به نام خدا

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تمرین شماره 4

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فهرست

چکیده

در این سری تمرین مباحثی مانند LDA ، PCA و FDA را بررسی کردیم و در سوال های ابتدایی به اثبات چندی از ویژگی های این مباحث پرداختیم. در مابقی سوالات به پیاده سازی این مباحث پرداختیم و مشاهده کردیم که نتایج همان‌طور که در تئوری اثبات کردیم ، منطبق بودند.

در سوال ۶ با استفاده از کتابخانه sklearn و با استفاده ار PCA به فشرده سازی تعدادی از عکس پرداختیم و مشاهده کردیم که کاهش بعد با استفاده از PCA برای ابعادی که مقدار ویژه نزدیک به ۰ دارند در خطای طبقه بندی تاثیری ندارند.

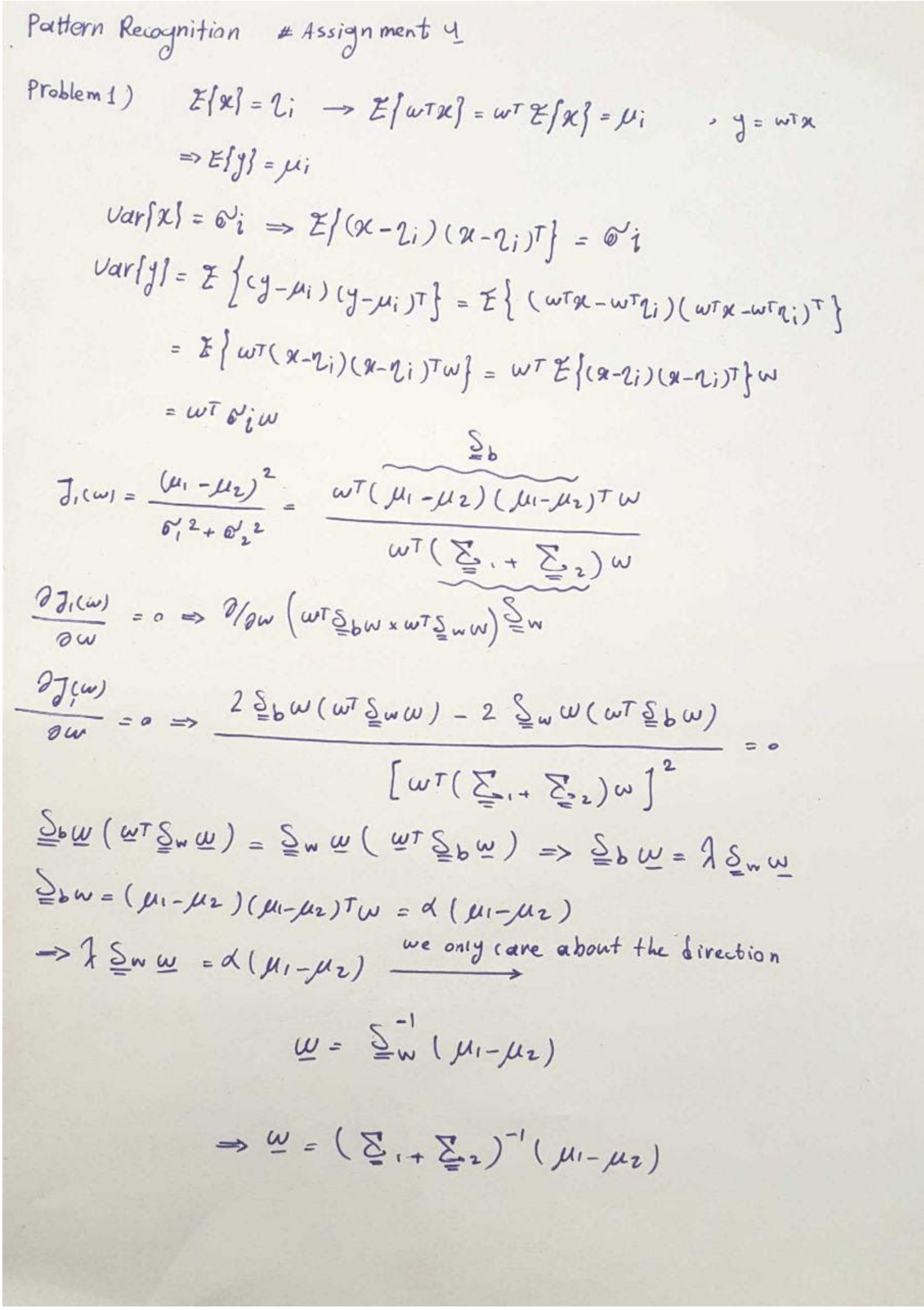
در سوال ۷ با استفاده از PCA که خودمان پیاده سازی کرده بودیم وابستگی های خطی در مجموعه داده fashion mnist را حذف کردیم و داده ها را نیز سفید کردیم.

در سوال ۸ الگوریتم forward selection را پیاده سازی کردیم و نحوه انتخاب ویژگی ها را با استفاده از acc روی train بدست آوردیم.

در سوال ۹ PCA را از پایه پیاده سازی کردیم و با استفاده از آن بهبود طبقه بندی را پس از اعمال PCA و کاهش بعد مشاهده نمودیم. در حقیقت مشاهده کردیم در صورتی که این کاهش بعد انجام شود generalization بهبود بخشیده می‌شود.

در سوال ۱۰ LDA را پیاده سازی کردیم و مشاهده کردیم که اگر LDA را با استفاده از eigenvector eigenvalue decomposition حل کنیم ، با توجه به اینکه در فرایند dimension reduction برچسب های داده ها را می‌دانستیم قابلیت generalization افزایش پیدا کرد.

