

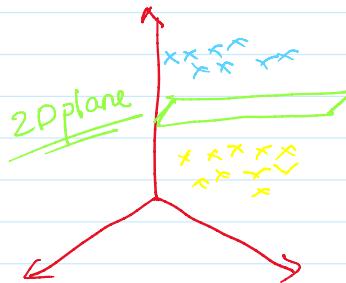
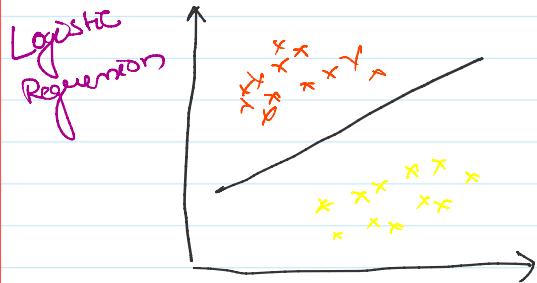
Support Vector Machines (Udemy KN)

13 June 2024 15:40

Support Vector Machine

① SVC - (Support Vector classifier)

② SVR - (Support Vector Regressor)



① SVC → classifier

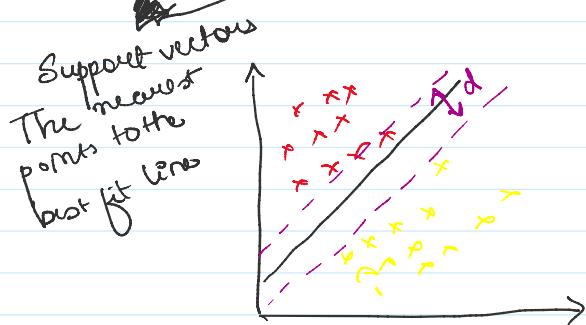
Marginal plane - They are both equidistant from best fit line



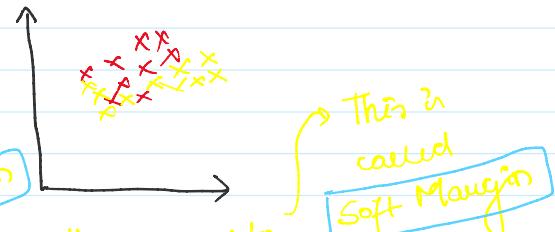
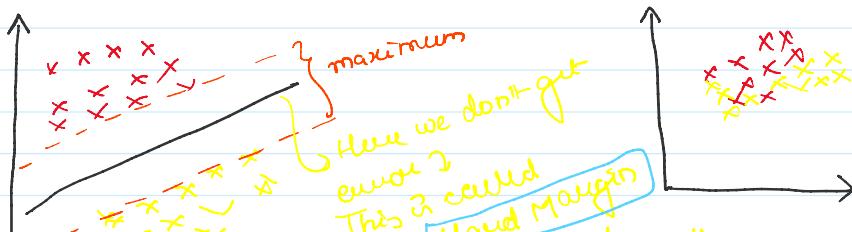
Aim is to ensure the 'd' is maximum

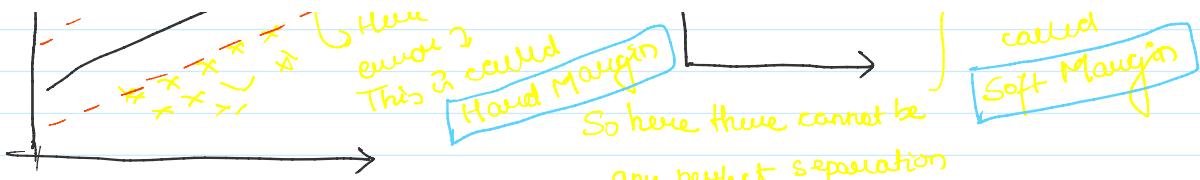
Which of the plane should we select?

- The plane with highest 'd' or distance between the marginal planes



⇒ Soft Margin and Hard Margin in SVM

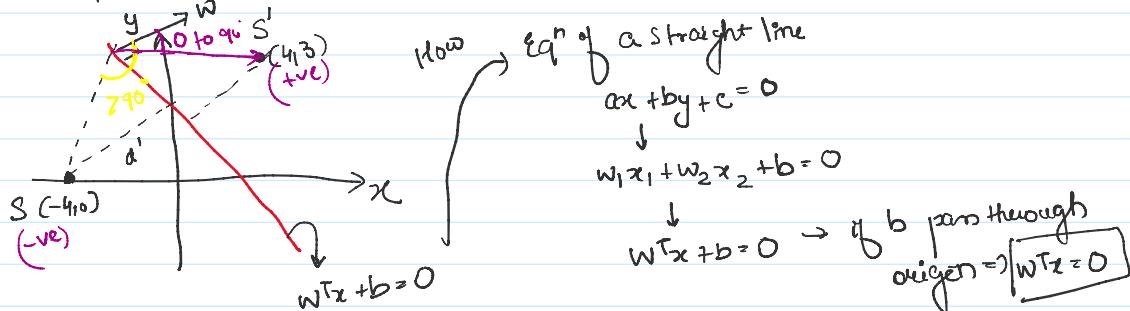




In this case we have a clear boundary but in real-life datapoints will be overlapping.

