1. Define simple and multigraph. What does isolated vertex mean? Consider two distinct paths from a vertex u to a vertex v of a graph G. prove that G contains a cycle.

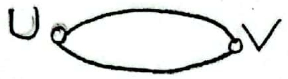
→ A graph G(V,E) is called a simple graph if it has no parallel edges and no loops. Eg:- 2 is a simple graph.

A graph G(V,E) is called a multigraph if it contains multiple parallel edges.

An isolated vertex is a vextex with degree zero; that is, a vertex that is not an endpoint of any edge.

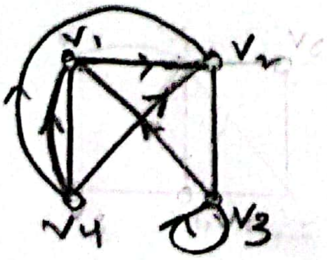
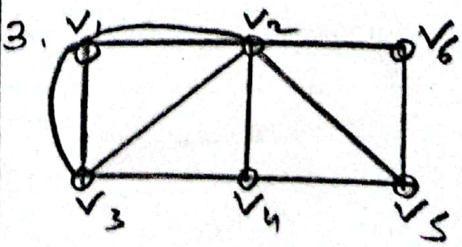
Given,

A graph G(U,V) with n=2 in which each vertex is connected to another and last vertex (V) is connected with first one (U). This form a closed path, which is a cycle.



2. Distinguish between Eulerian circuit and Hamiltonian cycle (circuit). Determine if there is an Eulerian circuit and (or) a Hamiltonian cycle in the following graph. Also find their length if exists.

→

	Eulerian Circuit	Hamiltonian Cycle.
1.	A circuit in a graph G is said to be an eulerian circuit if it contains all the edges of the graph.	1. A cycle in a graph G is said be a hamiltanian cycle if it contains all the vertices of graph G exactly once.
2.	A graph G which contains an eulerian circuit, is called a eulerian graph.	2. A graph G which contains a Hamilterian cycle is called a Hamilterian graph.
3.	 eg: ($V_1, V_2, V_3, V_3, V_1, V_4, V_2, V_4, V_1$) is an eulerian circuit.	3.  eg: ($V_1, V_3, V_4, V_5, V_6, V_2, V_1$) is an hamiltonian cycle.

3. Prove that the number of odd vertices in a graph is always even.

→ Case 1

Proof

Let $a = (V, E)$ be a graph.

If a contains no odd vertices there is nothing to prove.

Case 2

Let the graph G contain the number of odd vertices and n numbers of even vertices.

$[\deg v_1 + \deg v_2 + \ldots d[v_n]] + [\deg[u_1] + \deg[u_2] + \ldots + \deg[u_n]] . 2ne$

$[\deg[v_1] + \deg[v_2] + \ldots + \deg[v_n]] = 2ne - [\deg[u_1] + \deg[u_2] + \ldots + \deg[u_n]]$ -1)

Since RHS of eqⁿ (i) is even but the vertices of LHS are odd vertices.

∴ The number of vertices k must be even.

Case 3

If a graph G contains all odd vertices

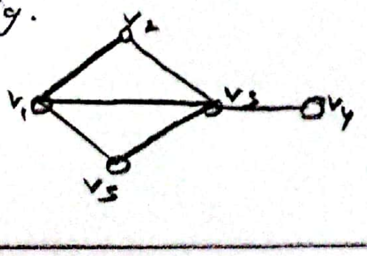
$\deg[v_1] + \deg[v_2] + \ldots + (\deg[v_k] = 2ne$

∴ k must be even.

4) When is a multigraph is said to be a traversable graph.

→ A multigraph where the graph G contains a trail called Eulerian trial. If it contains every edeges of G is called a traversable graph.

Eg.



$W = (v_1, v_2, v_3, v_1, v_5, v_3, v_4)$ is an eulerian trial.

∴ Given graph is traversable graph.

5) What do you mean by bipartite graph ? Prove that if there is a walk from any two vertices u and v of graph G, then there is a path from these vertices.

→ A graph G (V,E) is said to be bi-partite if the vertex set v is the union of two non-empty disjoint subsets V_1 and V_2 such that each edge in E is incident one vertex of V_1 and one vertex of V_2.

Eg:

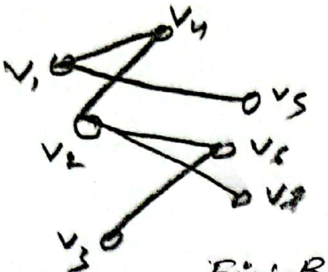
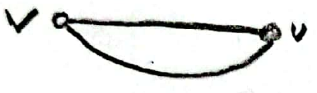


Fig:- Bi-partite graph.

→ Sol^n,



Here, A walk in a graph G has all the vertices and edges distinct. Therefore, the walk is the path,

Eg:- W = (v, u). This is a path.

W = (u, v, u). This is a cycle.

6. Construct the truth table of the proposition $(p \wedge q) \to \sim p$

→ Truth Table.

p	q	$p \wedge q$	$\sim p$	$(p \wedge q) \to \sim p$
T	T	T	F	F
F	T	F	T	T
T	F	F	F	T
F	F	F	T	T

7. Define ~~the~~ the terms :- tautology ; Contradiction ; Conditional compound statement and its types with examples.

→ A compound statement which is always true is called a tautology.

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Hence, $p \vee \sim p$ is tautology

→ A compound statement which is always false is called a contradiction.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Hence, $p \wedge \sim p$ is contradiction.

→ A statement or propositional variables can be combined by means of logical connectives (operators) to form a single statement called compound statement.
It's types are :-

1) Negation :- A proposition which denies a given proposition is negation.
Symbol : $\sim$, Connective : Not
Eg :- P : demand is increasing (T)
$\sim P$: demand is not increasing (F)

2) Conjunction : Two simple propositions combined by the word "and" to form a compound proposition. Symbol : $\wedge$, connective : AND.
Eg :- Nepal is in India and $1+3=4$

Symbol :- $\wedge$	connective: AND

$= F \wedge T$
$= F$

3) Disjunction : Two proposition combined by the word "OR" to form a compound proposition.
Symbol : V
Connective : OR
Eg :- Nepal is in India or $1+3=4$
$= F \vee T$
$= T$

4) Conditional : Two statements combined by the word "If.. then". to form a compound proposition.
Symbol : $\Rightarrow$ or $\rightarrow$
Connective : If .... then
Eg : If Afganistan is in pakistan then $5+6=10$.
$= F \Rightarrow F$
$= T$

5. Biconditional : Two simple proposition combined by the word "If" or If and only if to form a compound proposition.
Symbol : $\Leftrightarrow$ or $\leftrightarrow$
Connective : If and only if
Eg :- If Afganistan is in pakistan and only if $5+6=10$
$= F \leftrightarrow T$
$= F$