سوال 1



Pattern Recognition # Assignment 4

Problem 1) $E\{x\} = \eta_i \rightarrow E\{w^T x\} = w^T E\{x\} = \mu_i$, $y = w^T x$

$\Rightarrow E\{y\} = \mu_i$

$Var\{x\} = \sigma_i \Rightarrow E\{(x-\eta_i)(x-\eta_i)^T\} = \sigma_i$

$Var\{y\} = E\{(y-\mu_i)(y-\mu_i)^T\} = E\{(w^Tx - w^T\eta_i)(w^Tx - w^T\eta_i)^T\}$

$= E\{w^T(x-\eta_i)(x-\eta_i)^T w\} = w^T E\{(x-\eta_i)(x-\eta_i)^T\} w$

$= w^T \sigma_i w$

$$J_1(w) = \frac{(\mu_1-\mu_2)^2}{\sigma_1^2+\sigma_2^2} = \frac{\overbrace{w^T(\mu_1-\mu_2)(\mu_1-\mu_2)^T w}^{\underline{\underline{S}}_b}}{w^T \underbrace{(\underline{\underline{\Sigma}}_1 + \underline{\underline{\Sigma}}_2)}_{} w}$$

$$\frac{\partial J_1(w)}{\partial w} = 0 \Rightarrow \partial/\partial w \left(w^T \underline{\underline{S}}_b w \times w^T \underline{\underline{S}}_w w\right) \underline{\underline{S}}_w$$

$$\frac{\partial J_1(w)}{\partial w} = 0 \Rightarrow \frac{2\underline{\underline{S}}_b w (w^T \underline{\underline{S}}_w w) - 2\underline{\underline{S}}_w w (w^T \underline{\underline{S}}_b w)}{\left[w^T(\underline{\underline{\Sigma}}_1 + \underline{\underline{\Sigma}}_2) w\right]^2} = 0$$

$\underline{\underline{S}}_b \underline{w} (\underline{w}^T \underline{\underline{S}}_w \underline{w}) = \underline{\underline{S}}_w \underline{w} (\underline{w}^T \underline{\underline{S}}_b \underline{w}) \Rightarrow \underline{\underline{S}}_b \underline{w} = \lambda \underline{\underline{S}}_w \underline{w}$

$\underline{\underline{S}}_b w = (\mu_1-\mu_2)(\mu_1-\mu_2)^T w = \alpha(\mu_1-\mu_2)$

$\Rightarrow \lambda \underline{\underline{S}}_w \underline{w} = \alpha(\mu_1-\mu_2) \xrightarrow{\text{we only care about the direction}}$

$$\underline{w} = \underline{\underline{S}}_w^{-1}(\mu_1-\mu_2)$$

$$\Rightarrow \underline{w} = (\underline{\underline{\Sigma}}_1 + \underline{\underline{\Sigma}}_2)^{-1}(\mu_1-\mu_2)$$

سوال 2

Problem 2)

$$J = \frac{1}{n_1 n_2} \sum_{y_i \in Y_1} \sum_{y_j \in Y_2} (y_i - y_j)^2 = \frac{1}{n_1 n_2} \sum_i \sum_j (y_i^2 + y_j^2 - 2 y_i y_j)$$

$$= \frac{1}{n_1 n_2} \left[\sum_i \sum_j y_i^2 + \sum_i \sum_j y_j^2 - 2 \sum_i \sum_j y_i y_j \right]$$

$$= \frac{1}{n_1 n_2} \left[n_2 \sum_i y_i^2 + n_1 \sum_j y_j^2 - 2 \sum_i y_i \sum_j y_j \right]$$

$$= \frac{1}{n_1 n_2} \left[n_2 \sum_i y_i^2 + n_1 \sum_j y_j^2 - 2 n_1 m_1 n_2 m_2 \right]$$

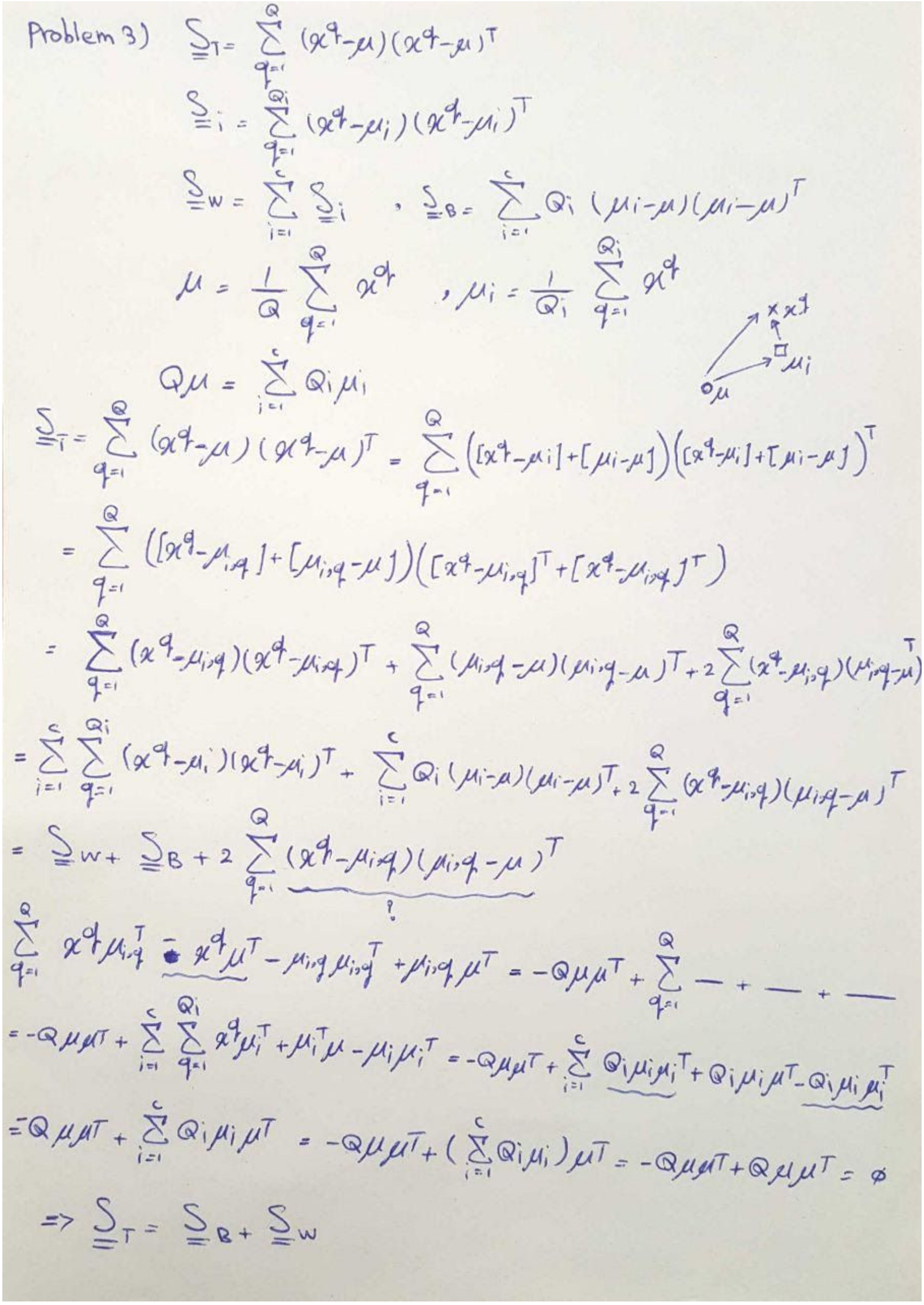
$$= -2 m_1 m_2 + \frac{1}{n_1 n_2} \left[n_2 \sum_i y_i^2 + n_1 \sum_j y_j^2 \right]$$

