1. Решение задачи 1

Задача:

$$C = \begin{pmatrix} 1 & 1/5 & 1/5 & 1 & 1/3 \\ 5 & 1 & 1/5 & 1/5 & 1 \\ 5 & 5 & 1 & 1/5 & 1 \\ 1 & 5 & 5 & 1 & 5 \\ 3 & 1 & 1 & 1/5 & 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 3 & 7 & 9 \\ 1/3 & 1 & 6 & 7 \\ 1/7 & 1/6 & 1 & 3 \\ 1/9 & 1/7 & 1/3 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 1/5 & 1/6 & 1/4 \\ 5 & 1 & 2 & 4 \\ 6 & 1/2 & 1 & 6 \\ 4 & 1/4 & 1/6 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 7 & 7 & 1/2 \\ 1/7 & 1 & 1 & 1/7 \\ 1/7 & 1 & 1 & 1/7 \\ 2 & 7 & 7 & 1 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 1 & 4 & 1/4 & 1/3 \\ 1/4 & 1 & 1/2 & 3 \\ 4 & 2 & 1 & 3 \\ 3 & 1/3 & 1/3 & 1 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} 1 & 1 & 7 & 4 \\ 1 & 1 & 6 & 3 \\ 1/7 & 1/6 & 1 & 1/4 \\ 1/4 & 1/3 & 4 & 1 \end{pmatrix}$$

Нужные степени матрицы C:

$$C^2 = \begin{pmatrix} 1 & 5 & 5 & 1 & 5 \\ 5 & 1 & 1 & 5 & 5/3 \\ 25 & 5 & 1 & 5 & 5 \\ 25 & 25 & 5 & 1 & 5 \\ 5 & 5 & 1 & 3 & 1 \end{pmatrix}$$

$$C^3 = \begin{pmatrix} 25 & 25 & 5 & 1 & 5 \\ 5 & 25 & 25 & 5 & 25 \\ 25 & 25 & 25 & 25 & 25 \\ 125 & 25 & 5 & 25 & 25 \\ 25 & 15 & 15 & 5 & 15 \end{pmatrix}$$

$$C^4 = \begin{pmatrix} 125 & 25 & 5 & 25 & 25 \\ 125 & 125 & 25 & 5 & 25 \\ 125 & 125 & 125 & 25 & 125 \\ 125 & 125 & 125 & 125 & 125 \\ 75 & 75 & 25 & 25 & 25 \end{pmatrix}$$

$$C^5 = \begin{pmatrix} 125 & 125 & 125 & 125 & 125 \\ 625 & 125 & 25 & 125 & 125 \\ 625 & 625 & 125 & 125 & 125 \\ 625 & 625 & 625 & 125 & 625 \\ 375 & 125 & 125 & 75 & 125 \end{pmatrix}$$

Спектральный радиус матрицы C:

$$\lambda_C = \mathrm{tr}C \oplus \cdots \oplus \mathrm{tr}^{1/5}(C^5) = (125)^{1/4} \approx 3.3437$$

Матрица $\lambda^{-1}C$ и ее степени:

$$(\lambda^{-1}C)^1 = \begin{pmatrix} (1/125)^{1/4} & (1/78125)^{1/4} & (1/78125)^{1/4} & (1/125)^{1/4} & (1/10125)^{1/4} \\ (5)^{1/4} & (1/125)^{1/4} & (1/78125)^{1/4} & (1/78125)^{1/4} & (1/125)^{1/4} \\ (5)^{1/4} & (5)^{1/4} & (1/125)^{1/4} & (1/78125)^{1/4} & (1/125)^{1/4} \\ (1/125)^{1/4} & (5)^{1/4} & (5)^{1/4} & (1/125)^{1/4} & (5)^{1/4} \\ (81/125)^{1/4} & (1/125)^{1/4} & (1/125)^{1/4} & (1/78125)^{1/4} & (1/125)^{1/4} \end{pmatrix}$$

$$(\lambda^{-1}C)^2 = \begin{pmatrix} (1/15625)^{1/4} & (1/25)^{1/4} & (1/25)^{1/4} & (1/15625)^{1/4} & (1/25)^{1/4} \\ (1/25)^{1/4} & (1/15625)^{1/4} & (1/15625)^{1/4} & (1/25)^{1/4} & (1/2025)^{1/4} \\ (25)^{1/4} & (1/25)^{1/4} & (1/15625)^{1/4} & (1/25)^{1/4} & (1/25)^{1/4} \\ (25)^{1/4} & (25)^{1/4} & (1/25)^{1/4} & (1/15625)^{1/4} & (1/25)^{1/4} \\ (1/25)^{1/4} & (1/25)^{1/4} & (1/15625)^{1/4} & (81/15625)^{1/4} & (1/15625)^{1/4} \end{pmatrix}$$

$$(\lambda^{-1}C)^3 = \begin{pmatrix} (1/5)^{1/4} & (1/5)^{1/4} & (1/3125)^{1/4} & (1/1953125)^{1/4} & (1/3125)^{1/4} \\ (1/3125)^{1/4} & (1/5)^{1/4} & (1/5)^{1/4} & (1/3125)^{1/4} & (1/5)^{1/4} \\ (1/5)^{1/4} & (1/5)^{1/4} & (1/5)^{1/4} & (1/5)^{1/4} & (1/5)^{1/4} \\ (125)^{1/4} & (1/5)^{1/4} & (1/3125)^{1/4} & (1/5)^{1/4} & (1/5)^{1/4} \\ (1/5)^{1/4} & (81/3125)^{1/4} & (81/3125)^{1/4} & (1/3125)^{1/4} & (81/3125)^{1/4} \end{pmatrix}$$

$$(\lambda^{-1}C)^4 = \begin{pmatrix} (1)^{1/4} & (1/625)^{1/4} & (1/390625)^{1/4} & (1/625)^{1/4} & (1/625)^{1/4} \\ (1)^{1/4} & (1)^{1/4} & (1/625)^{1/4} & (1/390625)^{1/4} & (1/625)^{1/4} \\ (1)^{1/4} & (1)^{1/4} & (1)^{1/4} & (1/625)^{1/4} & (1)^{1/4} \\ (1)^{1/4} & (1)^{1/4} & (1)^{1/4} & (1)^{1/4} & (1)^{1/4} \\ (81/625)^{1/4} & (81/625)^{1/4} & (1/625)^{1/4} & (1/625)^{1/4} & (1/625)^{1/4} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}C)^* = I \oplus (\lambda^{-1}C)^1 \oplus (\lambda^{-1}C)^2 \oplus (\lambda^{-1}C)^3 \oplus (\lambda^{-1}C)^4 =$$

$$= \begin{pmatrix} 1 & (1/5)^{1/4} & (1/25)^{1/4} & (1/125)^{1/4} & (1/25)^{1/4} \\ (5)^{1/4} & 1 & (1/5)^{1/4} & (1/25)^{1/4} & (1/5)^{1/4} \\ (25)^{1/4} & (5)^{1/4} & 1 & (1/5)^{1/4} & (1)^{1/4} \\ (125)^{1/4} & (25)^{1/4} & (5)^{1/4} & 1 & (5)^{1/4} \\ (81/125)^{1/4} & (81/625)^{1/4} & (81/3125)^{1/4} & (81/15625)^{1/4} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (1/25)^{1/4} \\ (5)^{1/4} & (1/5)^{1/4} \\ (25)^{1/4} & (1)^{1/4} \\ (125)^{1/4} & (5)^{1/4} \\ (81/125)^{1/4} & 1 \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (1/125)^{1/4} \\ (1/25)^{1/4} \\ (1/5)^{1/4} \\ (1)^{1/4} \\ (81/15625)^{1/4} \end{pmatrix} \qquad w_2 = \begin{pmatrix} (1/125)^{1/4} & (1/125)^{1/4} \\ (1/25)^{1/4} & (1/25)^{1/4} \\ (1/5)^{1/4} & (1/5)^{1/4} \\ (1)^{1/4} & (1)^{1/4} \\ (1/125)^{1/4} & (1/5)^{1/4} \end{pmatrix}$$

$$B = \begin{pmatrix} (1)^{1/4} & (2401/5)^{1/4} & (2401/5)^{1/4} & (6561/125)^{1/4} \\ (25)^{1/4} & (1)^{1/4} & (1296/125)^{1/4} & (81)^{1/4} \\ (256)^{1/4} & (16)^{1/4} & (1)^{1/4} & (81)^{1/4} \\ (81)^{1/4} & (2401/5)^{1/4} & (2401/5)^{1/4} & (1)^{1/4} \end{pmatrix}$$

$$D = \begin{pmatrix} (1)^{1/4} & (2401/5)^{1/4} & (2401/5)^{1/4} & (6561/125)^{1/4} \\ (25)^{1/4} & (1)^{1/4} & (1296/125)^{1/4} & (81)^{1/4} \\ (256)^{1/4} & (16)^{1/4} & (1)^{1/4} & (81)^{1/4} \\ (81)^{1/4} & (2401/5)^{1/4} & (2401/5)^{1/4} & (1)^{1/4} \end{pmatrix}$$

Нужные степени матрицы B:

$$B^2 = \begin{pmatrix} (614656/5)^{1/4} & (15752961/625)^{1/4} & (15752961/625)^{1/4} & (194481/5)^{1/4} \\ (6561)^{1/4} & (194481/5)^{1/4} & (194481/5)^{1/4} & (6561/5)^{1/4} \\ (6561)^{1/4} & (614656/5)^{1/4} & (614656/5)^{1/4} & (1679616/125)^{1/4} \\ (614656/5)^{1/4} & (194481/5)^{1/4} & (194481/5)^{1/4} & (194481/5)^{1/4} \end{pmatrix}$$

$$B^3 = \begin{pmatrix} (4032758016/625)^{1/4} & (1475789056/25)^{1/4} & (1475789056/25)^{1/4} & (4032758016/625)^{1/4} \\ (49787136/5)^{1/4} & (15752961/5)^{1/4} & (15752961/5)^{1/4} & (15752961/5)^{1/4} \\ (157351936/5)^{1/4} & (4032758016/625)^{1/4} & (4032758016/625)^{1/4} & (49787136/5)^{1/4} \\ (49787136/5)^{1/4} & (1475789056/25)^{1/4} & (1475789056/25)^{1/4} & (4032758016/625)^{1/4} \end{pmatrix}$$

$$B^4 = \begin{pmatrix} (377801998336/25)^{1/4} & (9682651996416/3125)^{1/4} & (9682651996416/3125)^{1/4} & (119538913536/2 \\ (4032758016/5)^{1/4} & (119538913536/25)^{1/4} & (119538913536/25)^{1/4} & (326653399296/62 \\ (1032386052096/625)^{1/4} & (377801998336/25)^{1/4} & (377801998336/25)^{1/4} & (1032386052096/6 \\ (377801998336/25)^{1/4} & (119538913536/25)^{1/4} & (119538913536/25)^{1/4} & (119538913536/2 \end{pmatrix}$$

