An Optimal Real-time Controller for Vertical Plasma Stabilization

N. Cruz, J.-M. Moret, S. Coda, B.P. Duval, H.B. Le, A.P. Rodrigues, C.A.F. Varandas, C.M.B.A. Correia and B. Gonçalves

***Abstract*—Modern tokamaks have evolved from the initial axisymmetric circular plasma shape to an elongated axisymmetric plasma shape that improves the energy confinement time and the triple product, which is a generally used figure of merit for the conditions needed for fusion reactor performance. However, the elongated plasma cross section introduces a vertical instability that demands a real-time feedback control loop to stabilize the plasma vertical position and velocity. At the Tokamak Configuration Variable (TCV) in-vessel poloidal field coils driven by fast switching power supplies are used to stabilize highly elongated plasmas. TCV plasma experiments have used a PID algorithm based controller to correct the plasma vertical position. In late 2013 experiments a new optimal real-time controller was tested improving the stability of the plasma.**

This contribution describes the new optimal real-time controller developed. The choice of the model that describes the plasma response to the actuators is discussed. The high order model that is initially implemented demands the application of a mathematical order reduction and the validation of the new reduced model. The lower order model is used to derive the time optimal control law. A new method for the construction of the switching curves of a bang-bang controller is presented that is based on the state-space trajectories that optimize the time to target of the system.

A closed loop controller simulation tool was developed to test different possible algorithms and the results were used to improve the controller parameters.

The final control algorithm and its implementation are described and preliminary experimental results are discussed.

***Index Terms*—Real-Time, Tokamak, Plasma Control, Optimal Control**

I. Introduction

MODERN tokamak devices [1] are designed to accommodate elongated cross-section plasmas [2][3] to improve fusion performance. A vertically elongated plasma presents important advantages since it allows the creation of divertor plasmas, the increase of the plasma current and density limit as well as providing plasma stability. However, an elongated plasma is unstable due to the forces that pull the plasma column upward or downward. The result of these forces is a plasma configuration that tends to be pushed up or down depending on the initial displacement disturbance. For example, a small displacement downwards results in the lower poloidal field coils pulling the plasma down, with increased strength as the plasma gets further from the equilibrium position. To compensate this instability, feedback controllers have been designed to correct the vertical position displacement [4][5][6].

The design of vertical stabilization feedback controllers has been based in simple models, resulting in experimentally tuned Single Input Single Output (SISO) Proportional Integral and Derivative (PID) regulators. This procedure requires an in-depth experimental treatment that is time consuming and demands a big number of experimental discharges to obtain the necessary gains optimization. This conference record presents an alternative method to design the vertical stabilization controller of a tokamak using a simple plasma model and the application of optimal control theory.

This conference record highlights the main parts of the paper with the same title that was submitted for publication at *IEEE Transaction on Nuclear Science* [7]. It is organized as follows: Section II presents the vertical observer developed to detect the plasma centroid vertical position and velocity in real-time; Section III briefly depicts the different methods that can be used to describe a tokamak plasma; Section IV describes the state-space plasma model that predicts the plasma response to the actuators and the model reduction performed to permit the application of the time optimal control theory that is presented in Section V; Section VI depicts the simulation tool that permits off line testing and parameter tunning of the controller; The controller results and future work is presented in Section VII.

II. Vertical Plasma Position Observer

The vertical position observer is a linear combination of the magnetic field measured using the magnetic diagnostics. A matrix containing the contribution weight of each magnetic probe is calculated before each plasma discharge, taking into account the planned plasma parameters such as shape and position. The contribution of each probe to the observer has in account the pre-planned plasma parameters, because the

Manuscript received May 19, 2014. This work was supported by EURATOM and carried out within the framework of the European Fusion Development Agreement. IST activities also received financial support from ”Fundação para a Ciência e Tecnologia” through project Pest-OE/SADG/LA0010/2013. The views and opinions expressed herein do not necessarily reflect those of the European Commission. This work was partly supported by the Swiss National Science Foundation.

N. Cruz is with the Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, P-1049-001 Lisboa, Portugal (Telephone: +351-239410108, e-mail: nunocruz@ipfn.ist.utl.pt).

J.-M. Moret, S. Coda, B. P. Duval and H. B. Le are with the Centre de Recherches en Physique des Plasmas, École Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland.

A. P. Rodrigues, C. A. F. Varandas and B. Gonçalves are with the Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, P-1049-001 Lisboa, Portugal.