$$= m_1^2 - 2 m_1 m_2 + m_2^2 + \frac{1}{n_1} \sum_i y_i^2 + \frac{1}{n_2} \sum_j y_j^2 - m_1^2 - m_2^2$$

$$= (m_1 - m_2)^2 + \underbrace{\frac{1}{n_1} \sum_i y_i^2 - m_1^2}_{\frac{1}{n_1} S_1} + \underbrace{\frac{1}{n_2} \sum_j y_j^2 - m_2^2}_{\frac{1}{n_2} S_2}$$

$$= (m_1 - m_2)^2 + \frac{1}{n_1} S_1 + \frac{1}{n_2} S_2$$

سوال 3



Problem 3)

$$\underline{\underline{S}}_T = \sum_{q=1}^{Q} (x^q - \mu)(x^q - \mu)^T$$

$$\underline{\underline{S}}_i = \sum_{q=1}^{Q_i} (x^q - \mu_i)(x^q - \mu_i)^T$$

$$\underline{\underline{S}}_W = \sum_{i=1}^{c} \underline{\underline{S}}_i \quad , \quad \underline{\underline{S}}_B = \sum_{i=1}^{c} Q_i (\mu_i - \mu)(\mu_i - \mu)^T$$

$$\mu = \frac{1}{Q} \sum_{q=1}^{Q} x^q \quad , \mu_i = \frac{1}{Q_i} \sum_{q=1}^{Q_i} x^q$$

$$Q\mu = \sum_{i=1}^{c} Q_i \mu_i$$

$$\underline{\underline{S}}_T = \sum_{q=1}^{Q} (x^q - \mu)(x^q - \mu)^T = \sum_{q=1}^{Q} \Big([x^q - \mu_i] + [\mu_i - \mu] \Big) \Big([x^q - \mu_i] + [\mu_i - \mu] \Big)^T$$

$$= \sum_{q=1}^{Q} \Big([x^q - \mu_{i,q}] + [\mu_{i,q} - \mu] \Big) \Big([x^q - \mu_{i,q}]^T + [x^q - \mu_{i,q}]^T \Big)$$

$$= \sum_{q=1}^{Q} (x^q - \mu_{i,q})(x^q - \mu_{i,q})^T + \sum_{q=1}^{Q} (\mu_{i,q} - \mu)(\mu_{i,q} - \mu)^T + 2 \sum_{q=1}^{Q} (x^q - \mu_{i,q})(\mu_{i,q} - \mu)^T$$

$$= \sum_{i=1}^{c} \sum_{q=1}^{Q_i} (x^q - \mu_i)(x^q - \mu_i)^T + \sum_{i=1}^{c} Q_i (\mu_i - \mu)(\mu_i - \mu)^T + 2 \sum_{q=1}^{Q} (x^q - \mu_{i,q})(\mu_{i,q} - \mu)^T$$

$$= \underline{\underline{S}}_W + \underline{\underline{S}}_B + 2 \sum_{q=1}^{Q} \underbrace{(x^q - \mu_{i,q})(\mu_{i,q} - \mu)^T}_{?}$$

$$\sum_{q=1}^{Q} x^q \mu_{i,q}^T = \underline{x^q \mu^T} - \mu_{i,q} \mu_{i,q}^T + \mu_{i,q} \mu^T = -Q\mu\mu^T + \sum_{q=1}^{Q} \_\_ + \_\_ + \_\_$$

$$= -Q\mu\mu^T + \sum_{i=1}^{c} \sum_{q=1}^{Q_i} x^q \mu_i^T + \mu_i^T \mu - \mu_i \mu_i^T = -Q\mu\mu^T + \sum_{i=1}^{c} \underline{Q_i \mu_i \mu_i^T} + Q_i \mu_i \mu^T - \underline{Q_i \mu_i \mu_i^T}$$

$$= Q\mu\mu^T + \sum_{i=1}^{c} Q_i \mu_i \mu^T = -Q\mu\mu^T + \Big(\sum_{i=1}^{c} Q_i \mu_i \Big) \mu^T = -Q\mu\mu^T + Q\mu\mu^T = \phi$$

$$\Rightarrow \underline{\underline{S}}_T = \underline{\underline{S}}_B + \underline{\underline{S}}_W$$

سوال 4

Problem 4) assumptions: $\underline{x} = \sum_{i=0}^{N-1} y_i e_i \rightarrow e_i \in E$ is orthogonal basis

$$y_i = \underline{e_i}^T \underline{x} \qquad \hat{\underline{x}} = \sum_{i=0}^{m-1} y_i e_i + \sum_{i=m}^{N-1} c_i e_i$$

$$\underline{x} - \hat{\underline{x}} = \sum_{i=m}^{N-1} y_i e_i - c_i e_i = \sum_{i=m}^{N-1} (y_i - c_i) e_i$$

$$\|\underline{x} - \hat{\underline{x}}\|^2 = (\underline{x} - \hat{\underline{x}})^T (\underline{x} - \hat{\underline{x}})$$

$$= \left[\sum_{i=m}^{N-1} (e_i^T x e_i - c_i e_i)^T\right]\left[\sum_{i=m}^{N-1} (e_i^T x e_i - c_i e_i)\right] \xrightarrow{e_i \perp e_j}$$

$$= \sum_{i=m}^{N-1} \left[(y_i - c_i) e_i^T\right]\left[(y_i - c_i) e_i\right] = \sum_{i=m}^{N-1} (y_i - c_i)^2$$

$$= \sum_{i=m}^{N-1} y_i^2 + c_i^2 - 2 y_i c_i = \sum_{i=m}^{N-1} e_i^T x x^T e_i + c_i^2 - 2 e_i^T x c_i$$

$$\Rightarrow E\{\|x - \hat{x}\|^2\} = \sum_{i=m}^{N-1} e_i^T \underbrace{E\{x x^T\}}_{R_x} e_i + c_i - 2 e_i^T E\{x\} c_i$$

