Plasma equilibrium in a Tokamak

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Review paper

PLASMA EQUILIBRIUM IN A TOKAMAK

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ABSTRACT. The paper summarizes the basic information on the equilibrium of a toroidal plasma column in systems of the Tokamak type. It considers the methods of maintaining a plasma in equilibrium with the help of a conducting casing, an external maintaining field and the iron core of a transformer. Attention is paid to the role of the inhomogeneity of the maintaining field. It is shown in particular how the shape of the column cross-section depends on the curvature of the lines of force of the maintaining field. For the case (which has practical importance) weak asymmetry of the field distribution in the transverse cross-section, this paper describes a uniform method of consideration, which takes into account the influence of different factors on the equilibrium position of the column. This method is used for calculating plasma equilibrium in a Tokamak model with a conducting casing. Account is here taken of the effect of gaps in the casing and of finite electrical conductivity. Some cases of plasma equilibrium which are outside the standard Tokamak scheme are also considered, such as equilibrium in a conducting shell having the shape of a racetrack, equilibrium where the whole current is transferred by relativistic runaway electrons and equilibrium at high plasma pressure $\beta_I \sim R/a$.

CONTENTS.

SELECTED NOTATION

a	Minor radius of the plasma column, i.e. radius of the cross-section of the toroidal magnetic surface, outside which current density is small in comparison with the average one.
b	Minor radius of the casing.
b_d	Radius of diaphragm aperture.
b_λ	Minor radius of the liner.
b_1	Minor radius of the toroidal surface on which the internal control conductors are located.
b_2	Same for the external control conductors.
b_3	Same for the primary winding of the transformer.
b_4	Minor radius of the internal surface of the toroidal iron core.
b_5	Minor radius of the external surface of the toroidal iron core.
d_λ	Thickness of the liner wall.
d_k	Thickness of the casing wall.
Δ	Displacement of the plasma column.
Δ_0	Displacement of the plasma column in an ideal casing (Eq. (60)).
H_φ	Toroidal (longitudinal) magnetic field.
H_ω	Poloidal magnetic field.
$H_\perp$	Magnetic field transverse to the torus plane.
$H_{\perp 0}$	Strength of the maintaining equilibrium magnetic field (Eq. (37)).
h_{ii}^∞	Transverse magnetic field of the internal control conductors in the discharge chamber ($\rho < b_i$) in the case of an ideal casing.

h_{ii}^0	Same in the absence of a casing.
h_{ei}^0	Transverse magnetic field generated in the discharge chamber by external sources in the absence of a casing.
I_p	Longitudinal current in the plasma column.
I_0	Current in the primary winding of the transformer.
I_1	Current in the control conductors.
I_λ	Current flowing through the liner.
$I_\perp$	Transverse current in the plasma column short-circuited through the diaphragm.
i_1	Amplitude of the dipole component of the surface density of the current in the internal control conductors.
i_2	Same in the external control conductors.
i_3	Same in the primary winding of the transformer.
i_ϵ	Amplitude of the quadrupole component of the surface density of the current in the control conductors.
R	Radius of the axial line of the quasi-cylindrical coordinate system ρ, φ, ω; major radius of the casing.
R_p	Major radius of the plasma column.
r, φ, z	Cylindrical coordinates.
ρ, ω	Polar coordinates in the cross-section φ = const. with origin at $r = R$.

1. INTRODUCTION

The first problem to be tackled in the study of the confinement of a high-temperature plasma in toroidal systems is that of equilibrium. A toroidal plasma column, like an inflated balloon, tends to extend. In a Tokamak, the extension force is balanced by the force of interaction between the longitudinal current I_p excited in the plasma and the external transverse magnetic field $H_\perp$. When I_p and $H_\perp$ are given, the position of a plasma column in the discharge chamber depends on the kinetic energy of the plasma particles and on the form of current distribution over the column cross-section. During heating of the plasma both these characteristics change, as a result of which the plasma column is displaced in the chamber. The fact that the equilibrium position of the plasma column depends on the plasma parameters has both a negative and a positive aspect. The negative aspect is that one has to ensure regulation of the equilibrium position of the column, the positive aspect being the possibility of using this dependence for diagnostic purposes. In order to deal with both these problems, it is necessary to know how the equilibrium position of the column is affected by the various specific features of particular devices.

The purpose of the present work is to systematically consider the basic factors affecting plasma equilibrium in a Tokamak with emphasis on the practical side of the problem. We give a number of calculational formulae and describe a simple uniform method of calculating the equilibrium position of a column, which can be used in considering cases not included in this work.

2. GENERAL RELATIONS

The basic macroscopic equations defining the conditions of equilibrium of a toroidal plasma column are the pressure-balance equation and the equation describing the changes in the major radius of the column. The form of these equations depends on the shape of the column cross-section and on the form of distribution of the current and plasma pressure over the column cross-section. The shape of the column cross-section is determined by the shape of the cross-section of the conducting casing or, where there is no casing, by the configuration of the external maintaining magnetic field. We shall mainly consider a plasma column with a circular cross-section and assume that the asymmetry of field distribution in the column cross-section resulting from toroidality is small. For this asymmetry to be small it is obviously essential that the ratio of the minor radius a to the major radius R_p of the column should be small ($a/R_p \ll 1$). This condition enables us to derive formulae defining the equilibrium position of a column without any assumptions about the form of distribution of the current and pressure of the plasma over the column cross-section.

2.1. PRESSURE-BALANCE EQUATION

As is known, Tokamak has a longitudinal, toroidal magnetic field and a transverse, poloidal magnetic field. The toroidal magnetic field outside the plasma changes according to the law

$$H_{\varphi e} = H_{e0}\frac{R_p}{r} \qquad (1)$$

where R_p is the radius of the geometrical axis of the plasma column, and r is the distance from the axis of symmetry (Fig.1). The toroidal magnetic field stabilizes disturbances of the plasma column. Furthermore, this field together with the field of the current flowing in a plasma

$$H_I = \frac{2I_p}{ca} \qquad (2)$$

takes part in balancing the plasma over the minor radius.

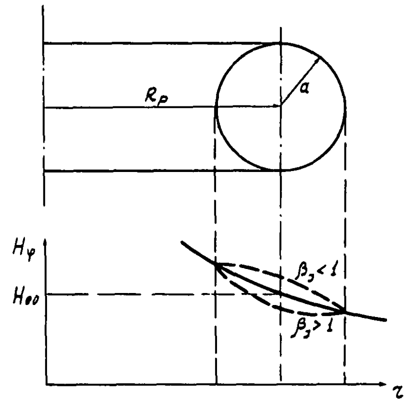

FIG.1. Distribution of a toroidal magnetic field.

The pressure-balance equation, when the curvature of the torus is neglected, takes the form [1]

$$8\pi\langle p\rangle = \frac{4I_p^2}{c^2a^2} + H_{e0}^2 - \langle H_{\varphi i}^2\rangle \tag{3}$$

where $H_{\varphi i}$ is the strength of the longitudinal magnetic field inside the column and the angle brackets denote averaging over the plasma column cross-section. For a given plasma current I_p and pressure $\langle p\rangle$, the difference $H_{e0}^2 - \langle H_{\varphi i}^2\rangle$ is adjusted in such a way that the pressure-balance equation is satisfied. This occurs because of the appearance of a corresponding diamagnetic or paramagnetic current in the plasma. In a Tokamak $H_{\varphi e}^2 \gg H_I^2$; therefore, the relative magnitude of diamagnetism (or paramagnetism) is small and the difference $H_{e0}^2 - \langle H_{\varphi i}^2\rangle$ can be represented as

$$H_{e0}^2 - \langle H_{\varphi i}^2\rangle \simeq 2H_{e0}\langle H_{e0} - H_{\varphi i}\rangle = \frac{2H_{e0}\delta\Phi}{\pi a^2} \tag{4}$$

where $\delta\Phi = \pi a^2\langle H_{e0} - H_{\varphi i}\rangle$ is the diamagnetic flux of the longitudinal magnetic field in the plasma column. Substituting this expression into the pressure-balance equation, we obtain a convenient formula for determining the parameter

$$\beta_I = \frac{8\pi\langle p\rangle}{H_I^2} \tag{5}$$

from the values of I_p, $\delta\Phi$ and H_{e0} measured experimentally. This formula is

$$\beta_I = 1 + \frac{c^2}{2\pi I_p^2} H_{e0}\delta\Phi \tag{6}$$

If the torus curvature is taken into account, corrections of the type $(a/R)^2\beta_I$, $(a/R)^2\beta_I^2$ appear in the right-hand part of this relation [2-3]. When $\beta_I \ll R/a$, these corrections are insignificant. The only peculiarity of using formula (6) in toroidal geometry is that H_{e0} should be taken as denoting the value of the longitudinal magnetic field on line $r = R_p$ passing through the centre of the plasma column cross-section. Thus, for correct determination of $(\beta_I - 1)$ it is necessary to know the position of the plasma column inside the chamber. When the value of the longitudinal field at the centre of the chamber is used as H_{e0}, the difference $(\beta - 1)$ is determined with relative error (Δ/R), where Δ is the amount of displacement of the centre of the plasma column cross-section relative to the centre of the chamber cross-section.

2.2. POLOIDAL MAGNETIC FIELD (GENERAL FORMULAE)

A poloidal magnetic field can be conveniently expressed with the help of the transverse-flux function ψ

$$\vec{H} = \frac{1}{2\pi r}[\nabla\psi\vec{e}_\varphi] \tag{7}$$

Here $\vec{e}_\varphi$ is a unit vector tangent to the axial line of the torus. The equation

$$\psi(r, z) = \text{const.} \tag{8}$$

determines the form of the magnetic surfaces. Outside the toroidal loop, along which the longitudinal current I flows, this function expressed in quasicyclindrical coordinates ρ, φ, ω (Fig.2) in the first approximation of the expansion for ρ/R has the form [4]

$$\psi = -\frac{4\pi R}{c} I\left(\ln\frac{8R}{\rho} - 2\right) + \left[-\frac{2\pi I}{c}\left(\ln\frac{8R}{\rho} - 1\right)\rho + \frac{C_1}{\rho} + C_2\rho\right]\cos\omega \tag{9}$$

where C_1 and C_2 are constants determined by the boundary conditions.

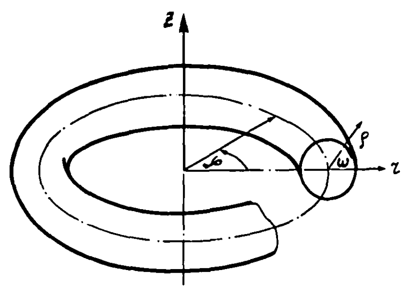

FIG.2. Quasicylindrical coordinate system ρ, φ, ω.

In this formula, the expression in brackets containing $\ln 8R/\rho$ is an approximate value, valid for $\rho \ll R$, of the function specific for toroidal geometry, which becomes zero when $\rho \to \infty$. Thus, at infinity the function ψ takes the form $\psi = C_2\rho\cos\omega$, i.e. represents the homogeneous magnetic field of the external sources

$$H_z = \frac{1}{2\pi R} C_2 \tag{10}$$

If there is no external field ($C_2 = 0$), then at infinity $\psi = 0$. This normalization of ψ corresponds to considering infinity as the reference point for reading off the magnetic flux.

The cross-sections of the magnetic surfaces determined by equation

$$\psi(\rho, \omega) = \text{const.} \tag{11}$$

in the approximation considered are circles, the centres of which are displaced in relation to the axis of the coordinate system $\rho = 0$ by the distance

$$\Delta(\rho) = -\frac{\rho^2}{2R}\left(\ln\frac{8R}{\rho} - 1\right) + \frac{c}{4\pi RI}(C_1 + C_2\rho^2) \tag{12}$$

The positive values of $\Delta(\rho)$ correspond to a displacement towards the symmetry axis.

