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Control of Unknown MIMO Systems using Self-Tuning PID Controllers

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Abstract

In this paper, new theoretical and simulated results of a self-tuning proportional-integral-derivative (PID) robust servomechanism (for the class of constant reference and disturbance signals) are given, for cases involving both known and unknown estimates of the steady-state gain matrix $\mathcal{T}$.

1. Introduction

In the conventional design of controllers for multivariable systems, the general approach often adopted is to find a model for the plant, and thence design a controller based on this model. The difficulty of this approach, however, lies in the fact that it is generally difficult and often impossible to obtain an accurate model representation of an actual industrial plant. While conventional adaptive control techniques reduce the amount of information required, detailed plant information (eg. a knowledge of the upper bound on the order of the plant, the relative degree of the plant, etc.) is still usually needed. During the past several years, there has therefore been a considerable amount of interest in developing control design methods which require using as little *a priori* plant information as possible (e.g. see [1], [5], [6], [7], [8]). In Miller and Davison [7] and Chang and Davison [2], a potential controller design method for LTI open loop asymptotically stable plants is outlined, which appears to be quite attractive in that no additional information about the plant is required. The controller proposed in [7], [2] is a self-tuning switching integral [7] or proportional-integral [2] type controller which has the property that after a finite time, the controller stops switching and simplifies to a LTI controller.

In this paper, an extension of [2] is made to include 'derivative' type terms in the controller for instances involving both known and unknown estimates of the steady-state gain matrix $\mathcal{T}$. Simulated output responses comparing this new controller with previous results are also presented in this paper.

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2. Notation

Let $\mathbf{R}$, $\mathbf{R}^+$, $\mathbf{N}$ and $\mathbf{C}$ denote respectively the set of real, positive real, natural, and complex numbers; $\mathbf{R}^n$ will be the n-dimensional real vector space, $\mathbf{R}^{m\times n}$ the set of $m \times n$ real matrices, $\mathbf{C}^-$ ($\mathbf{C}^+$) the set of complex numbers with strictly negative (positive) real parts, and $\mathbf{C}^0$ the set of complex numbers lying strictly on the imaginary axis. For any $x, y \in \mathbf{N}$,

$$x \bmod y := x - \text{floor}\left(\frac{x}{y}\right) y$$

where floor$(\cdot)$ rounds the expression $(\cdot)$ down to the nearest integer.

With $x \in \mathbf{R}^n$, denote its ∞-norm to be

$$\|x\| := \max_{1\le i\le n} |x_i|.$$

For any arbitrary $A \in \mathbf{R}^{n\times n}$, let $\lambda(A)$ denote the eigenvalues of A, and let $\mathbf{Re}(\lambda)$ and $\mathbf{Im}(\lambda)$ be the real and imaginary parts of λ respectively. A is said to be *stable* if $\mathbf{Re}(\lambda(A)) < 0$ and *unstable* otherwise.

For the more general case when $A \in \mathbf{R}^{m\times n}$, A^T denotes its matrix transpose, rank(A) its rank, and, if A has full row rank, $A^\dagger := A^T(AA^T)^{-1}$ denotes its pseudo-inverse. In addition, the corresponding induced ∞-norm of A is defined by

$$\|A\| := \max_{1\le i\le m} \sum_{j=1}^{n} |a_{ij}|.$$

3. Assumptions

Assume that the finite dimensional LTI plant to be controlled can be described by the equations

$$\begin{aligned} \dot{x} &= Ax + Bu + Ew, \\ y &= Cx + Fw, \\ e &:= y_{ref} - y \end{aligned} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^m$ is the control input, $y \in \mathbf{R}^r$ is the output to be regulated, $w \in \mathbf{R}^q$ is the disturbance, and $e \in \mathbf{R}^r$ is the difference between the specified reference input y_{ref} and the output y. Assume too that A is stable, that n, A, B, C, E, and F are unknown, and define $\mathcal{T} := -CA^{-1}B$.

