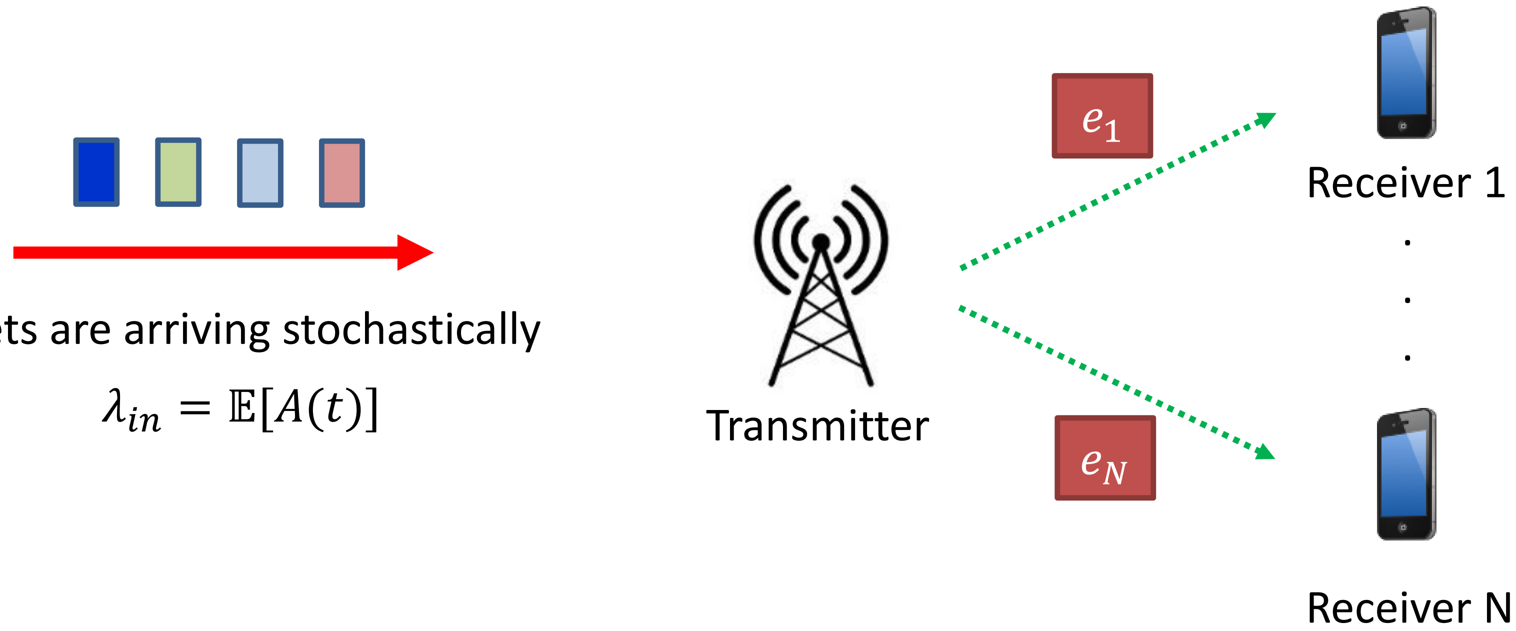
LTE Cellular Network

Deliver packets to all users

- **Emergency Alert**



Multicast Transmission Model



$e_1 = 0.1$
# of packets $= 1000$

User 1 receives 900 packets successfully

Our Goal
Deliver packets to all receivers without loss $\lambda_{out} = \lambda_{in} = \lambda$

