# ImageObject Kalman Filtering

## Location

State vector s:

$$s = \begin{bmatrix} x \\ y \\ vx \\ vy \\ ax \\ ay \end{bmatrix}$$

where

x = horizontal location in the image
y= vertical location in the image
vx = horizontal velocity in the image
vy= vertical velocity in the image
ax = horizontal acceleration in the image
ay= vertical acceleration in the image

Coordinate system:

```
0/0 --- x --- >
|
|
```

y

```
|
|
v
```

State equation in differential form:

$$\frac{ds(t)}{dt} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * s(t) + \epsilon(t) = A_1 * s$$

State equation in difference form:

$$s(k+1) = (I + \Delta * A_1) * s(k) + \epsilon(k) = \begin{bmatrix} 1 & 0 & \Delta & 0 & 0 & 0 \\ 0 & 1 & 0 & \Delta & 0 & 0 \\ 0 & 0 & 1 & 0 & \Delta & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * s(k) + \varepsilon(k) = A * s(k) + \varepsilon(k)$$

where $\Delta$ is the time increment and $\varepsilon$ Gaussian noise with covariance R.

Measurement equation

$$z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} * s(k) + \delta(k) = C * s(k) + \delta(k)$$

Where $\delta$ is Gaussian noise with covariance matrix Q.

Kalman filter initialization:

$$\mu(0) = \begin{bmatrix} x(0) \\ y(0) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where x(0) and y(0) are the first location measurements.

$$\Sigma(0) = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma \end{bmatrix}$$

where α, β and γ are believed variances of location, velocity and acceleration, for example 1.

$$R = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_3 \end{bmatrix}$$

where $r_1$, $r_2$ and $r_3$ are believed variances of location, velocity and acceleration, for example 1.

$$Q = \begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix}$$

Where q is the believed measurement variance. It is larger than system variance because the objects have tendency to move smoothly but the bounding boxes exhibit more random behaviour. Q can be set to 10, for example.

Kalman filter update:

$$\mu_1(k) = A * \mu(k-1)$$

$$\Sigma_1(k) = A * \Sigma(k-1) * A^T + R$$

$$K(k) = \Sigma_1(k) * C^T(C * \Sigma_1(k) * C^T + Q)^{-1}$$

$$\mu(k) = \mu_1(k) + K(k) * (z(k) - C * \mu_1(k))$$

$$\Sigma(k) = (I - K(k) * C) * \Sigma_1(k)$$

## Size

State vector s:

$$s = \begin{bmatrix} sx \\ sy \\ vsx \\ vsy \\ asx \\ asy \end{bmatrix}$$

where

sx = horizontal size of the image
sy= vertical size of the image
vsx = horizontal size change of the image
vsy= vertical size change of the image
asx = horizontal size change acceleration in the image
asy= vertical size change acceleration in the image

Coordinate system:

```
0/0 --- x --- >
|
|
```

y

```
|
|
v
```

State equation in differential form:

$$\frac{ds(t)}{dt} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * s(t) + \epsilon(t) = A_1 * s$$

State equation in difference form:

$$s(k+1) = (I + \Delta * A_1) * s(k) + \epsilon(k) = \begin{bmatrix} 1 & 0 & \Delta & 0 & 0 & 0 \\ 0 & 1 & 0 & \Delta & 0 & 0 \\ 0 & 0 & 1 & 0 & \Delta & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * s(k) + \varepsilon(k) = A * s(k) + \varepsilon(k)$$

where $\Delta$ is the time increment and $\varepsilon$ Gaussian noise with covariance R.

Measurement equation

$$z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} * s(k) + \delta(k) = C * s(k) + \delta(k)$$

Where $\delta$ is Gaussian noise with covariance matrix Q.

Kalman filter initialization:

$$\mu(0) = \begin{bmatrix} sx(0) \\ sy(0) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where sx(0) and sy(0) are the first size measurements.

$$\Sigma(0) = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma \end{bmatrix}$$

where α, β and γ are believed variances of size, change and acceleration, for example 1.

$$R = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_3 \end{bmatrix}$$

where $r_1$, $r_2$ and $r_3$ are believed variances of location, velocity and acceleration, for example 1.

$$Q = \begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix}$$

Where q is the believed measurement variance. It is larger than system variance because the objects have tendency to change smoothly but the bounding boxes exhibit more random behaviour. Q can be set to 10, for example.

Kalman filter update:

$$\mu_1(k) = A * \mu(k-1)$$

$$\Sigma_1(k) = A * \Sigma(k-1) * A^T + R$$

$$K(k) = \Sigma_1(k) * C^T (C * \Sigma_1(k) * C^T + Q)^{-1}$$

$$\mu(k) = \mu_1(k) + K(k) * (z(k) - C * \mu_1(k))$$

$$\Sigma(k) = (I - K(k) * C) * \Sigma_1(k)$$