4. Proportional-integral-derivative (PID) self-tuning control

In this section, self-tuning proportional-integral-derivative (PID) controllers will be described for situations involving both known and unknown estimates of $\mathcal{T}$. Assume now that the parameters $\epsilon > 0$, $\epsilon_1 > 0$, $\epsilon_2 > 0$, $N > 0$, $\rho \geq 0$, and K are all constant, and consider the following controller:

$$\begin{aligned} u &= K(\eta + \rho\epsilon_1 e + \epsilon_2 d), \\ \dot{\eta} &= \epsilon e, \\ d(s) &= \frac{-s}{\left(\frac{s}{N} + 1\right)} y(s) \end{aligned}$$

which, for large N, approximates the behaviour of a multivariable PID controller. On applying this controller to (1), the closed loop system can be expressed in the form

$$\dot{z} = \bar{A}_{PID} z + \bar{\mathcal{B}}\bar{u}, \tag{2}$$

$$\begin{bmatrix} y \\ \eta \\ a \end{bmatrix} = \begin{bmatrix} C & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \underbrace{\begin{bmatrix} x \\ \eta \\ a \end{bmatrix}}_{z} + \begin{bmatrix} 0 & F \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} y_{ref} \\ w \end{bmatrix}}_{\bar{u}}$$

where $[\ \bar{A}_{PID} \mid \mathcal{B}\ ] :=$

$$\left[\begin{array}{ccc|cc} \hat{A} & BK & -\epsilon_2 N^2 BK & \rho\epsilon_1 BK & \hat{E} \\ -\epsilon C & 0 & 0 & \epsilon I & -\epsilon F \\ -C & 0 & -NI & 0 & -F \end{array}\right],$$

and

$$\begin{aligned} \hat{A} &:= A - \rho\epsilon_1 BKC - \epsilon_2 NBKC, \\ \hat{E} &:= E - \rho\epsilon_1 BKF - \epsilon_2 NBKF. \end{aligned}$$

4.1. Preliminary results and definitions

The following results and definitions will be used in our later developments.

Corollary 4.1 Consider the closed loop system (2), and assume that A and $-\mathcal{T}K$ are stable. Then there exist constants $\epsilon^* > 0$, $\alpha > 0$, and $\beta > 0$ with the property that for every initial condition and every pair of piecewise continuous and bounded reference and disturbance signals,

$$\|z(t)\| \leq \alpha\|z(0)\| + \beta \sup_{\tau \geq 0}(\|y_{ref}(\tau)\| + \|w(\tau)\|)$$

for $\epsilon \in (0, \epsilon^*)$, $\epsilon_1 \in (0, \epsilon^*)$, $\epsilon_2 \in (0, \epsilon^*)$, $t \geq 0$.

Proof The proof of this result is omitted since it closely follows the approach used to prove corollary 5.3 in [4, pg. 102]. □

Definition 4.1 A function $f : \mathbf{N} \to \mathbf{R}^+$ is a *bounding function* ($f \in$ BF) if it is strictly increasing and if, for every $c \in \mathbf{R}^+$ and $n \in \mathbf{N}$, there exists a $k > n$ such that $\frac{f(k+1)}{f(k)} > c$. If $f \in$ BF, and if $\lim_{k\to\infty} \frac{f(k+1)}{f(k)} \to \infty$, then define $f \in$ BF$'$ to be a *modified bounding function.* Furthermore, define $f \in$ MSBF to be a *modified strong bounding function* if $f \in$ BF$'$ and if, for all fixed constants $(c_0, c_1, c_3) \in \mathbf{R}^+ \times \mathbf{R}^+ \times \mathbf{R}^+$,

$$\frac{f(i)}{c_0 + c_1(i-1) + c_3 \sum_{j=1}^{i-1} f(i-j)} \to \infty \tag{3}$$

as $i \to \infty$.

Proposition 4.1 There exists a MSBF (e.g. $f(i) = i\exp(i^2)$).

Proof The proof of this result is given in [3]. □

Definition 4.2 A function $g : \mathbf{N} \to \mathbf{R}^+$ is a *tuning function* ($g \in$ TF) if $\lim_{k\to\infty} g(k) = 0$. If $g \in$ TF and there exist constants $\epsilon_0 > 0$ and $\tau > 1$ so that $g(k) = \frac{\epsilon_0}{\tau^k}$ for $k \in \mathbf{N}$, then define $g \in$ TF$'$ to be a *modified tuning function.*

4.2. Known estimate of $\mathcal{T}$

For the noise tolerant self-tuning PID controller using a known estimate of $\mathcal{T}$, given by $\hat{\mathcal{T}}$, the following assumption is made:

Assumption PID1

i) $\|[\eta(0)^T\ a(0)^T]^T\| < f(1)$;

ii) $\|e(0)\| < f(1)$; and

iii) $-\mathcal{T}\hat{\mathcal{T}}^\dagger$ is stable.