The magnetic-field components corresponding to function ψ are

$$H_\omega = \frac{1}{2\pi r}\frac{\partial\omega}{\partial\rho} = \frac{2I}{c\rho} + \left[-\frac{I}{cR}\ln\frac{8R}{\rho} + \frac{1}{2\pi R}(C_2 - C_1/\rho^2)\right]\cos\omega \tag{13}$$

$$H_\rho = -\frac{1}{2\pi r}\frac{\partial\psi}{\rho\partial\omega} = \left[-\frac{I}{cR}\left(\ln\frac{8R}{\rho} - 1\right) + \frac{1}{2\pi R}(C_2 + C_1/\rho^2)\right]\sin\omega \tag{14}$$

2.3. SUPERCONDUCTING RING WITH CURRENT

Let us apply these formulae to a superconducting ring with current [5]. The condition that external sources are absent requires that constant C_2 be equal to zero. Constant C_1 is determined from the condition that the normal component of the field on the superconductor surface should be equal to zero. If the coordinate axis coincides with the geometrical axis of the superconducting ring, the minor radius of which is a, then

$$C_1 = \frac{2\pi I}{c}a^2\left(\ln\frac{8R}{a} - 1\right) \tag{15}$$

The magnetic flux generated by the current flowing through the superconducting ring equals

$$\frac{1}{c}L_eI = \psi(\infty) - \psi(a) = \frac{4\pi R}{c}I\left(\ln\frac{8R}{a} - 2\right) \tag{16}$$

Hence, for the external inductance of the ring we obtain the well-known approximate expression

$$L_e = 4\pi R\left(\ln\frac{8R}{a} - 2\right) \tag{17}$$

From a comparison of this value with the accurate value of L_e (Fig.3a) taken from Ref.[6], we can conclude that the approximation $\rho/R \ll 1$ used for determining the integral characteristics of the configuration is accurate. As can be seen from the figure, the approximate formula gives an error smaller than 30% for $R/a > 2$; when $R/a = 3$, the error is only $\simeq 10\%$.

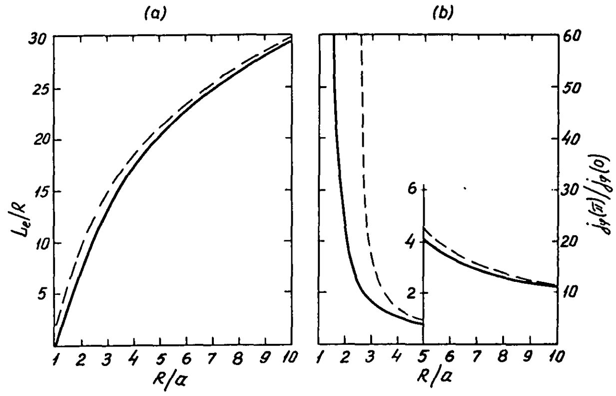

FIG.3. Curves illustrating the accuracy of the approximation $\rho/R \ll 1$ used in the work.
(a) Inductance L_e of the superconducting ring. The solid line represents the accurate value and the broken line the approximate value determined by formula (17).
(b) Relationship between current densities on the internal and external surfaces of the superconducting ring. The solid line represents the accurate value and the broken line the approximate value given by formula (23).

The magnetic field and the amount of displacement of the external magnetic surface of radius ρ are determined by the formulae

$$H_\omega = \frac{2I}{c\rho}\left[1 - \frac{\rho}{2R}\left(\ln\frac{8R}{\rho} + \frac{a^2}{\rho^2}\ln\frac{8R}{a} - \frac{a^2}{\rho^2}\right)\cos\omega\right] \tag{18}$$

$$H_\rho = -\frac{I}{cR}\left[\left(\ln\frac{8R}{\rho} - 1\right) - \frac{a^2}{\rho^2}\left(\ln\frac{8R}{a} - 1\right)\right]\sin\omega \tag{19}$$

$$\Delta(\rho) = -\frac{\rho^2}{2R}\left(\ln\frac{8R}{\rho} - 1\right) + \frac{a^2}{2R}\left(\ln\frac{8R}{a} - 1\right) \tag{20}$$

In particular, on the ring surface

$$H_\rho(a) = 0 \tag{21}$$

$$H_\omega(a) = \frac{2I}{ca}\left[1 - \frac{a}{R}\left(\ln\frac{8R}{a} - \frac{1}{2}\right)\cos\omega\right] \tag{22}$$

The field H_ω and, therefore, the density of the longitudinal surface current $j_\varphi = (c/4\pi)H_\omega(a)$ are greater on the internal surface ($\omega = \pi$) than on the external surface ($\omega = 0$) in a relation

$$\frac{H_\omega(a,\pi)}{H_\omega(a,0)} = \frac{j_\varphi(a,\pi)}{j_\varphi(a,0)} = \frac{1 + \frac{a}{R}\left(\ln\frac{8R}{a} - \frac{1}{2}\right)}{1 - \frac{a}{R}\left(\ln\frac{8R}{a} - \frac{1}{2}\right)} \tag{23}$$

As is to be expected, the distribution of current density, unlike inductance, is described less satisfactorily in the approximation of the expansion for ρ/R used here. Formula (23) gives satisfactory accuracy (error not greater than 30%) only if $R/a > 4$ (Fig.3b). When $R/a < 3$, this formula is not generally applicable (error exceeds 100%)[1].

The example of current distribution in a superconducting ring helps one to understand the need for external fields for confining a plasma in equilibrium. In one respect a plasma column is similar to a superconductor. Since the plasma pressure is established as constant along the magnetic lines of force, the normal component of the magnetic field equals zero both at the column boundary ($p = 0$) and on the surface of the superconducting ring, i.e. the plasma boundary coincides with the magnetic surface. If there are no volume currents in the plasma, the plasma pressure on the column surface registers a jump, $p = \text{const.}$, which should be balanced by a corresponding jump in the magnetic field pressure. However, the pressure of the magnetic field of the current

$$\frac{H_\omega^2(a)}{8\pi} = \frac{1}{8\pi}\left(\frac{2I}{ca}\right)^2\left[1 - 2\,\frac{a}{R}\left(\ln\frac{8R}{a} - \frac{1}{2}\right)\cos\omega\right] \qquad (24)$$

is not constant on the ring surface, being greater on the internal side. In order to balance it, we need to add a field transverse to the plane of the ring so as to strengthen the field of the ring current outside the ring and weaken it inside (Fig.4).

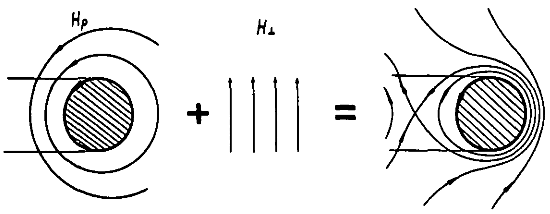

FIG.4. Diagram of the combination of the proper magnetic field of a ring current with transverse balancing magnetic field.

2.4. MAGNETIC FIELD OF EQUILIBRIUM CONFIGURATION

Thus, the total poloidal magnetic field of a toroidal plasma column in equilibrium should be the sum of the proper field of the current flowing through the plasma and of the field of the external sources (in the presence of a conducting casing, this is the field of the image currents). The distribution of the total equilibrium poloidal field depends on the distribution of the plasma pressure and the density of the longitudinal current in the plasma column. It is, however, important that in the first approximation of the expansion in ρ/R the distribution of this field outside the plasma does not depend on the details of pressure and current distribution but is determined solely by the integral characteristics of the column, such as β_I introduced earlier and the value of the internal inductance of the unit length of the column

$$\ell_i = \frac{\int H_\omega^2 \rho\, d\rho\, d\omega}{\pi a^2 H_I^2} \qquad (25)$$

Let us take the plasma column axis as the coordinate axis. The distribution of the poloidal field on the outermost toroidal magnetic surface of the plasma column, which determines the whole configuration of the external equilibrium field, has the form

$$H_\omega(a, \omega) = H_I\left(1 + \frac{a}{R_p}\,\Lambda\cos\omega\right) \qquad (26)$$

where Λ is the coefficient of asymmetry of the poloidal field determined by the formula [4, 7, 8]

$$\Lambda = \frac{8\pi\langle p\rangle}{H_I^2} + \frac{\ell_i}{2} - 1 \qquad (27)$$

It was assumed above that the plasma pressure is isotropic. But if the plasma pressure $p_\parallel$ along the lines of force is not equal to the transverse pressure $p_\perp$, then, in the case $H_\varphi^2 \gg H_I^2$, $\langle p\rangle$ is replaced by $\langle p_\perp\rangle$ in the pressure-balance equation and the coefficient of asymmetry Λ takes the form

$$\Lambda = \frac{4\pi(\langle p_\parallel\rangle + \langle p_\perp\rangle)}{H_I^2} + \frac{\ell_i}{2} - 1 \qquad (28)$$

We write the flux function ψ (9) in the form

$$\psi = \psi_p + \psi_e \qquad (29)$$

where $\psi_e = C_2\rho\cos\omega$ corresponds to the field of the external sources. Determining constants C_1 and C_2 from the condition that the normal field component $(\vec{H}\cdot\vec{n}) = 0$ and the condition of continuity of the tangential field component $H_\omega^e(a, \omega) = H_\omega^i(a, \omega) = H_I(1 + (a/R_p)\Lambda\cos\omega)$ on the plasma surface,

$$C_1 = -\frac{2\pi I_p}{c}\,a^2\left(\Lambda + \frac{1}{2}\right) \qquad (30)$$

$$C_2 = \frac{2\pi I_p}{c}\left(\ln\frac{8R_p}{a} + \Lambda - \frac{1}{2}\right) \qquad (31)$$

we find the flux function of the equilibrium configuration

$$\psi = -\frac{4\pi R_p}{c} I_p\left(\ln\frac{8R_p}{\rho} - 2\right) + \frac{2\pi I_p}{c}\left[\ln\frac{\rho}{a} + \left(\Lambda + \frac{1}{2}\right)\left(1 - \frac{a^2}{\rho^2}\right)\right]\rho\cos\omega \qquad (32)$$

$$\psi_e = \frac{2\pi I_p}{c}\left(\ln\frac{8R_p}{a} + \Lambda - \frac{1}{2}\right)\rho\cos\omega \qquad (33)$$

[1] In the case of a plasma column, this restriction is less rigorous. The distribution of the magnetic field outside the plasma is described satisfactorily by the approximate formulae, even when $a/R \simeq 0.7$ (see Section 5.3).

$$\psi_p = -\frac{4\pi R_p}{c} I_p \left(\ln\frac{8R_p}{\rho} - 2\right) + \frac{2\pi I_p}{c}\left[-\ln\frac{8R_p}{\rho} + 1 - \frac{a^2}{\rho^2}\left(\Lambda + \frac{1}{2}\right)\right]\rho\cos\omega \tag{34}$$

Considering that $\rho\cos\omega = r - R_p$, from the expression for ψ_e we obtain the components of the external maintaining field

$$H_r = -\frac{1}{2\pi r}\frac{\partial\psi_e}{\partial z} = 0 \tag{35}$$

$$H_z = \frac{1}{2\pi r}\frac{\partial\psi_e}{\partial r} = \frac{I_p}{cR_p}\left(\ln\frac{8R_p}{a} + \Lambda - \frac{1}{2}\right) \tag{36}$$

These formulae show that the plasma column is maintained in equilibrium with the help of an approximately homogeneous magnetic field transverse to the plane of the torus and having an intensity [3, 9, 10]

$$H_{\perp 0} = \frac{I_p}{cR_p}\left(\ln\frac{8R_p}{a} + \Lambda - \frac{1}{2}\right) \tag{37}$$

3. METHODS OF MAINTAINING A PLASMA COLUMN IN EQUILIBRIUM

The transverse magnetic field $H_{\perp 0}$ necessary for plasma equilibrium can be generated with the help of external conductors with current. If the plasma column is placed inside an ideally conducting casing, this field is generated automatically by the Foucault currents in the casing walls. The equilibrium of the plasma column can be ensured also by the interaction of the current in the plasma with the transformer core.