Solution

Queue-based packet management

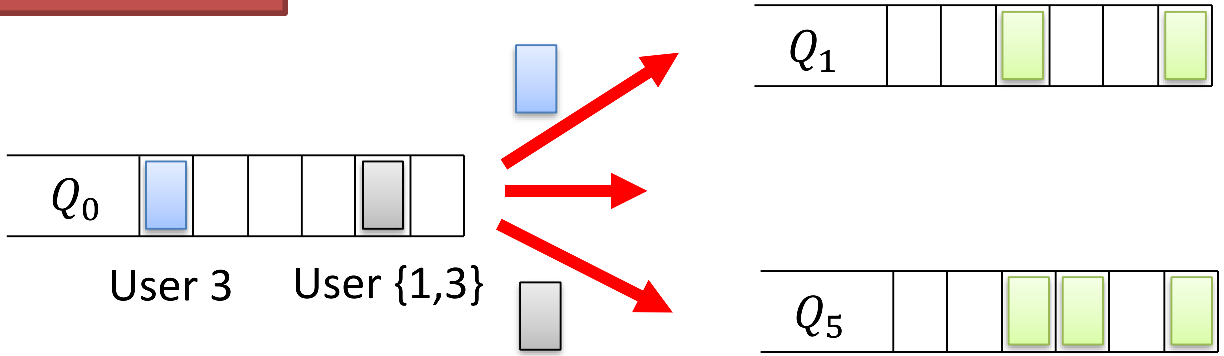
Details Of Queue-based strategy

$N = 3$

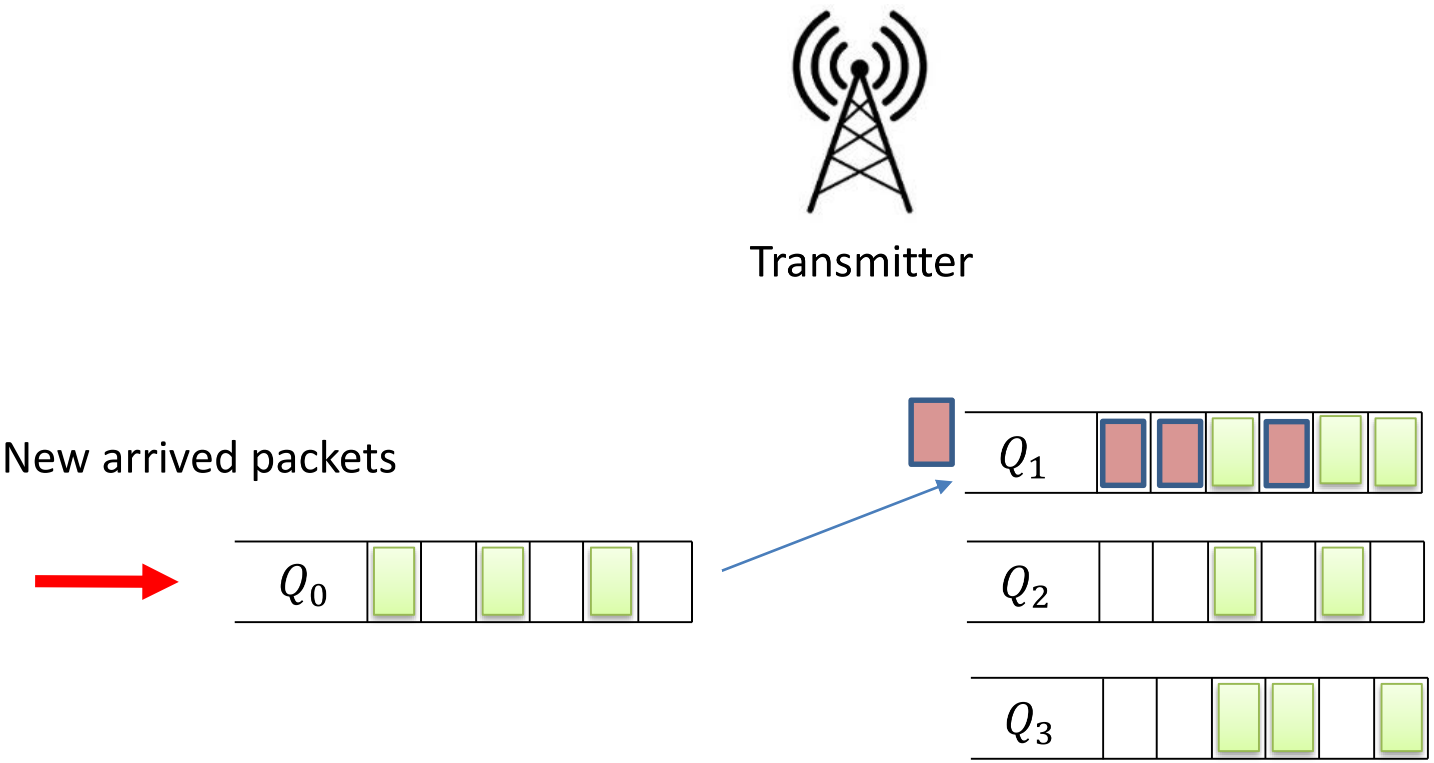
Queue	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Index set	{1,2,3}	{1,2}	{2,3}	{1,3}	{1}	{2}	{3}

Deliver packets by re-transmission

Packet Movement Rules



Problem of Queue-based strategy



Problem → Backlogged packets → Need to stabilize the system

Queue Stability Analysis

$$U_i(t+1) = \max[U_i(t) - \mu_i(t), 0] + A_i(t)]$$

# of packets inside the Q_i at time t+1

# of packets arrived at Q_i

# of packets left the Q_i

Network Stability
A network is strongly stable if all individual queues of the network are strongly stable.

Lemma
If a queue is strongly stable we should have

$\mathbb{E}[A_i(t)] \leq A_{max}$ or $\mathbb{E}[\mu_i(t) - A_i(t)] \leq D_{max}$ and then,

$$\lim_{t \to \infty} \frac{\mathbb{E}[U_i(t)]}{t} = 0$$

Stability Constraints

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Solution → Incoming packets $\leq$ outcoming packets

Assign Network Coding to Queue-based strategy

Queue	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Index set	{1,2,3}	{1,2}	{2,3}	{1,3}	{1}	{2}	{3}

Random Coding	Probability Of Use	Responsible Queues
Network Coding # 0	P0	Q_0
Network Coding # 1	P1	Q_4, Q_5, Q_6
Network Coding # 2	P2	Q_1, Q_6
Network Coding # 3	P3	Q_2, Q_4
Network Coding # 4	P4	Q_3, Q_5

Linear Programming

maximize λ

Subject to: stability constraints in each queue

$$\sum_i P_i = 1$$

$$P_i \geq 0$$

Stability Constraints

Queue	Q_1
Index set	{1,2}

A packet leaving Q_1 has 4 possible outcomes:

Outcome 1: the packet is received by R_1 and R_2, so the packet is received by all three receivers and leaves the queuing system with transition probability $P_2(1-e_1)\,(1-e_2)$.