Спектральный радиус матрицы B:

$$\lambda_B = \mathrm{tr}B \oplus \cdots \oplus \mathrm{tr}^{1/4}(B^4) = (614656/5)^{1/8} \approx 4.32721$$

Матрица $\lambda^{-1}B$ и ее степени:

$$(\lambda^{-1}B)^1 = \begin{pmatrix} (5/614656)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (43046721/1920800000)^{1/8} \\ (3125/614656)^{1/8} & (5/614656)^{1/8} & (6561/7503125)^{1/8} & (32805/614656)^{1/8} \\ (1280/2401)^{1/8} & (5/2401)^{1/8} & (5/614656)^{1/8} & (32805/614656)^{1/8} \\ (32805/614656)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (5/614656)^{1/8} \end{pmatrix}$$

$$(\lambda^{-1}B)^2 = \begin{pmatrix} (1)^{1/8} & (43046721/1024000000)^{1/8} & (43046721/1024000000)^{1/8} & \\ (1076168025/377801998336)^{1/8} & (6561/65536)^{1/8} & (6561/65536)^{1/8} & (43 \\ (1076168025/377801998336)^{1/8} & (1)^{1/8} & (1)^{1/8} & (4 \\ (1)^{1/8} & (6561/65536)^{1/8} & (6561/65536)^{1/8} & \end{pmatrix}$$

$$(\lambda^{-1}B)^3 = \begin{pmatrix} (43046721/1920800000)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (4 \\ (32805/614656)^{1/8} & (215233605/40282095616)^{1/8} & (215233605/40282095616)^{1/8} & (21 \\ (1280/2401)^{1/8} & (43046721/1920800000)^{1/8} & (43046721/1920800000)^{1/8} & \\ (32805/614656)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (4 \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}B)^* = I \oplus (\lambda^{-1}B)^1 \oplus (\lambda^{-1}B)^2 \oplus (\lambda^{-1}B)^3 =$$

$$= \begin{pmatrix} 1 & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (6561/65536)^{1/8} \\ (32805/614656)^{1/8} & 1 & (6561/65536)^{1/8} & (32805/614656)^{1/8} \\ (1280/2401)^{1/8} & (1)^{1/8} & 1 & (32805/614656)^{1/8} \\ (1)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (2401/1280)^{1/8} & (6561/65536)^{1/8} \\ (32805/614656)^{1/8} & 1 & (32805/614656)^{1/8} \\ (1280/2401)^{1/8} & (1)^{1/8} & (32805/614656)^{1/8} \\ (1)^{1/8} & (2401/1280)^{1/8} & 1 \end{pmatrix}$$

$$w_1 = \begin{pmatrix} 1 & (6561/65536)^{1/8} \\ (32805/614656)^{1/8} & (32805/614656)^{1/8} \\ (1280/2401)^{1/8} & (32805/614656)^{1/8} \\ (1)^{1/8} & (1)^{1/8} \end{pmatrix} \qquad w_2 = \begin{pmatrix} 1 \\ (1280/2401)^{1/8} \\ (1280/2401)^{1/8} \\ (1)^{1/8} \end{pmatrix}$$

Нужные степени матрицы D:

$$D^2 = \begin{pmatrix} (614656/5)^{1/4} & (15752961/625)^{1/4} & (15752961/625)^{1/4} & (194481/5)^{1/4} \\ (6561)^{1/4} & (194481/5)^{1/4} & (194481/5)^{1/4} & (6561/5)^{1/4} \\ (6561)^{1/4} & (614656/5)^{1/4} & (614656/5)^{1/4} & (1679616/125)^{1/4} \\ (614656/5)^{1/4} & (194481/5)^{1/4} & (194481/5)^{1/4} & (194481/5)^{1/4} \end{pmatrix}$$

$$D^3 = \begin{pmatrix} (4032758016/625)^{1/4} & (1475789056/25)^{1/4} & (1475789056/25)^{1/4} & (4032758016/625)^{1/4} \\ (49787136/5)^{1/4} & (15752961/5)^{1/4} & (15752961/5)^{1/4} & (15752961/5)^{1/4} \\ (157351936/5)^{1/4} & (4032758016/625)^{1/4} & (4032758016/625)^{1/4} & (49787136/5)^{1/4} \\ (49787136/5)^{1/4} & (1475789056/25)^{1/4} & (1475789056/25)^{1/4} & (4032758016/625)^{1/4} \end{pmatrix}$$

$$D^4 = \begin{pmatrix} (377801998336/25)^{1/4} & (9682651996416/3125)^{1/4} & (9682651996416/3125)^{1/4} & (119538913536/2 \\ (4032758016/5)^{1/4} & (119538913536/25)^{1/4} & (119538913536/25)^{1/4} & (326653399296/6 \\ (1032386052096/625)^{1/4} & (377801998336/25)^{1/4} & (377801998336/25)^{1/4} & (1032386052096/6 \\ (377801998336/25)^{1/4} & (119538913536/25)^{1/4} & (119538913536/25)^{1/4} & (119538913536/2 \end{pmatrix}$$

Спектральный радиус матрицы D:

$$\lambda_D = \mathrm{tr}D \oplus \cdots \oplus \mathrm{tr}^{1/4}(D^4) = (614656/5)^{1/8} \approx 4.32721$$

Матрица $\lambda^{-1}D$ и ее степени:

$$(\lambda^{-1}D)^1 = \begin{pmatrix} (5/614656)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (43046721/1920800000)^{1/8} \\ (3125/614656)^{1/8} & (5/614656)^{1/8} & (6561/7503125)^{1/8} & (32805/614656)^{1/8} \\ (1280/2401)^{1/8} & (5/2401)^{1/8} & (5/614656)^{1/8} & (32805/614656)^{1/8} \\ (32805/614656)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (5/614656)^{1/8} \end{pmatrix}$$

$$(\lambda^{-1}D)^2 = \begin{pmatrix} (1)^{1/8} & (43046721/1024000000)^{1/8} & (43046721/1024000000)^{1/8} & \\ (1076168025/377801998336)^{1/8} & (6561/65536)^{1/8} & (6561/65536)^{1/8} & (43 \\ (1076168025/377801998336)^{1/8} & (1)^{1/8} & (1)^{1/8} & (4 \\ (1)^{1/8} & (6561/65536)^{1/8} & (6561/65536)^{1/8} & \end{pmatrix}$$

$$(\lambda^{-1}D)^3 = \begin{pmatrix} (43046721/1920800000)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (4 \\ (32805/614656)^{1/8} & (215233605/40282095616)^{1/8} & (215233605/40282095616)^{1/8} & (21 \\ (1280/2401)^{1/8} & (43046721/1920800000)^{1/8} & (43046721/1920800000)^{1/8} & \\ (32805/614656)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (4 \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}D)^* = I \oplus (\lambda^{-1}D)^1 \oplus (\lambda^{-1}D)^2 \oplus (\lambda^{-1}D)^3 =$$

$$= \begin{pmatrix} 1 & (2401/1280)^{1/8} & (2401/1280)^{1/8} & (6561/65536)^{1/8} \\ (32805/614656)^{1/8} & 1 & (6561/65536)^{1/8} & (32805/614656)^{1/8} \\ (1280/2401)^{1/8} & (1)^{1/8} & 1 & (32805/614656)^{1/8} \\ (1)^{1/8} & (2401/1280)^{1/8} & (2401/1280)^{1/8} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (2401/1280)^{1/8} & (6561/65536)^{1/8} \\ (32805/614656)^{1/8} & 1 & (32805/614656)^{1/8} \\ (1280/2401)^{1/8} & (1)^{1/8} & (32805/614656)^{1/8} \\ (1)^{1/8} & (2401/1280)^{1/8} & 1 \end{pmatrix}$$

$$w_1 = \begin{pmatrix} 1 & (6561/65536)^{1/8} \\ (32805/614656)^{1/8} & (32805/614656)^{1/8} \\ (1280/2401)^{1/8} & (32805/614656)^{1/8} \\ (1)^{1/8} & (1)^{1/8} \end{pmatrix} \qquad w_2 = \begin{pmatrix} 1 \\ (1280/2401)^{1/8} \\ (1280/2401)^{1/8} \\ (1)^{1/8} \end{pmatrix}$$

$$w_{best} = \begin{pmatrix} 1.000000 & 0.750000 \\ 0.693288 & 0.693288 \\ 0.924384 & 0.693288 \\ 1.000000 & 1.000000 \end{pmatrix}$$

$$w_{worst} = \begin{pmatrix} 1.000000 \\ 0.924384 \\ 0.924384 \\ 1.000000 \end{pmatrix}$$

2. Решение задачи 2

Задача:

$$C = \begin{pmatrix} 1 & 1/4 & 1/5 & 1/4 & 6 & 1/6 \\ 4 & 1 & 1/3 & 3 & 6 & 1/2 \\ 5 & 3 & 1 & 4 & 7 & 3 \\ 4 & 1/3 & 1/4 & 1 & 5 & 1/5 \\ 1/6 & 1/6 & 1/7 & 1/5 & 1 & 1/7 \\ 6 & 2 & 1/3 & 5 & 7 & 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 5 & 8 \\ 1/5 & 1 & 5 \\ 1/8 & 1/5 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 7 & 9 \\ 1/7 & 1 & 7 \\ 1/9 & 1/7 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 1/7 & 1/9 \\ 7 & 1 & 1/7 \\ 9 & 7 & 1 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 4 \\ 1/5 & 1/4 & 1 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 4 \\ 1/5 & 1/4 & 1 \end{pmatrix}$$

$$A_6 = \begin{pmatrix} 1 & 7 & 9 \\ 1/7 & 1 & 7 \\ 1/9 & 1/7 & 1 \end{pmatrix}$$