3.1. MAINTAINING OF A COLUMN BY THE MAGNETIC FIELD OF EXTERNAL CONDUCTORS

3.1.1. Stability relative to displacement along the axis of symmetry

Let us consider the equilibrium of a plasma column in a given external field $H_\perp$. If this field is strictly homogeneous, the equilibrium of the plasma column will be neutral relative to its displacement as a whole along the axis of symmetry. However, if the lines of force of the confining field are curved, then on accidental displacement from the equatorial plane the column will encounter the r component of the magnetic field, the interaction of which with the current will tend either to increase the displacement or to bring the column back to its previous position, depending on the direction of convexity of the lines of force. The curving of the lines of force results from superposition on the homogeneous magnetic field of a quadrupole magnetic field generated, for example, by two pairs of conductors with

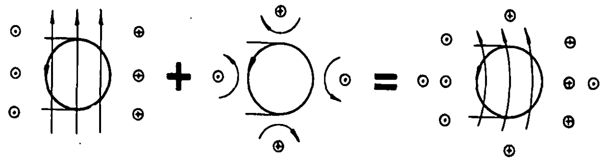

FIG.5. Diagram of the combination of a homogeneous magnetic field with a quadrupole field.

current, as is shown in Fig.5. This figure schematically shows how the superposition of a quadrupole on a homogeneous field gives a barrel field with convex lines of force. The curved arrow shows the direction of the field generated by the current in the plasma. It is evident from the figure that for the chosen sign of the quadrupole field, when the lines of force of the total maintaining field are convex, the direction of the currents in the loops in a vertical plane is inverse to that of the current in the plasma. Consequently, the position of the plasma loop is stable relative to vertical displacement.

The decay index

$$n = -\frac{r}{H_\perp}\frac{\partial H_\perp}{\partial r} \tag{38}$$

is generally used to characterize the barrel shape of the field. The condition of stability of the equilibrium position of the plasma column relative to vertical displacements can be written as [11, 12]

$$n > 0 \tag{39}$$

3.1.2. Stability relative to horizontal displacement

The stability of the equilibrium position of a plasma column in a transverse field $H_\perp$ in relation to an accidental change in the major radius R_p depends on the law of radial variation of the total radial force F_R acting on the plasma column. This force can be written as

$$F_R = -\frac{2\pi R_p I_p}{c}(H_\perp - H_{\perp 0}) \tag{40}$$

where $H_{\perp 0}$ is determined by formula (37). The stability condition reduces to the requirement

$$\frac{\partial F_R}{\partial R_p} < 0$$

When quantity $[\ln(8R_p/a) + \Lambda - (1/2)]^{-1}$ is neglected in comparison with unity and provided $\beta_I \lesssim 1$, the stability depends only on the law of radial variation of the containing force $-(2\pi/c) I_p R_p H_\perp$

$$\frac{\partial F_R}{\partial R_p} = \frac{2\pi}{c} I_p H_\perp \left(n - 1 + \frac{R_p}{I_p}\frac{\partial I_p}{\partial R_p}\right)$$

Considering that during rapid displacements of the column the magnetic flux enclosed in the column remains unchanged, i.e. assuming $(\partial/\partial R_p)(L_e I_p) + 2\pi R_p c H_{\perp 0} = 0$, we obtain the stability condition in the form [11]

$$n < \frac{3}{2} \tag{41}$$

3.1.3. Evaluation of distortion in the shape of the column resulting from inhomogeneity of the maintaining field

A change in the magnitude of the quadrupole field is inevitably reflected in the shape of the plasma column cross-section; in particular, the latter is flattened in the vertical direction in a field stabilizing vertical displacement, and is elongated in a destabilizing field (Fig. 6).

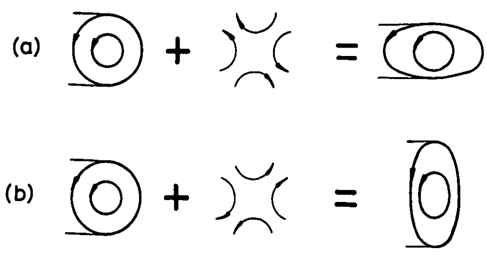

Fig. 6. Change in the shape of a plasma column cross-section on application of a quadrupole magnetic field.
(a) Quadrupole field corresponding to the convex lines of force of the balancing field.
(b) Quadrupole field corresponding to the concave lines of force of the maintaining field.

In order to evaluate the extent to which the shape of the column cross-section depends on the magnitude of the quadrupole magnetic field, we may consider a straight plasma column of elliptical cross-section with homogeneous current density. Calculation shows that in order to confine in equilibrium such a column with cross-section semi-axes ℓ_z and ℓ_x, it is necessary to place it in a quadrupole field described by the scalar potential

$$\varphi = A x z \tag{42}$$

The ellipticity parameter of the cross-section

$$\epsilon \equiv \frac{\ell_z^2 - \ell_x^2}{\ell_z^2 + \ell_x^2} \tag{43}$$

is determined by the formula

$$\epsilon = \frac{cA}{4I_p} (\ell_z + \ell_x)^2 \tag{44}$$

If the quadrupole field is generated by currents flowing along the surface $x^2 + z^2 = b_1^2$, the surface density of which equals $i = i_\epsilon \cos 2\omega$, then

$$A = -\frac{2\pi}{c} \frac{i_\epsilon}{b_1} \tag{45}$$

In the case of four conductors, each with current strength I_ϵ, placed as shown in Fig. 5,

$$i_\epsilon = \frac{4I_\epsilon}{\pi b_1} \tag{46}$$

and the ellipticity parameter

$$\epsilon = -\frac{2I_\epsilon}{I_p} \frac{\ell_z^2 + \ell_x^2}{b_1^2} \tag{47}$$

If the lines of force of the maintaining field have different curvatures, then this is equivalent to superposing a hexapolar magnetic field on the homogeneous field (Fig. 7). In a hexapolar magnetic field the plasma column cross-section becomes triangular (Fig. 8).

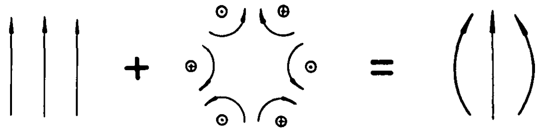

FIG. 7. Diagram of the combination of a homogeneous magnetic field with a hexapolar magnetic field.

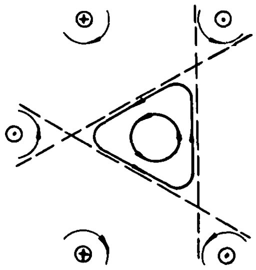

FIG. 8. The shape of a plasma column cross-section in the presence of a hexapolar magnetic field.

Thus, with the help of external fields we can, in principle, choose the desired shape for the plasma column cross-section.

If the magnetic potential of the m-th harmonic component of the applied field $\vec{h}$ is written in the form

$$\varphi_m = \frac{a}{m} h_{m0} \left(\frac{\rho}{a}\right)^m \sin m\omega \tag{48}$$

so that $h_{\omega m} = h_{m0} (\rho/a)^{m-1} \cos m\omega$, $h_{\rho m} = h_{m0} (\rho/a)^{m-1} \sin m\omega$, the corresponding displacement Δ_m of the plasma column boundary, $\rho = a + \sum_m \Delta_m$, is expressed in terms of the amplitude h_{m0} of the disturbing field at the plasma boundary by the formula

$$\frac{\Delta_m}{a} = -\xi \frac{h_{m0}}{m H_\omega (a)} \cos m\omega \tag{49}$$

where ξ is a numerical coefficient depending on the current distribution in the plasma. In the case of a quadrupole field m = 2, h_{20} = Aa, in the case of a hexapolar field m = 3 and so on.

As is seen from formula (49), to cause a noticeable change in the shape of the plasma column cross-section, fields comparable in magnitude with the current field are required.

The situation may change if the field applied is inhomogeneous with respect not only to ω but also to φ. Such a field can be represented as superposition of screw harmonic components of form $\exp(i(m\omega - n\varphi))$ having a longitudinal and an azimuthal period

$$\lambda_\varphi = \frac{2\pi R}{n}, \quad \lambda_\omega = \frac{2\pi\rho}{m} \tag{50}$$

Let the configuration contain a resonance magnetic surface, on which the magnetic line of force is closed, making Nm rounds along the torus and Nn rounds around the magnetic axis (N is a whole number). In this case, even when the magnitude of the applied field is small, a noticeable distortion of the configuration may occur, the magnetic surfaces acquiring a fibrous structure [13]. The smaller the derivative $dq/d\rho$, where $q = \rho H_\varphi(\rho)/RH_\omega(\rho)$, the bigger are the fibres. The 'resonance' condition takes the form

$$\frac{\lambda_\varphi}{\lambda_\omega} = \frac{H_\varphi(\rho)}{H_\omega(\rho)} \tag{51}$$

or

$$n\,q(\rho) = m \tag{52}$$

If $\lambda_\varphi/\lambda_\omega \gg H_\varphi/H_\omega$, i.e. $nq \ll m$, the longitudinal field has no effect on the distortion of the magnetic surfaces. In the opposite limiting case, on the other hand, we can neglect the effect of field H_ω. Here it appears that the action of disturbance is reflected mainly in the shape of the lines of force of the magnetic field. However, the plasma column axis is distorted in accordance with the distortion of the line of force of the longitudinal magnetic field. Let, for example, the transverse vertical field $H_\perp$ have, apart from a constant component which is balanced by the displacement of the magnetic surfaces, a variable component $h(\varphi)$, such that

$$\int_0^{2\pi} h(\varphi)\,d\varphi = 0 \tag{53}$$

We can say that under the action of this variable component the lines of force of the longitudinal field are deformed in accordance with the equation

$$\frac{dz}{h(\varphi)} = \frac{R\,d\varphi}{H_\varphi} \tag{54}$$

Hence, for vertical displacement of the plasma column axis we obtain

$$z(\varphi) = \frac{R}{H_\varphi}\int h(\varphi)\,d\varphi \tag{55}$$

If field $h(\varphi)$ has N periods, the amplitude of vertical displacement of the column will be

$$\delta z_{\text{max.}} \simeq \frac{Rh_{\text{max.}}}{H_\varphi N} \tag{56}$$

3.1.4. Relationship between the stability of the equilibrium position and the shape of the plasma column cross-section

If the plasma column cross-section has the shape of an ellipse with small eccentricity, then, with homogeneous current density, the maintaining transverse field in the equatorial plane is determined by the formula[2]

$$H_\perp = \frac{I_p}{cR_p}\left\{\ln\frac{8R_p}{a} + \beta_I - \frac{5}{4} - \frac{r - R_p}{R_p}\left[\frac{3}{4}\ln\frac{8R_p}{a} - \frac{17}{16} + \frac{R_p^2}{a^2}\left(1 - \frac{\ell_z}{\ell_r}\right)\right]\right\} \tag{57}$$

Hence there ensues the following relationship between the ratio of the semi-axes ℓ_z/ℓ_r of the ellipse and the decay index n of the maintaining field

$$\frac{\ell_z}{\ell_r} = 1 + \frac{3}{4}\frac{a^2}{R_p^2}\left(\ln\frac{8R_p}{a} - \frac{17}{12}\right) - \frac{a^2}{R_p^2}\left(\ln\frac{8R_p}{a} + \beta_I - \frac{5}{4}\right)n \tag{58}$$

In a purely homogeneous field (n = 0, neutral position of equilibrium relative to displacement along z) the cross-section of a plasma column in equilibrium is slightly elongated in the vertical direction ($\ell_z/\ell_r > 1$). To the equilibrium of a column with circular cross-section, $\ell_z/\ell_r = 1$ there corresponds a maintaining barrel field with decay index

$$n_0 = \frac{3}{4}\,\frac{\ln\frac{8R_p}{a} - \frac{17}{12}}{\ln\frac{8R_p}{a} + \beta_I - \frac{5}{4}} \tag{59}$$

satisfying stability conditions (39) and (41) in regard to both vertical and horizontal displacements. When $n > n_0$, the plasma column cross-section undergoes a slight flattening.

Figure 9 shows the ratio of the dimensions of the column cross-section along the vertical line ℓ_z and along the horizontal line ℓ_r as a function of the decay index of the maintaining magnetic field. The continuous part of the curve corresponds to stable equilibrium. When $n < 0$, the equilibrium is unstable relative to vertical displacements. When $n > 3/2$, the equilibrium is unstable relative to horizontal displacements. A thin plasma column ($\ln 8R_p/a \gg 1$; β_I) having a circular cross-section, occupies an approximately

[2] This formula has been derived by Zakharov [14].