Let $\mathcal{S} := \{(\epsilon_0, \tau) : \epsilon_0 > 0, \tau > 1\}$ and define the set of admissible controller parameters as

$$\Omega_{PID} := \{(f, g, g_1, g_2, \rho, N) : f \in \text{MSBF}, g \in \text{TF}', g_1 \in \text{TF}', g_2 \in \text{TF}', \rho \geq 0, N > 0\}.$$

With $\sigma_{PID} = (f, g, g_1, g_2, \rho, N) \in \Omega_{PID}$ and $\xi(t) := [\eta(t)^T \ a(t)^T]^T$, define **Controller PID1** as follows:

Controller PID1

$$\begin{aligned}
\eta(t) &= \int_{t_k}^{t} \epsilon(\tau)e(\tau)d\tau, \ \eta(t_k^+) \equiv 0, \ t \in (t_k, t_{k+1}] \\
\dot{a}(t) &= -Na(t) - y(t), \\
d(t) &= -N^2 a(t) - Ny(t), \\
u(t) &= K\left(\eta(t) + \rho\epsilon_1(t)e(t) + \epsilon_2(t)d(t)\right)
\end{aligned}$$

where $k \in \{1, 2, 3, \ldots\}$, $K := \hat{\mathcal{T}}^{\dagger}$,

$$(\epsilon(t), \epsilon_1(t), \epsilon_2(t)) = (g(k), g_1(k), g_2(k))$$

for $t \in (t_k, t_{k+1}]$, $t_1 := 0$, and where, for each $k \geq 2$ such that $t_{k-1} \neq \infty$, t_k is defined by

$$t_k := \left\{ \begin{array}{ll} \min(t) \ni & \text{if } \min(t) \text{ exists} \\ \quad \text{i)} \quad t > t_{k-1}, \text{ and} & \\ \quad \text{ii)} \quad \|\xi(t)\| = f(k-1) \text{ and/or} & \\ \qquad\quad \|e(t)\| = f(k-1) & \\ \infty & \text{otherwise} \end{array} \right\}.$$

Corollary 4.2 Consider matrix $\bar{A}_{PID}$ (ρ, ϵ, ϵ_1, ϵ_2, K, N) given in (2), and assume that A is stable, $\mathcal{T}$ has full row rank, and $\rho \geq 0$, $N > 0$ are fixed; then if $-\mathcal{T}K$ is stable for $K := \hat{\mathcal{T}}^{\dagger}$, for almost all ($\epsilon_0$, τ, ϵ_{0_1}, τ_1, ϵ_{0_2}, τ_2) $\in \mathcal{S} \times \mathcal{S} \times \mathcal{S}$,

$$\left(\bigcup_{h=1}^{\infty} \lambda(\bar{A}_{PID}(\rho, \frac{\epsilon_0}{\tau^h}, \frac{\epsilon_{0_1}}{\tau_1^h}, \frac{\epsilon_{0_2}}{\tau_2^h}, K, N)) \right) \cap \mathbf{C}^0 = \emptyset.$$

Proof The proof of this result is omitted since it closely follows the method used to prove lemma 4 in [7, pp. 521-522]. □

This controller has the following property:

Theorem 4.1 Consider the stable plant (1) with Controller PID1 applied at $t = 0$; then for every $\sigma_{PID} \in \Omega_{PID}$, every bounded constant reference and disturbance signal, and every initial condition $z(0) := [x(0)^T \ \eta(0)^T \ a(0)^T]^T$ for which assumption PID1 holds, the closed loop system has the properties that:

i.) there exist a finite time $t_{ss} \geq 0$ and constants $\epsilon_{ss} > 0$, $\epsilon_{ss_1} > 0$, $\epsilon_{ss_2} > 0$ such that $\epsilon(t) = \epsilon_{ss}$, $\epsilon_1(t) = \epsilon_{ss_1}$, $\epsilon_2(t) = \epsilon_{ss_2}$ for all $t \geq t_{ss}$;

ii.) the controller states $\eta(t)$, $a(t)$, and the plant state $x(t)$ remain bounded for all $t \geq 0$; and

iii.) if the reference and disturbance inputs are constant signals and $g \in \text{TF}'$, $g_1 \in \text{TF}'$, $g_2 \in \text{TF}'$, then for almost all $(\epsilon_0, \tau, \epsilon_{0_1}, \tau_1, \epsilon_{0_2}, \tau_2) \in \mathcal{S} \times \mathcal{S} \times \mathcal{S}$, $e(t) \to 0$ as $t \to \infty$.