Minimize $E\{\|\varepsilon\|^2\}$ s.t $e_i^T e_i = 1$ $i = 0, \dots, N-1$

*using Lagrange mutipiyer method

$$L = \sum_{i=m}^{N-1} \left(e_i^T R_x e_i + c_i^2 - 2 c_i c_i^T E\{x\}\right) - \sum_{i=0}^{N-1} \lambda_i (e_i^T e_i - 1)$$

$$\frac{\partial L}{\partial e_i} = 2 E\{x x^T\} e_i - 2 c_i E\{x\} - 2 \lambda_i e_i = 0$$

ⓑ

$$\Rightarrow R_x e_i = \lambda_i e_i + c_i E\{\underline{x}\} \Rightarrow \boxed{e_i \text{ is generalized eigen vector}}$$

$$\frac{\partial L}{\partial c_i} = 2c_i - 2e_i^T E\{x\} = 0 \Rightarrow c_i = e_i^T E\{x\}$$

$$\Rightarrow \boxed{c_i = E\{e_i^T x\} = E\{y_i\}} \quad \text{(الف)}$$

$$E\{\|\varepsilon\|^2\} = \sum_{i=m}^{N-1} \left(e_i^T R_x e_i + c_i^2 - 2c_i e_i^T E\{x\} \right)$$

$$= \sum_{i=m}^{N-1} e_i^T \lambda_i e_i + e_i^T c_i E\{x\} + c_i^2 - 2c_i e_i^T E\{x\}$$

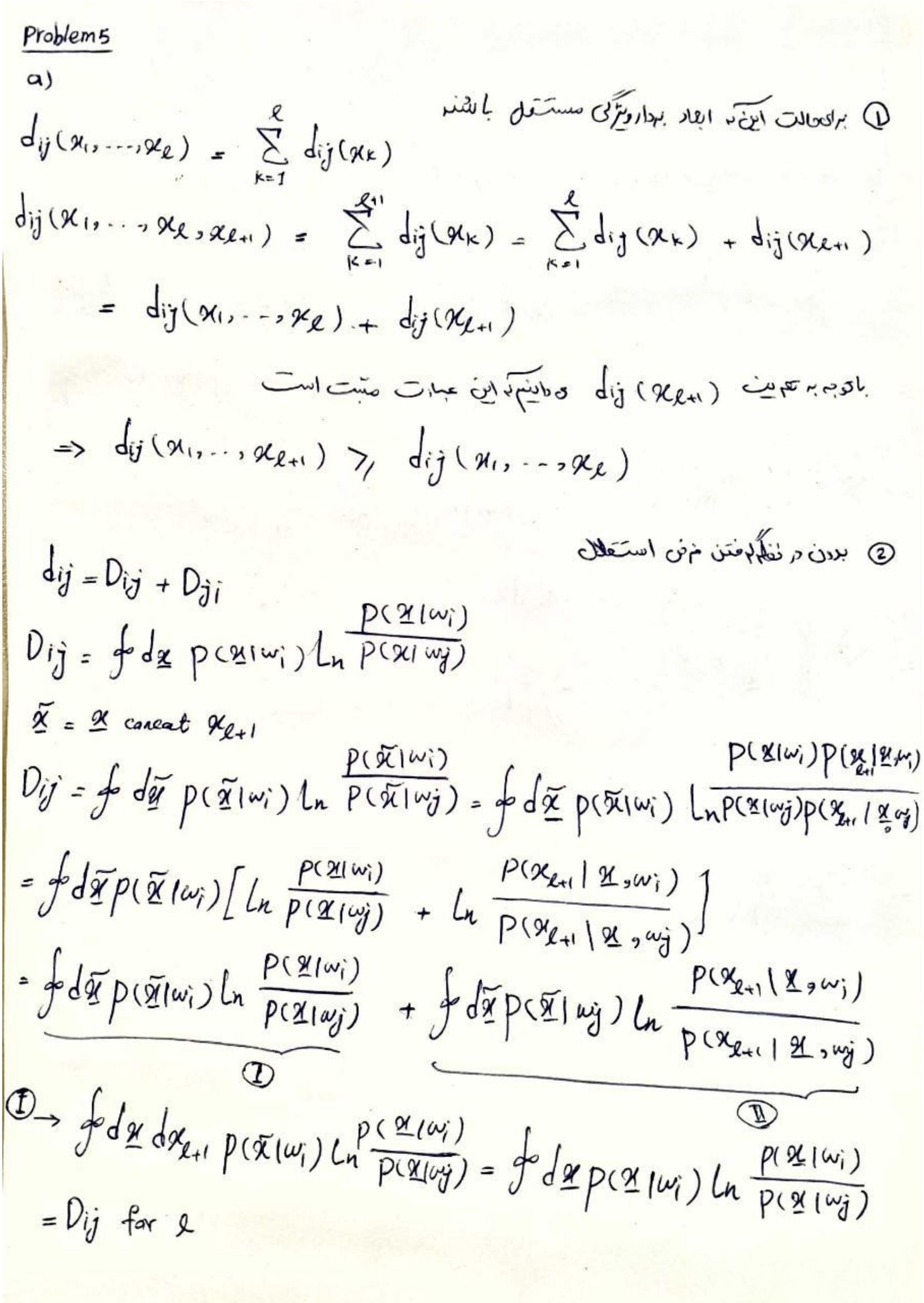
$$= \sum_{i=m}^{N-1} e_i^{\dagger} \lambda_i e_i - c_i e_i^T E\{x\} + c_i^2$$

$$= \sum_{i=m}^{N-1} e_i \lambda_i e_i + c_i \underbrace{(c_i - e_i^T E\{x\})}_{0} = \sum_{i=m}^{N-1} \lambda_i$$

(ج)

* پس e_i ها برای $i = m \ldots N-1$ باید متناسب با کوچک‌ترین λ_i ها باشند

سوال 5



Problem5

a)