Нужные степени матрицы C:

$$C^2 = \begin{pmatrix} 1 & 1 & 6/7 & 6/5 & 6 & 6/7 \\ 12 & 1 & 6/7 & 3 & 24 & 1 \\ 18 & 6 & 1 & 15 & 30 & 3 \\ 4 & 1 & 4/5 & 1 & 24 & 3/4 \\ 6/7 & 3/7 & 1/7 & 5/7 & 1 & 3/7 \\ 20 & 2 & 5/4 & 6 & 36 & 1 \end{pmatrix}$$

$$C^3 = \begin{pmatrix} 36/7 & 18/7 & 6/7 & 30/7 & 6 & 18/7 \\ 12 & 4 & 24/7 & 5 & 72 & 24/7 \\ 60 & 6 & 30/7 & 18 & 108 & 30/7 \\ 9/2 & 4 & 24/7 & 24/5 & 24 & 24/7 \\ 20/7 & 6/7 & 5/28 & 15/7 & 36/7 & 3/7 \\ 24 & 6 & 36/7 & 36/5 & 120 & 36/7 \end{pmatrix}$$

$$C^4 = \begin{pmatrix} 120/7 & 36/7 & 15/14 & 90/7 & 216/7 & 18/7 \\ 144/7 & 12 & 72/7 & 120/7 & 72 & 72/7 \\ 72 & 18 & 108/7 & 108/5 & 360 & 108/7 \\ 144/7 & 72/7 & 24/7 & 120/7 & 27 & 72/7 \\ 60/7 & 6/7 & 36/49 & 18/7 & 120/7 & 36/49 \\ 216/7 & 20 & 120/7 & 180/7 & 144 & 120/7 \end{pmatrix}$$

$$C^5 = \begin{pmatrix} 360/7 & 36/7 & 216/49 & 108/7 & 720/7 & 216/49 \\ 480/7 & 216/7 & 72/7 & 360/7 & 864/7 & 216/7 \\ 648/7 & 60 & 360/7 & 540/7 & 432 & 360/7 \\ 480/7 & 144/7 & 30/7 & 360/7 & 864/7 & 72/7 \\ 72/7 & 20/7 & 120/49 & 180/49 & 360/7 & 120/49 \\ 720/7 & 360/7 & 144/7 & 600/7 & 1296/7 & 360/7 \end{pmatrix}$$

$$C^6 = \begin{pmatrix} 432/7 & 120/7 & 720/49 & 1080/49 & 2160/7 & 720/49 \\ 1440/7 & 432/7 & 864/49 & 1080/7 & 2880/7 & 216/7 \\ 2160/7 & 1080/7 & 432/7 & 1800/7 & 3888/7 & 1080/7 \\ 1440/7 & 144/7 & 864/49 & 432/7 & 2880/7 & 864/49 \\ 720/49 & 60/7 & 360/49 & 600/49 & 432/7 & 360/49 \\ 2400/7 & 720/7 & 1296/49 & 1800/7 & 4320/7 & 432/7 \end{pmatrix}$$

Спектральный радиус матрицы C:

$$\lambda_C = \mathrm{tr}C \oplus \cdots \oplus \mathrm{tr}^{1/6}(C^6) = (360/7)^{1/5} \approx 2.19908$$

Матрица $\lambda^{-1}C$ и ее степени:

$$(\lambda^{-1}C)^1 = \begin{pmatrix} (7/360)^{1/5} & (7/368640)^{1/5} & (7/1125000)^{1/5} & (7/368640)^{1/5} & (756/5)^{1/5} & (7/2799 \\ (896/45)^{1/5} & (7/360)^{1/5} & (7/87480)^{1/5} & (189/40)^{1/5} & (756/5)^{1/5} & (7/115 \\ (4375/72)^{1/5} & (189/40)^{1/5} & (7/360)^{1/5} & (896/45)^{1/5} & (117649/360)^{1/5} & (189/4 \\ (896/45)^{1/5} & (7/87480)^{1/5} & (7/368640)^{1/5} & (7/360)^{1/5} & (4375/72)^{1/5} & (7/1125 \\ (7/2799360)^{1/5} & (7/2799360)^{1/5} & (1/864360)^{1/5} & (7/1125000)^{1/5} & (7/360)^{1/5} & (1/8643 \\ (756/5)^{1/5} & (28/45)^{1/5} & (7/87480)^{1/5} & (4375/72)^{1/5} & (117649/360)^{1/5} & (7/36 \end{pmatrix}$$

$$(\lambda^{-1}C)^2 = \begin{pmatrix} (49/129600)^{1/5} & (49/129600)^{1/5} & (3/17150)^{1/5} & (147/156250)^{1/5} & (147/50)^{1/5} & (\\ (2352/25)^{1/5} & (49/129600)^{1/5} & (3/17150)^{1/5} & (147/1600)^{1/5} & (75264/25)^{1/5} & (4 \\ (35721/50)^{1/5} & (147/50)^{1/5} & (49/129600)^{1/5} & (18375/64)^{1/5} & (18375/2)^{1/5} & (1 \\ (784/2025)^{1/5} & (49/129600)^{1/5} & (784/6328125)^{1/5} & (49/129600)^{1/5} & (75264/25)^{1/5} & (14 \\ (3/17150)^{1/5} & (3/548800)^{1/5} & (1/44452800)^{1/5} & (125/1778112)^{1/5} & (49/129600)^{1/5} & (\\ (98000/81)^{1/5} & (49/4050)^{1/5} & (6125/5308416)^{1/5} & (147/50)^{1/5} & (571536/25)^{1/5} & (4 \end{pmatrix}$$

$$(\lambda^{-1}C)^3 = \begin{pmatrix} (162/6125)^{1/5} & (81/98000)^{1/5} & (1/294000)^{1/5} & (25/2352)^{1/5} & (343/600 \\ (686/375)^{1/5} & (686/91125)^{1/5} & (64/18375)^{1/5} & (8575/373248)^{1/5} & (1778112/ \\ (17150/3)^{1/5} & (343/6000)^{1/5} & (25/2352)^{1/5} & (27783/2000)^{1/5} & (13502538/ \\ (27783/2048000)^{1/5} & (686/91125)^{1/5} & (64/18375)^{1/5} & (21952/1171875)^{1/5} & (21952/3 \\ (50/35721)^{1/5} & (1/294000)^{1/5} & (25/18728091648)^{1/5} & (25/75264)^{1/5} & (162/612 \\ (21952/375)^{1/5} & (343/6000)^{1/5} & (162/6125)^{1/5} & (55566/390625)^{1/5} & (548800/ \end{pmatrix}$$

$$(\lambda^{-1}C)^4 = \begin{pmatrix} (40/189)^{1/5} & (9/17500)^{1/5} & (5/24772608)^{1/5} & (45/896)^{1/5} & (17496/43 \\ (2304/4375)^{1/5} & (2401/67500)^{1/5} & (72/4375)^{1/5} & (40/189)^{1/5} & (172872/6 \\ (172872/625)^{1/5} & (21609/80000)^{1/5} & (2187/17500)^{1/5} & (5250987/7812500)^{1/5} & (864360 \\ (2304/4375)^{1/5} & (72/4375)^{1/5} & (8/118125)^{1/5} & (40/189)^{1/5} & (5250987/256 \\ (5/756)^{1/5} & (1/15120000)^{1/5} & (9/294122500)^{1/5} & (9/560000)^{1/5} & (40/189 \\ (17496/4375)^{1/5} & (12005/26244)^{1/5} & (40/189)^{1/5} & (45/28)^{1/5} & (5531904/6 \end{pmatrix}$$

$$(\lambda^{-1}C)^5 = \begin{pmatrix} (1)^{1/5} & (1/100000)^{1/5} & (243/52521875)^{1/5} & (243/100000)^{1/5} & (32)^{1/5} \\ (1024/243)^{1/5} & (243/3125)^{1/5} & (1/3125)^{1/5} & (1)^{1/5} & (248832/3125)^{1/5} \\ (59049/3125)^{1/5} & (16807/7776)^{1/5} & (1)^{1/5} & (243/32)^{1/5} & (130691232/3125)^{1} \\ (1024/243)^{1/5} & (32/3125)^{1/5} & (1/248832)^{1/5} & (1)^{1/5} & (248832/3125)^{1/5} \\ (1/3125)^{1/5} & (1/1889568)^{1/5} & (1/4084101)^{1/5} & (1/537824)^{1/5} & (1)^{1/5} \\ (32)^{1/5} & (1)^{1/5} & (32/3125)^{1/5} & (3125/243)^{1/5} & (1889568/3125)^{1/5} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}C)^* = I \oplus (\lambda^{-1}C)^1 \oplus (\lambda^{-1}C)^2 \oplus (\lambda^{-1}C)^3 \oplus (\lambda^{-1}C)^4 \oplus (\lambda^{-1}C)^5 =$$

$$= \begin{pmatrix} 1 & (81/98000)^{1/5} & (3/17150)^{1/5} & (45/896)^{1/5} & (756/5)^{1/5} & (81/98000)^{1/5} \\ (2352/25)^{1/5} & 1 & (72/4375)^{1/5} & (189/40)^{1/5} & (1778112/125)^{1/5} & (243/3125)^{1/5} \\ (17150/3)^{1/5} & (189/40)^{1/5} & 1 & (18375/64)^{1/5} & (864360)^{1/5} & (189/40)^{1/5} \\ (896/45)^{1/5} & (72/4375)^{1/5} & (64/18375)^{1/5} & 1 & (75264/25)^{1/5} & (72/4375)^{1/5} \\ (5/756)^{1/5} & (3/548800)^{1/5} & (1/864360)^{1/5} & (25/75264)^{1/5} & 1 & (3/548800)^{1/5} \\ (98000/81)^{1/5} & (1)^{1/5} & (40/189)^{1/5} & (4375/72)^{1/5} & (548800/3)^{1/5} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (81/98000)^{1/5} \\ (2352/25)^{1/5} & 1 \\ (17150/3)^{1/5} & (189/40)^{1/5} \\ (896/45)^{1/5} & (72/4375)^{1/5} \\ (5/756)^{1/5} & (3/548800)^{1/5} \\ (98000/81)^{1/5} & (1)^{1/5} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (3/17150)^{1/5} & (3/17150)^{1/5} \\ (72/4375)^{1/5} & (40/189)^{1/5} \\ (1)^{1/5} & (1)^{1/5} \\ (64/18375)^{1/5} & (64/18375)^{1/5} \\ (1/864360)^{1/5} & (1/864360)^{1/5} \\ (40/189)^{1/5} & (40/189)^{1/5} \end{pmatrix} \qquad w_2 = \begin{pmatrix} (3/17150)^{1/5} & (3/17150)^{1/5} \\ (72/4375)^{1/5} & (40/189)^{1/5} \\ (1)^{1/5} & (1)^{1/5} \\ (64/18375)^{1/5} & (64/18375)^{1/5} \\ (1/864360)^{1/5} & (1/864360)^{1/5} \\ (40/189)^{1/5} & (40/189)^{1/5} \end{pmatrix}$$

$$B = \begin{pmatrix} (1)^{1/5} & (96040/27)^{1/5} & (87480/7)^{1/5} \\ (16807)^{1/5} & (1)^{1/5} & (96040/27)^{1/5} \\ (59049)^{1/5} & (16807)^{1/5} & (1)^{1/5} \end{pmatrix}$$

$$D = \begin{pmatrix} (1)^{1/5} & (96040/27)^{1/5} & (87480/7)^{1/5} \\ (16807)^{1/5} & (1)^{1/5} & (96040/27)^{1/5} \\ (59049)^{1/5} & (16807)^{1/5} & (1)^{1/5} \end{pmatrix}$$

