Collision Detection

Collision probability matrix:

$$
\begin{bmatrix}
0 & p_{12} & p_{13} & p_{14} & \cdots & p_{1n} \\
p_{21} & 0 & p_{23} & p_{24} & \cdots & p_{2n} \\
p_{31} & p_{32} & 0 & p_{34} & \cdots & p_{3n} \\
p_{41} & p_{42} & p_{43} & 0 & \cdots & p_{4n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
p_{n1} & p_{n2} & p_{n3} & p_{n4} & \cdots & 0
\end{bmatrix}
$$

$$
\begin{bmatrix} x_i(t) \\ y_i(t) \\ z_i(t) \end{bmatrix} = \begin{bmatrix} x_i(0) \\ y_i(0) \\ z_i(0) \end{bmatrix} + t * \begin{bmatrix} v_{i,x}(0) \\ v_{i,y}(0) \\ v_{i,z}(0) \end{bmatrix}
$$

$$
d_{ij}(t) = \sqrt{(x(t)_i - x(t)_j)^2 + (y(t)_i - y(t)_j)^2 + (z(t)_i - z(t)_j)^2}
$$

$$
r = \begin{bmatrix}
x_1(0) \\
y_1(0) \\
z_1(0) \\
v_{1,x}(0) \\
v_{1,y}(0) \\
v_{1,z}(0) \\
r_1 \\
\cdots \\
x_n(0) \\
y_n(0) \\
z_n(0) \\
v_{n,x}(0) \\
v_{n,y}(0) \\
v_{n,z}(0) \\
r_n \\
t
\end{bmatrix}
$$

$$
p_{ij} = E\{C_{ij}\} = \sum_{k=1}^{m} \frac{\delta(C_{i,j})}{m}
$$

$$r = \begin{bmatrix} x_i(0) \\ y_i(0) \\ z_i(0) \\ v_{i,x}(0) \\ v_{i,y}(0) \\ v_{i,z}(0) \\ r_i \\ r \end{bmatrix}$$

$$t = \frac{(P(0)-0)\cdot(P(0)+V(0)-P(0))}{|P(0)-V(0)-P(0)|^2} = \frac{P(0)\cdot V(0)}{|V(0)|^2}$$

$$d = \frac{\left|(0-P(0))\times\left(0-\left(P(0)+V(0)\right)\right)\right|}{\left|\left(P(0)+V(0)\right)-P(0)\right|} = \frac{|-P(0)\times(-P(0)-V(0))|}{|V(0)|}$$