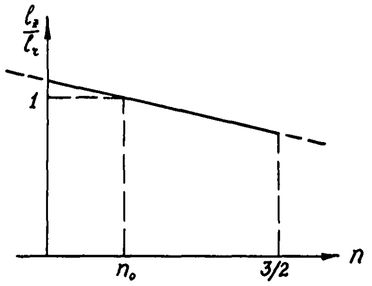

FIG.9. The shape of the plasma column cross-section as a function of the decay index $n = -(R_p/H_\perp)(\partial H_\perp/\partial R_p)$ of the maintaining field. The continuous area of the curve corresponds to stable equilibrium in relation to horizontal and vertical displacements.

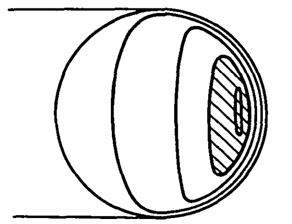

FIG.10. The shape of a plasma column cross-section in an ideally conducting casing at high plasma pressure.

central position in the stability region, as follows from formula (59). With increasing plasma pressure the position of the circular plasma column, $\ell_z/\ell_r = 1$, is displaced in this graph towards smaller n.

The method of maintaining a column by the magnetic field of external conductors discussed in this section has one disadvantage. The change in the parameters of the plasma column with time calls for continuous regulation of the maintaining field, which is a quite complicated technical problem, especially if the discharge parameters change rapidly. The method, however, can be very effective in systems designed for prolonged plasma maintaining if it is used in combination with a thin conducting casing (see Section 4).

3.2. MAINTAINING BY AN IDEALLY CONDUCTING CASING

If a plasma column is surrounded by an ideally conducting casing, the transverse field $H_{\perp 0}$, which is needed for ensuring equilibrium, is generated by the Foucault currents in casing walls when the centre of the column cross-section is displaced outwards in relation to the centre of the casing cross-section by distance Δ_0. The magnitude of Δ_0 is determined by formula (12) when $\rho = b$, where b is the radius of the casing cross-section. Substituting into this formula constants C_1 and C_2 expressed by formulae (30) and (31), we obtain

$$\Delta_0 = \Delta(b) = \frac{b^2}{2R}\left[\ln\frac{b}{a} + \left(1 - \frac{a^2}{b^2}\right)\left(\Lambda + \frac{1}{2}\right)\right] \qquad (60)$$

This formula for column displacement is applicable when $\Delta_0 \ll b$. If on heating the plasma pressure increases so much that Δ_0 becomes $\gtrsim b$, the shape of the plasma column cross-section will be distorted as shown qualitatively in Fig.10. It is difficult to perform the calculation for equilibrium in the general form. If, however, the transverse dimension of the column is small in comparison with the radius of the casing, the shape of the column cross-section will have little effect on the magnitude of its displacement Δ. In this case, displacement Δ can be calculated by equating the expanding force of the plasma column to the force of interaction between the current in the plasma and the image current in the casing. The latter current is found at a distance of $(b^2/\Delta) - \Delta$ from the plasma current and generates a field $H_\perp = 2I\Delta/c(b^2 - \Delta^2)$. Equating the forces gives an equation for Δ [15]

$$\frac{\Delta b^2}{b^2 - \Delta^2} = \Delta_0 = \frac{b^2}{2R}\left[\ln\frac{b}{a} + \beta_I + \frac{\ell_i - 1}{2}\right] \qquad (61)$$

Hence

$$\Delta = \sqrt{b^2 + (b^4/4\Delta_0^2)} - (b^2/2\Delta_0) \qquad (62)$$

When $\Delta_0/b \ll 1$, $\Delta = \Delta_0$; if, however, $\Delta_0/b \gg 1$, then $\Delta \approx b$.

The equilibrium of a plasma column in an ideally conducting casing is obviously stable. If the equilibrium position of the column is changed by means of some force, then the column, after the force stops acting, will perform oscillations near the original equilibrium position. These oscillations can be of interest from the diagnostic point of view, since the frequency of natural oscillations ω depends on the total mass M of the column's unit length.

Let us first consider the oscillations of a plasma column with a free boundary. When the column is displaced rapidly, the casing can be regarded as ideally conducting, while dissipation of the induced currents may occur in the liner. Thus, in the presence of a liner the oscillations of the column as a whole will be damped. We use δ to denote displacement of the column from the equilibrium position and b_λ, d_λ and σ_λ to denote the radius, wall thickness, and electrical conductivity of the liner.

Calculations give the following oscillation equations

$$\frac{d^2\delta}{dt^2} + \omega_0^2\,\delta = \frac{2\pi I}{Mc}\,\frac{b^2 - b_\lambda^2}{b^2 - a^2}\, i_\lambda \qquad (63)$$

$$\frac{di_\lambda}{dt} + \gamma_0 i_\lambda = -\frac{I}{b_\lambda^2 - a^2}\,\frac{d\delta}{dt} \qquad (64)$$

Here $i_\lambda(t)$ is the amplitude of the dipole component of the surface density of the current in the liner,

$$\omega_0^2 = \frac{2I^2}{Mc^2(b^2 - a^2)} \qquad (65)$$

$$\gamma_0 = \frac{c^2 b_\lambda}{2\pi\sigma_\lambda d_\lambda(b - b_\lambda)} A^2 \qquad (66)$$

$$A^2 = \frac{b^2 - a^2}{b_\lambda^2 - a^2} \qquad (67)$$

The solution of the oscillation equations takes the form $\delta \sim$ const. $\exp(\kappa t)$. For $\kappa = \pm i\omega - \gamma$, we obtain the equation

$$\kappa^3 + \gamma_0\kappa^2 + A^2\omega_0^2\kappa + \gamma_0\omega_0^2 = 0 \qquad (68)$$

or in dimensionless variables $y = \kappa/\omega_0$, $\gamma_1 = \gamma_0/\omega_0$

$$y^3 + \gamma_1 y^2 + A^2 y + \gamma_1 = 0 \qquad (69)$$

We now consider the limiting cases $\gamma_0 \gg \omega_0$ ('thin' liner) and $\gamma_0 \ll \omega_0 A$ ('thick' liner). In the first case, the oscillation frequency does not depend on the liner

$$\omega^2 = \omega_0^2 = \frac{2I^2}{Mc^2(b^2 - a^2)} \qquad (70)$$

and the damping decrement of the oscillations γ is proportional to σ_λ

$$\gamma = \frac{\omega_0^2}{2\gamma_0}(A^2 - 1) \qquad (71)$$

In the second case, the liner acts as an ideal casing

$$\omega^2 = \omega_0^2 A^2 = \frac{2I^2}{Mc^2(b_\lambda^2 - a^2)} \qquad (72)$$

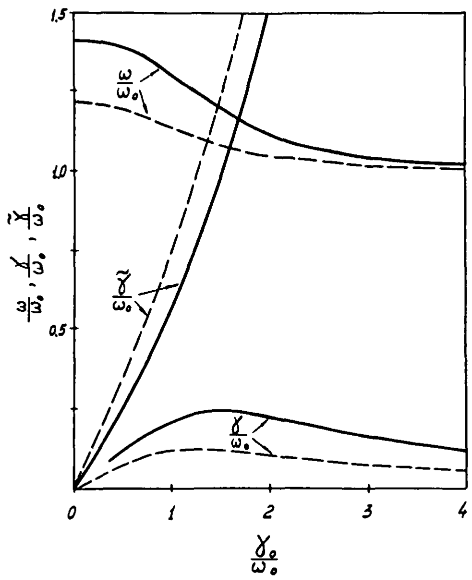

FIG.11. Frequency ω and damping decrements γ and $\tilde{\gamma}$ of the oscillations of a plasma column in an ideally conducting casing in the presence of a liner. The expressions for γ_0 and ω_0 and for the parameter of curves A^2 are given by formulae (65) - (67). The solid lines correspond to $A^2 = 2$ and the broken lines to $A^2 = 1.5$.

The damping decrement is now inversely proportional to σ_λ

$$\gamma = \gamma_0 \frac{A^2 - 1}{2A^2} \qquad (73)$$

Apart from the solutions given, Eq. (69) has a purely damped solution with damping decrement $\tilde{\gamma} \approx \gamma_0$. This solution is not of interest. The roots of Eq. (69) $y_{1,2} = \pm i(\omega/\omega_0) - (\gamma/\omega_0)$ and $y_3 = -\tilde{\gamma}/\omega_0$ for the intermediate values of parameter γ_0, when $A^2 = 1.5$ and $A^2 = 2$, are given in Fig. 11.

In a plasma extending to the casing (column with a fixed boundary), displacements of the column as a whole are impossible (when $a \to b$, as is seen from formula (70), $\omega \to \infty$). In this case, the chief role is played by the hydromagnetic oscillations of the internal part of the plasma column, which are associated with displacement of the magnetic axis (the external boundary being fixed). The oscillation frequency is determined by the relation

$$\omega^2 = 2I^2/\mu c^2 b^2 \qquad (74)$$

Obviously, this oscillation mode also exists in a plasma column with a free boundary.

In Section 4 we shall consider the equilibrium of a plasma column, taking into account the effects resulting from the non-ideal nature of the casing.

3.3. MAINTAINING BY THE IRON CORE OF A TRANSFORMER

The equilibrium of a plasma column can also be ensured by the interaction between the current flowing in the plasma and a transformer core [16]. A core with magnetic permeability $\mu \gg 1$ attracts the magnetic lines of force surrounding the plasma column, resulting in the generation of a force which attracts the plasma loop to the core. If the core is an infinite cylinder of radius r_c and the radius of the ring with current is equal to R, the force of attraction of the ring to the core is determined by the expression

$$F_c = -\frac{4\pi I^2}{c^2} f\left(\frac{R}{r_c}\right) \qquad (75)$$

where the function $f(R/r_c)$ is expressed in terms of the integral of modified Bessel functions

$$f\left(\frac{R}{r_c}\right) = \frac{2R^2}{r_c^2}\int_0^\infty \frac{I_0(\kappa)}{K_0(\kappa)} K_1\left(\frac{\kappa R}{r_c}\right) K_0\left(\frac{\kappa R}{r_c}\right)\kappa\, d\kappa \qquad (76)$$

Function $f(R/r_c)$ is plotted in Fig. 12.

Since the force of attraction increases on accidental displacement of the column from the equilibrium position towards the core and, inversely, decreases on displacement away from it, the equilibrium created by the core alone is unstable. It is not impossible, however, that the force of attraction of the core can be used in

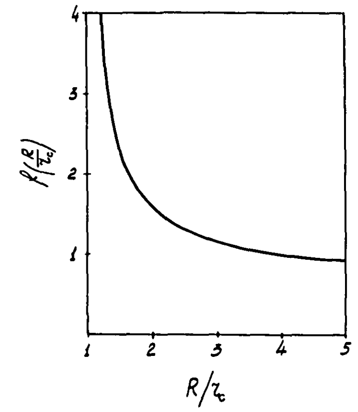

FIG.12. The force of attraction of a ring current to a cylindrical iron core as a function of the distance between them.

combination with some means of stabilization of the equilibrium. There are experimental data [17] to show that in existing Tokamak devices the effect of the core cannot be neglected because of the presence of gaps in the casing and the latter's finite electrical conductivity. Since the question of maintaining a plasma column by an iron core can be of interest in future studies, we give the formula describing the amount of displacement of a plasma column placed in an axisymmetric iron casing.

When $\mu = \infty$, the normal component of the magnetic field on the surface inside a magnet is equal to zero, while the tangential component is continuous. This being so, we can easily obtain the expression describing displacement of a column in a magnetic casing. If the casing is continuous with an aperture of radius b_4, then

$$\Delta = -\frac{b_4^2}{2R}\left[2\ln\frac{8R}{b_4} + \ln\frac{b_4}{a} + \left(\Lambda + \frac{1}{2}\right)\left(1 + \frac{a^2}{b_4^2}\right) - 2\right] \tag{77}$$

In contrast with the case of an ideally conducting casing, the equilibrium position of the plasma column in an iron casing with $\mu = \infty$ is displaced towards the internal side of the casing. The absolute amount of displacement is greater than the amount in an ideally conducting casing.

