ICS Answer Sheet #7

Sakar Gopal Gurubacharya
s.gurubacharya@jacobs-university.de

Problem 7.1: quine-mccluskey algorithm

A Boolean function ϕ is defined using the following sum of minterms: $\phi(A, B, C, D, E) = m_0+m_2+ m_4+m_6+ m_9+m_{10}+ m_{13}+m_{14}+ m_{15}+m_{16}+ m_{17}+m_{21}+ m_{26}+m_{28}+ m_{30}+m_{31}$

a. **Calculate the prime implicants of ϕ.**

Step Q_0:

$m_0+m_2+ m_4+m_6+ m_9+m_{10}+ m_{13}+m_{14}+ m_{15}+m_{16}+ m_{17}+m_{21}+ m_{26}+m_{28}+ m_{30}+m_{31}$

Since we already have it in implicant form, we basically already have step Q_0 solved.

The **cost** of this initial expression is: 16 * 4 +15 = **79**

Step Q_1: Identify prime implicants

I

Minterm	Pattern	Used	Minterm	Pattern	Used
m_0	00000	✓	m_0, 2	000-0	
			m_0, 4	00-00	
			m_0, 16	-0000	
--					
m_2	00010	✓	m_2, 6	00-10	
			m_2, 10	0-010	
m_4	00100	✓	m_4, 6	001-0	
m_16	10000	✓	m_16, 17	1000-	
--					

m_6	00110	√	m_6, 14	0-110	
m_9	01001	√	m_9, 13	01-01	
m_10	01010	√	m_10, 14	01-10	
			m_10, 26	-1010	
m_17	10001	√	m_17, 21	10-01	
--					
m_13	01101	√	m_13, 15	011-1	
m_14	01110	√	m_14, 15	0111-	
			m_14, 30	-1110	
m_21	10101	√			
m_26	11010	√	m_26, 30	11-10	
m_28	11100	√	m_28, 30	111-0	
--					
m_15	01111	√	m_15, 31	-1111	
m_30	11110	√	m_30, 31	1111-	
--					
m_31	11111	√			
--					

II

Minterm	Pattern	Used	Minterm	Pattern	Used
m_0, 2	000-0	✓	m_0, 2, 4, 6	00--0	
m_0, 4	00-00	✓	m_0, 4, 2, 6	00--0	
m_0, 16	-0000				
--					
m_2, 6	00-10	✓	m_2, 6, 10, 14	0--10	
m_2, 10	0-010	✓	m_2, 10, 6, 14	0--10	
m_4, 6	001-0	✓			
m_16, 17	1000-				
--					
m_6, 14	0-110	✓			
m_9, 13	01-01				
m_10, 14	01-10	✓	m_10, 14, 26, 30	-1-10	
m_10, 26	-1010	✓	m_10, 26, 14, 30	-1-10	
m_17, 21	10-01				
--					
m_13, 15	011-1				
m_14, 15	0111-	✓	m_14, 15, 30, 31	-111-	
m_14, 30	-1110	✓	m_14, 30, 15, 31	-111-	

m_26, 30	11-10	✓			
m_28, 30	111-0				
--					
m_15, 31	-1111	✓			
m_30, 31	1111-	✓			
--					

III

(Nothing done in step III, check step IV)

Minterm	**Pattern**	**Used**
m_0, 2, 4, 6	00--0	
m_0, 4, 2, 6	00--0	
m_2, 6, 10, 14	0--10	
m_2, 10, 6, 14	0--10	
m_10, 14, 26, 30	-1-10	
m_10, 26, 14, 30	-1-10	
m_14, 15, 30, 31	-111-	
m_14, 30, 15, 31	-111-	

IV

Since patterns within a block match, we can ignore one of them.

Minterm	Pattern	Used
m_0, 2, 4, 6	00--0	
m_2, 6, 10, 14	0--10	
m_10, 14, 26, 30	-1-10	
m_14, 15, 30, 31	-111-	

From Step IV, we can see that now no more pattern matching can be done and so the Q_1 ends here with prime implicants as:

m_0, 16
m_16, 17
m_9, 13
m_17, 21
m_13, 15
m_28, 30
m_0, 2, 4, 6
m_2, 6, 10, 14
m_10, 14, 26, 30
m_14, 15, 30, 31

b. Construct the prime implicant chart and identify the essential prime implicants.

