# Design of the Plasma Position and Shape Control in the ITER Tokamak Using In-Vessel Coils

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***Abstract*—The International Thermonuclear Experimental Reactor (ITER) is the next step toward the realization of electricity-producing fusion power plants, and it is planned to be in operation in 2016. ITER has been designed so as to reach the plasma burning condition and to operate with high-elongated unstable plasmas. However, due to the constraints that affect the machine realization, these open-loop unstable high-performance plasmas can hardly be stabilized using the poloidal field coils placed outside the tokamak vessel. For this reason, during the *ITER design review phase*, it has been proposed to investigate the possibility of using in-vessel coils in order to improve the best achievable performance of the vertical stabilization (VS) system. This paper proposes a new approach for the plasma current, position, and shape control design in the presence of in-vessel coils. Two control loops are designed: a first loop that guarantees the VS driving the voltage applied to in-vessel coils, and a second loop that controls the plasma current and up to 32 geometrical shape descriptors as close as possible to the reference values. The performance of the proposed control system is shown by means of simulations of some cases of interest.**

***Index Terms*—Plasma shape control, plasma vertical stabilization (VS), tokamaks control.**

## I. Introduction

THE NEED of achieving always better performance in present and future tokamak devices [1] has pushed plasma control to gain more and more importance in tokamak engineering (see the recent book [2]). High performance in tokamaks is achieved by plasmas with elongated poloidal cross section (see Fig. 1). Since such elongated plasmas are vertically unstable, position control is clearly an essential feature of all machines. Beyond this, a strong motivation to improve plasma control is that, in order to obtain the best performance out of a device, it is always necessary to maximize the plasma volume within the available space; hence, the ability to control the shape of the plasma while ensuring good clearance between the plasma and the facing components is an essential feature of any plasma position and shape control system.

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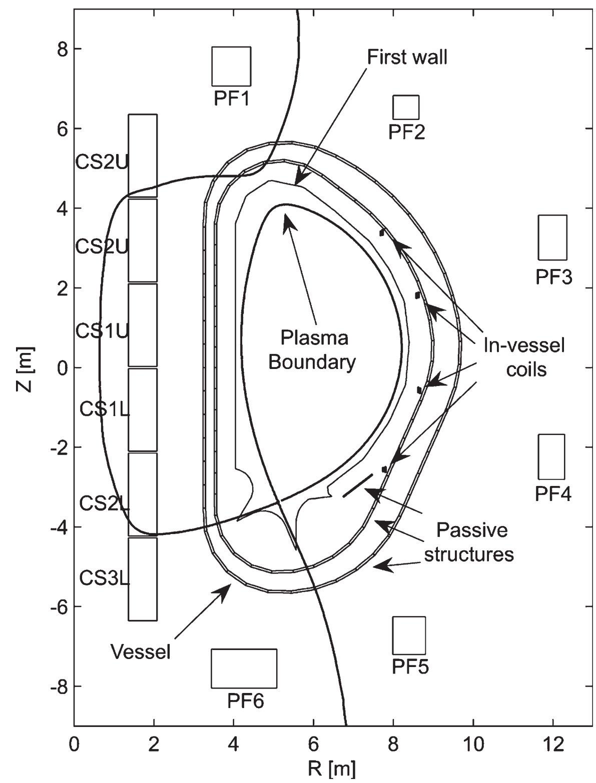


Fig. 1. ITER cross section and PF coil system. Note that the CS coils are part of the PF system.

The International Thermonuclear Experimental Reactor (ITER) is the next step toward the realization of electricity-producing fusion power plants, and it is planned to be in operation in 2016. The main goal of the ITER tokamak is to attain the plasma burning condition and produce about 500 MW of fusion generated power for more than 400 s [3]. To estimate the effort required to build ITER, it may help to note that its major radius is twice the one of the Joint European Torus (JET) [4], which is the world's largest fusion reactor today. Furthermore, the plasma current in ITER is about 15 MA, which is more than twice the maximum current obtained during JET operation in the last 20 years.

Although high performance is needed to reach the desired objectives, the design of the ITER plasma position and shape control system is affected by a number of constraints that are strongly related to the effective realization of the facility. In particular, while elongated and vertical unstable plasmas, with *poloidal beta* $\beta_p$ up to 1.9 and *internal inductance* $l_i$ up to 1.2, are envisaged so as to guarantee the needed particle

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and energy confinement,[1] the power available to control such plasmas is limited, so the saturation levels of actuators are present. Moreover, the passive structures of the ITER vessel introduce a nonnegligible delay on the control action when poloidal field (PF) coils are used to perform plasma vertical stabilization (VS).

Recently, during the *design review phase*, it turned out that the high-elongated and unstable plasmas needed for ITER operations can hardly be stabilized using the superconducting PF coils placed outside the tokamak vessel. For this reason, it has been proposed to investigate the possibility of using in-vessel coils (see Fig. 1) so as to improve the best achievable performance of the VS system.

The problem of designing a plasma current, position, and shape controller using in-vessel coils is tackled in this paper. It is worth noticing that in-vessel coils, which cannot be superconductive, do not permit one to use a simple derivative action on the plasma vertical position, i.e., a proportional action on the plasma vertical position, to stabilize the plasma. It follows that a further control action must be added so as to vertically stabilize the plasma.

Although a possible solution is obtained by adding a proportional action on the plasma position, such an approach would determine a strong interaction between the VS system and the shape controller, which controls the plasma shape and position.

In order to avoid such an interaction, in the proposed approach, we replace the proportional action on the vertical position with a proportional action on the current flowing in the in-vessel coils. In particular, the following two control loops are designed:

1) the VS *system*, which stabilizes the plasma vertical position;
2) the *plasma current and shape control system*, which drives the plasma current error to zero and minimizes the error between the actual plasma boundary and the desired shape reference.

As far as plasma shape control is concerned, the proposed approach combines the solutions previously presented by some of the authors in [5] and [6], allowing us to control extremely shaped plasmas. The proposed VS system consists of a first-order multivariable controller, whose output is the voltage applied to the in-vessel coils. The simple structure of the proposed vertical controller permits one to envisage effective adaptive algorithms and to mitigate the effect of measurement noise on control performance.