If the iron casing has the shape of a toroidal shell, the external surface of which with radius b_5 is displaced outwards by distance Δ_5, then

$$\Delta = \frac{-b_4^2}{2R}\left[\ln\frac{b_4}{a} + \left(\Lambda + \frac{1}{2}\right)\left(1 + \frac{a^2}{b_4^2}\right) + \frac{2b_5^2}{b_5^2 - b_4^2}\left(\ln\frac{b_5}{b_4} + \frac{2R\Delta_5}{b_5^2}\right)\right] \tag{78}$$

4. EQUILIBRIUM WITH ALLOWANCE FOR VARIOUS COMPLICATING FACTORS

At present, Tokamak devices are generally made with a conducting casing. The casing shields the discharge volume from undesirable external fields and ensures a stable equilibrium. The equilibrium position of the plasma column in an ideal casing is determined by formula (60).

In actual devices the casing is not ideal. It has to have transverse gaps so that an electrical field can be applied to generate a current in the plasma. Longitudinal slits are made in the casing in order to apply a longitudinal magnetic field. The conductivity of the casing can be assumed to be infinite if the depth of penetration of the magnetic field into the casing walls is small in comparison with the amount of displacement Δ and the wall thickness d_k, i.e. if

$$t \ll \frac{4\pi\sigma_k \Delta^2}{c^2} \tag{79}$$

$$t \ll \frac{4\pi\sigma_k d_k^2}{c^2} \tag{80}$$

where t is the duration of the discharge pulse.

The gaps in the casing and the finite electrical conductivity of the latter weaken its confinement properties and result in the penetration of external stray magnetic fields into the discharge chamber. Under these conditions, there also occurs an interaction between the plasma column and the transformer core.

It is naturally complicated to take account simultaneously of all the factors that affect the equilibrium of the plasma column. We consider the effects of some of the most important factors separately.

Let us consider the following layout of a device (Fig.13). A plasma column of radius a is confined inside a conducting casing of radius b. The internal control conductors are located on a toroidal surface of radius b_1 between the plasma and the casing. It is assumed that the transformer core has the form of an axisymmetric toroidal shell with minor radii of cross-sections b_4 and b_5. The centres of these cross-sections are displaced outwards in relation to the centre of the casing cross-section by distances Δ_4 and Δ_5 respectively. The primary winding of the transformer is situated on a surface of radius b_3 in the space between the core and the external control loops.

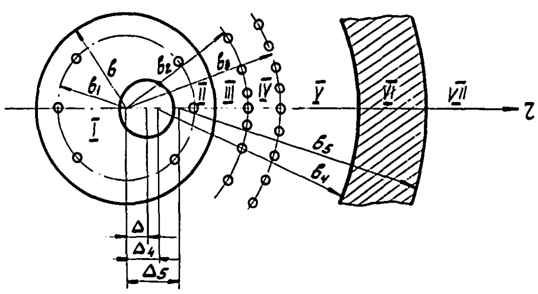

FIG.13. Layout of the device considered in the text. The internal circle of radius a is the cross-section of the plasma column surface; on a circle of radius b_1 are located the internal control conductors; the casing is represented by radius b, the external control conductors by radius b_2, and the primary winding by radius b_3; b_4 and b_5 are the radii of the internal and external surfaces of the toroidal iron core.

Apart from the elements mentioned here, Tokamak devices usually contain a liner between the plasma and the casing. It serves a double purpose – it improves the vacuum conditions in the discharge volume and balances the electrical vorticity field along the torus length. The effect of the currents generated in the liner on the equilibrium of the plasma column can easily be included in the general pattern of consideration. However, since this effect is usually not very substantial, we shall not consider it here. The role of the liner is evaluated separately (section 4.4).

In calculating equilibrium it is convenient to take the casing axis as the coordinate axis. The formulae for the flux function ψ in region I between the plasma and the control loops and also in regions II, III, etc. as well as the formulae for the magnetic field will be determined by expressions (9), (13) and (14) with the corresponding constants C_1 and C_2, to which we shall assign superscripts indicating the number of the region. In region I these constants are

$$C_1^I = -\frac{2\pi I_p}{c} a^2\left(\frac{2R\Delta}{a^2} + \Lambda + \frac{1}{2}\right) \tag{81}$$

$$C_2^I = \frac{2\pi I_p}{c}\left(\ln\frac{8R}{a} + \Lambda - \frac{1}{2}\right) \tag{82}$$

Here I_p is the current in the plasma column and Δ the as yet unknown amount of displacement of the plasma column to be determined, together with the other constants C_1^{II}, C_2^{II}, etc., from the conditions on the external surfaces.

4.1. EQUILIBRIUM IN AN IDEAL CASING IN THE PRESENCE OF AN ADDITIONAL CONTROL FIELD

We shall first consider the equilibrium of a plasma column in an ideally conducting casing in the presence of an external transverse magnetic field and with open (or no) control loops.

If an additional maintaining field $H_\perp$ penetrating the casing walls is generated in the volume occupied by the plasma column, the latter will be displaced in such a way that on the casing, $\rho = b$, the normal field component will assume the given value

$$(\vec{H}\vec{n}) = H_\perp(\vec{e}_z\,\vec{n}) \tag{83}$$

or

$$H_\rho^I(b) = H_\perp \sin\omega \tag{84}$$

Substituting into this condition the value of H_ρ^I from formula (14) with constants C_1^I and C_2^I determined by formulae (81) and (82), we obtain the expression for displacement

$$\Delta = \Delta_0 + \Delta_H \tag{85}$$

where Δ_0 is displacement in an ideal casing without a transverse field (60), and Δ_H is displacement due to the transverse field present

$$\Delta_H = -\frac{cb^2}{2I_p} H_\perp \tag{86}$$

Calculations show that formula (85) is valid for an arbitrary method of generation of field $H_\perp$ in the region occupied by the plasma (by external or internal loops), although the actual conditions on the casing can change. $H_\perp$ is taken to be positive if its direction coincides with that of the current field on the outer side of the plasma column

4.2. FIELD OF CONTROL LOOPS

The control loops are used for regulating the equilibrium position of the plasma column. To eliminate the effect of currents in the control loops on the value of the current in the plasma, the algebraic sum of the currents in the loops must be equal to zero. In this case, the equilibrium position of the plasma column is affected only by the dipole component of the surface density of current i_1 connected with $i(\omega)$ by the relation

$$i_1 = \frac{1}{\pi}\int_0^{2\pi} i(\omega)\cos\omega\, d\omega \tag{87}$$

If the loops with current of one direction are situated on surface $\rho = b_1$ when $\omega = 0$, $\pm\pi/4$ and those with current of another direction when $\omega = \pi$, $\pm 3\pi/4$, then

$$i(\omega) = \frac{I_1}{b_1}\left[\delta(\omega) + \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) - \delta(\omega - \pi) - \delta\left(\omega - \frac{3\pi}{4}\right) - \delta\left(\omega + \frac{3\pi}{4}\right)\right] \tag{88}$$

where I_1 is the absolute value of current strength in each loop. Hence it follows that

$$i_1 = \frac{2(1+\sqrt{2})}{\pi b_1} I_1 \tag{89}$$

With four loops situated at $\omega = \pm\pi/4$ and $\omega = \pm 3\pi/4$,

$$i(\omega) = \frac{I_1}{b_1}\left[\delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right) - \delta\left(\omega - \frac{3\pi}{4}\right) - \delta\left(\omega + \frac{3\pi}{4}\right)\right] \tag{90}$$

Hence

$$i_1 = \frac{2\sqrt{2}}{\pi b_1} I_1 \tag{91}$$

The field of the control loops is described by the flux function

$$\psi^{I} = C_2^{I}\rho \cos\omega \qquad (\rho < b_1) \qquad (92)$$

$$\psi^{II} = \left(\frac{C_1^{II}}{\rho} + C_2^{II}\rho\right)\cos\omega \qquad (\rho > b_1) \qquad (93)$$

In the absence of a conducting casing $C_2^{II} = 0$. Constants C_2^{I} and C_1^{II} are determined from the conditions: $H_\rho^{I}(b_1) = H_\rho^{II}(b_1)$ and $H_\omega^{II}(b_1) - H_\omega^{I}(b_1) = (4\pi/c)i_1\cos\omega$. Thus, the vertical component of the field of the control loops $h_{ii}^{0} = C_2^{I}/2\pi R$ affecting the position of the plasma column is obtained in the form

$$h_{ii}^{0} = -\frac{2\pi}{c}i_1 \qquad (94)$$

When $\rho > b_1$, the field of the loops in the equatorial plane changes according to the law

$$h_{ie}^{0} = \frac{2\pi}{c}i_1\frac{b_1^2}{\rho^2} \qquad (95)$$

In the presence of a conducting casing situated at $\rho = b > b_1$, constants C_2^{I}, C_1^{II} and C_2^{II} determined from conditions $H_\rho^{I}(b_1) = H_\rho^{II}(b_1)$, $H_\omega^{II}(b_1) - H_\omega^{I}(b_1) = (4\pi/c)i_1\cos\omega$, $H_\rho^{II}(b) = 0$ are equal to

$$C_2^{I} = -2\pi R\frac{2\pi}{c}i_1\left(1 - \frac{b_1^2}{b^2}\right) \qquad (96)$$

$$C_1^{II} = -2\pi R\frac{2\pi}{c}i_1 b_1^2 \qquad (97)$$

$$C_2^{II} = 2\pi R\frac{2\pi}{c}i_1\frac{b_1^2}{b^2} \qquad (98)$$

The magnetic field of the control loops in the internal and external regions is determined respectively by the expressions

$$h_{ii}^{\infty} = -\frac{2\pi}{c}i_1\left(1 - \frac{b_1^2}{b^2}\right) \qquad (99)$$

$$h_{ie}^{\infty} = \frac{2\pi}{c}i_1\left(\frac{b_1^2}{b^2} + \frac{b_1^2}{\rho^2}\right) \qquad (100)$$

The field in the internal region is smaller than that in the absence of a casing, since it is weakened by the image currents in the casing. Between the loops and the casing the image currents increase the field almost by a factor of two.

Similarly we can calculate the quadrupole component of the surface density of the current i_ϵ

$$i_\epsilon = \frac{1}{\pi}\int_0^{2\pi} i(\omega)\cos 2\omega\, d\omega \qquad (101)$$

affecting the curvature of the lines of force and, consequently, the decay index n of the maintaining field. The magnitude of the total maintaining field in the equatorial plane is expressed in terms of i_1 and i_ϵ by the formula

$$H_\perp = -\frac{2\pi}{c}i_1 - \frac{2\pi}{c}i_\epsilon\frac{r - R}{b_1} \qquad (102)$$

so that the decay index

$$n = \frac{i_\epsilon}{i_1}\frac{R}{b_1} \qquad (103)$$

This formula enables us, on the basis of the decay index n given by the stability condition, to evaluate the necessary current strength in the quadrupole conductors in the absence of a conducting casing. Let, for example, the quadrupole conductors be situated at $\omega = 0$, π and $\omega = \pm\pi/2$. In this case

$$i(\omega) = \frac{I_\epsilon}{b_1}\left[-\delta(\omega) + \delta\left(\omega - \frac{\pi}{2}\right) - \delta(\omega - \pi) + \delta\left(\omega + \frac{\pi}{2}\right)\right] \qquad (104)$$

Hence

$$i_\epsilon = \frac{4I_\epsilon}{\pi b_1} \qquad (105)$$

Since $i_1 \sim I_1/\pi b_1$, while the decay index should not be large, the current in the quadrupole conductors should be $\xi R/b_1$ times smaller than that in the dipole conductors, ξ being a numerical coefficient.