Нужные степени матрицы B:

$$B^2 = \begin{pmatrix} (5165606520/7)^{1/5} & (210039480)^{1/5} & (9223681600/729)^{1/5} \\ (210039480)^{1/5} & (1614144280/27)^{1/5} & (210039480)^{1/5} \\ (282475249)^{1/5} & (210039480)^{1/5} & (5165606520/7)^{1/5} \end{pmatrix}$$

$$B^3 = \begin{pmatrix} (3530133540360)^{1/5} & (2624893387200)^{1/5} & (451887258369600/49)^{1/5} \\ (12402621254520)^{1/5} & (3530133540360)^{1/5} & (2624893387200)^{1/5} \\ (305023899399480/7)^{1/5} & (12402621254520)^{1/5} & (3530133540360)^{1/5} \end{pmatrix}$$

Спектральный радиус матрицы B:

$$\lambda_B = \mathrm{tr}B \oplus \cdots \oplus \mathrm{tr}^{1/3}(B^3) = (5165606520/7)^{1/10} \approx 7.70553$$

Матрица $\lambda^{-1}B$ и ее степени:

$$(\lambda^{-1}B)^1 = \begin{pmatrix} (7/5165606520)^{1/10} & (1614144280/94143178827)^{1/10} & (40/189)^{1/10} \\ (1977326743/5165606520)^{1/10} & (7/5165606520)^{1/10} & (1614144280/94143178827)^{1/} \\ (189/40)^{1/10} & (1977326743/5165606520)^{1/10} & (7/5165606520)^{1/10} \end{pmatrix}$$

$$(\lambda^{-1}B)^2 = \begin{pmatrix} (1)^{1/10} & (282475249/3486784401)^{1/10} \\ (282475249/3486784401)^{1/10} & (79792266297612001/121576654590569 \\ (3909821048582988049/26683490719466510400)^{1/10} & (282475249/3486784401)^{1/10} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}B)^* = I \oplus (\lambda^{-1}B)^1 \oplus (\lambda^{-1}B)^2 =$$

$$= \begin{pmatrix} 1 & (282475249/3486784401)^{1/10} & (40/189)^{1/10} \\ (1977326743/5165606520)^{1/10} & 1 & (282475249/3486784401)^{1/10} \\ (189/40)^{1/10} & (1977326743/5165606520)^{1/10} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (282475249/3486784401)^{1/10} \\ (1977326743/5165606520)^{1/10} & 1 \\ (189/40)^{1/10} & (1977326743/5165606520)^{1/10} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (40/189)^{1/10} & (282475249/3486784401)^{1/10} \\ (282475249/3486784401)^{1/10} & (1)^{1/10} \\ (1)^{1/10} & (1977326743/5165606520)^{1/10} \end{pmatrix} \qquad w_2 = \begin{pmatrix} (40/189)^{1/10} & (40/189)^{1/} \\ (40/189)^{1/10} & 1 \\ (1)^{1/10} & (1)^{1/10} \end{pmatrix}$$

Нужные степени матрицы D:

$$D^2 = \begin{pmatrix} (5165606520/7)^{1/5} & (210039480)^{1/5} & (9223681600/729)^{1/5} \\ (210039480)^{1/5} & (1614144280/27)^{1/5} & (210039480)^{1/5} \\ (282475249)^{1/5} & (210039480)^{1/5} & (5165606520/7)^{1/5} \end{pmatrix}$$

$$D^3 = \begin{pmatrix} (3530133540360)^{1/5} & (2624893387200)^{1/5} & (451887258369600/49)^{1/5} \\ (12402621254520)^{1/5} & (3530133540360)^{1/5} & (2624893387200)^{1/5} \\ (305023899399480/7)^{1/5} & (12402621254520)^{1/5} & (3530133540360)^{1/5} \end{pmatrix}$$

Спектральный радиус матрицы D:

$$\lambda_D = \mathrm{tr}D \oplus \cdots \oplus \mathrm{tr}^{1/3}(D^3) = (5165606520/7)^{1/10} \approx 7.70553$$

Матрица $\lambda^{-1}D$ и ее степени:

$$(\lambda^{-1}D)^1 = \begin{pmatrix} (7/5165606520)^{1/10} & (1614144280/94143178827)^{1/10} & (40/189)^{1/10} \\ (1977326743/5165606520)^{1/10} & (7/5165606520)^{1/10} & (1614144280/94143178827)^{1/} \\ (189/40)^{1/10} & (1977326743/5165606520)^{1/10} & (7/5165606520)^{1/10} \end{pmatrix}$$

$$(\lambda^{-1}D)^2 = \begin{pmatrix} (1)^{1/10} & (282475249/3486784401)^{1/10} \\ (282475249/3486784401)^{1/10} & (79792266297612001/121576654590569 \\ (3909821048582988049/26683490719466510400)^{1/10} & (282475249/3486784401)^{1/10} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}D)^* = I \oplus (\lambda^{-1}D)^1 \oplus (\lambda^{-1}D)^2 =$$

$$= \begin{pmatrix} 1 & (282475249/3486784401)^{1/10} & (40/189)^{1/10} \\ (1977326743/5165606520)^{1/10} & 1 & (282475249/3486784401)^{1/10} \\ (189/40)^{1/10} & (1977326743/5165606520)^{1/10} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (282475249/3486784401)^{1/10} \\ (1977326743/5165606520)^{1/10} & 1 \\ (189/40)^{1/10} & (1977326743/5165606520)^{1/10} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (40/189)^{1/10} & (282475249/3486784401)^{1/10} \\ (282475249/3486784401)^{1/10} & (1)^{1/10} \\ (1)^{1/10} & (1977326743/5165606520)^{1/10} \end{pmatrix} \qquad w_2 = \begin{pmatrix} (40/189)^{1/10} & (40/189)^{1/} \\ (40/189)^{1/10} & 1 \\ (1)^{1/10} & (1)^{1/10} \end{pmatrix}$$

$$w_{best} = \begin{pmatrix} 0.856170 & 0.777778 \\ 0.777778 & 1.000000 \\ 1.000000 & 0.908439 \end{pmatrix}$$

$$w_{worst} = \begin{pmatrix} 0.856170 & 0.856170 \\ 0.856170 & 1.000000 \\ 1.000000 & 1.000000 \end{pmatrix}$$

3. Решение задачи 9

Задача:

$$C = \begin{pmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/2 \\ 2 & 1 & 1/2 & 1/2 & 1 \\ 3 & 2 & 1 & 1/2 & 2 \\ 4 & 2 & 2 & 1 & 2 \\ 2 & 1 & 1/2 & 1/2 & 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/2 \\ 1 & 2 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 1/2 & 3 \\ 2 & 1 & 4 \\ 1/3 & 1/4 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 1/3 & 1/5 \\ 3 & 1 & 1/2 \\ 5 & 2 & 1 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 1 & 1/2 & 1/2 \\ 2 & 1 & 2 \\ 2 & 1/2 & 1 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} 1 & 5 & 1 \\ 1/5 & 1 & 1/2 \\ 1 & 2 & 1 \end{pmatrix}$$

Нужные степени матрицы C:

$$C^2 = \begin{pmatrix} 1 & 2/3 & 1/2 & 1/4 & 2/3 \\ 2 & 1 & 1 & 1/2 & 1 \\ 4 & 2 & 1 & 1 & 2 \\ 6 & 4 & 2 & 1 & 4 \\ 2 & 1 & 1 & 1/2 & 1 \end{pmatrix}$$

$$C^3 = \begin{pmatrix} 3/2 & 1 & 1/2 & 1/3 & 1 \\ 3 & 2 & 1 & 1/2 & 2 \\ 4 & 2 & 2 & 1 & 2 \\ 8 & 4 & 2 & 2 & 4 \\ 3 & 2 & 1 & 1/2 & 2 \end{pmatrix}$$

$$C^4 = \begin{pmatrix} 2 & 1 & 2/3 & 1/2 & 1 \\ 4 & 2 & 1 & 1 & 2 \\ 6 & 4 & 2 & 1 & 4 \\ 8 & 4 & 4 & 2 & 4 \\ 4 & 2 & 1 & 1 & 2 \end{pmatrix}$$

$$C^5 = \begin{pmatrix} 2 & 4/3 & 1 & 1/2 & 4/3 \\ 4 & 2 & 2 & 1 & 2 \\ 8 & 4 & 2 & 2 & 4 \\ 12 & 8 & 4 & 2 & 8 \\ 4 & 2 & 2 & 1 & 2 \end{pmatrix}$$

Спектральный радиус матрицы C:

$$\lambda_C = \mathrm{tr}C \oplus \cdots \oplus \mathrm{tr}^{1/5}(C^5) = (2)^{1/3} \approx 1.25992$$

Матрица $\lambda^{-1}C$ и ее степени:

$$(\lambda^{-1}C)^1 = \begin{pmatrix} (1/2)^{1/3} & (1/16)^{1/3} & (1/54)^{1/3} & (1/128)^{1/3} & (1/16)^{1/3} \\ (4)^{1/3} & (1/2)^{1/3} & (1/16)^{1/3} & (1/16)^{1/3} & (1/2)^{1/3} \\ (27/2)^{1/3} & (4)^{1/3} & (1/2)^{1/3} & (1/16)^{1/3} & (4)^{1/3} \\ (32)^{1/3} & (4)^{1/3} & (4)^{1/3} & (1/2)^{1/3} & (4)^{1/3} \\ (4)^{1/3} & (1/2)^{1/3} & (1/16)^{1/3} & (1/16)^{1/3} & (1/2)^{1/3} \end{pmatrix}$$

$$(\lambda^{-1}C)^2 = \begin{pmatrix} (1/4)^{1/3} & (2/27)^{1/3} & (1/32)^{1/3} & (1/256)^{1/3} & (2/27)^{1/3} \\ (2)^{1/3} & (1/4)^{1/3} & (1/4)^{1/3} & (1/32)^{1/3} & (1/4)^{1/3} \\ (16)^{1/3} & (2)^{1/3} & (1/4)^{1/3} & (1/4)^{1/3} & (2)^{1/3} \\ (54)^{1/3} & (16)^{1/3} & (2)^{1/3} & (1/4)^{1/3} & (16)^{1/3} \\ (2)^{1/3} & (1/4)^{1/3} & (1/4)^{1/3} & (1/32)^{1/3} & (1/4)^{1/3} \end{pmatrix}$$

$$(\lambda^{-1}C)^3 = \begin{pmatrix} (27/64)^{1/3} & (1/8)^{1/3} & (1/64)^{1/3} & (1/216)^{1/3} & (1/8)^{1/3} \\ (27/8)^{1/3} & (1)^{1/3} & (1/8)^{1/3} & (1/64)^{1/3} & (1)^{1/3} \\ (8)^{1/3} & (1)^{1/3} & (1)^{1/3} & (1/8)^{1/3} & (1)^{1/3} \\ (64)^{1/3} & (8)^{1/3} & (1)^{1/3} & (1)^{1/3} & (8)^{1/3} \\ (27/8)^{1/3} & (1)^{1/3} & (1/8)^{1/3} & (1/64)^{1/3} & (1)^{1/3} \end{pmatrix}$$

