Sankar Gopal Gurubachagar | Algorithms and Data Structures
Homework 3

Problem 3.1

a. $f(n) = 9n$
$g(n) = 5n^3$

$\lim_{n\to\infty} \left(\frac{9n}{5n^3}\right) = 0 \quad , f \in o(g)$

$\lim_{n\to\infty} \left(\frac{9n}{5n^3}\right) < \infty \quad , f \in O(g)$

$\lim_{n\to\infty} \left(\frac{5n^3}{9n}\right) = \infty \quad , g \in \omega(f)$

$\lim_{n\to\infty} \left(\frac{5n^3}{9n}\right) > 0 \quad , g \in \Omega(f)$

b. $f(n) = 9n^{0.8} + 2n^{0.3} + 14 \log n$
$g(n) = \sqrt{n}$

$\lim_{n\to\infty} \left(\frac{9n^{0.8} + 2n^{0.3} + 14\log n}{\sqrt{n}}\right) = \infty \quad , f \in \omega(g)$

$\lim_{n\to\infty} \left(\frac{9n^{0.8} + 2n^{0.3} + 14\log n}{\sqrt{n}}\right) > 0 \quad , f \in \Omega(g)$

$\lim_{n\to\infty} \left(\frac{\sqrt{n}}{9n^{0.8} + 2n^{0.3} + 14\log n}\right) = 0 \quad g \in o(f)$

$\lim_{n\to\infty} \left(\frac{\sqrt{n}}{9n^{0.8} + 2n^{0.3} + 14\log n}\right) < \infty \quad , g \in O(g)$

c. $f(n) = \frac{n^2}{\log n}$

$g(n) = n \log n$

$\lim_{n \to \infty} \left(\frac{n^2/\log n}{n \log n} \right) = \infty$, $f \in \omega(g)$

$\lim_{n \to \infty} \left(\frac{n^2/\log n}{n \log n} \right) > 0$, $f \in \Omega(g)$

$\lim_{n \to \infty} \left(\frac{n \log n}{n^2/\log n} \right) = 0$, $g \in o(f)$

$\lim_{n \to \infty} \left(\frac{n \log n}{n^2/\log n} \right) < \infty$, $g \in O(f)$

d. $f(n) = (\log(3n))^3$

$g(n) = 9 \log n$

$\lim_{n \to \infty} \left(\frac{(\log(3n))^3}{9 \log n} \right) = \infty$, $f \in \omega(g)$

$\lim_{n \to \infty} \left(\frac{(\log(3n))^3}{9 \log n} \right) > 0$, $f \in \Omega(g)$

$\lim_{n \to \infty} \left(\frac{9 \log n}{(\log(3n))^3} \right) = 0$, $g \in o(f)$

$\lim_{n \to \infty} \left(\frac{9 \log n}{(\log(3n))^3} \right) < \infty$, $g \in O(f)$

b. Show that Selection Sort is correct (Hint: consider the loop invariant).

To prove the selection sort is correct, we must show the loop invariant has 3 conditions.

Initialization:

Does it hold before the program runs, i.e. before the first loop?
And it does because initially there are no elements in the array and hence is sorted.

Maintenance:

Does each iteration maintain the invariant?
With this sorting algorithm, everything until the current position, i.e. n, is always sorted. And after each loop, the n+1th item is added and still maintains the loop invariant.

Termination:

Does the loop invariant show properties and correctness when it terminates?
At the end, there will be no other elements to sort, no other elements to add to the left sub array and hence terminates correctly.