Proof The proof is given in [3]. □

Remark 4.1 We label the above controller "noise tolerant" due to the lack of a pure derivative term, and due to the fact that the switching criteria on $\|e(t)\|$ is not an exponentially decaying bound (as opposed to controllers 1 and 1′ given originally in [7]). ◇

4.3. Unknown estimate of $\mathcal{T}$

For the noise tolerant self-tuning PID controller using no known estimate of $\mathcal{T}$, define the set of admissible controller parameters as

$$\Omega'_{PID} := \{(f, g, g_1, g_2, \rho, N, U) : f \in \text{MSBF}, g \in \text{TF}', g_1 \in \text{TF}', g_2 \in \text{TF}', \rho \geq 0, N > 0, U \in \mathbf{R}^{m \times m} \text{ and is nonsingular}\};$$

let $\mathcal{S}' := \{(\epsilon_0, \tau, U) : \epsilon_0 > 0, \tau > 1, U \in \mathbf{R}^{m \times m}$ and is nonsingular$\}$, and define **assumption PID1′** to be assumption PID1 with condition (iii) removed.

With $\sigma'_{PID} = (f, g, g_1, g_2, \rho, N, U) \in \Omega'_{PID}$ and $\xi(t) := [\eta(t)^T \ a(t)^T]^T$, define **Controller PID1′** as

Controller PID1′:

$$\begin{aligned}
\eta(t) &= \int_{t_k}^{t} \epsilon(\tau)e(\tau)d\tau, \ \eta(t_k^+) \equiv 0, \ t \in (t_k, t_{k+1}] \\
\dot{a}(t) &= -Na(t) - y(t), \\
d(t) &= -N^2 a(t) - Ny(t), \\
u(t) &= K(t)\left(\eta(t) + \rho\epsilon_1(t)e(t) + \epsilon_2(t)d(t)\right)
\end{aligned}$$

where $k \in \{1, 2, 3, \ldots\}$,

$$(\epsilon(t), \epsilon_1(t), \epsilon_2(t)) = (g(k), g_1(k), g_2(k))$$

for $t \in (t_k, t_{k+1}]$, $t_1 := 0$, and where, for each $k \geq 2$ such that $t_{k-1} \neq \infty$, t_k is defined by

$$t_k := \left\{ \begin{array}{ll} \min(t) \ni & \text{if } \min(t) \text{ exists} \\ \quad \text{i)} \quad t > t_{k-1}, \text{ and} & \\ \quad \text{ii)} \quad \|\xi(t)\| = f(k-1) \text{ and/or} & \\ \qquad\quad \|e(t)\| = f(k-1) & \\ \infty & \text{otherwise} \end{array} \right\}$$

with

$$K(t) := UW_i, \quad i \in \{1, 2, \ldots, s\}$$

p. 3

for $i := ((k-1) \bmod s) + 1$, $t \in (t_k, t_{k+1}]$. An explicit method for constructing the $K_j := UW_j$ for the case when $m \geq r$ is given in [9].

Remark 4.2 For a two-input two-output system, with $U = I$, $K \in$

$$\left\{ \begin{array}{ccc} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{W_1}, & \underbrace{\frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}}_{W_2}, & \underbrace{\frac{1}{2}\begin{bmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{bmatrix}}_{W_3}, \\ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{W_4}, & \underbrace{\frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}}_{W_5}, & \underbrace{\frac{1}{2}\begin{bmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}}_{W_6} \end{array} \right\}.$$