$$(\lambda^{-1}C)^4 = \begin{pmatrix} (1/2)^{1/3} & (1/16)^{1/3} & (1/54)^{1/3} & (1/128)^{1/3} & (1/16)^{1/3} \\ (4)^{1/3} & (1/2)^{1/3} & (1/16)^{1/3} & (1/16)^{1/3} & (1/2)^{1/3} \\ (27/2)^{1/3} & (4)^{1/3} & (1/2)^{1/3} & (1/16)^{1/3} & (4)^{1/3} \\ (32)^{1/3} & (4)^{1/3} & (4)^{1/3} & (1/2)^{1/3} & (4)^{1/3} \\ (4)^{1/3} & (1/2)^{1/3} & (1/16)^{1/3} & (1/16)^{1/3} & (1/2)^{1/3} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}C)^* = I \oplus (\lambda^{-1}C)^1 \oplus (\lambda^{-1}C)^2 \oplus (\lambda^{-1}C)^3 \oplus (\lambda^{-1}C)^4 =$$

$$= \begin{pmatrix} 1 & (1/8)^{1/3} & (1/32)^{1/3} & (1/128)^{1/3} & (1/8)^{1/3} \\ (4)^{1/3} & 1 & (1/4)^{1/3} & (1/16)^{1/3} & (1)^{1/3} \\ (16)^{1/3} & (4)^{1/3} & 1 & (1/4)^{1/3} & (4)^{1/3} \\ (64)^{1/3} & (16)^{1/3} & (4)^{1/3} & 1 & (16)^{1/3} \\ (4)^{1/3} & (1)^{1/3} & (1/4)^{1/3} & (1/16)^{1/3} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (1/8)^{1/3} \\ (4)^{1/3} & 1 \\ (16)^{1/3} & (4)^{1/3} \\ (64)^{1/3} & (16)^{1/3} \\ (4)^{1/3} & (1)^{1/3} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (1/128)^{1/3} \\ (1/16)^{1/3} \\ (1/4)^{1/3} \\ (1)^{1/3} \\ (1/16)^{1/3} \end{pmatrix} \qquad w_2 = \begin{pmatrix} (1/64)^{1/3} \\ (1/16)^{1/3} \\ (1/4)^{1/3} \\ (1)^{1/3} \\ (1/16)^{1/3} \end{pmatrix}$$

$$B = \begin{pmatrix} (1)^{1/3} & (125/16)^{1/3} & (27/16)^{1/3} \\ (8)^{1/3} & (1)^{1/3} & (8)^{1/3} \\ (125/4)^{1/3} & (2)^{1/3} & (1)^{1/3} \end{pmatrix}$$

$$D = \begin{pmatrix} (1)^{1/3} & (125/16)^{1/3} & (27/16)^{1/3} \\ (8)^{1/3} & (1)^{1/3} & (8)^{1/3} \\ (125/4)^{1/3} & (2)^{1/3} & (1)^{1/3} \end{pmatrix}$$

Нужные степени матрицы B:

$$B^2 = \begin{pmatrix} (125/2)^{1/3} & (125/16)^{1/3} & (125/2)^{1/3} \\ (250)^{1/3} & (125/2)^{1/3} & (27/2)^{1/3} \\ (125/4)^{1/3} & (15625/64)^{1/3} & (3375/64)^{1/3} \end{pmatrix}$$

$$B^3 = \begin{pmatrix} (15625/8)^{1/3} & (15625/32)^{1/3} & (3375/32)^{1/3} \\ (500)^{1/3} & (15625/8)^{1/3} & (500)^{1/3} \\ (15625/8)^{1/3} & (15625/64)^{1/3} & (15625/8)^{1/3} \end{pmatrix}$$

Спектральный радиус матрицы B:

$$\lambda_B = \mathrm{tr}B \oplus \cdots \oplus \mathrm{tr}^{1/3}(B^3) = (15625/8)^{1/9} \approx 2.32079$$

Матрица $\lambda^{-1}B$ и ее степени:

$$(\lambda^{-1}B)^1 = \begin{pmatrix} (8/15625)^{1/9} & (125/512)^{1/9} & (19683/8000000)^{1/9} \\ (4096/15625)^{1/9} & (8/15625)^{1/9} & (4096/15625)^{1/9} \\ (125/8)^{1/9} & (64/15625)^{1/9} & (8/15625)^{1/9} \end{pmatrix}$$

$$(\lambda^{-1}B)^2 = \begin{pmatrix} (8/125)^{1/9} & (1/8000)^{1/9} & (8/125)^{1/9} \\ (512/125)^{1/9} & (8/125)^{1/9} & (157464/244140625)^{1/9} \\ (1/125)^{1/9} & (15625/4096)^{1/9} & (19683/512000)^{1/9} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}B)^* = I \oplus (\lambda^{-1}B)^1 \oplus (\lambda^{-1}B)^2 =$$

$$= \begin{pmatrix} 1 & (125/512)^{1/9} & (8/125)^{1/9} \\ (512/125)^{1/9} & 1 & (4096/15625)^{1/9} \\ (125/8)^{1/9} & (15625/4096)^{1/9} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 \\ (512/125)^{1/9} \\ (125/8)^{1/9} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (8/125)^{1/9} \\ (4096/15625)^{1/9} \\ (1)^{1/9} \end{pmatrix} \qquad w_2 = \begin{pmatrix} (8/125)^{1/9} \\ (4096/15625)^{1/9} \\ (1)^{1/9} \end{pmatrix}$$

Нужные степени матрицы D:

$$D^2 = \begin{pmatrix} (125/2)^{1/3} & (125/16)^{1/3} & (125/2)^{1/3} \\ (250)^{1/3} & (125/2)^{1/3} & (27/2)^{1/3} \\ (125/4)^{1/3} & (15625/64)^{1/3} & (3375/64)^{1/3} \end{pmatrix}$$

$$D^3 = \begin{pmatrix} (15625/8)^{1/3} & (15625/32)^{1/3} & (3375/32)^{1/3} \\ (500)^{1/3} & (15625/8)^{1/3} & (500)^{1/3} \\ (15625/8)^{1/3} & (15625/64)^{1/3} & (15625/8)^{1/3} \end{pmatrix}$$

Спектральный радиус матрицы D:

$$\lambda_D = \mathrm{tr} D \oplus \cdots \oplus \mathrm{tr}^{1/3}(D^3) = (15625/8)^{1/9} \approx 2.32079$$

Матрица $\lambda^{-1}D$ и ее степени:

$$(\lambda^{-1}D)^1 = \begin{pmatrix} (8/15625)^{1/9} & (125/512)^{1/9} & (19683/8000000)^{1/9} \\ (4096/15625)^{1/9} & (8/15625)^{1/9} & (4096/15625)^{1/9} \\ (125/8)^{1/9} & (64/15625)^{1/9} & (8/15625)^{1/9} \end{pmatrix}$$

$$(\lambda^{-1}D)^2 = \begin{pmatrix} (8/125)^{1/9} & (1/8000)^{1/9} & (8/125)^{1/9} \\ (512/125)^{1/9} & (8/125)^{1/9} & (157464/244140625)^{1/9} \\ (1/125)^{1/9} & (15625/4096)^{1/9} & (19683/512000)^{1/9} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}D)^* = I \oplus (\lambda^{-1}D)^1 \oplus (\lambda^{-1}D)^2 =$$

$$= \begin{pmatrix} 1 & (125/512)^{1/9} & (8/125)^{1/9} \\ (512/125)^{1/9} & 1 & (4096/15625)^{1/9} \\ (125/8)^{1/9} & (15625/4096)^{1/9} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 \\ (512/125)^{1/9} \\ (125/8)^{1/9} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (8/125)^{1/9} \\ (4096/15625)^{1/9} \\ (1)^{1/9} \end{pmatrix} \qquad w_2 = \begin{pmatrix} (8/125)^{1/9} \\ (4096/15625)^{1/9} \\ (1)^{1/9} \end{pmatrix}$$

$$w_{best} = \begin{pmatrix} 0.736806 \\ 0.861774 \\ 1.000000 \end{pmatrix}$$

$$w_{worst} = \begin{pmatrix} 0.736806 \\ 0.861774 \\ 1.000000 \end{pmatrix}$$

4. Решение задачи 1 из статьи стр 9

Задача:

$$C = \begin{pmatrix} 1 & 4 & 2 & 2 \\ 1/4 & 1 & 1/3 & 1/4 \\ 1/2 & 3 & 1 & 3 \\ 1/2 & 4 & 1/3 & 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 4 & 2 & 2 \\ 1/4 & 1 & 1/3 & 1/4 \\ 1/2 & 3 & 1 & 3 \\ 1/2 & 4 & 1/3 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 4 & 2 & 2 \\ 1/4 & 1 & 1/3 & 1/4 \\ 1/2 & 3 & 1 & 3 \\ 1/2 & 4 & 1/3 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 1/4 & 1/4 & 1/3 \\ 4 & 1 & 2 & 2 \\ 4 & 1/2 & 1 & 1/3 \\ 3 & 1/2 & 3 & 1 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 1 & 1/4 & 1/4 & 1/3 \\ 4 & 1 & 2 & 2 \\ 4 & 1/2 & 1 & 1/3 \\ 3 & 1/2 & 3 & 1 \end{pmatrix}$$

Нужные степени матрицы C:

$$C^2 = \begin{pmatrix} 1 & 8 & 2 & 6 \\ 1/4 & 1 & 1/2 & 1 \\ 3/2 & 12 & 1 & 3 \\ 1 & 4 & 4/3 & 1 \end{pmatrix}$$

$$C^3 = \begin{pmatrix} 3 & 24 & 8/3 & 6 \\ 1/2 & 4 & 1/2 & 3/2 \\ 3 & 12 & 4 & 3 \\ 1 & 4 & 2 & 4 \end{pmatrix}$$

$$C^4 = \begin{pmatrix} 6 & 24 & 8 & 8 \\ 1 & 6 & 4/3 & 3/2 \\ 3 & 12 & 6 & 12 \\ 2 & 16 & 2 & 6 \end{pmatrix}$$

Спектральный радиус матрицы C:

$$\lambda_C = \mathrm{tr}C \oplus \cdots \oplus \mathrm{tr}^{1/4}(C^4) = (4)^{1/3} \approx 1.5874$$