① برای حالت این که ابعاد بردار ویژگی مستقل باشند

$$d_{ij}(x_1,\dots,x_\ell) = \sum_{k=1}^{\ell} d_{ij}(x_k)$$

$$d_{ij}(x_1,\dots,x_\ell,x_{\ell+1}) = \sum_{k=1}^{\ell+1} d_{ij}(x_k) = \sum_{k=1}^{\ell} d_{ij}(x_k) + d_{ij}(x_{\ell+1})$$

$$= d_{ij}(x_1,\dots,x_\ell) + d_{ij}(x_{\ell+1})$$

با توجه به تعریف $d_{ij}(x_{\ell+1})$ می‌دانیم که این عبارت مثبت است

$$\Rightarrow d_{ij}(x_1,\dots,x_{\ell+1}) \geq d_{ij}(x_1,\dots,x_\ell)$$

② بدون در نظر گرفتن فرض استقلال

$$d_{ij} = D_{ij} + D_{ji}$$

$$D_{ij} = \int d\underline{x}\, p(\underline{x}|\omega_i) \ln \frac{p(\underline{x}|\omega_i)}{p(\underline{x}|\omega_j)}$$

$\underline{\tilde{x}} = \underline{x}$ concat $x_{\ell+1}$

$$D_{ij} = \int d\underline{\tilde{x}}\, p(\underline{\tilde{x}}|\omega_i) \ln \frac{p(\underline{\tilde{x}}|\omega_i)}{p(\underline{\tilde{x}}|\omega_j)} = \int d\underline{\tilde{x}}\, p(\underline{\tilde{x}}|\omega_i) \ln \frac{p(\underline{x}|\omega_i)\,p(x_{\ell+1}|\underline{x},\omega_i)}{p(\underline{x}|\omega_j)\,p(x_{\ell+1}|\underline{x},\omega_j)}$$

$$= \int d\underline{\tilde{x}}\, p(\underline{\tilde{x}}|\omega_i)\left[\ln \frac{p(\underline{x}|\omega_i)}{p(\underline{x}|\omega_j)} + \ln \frac{p(x_{\ell+1}|\underline{x},\omega_i)}{p(x_{\ell+1}|\underline{x},\omega_j)}\right]$$

$$= \underbrace{\int d\underline{\tilde{x}}\, p(\underline{\tilde{x}}|\omega_i) \ln \frac{p(\underline{x}|\omega_i)}{p(\underline{x}|\omega_j)}}_{\text{Ⓘ}} + \underbrace{\int d\underline{\tilde{x}}\, p(\underline{\tilde{x}}|\omega_j) \ln \frac{p(x_{\ell+1}|\underline{x},\omega_i)}{p(x_{\ell+1}|\underline{x},\omega_j)}}_{\text{Ⓘⓘ}}$$

$$\text{Ⓘ} \rightarrow \int d\underline{x}\, dx_{\ell+1}\, p(\underline{\tilde{x}}|\omega_i) \ln \frac{p(\underline{x}|\omega_i)}{p(\underline{x}|\omega_j)} = \int d\underline{x}\, p(\underline{x}|\omega_i) \ln \frac{p(\underline{x}|\omega_i)}{p(\underline{x}|\omega_j)}$$

$= D_{ij}$ for ℓ

$$(II) \rightarrow \oint d\underline{\tilde{x}}\, p(\tilde{x}|\omega_j)\, Ln \frac{P(x_{\ell+1}|\underline{x},\omega_i)}{P(x_{\ell+1}|\underline{x},\omega_j)}$$

$$\oint dx_{\ell+1}\, P(x_{\ell+1}|\omega_i)\, Ln \frac{P(x_{\ell+1}|\underline{x},\omega_i)}{P(x_{\ell+1}|\underline{x},\omega_j)} = d_{ij\,\ell+1} \geq 0$$

$$\Rightarrow D_{ij\,\ell+1} \geq D_{ij\,\ell} \quad :)$$

Problem 5 b)

$$d_{ij}(x_1, x_2, \dots, x_\ell) = \sum_{m=1}^{\ell} d_{ij}(x_m)$$

$$d_{ij}(\underline{x}) = \oint [p(\underline{x}|\omega_i) - p(\underline{x}|\omega_j)]\, Ln \frac{p(\underline{x}|\omega_i)}{p(\underline{x}|\omega_j)}\, d\underline{x}$$

$$\xrightarrow{\text{independent}} \oint [\prod_{i=1}^{\ell} P(x_i|\omega_i) - \prod_{j=1}^{\ell} p(x_j|\omega_j)]\, Ln \frac{\prod_{i=1}^{\ell} P(x_\ell|\omega_i)}{\prod_{j=1}^{\ell} P(x_j|\omega_j)}\, d\underline{x}$$

$$= \oint [\prod_{i=1}^{\ell} p(x_i|\omega_i) - \prod_{j=1}^{\ell} p(x_j|\omega_j)] \times \left[Ln \frac{P_{x_1|\omega_i)}}{P(x_1|\omega_j)} + \dots + Ln \frac{p(x_n|\omega_i)}{p(x_n|\omega_j)} \right] d\underline{x}$$

$$= \sum_{K=1}^{\ell} \oint [\prod_{i=1}^{\ell} p(x_i|\omega_i) - \prod_{j=1}^{\ell} p(x_j|\omega_j)] \times Ln \frac{p(x_K|\omega_i)}{p(x_K|\omega_j}\, d\underline{x}$$

از آنجایی‌که می‌توان $d\underline{x}$ را به صورت $d\underline{x}'\, dx_K$ نوشت، پس انتگرال بالا ساده‌تر می‌شود.

$$= \sum_{K=1}^{\ell} \oint_{\underline{x}} d\underline{x}' [\prod_{i=1}^{\ell} p(x_i|\omega_i) - \prod_{j=1}^{\ell} p(x_j|\omega_j)]\, Ln \frac{P(x_K|\omega_i)}{p(x_K|\omega_j)}\, dx_K$$