$\diamond$

Corollary 4.3 Consider matrix $\bar{A}_{PID}$ $(\rho, \epsilon, \epsilon_1, \epsilon_2, K, N)$ given in (2), and assume that A is stable, $\mathcal{T}$ has full row rank, and $\rho \geq 0$, $N > 0$ are fixed; then with $\mathbf{K} := \{K_j : j \in \{1, 2, \ldots, s\}\}$, for almost all $(\epsilon_0, \tau, \epsilon_{0_1}, \tau_1, \epsilon_{0_2}, \tau_2, U) \in \mathcal{S} \times \mathcal{S} \times \mathcal{S}'$,

$$\left(\bigcup_{i=1}^{s} \bigcup_{h=1}^{\infty} \lambda(\bar{A}_{PID}(\rho, \frac{\epsilon_0}{\tau^h}, \frac{\epsilon_{0_1}}{\tau_1^h}, \frac{\epsilon_{0_2}}{\tau_2^h}, K_i, N)) \right) \cap \mathbf{C}^0 = \emptyset.$$

Proof The proof of this result is omitted since it closely follows the method used to prove lemma 4 in [7, pp. 521-522]. $\square$

This controller has the following property:

Theorem 4.2 Consider the stable plant (1) with Controller PID1$'$ applied at $t = 0$; then for every $\sigma'_{PID} \in \Omega'_{PID}$, every bounded constant reference and disturbance signal, and every initial condition $z(0) := [x(0)^T \ \eta(0)^T \ a(0)^T]^T$ for which assumption PID1$'$ holds, the closed loop system has the properties that:

i.) there exist a finite time $t_{ss} \geq 0$, a matrix K_{ss}, and constants $\epsilon_{ss} > 0$, $\epsilon_{ss_1} > 0$, $\epsilon_{ss_2} > 0$ such that $K(t) = K_{ss}$, $\epsilon(t) = \epsilon_{ss}$, $\epsilon_1(t) = \epsilon_{ss_1}$, $\epsilon_2(t) = \epsilon_{ss_2}$ for all $t \geq t_{ss}$;

ii.) the controller states $\eta(t)$, $a(t)$, and the plant state $x(t)$ remain bounded for all $t \geq 0$; and

iii.) if the reference and disturbance inputs are constant signals and $g \in \mathrm{TF}'$, $g_1 \in \mathrm{TF}'$, $g_2 \in \mathrm{TF}'$, then for almost all $(\epsilon_0, \tau, \epsilon_{0_1}, \tau_1, \epsilon_{0_2}, \tau_2, U) \in \mathcal{S} \times \mathcal{S} \times \mathcal{S}'$, $e(t) \to 0$ as $t \to \infty$.

Proof The proof is given in [3]. $\square$

5. Simulation results

Consider the following example (Rosenbrock problem) taken from [7, pg. 517]:

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} -1/6 & 0 \\ 2/3 & 1 \\ 0 & 1/2 \end{bmatrix} u + Ew,$$

$$y = \begin{bmatrix} 3 & -3/4 & -1/2 \\ 2 & -1 & 0 \end{bmatrix} x,$$

which has a DC gain given by

$$\mathcal{T} = \begin{bmatrix} 1 & 2 \\ 1/3 & 1 \end{bmatrix}.$$

Assume that the following estimate of $\mathcal{T}$ is known:

$$\hat{\mathcal{T}} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix};$$

let

$$f(i) := \left\{ \begin{array}{l} 4i, \ 1 \leq i \leq 10 \\ 20(i-10)\exp((i-10)^2), \ i > 10 \end{array} \right\} \quad (4)$$

with $g(i) := \frac{10}{2^i}$, $y_{ref}(t)^T := [-2 \ -2]$, $w(t) := 0$, $i \in \mathbf{N}$, and set all initial conditions to be equal to zero at $t = 0$.

Using controller 2 defined in [7], the output response shown in figure 1 is obtained. For comparative purposes, the response obtained by using controller PID1 with $g_1(i) = g_2(i) := g(i)$, $\rho := 1$, and $N := 1$ is given in figure 2; in this case, all initial conditions were also set to be equal to zero at $t = 0$, the states of $\eta(t)$ were *not* reset to zero after each controller switch, and $f(i)$ as defined in (4) was used.