Матрица $\lambda^{-1}C$ и ее степени:

$$(\lambda^{-1}C)^1 = \begin{pmatrix} (1/4)^{1/3} & (16)^{1/3} & (2)^{1/3} & (2)^{1/3} \\ (1/256)^{1/3} & (1/4)^{1/3} & (1/108)^{1/3} & (1/256)^{1/3} \\ (1/32)^{1/3} & (27/4)^{1/3} & (1/4)^{1/3} & (27/4)^{1/3} \\ (1/32)^{1/3} & (16)^{1/3} & (1/108)^{1/3} & (1/4)^{1/3} \end{pmatrix}$$

$$(\lambda^{-1}C)^2 = \begin{pmatrix} (1/16)^{1/3} & (32)^{1/3} & (1/2)^{1/3} & (27/2)^{1/3} \\ (1/1024)^{1/3} & (1/16)^{1/3} & (1/128)^{1/3} & (1/16)^{1/3} \\ (27/128)^{1/3} & (108)^{1/3} & (1/16)^{1/3} & (27/16)^{1/3} \\ (1/16)^{1/3} & (4)^{1/3} & (4/27)^{1/3} & (1/16)^{1/3} \end{pmatrix}$$

$$(\lambda^{-1}C)^3 = \begin{pmatrix} (27/64)^{1/3} & (216)^{1/3} & (8/27)^{1/3} & (27/8)^{1/3} \\ (1/512)^{1/3} & (1)^{1/3} & (1/512)^{1/3} & (27/512)^{1/3} \\ (27/64)^{1/3} & (27)^{1/3} & (1)^{1/3} & (27/64)^{1/3} \\ (1/64)^{1/3} & (1)^{1/3} & (1/8)^{1/3} & (1)^{1/3} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}C)^* = I \oplus (\lambda^{-1}C)^1 \oplus (\lambda^{-1}C)^2 \oplus (\lambda^{-1}C)^3 =$$

$$= \begin{pmatrix} 1 & (216)^{1/3} & (2)^{1/3} & (27/2)^{1/3} \\ (1/256)^{1/3} & 1 & (1/108)^{1/3} & (1/16)^{1/3} \\ (27/64)^{1/3} & (108)^{1/3} & 1 & (27/4)^{1/3} \\ (1/16)^{1/3} & (16)^{1/3} & (4/27)^{1/3} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (216)^{1/3} \\ (1/256)^{1/3} & 1 \\ (27/64)^{1/3} & (108)^{1/3} \\ (1/16)^{1/3} & (16)^{1/3} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} 1 \\ (1/256)^{1/3} \\ (27/64)^{1/3} \\ (1/16)^{1/3} \end{pmatrix} \qquad w_2 = \begin{pmatrix} 1 \\ (1/216)^{1/3} \\ (1/2)^{1/3} \\ (2/27)^{1/3} \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 4 & 2 & 2 \\ (27)^{1/3} & 1 & (27/8)^{1/3} & (27/8)^{1/3} \\ (27)^{1/3} & 3 & 1 & 3 \\ (729/64)^{1/3} & 4 & (729/64)^{1/3} & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 4 & 2 & 2 \\ (32)^{1/3} & 1 & (4)^{1/3} & (4)^{1/3} \\ (32)^{1/3} & 3 & 1 & 3 \\ (27/2)^{1/3} & 4 & (27/2)^{1/3} & 1 \end{pmatrix}$$

Нужные степени матрицы B:

$$B^2 = \begin{pmatrix} (1728)^{1/3} & 8 & (216)^{1/3} & (216)^{1/3} \\ (729/8)^{1/3} & (1728)^{1/3} & (216)^{1/3} & (216)^{1/3} \\ (729)^{1/3} & (1728)^{1/3} & (19683/64)^{1/3} & (216)^{1/3} \\ (1728)^{1/3} & (729)^{1/3} & (216)^{1/3} & (19683/64)^{1/3} \end{pmatrix}$$

$$B^3 = \begin{pmatrix} (13824)^{1/3} & (110592)^{1/3} & (13824)^{1/3} & (13824)^{1/3} \\ (46656)^{1/3} & (13824)^{1/3} & (5832)^{1/3} & (5832)^{1/3} \\ (46656)^{1/3} & (46656)^{1/3} & (5832)^{1/3} & (531441/64)^{1/3} \\ (19683)^{1/3} & (110592)^{1/3} & (13824)^{1/3} & (13824)^{1/3} \end{pmatrix}$$

$$B^4 = \begin{pmatrix} (2985984)^{1/3} & (884736)^{1/3} & (373248)^{1/3} & (373248)^{1/3} \\ (373248)^{1/3} & (2985984)^{1/3} & (373248)^{1/3} & (373248)^{1/3} \\ (1259712)^{1/3} & (2985984)^{1/3} & (373248)^{1/3} & (373248)^{1/3} \\ (2985984)^{1/3} & (1259712)^{1/3} & (373248)^{1/3} & (373248)^{1/3} \end{pmatrix}$$

Спектральный радиус матрицы B:

$$\lambda_B = \mathrm{tr}B \oplus \cdots \oplus \mathrm{tr}^{1/4}(B^4) = (1728)^{1/6} \approx 3.4641$$

Матрица $\lambda^{-1}B$ и ее степени:

$$(\lambda^{-1}B)^1 = \begin{pmatrix} (1/1728)^{1/6} & (64/27)^{1/6} & (1/27)^{1/6} & (1/27)^{1/6} \\ (27/64)^{1/6} & (1/1728)^{1/6} & (27/4096)^{1/6} & (27/4096)^{1/6} \\ (27/64)^{1/6} & (27/64)^{1/6} & (1/1728)^{1/6} & (27/64)^{1/6} \\ (19683/262144)^{1/6} & (64/27)^{1/6} & (19683/262144)^{1/6} & (1/1728)^{1/6} \end{pmatrix}$$

$$(\lambda^{-1}B)^2 = \begin{pmatrix} (1)^{1/6} & (64/729)^{1/6} & (1/64)^{1/6} & (1/64)^{1/6} \\ (729/262144)^{1/6} & (1)^{1/6} & (1/64)^{1/6} & (1/64)^{1/6} \\ (729/4096)^{1/6} & (1)^{1/6} & (531441/16777216)^{1/6} & (1/64)^{1/6} \\ (1)^{1/6} & (729/4096)^{1/6} & (1/64)^{1/6} & (531441/16777216)^{1/6} \end{pmatrix}$$

$$(\lambda^{-1}B)^3 = \begin{pmatrix} (1/27)^{1/6} & (64/27)^{1/6} & (1/27)^{1/6} & (1/27)^{1/6} \\ (27/64)^{1/6} & (1/27)^{1/6} & (27/4096)^{1/6} & (27/4096)^{1/6} \\ (27/64)^{1/6} & (27/64)^{1/6} & (27/4096)^{1/6} & (14348907/1073741824)^{1/6} \\ (19683/262144)^{1/6} & (64/27)^{1/6} & (1/27)^{1/6} & (1/27)^{1/6} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}B)^* = I \oplus (\lambda^{-1}B)^1 \oplus (\lambda^{-1}B)^2 \oplus (\lambda^{-1}B)^3 =$$

$$= \begin{pmatrix} 1 & (64/27)^{1/6} & (1/27)^{1/6} & (1/27)^{1/6} \\ (27/64)^{1/6} & 1 & (1/64)^{1/6} & (1/64)^{1/6} \\ (27/64)^{1/6} & (1)^{1/6} & 1 & (27/64)^{1/6} \\ (1)^{1/6} & (64/27)^{1/6} & (19683/262144)^{1/6} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (1/27)^{1/6} & (1/27)^{1/6} \\ (27/64)^{1/6} & (1/64)^{1/6} & (1/64)^{1/6} \\ (27/64)^{1/6} & 1 & (27/64)^{1/6} \\ (1)^{1/6} & (19683/262144)^{1/6} & 1 \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (1/27)^{1/6} & (1/27)^{1/6} & (1/27)^{1/6} \\ (1/64)^{1/6} & (1/64)^{1/6} & (1/64)^{1/6} \\ (1)^{1/6} & (1)^{1/6} & (27/64)^{1/6} \\ (19683/262144)^{1/6} & (1)^{1/6} & (1)^{1/6} \end{pmatrix} \qquad w_2 = \begin{pmatrix} 1 & (1)^{1/6} \\ (27/64)^{1/6} & (27/64)^{1/6} \\ (27/64)^{1/6} & (1)^{1/6} \\ (1)^{1/6} & (1)^{1/6} \end{pmatrix}$$

Нужные степени матрицы D:

$$D^2 = \begin{pmatrix} (2048)^{1/3} & 8 & (256)^{1/3} & (256)^{1/3} \\ (128)^{1/3} & (2048)^{1/3} & (256)^{1/3} & (256)^{1/3} \\ (864)^{1/3} & (2048)^{1/3} & (729/2)^{1/3} & (256)^{1/3} \\ (2048)^{1/3} & (864)^{1/3} & (256)^{1/3} & (729/2)^{1/3} \end{pmatrix}$$

$$D^3 = \begin{pmatrix} (16384)^{1/3} & (131072)^{1/3} & (16384)^{1/3} & (16384)^{1/3} \\ (65536)^{1/3} & (16384)^{1/3} & (8192)^{1/3} & (8192)^{1/3} \\ (65536)^{1/3} & (55296)^{1/3} & (8192)^{1/3} & (19683/2)^{1/3} \\ (27648)^{1/3} & (131072)^{1/3} & (16384)^{1/3} & (16384)^{1/3} \end{pmatrix}$$

$$D^4 = \begin{pmatrix} (4194304)^{1/3} & (1048576)^{1/3} & (524288)^{1/3} & (524288)^{1/3} \\ (524288)^{1/3} & (4194304)^{1/3} & (524288)^{1/3} & (524288)^{1/3} \\ (1769472)^{1/3} & (4194304)^{1/3} & (524288)^{1/3} & (524288)^{1/3} \\ (4194304)^{1/3} & (1769472)^{1/3} & (524288)^{1/3} & (524288)^{1/3} \end{pmatrix}$$

Спектральный радиус матрицы D:

$$\lambda_D = \mathrm{tr}D \oplus \cdots \oplus \mathrm{tr}^{1/4}(D^4) = (2048)^{1/6} \approx 3.56359$$

