Image Object Kalman Filtering

Bounding box line location

State vector s:

$$s = \begin{bmatrix} l \\ v \\ a \end{bmatrix}$$

where

l = location of the bounding box line in the image (x_{min} , x_{max} , y_{min} , y_{max})
v = velocity of the line in the image
a = acceleration of the line in the image

State equation in differential form:

$$\frac{ds(t)}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} * s(t) + \epsilon(t) = A_1 * s$$

State equation in difference form:

$$s(k+1) = (I + \Delta * A_1) * s(k) + \epsilon(k) = \begin{bmatrix} 1 & \Delta & 0 \\ 0 & 1 & \Delta \\ 0 & 0 & 1 \end{bmatrix} * s(k) + \varepsilon(k) = A * s(k) + \varepsilon(k)$$

where Δ is the time increment and ε Gaussian noise with covariance R.

Measurement equation

$$z(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} * s(k) + \delta(k) = C * s(k) + \delta(k)$$

Where δ is Gaussian noise with covariance matrix Q.

Kalman filter initialization:

$$\mu(0) = \begin{bmatrix} x(0) \\ 0 \\ 0 \end{bmatrix}$$

where x(0) is the first location measurement.

$$\Sigma(0) = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix}$$

where α, β and γ are believed variances of location, velocity and acceleration, for example 1.

$$R = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}$$

where r_1, r_2 and r_3 are believed variances of location, velocity and acceleration, for example 1.

$$Q = [q]$$

Where q is the believed measurement variance. It is larger than system variance because the objects have tendency to move smoothly, but the bounding boxes exhibit more random behaviour. Q can be set to 10, for example.

Kalman filter update:

$$\mu_1(k) = A * \mu(k-1)$$

$$\Sigma_1(k) = A * \Sigma(k-1) * A^T + R$$

$$K(k) = \Sigma_1(k) * C^T (C * \Sigma_1(k) * C^T + Q)^{-1}$$

$$\mu(k) = \mu_1(k) + K(k) * (z(k) - C * \mu_1(k))$$

$$\Sigma(k) = (I - K(k) * C) * \Sigma_1(k)$$