So here there cannot be any perfect separation and there will be errors.

① Support Vector Machine Maths intuition



∴ Whenever for a point, the angle b/w \vec{w} and the point connecting w to S is greater than 90° , the distance of the point S is (-ve) from the plane. (points below line)

$d = -vc$ below plane
 $d = +ve$ above plane

∴ In case it is b/w 0 to 90° , the distance of pt from line is (+ve). (points above line)

- Whenever we go up from the hyperplane towards the pt. above it, we are going to get positive values always

$$\therefore w^T x + b = +1$$

- Similarly, if we go down the hyperplane, we get negative values.

$$\therefore w^T x + b = -1.$$

We need the distance b/w marginal planes

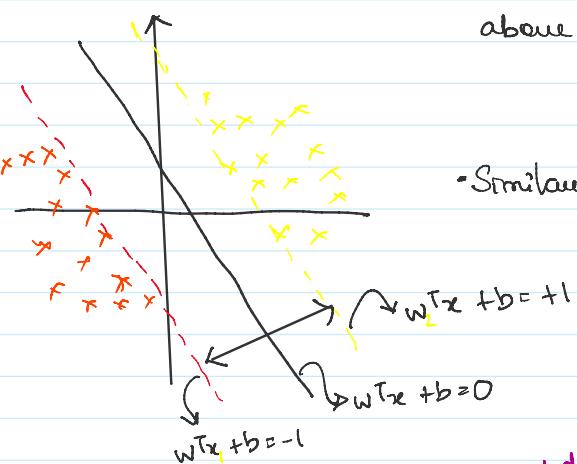
$$w^T x + b = +1$$

$$\underline{\underline{w^T x_1 + b = -1}} \quad \underline{\underline{w^T x_2 + b = +1}}$$

$$w^T(x_1 - x_2) = 2$$

A vector whose magnitude

We convert the vector to the **unit vector** → divide the vector by its magnitude



$$\frac{w^T(x_1 - x_2)}{\|w\|} = \frac{2}{\|w\|}$$

Cost function
 \Rightarrow Maximize $\frac{2}{\|w\|}$
 $\therefore w, b$ under constraint

Convexity of classified points

under constraint $y_i = \begin{cases} +1 & w^T x + b \geq 1 \\ -1 & w^T x + b \leq -1 \end{cases}$

For all correct points

$$\text{constraint} \Rightarrow y_i \cdot (w^T x + b) \geq 1$$

SVC Cost function

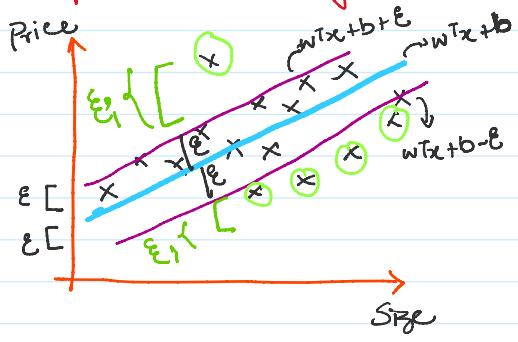
$$\underset{w, b}{\text{maximize}} \frac{2}{\|w\|} \Rightarrow \underset{(w, b)}{\text{minimize}} \frac{\|w\|}{2}$$

final cost fn for SVC $\rightarrow \min_{w, b} \frac{\|w\|}{2} + C \sum_{i=1}^n \xi_i$

Hinge Loss
 \downarrow
 Soft Margin
 (because in soft margin we expect overlapping classes and misclassification)
 How many points we want to avoid misclassification?
 (Basically if max ξ I can ignore C misclassified points)

Summation of the distance of the incorrect data points from the marginal plane

Support Vector Regressor (SVR)