C. M. B. A. Correia is with the Centro de Instrumentação, Departamento de Física, Universidade de Coimbra, P-3004-516 Coimbra, Portugal

978-1-4799-3659-5/14/$31.00 ©2014 IEEE

probes closer to the plasma are more efficient estimating its position and will be given more weight in the observer.

A set of coefficients are calculated to define the observer from a finite element set of plasma current filaments, using Green's functions, thus it is possible to calculate the magnetic field produced in the probes. The matrix is built with the set of probes that are going to be used in the measurements and inverted to obtain the observer coefficients [8][9].

III. Plasma Description

The modeling of a tokamak plasma demands complex mathematical calculation, in depth physical knowledge and computational power for numerical calculation during simulation phase. Different paths have been tried to accomplish this mission.

The simpler models consider the plasma as a filament or non-deformable matrix of conducting filaments. The more complex models include nonlinear codes, which permit the simulation of nonlinear behaviors such as large vertical position displacements. Some important plasma model and reconstruction codes include [10]:*PET* [11], *ASTRA* [12], *TSC* [13], *EFIT* [14], *FBT* [15], *PROTEUS* [16], *CREATE-L* [17][18], *DINA* [19] and *RZIP* [20].

Some of these codes are accurate for plasma simulation and reconstruction but due to its complex structure are not suitable for controller design. This action is based on simpler linear models that ensure the stability, robustness and performance of the controller, provided that the states are not too far from equilibrium. Controllers are thus usually designed based on the linear model of the flat top phase, achieving good performance through the whole discharge due to its robustness.

Linear models for control design purposes use the electrical circuit equations to calculate the time evolution of the plasma current. Two such models are the CREATE-L and the RZIP models. CREATE-L considers the plasma deformation through the calculation of the plasma current distribution equilibrium. On the other hand, RZIP is an enhanced non deformable model that may vary its vertical and radial position, as well as its total plasma current. The RZIP model is presented in the next section, to be used for the design of the new plasma stabilization controller.

IV. Plasma Model for Control

A. RZIP Plasma Model

The use of the RZIP plasma model aims at finding the transfer function between the currents in the poloidal field coils, internal to the TCV structure close to the plasma, and the vertical plasma displacement [21][22][23][24].

The RZIP model gets its name from the simplifications assumed to build the circuit equations, with the following characteristics: (i) the current has constant distribution, rigid model, as the plasma shape is assumed not to change; (ii) the center of the vertical position can change: plasma is free to move vertically; (iii) the center of the radial position can change: plasma is free to move radially; (iv) the integral of the plasma filaments current can change: the total plasma current is free to change.

The model design simplifications give important advantages over more complex plasma models, maintaining an overall accuracy: (i) A simple model that is easier to implement; (ii) No need to calculate the complete plasma equilibrium; (iii) More explicit model to the quantities that define plasma response to the control variables (a better control model).

B. Step Response to a Voltage Change on the Fast Coil

From the complete RZIP model described with some state variables that can be neglected in the vertical stabilization problem, the model was simplified aiming at calculating the transfer function from the current on the internal FPS coils to the plasma vertical position. This is the mathematical method that describes the influence of the currents in the fast coils in the plasma vertical position.

The simplification of the full plasma model for the particular case of the plasma vertical stability using the in-vessel fast coils presents a difficulty from the fact that the multiple input multiple output (MIMO) system that is obtained from the plasma model must be diagonalized to obtain a single input single output (SISO) system, independent from the remain system inputs and outputs. This is not always possible and some constraints must be analyzed to make them independent.

This simplification is possible for the present case because the vertical stabilization operates in a different time scale from the other plasma control variables such as position, shape or current. Moreover the vertical position that is also controlled by the poloidal coils outside the vessel can be considered independent of the internal poloidal coils, because of the same reason. While the poloidal coils outside the vessel control the slow vertical displacement of the plasma, the in-vessel coils act on a much faster time-scale, reacting to fast plasma disturbances.

The state space system is diagonalized to obtain the independent influence of the coil currents over the plasma vertical position. Then the equation of the fast coil is taken by neglecting the influence of the other coils. This is possible due to the different time scale of the actuation of the coils. The typical way to address the vertical stabilization problem is to independently control the vertical plasma position from the plasma current and shape controllers [2], which are designed on the basis that the system is vertically stable due to the controller already implemented. This double loop arrangement simplifies the design of the controllers, based on the assumption and later confirmation that the controllers act on different time scales. Different frequencies in the controllers permit the treatment of some parameters as disturbances to the next stage of the global controller.