This paper is structured as follows. The next section presents the model of the ITER tokamak plant, while the whole control system architecture is described in Section III. The proposed design techniques for the plasma VS system and the plasma position and shape control system are described in Sections IV and V, respectively. Eventually, closed-loop simulations are carried out using the CREATE-L model [7] for the ITER plant so as to illustrate the performance of the proposed control system.

[1] $\beta_p$ is a normalized measure of the mean of the plasma pressure, while $l_i$ measures the plasma current internal distribution.

## II. Plant Model

A tokamak device is a rather complex system, including the plasma, the active coils, and the metallic structures (hereinafter named passive conductors). What we are mainly interested in are the electromagnetic interaction of the plasma with the surrounding coils and the control of the plasma current, position, and shape. For these purposes, it is possible to approximate the plasma behavior using a state-space model.

In particular, the plasma linearized state-space model [7] presented in this section describes the behavior of the plasma shape around a given equilibrium, which is specified in terms of nominal values of plasma current, PF coil currents, poloidal beta, and internal inductance[2]

$$\begin{pmatrix} \mathbf{L}_{11}^* & \mathbf{L}_{12}^* & \mathbf{L}_{13}^* \\ \mathbf{l}_{21}^{*\mathrm{T}} & l_{22}^* & \mathbf{l}_{23}^{*\mathrm{T}} \\ \mathbf{L}_{31}^* & \mathbf{L}_{32}^* & \mathbf{L}_{33}^* \end{pmatrix} \begin{pmatrix} \delta\dot{\mathbf{x}}_{\mathrm{pf}}(t) \\ \delta\dot{x}_{\mathrm{ic}}(t) \\ \delta\dot{\mathbf{x}}_{\mathrm{ec}}(t) \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & r_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_{33} \end{pmatrix} \begin{pmatrix} \delta\mathbf{x}_{\mathrm{pf}}(t) \\ \delta x_{\mathrm{ic}}(t) \\ \delta\mathbf{x}_{\mathrm{ec}}(t) \end{pmatrix}$$

$$= \mathbf{L}^*\delta\dot{\mathbf{x}}(t) + \mathbf{R}\delta\mathbf{x}(t) = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & 1 \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \delta\mathbf{u}_{\mathrm{pf}}(t) \\ \delta u_{\mathrm{ic}}(t) \end{pmatrix} \tag{1a}$$

$$\delta z_p(t) = \begin{pmatrix} \mathbf{c}_{1z}^{\mathrm{T}} & c_{2z} & \mathbf{c}_{3z}^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \delta\mathbf{x}_{\mathrm{pf}}(t) \\ \delta x_{\mathrm{ic}}(t) \\ \delta\mathbf{x}_{\mathrm{ec}}(t) \end{pmatrix} = \mathbf{c}_z^{\mathrm{T}}\delta\mathbf{x}(t) \tag{1b}$$

$$\delta\mathbf{y}(t) = \begin{pmatrix} \mathbf{C}_1 & \mathbf{c}_2 & \mathbf{C}_3 \end{pmatrix} \begin{pmatrix} \delta\mathbf{x}_{\mathrm{pf}}(t) \\ \delta x_{\mathrm{ic}}(t) \\ \delta\mathbf{x}_{\mathrm{ec}}(t) \end{pmatrix} = \mathbf{C}\delta\mathbf{x}(t) \tag{1c}$$

where:

- $\mathbf{L}^*$, $\mathbf{R}$, $\mathbf{c}_z^{\mathrm{T}}$, and $\mathbf{C}$ are the model matrices.
- $\delta\mathbf{x}(t) = (\delta\mathbf{x}_{\mathrm{pf}}^{\mathrm{T}}(t)\, \delta x_{\mathrm{ic}}(t)\, \delta\mathbf{x}_{\mathrm{ec}}^{\mathrm{T}}(t))^{\mathrm{T}} \in \mathbb{R}^{n_{\mathrm{PF}}+n_{\mathrm{EC}}+1}$ is the state-space vector, which includes the current variations $\delta\mathbf{x}_{\mathrm{pf}}(t)$ in the $n_{\mathrm{PF}}$ PF coils used for plasma shape control, the variation of the in-vessel coil current $\delta x_{\mathrm{ic}}(t)$ used for plasma VS,[3] and the variations of the passive currents $\delta\mathbf{x}_{\mathrm{ec}}(t)$ (eddy currents), reduced to $n_{\mathrm{EC}}$ shortcut circuits with the aid of a finite-element approximation.
- $\delta\mathbf{u}(t) = (\delta\mathbf{u}_{\mathrm{pf}}^{\mathrm{T}}(t)\, \delta u_{\mathrm{ic}}(t))^{\mathrm{T}} \in \mathbb{R}^{n_{\mathrm{PF}}+1}$ is the input-voltage variation vector, which includes the voltages $\delta\mathbf{u}_{\mathrm{pf}}(t)$ applied to the PF coils and the voltage $\delta u_{\mathrm{ic}}(t)$ applied to the in-vessel coils.
- $\delta z_p(t)$ is the variation of the plasma vertical position.
- The $\delta\mathbf{y}(t) = (\delta\mathbf{g}^{\mathrm{T}}(t)\delta I_p(t))^{\mathrm{T}} \in \mathbb{R}^{n_G+1}$ vector includes the variations $\delta\mathbf{g}(t)$ of $n_G$ plasma shape geometrical descriptors and the plasma current variation $\delta I_p(t)$.

Notice that (1) is obtained under the assumption of nonresistive and massless plasma. In [8], it has been shown that neglecting plasma mass may lead to erroneous conclusion on closed-loop stability, when simple derivative or proportional vertical controllers are adopted, which is not the case considered in this paper.

Furthermore, in (1), the effects of plasma profile parameters $\delta\beta_p(t)$ and $\delta l_i(t)$, which can be regarded as disturbances,

[2] The superscript $^{\mathrm{T}}$ denotes the vector/matrix transpose.

