

Introduction to data-parallelism and OpenCL

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Longest common subsequence

- LCSS(S, Q)
 - *Length of* longest sub-sequence
- Example:
 - ABC**DE**FGH
 - ZZ**C**ZFZH
 - LCCS = 3
 - (there can be multiple sub-seq of same length)
- Notion of prefix:
 - $S = (ABCDEFGH)$
 - $S = ((ABCDEFG), H)$
 - $S_n = (S_{n-1}, H)$

Longest common subsequence

- Property 1
 - $S_n = (\text{BONT}\textcolor{red}{O}) = (\text{BONT}) , \textcolor{red}{O}$
 - $Q_n = (\text{BANJ}\textcolor{red}{O}) = (\text{BANJ}) , \textcolor{red}{O}$
- Consider
 - $\text{LCSS}(S_{n-1}, Q_{n-1})$
 - $\text{LCSS}(S_{n-1}, \mathbf{x}), (Q_{n-1}, \mathbf{x}) = 1 + \text{LCSS}(S_{n-1}, Q_{n-1})$

Longest common subsequence

- Property 2

- $S_n = (\text{BONT}) = (\text{BON}), \text{T}$

- $Q_n = (\text{BANJ}) = (\text{BAN}), \text{J}$

- Consider

- $\text{LCSS}(S_{n-1}, Q_n), \text{LCSS}(S_n, Q_{n-1})$

$$\begin{aligned} \rightarrow \text{LCSS}((S_{n-1}, \mathbf{x}), (Q_{n-1}, \mathbf{y})) = \\ \mathbf{x} \neq \mathbf{y} \quad \max(\text{LCSS}(S_{n-1}, Q_n) , \text{LCSS}(S_n, Q_{n-1})) \end{aligned}$$

Algorithm

LCSS(S_n , Q_m)

if ($S_n = \emptyset$ or $Q_m = \emptyset$) { return 0 }

(S_{n-1} , x) = S_n

(Q_{m-1} , y) = Q_m

if (x == y) {

 return 1 + LCSS(S_{n-1} , Q_{m-1})

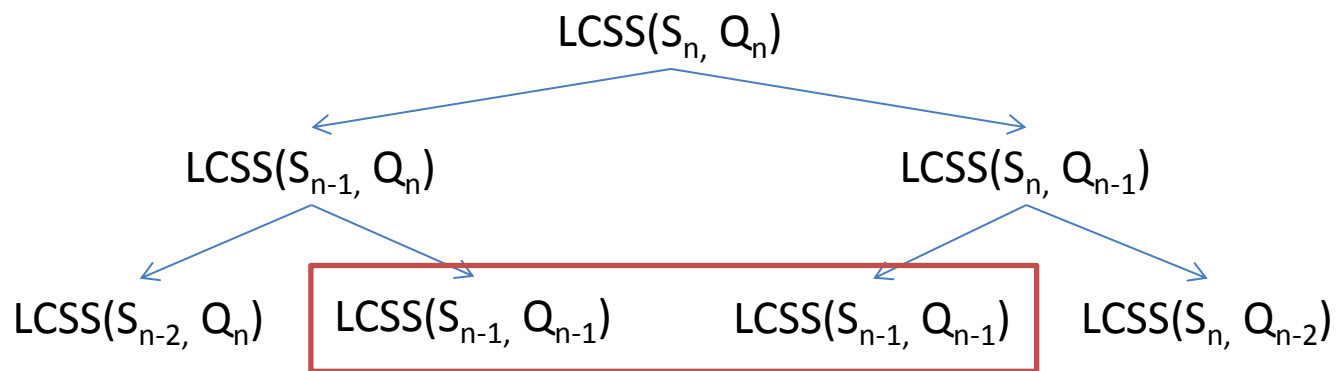
} else {

 return max(LCSS(S_{n-1} , Q_m) , LCSS(S_n , Q_{m-1}))

}

Dynamic programming

- General method
- Key observation:
 - Recursive calls often overlap
 - Memorize them, visit in order



Dynamic programming table

$\text{LCSS}(S_0, Q_0)$	$\text{LCSS}(S_1, Q_0)$	$\text{LCSS}(S_2, Q_0)$	$\text{LCSS}(S_3, Q_0)$	$\text{LCSS}(S_4, Q_0)$
$\text{LCSS}(S_0, Q_1)$	$\text{LCSS}(S_1, Q_1)$	$\text{LCSS}(S_2, Q_1)$	$\text{LCSS}(S_3, Q_1)$	$\text{LCSS}(S_4, Q_1)$
$\text{LCSS}(S_0, Q_2)$	$\text{LCSS}(S_1, Q_2)$	$\text{LCSS}(S_2, Q_2)$	$\text{LCSS}(S_3, Q_2)$	$\text{LCSS}(S_4, Q_2)$
$\text{LCSS}(S_0, Q_3)$	$\text{LCSS}(S_1, Q_3)$	$\text{LCSS}(S_2, Q_3)$	$\text{LCSS}(S_3, Q_3)$	$\text{LCSS}(S_4, Q_3)$
$\text{LCSS}(S_0, Q_4)$	$\text{LCSS}(S_1, Q_4)$	$\text{LCSS}(S_2, Q_4)$	$\text{LCSS}(S_3, Q_4)$	$\text{LCSS}(S_4, Q_4)$

Dynamic programming table

0	0	0	0	0
0	$\text{LCSS}(S_1, Q_1)$	$\text{LCSS}(S_2, Q_1)$	$\text{LCSS}(S_3, Q_1)$	$\text{LCSS}(S_4, Q_1)$
0	$\text{LCSS}(S_1, Q_2)$	$\text{LCSS}(S_2, Q_2)$	$\text{LCSS}(S_3, Q_2)$	$\text{LCSS}(S_4, Q_2)$
0	$\text{LCSS}(S_1, Q_3)$	$\text{LCSS}(S_2, Q_3)$	$\text{LCSS}(S_3, Q_3)$	$\text{LCSS}(S_4, Q_3)$
0	$\text{LCSS}(S_1, Q_4)$	$\text{LCSS}(S_2, Q_4)$	$\text{LCSS}(S_3, Q_4)$	$\text{LCSS}(S_4, Q_4)$

Pseudo-code

```
LCSS (  $S_N, Q_M$  )  
  DP = [0.. $N$ ][0.. $M$ ]  
  for  $j = 0 \dots M$   
    DP[0][ $j$ ] = 0  
  end  
  for  $i = 1 \dots N$   
    DP[ $i$ ][0] = 0  
    for  $j = 1 \dots M$   
      if  $S[i] == Q[j]$   
        DP[ $i$ ][ $j$ ] = 1 + DP[ $i-1$ ][ $j-1$ ]  
      else  
        DP[ $i$ ][ $j$ ] = max( DP[ $i-1$ ][ $j$ ] , DP[ $i$ ][ $j-1$ ] )  
      end  
    end  
  end  
end
```