Матрица $\lambda^{-1}D$ и ее степени:

$$(\lambda^{-1}D)^1 = \begin{pmatrix} (1/2048)^{1/6} & (2)^{1/6} & (1/32)^{1/6} & (1/32)^{1/6} \\ (1/2)^{1/6} & (1/2048)^{1/6} & (1/128)^{1/6} & (1/128)^{1/6} \\ (1/2)^{1/6} & (729/2048)^{1/6} & (1/2048)^{1/6} & (729/2048)^{1/6} \\ (729/8192)^{1/6} & (2)^{1/6} & (729/8192)^{1/6} & (1/2048)^{1/6} \end{pmatrix}$$

$$(\lambda^{-1}D)^2 = \begin{pmatrix} (1)^{1/6} & (1/16)^{1/6} & (1/64)^{1/6} & (1/64)^{1/6} \\ (1/256)^{1/6} & (1)^{1/6} & (1/64)^{1/6} & (1/64)^{1/6} \\ (729/4096)^{1/6} & (1)^{1/6} & (531441/16777216)^{1/6} & (1/64)^{1/6} \\ (1)^{1/6} & (729/4096)^{1/6} & (1/64)^{1/6} & (531441/16777216)^{1/6} \end{pmatrix}$$

$$(\lambda^{-1}D)^3 = \begin{pmatrix} (1/32)^{1/6} & (2)^{1/6} & (1/32)^{1/6} & (1/32)^{1/6} \\ (1/2)^{1/6} & (1/32)^{1/6} & (1/128)^{1/6} & (1/128)^{1/6} \\ (1/2)^{1/6} & (729/2048)^{1/6} & (1/128)^{1/6} & (387420489/34359738368)^{1/6} \\ (729/8192)^{1/6} & (2)^{1/6} & (1/32)^{1/6} & (1/32)^{1/6} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}D)^* = I \oplus (\lambda^{-1}D)^1 \oplus (\lambda^{-1}D)^2 \oplus (\lambda^{-1}D)^3 =$$

$$= \begin{pmatrix} 1 & (2)^{1/6} & (1/32)^{1/6} & (1/32)^{1/6} \\ (1/2)^{1/6} & 1 & (1/64)^{1/6} & (1/64)^{1/6} \\ (1/2)^{1/6} & (1)^{1/6} & 1 & (729/2048)^{1/6} \\ (1)^{1/6} & (2)^{1/6} & (729/8192)^{1/6} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (1/32)^{1/6} & (1/32)^{1/6} \\ (1/2)^{1/6} & (1/64)^{1/6} & (1/64)^{1/6} \\ (1/2)^{1/6} & 1 & (729/2048)^{1/6} \\ (1)^{1/6} & (729/8192)^{1/6} & 1 \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (1/32)^{1/6} & (1/32)^{1/6} & (1/32)^{1/6} \\ (1/64)^{1/6} & (1/64)^{1/6} & (1/64)^{1/6} \\ (1)^{1/6} & (1)^{1/6} & (729/2048)^{1/6} \\ (729/8192)^{1/6} & (1)^{1/6} & (1)^{1/6} \end{pmatrix} \qquad w_2 = \begin{pmatrix} 1 & (1)^{1/6} \\ (1/2)^{1/6} & (1/2)^{1/6} \\ (1/2)^{1/6} & (1)^{1/6} \\ (1)^{1/6} & (1)^{1/6} \end{pmatrix}$$

$$w_{best} = \begin{pmatrix} 0.577350 & 0.577350 & 0.577350 \\ 0.500000 & 0.500000 & 0.500000 \\ 1.000000 & 1.000000 & 0.866025 \\ 0.649519 & 1.000000 & 1.000000 \end{pmatrix}$$

$$w_{worst} = \begin{pmatrix} 1.000000 & 1.000000 \\ 0.890899 & 0.890899 \\ 0.890899 & 1.000000 \\ 1.000000 & 1.000000 \end{pmatrix}$$

5. Решение задачи о выборе места работы из статьи стр 20

Задача:

$$C = \begin{pmatrix} 1 & 1 & 1 & 4 & 1 & 1/2 \\ 1 & 1 & 2 & 4 & 1 & 1/2 \\ 1 & 1/2 & 1 & 5 & 3 & 1/2 \\ 1/4 & 1/4 & 1/5 & 1 & 1/3 & 1/3 \\ 1 & 1 & 1/3 & 3 & 1 & 1 \\ 2 & 2 & 2 & 3 & 1 & 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 1/4 & 1/2 \\ 4 & 1 & 3 \\ 2 & 1/3 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 1/4 & 1/5 \\ 4 & 1 & 1/2 \\ 5 & 2 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 3 & 1/3 \\ 1/3 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 1 & 1/3 & 5 \\ 3 & 1 & 7 \\ 1/5 & 1/7 & 1 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} 1 & 1 & 7 \\ 1 & 1 & 7 \\ 1/7 & 1/7 & 1 \end{pmatrix}$$

$$A_6 = \begin{pmatrix} 1 & 7 & 9 \\ 1/7 & 1 & 5 \\ 1/9 & 1/5 & 1 \end{pmatrix}$$

Нужные степени матрицы C:

$$C^2 = \begin{pmatrix} 1 & 1 & 2 & 5 & 3 & 4/3 \\ 2 & 1 & 2 & 10 & 6 & 4/3 \\ 3 & 3 & 1 & 9 & 3 & 3 \\ 2/3 & 2/3 & 2/3 & 1 & 3/5 & 1/3 \\ 2 & 2 & 2 & 4 & 1 & 1 \\ 2 & 2 & 4 & 10 & 6 & 1 \end{pmatrix}$$

$$C^3 = \begin{pmatrix} 3 & 3 & 8/3 & 10 & 6 & 3 \\ 6 & 6 & 8/3 & 18 & 6 & 6 \\ 6 & 6 & 6 & 12 & 3 & 3 \\ 2/3 & 2/3 & 4/3 & 10/3 & 2 & 3/5 \\ 2 & 2 & 4 & 10 & 6 & 4/3 \\ 6 & 6 & 4 & 20 & 12 & 6 \end{pmatrix}$$

$$C^4 = \begin{pmatrix} 6 & 6 & 6 & 18 & 8 & 6 \\ 12 & 12 & 12 & 24 & 8 & 6 \\ 6 & 6 & 12 & 30 & 18 & 4 \\ 2 & 2 & 4/3 & 20/3 & 4 & 2 \\ 6 & 6 & 4 & 20 & 12 & 6 \\ 12 & 12 & 12 & 36 & 12 & 12 \end{pmatrix}$$

$$C^5 = \begin{pmatrix} 12 & 12 & 12 & 30 & 18 & 8 \\ 12 & 12 & 24 & 60 & 36 & 8 \\ 18 & 18 & 12 & 60 & 36 & 18 \\ 4 & 4 & 4 & 12 & 4 & 4 \\ 12 & 12 & 12 & 36 & 12 & 12 \\ 24 & 24 & 24 & 60 & 36 & 12 \end{pmatrix}$$

$$C^6 = \begin{pmatrix} 18 & 18 & 24 & 60 & 36 & 18 \\ 36 & 36 & 24 & 120 & 72 & 36 \\ 36 & 36 & 36 & 108 & 36 & 36 \\ 8 & 8 & 8 & 20 & 12 & 4 \\ 24 & 24 & 24 & 60 & 36 & 12 \\ 36 & 36 & 48 & 120 & 72 & 36 \end{pmatrix}$$

Спектральный радиус матрицы C:

$$\lambda_C = \mathrm{tr}C \oplus \cdots \oplus \mathrm{tr}^{1/6}(C^6) = (12)^{1/4} \approx 1.86121$$

Матрица $\lambda^{-1}C$ и ее степени:

$$(\lambda^{-1}C)^1 = \begin{pmatrix} (1/12)^{1/4} & (1/12)^{1/4} & (1/12)^{1/4} & (64/3)^{1/4} & (1/12)^{1/4} & (1/192)^{1/4} \\ (1/12)^{1/4} & (1/12)^{1/4} & (4/3)^{1/4} & (64/3)^{1/4} & (1/12)^{1/4} & (1/192)^{1/4} \\ (1/12)^{1/4} & (1/192)^{1/4} & (1/12)^{1/4} & (625/12)^{1/4} & (27/4)^{1/4} & (1/192)^{1/4} \\ (1/3072)^{1/4} & (1/3072)^{1/4} & (1/7500)^{1/4} & (1/12)^{1/4} & (1/972)^{1/4} & (1/972)^{1/4} \\ (1/12)^{1/4} & (1/12)^{1/4} & (1/972)^{1/4} & (27/4)^{1/4} & (1/12)^{1/4} & (1/12)^{1/4} \\ (4/3)^{1/4} & (4/3)^{1/4} & (4/3)^{1/4} & (27/4)^{1/4} & (1/12)^{1/4} & (1/12)^{1/4} \end{pmatrix}$$

$$(\lambda^{-1}C)^2 = \begin{pmatrix} (1/144)^{1/4} & (1/144)^{1/4} & (1/9)^{1/4} & (625/144)^{1/4} & (9/16)^{1/4} & (16/729)^{1/4} \\ (1/9)^{1/4} & (1/144)^{1/4} & (1/9)^{1/4} & (625/9)^{1/4} & (9)^{1/4} & (16/729)^{1/4} \\ (9/16)^{1/4} & (9/16)^{1/4} & (1/144)^{1/4} & (729/16)^{1/4} & (9/16)^{1/4} & (9/16)^{1/4} \\ (1/729)^{1/4} & (1/729)^{1/4} & (1/729)^{1/4} & (1/144)^{1/4} & (9/10000)^{1/4} & (1/11664)^{1/4} \\ (1/9)^{1/4} & (1/9)^{1/4} & (1/9)^{1/4} & (16/9)^{1/4} & (1/144)^{1/4} & (1/144)^{1/4} \\ (1/9)^{1/4} & (1/9)^{1/4} & (16/9)^{1/4} & (625/9)^{1/4} & (9)^{1/4} & (1/144)^{1/4} \end{pmatrix}$$

$$(\lambda^{-1}C)^3 = \begin{pmatrix} (3/64)^{1/4} & (3/64)^{1/4} & (64/2187)^{1/4} & (625/108)^{1/4} & (3/4)^{1/4} & (3/64)^{1/4} \\ (3/4)^{1/4} & (3/4)^{1/4} & (64/2187)^{1/4} & (243/4)^{1/4} & (3/4)^{1/4} & (3/4)^{1/4} \\ (3/4)^{1/4} & (3/4)^{1/4} & (3/4)^{1/4} & (12)^{1/4} & (3/64)^{1/4} & (3/64)^{1/4} \\ (1/8748)^{1/4} & (1/8748)^{1/4} & (4/2187)^{1/4} & (625/8748)^{1/4} & (1/108)^{1/4} & (3/40000)^{1/4} \\ (1/108)^{1/4} & (1/108)^{1/4} & (4/27)^{1/4} & (625/108)^{1/4} & (3/4)^{1/4} & (4/2187)^{1/4} \\ (3/4)^{1/4} & (3/4)^{1/4} & (4/27)^{1/4} & (2500/27)^{1/4} & (12)^{1/4} & (3/4)^{1/4} \end{pmatrix}$$

$$(\lambda^{-1}C)^4 = \begin{pmatrix} (1/16)^{1/4} & (1/16)^{1/4} & (1/16)^{1/4} & (81/16)^{1/4} & (16/81)^{1/4} & (1/16)^{1/4} \\ (1)^{1/4} & (1)^{1/4} & (1)^{1/4} & (16)^{1/4} & (16/81)^{1/4} & (1/16)^{1/4} \\ (1/16)^{1/4} & (1/16)^{1/4} & (1)^{1/4} & (625/16)^{1/4} & (81/16)^{1/4} & (1/81)^{1/4} \\ (1/1296)^{1/4} & (1/1296)^{1/4} & (1/6561)^{1/4} & (625/6561)^{1/4} & (1/81)^{1/4} & (1/1296)^{1/4} \\ (1/16)^{1/4} & (1/16)^{1/4} & (1/81)^{1/4} & (625/81)^{1/4} & (1)^{1/4} & (1/16)^{1/4} \\ (1)^{1/4} & (1)^{1/4} & (1)^{1/4} & (81)^{1/4} & (1)^{1/4} & (1)^{1/4} \end{pmatrix}$$