$\epsilon \rightarrow$ marginal error

Cost fn

$\min_{w, b} \frac{\|w\|}{2} + C \sum_{i=1}^n \xi_i$

Hinge Loss

The marginal error we take to accommodate the points outside marginal plane

Constraints

$|y_i - w^T x_i| \leq \epsilon + \xi_i$

Margin Error

Loss fn

Basically all points should fall below marginal plane and the difference should be as low as possible

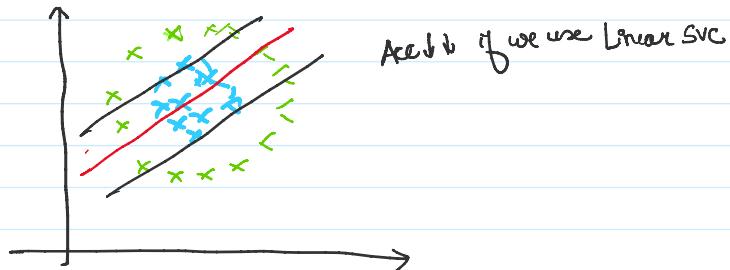
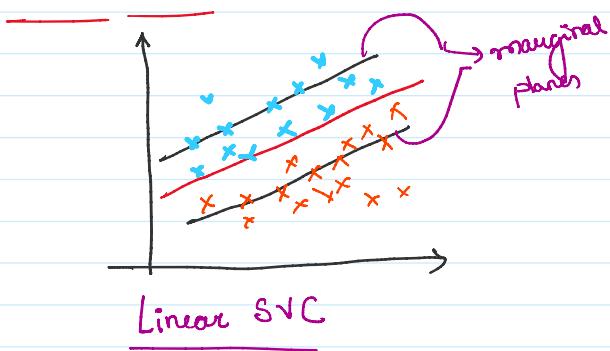
$$|y_i - \hat{y}_i| \leq \epsilon + \hat{\epsilon}$$

↑ true ↑ predicted
error above the margin

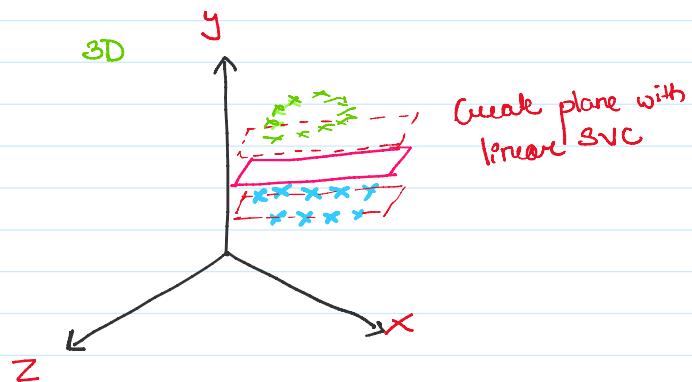
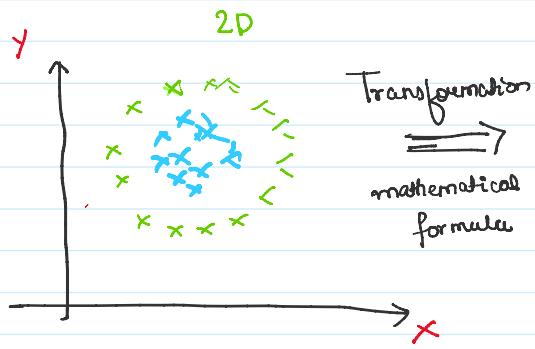
$C \uparrow$
 $\text{Lossf} \downarrow$



SVM Kernels



So in these scenarios, where our data is not linearly separable, we use SVM Kernels.

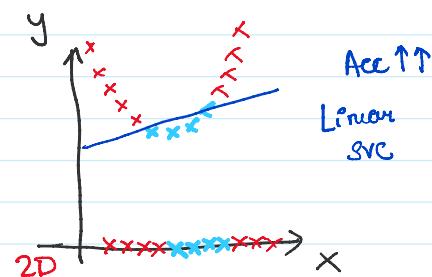


Dataset : 1D

Cannot be solved by linear SVC



\Rightarrow Transformation
 $y = x^2$



- ① Polynomial Kernel
 - ② RBF Kernel
 - ③ Sigmoid Kernel
- major SVM Kernels

Linear Kernel

used for text classification

Advantages:

— Faster
— Only one hyperparameter
so can be easily obtained

Polynomial Kernel

formula: $(\alpha \cdot b + r)^d$ d = degree of polynomial

Observation
from data

coefficient of polynomial

r, d can be found
using cross validation

RBF Kernel

Radial Basis function

formula: $e^{-r(x-l)^2}$ 2 data points

r → scaling factor