Body Kalman Filtering

Body center point location

State vector s:

$$s = \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \\ a_x \\ a_y \\ a_z \end{bmatrix}$$

where

(x, y, z) = location of the world object center point
(v_x , v_y , v_z) = velocity of the object
(a_x , a_y , a_z) = acceleration of the object

State equation in differential form:

$$\frac{ds(t)}{dt} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * s(t) + \epsilon(t) = A_1 * s$$

State equation in difference form:

$$s(k+1) = (I + \Delta * A_1) * s(k) + \epsilon(k)$$

$$= \begin{bmatrix} 1 & 0 & 0 & \Delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \Delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * s(k) + \varepsilon(k) = A * s(k) + \varepsilon(k)$$

where Δ is the time increment and ε Gaussian noise with covariance R.

Measurement equation

$$z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * s(k) + \delta(k) = C * s(k) + \delta(k)$$

Where δ is Gaussian noise with covariance matrix Q.

Kalman filter initialization:

$$\mu(0) = \begin{bmatrix} x(0) \\ y(0) \\ z(0) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where x(0), y(0), z(0) is the first location measurement.

$$\Sigma(0) = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma \end{bmatrix}$$

where α, β and γ are believed variances of location, velocity and acceleration.

$$R = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_3 \end{bmatrix}$$

where r_1, r_2 and r_3 are believed variances of location, velocity and acceleration.

$$Q = \begin{bmatrix} q & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & q \end{bmatrix}$$

Where q is the believed measurement variance.

Kalman filter update:

$$\mu_1(k) = A * \mu(k-1)$$

$$\Sigma_1(k) = A * \Sigma(k-1) * A^T + R$$

$$K(k) = \Sigma_1(k) * C^T(C * \Sigma_1(k) * C^T + Q)^{-1}$$

$$\mu(k) = \mu_1(k) + K(k) * (z(k) - C * \mu_1(k))$$

$$\Sigma(k) = (I - K(k) * C) * \Sigma_1(k)$$