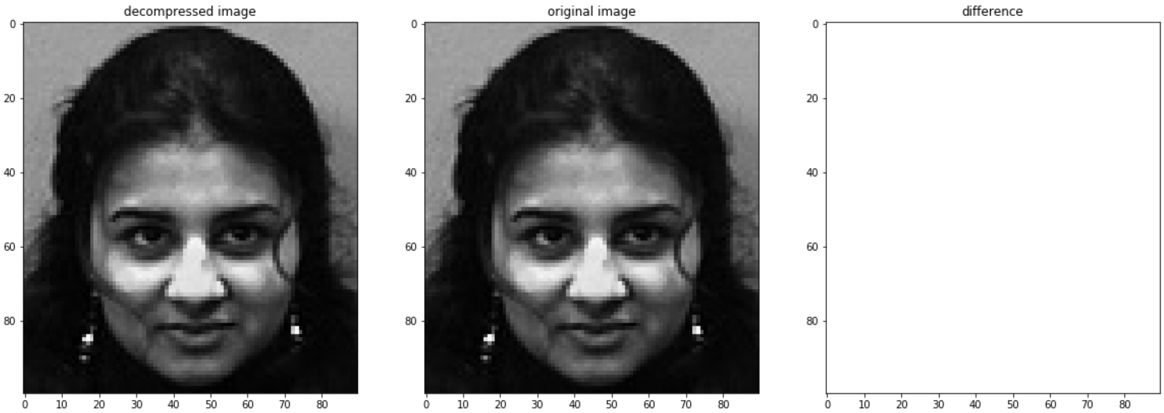
$$= \sum_{K=1}^{\ell} \oint_{x_K} dx_K [p(x_K|\omega_i) - p(x_K|\omega_j)]\, Ln \frac{p(x_K|\omega_i)}{p(x_K|\omega_j)}$$

$$= \sum_{K=1}^{\ell} d_{ij}(x_K)$$

سوال 6

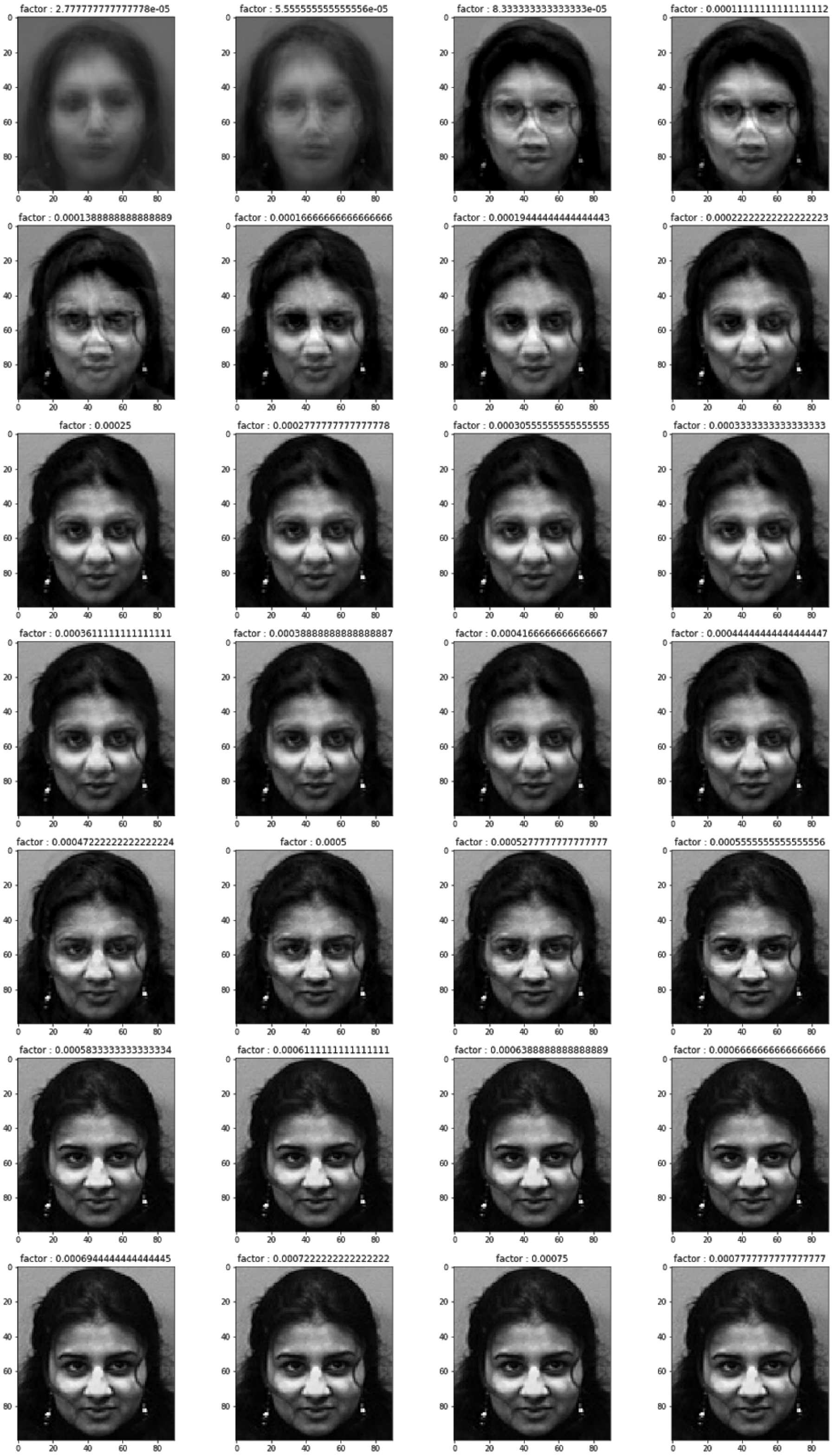
a)

تعداد ویژگی های هر عکس ۳۶۰۰۰ است که پس از انجام pca به مقدار ۲۸ عدد می رسد بنابراین مقدار فشرده‌سازی برابر 1285.71 است.

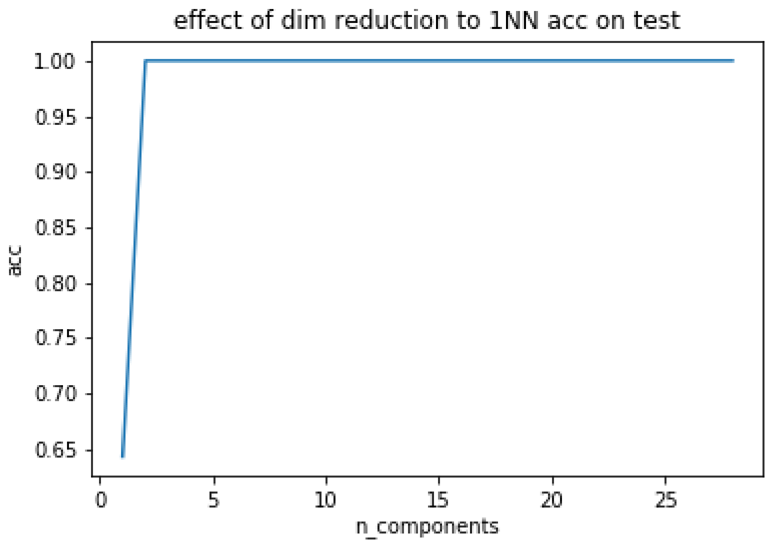


b)

ماژول pca تعداد بعد خروجی را ورودی می‌گیرد . با تغییر این پارامتر در ابتدا تصویر را نمایش می‌دهیم و مشاهده می‌شود که هر چه تصویر فشرده‌تر شود و یا به عبارتی تعداد component ها کم شود کیفیت عکس کاهش می‌یابد.