4.3. EFFECT OF GAPS IN AN IDEALLY CONDUCTING CASING

In order to introduce inside the casing an electrical field to excite the discharge, the casing is made out of separate electrically insulated sections. The Foucault currents excited in the casing form a circuit inside each section. The part of the current, which is symmetrical and independent of ω forms a circuit along the external

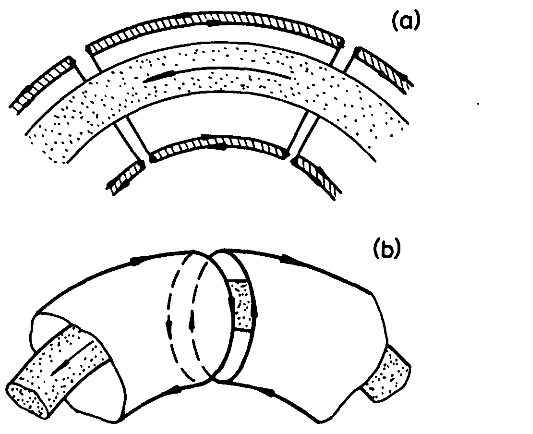

FIG.14. Circuit formation by the symmetrical (a) and non-symmetrical (b) components of the current in the casing in the presence of transverse gaps.

surface of the sections (Fig.14(a)). The non-symmetrical current component responsible for the maintaining field forms a circuit near the gap both on the internal and on the external surface of the sections. The circuit-forming currents of neighbouring sections flow along the gap in opposite directions (Fig.14(b)). Therefore, an additional transverse field appears in the gap. The sign of this field is the opposite of that of the maintaining field. The gaps thus weaken the confinement properties of the casing. Furthermore, they lead to the penetration of the discharge chamber by external stray magnetic fields, which can be generated by sources of the most varied kinds (for example, the stray field of the transformer core). As follows from the work of Leontovich[18], the gaps distort the magnetic field at a distance of the order of the casing radius. For this reason the action of the gaps can be considered independent if the length of each casing section is greater than the casing diameter.

4.3.1. Calculation of a maintaining field in the presence of gaps

In the presence of gaps in the casing, the normal component of the magnetic field, for $\rho = b$, is not equal to zero but is a function of φ. In the approximation considered (smallness of b/R)

$$(\vec{H}\vec{n}) = f(\varphi)\sin\omega \tag{106}$$

Quantity $f(\varphi)$ (in the absence of any shields), with logarithmic accuracy, equals[3]

$$f(\varphi) = \begin{cases} -\sum_{k=1}^{N} \dfrac{\Phi_i(b) - \Phi_e(b)}{2R\sqrt{(\varphi_k^2 - \varphi^2)}\ln(\alpha b/h_k)} & \left(\varphi_k - \dfrac{h_k}{R} < \varphi < \varphi_k + \dfrac{h_k}{R}\right) \\ 0 & \left(\varphi < \varphi_k - \dfrac{h_k}{R}\,;\ \varphi > \varphi_k + \dfrac{h_k}{R}\right) \end{cases} \tag{107}$$

Here N is the total number of gaps, $2h_k$ the width of the gap located at $\varphi = \varphi_k$, α a coefficient of the order of unity and $\Phi_i \sin\omega$ and $\Phi_e \sin\omega$ are the dipole components of the magnetic potentials respectively on the internal and the external surface of the casing. Considering that $\vec{H} = \nabla\Phi$, the difference $\Phi_i(b) - \Phi_e(b)$ can be written as

$$\Phi_i(b) - \Phi_e(b) = b[H_\omega^{II}(b) - H_\omega^{III}(b)] \tag{108}$$

[3] In Refs [4,19], where the effect of gaps in the casing on the equilibrium of a toroidal plasma column was evaluated, it was erroneously assumed that $\Phi_e(b) = 0$. Hence formula (61) obtained in Ref.[4] and the corresponding formula in Ref.[19] contain errors.

We expand $f(\varphi)$ into a Fourier series

$$f(\varphi) = f_0 + 2\sum_{m=1}^{\infty} f_m \cos m\varphi \tag{109}$$

The harmonic components with $m \geq 1$ lead to distortion of the shape of the plasma column along its length. In a strong longitudinal field $(H_{e0}a/H_I R \gg 1)$ this distortion is small (see below). The zero harmonic component determines the effective transverse field h^g associated with the gaps

$$h^g = f_0 = -\frac{H_\omega^{II}(b) - H_\omega^{III}(b)}{2\pi R} b\mu \tag{110}$$

where

$$\mu = \sum_{k=1}^{N} \mu_k = \sum_{k=1}^{N} \frac{\pi}{2\ln\dfrac{\alpha b}{h_k}} \tag{111}$$

Substituting H_ω^{II} and H_ω^{III} from formula (13), we obtain an expression for h^g expressed in terms of constants C_1, C_2

$$h^g = \frac{(C_2^{III} - C_2^{II})b^2 - (C_1^{III} - C_1^{II})}{2\pi R b}\frac{\mu}{2\pi R} \tag{112}$$

Determining constants C_1, C_2 from the conditions on the surface $\rho = b_1$ ($H_\rho^I = H_\rho^{II}$, $H_\rho^{II} - H_\omega^I = (4\pi/c)i_1\cos\omega$) and on the casing ($H_\rho^{II}(b) = 0$), we obtain

$$C_1^{II} = -\frac{2\pi I_p}{c}a^2\left(\frac{2R\Delta}{a^2} + \Lambda + \frac{1}{2}\right) - \frac{4\pi^2 R}{c} i_1 b_1^2 \tag{113}$$

$$C_2^{II} = \frac{2\pi I_p}{c}\left(\ln\frac{8R}{a} + \Lambda - \frac{1}{2}\right) + \frac{4\pi^2 R}{c} i_1 \tag{114}$$

If the external sources generate, in the absence of a casing, a transverse field h_{ei}^0 in the discharge volume, then this is equivalent, when $\rho \to \infty$, to $H_z^{III} = h_{ei}^0$ outside the casing and to $H_\rho^{III} = 0$ on the casing. From this we obtain two other constants

$$C_1^{III} = -2\pi R h_{ei}^0 b^2 + \frac{2\pi I_p}{c} b^2\left(\ln\frac{8R}{b} - 1\right) \tag{115}$$

$$C_2^{III} = 2\pi R h_{ei}^0 \tag{116}$$

The expression for displacement Δ in the presence of the field of control loops (99) and the field h^g due to gaps has, according to formula (85), the following form

$$\Delta = \frac{b^2}{2R}\left[\ln\frac{b}{a} + \left(1 - \frac{a^2}{b^2}\right)\left(\Lambda + \frac{1}{2}\right)\right] + \frac{cb^2}{2I_p}\frac{2\pi i_1}{c}\left(1 - \frac{b_1^2}{b^2}\right) - \frac{cb^2}{2I_p}h^g \tag{117}$$

Substituting this expression into C_1^{II}, we obtain from formula (112) the following expression for h^g

$$h^g = (h_{ii}^0 + h_{ei}^0 - H_{\perp 0}) \frac{2b\mu}{2\pi R - b\mu} \qquad (118)$$

Here h_{ii}^0 is the field of the internal control conductors in the absence of a casing (94), h_{ei}^0 the field of the external sources in the absence of a casing and $H_{\perp 0}$ the magnitude of the equilibrium-maintaining transverse field (37).

The field generated inside the casing by the external sources is connected with h_{ei}^0 by the relation

$$h_{ei} = \frac{\mu b}{\pi R} h_{ei}^0 \qquad (119)$$

which is obtained from formula (112) when $C_1^{II} = C_2^{II} = 0$ and $C_1^{III} = -2\pi R b^2 h_{ei}^0$, $C_2^{III} = 2\pi R h_{ei}^0$. It can be seen from this formula that the external transverse field penetrates the gaps in a manner which is similar to the homogeneous flux of a field passing through a slit with effective width

$$\delta_k = 2b\mu_k = \pi b / \ln \frac{\alpha b}{h_k} \qquad (120)$$

The field of the internal control conductors in the presence of gaps in the casing is obtained from formulae (97), (98) and (112) in the form

$$h_{ii} = -\frac{2\pi}{c} i_1 \left(1 - \frac{b_1^2}{b^2}\right) - \frac{2\pi}{c} i_1 \frac{b_1^2}{b^2} \frac{\mu b}{\pi R} \qquad (121)$$

The magnitude of the transverse magnetic flux ψ_g passing through a gap in the casing can be easily measured with the help of a magnetic loop on the surface $\rho = b$ along lines

$$\left.\begin{aligned} \varphi &= \varphi_k + \frac{h_k}{R} \\ \varphi &= \varphi_k - \frac{h_k}{R} \end{aligned}\right\} \; 0 \leq \omega \leq \pi \qquad (122)$$

$$\left.\begin{aligned} \omega &= 0 \\ \omega &= \pi \end{aligned}\right\} \; \varphi - \frac{h_k}{R} \leq \varphi \leq \varphi_k + \frac{h_k}{R}$$

Figure 15 gives the measurements of flux ψ_g performed on T-5 and TM-3 devices[4]. In these experiments the plasma column was modelled with the help of a thin ring conductor. In the graph the value of ratio $h^g/H_\omega(b) = cN\psi_g/8\pi RI$ is plotted as a function of displacement of the conductor in relation to the geometric axis of the casing. The continuous straight lines in this figure represent the theoretical dependence of $h^g/H_\omega(b)$ on Δ, which follows from formula (118). It is evident that the experimental data agree satisfactorily with the calculated results.

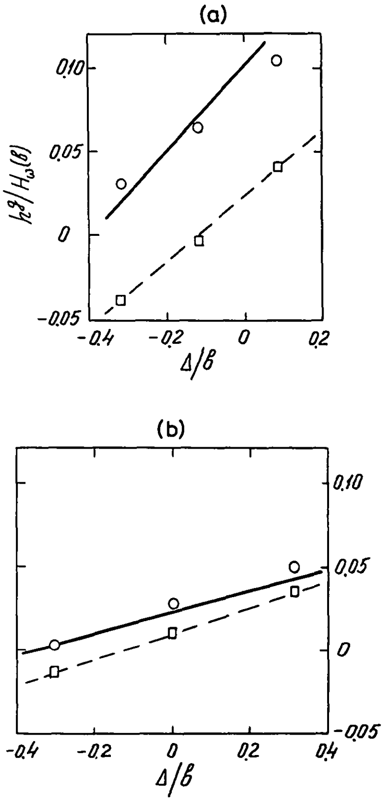

FIG.15. The magnitude of the additional transverse magnetic field h^g generated by transverse gaps in the casing as a function of displacement Δ: (a) in T-5 and (b) in TM-3 devices. The solid line represents the theoretical dependence when $h_{ei} = h_{ii} = 0$. The circles represent the experimental data corresponding to this case (absence of extraneous fields and iron core) and the broken lines the experimental data in the presence of an iron core.

It is to be noted that the currents passing along the gaps generate an additional longitudinal field, the magnitude of which on the internal surface of the casing equals

$$h_{\varphi i} = \frac{H_\omega^{II}(b) - H_\omega^{III}(b)}{2R \ln \frac{\alpha b}{h_k}} \frac{b}{\sqrt{(\varphi - \varphi_k)^2 + \frac{h_k^2}{R^2}}}$$

$$= \frac{h^g \mu_k}{\sqrt{(\varphi - \varphi_k)^2 + \frac{h_k^2}{R^2}}} \qquad (123)$$

The additional field on the external side has the same absolute value but the opposite sign. Thus, the pressure of the longitudinal magnetic field is not the same on the internal and external surfaces of the casing. This leads to the appearance of a force

$$F_z = \frac{b^2 H_{e0}}{2} (H_{\perp 0} - h_{ei}^0 - h_{ii}^0) \qquad (124)$$

acting on the casing near the gaps in the direction of the symmetry axis of the torus. The sign of

[4] The measurements on TM-3 were performed by Yu.A. Sokolov.

force F_z is different on different sides of the gaps. In large devices, with large H_{e0} and I_p, the magnitude of this force is considerable and it must be taken into consideration in designing the device.