Outcome 2: the packet is received by R_1 but not R_2, so the packet moves to Q_5 with transition probability $P_2(1-e_1)\,e_2$

Outcome 3: the packet is received by R_2 but not R_1, so the packet moves to Q_4 with transition probability $P_2 e_1(1-e_2)$.

Outcome 4: The packet is received by none of the receivers and it stays in Q_1 with transition probability $P_2 e_1 e_2$.

$$Q_1 : P_0 e_1 e_2 (1-e_3) \leq P_2[(1-e_1)\,(1-e_2) + (1-e_1)\,e_2 + e_1(1-e_2)]$$

Linear Programming

maximize λ

Subject to:

$$Q_0\text{:}\ \ \lambda \leq P_0[e_1\, e_2(1-e_3) + (1-e_3)\,(1-e_2)e_1 + (1-e_1)\,(1-e_2)\,(1-e_3) + (1-e_3)\,(1-e_1)e_2 + (1-e_1)\,e_2 e_3 + (1-e_1)\,(1-e_2)e_3 + e_3\,(1-e_2)e_1]$$

$$Q_1\text{:}\ \ P_0 e_1 e_2(1-e_3) \leq P_2[(1-e_1)\,(1-e_2) + (1-e_1)\,e_2 + e_1(1-e_2)]$$

$$Q_2\text{:}\ \ P_0 e_2 e_3(1-e_1) \leq P_3[(1-e_3)\,(1-e_2) + (1-e_3)\,e_2 + e_3(1-e_2)]$$

$$Q_3\text{:}\ \ P_0 e_1 e_3(1-e_2) \leq P_4[(1-e_3)\,(1-e_1) + e_3\,e_1 + e_3(1-e_1)]$$

$$Q_4\text{:}\ \ P_2 e_1(1-e_2) + P_0 e_1(1-e_2)\,(1-e_3) + P_4 e_1(1-e_3) \leq (P_1+P_3)\,(1-e_1)$$

$$Q_5\text{:}\ \ P_2 e_2(1-e_1) + P_0 e_2(1-e_1)\,(1-e_3) + P_3 e_2(1-e_3) \leq (P_1+P_4)\,(1-e_2)$$

$$Q_6\text{:}\ \ P_3 e_3(1-e_2) + P_0 e_3(1-e_2)\,(1-e_1) + P_4 e_3(1-e_1) \leq (P_1+P_2)\,(1-e_3)$$

$$\sum_i P_i = 1 \qquad\qquad P_i \geq 0$$

Simulation Results

e_1	e_2	e_3	λ_{max}	P_0	P_1	P_2	P_3	P_4
0.1	0.1	0.1	0.900118	0.9002	0.0745	0.0082	0.0082	0.0089
0.2	0.2	0.2	0.79633	0.8028	0.113	0.0268	0.0268	0.0306
0.1	0.1	0.2	0.79906	0.8007	0.1625	0.0064	0.0148	0.0156
0.1	0.2	0.3	0.699274	0.7035	0.2279	0.0100	0.0404	0.0182

Max error probability

Backlog description for $\lambda > \lambda_{max}$

$e_1 = 0.1 \quad e_2 = 0.2 \quad e_3 = 0.3$

Constraint	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Dual Price	1	0	0.998963	1.04193	0	0	0.998963

Backlog

Complexity for N>3

Number of NC schemes

Bell Number

$$B(n) = \sum_{k=0}^{n} S(n,k)$$

$$S(n,k) = \frac{1}{k!} T(n,k)$$

$$T(n,k) = k^n - C(k,1)(k-1)^n + C(k,2)(k-2)^n - \cdots + (-1)^{(k-1)} C(k,k-1)1^n$$

$$B(3) = \sum_{k=0}^{3} S(3,k) = 0 + 1 + 3 + 1 = 5$$

Complexity for N>3

Number of NC schemes

$$B(n) = \sum_{k=0}^{n} S(n,k)$$

For a multicast network with N users, the number of network coding schemes is $S_N = B(N)$

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1(2^N-1)} \\ \vdots & \ddots & \vdots \\ x_{S_N 1} & \cdots & x_{S_N(2^N-1)} \end{bmatrix}_{S_N \times (2^N-1)}$$

$$x_{ij} = \{0,1\}$$

$$O(S_N)$$

Complexity for N>3

Queues' inter-connection

- $N = 3$
- Number of queue: $2^3 - 1 = 7$

- $N = \mathbf{10}$
- Number of queue: $2^{10} - 1 = 1023$

$O(exponential)$

Thanks for your attention