Now, we identify the essential prime implicants and its coverage.

PI	m_0	m_2	m_4	m_6	m_9	m_{10}	m_{13}	m_{14}	comment
m_0, 16	✓								
m_16, 17									
m_9, 13					✓		✓		essential
m_17, 21									essential
m_13, 15							✓		
m_28, 30									essential
m_0, 2, 4, 6	✓	✓	✓	✓					essential
m_2, 6, 10, 14		✓		✓		✓		✓	
m_10, 14, 26, 30						✓		✓	essential
m_14, 15, 30, 31								✓	essential
Coverage	✓	✓	✓	✓	✓	✓	✓	✓	

PI	m_{15}	m_{16}	m_{17}	m_{21}	m_{26}	m_{28}	m_{30}	m_{31}	comment
m_0, 16		✓							
m_16, 17		✓	✓						
m_9, 13									essential
m_17, 21			✓	✓					essential
m_13, 15	✓								
m_28, 30						✓	✓		essential
m_0, 2, 4, 6									essential
m_2, 6, 10, 14									
m_10, 14, 26, 30					✓		✓		essential
m_14, 15, 30, 31	✓						✓	✓	essential
Coverage	✓		✓	✓	✓	✓	✓	✓	

Therefore,

Essential prime implicants are:

m_9, 13 | m_17, 21 | m_28, 30 | m_0, 2, 4, 6 | m_10, 14, 26, 30 | m_14, 15, 30, 31

Since the essential prime implicants do not cover all implicants, i.e. we are missing $\mathbf{m_{16}}$, we have to assign one more prime implicant as essential to obtain the following solutions.

In the table above, only two prime implicants have 16 in them and since they both have the same costs, we can take either one of them

We can add either m_16, 17 or m_0, 16 as an essential prime implicant.

Final list of essential prime implicants are:

m_0, 2, 4, 6
m_10, 14, 26, 30
m_14, 15, 30, 31
m_9, 13
m_16, 17 OR m_0, 16
m_17, 21
m_28, 30

c. **Write out all minimal boolean expressions defining ϕ using the mathematical logic notation.**

Therefore, finally we have:

m_0, 2, 4, 6 = 00--0
= (¬ A ∧ ¬ B ∧ ¬ E)

m_10, 14, 26, 30 = -1-10
= (B ∧ D ∧ ¬ E)

m_14, 15, 30, 31 = -111-
= (B ∧ C ∧ D)

m_9, 13 = 01-01
= (¬ A ∧ B ∧ ¬ D ∧ E)

m_16, 17 = 1000-
= (A ∧ ¬ B ∧ ¬ C ∧ ¬ D)

OR

m_0, 16 = -0000
= (¬ B ∧ ¬ C ∧ ¬ D ∧ ¬ E)

m_17, 21 = 10-01
= (A ∧ ¬ B ∧ ¬ D ∧ E)

m_28, 30 = 111-0
= (A ∧ B ∧ C ∧ ¬ E)

Combining all of these, we get,

$(\neg A \land \neg B \land \neg E) \lor (B \land D \land \neg E) \lor (B \land C \land D) \lor (\neg A \land B \land \neg D \land E)$
$\lor (A \land \neg B \land \neg C \land \neg D) \lor (A \land \neg B \land \neg D \land E) \lor (A \land B \land C \land \neg E)$

OR

$(\neg A \land \neg B \land \neg E) \lor (B \land D \land \neg E) \lor (B \land C \land D) \lor (\neg A \land B \land \neg D \land E)$
$\lor (\neg B \land \neg C \land \neg D \land \neg E) \lor (A \land \neg B \land \neg D \land E) \lor (A \land B \land C \land \neg E)$

The **cost** for both of these final expressions is: **24**

Therefore, with the help of the **Quine-McCluskey Algorithm**, the cost of the expression dropped from **79 to 24** and resulted in a **well-simplified expression.**