[3] Note that the in-vessel coils shown in Fig. 1 are connected in series, so a single voltage and current are considered for the in-vessel circuit.

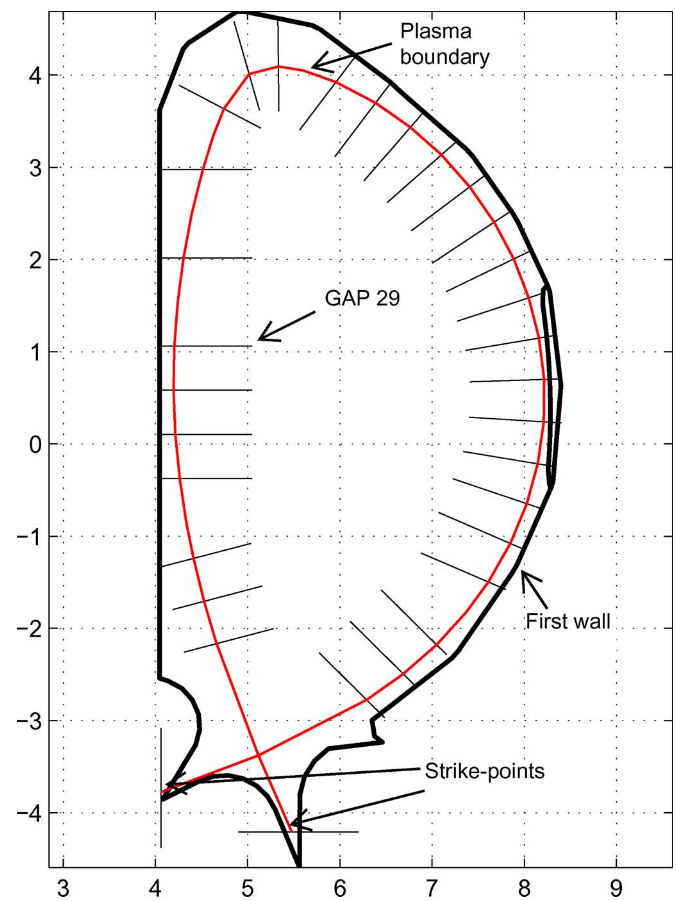



Fig. 2. Plasma boundary geometrical descriptors. The figure shows both gaps, which are plasma-to-wall distances along given directions, and strike points.

are neglected. To control the plasma shape with the needed accuracy, the boundary is usually described by plasma–wall distances, which are called *gaps*, and by the locations of the strike points on the divertor tiles (see Fig. 2).

The elements of matrices $\mathbf{L}^*$ and $\mathbf{R}$ in (1) have the formal role of inductances and resistances. The upper left-hand entry of the $\mathbf{R}$ matrix reflects the fact that the PF coils are superconductive.

The plasma model (1) has to be completed with the models of the power supply and of the diagnostic systems. As far as the power supply system is concerned, its inputs are the voltages demanded by the feedback controller and its outputs are the actual voltages applied to both the PF and in-vessel coils. For our purposes, it is enough to approximate the power supply system as a saturation plus a pure time delay $\tau_1$, and a first-order dynamic characterized by a pole at $1/\tau_2$. The following two types of converters are used: the main converters for the PF coils, i.e., PF1-6, central solenoid CS1U, CS1L, CS2U, CS2L, CS3U, and CS3L (note that the CS1U and the CS1L coils are connected in series), and a fast converter, which is linked to the in-vessel coil, for VS. Table I(a) gives the time parameters $\tau_1$ and $\tau_2$, together with the saturation voltages $u_{\max}$'s, for each converter. The dynamic response of the sensors has been approximated as a first-order system characterized by a pole at $1/\tau_3$. Table I(b) shows the value of $\tau_3$ for all the measurements needed for plasma position and shape control.

TABLE I
ACTUATOR AND SENSOR PARAMETERS FOR THE ITER TOKAMAK. (a) MAIN AND VS CONVERTER PARAMETERS. (b) PLASMA POSITION AND SHAPE SENSOR PARAMETERS

(a)

| Converter | $\tau_1$ | $\tau_2$ | $u_{max}$ |
|---|---|---|---|
| CS1U&L | 15 ms | 15 ms | 3.00 kV |
| CS2U | 15 ms | 15 ms | 1.50 kV |
| CS2L | 15 ms | 15 ms | 1.50 kV |
| CS3U | 15 ms | 15 ms | 1.50 kV |
| CS3L | 15 ms | 15 ms | 1.50 kV |
| PF1 | 15 ms | 15 ms | 1.50 kV |
| PF2 | 15 ms | 15 ms | 1.50 kV |
| PF3 | 15 ms | 15 ms | 1.50 kV |
| PF4 | 15 ms | 15 ms | 1.50 kV |
| PF5 | 15 ms | 15 ms | 1.50 kV |
| PF6 | 15 ms | 15 ms | 1.50 kV |
| VS | 2.5 ms | 7.5 ms | 0.3 kV |

(b)

| Measurement | $\tau_3$ |
|---|---|
| $z_p$ | 7 ms |
| $I_p$ | 150 ms |
| $\mathbf{x}_{pf}$ | 150 ms |
| $x_{ic}$ | 7 ms |
| $gaps$ | 150 ms |

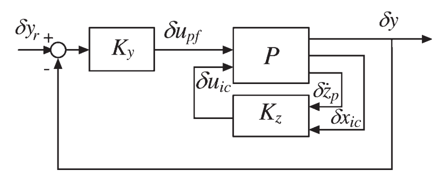



Fig. 3. Control system architecture.