C. Model Reduction and Validation

In control engineering, the best model is not always the most accurate, but the one that permits the construction of a robust stable controller, according to the necessary performance and specifications.

For the purpose of applying optimal control theory to the plasma model obtained a model reduction was necessary to permit the mathematical treatment presented in the next section. The transfer function that was obtained is of 52^{nd} order, while optimal control theory is usually applied to systems with second or third order at most. This led to the application of model reduction techniques.

Model reduction techniques are a powerful tool that uses methods based on the idea of projecting the state space to a much lower dimension, obtaining a reduced system that may be solved more efficiently. For control design purposes, it is possible to approximate the model with another model of reduced order that preserves the original transfer function as much as possible.

The method of balanced realization was applied to reduce the transfer function [25], by eliminating the states with small σ_i, i.e. with small influence in the behavior of the transfer function. This method permits the model reduction to a second order transfer function with difference results that could not be detected by the plot of the step response of both transfer functions.

V. Optimal Control

A. System Definition

Time optimal control was applied to the second order model to obtain a control law, the switching time and the final time of the bang-bang controller [26][27][28][29].

The second order transfer-function that describes the plasma model has the form:

$$\frac{X_s(s)}{U(s)} = \frac{n_1 s + n_2}{s^2 + d_1 s + d_2} \tag{1}$$

This transfer function represents the following controllable state space model:

$$\dot{X} = AX + Bu \tag{2}$$

with $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 \\ -d_2 & -d_1 \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, where

$$x_1 = x_s \tag{3}$$

and

$$x_2 = x_1 + b_1 u \tag{4}$$

are the system variables.

Having defined the system model and given the initial system state X_0, the aim is finding the control law and parameters that take the system from the initial state X_0 to a target state X_1, minimizing the time to target.

B. Control Law

The problem of finding the control law that drives the plasma position from an initial position X_0 to a final position X_1 in the minimum amount of time, is easier with the definition of a new system state X_N and the redefinition of the state system equations. In this state system the set point becomes the origin, thus simplifying the problem.

Using Pontryagin's Minimum Principle (PMP) to minimize the cost function given by the time to achieve the set point, the control must minimize the Hamiltonian of the system given by:

$$H = 1 + \lambda^T (AX_N + Bu + AX_1) \tag{5}$$

where λ is the state of the adjoint system, representing the system as a linear transformation using the vector space defined by the eigenvectors. The minimization of the Hamiltonian yields the optimal time control law

$$\begin{aligned} \lambda^T B > 0 \Rightarrow u = u_- \\ \lambda^T B < 0 \Rightarrow u = u_+ \end{aligned} \tag{6}$$

The bang-bang control law is complete with an arbitrary value of u for $\lambda^T B = 0$, which might also be a dead zone where no control is applied to avoid unnecessary switching due to hysteresis.

C. Predictive Control and Construction of Switching Curves

This subsection presents the method to predict the action ahead, preventing situations when the observer becomes temporarily unavailable, for example in the presence of Edge Localized Modes (ELMs). By the use of this method, it is possible to keep the system stable, by predicting the control action needed, provided the time the observer is not available is shorter than the final control time calculated and no other major unpredicted disturbance affects the system.

This method is based on the a-priori calculation of the switching time and final time for the optimal time control law of the system. This application uses some of the deduction and results already presented [29], but a new simpler and more generic algorithm was developed. The idea is to find the position where the following two paths cross each other. From the initial system state is applied the maximum control possible in the direction of the set point tracing this path. Also trace the path from the set point applying the opposite control backward in time. The state-space point where both paths cross is the place where the controller should switch.

Based on the idea presented a fully computational algorithm was developed and implemented [7].

VI. Controller Simulations

A. Simulator Tool

The plasma model was used to build a system simulation tool using Matlab Simulink [31].

The plasma model includes the transfer function between the currents in the internal poloidal field coils and the plasma position, but lacks the mathematical model of the fast power supplies. This transfer function was studied and validated and was also taken into account in the Simulink model.

B. Controller Simulations

The controller algorithm was tested and tuned based on simulation analysis. The decision for the best controller based on these analysis, resulted in a controller that adapts its force to the initial velocity detected.