$$(\lambda^{-1}C)^5 = \begin{pmatrix} (1/12)^{1/4} & (1/12)^{1/4} & (1/12)^{1/4} & (625/192)^{1/4} & (27/64)^{1/4} & (4/243)^{1/4} \\ (1/12)^{1/4} & (1/12)^{1/4} & (4/3)^{1/4} & (625/12)^{1/4} & (27/4)^{1/4} & (4/243)^{1/4} \\ (27/64)^{1/4} & (27/64)^{1/4} & (1/12)^{1/4} & (625/12)^{1/4} & (27/4)^{1/4} & (27/64)^{1/4} \\ (1/972)^{1/4} & (1/972)^{1/4} & (1/972)^{1/4} & (1/12)^{1/4} & (1/972)^{1/4} & (1/972)^{1/4} \\ (1/12)^{1/4} & (1/12)^{1/4} & (1/12)^{1/4} & (27/4)^{1/4} & (1/12)^{1/4} & (1/12)^{1/4} \\ (4/3)^{1/4} & (4/3)^{1/4} & (4/3)^{1/4} & (625/12)^{1/4} & (27/4)^{1/4} & (1/12)^{1/4} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}C)^* = I \oplus (\lambda^{-1}C)^1 \oplus (\lambda^{-1}C)^2 \oplus (\lambda^{-1}C)^3 \oplus (\lambda^{-1}C)^4 \oplus (\lambda^{-1}C)^5 =$$

$$= \begin{pmatrix} 1 & (1/12)^{1/4} & (1/9)^{1/4} & (64/3)^{1/4} & (3/4)^{1/4} & (1/16)^{1/4} \\ (1)^{1/4} & 1 & (4/3)^{1/4} & (625/9)^{1/4} & (9)^{1/4} & (3/4)^{1/4} \\ (3/4)^{1/4} & (3/4)^{1/4} & 1 & (625/12)^{1/4} & (27/4)^{1/4} & (9/16)^{1/4} \\ (1/729)^{1/4} & (1/729)^{1/4} & (4/2187)^{1/4} & 1 & (1/81)^{1/4} & (1/972)^{1/4} \\ (1/9)^{1/4} & (1/9)^{1/4} & (4/27)^{1/4} & (625/81)^{1/4} & 1 & (1/12)^{1/4} \\ (4/3)^{1/4} & (4/3)^{1/4} & (16/9)^{1/4} & (2500/27)^{1/4} & (12)^{1/4} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (1/12)^{1/4} & (64/3)^{1/4} \\ (1)^{1/4} & 1 & (625/9)^{1/4} \\ (3/4)^{1/4} & (3/4)^{1/4} & (625/12)^{1/4} \\ (1/729)^{1/4} & (1/729)^{1/4} & 1 \\ (1/9)^{1/4} & (1/9)^{1/4} & (625/81)^{1/4} \\ (4/3)^{1/4} & (4/3)^{1/4} & (2500/27)^{1/4} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} (3/4)^{1/4} & (1/16)^{1/4} \\ (3/4)^{1/4} & (3/4)^{1/4} \\ (9/16)^{1/4} & (9/16)^{1/4} \\ (1/972)^{1/4} & (1/972)^{1/4} \\ (1/12)^{1/4} & (1/12)^{1/4} \\ (1)^{1/4} & (1)^{1/4} \end{pmatrix} \qquad w_2 = \begin{pmatrix} (3/4)^{1/4} & (144/625)^{1/4} \\ (3/4)^{1/4} & (3/4)^{1/4} \\ (9/16)^{1/4} & (9/16)^{1/4} \\ (27/2500)^{1/4} & (27/2500)^{1/4} \\ (1/12)^{1/4} & (1/12)^{1/4} \\ (1)^{1/4} & (1)^{1/4} \end{pmatrix}$$

$$B = \begin{pmatrix} (1)^{1/4} & (2401)^{1/4} & (6561)^{1/4} \\ (192)^{1/4} & (1)^{1/4} & (625)^{1/4} \\ (1875/4)^{1/4} & (12)^{1/4} & (1)^{1/4} \end{pmatrix}$$

$$D = \begin{pmatrix} (1)^{1/4} & (2401)^{1/4} & (6561)^{1/4} \\ (192)^{1/4} & (1)^{1/4} & (625)^{1/4} \\ (1875/4)^{1/4} & (12)^{1/4} & (1)^{1/4} \end{pmatrix}$$

Нужные степени матрицы B:

$$B^2 = \begin{pmatrix} (12301875/4)^{1/4} & (78732)^{1/4} & (1500625)^{1/4} \\ (1171875/4)^{1/4} & (460992)^{1/4} & (1259712)^{1/4} \\ (2304)^{1/4} & (4501875/4)^{1/4} & (12301875/4)^{1/4} \end{pmatrix}$$

$$B^3 = \begin{pmatrix} (2813671875/4)^{1/4} & (29536801875/4)^{1/4} & (80712601875/4)^{1/4} \\ (590490000)^{1/4} & (2813671875/4)^{1/4} & (7688671875/4)^{1/4} \\ (23066015625/16)^{1/4} & (36905625)^{1/4} & (2813671875/4)^{1/4} \end{pmatrix}$$

Спектральный радиус матрицы B:

$$\lambda_B = \mathrm{tr}B \oplus \cdots \oplus \mathrm{tr}^{1/3}(B^3) = (12301875/4)^{1/8} \approx 6.47126$$

Матрица $\lambda^{-1}B$ и ее степени:

$$(\lambda^{-1}B)^1 = \begin{pmatrix} (4/12301875)^{1/8} & (23059204/12301875)^{1/8} & (8748/625)^{1/8} \\ (16384/1366875)^{1/8} & (4/12301875)^{1/8} & (2500/19683)^{1/8} \\ (625/8748)^{1/8} & (64/1366875)^{1/8} & (4/12301875)^{1/8} \end{pmatrix}$$

$$(\lambda^{-1}B)^2 = \begin{pmatrix} (1)^{1/8} & (256/390625)^{1/8} & (92236816/387420489)^{1/8} \\ (390625/43046721)^{1/8} & (377801998336/16815125390625)^{1/8} & (65536/390625)^{1/8} \\ (1048576/1868347265625)^{1/8} & (5764801/43046721)^{1/8} & (1)^{1/8} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}B)^* = I \oplus (\lambda^{-1}B)^1 \oplus (\lambda^{-1}B)^2 =$$

$$= \begin{pmatrix} 1 & (23059204/12301875)^{1/8} & (8748/625)^{1/8} \\ (16384/1366875)^{1/8} & 1 & (65536/390625)^{1/8} \\ (625/8748)^{1/8} & (5764801/43046721)^{1/8} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (23059204/12301875)^{1/8} \\ (16384/1366875)^{1/8} & 1 \\ (625/8748)^{1/8} & (5764801/43046721)^{1/8} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} 1 \\ (16384/1366875)^{1/8} \\ (625/8748)^{1/8} \end{pmatrix} \qquad w_2 = \begin{pmatrix} 1 & (1)^{1/8} \\ (625/8748)^{1/8} & (12301875/23059204)^{1/8} \\ (625/8748)^{1/8} & (625/8748)^{1/8} \end{pmatrix}$$

Нужные степени матрицы D:

$$D^2 = \begin{pmatrix} (12301875/4)^{1/4} & (78732)^{1/4} & (1500625)^{1/4} \\ (1171875/4)^{1/4} & (460992)^{1/4} & (1259712)^{1/4} \\ (2304)^{1/4} & (4501875/4)^{1/4} & (12301875/4)^{1/4} \end{pmatrix}$$

$$D^3 = \begin{pmatrix} (2813671875/4)^{1/4} & (29536801875/4)^{1/4} & (80712601875/4)^{1/4} \\ (590490000)^{1/4} & (2813671875/4)^{1/4} & (7688671875/4)^{1/4} \\ (23066015625/16)^{1/4} & (36905625)^{1/4} & (2813671875/4)^{1/4} \end{pmatrix}$$

Спектральный радиус матрицы D:

$$\lambda_D = \mathrm{tr}D \oplus \cdots \oplus \mathrm{tr}^{1/3}(D^3) = (12301875/4)^{1/8} \approx 6.47126$$

Матрица $\lambda^{-1}D$ и ее степени:

$$(\lambda^{-1}D)^1 = \begin{pmatrix} (4/12301875)^{1/8} & (23059204/12301875)^{1/8} & (8748/625)^{1/8} \\ (16384/1366875)^{1/8} & (4/12301875)^{1/8} & (2500/19683)^{1/8} \\ (625/8748)^{1/8} & (64/1366875)^{1/8} & (4/12301875)^{1/8} \end{pmatrix}$$

$$(\lambda^{-1}D)^2 = \begin{pmatrix} (1)^{1/8} & (256/390625)^{1/8} & (92236816/387420489)^{1/8} \\ (390625/43046721)^{1/8} & (377801998336/16815125390625)^{1/8} & (65536/390625)^{1/8} \\ (1048576/1868347265625)^{1/8} & (5764801/43046721)^{1/8} & (1)^{1/8} \end{pmatrix}$$

Матрица клини:

$$(\lambda^{-1}D)^* = I \oplus (\lambda^{-1}D)^1 \oplus (\lambda^{-1}D)^2 =$$

$$= \begin{pmatrix} 1 & (23059204/12301875)^{1/8} & (8748/625)^{1/8} \\ (16384/1366875)^{1/8} & 1 & (65536/390625)^{1/8} \\ (625/8748)^{1/8} & (5764801/43046721)^{1/8} & 1 \end{pmatrix}$$

Линейно независимые столбцы:

$$P = \begin{pmatrix} 1 & (23059204/12301875)^{1/8} \\ (16384/1366875)^{1/8} & 1 \\ (625/8748)^{1/8} & (5764801/43046721)^{1/8} \end{pmatrix}$$

$$w_1 = \begin{pmatrix} 1 \\ (16384/1366875)^{1/8} \\ (625/8748)^{1/8} \end{pmatrix} \qquad w_2 = \begin{pmatrix} 1 & (1)^{1/8} \\ (625/8748)^{1/8} & (12301875/23059204)^{1/8} \\ (625/8748)^{1/8} & (625/8748)^{1/8} \end{pmatrix}$$

$$w_{best} = \begin{pmatrix} 1.000000 \\ 0.575223 \\ 0.719029 \end{pmatrix}$$

$$w_{worst} = \begin{pmatrix} 1.000000 & 1.000000 \\ 0.719029 & 0.924466 \\ 0.719029 & 0.719029 \end{pmatrix}$$