## III. CONTROL SYSTEM ARCHITECTURE

The plasma current, position, and shape control system has a twofold task. On the one hand, it has to vertically stabilize the plasma vertical position, and on the other hand, it has to drive the plasma current and the geometrical shape descriptors as close as possible to the reference values. It is important to note that these two tasks can be performed on different time scales; indeed, while the time constant of the unstable mode in the ITER tokamak is about 100 ms, the settling time in the response to the reference signals can vary between 15 and 25 s. For these reasons, it is convenient to use the feedback control structure shown in Fig. 3. In this scheme, the controller $K_z$ aims to stabilize the plasma by applying a voltage signal to the VS converter, while the controller $K_y$ aims to control the output vector $\delta\mathbf{y}(t)$ (whose components are the plasma current plus the plasma geometrical descriptors shown in Fig. 2). Note that the VS loop is a multi-input–single-output controller, while $K_y$ is a multi-input–multi-output controller acting on the stabilized plant.

## IV. VERTICAL STABILIZATION SYSTEM

In principle, using as actuators the superconductive PF coils, it is possible to perform plasma VS by using a simple derivative feedback control of the vertical position $z_p$. Although this approach had been proposed in the past [5], recent investigation pointed out that such an approach implies extremely low closed-loop stability margins, in particular for plasma equilibria with internal inductance $l_i$ exceeding the value of 1.0 (the higher the $l_i$ is, the more unstable the plasma becomes). As an example, two planned ITER equilibria have been considered [9], showing that, using some of the PF coils as actuators of the VS system, the maximum achievable phase margin (when a proportional controller is supposed to be used) is about 22° when $l_i = 0.85$, while this value drops to 6° when $l_i$ increases to 1.1. It is worth noticing that, in current operating tokamaks, as in the JET tokamak, the phase margin ranges from 45° to 60° [10].

In order to achieve higher values of phase margin, it is possible to resort to a higher order controller so as to introduce the needed phase lead in the neighborhood of the crossover frequency. However, in the authors' opinion, this solution is not recommendable for the following reasons.

1) Both operational experience on operating tokamaks [10] and simulation studies show that the VS controller parameters must be adaptively changed against the large variations of plasma parameters—geometrical shape and current distribution—occurring during the various phases of the scenario (current ramp-up, limiter/divertor transition, flattop, heating, and ramp-down). The reliability of such a stabilizing adaptive controller strongly depends upon the number of parameters to be adapted, so the use of high-order controllers should be avoided.
2) High-order controllers, which are able to significantly improve the phase margin, have the drawback of amplifying the unavoidable noise on the vertical velocity measurement. This amplification could even lead to voltage saturation of power supplies and could hence cause the loss of the closed-loop stability.

For these reasons, a VS controller with a simple structure would be preferable in order to envisage effective adaptive algorithms and to mitigate the effect of measurement noise on control performance.

A solution envisaged in ITER to obtain such a simple controller provides for the installation of in-vessel coils to be used for VS. In particular, using in-vessel coils to stabilize the plasma, it is possible to eliminate the delay introduced by the shielding effect of the vacuum vessel conductors.

On the other hand, since the in-vessel coils are close to the plasma, they cannot be superconductive, so the plasma equilibrium cannot be stabilized by means only of a derivative action on $z_p$. It is then necessary to add a proportional action on either the plasma vertical position [11] or the current flowing in the in-vessel coils.

In the approach proposed here, we resort to the latter option in order to let $z_p$ to vary according to the request of the plasma shape controller. Let the in-vessel $\delta u_{\mathrm{ic}}(t)$ voltage be equal to

$$\begin{aligned}\delta u_{\mathrm{ic}}(t) &= k_D \delta \dot{z}_p(t) + k_I \delta x_{\mathrm{ic}}(t) \\ &= k_D \left( \mathbf{c}_{1z}^{\mathrm{T}} \delta \dot{\mathbf{x}}_{\mathrm{pf}}(t) + c_{2z} \delta \dot{x}_{\mathrm{ic}}(t) + \mathbf{c}_{3z}^{\mathrm{T}} \delta \dot{\mathbf{x}}_{\mathrm{ec}} \right) \\ &\quad + k_I \delta x_{\mathrm{ic}}(t) \end{aligned} \tag{2}$$

where $\delta \dot{z}_p(t)$ is obtained straightforwardly from (1b), while $k_I \delta x_{\mathrm{ic}}(t)$ is the proportional action on the in-vessel coil currents, which is needed to control to zero such currents. The closed-loop state equation reads

$$\begin{pmatrix} \mathbf{L}_{11}^* & \mathbf{L}_{12}^* & \mathbf{L}_{13}^* \\ \widetilde{\mathbf{l}}_{21}^{*\mathrm{T}} & \widetilde{l}_{22}^* & \widetilde{\mathbf{l}}_{23}^{*\mathrm{T}} \\ \mathbf{L}_{31}^* & \mathbf{L}_{32}^* & \mathbf{L}_{33}^* \end{pmatrix} \begin{pmatrix} \delta \dot{\mathbf{x}}_{\mathrm{pf}}(t) \\ \delta \dot{x}_{\mathrm{ic}}(t) \\ \delta \dot{\mathbf{x}}_{\mathrm{ec}}(t) \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \widetilde{r}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_{33} \end{pmatrix} \begin{pmatrix} \delta \mathbf{x}_{\mathrm{pf}}(t) \\ \delta x_{\mathrm{ic}}(t) \\ \delta \mathbf{x}_{\mathrm{ec}}(t) \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \delta \mathbf{u}_{\mathrm{pf}}(t) \tag{3}$$

where

$$\begin{aligned} \widetilde{\mathbf{l}}_{21}^{*\mathrm{T}} &= \mathbf{l}_{21}^{*\mathrm{T}} - k_D \mathbf{c}_{1z}^{\mathrm{T}} \quad \widetilde{l}_{22}^* = l_{22}^* - k_D c_{2z} \\ \widetilde{\mathbf{l}}_{23}^{*\mathrm{T}} &= \mathbf{l}_{23}^{*\mathrm{T}} - k_D \mathbf{c}_{3z}^{\mathrm{T}} \quad \widetilde{r}_{22} = r_{22} - k_I. \end{aligned}$$