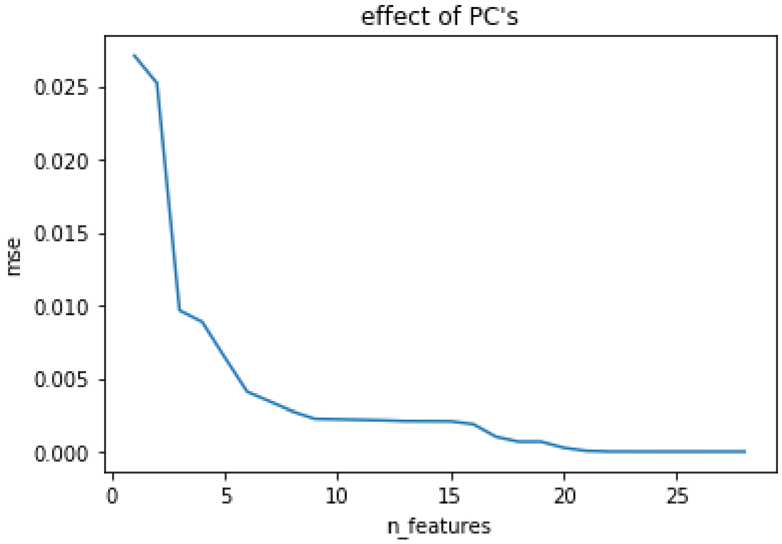
در ادامه تاثیر فشرده سازی را در recognition مشاهده می‌کنیم.



مشاهده می‌شود که در صورتی که بازیابی کامل اطلاعات برای ما مهم نباشد و تنها طبقه بندی برای ما مهم باشد در این صورت حتی می‌توان تعداد component ها را تا ۴ و یا ۳ کاهش داد بدون این که در خطای طبقه‌بندی کاهش شدیدی مشاهده شود.

c)

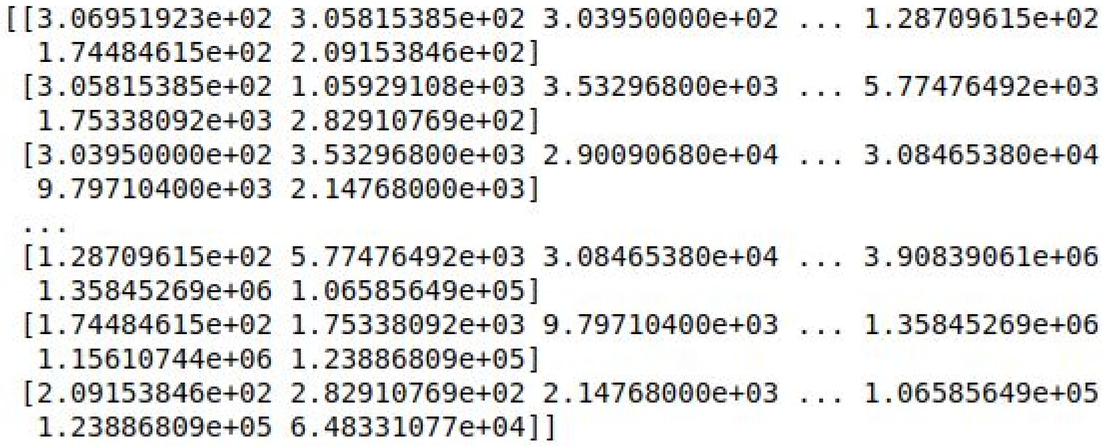
در این قسمت خطای موجود در عکس های بازیابی شده را بررسی می‌کنیم.



مشاهده می‌شود که با افزایش تعداد component ها خطای بازیابی کاهش می‌یابد و پس از این که در حدود ۲۸ component وجود داشت خطا به حدود ۰ می‌رسد که این نشان دهنده این است که بیشتر اطلاعات در همین ۲۸ بعد بوده اند.

سوال 7

برای حل این سوال از روشی مشابه pca استفاده شده است به این معنی که ابتدا ماتریس Scatter total ساخته می‌شود. ماتریس cov نشان دهنده ی وابستگی بین ابعاد مختلف داده ها است و اگر این ماتریس قطری شودبه معنی uncorolated بودن ابعاد است. در مثال ما این ماتریس به شکل زیر است که قطری نمی‌باشد.



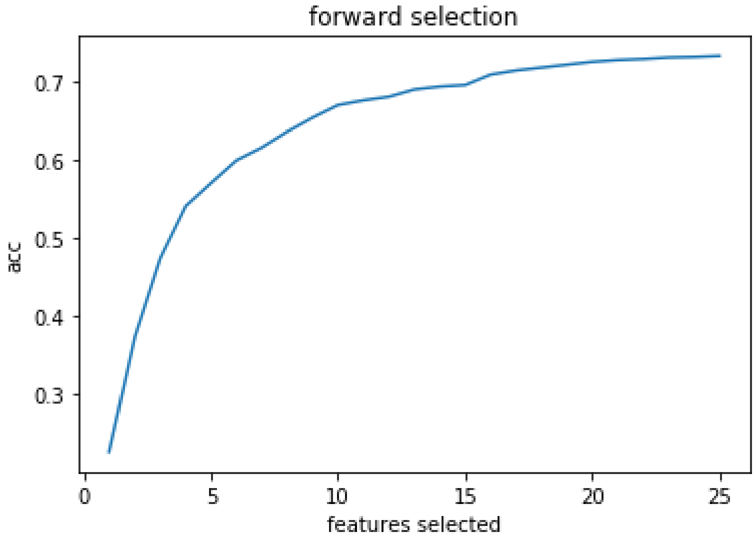
```
[[3.06951923e+02 3.05815385e+02 3.03950000e+02 ... 1.28709615e+02
  1.74484615e+02 2.09153846e+02]
 [3.05815385e+02 1.05929108e+03 3.53296800e+03 ... 5.77476492e+03
  1.75338092e+03 2.82910769e+02]
 [3.03950000e+02 3.53296800e+03 2.90090680e+04 ... 3.08465380e+04
  9.79710400e+03 2.14768000e+03]
 ...
 [1.28709615e+02 5.77476492e+03 3.08465380e+04 ... 3.90839061e+06
  1.35845269e+06 1.06585649e+05]
 [1.74484615e+02 1.75338092e+03 9.79710400e+03 ... 1.35845269e+06
  1.15610744e+06 1.23886809e+05]
 [2.09153846e+02 2.82910769e+02 2.14768000e+03 ... 1.06585649e+05
  1.23886809e+05 6.48331077e+04]]
```