A true bang-bang controller that always applies the maximum restore signal would exhibit a big oscillation in the plasma position. On the opposite side, a bang-bang controller that was limited to use a small control signal avoiding to exhibit oscillations, would be limited to the control of small perturbations. Thus, a weighted bang-bang controller that increases its restore signal according to the initial plasma velocity demonstrated to be much more efficient, resulting in a more stable controller.

From the analysis of simulations it was seen that a big disturbance can be controlled using a high control signal for higher displacements and a smaller control signal for smaller displacements. The implemented controller is a weighted bang-bang controller, that is similar to use an adaptive bang-bang controller that reconfigures based on system state velocity limits.

VII. Controller Validation and Results

The controller was implemented based on the simulation results and tested during plasma discharges at TCV, with improvement in the overall stability of the plasma. Figure 1 and figure 2 depict the stability improvement using the new controller. The plasma discharges were designed to test the limits of the controllers by increasing the plasma elongation from 0.5 seconds.

The increased instability limit using the new controller can be confirmed by the improvement in discharge time for the same conditions. The current PID controller was not able to cope with the vertical instability finishing the discharge with a vertical disruption at approximately 0.65 s (0.15 s after starting the linear increase in plasma elongation). On the other hand the new bang-bang controller maintained the plasma discharge up to approximately 0.8 s (0.3 s after starting the linear increase in plasma elongation).

The vertical stabilization controller was implemented and tested using one of the hardware modules with parallel digital signal processing capabilities of the Advanced Plasma Control System [32]. For further testing of the controller it is envisaged the use of an ELM detector [33] capable of signaling the error and unavailability of plasma position observer. It is also planned the controller implementation in a newer control hardware based on FPGA [34] to study and compare the performance of both systems.

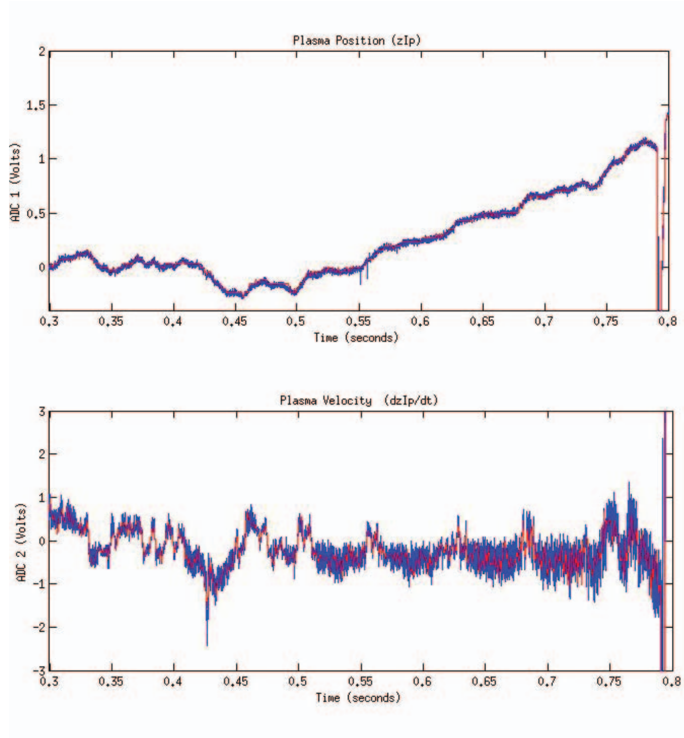

Fig. 1. Plasma position and velocity for discharge 49564 using the new bang-bang controller