Letting $\mathbf{k}^{\mathrm{T}} = (k_D \quad k_I)$, the two gains of the VS system can be chosen so as to fix the closed-loop decay rate in the $[\theta_{\min}, \theta_{\max}]$ range by solving the following bilinear matrix inequality (BMI) [12]:

$$(\mathbf{A} + \mathbf{b}\mathbf{k}^{\mathrm{T}}\mathbf{C})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\mathbf{A} + \mathbf{b}\mathbf{k}^{\mathrm{T}}\mathbf{C}) < -2\theta\mathbf{P} \tag{4}$$

where $\mathbf{P}$ is a symmetric positive definite matrix and

$$\mathbf{A} = -(\mathbf{L}^*)^{-1}\mathbf{R} \quad \mathbf{b} = (\mathbf{L}^*)^{-1} \begin{pmatrix} \mathbf{0} \\ 1 \\ \mathbf{0} \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} \mathbf{c}_{1z}^{\mathrm{T}} & c_{2z} & \mathbf{c}_{3z}^{\mathrm{T}} \\ \mathbf{0} & 1 & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{A} \\ \mathbf{I} \end{pmatrix}$$

with $\theta_{\min} < \theta < \theta_{\max}$ and $\theta_{\min}, \ \theta_{\max} > 0$. Note that the larger $\theta$ is, the faster the closed-loop system results (see [13, Sec. 5.1.3]).

*Remark 1:* Although plasma velocity is given by

$$\delta \dot{z}_p(t) = \mathbf{c}_z^{\mathrm{T}} \delta \dot{\mathbf{x}}(t) = \mathbf{c}_z^{\mathrm{T}} \left( \mathbf{A}\delta \mathbf{x}(t) + \mathbf{b}\delta \mathbf{u}(t) \right)$$

it turns out that, in the ITER tokamak, the feedthrough matrix is negligible, i.e., $d = \mathbf{c}_z^{\mathrm{T}}\mathbf{b} \cong 0$, so the direct link between the plasma velocity $\delta \dot{z}_p(t)$ and the input voltage vector $\delta \mathbf{u}(t)$ can be neglected. This allows us to find the static output feedback matrix $\mathbf{k}^{\mathrm{T}}$ by solving the BMI (4). ♢

## V. Plasma Current and Shape Controller

As it has already been noted, the plasma current and shape control system can act on a slow time scale; as discussed in [5] for the ITER tokamak, this time scale is slow also with respect to the decay time of the currents in the metallic structures.[4] Consequently, the eddy current dynamics can be neglected in the design of the controller $K_y$; this is equivalent to equal to zero both $\delta \dot{x}_{ic}$ and $\delta \dot{x}_{ec}$ in (3), obtaining

$$\delta \dot{\mathbf{x}}_{\mathrm{pf}}(t) = \left(\mathbf{L}_{11}^*\right)^{-1} \delta \mathbf{u}_{\mathrm{pf}}(t) \tag{5a}$$

$$\delta \mathbf{y}(t) = \mathbf{C}_1 \delta \mathbf{x}_{\mathrm{pf}}(t). \tag{5b}$$

[4]The typical decay time of the eddy currents in the ITER tokamak ranges from 1 to 200 ms.

In particular, vector $\delta\mathbf{y}(t)$ contains the plasma current plus a set of geometrical descriptors that completely characterize the plasma shape (see Section II).

In this case, we consider as geometrical descriptor 30 gaps plus the two strike points shown in Fig. 2. On the other hand, the number of actuators is equal to $n_{\mathrm{PF}} = 11$. Hence, the plant (5) is nonright invertible. For such a plant, it is not possible to track a generic set of references with zero steady-state error [14]. For this reason, we resort to an approach similar to the one proposed in [6], which has successfully been tested and is currently adopted on the JET tokamak [15].

Let us consider the following partition of the output vector $\delta\mathbf{y}(t) = (\,\delta\mathbf{g}^{\mathrm{T}}(t) \quad \delta I_p(t)\,)^{\mathrm{T}}$, where $\delta\mathbf{g}(t)$ is the plasma geometrical descriptor vector and $\delta I_p(t)$ is the plasma current. If $\delta\mathbf{g}(t) = \mathbf{C}_{1g}\delta\mathbf{x}_{\mathrm{pf}}(t)$, let us consider the following singular-value decomposition (SVD):

$$\mathbf{C}_{1g} = \mathbf{U}_g\mathbf{\Sigma}_g\mathbf{V}_g^{\mathrm{T}} \tag{6}$$

with $\mathbf{U}_g \in \mathbb{R}^{n_G \times n_{\mathrm{PF}}}$, $\mathbf{\Sigma}_g \in \mathbb{R}^{n_{\mathrm{PF}} \times n_{\mathrm{PF}}}$, $\mathbf{V}_g \in \mathbb{R}^{n_{\mathrm{PF}} \times n_{\mathrm{PF}}}$, where $n_G$ is the number of plasma boundary geometrical descriptors (gaps and strike points).

The control law is chosen as

$$\begin{aligned}\delta\mathbf{u}_{\mathrm{pf}}(t) =& \mathbf{K}_{\mathrm{SF}}\delta\mathbf{x}_{\mathrm{pf}}(t) + \mathbf{K}_{P_1}\mathbf{\Sigma}_g^{-1}\mathbf{U}_g^{\mathrm{T}}\delta\mathbf{g}(t)\\ &+ \mathbf{K}_{I_1}\mathbf{\Sigma}_g^{-1}\mathbf{U}_g^{\mathrm{T}}\int_0^{\mathrm{T}}\left(\delta\mathbf{g}(t) - \delta\mathbf{g}_r(t)\right)dt\\ &+ k_{P_2}\delta I_p(t) + k_{I_2}\int_0^{\mathrm{T}}\left(\delta I_p(t) - \delta I_{p_r}(t)\right)dt\end{aligned} \tag{7}$$

where $\delta\mathbf{g}_r(t)$ and $\delta I_{p_r}(t)$ are the reference on the plasma geometrical descriptors and the plasma current, respectively. In [14], it has be shown that (7) tracks the reference of plasma current and minimizes the following steady-state performance index:

$$J = \lim_{t\to+\infty}\|\delta\bar{\mathbf{g}}_r - \delta\mathbf{g}(t)\|^2 \tag{8}$$

with $\delta\bar{\mathbf{g}}_r$ constant references to the geometrical descriptors. Although, in principle, the term $\mathbf{K}_{\mathrm{SF}}\delta\mathbf{x}_{\mathrm{pf}}(t)$ is not needed to minimize (8), it has been added in (7) so as to avoid the divergence of currents in the superconductive PF coils.