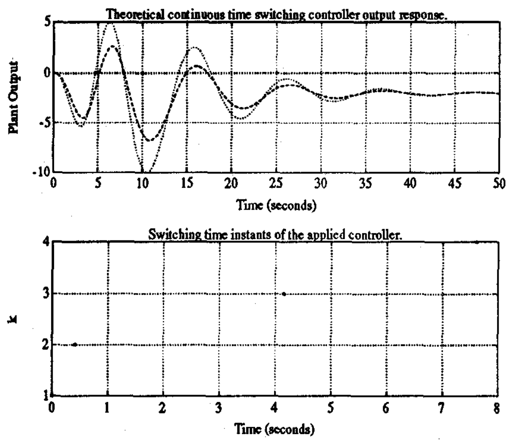

Figure 1: Simulated results of y_1 (dotted) and y_2 (dashed) with controller 2 given by Miller and Davison [7] applied to Rosenbrock's problem.

As can be seen, in this instance, the output transient response is noticeably improved over that shown in figure 1. For further comparison, the plant output obtained using controller PID1$'$ is shown in figure 3, where the same initial conditions and parameter functions/values as defined earlier were used, and the cyclic switching action as summarized in table 1 was implemented. (In this case, four cycles through each of the six possible feedback matrices was required before switch-

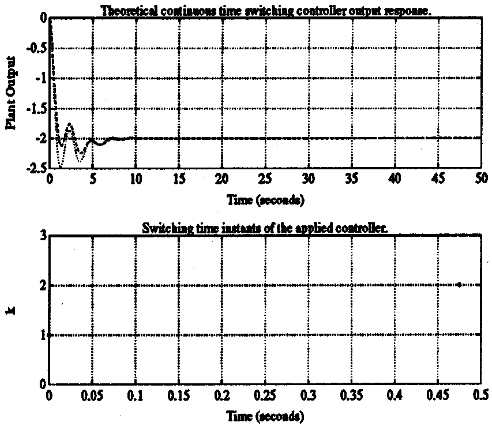

Figure 2: Simulated results of y_1 (dotted) and y_2 (dashed) with controller PID1 applied to Rosenbrock's problem.

ing stopped; once again, the states of $\eta(t)$ were *not* reset to zero after each controller switch.)

k	1	2	$\cdots$	6	7
t	$(t_1, t_2]$	$(t_2, t_3]$	$\cdots$	$(t_6, t_7]$	$(t_7, t_8]$
K	W_1	W_2	$\cdots$	W_6	W_1
f	$f(1)$	$f(1)$	$\cdots$	$f(1)$	$f(2)$
g	$g(1)$	$g(1)$	$\cdots$	$g(1)$	$g(2)$
k	8	9	10	11	$\cdots$
t	$(t_8, t_9]$	$(t_9, t_{10}]$	$(t_{10}, t_{11}]$	$(t_{11}, t_{12}]$	$\cdots$
K	W_2	W_3	W_4	W_5	$\cdots$
f	$f(2)$	$f(2)$	$f(2)$	$f(2)$	$\cdots$
g	$g(2)$	$g(2)$	$g(2)$	$g(2)$	$\cdots$

Table 1: Summary of the cyclic switching behaviour used for figure 3.

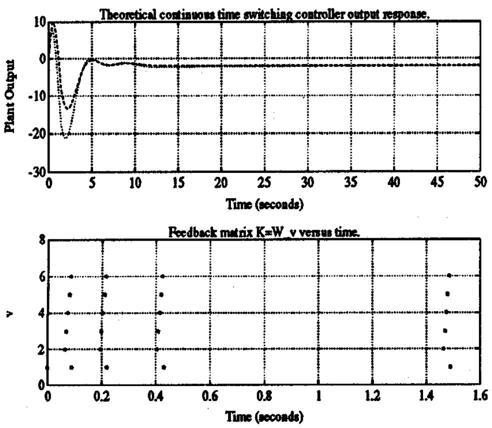

Figure 3: Simulated results of y_1 (dotted) and y_2 (dashed) with controller PID1$'$ applied to Rosenbrock's problem.

6. Conclusions

In this paper, we have shown how the noise tolerant controllers given in [1], [2], [7] can be modified to yield corresponding noise insensitive self-tuning proportional-integral-derivative (PID) controllers. The simulated results presented here indicate that a much more general controller structure (PID) than originally given by Miller and Davison [7] (I) can be implemented using a switching criteria with very little *a priori* system information, and therefore give encouragement as to the development of a full theory of self-tuning controllers of this form.

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