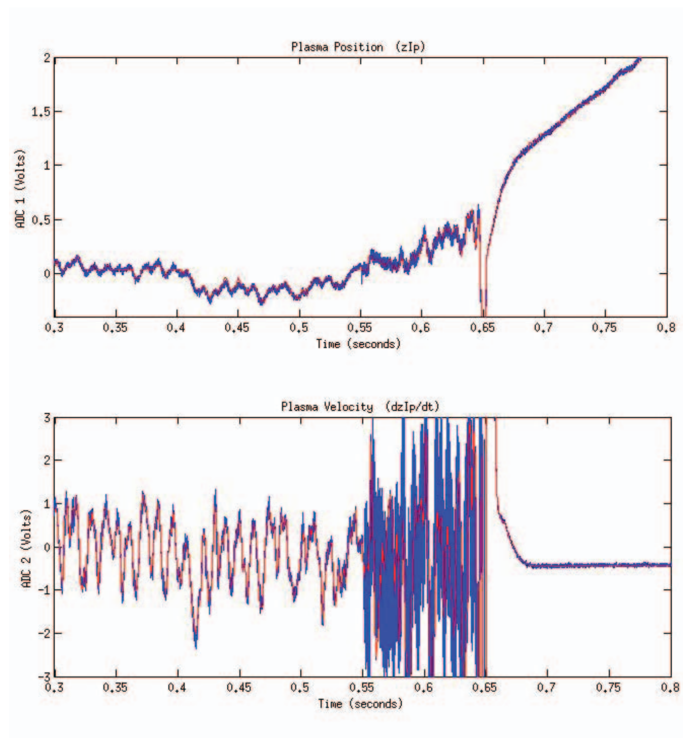

Fig. 2. Plasma position and velocity for reference discharge 49567 using the standard controller in the same plasma conditions as discharge 49564

References

[1] J. Wesson, *Tokamaks*, Oxford University Press (2004).

[2] M. Ariola and A. Pironti, *Magnetic control of tokamak plasmas*, Springer-Verlag (2008).

[3] G. Ambrosino and R. Albanese, *Magnetic control of plasma current, position, and shape in tokamaks - A survey of modeling and control approaches*, IEEE Control Systems Magazine, vol.25, no.5, pp. 76- 92, Oct. 2005.

[4] F. Sartori, A. Barbalace, A.J.N. Batista, T. Bellizio, P. Card, G. De Tommasi, P. Mc Cullen, A. Neto, F. Piccolo, R. Vitelli, L. Zabeo and JET-EFDA contributors, *The PCU JET plasma vertical stabilization control system*, Fusion Engineering and Design, Volume 85, Issues 3-4, Proceedings of the 7th IAEA Technical Meeting on Control, Data Acquisition, and Remote Participation for Fusion Research, pp. 438-442, July 2010.

[5] P. Vyas, D. Mustafa and A.W. Morris, *Vertical position control on COMPASS-D*, Fusion Technology, 33(2), 97-105 (1998)

[6] M.L. Walker, D.A. Humphreys, *On feedback stabilization of the tokamak plasma vertical instability*, Automatica, Volume 45, Issue 3, pp.665-674, March 2009.

[7] N. Cruz, J.-M. Moret, S. Coda, B.P. Duval, H.B. Le, A.P. Rodrigues, C.A.F. Varandas, C.M.B.A. Correia, *An optimal real-time controller for vertical plasma stabilization*, submitted to IEEE Transactions on Nuclear Science, preprint available at ArXiv.org http://arxiv.org/abs/1406.4436

[8] F. Hofmann , M.J. Dutch , A. Favre , Y. Martin , J-M. Moret and D.J. Ward, *Feedback stabilization of axisymmetric modes in the TCV tokamak using active coils inside and outside the vacuum vessel*, 1998 Nucl. Fusion 38 399.

[9] N. Cruz, J-M. Moret, S. Coda, J. I. Paley, B.P. Duval, A. P. Rodrigues, F. Piras, F. Felici, C. M. B. A. Correia, C. A. F. Varandas, *Using APCS for Plasma Vertical Control at TCV*, IEEE Transactions on Nuclear Science, 58 Issue 4, 1570-1575, 2011.

[10] A.J. Garrido, I. Garrido, M.G. Sevillano, M. Alberdi, M. Amundarain, O. Barambones, M. De La Sen, *Nuclear fusion control-oriented plasma current linear models*, ICS'10 Proceedings of the 14th WSEAS international conference on Systems: part of the 14th WSEAS CSCC multiconference - Volume I, Pages 145-150, July (2010).

[11] S.A. Galkin, A.A. Ivanov, S.Yu. Medvedev and Yu.Yu. Poshekhonov, *Comparison of tokamak axisymmetric mode growth rates from linear MHD and equilibrium evolution approaches*, Nuclear Fusion, 37, 1455, (1997).

[12] G.V. Pereverzev, P.N. Yushmanov, *ASTRA - Automated System for TRansport Analysis*, Max-Planck-Institut Fur Plasmaphysik, IPP-Report, IPP 5/98, February (2002).

[13] S.C. Jardin , N. Pomphrey and J. De Lucia, *Dynamic Modeling of Transport and Positional Control of Tokamaks*, Journal of Computational Physics, 66, 481-507, (1986).

[14] L.L. Lao, J.R. Ferron, R.J. Groebner, W. Howl, H. St. John, E.J. Strait and T.S. Taylor, *Equilibrium analysis of current profiles in tokamaks*, Nuclear Fusion, 30, 1035, (1990).