As a matter of fact, using the ITER linearized models, it turned out that some singular values—depending on the configuration—are one order of magnitude smaller than the others. This fact implies that minimizing the performance index (8) retaining all the singular values results in a high control effort at steady state in terms of PF coil currents. In order to achieve a tradeoff condition, we control to zero only the error for the $\bar{n} < n_{\mathrm{PF}}$ linear combination related to the largest singular values. Considering the matrices $\mathbf{\Sigma}_M$ and $\mathbf{U}_M^{\mathrm{T}}$ that correspond to the $\bar{n}$ largest singular values of the SVD in (6) and

$$\mathbf{K}_p = \left(\mathbf{K}_{P_1}\mathbf{\Sigma}_M^{-1}\mathbf{U}_M^{\mathrm{T}}\;k_{P_2}\right) \quad \mathbf{K}_I = \left(\mathbf{K}_{I_1}\mathbf{\Sigma}_M^{-1}\mathbf{U}_M^{\mathrm{T}}\;k_{I_2}\right)$$

we obtain the following closed-loop state equation:

$$\begin{aligned}\delta\dot{\mathbf{x}}_{\mathrm{pf}}(t) &= \left(\mathbf{L}_{11}^*\right)^{-1}(\mathbf{K}_{\mathrm{SF}}+\mathbf{K}_p\mathbf{C}_1)\delta\mathbf{x}_{\mathrm{pf}}(t) + \left(\mathbf{L}_{11}^*\right)^{-1}\mathbf{K}_I\boldsymbol{\epsilon}(t)\\ \dot{\boldsymbol{\epsilon}}(t) &= \mathbf{C}_1\delta\mathbf{x}_{\mathrm{pf}}(t) - \delta\mathbf{y}_r(t)\\ \mathbf{y}(t) &= \mathbf{C}_1\delta\mathbf{x}_{\mathrm{pf}}(t)\end{aligned}$$

where $\delta\mathbf{y}_r(t) = (\delta\mathbf{g}_r(t)^{\mathrm{T}}\delta I_{p_r}(t))^{\mathrm{T}}$. Now, let $\mathbf{K}_{\mathrm{SF}}$ and $\mathbf{K}_P$ satisfy

$$\left(\mathbf{L}_{11}^*\right)^{-1}\left(\mathbf{K}_{\mathrm{SF}} + \mathbf{K}_p\mathbf{C}_1\right) = -\alpha\mathbf{I} \tag{9}$$

with $\alpha > 0$, then the output vector derivative is given by

$$\begin{aligned}\delta\dot{\mathbf{y}}(t) &= -\alpha\delta\mathbf{y} + \mathbf{C}_1\left(\mathbf{L}_{11}^*\right)^{-1}\mathbf{K}_I\boldsymbol{\epsilon}(t)\\ \dot{\boldsymbol{\epsilon}}(t) &= \delta\mathbf{y}(t) - \delta\mathbf{y}_r(t).\end{aligned}$$

As far as integral gain is concerned, it is convenient to choose $\mathbf{K}_I$ such that

$$\mathbf{C}_1\left(\mathbf{L}_{11}^*\right)^{-1}\mathbf{K}_I = -\beta\mathbf{I} \tag{10}$$

with $\beta > 0$. In this way, the closed-loop transfer function that relates the $i$th component of $\delta\mathbf{y}_r(t)$ and $\delta\mathbf{y}(t)$ is the following:

$$\delta Y_i(s) = \frac{\beta}{s^2 + \alpha s + \beta}\delta Y_{r_i}(s)$$

which is asymptotically stable since $\alpha$, $\beta > 0$. Moreover, by choosing suitable values to $\alpha$ and $\beta$, it is possible to assign the desired input/output behavior.

*Remark 2:* Note that (9) and (10) can be satisfied in different ways, for example, letting $\mathbf{K}_P = 0$, $\mathbf{K}_{\mathrm{SF}} = -\alpha(\mathbf{L}_{11}^*)^{-1}$, and $\mathbf{K}_I = -\beta\mathbf{L}_{11}^*\mathbf{C}_1^{\dagger}$, where $^{\dagger}$ indicates the Moore–Penrose pseudoinverse. ♢

## VI. Simulation Results

The proposed design approach has been proven to be effective in designing the plasma current and shape controller for ITER. In this section, some simulation results are discussed.