4.3.2. Consideration of the effect of the transformer core

In the presence of gaps the transformer core affects the position of the plasma column, making a contribution to the expression for field h^g. In order to evaluate this contribution, we require to calculate constants C_2^{III} and C_1^{III}. Constant C_2^{III} gives us the effective field $h_{ei}^0 = C_2^{III}/2\pi R$ generated by the interaction between the currents and the core. The general expression for h_{ei}^0, which simultaneously takes into account the current in the plasma and the currents in the primary circuit and control loops, is obtained from the solution of a system of six algebraic equations obtained from the boundary conditions

$$\psi^{III}(b) = \text{const.} \tag{125}$$

$$\psi^{III}(b_2) = \psi^{IV}(b_2) \tag{126}$$

$$H_\omega^{IV}(b_2) - H_\omega^{III}(b_2) = \frac{4\pi}{c} i_2 \cos\omega \tag{127}$$

$$\psi^{IV}(b_3) = \psi^{V}(b_3) \tag{128}$$

$$H_\omega^{V}(b_3) - H_\omega^{IV}(b_3) = -\frac{2I_0}{cb_3} + \frac{4\pi}{c} i_3 \cos\omega \tag{129}$$

$$H_\omega^{VI}(b_4) = H_\omega^{V}(b_4) \tag{130}$$

$$(\vec{H}\vec{n})^{VI} = 0; \qquad C_2^{VI} = 0 \tag{131}$$

Here we make the surface density of the current in the primary winding equal to $i_0 = -(I_0/2\pi b_3) + i_3 \cos\omega$.

Let us consider the effect of the different field sources separately.

(a) The field due to interaction between the current I_p in the plasma and the core. Here we consider regions III, IV and V as a single region III. From the boundary conditions

$$\psi^{III}(b) = \text{const.} \tag{132}$$

$$H_\omega^{III}(b_4 + \Delta_4 \cos\omega) = H_\omega^{VI}(b_4 + \Delta_4 \cos\omega) \tag{133}$$

$$(\vec{H}\vec{n})^{VI} = 0; \qquad C_2^{VI} = 0 \tag{134}$$

we find

$$h_{ei}^0 = -\frac{I_p}{cR(b^2 + b_4^2)}\left[(b_4^2 - b^2)\left(\ln\frac{8R}{b_4} - 1\right) - b^2 \ln\frac{b_4}{b} - 2R\Delta_4\right] \tag{135}$$

(b) The field due to the interaction between the primary-winding current I_0 and the core.

Here we assume $I_p = 0$ and consider regions III and IV as a single region III. The boundary conditions have the form

$$\psi^{III}(b) = \text{const.} \tag{136}$$

$$\psi^{IV}(b_3) = \psi^{III}(b_3) \tag{137}$$

$$H_\omega^{V}(b_3) - H_\omega^{III}(b_3) = -\frac{2I_0}{cb_3} + \frac{4\pi}{c} i_3 \cos\omega \tag{138}$$

$$(\vec{H}\vec{n})^{VI} = 0; \qquad C_2^{VI} = 0 \tag{139}$$

Solving these equations, we obtain

$$h_{ei}^0 = \frac{I_0 b_4^2}{cR(b^2 + b_4^2)}\left[\ln\frac{8R}{b_3} - \frac{1}{2}\left(1 - \frac{b_3^2}{b_4^2}\right) + \ln\frac{8R}{b_4} - 1 - \frac{2R\Delta_4}{b_4^2}\right] - \frac{2\pi i_3}{c}\,\frac{b_3^2 + b_4^2}{b^2 + b_4^2} \tag{140}$$

(c) The field due to the interaction between the currents in the control winding and the core.

Let there be current only in the external control loops. We consider regions IV and V as a single region V. The boundary conditions for determining the required constant C_2^{III} are

$$\psi^{III}(b) = \text{const.} \tag{141}$$

$$\psi^{V}(b_2) = \psi^{III}(b_2) \tag{142}$$

$$H_\omega^{V}(b_2) - H_\omega^{III}(b_2) = \frac{4\pi}{c} i_2 \cos\omega \tag{143}$$

$$H_\omega^{V}(b_4) = 0 \tag{144}$$

Solving these equations we find

$$h_{ei}^0 = -\frac{2\pi}{c} i_2 \frac{b_4^2}{b^2 + b_4^2}\left(1 + \frac{b_2^2}{b_4^2}\right) \tag{145}$$

4.3.3. Effect of gaps on the shape of the plasma column

As is seen from formula (106), the gaps generate a variable field component

$$h(\varphi) = f(\varphi) - \overline{f}(\varphi) \tag{146}$$

If N gaps are situated at $\varphi = 0$, $2\pi/N$, $4\pi/N$, ..., $2\pi(N-1)/N$, then

$$h(\varphi) = 2\sum_{n=1}^{\infty} h_n \cos nN\varphi \tag{147}$$

where

$$h_n = -\frac{[H_\omega^{II}(b) - H_\omega^{III}(b)]_b}{2\pi R} \sum_{m=1}^{N} \frac{\pi}{2\ln\frac{\alpha b}{h_m}} I_0\left(\frac{mNh_m}{R}\right) \tag{148}$$

The effect of the inhomogeneity of the maintaining field on the distortion of the shape of the plasma column was considered in Section 3.1.3. In accordance with formula (56), the amplitude of vertical displacement of the column axis will be

$$\delta z_{max.} \simeq \frac{2Rh^g}{H_\varphi N} \simeq b\mu_k \frac{H_{\perp 0}}{H} \sim \frac{a^2}{R^2} \frac{b\mu_k}{q} \qquad (149)$$

The amplitude of the variable component of horizontal displacement is Nq times smaller than δz_{max}.

4.3.4. Effect of longitudinal gaps

As has already been pointed out, the dipole currents flowing through the casing in the presence of transverse gaps form circuits along the upper and the lower half of the casing. The longitudinal gaps situated at $\omega = 0, \pi$ do not hinder this circuit formation and have practically no effect on the magnitude of the maintaining magnetic field (Fig.16a). However, if the longitudinal gaps are situated at $\omega = \pm\pi/2$, the circuit-forming currents are compelled to flow along these gaps in the opposite direction (Fig.16b). These reverse currents flowing along longitudinal gaps weaken the maintaining field by

$$\Delta H \simeq \frac{\delta}{2b} (h^0_{ii} + h^0_{ei} - H_{\perp 0})$$

where δ is the effective width of the slit. For example, in the case of the T-5 device, $\delta/b \simeq 0.8$. Thus, the longitudinal gaps situated at $\omega = \pm\pi/2$ noticeably reduce the maintaining property of the casing.

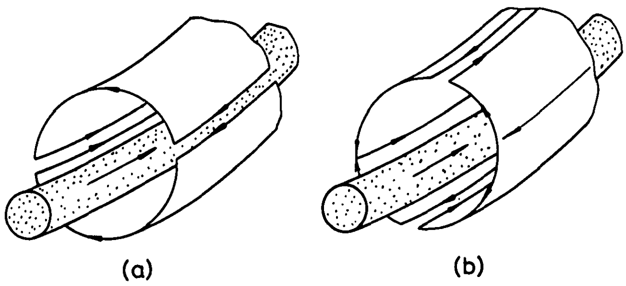

FIG.16. Circuit formation by the non-symmetrical component of the current in the casing in the presence of longitudinal and transverse gaps: (a) longitudinal gaps causing no distortions of the transverse field; (b) longitudinal gaps distorting the transverse field.

4.4. FIELD OF CURRENTS FLOWING ALONG THE LINER

Let b_λ be the radius of the cross-section of the liner ($b_\lambda < b$), and Δ_λ the displacement of the centre of the liner cross-section in relation to the centre of the casing cross-section towards the symmetry axis. Let us also assume that the resistance of the liner in the cross-section varies according to the law $1 + \lambda \cos\omega$ (for a sylphon liner $\lambda = 0$ and for a smooth liner $\lambda \simeq b_\lambda/R$). Under these conditions, and on the assumption that the casing is ideal, the additional transverse field generated in the discharge volume by the current I_λ flowing through the liner is determined by the expression

$$h^\infty_\lambda = -\frac{I_\lambda}{cR}\left[\ln\frac{b}{b_\lambda} + \left(1 - \frac{b_\lambda^2}{b^2}\right)\left(1 - \frac{R\lambda}{b_\lambda}\right) - \frac{2R\Delta_\lambda}{b^2}\right] \qquad (150)$$

When there are gaps in the casing, the transverse field equals

$$h_\lambda = h^\infty_\lambda + h^g_\lambda$$

where h^g_λ is the effective field associated with the gaps

$$h^g_\lambda = -\frac{\mu b}{\pi R}\frac{I_\lambda}{cR}\left[\ln\frac{8R}{b} - 1 + \frac{b_\lambda^2}{b^2} + \frac{2R\Delta_\lambda}{b^2} - \frac{\lambda b_\lambda R}{b^2}\right] \qquad (151)$$

In the absence of a casing, current I_λ generates inside the liner the field

$$h^0_\lambda = -\frac{I_\lambda}{cR}\left[\ln\frac{8R}{b_\lambda} - \frac{1}{2} - \frac{\lambda R}{b_\lambda}\right] \qquad (152)$$

When $\lambda = (b_\lambda/R)(\ln(8R/b_\lambda) - 1/2)$, the current in the liner is distributed in the same manner as in a superconducting ring (see Section 2.3) and the field h^0_λ is equal to zero.

4.5. EQUILIBRIUM IN A CONDUCTING CASING IN THE PRESENCE OF A LIMITER

In Tokamak devices, a circular aperture limiter is usually fitted inside the liner. The main purpose of the limiter is to protect the liner walls from being burnt by run-away electrons which are formed, as a rule, in each discharge pulse. The interaction between the plasma and the limiter inevitably affects the equilibrium position of the plasma column. This interaction may be of two kinds. First, the plasma may move away from the magnetic surfaces which intersect the limiter. In this case, the equilibrium position is established in such a way that the plasma column will be in contact with the external or (when the magnitude of the control field is sufficiently high) the internal edge of the limiter. This condition can be written in the form of an equality

$$b_d - a = |\Delta - \Delta_d| \qquad (153)$$

where b_d is the radius of the limiter opening and Δ_d the amount of displacement of the centre of the limiter opening in relation to the centre of the casing cross-section. Second, the transverse current (transverse to the torus plane) generated by the toroidal drift of the charges may form a circuit through the conducting limiter. The interaction between this current and the longitudinal magnetic field balances the expanding force of the plasma ring and can, in principle, ensure equilibrium of the plasma even in the absence of longitudinal current I_p [20]. In experiments one may observe both types of interaction between the plasma and the limiter.

4.5.1. Diagram of the equilibrium radii of the column

The first type of interaction between the plasma and the limiter is the one which usually occurs. In this case, when the values of parameters β_I, ℓ_i and Δ_H are fixed, formula (153) determines the equilibrium value of the radius a of the plasma column in contact with the limiter. The curves representing the equilibrium radius as a function of the magnitude of the maintaining field (Fig.17) limit the range of values of the radius a at which the plasma column is in equilibrium without touching the limiter. If for a given control field the plasma column radius is above the shaded area in the diagram, the plasma begins to move away from the column surface and the column radius diminishes to its own equilibrium value (boundary of the shaded area). However, not all equilibrium states are stable. If the current in the plasma diminishes with decreasing radius of the column cross-section, the lower parts of the curves correspond to unstable equilibrium states [21].

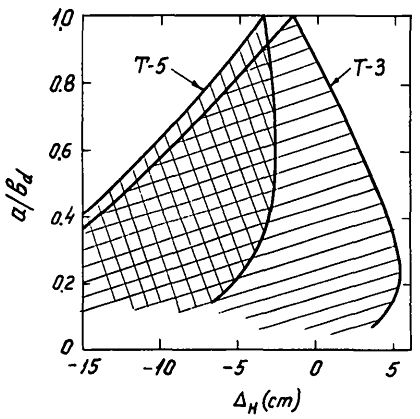

FIG.17. Equilibrium diagram for the T-3 and T-5 devices when $\beta_I + (\ell_i/2) = 0.75$. Inside the shaded area the plasma column is in equilibrium without touching the diaphragm.

The width of the Δ_H equilibrium diagram diminishes with increasing curvature of the torus and with the number of gaps in the casing. In particular, in Tokamak-5 where $R/b = 2.5$ and the longitudinal gaps are situated at $\omega = \pm\pi/2$, the width is noticeably smaller than in Tokamak-3. For this reason, Tokamak-5 shows a fairly clear correlation between the energy lifetime of the plasma τ_E and the presence of the region of equilibrium radii (Fig.18).