[15] F. Hofmann, *FBT - a free-boundary tokamak equilibrium code for highly elongated and shaped plasmas*, Computer Physics Communications, Volume 48, Issue 2, p. 207-221, (1988).

[16] R. Albanese, J. Blum, and O. De Barbieri, *Numerical studies of the NEXT European Torus via the PROTEUS Code*, Proceedings of the 12th Conference on the Numerical Simulation of Plasmas, San Francisco CA (USA), (1987).

[17] R. Albanese and F. Villone *The linearized CREATE-L plasma response model for the control of current, position and shape in tokamaks*, Nuclear Fusion, vol. 38, 723, (1998).

[18] P. Vyas, F. Villone, J.B. Lister, and R. Albanese. *The separatrix response of diverted TCV plasmas compared with the predictions of the CREATE-L model*, Nuclear Fusion, vol. 38, no. 7, pp. 1043-1053, (1998).

[19] R. Khayrutdinov, *Studies of Plasma Equilibrium and Transport in a Tokamak Fusion Device with the Inverse-Variable Technique*, Journal of Computational Physics, vol. 109, pp. 193-201, (1993).

[20] A. Coutlis, I. Bandyopadhyay, J.B. Lister, P. Vyas, R. Albanese, D.J.N. Limebeer, F. Villone, J.P. Wainwright, *Measurement of the open loop plasma equilibrium response in TCV*, Nuclear Fusion 39 (1999) 663.

[21] E.A. Lazarus, J.B. Lister, G.H. Neilson, *Control of the vertical instability in Tokamaks*, Nuclear Fusion 30 (1990) 111.

[22] J.B. Lister, et al., *Experimental study of the vertical stability of high decay index plasmas in the DIII-D Tokamak*, Nuclear Fusion 30 (1990) 2349.

[23] J.B. Lister, Y. Martin, J-M. Moret, *On locating the poloidal field coils for tokamak vertical position control*, Nuclear Fusion 36 (1996) 1547.

[24] F. Hofmann, et al., *Vertical instability in TCV: comparison of experimental and theoretical growth rates*, Nuclear Fusion 37 (1997) 681.

[25] A.J. Laub, M.T. Heath, C.C. Paige, R.C. Ward, *Computation of System Balancing Transformations and Other Applications of Simultaneous Diagonalization Algorithms*, IEEE Trans. Automatic Control, vol. 32, pp. 115-122, (1987).

[26] M. Athans, P. L. Falb, *Optimal Control: An Introduction to the Theory and its Applications*, McGraw-Hill, New York (1966).

[27] I. Vakilzadeh, *Bang-bang control of a plant with one positive and one negative real pole* J. Optim. Theory Appl. 24 (1978), 2, 315-324.

[28] I. Vakilzadeh, A. A. Keshavarz, *Bang-bang control of a second-order non-linear stable plant with second-order nonlinearity*, Kybernetika- Volume 18 (1982), Number 1.

[29] Z. Shen and S.B. Andersson, *Minimum Time Control of A Second-Order System* 49th IEEE Conference on Decision and Control, Hilton Atlanta Hotel, Atlanta, GA, USA, December 15-17, 2010.

[30] A. Favre, J-M. Moret, R. Chavan, A. Elkjaer, D. Fasel, F. Hofmann, J.B. Lister, J-M. Mayor, A. Perez, *Control of Highly Vertically Unstable Plasmas in TCV with Internal Coils and Fast Power Supply*, 19th Symposium on Fusion Technology, SOFT, September 16-20, Lisbon, Portugal, (1996).

[31] http://www.mathworks.com/products/simulink/

[32] N. Cruz, A.P. Rodrigues, B. Santos, C.A.F. Varandas, B.P. Duval, J-M. Moret, J. Berrino, Y. Martin, X. Llobet, *The integration of the new advanced digital plasma control system in TCV*, Fusion Engineering and Design 83 (2008) 215-219.

[33] F. Felici, J.X. Rossel, B.P. Duval, S. Coda, T.P. Goodman, Y. Martin, J-M. Moret, O. Sauter and the TCV Team, *Real-time control of the period of individual ELMs by EC power on TCV*, Nuclear Fusion 53 (2013) 113018.

[34] H.B. Le, B.P. Duval, J-M. Moret, N. Cruz and the TCV team, *New Developments for Real Time Plasma Control System of TCV Tokamak based on FPGA*, this conference proceedings.