برای سفید سازی داده ها از روش eigenvalue eigenvector decomposition استفاده میکنیم

به این صورت که

سوال 8

در این سوال به پیاده سازی الگوریتم forward selection و برای انتخاب ویژگی بهتر از naive bayes classifier استفاده کردیم. این الگورتم را تا ۲۵ iteration انجام دادیم و دیده می‌شوید که پس از این عدد تقریبا دقت طبقه بندی افزایش چشمگیری نخواهد داشت پس ۲۵ ویژگی را در کل انتخاب خواهیم کرد.



با توجه به این نمودار ۲۵ ویژگی برای انتخاب شدن مناسب هستند.

سوال 9

a)

در این سوال pca را پیاده سازی کردیم. در ابتدا مقادیر ویژه ماتریس cov را مشاهده می‌کنیم. که به صورت نزولی مرتب شده اند. این اعداد در بازه e^10 تا e^2 هستند و به این معنی است که از تعداد خوبی از آن‌ها میتوان صرف‌نظر کرد.

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,1.59933253e+04, 1.08861571e+04, 8.69090241e+03, 6.33770978e+03
([5.72638823e+03, 9.08442656e+02, 5.60101113e+02, 1.09375097e+02
```

b)

با استفاده از condition number تعداد ۵۳ ویژگی با استفاده از pca بدست آمد. سپس طبقه‌بند naive bayes را روی فضای جدید آموزش دادم و خطای طبقه بندی برابر ۷۶.۶ درصد شد.

c)

طبقه بند naive bayes را قبل از اعمال pca بر روی داده ها آموزش دادم و خطای طبقه‌بندی برابر 56.6 است.

سوال 10

a)

در این سوال می‌خواهیم که LDA را پیادەسازی کنیم. در زیر مقدار ویژه های separability matrix را به صورت کاهنده چاپ کرده ایم.

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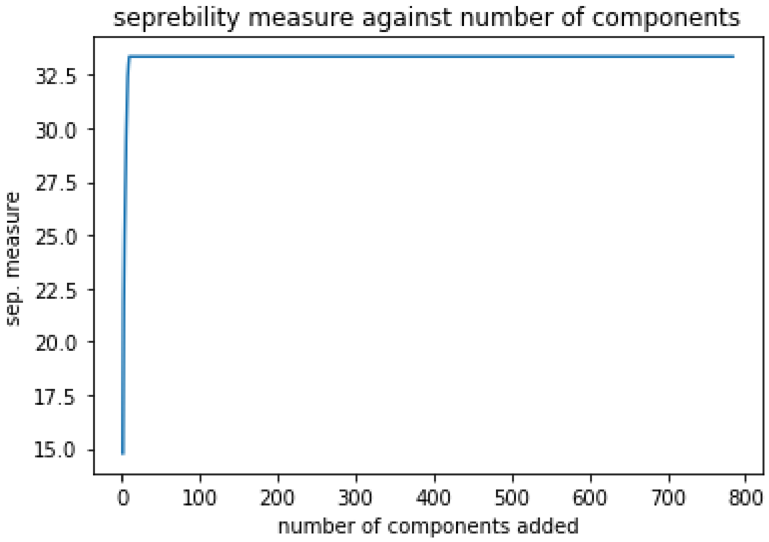
مشاهده می‌شود که تعداد زیادی از این مقادیر ویژه بسیار کوچک هستند که این به معنی صفر بودن آن ها است. و دقیقا c-1 مقدار ویژه وجود دارند که نزدیک به ۰ نباشند.

b)

بدون حذف ابعاد مقدار جداپذیری برابر با ۳۳.۳۶ می‌باشد. سپس با انتخاب بیشترین مقادیر ویژه (همان انتخاب ابعاد با بزرگترین مقدار ویژه) معیار جداپذیری را می‌سنجیم و پلات زیر بدست می‌آید.



این نمودار نشان می‌دهد که پس از ۹ ویژگی که همان مقدار C -1 است مقدار جداپذیری تقریبا ثابت است.

c)

با توجه به نتایج بدست آمده ما ۹ ویژگی با بیشترین مقدار ویژه را انتخاب کردیم و طبقه بند naive bayes را روی آن آموزش دادیم.

دقت آن ۸۱.۲۳ درصد شد.

d)

بدون اعمال lda دقت طبقه بند naive bayes روی مجموعه داده برابر با ۵۶.۶ بود

e)

بین pca , lda در هدف ما که طبقه بندی بود lda عملکرد بهتری روی داده های تست داشت به این معنی که lda به generalization بهتری دست‌می‌یابد.

دقت lda برابر ۸۱ درصد بود در صورتی که دقت pca برابر ۷۶ درصد بود پس از نظر طبقه بندی نیز lda به نتیجه مطلوب تری رسید.

در lda بعد داده ها به ۹ ویژگی کاهش یافت در صورتی که در pca بعد ویژگی به ۵۳ ویژگی کاهش یافت پس از نظر محاسباتی lda عملکرد بهتری از pca نشان داد.

البته این نتایج منطقی به نظر می‌رسد به این دلیل که در اجرا کردن lda برچسب ها به الگوریتم داده شده است به این معنی که اطلاعات بیشتری به الگوریتم داده شده است در صورتی که pca کاملا به صورت unsupervised عمل می‌کند.

پیوست 1: روند اجرای برنامه

کد های هر سوال در یک jupyter notebook جداگانه با نام سوال در پیوست قرار داده شده است و برای این که dataset ها بارگیری شوند باید دقت کنید که باید در directory یکسانی حضور داشته باشند..