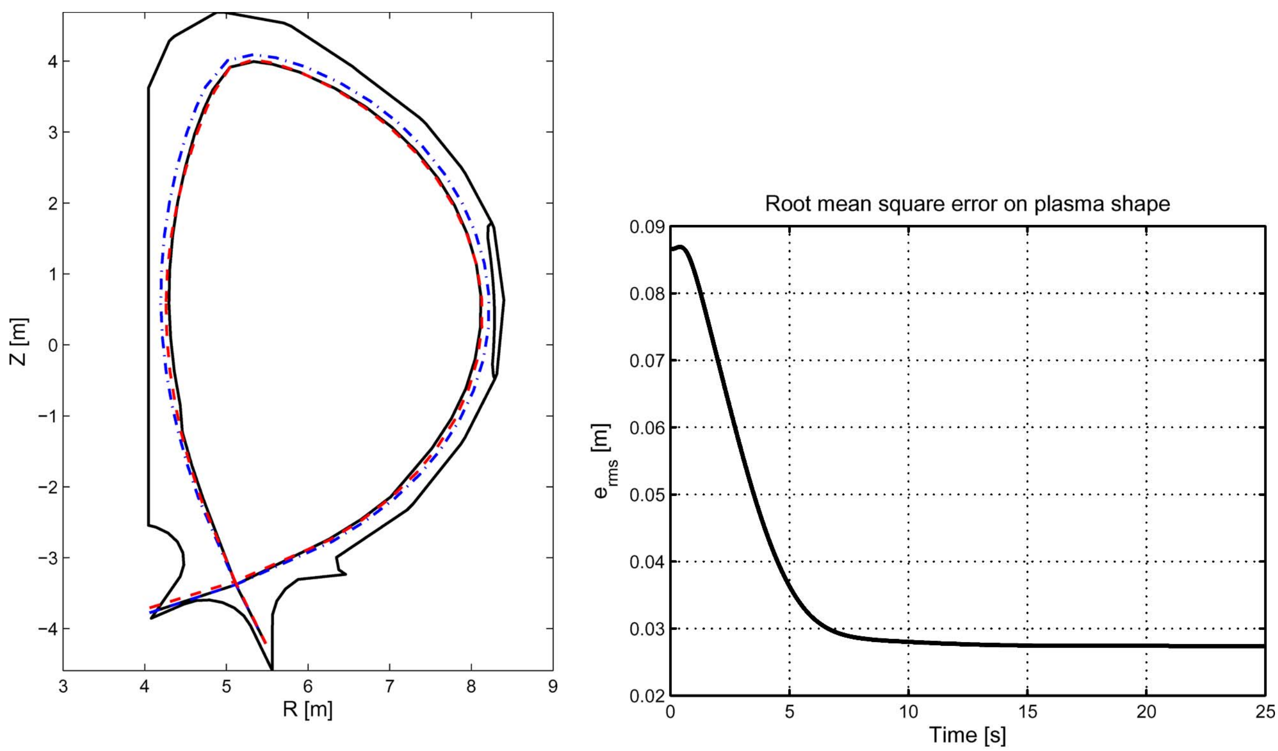
The plasma position and shape control system has been designed using a plasma linearized model around the equilibrium with $I_p = 15$ MA, $\beta_p = 0.1$, and $l_i = 1.0$, which corresponds to the ITER operational *Scenario 2* at the *start-of-the-flattop* (SOF) phase (see [16]). The design parameters are reported as follows:

$$\begin{aligned}\theta_{\min} &= 5 \quad \theta_{\max} = 12 \quad \theta = 8\\ \bar{n} &= 6 \quad \alpha = 0.57 \quad \beta = 0.15.\end{aligned}$$

The inequality (4) has been solved with PENBMI [17] using a fifth-order reduced model of the plasma.

In order to show the performance of the proposed control algorithm, the three cases reported in the following are considered:

1) tracking of a given plasma shape;
2) analysis of the performance during an H–L transition;
3) analysis of the VS system in the presence of noise.




(a) (b)

Fig. 4. Tracking of a given shape. Simulation results. (a) Tracking of a given shape. The reference shape is shown as solid black line, while the dash–dot blue line and the red dash line show the initial and final (after 25 s) shapes, respectively. The plasma–wall distance on the outboard varies from ∼6 to ∼15 cm. (b) Tracking of a given shape. Mean square error on the controller plasma shape descriptors.

Simulations 1) and 3) have been performed considering the plasma equilibrium at $I_p = 15$ MA, $\beta_p = 0.1$, and $l_i = 1.2$, which is the ITER Scenario 2 at SOF with slightly increased $l_i$, hence corresponding to a slightly more unstable plasma.

Moreover, the plasma equilibrium at $I_p = 15$ MA, $\beta_p = 0.65$, and $l_i = 0.85$ has been considered to evaluate the performance during an H–L transition. Such a plasma model corresponds to the ITER operational Scenario 2 at the *start-of-burn (SOB)* phase.

Figs. 4 and 5 show the behavior of the proposed control system when it tracks the shape reference reported as solid line in Fig. 4(a). Note that the required PF coil currents for shape control are within the envisaged allowable limits of ∼8 kA [18] and that the maximum required in-vessel coil current (∼12 kA) is well within the limit of ∼75 kA [19] foreseen by the ITER design review.

The PF current limits take into account the envisaged current available for shape control, despite the current needed to obtain the scenario, and the current used to compensate the flux consumption.[5]

The H–L transition is a plasma disturbance that can be modeled as a variation of $\beta_p(t)$ and $l_i(t)$. According to [9], the H–L transition has been modeled as follows.

Phase 1) $\beta_p$ decreases by 0.05, and $l_i$ increases by 0.05 with an exponential time behavior characterized by a time constant of 25 ms (settling time of about 100 ms).

[5]The flux consumption is due to plasma resistance, which is not taken into account in the considered plasma model.

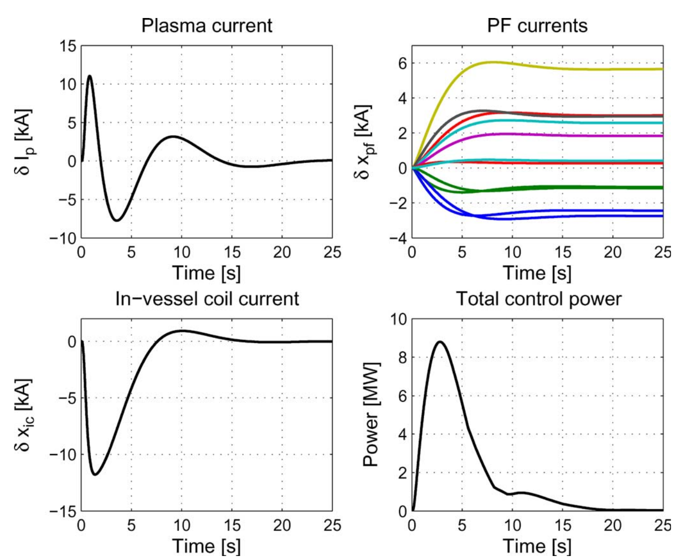



Fig. 5. Tracking of a given shape. This figure shows the time traces of the plasma current variation $\delta I_p(t)$, of the control currents $\delta \mathbf{x}_{\rm pf}(t)$ and $\delta x_{\rm ic}(t)$, and the total power required to track the desired shape reference.

Phase 2) $\beta_p$ decreases by 0.4 with a time constant of 0.36 s (settling time of about 1.5 s), and $l_i$ increases by 0.1 with a time constant of 1.8 s (settling time of about 7 s).

Fig. 6 shows the time traces of $\delta\beta_p(t)$ and $\delta l_i(t)$ during an H–L transition.

The results of H–L transition analysis are shown in Figs. 7 and 8. It is important to note that, as far as plasma shape control is concerned, the H–L transition represents the worst

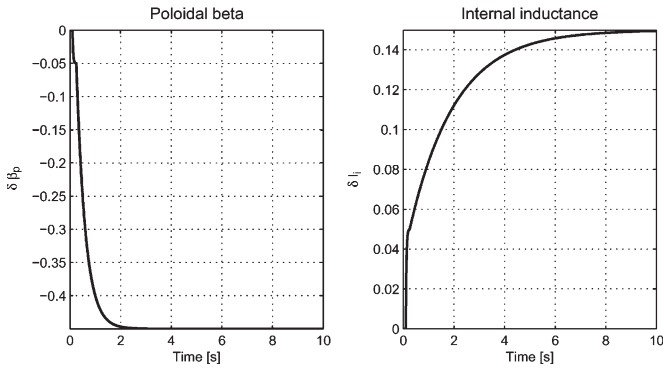



Fig. 6. Analysis of the performance during an H–L transition. Time traces of $\delta\beta_p(t)$ and $\delta l_i(t)$.

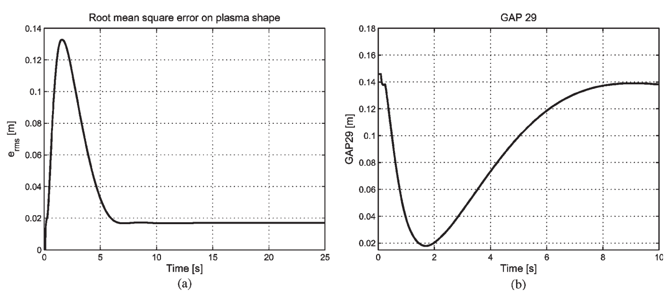



Fig. 7. Analysis of the performance during an H–L transition. Simulation results. (a) Analysis of the performance during an H–L transition. Mean square error on the controller plasma shape descriptors. (b) Analysis of the performance during an H–L transition. Time trace of GAP 29 (see Fig. 2); note that the minimum clearance between the plasma and the first wall in the inboard is about 2 cm.

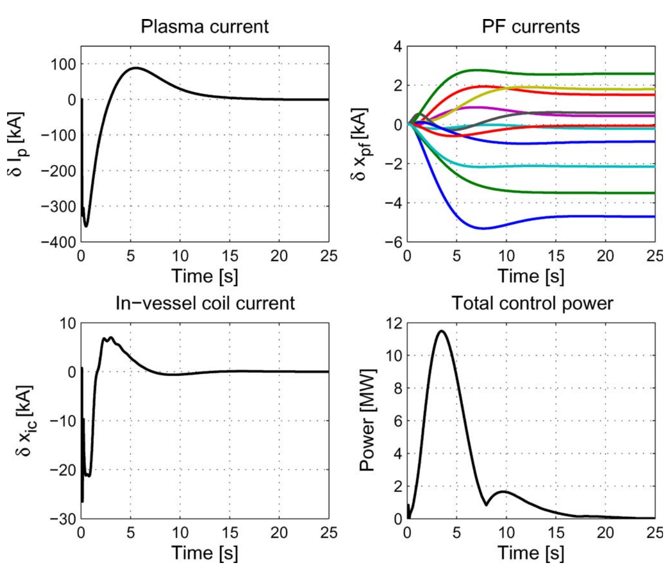



Fig. 8. Analysis of the performance during an H–L transition. This figure shows the time traces of the plasma current variation $\delta I_p(t)$, of the control currents $\delta\mathbf{x}_{\mathrm{pf}}(t)$ and $\delta x_{\mathrm{ic}}(t)$, and the total power required to track the desired shape reference.

case disturbance for the considered equilibrium. Indeed, the time trace of GAP 29 (see Fig. 2) is shown in Fig. 7(b), showing that the minimum distance between the plasma and the first wall is about 2 cm.

TABLE II

ANALYSIS OF THE VS SYSTEM IN THE PRESENCE OF NOISE. MAIN PERFORMANCE INDICATORS. NOTE THAT SINCE, AT THE EQUILIBRIUM, BOTH THE IN-VESSEL COIL VOLTAGE AND CURRENT ARE EQUAL TO ZERO, THE VARIATIONS $\delta x_{\mathrm{in}}$ AND $\delta u_{\mathrm{in}}$ COINCIDE WITH THEIR ABSOLUTE VALUES $x_{\mathrm{in}}$ AND $u_{\mathrm{in}}$, RESPECTIVELY

| $\lvert u_{in}\rvert_{max}$ [V] | $\lvert x_{in}\rvert_{max}$ [kA] | $\delta\lvert z_p\rvert_{max}$ [cm] |
|---|---|---|
| 100 | 44 | 3 |

The performance of the control system in the presence of noise is summarized in Table II. In this case, we have considered the behavior of the vertical control system in the presence of the maximum expected noise level, i.e., white noise with 1-kHz bandwidth, zero mean, and standard deviation $\sigma = 0.58$ m/s [9].

## VII. CONCLUSION

A solution to the problem of controlling the plasma current, position, and shape in the ITER tokamak has been presented in this paper. The proposed approach is based on two separate loops: the first one accomplishes VS, while the second one tracks both plasma current and shape. Since the VS loop makes use of in-vessel coils, a reduction of delay due to the shielding effect of the passive structures is attained, permitting one to improve the system performance. Thanks to its structure, the proposed solution for VS permits one to envisage effective adaptive algorithms and to mitigate the effect of measurement noise on control performance. Furthermore, the proposed plasma shape and current controller allows us to track extremely shaped plasmas, minimizing the error between the actual plasma boundary and the desired shape over up to 32 plasma geometrical descriptors. Simulation results have proven the effectiveness of the proposed approach.

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