When the opening in the limiter is not properly situated, the equilibrium may lie wholly in a

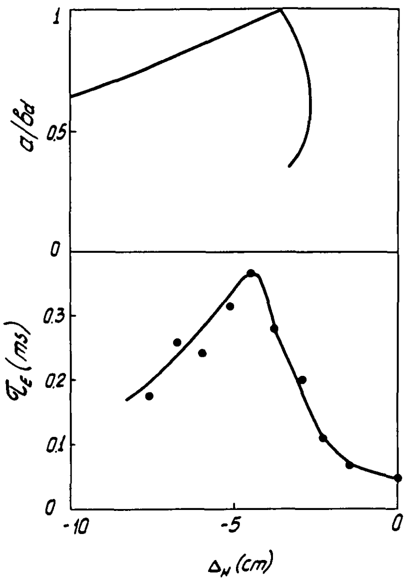

FIG.18. Comparison of the energy lifetime of the plasma τ_E with the equilibrium diagram for the T-5 device.

region with $\Delta_H < 0$. In this case, without introducing the control field, it is not possible to expect a plasma with good parameters.

4.5.2. Effect of currents forming a circuit through the limiter on the equilibrium of the plasma column

The equilibrium position of a plasma column in the presence of current $I_\perp$ forming a circuit along the limiter has been calculated in Ref. [22]. The displacement of the column axis is not uniform along its length and can be written in the form of an expansion into a Fourier series along the longitudinal coordinate. The amplitude of the n-th harmonic component is proportional to $1/n^2$; we can, therefore, confine ourselves with sufficient accuracy to the first term of the expansion only. In this case, for displacement ξ_R in the horizontal plane we obtain

$$\xi_R = \xi_0 + 2k\frac{I_\perp}{I_p^2} f_1 \cos\frac{S}{R} \tag{154}$$

and for vertical displacement

$$\xi_z = -2k\frac{I_\perp}{I_p^2}\frac{f_2}{q}\sin\frac{S}{R} \tag{155}$$

Here

$$f_1 = \frac{a^2}{b^2}\frac{q^2 - 2 - \frac{a^2}{b^2}}{(q^2-1)\left(q^2 - \frac{a^4}{b^4}\right)}$$

$$f_2 = \frac{a^2}{b^2}\frac{q^2\left(1 - \frac{a^2}{b^2}\right) + 2\frac{a^2}{b^2}}{(q^2-1)\left(q^2 - \frac{a^4}{b^4}\right)} \tag{156}$$

$$k = \frac{a(b^2 - a^2)}{8R} cH_\varphi \tag{157}$$

Quantity ξ_0 is the average displacement of the column, which is connected with $I_\perp$ by the relation

$$\xi_0 = \Delta + k\frac{I_\perp}{I_p^2} \tag{158}$$

where Δ is determined by formula (85). As follows from formulae (154) and (156), when $q^2 \gg 1$ the inhomogeneity of displacement along the length can be neglected.

The azimuthal magnetic field averaged over the column length outside the plasma for $I_\perp \neq 0$ is determined by the expression

$$H_\omega(\rho,\omega) = \frac{2I_p}{c\rho}\left\{1 + \frac{\rho}{2R}\left[\ln\frac{\rho}{a} - 1 + \left(\beta_I + \frac{\ell_i - 1}{2} + \frac{acH_\varphi}{4}\frac{I_\perp}{I_p^2}\right)\left(1+\frac{a^2}{\rho^2}\right) + \frac{2R\xi_0}{\rho^2}\right]\cos\omega\right\} \tag{159}$$

whence it follows that the presence of current $I_\perp$ forming a circuit along the diaphragm corresponds to replacement of the asymmetry coefficient Λ(27) by the effective value

$$\Lambda_{\text{eff.}} = \Lambda + \frac{acH_\varphi}{4}\frac{I_\perp}{I_p^2} \tag{160}$$

The effect of the current forming a circuit through the limiter on the equilibrium of the plasma was studied experimentally on the Tokamak-5 device [23]. For values of Δ_H corresponding to values of a/b_d close to unity in the equilibrium diagram (see Fig. 18), the magnitude of this current was small and its effect on the degree of displacement of the column in the main discharge phase could be neglected. For the values of Δ_H at which, according to the equilibrium diagram, the column should have no equilibrium states, there was a sharp rise in the current $I_\perp$. Figure 19 shows the dependence of displacement of the column ξ_R on the ratio $I_\perp/I_p^2$ obtained in special experiments, during which two metal plates were introduced into the upper and lower parts of the plasma column, a voltage being applied between these plates from an external source. This dependence for H_φ = const. is linear in accordance with formula (158), the coefficient of proportionality k between ξ_R and $I_\perp/I_p^2$ being close to that calculated.

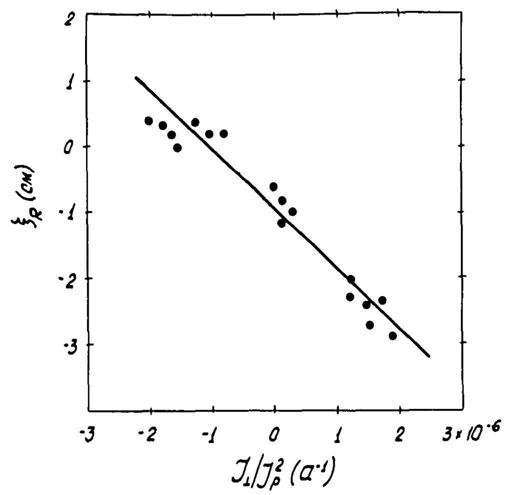

FIG.19. Column displacement ξ_R as a function of the transverse current $I_\perp$ forming a circuit through the diaphragm.

It is interesting to consider the case of a small longitudinal current $8\pi p/H_I^2 \gg 1$. In this case $f_1 = a^2/b^2q^2$, and formula (158) gives

$$k\frac{I_\perp}{I_p^2} = -\frac{b^2}{2R}\left(1 - \frac{a^2}{b^2}\right)\frac{8\pi p}{H_I^2} \tag{161}$$

The expression for ξ_R (154) reduces to the form

$$\xi_R = \xi_0 - R\frac{8\pi p}{H_\varphi^2}\left(1 - \frac{a^2}{b^2}\right)\cos\frac{S}{R} \tag{162}$$

This formula shows that, when $8\pi p/H_\varphi^2 \ll 1$, circuit formation by currents through the limiter (or through the liner walls) can ensure equilibrium of the plasma without a longitudinal current. This containment mechanism cannot be satisfactory for a high-temperature plasma, since it is invariably associated with plasma loss on the limiter. However, it appears to play a substantial role in the initial stage of development of the discharge in a Tokamak, when the longitudinal current is small and does not ensure maintaining.

4.6. EQUILIBRIUM IN THE APPROXIMATION OF A THIN CASING

After the dipole component of a magnetic field diffuses into the thickness of the casing wall, a homogeneous conduction current with the following equivalent surface density is generated in the wall:

$$i_k = \sigma_k E_\varphi d_k = \frac{\sigma_k d_k}{2\pi Rc}\frac{\partial\psi_1^{II}(b)}{\partial t} \tag{163}$$

where ψ_1^{II} is the non-symmetrical part of the flux function ψ^{II}.

The corresponding additional transverse field and, consequently, column displacement even at invariable plasma parameters will be functions of time.

We obtain the equation describing the behaviour of the plasma column through determining constants C_1 and C_2 from the conditions of continuity of the normal component on surfaces $\rho = b_1$, $\rho = b$, $\rho = b_2$, $\rho = b_3$, from the conditions for tangential components (their difference is due to the corresponding surface current) and from the conditions on the core, which we referred to earlier - continuity of the tangential component and the fact that the normal component inside the core becomes zero.

The values of constants C_1^I, C_2^I and C_1^{II}, C_2^{II} are given above (see formulae (81), (82), (113) and (114)). The constants for regions III, IV, and V are obtained in the form

$$C_1^{III} = -\frac{2\pi I_p}{c} a^2\left(\frac{2R\Delta}{a^2} + \Lambda + \frac{1}{2}\right) - \frac{4\pi^2 R}{c}(i_1 b_1^2 + i_k b^2) \quad (164)$$

$$C_2^{III} = \frac{2\pi I_p}{c}\left(\ln\frac{8R}{a} + \Lambda - \frac{1}{2}\right) + \frac{4\pi^2 R}{c}(i_1 + i_k) \quad (165)$$

$$C_1^{IV} = -\frac{2\pi I_p}{c} a^2\left(\frac{2R\Delta}{a^2} + \Lambda + \frac{1}{2}\right) - \frac{4\pi^2 R}{c}$$

$$\times (i_1 b_1^2 + i_k b^2 + i_2 b_2^2) \quad (166)$$

$$C_2^{IV} = \frac{2\pi I_p}{c}\left(\ln\frac{8R}{a} + \Lambda - \frac{1}{2}\right) + \frac{4\pi^2 R}{c}(i_1 + i_k + i_2) \quad (167)$$

$$C_1^{V} = -\frac{2\pi I_p}{c} a^2\left(\frac{2R\Delta}{a^2} + \Lambda + \frac{1}{2}\right) + \frac{\pi I_0}{c} b_3^2$$

$$-\frac{4\pi^2 R}{c}\left(i_k b^2 + \sum_{e=1}^{3} i_e b_e^2\right) \quad (168)$$

$$C_2^{V} = \frac{2\pi I_p}{c}\left(\ln\frac{8R}{a} + \Lambda - \frac{1}{2}\right) - \frac{2\pi I_0}{c}\left(\ln\frac{8R}{b_3} - \frac{1}{2}\right)$$

$$+\frac{4\pi^2 R}{c}\left(i_k + \sum_{e=1}^{3} i_e\right) \quad (169)$$

If the effect of the core can be neglected, constant C_2^V should be made equal to zero (since C_2^V characterizes the field at infinity). Substituting the expression for i_k into condition $C_2^V = 0$, we obtain the equation describing the behaviour of the plasma column in a thin casing

$$\frac{d}{dt} I_p(\Delta - \Delta_1^{**}) = \frac{1}{\tau_k}\frac{cb^2}{2}(H_{\perp 0} - h_{ei}^0 - h_{ii}^0) \quad (170)$$

where

$$\Delta_1^{**} = \Delta_0 - \frac{cb^2}{2I_p} h_{ii}^{\infty} \quad (171)$$

h_{ei}^0 is the field of the external sources in the absence of a casing, $H_{\perp 0}$, h_{ii}^0 and h_{ii}^{∞} are determined by formulae (37), (94) and (99), respectively, and τ_k is the characteristic time of leakage of the magnetic field through the casing

$$\tau_k = \frac{2\pi\sigma_k d_k b}{c^2} \quad (172)$$

In the displacement equation the sign of the derivative d/dt is considered to cover the field h_{ii}^{∞} of the internal control conductors on the assumption of ideal conductivity of the casing, while the right-hand part contains the sum of the control fields $h_i^0 = h_{ii}^0 + h_{ei}^0$, which would occur in the absence of a casing (when they have the same sources).

Formula (170) shows that a thin casing can be equivalent to an ideally conducting one if the total control field in the absence of a casing h_i^0 is chosen equal in accuracy to the equilibrium field $H_{\perp 0}$.

The penetration of the field of the external conductors inside the casing is described by the equation

$$\frac{dh_{ei}}{dt} + \frac{h_{ei}}{\tau_k} = \frac{h_{ei}^0}{\tau_k} \quad (173)$$

The control field of the internal conductors is governed by the equation

$$\frac{dh_{ii}}{dt} + \frac{h_{ii}}{\tau_k} = \frac{dh_{ii}^{\infty}}{dt} + \frac{h_{ii}^0}{\tau_k} \quad (174)$$

Using these relations, we can re-write the equation of motion of a plasma column (170) in other equivalent forms

$$\frac{d}{dt} I_p(\Delta - \Delta_1^{*}) = \frac{1}{\tau_k}\frac{cb^2}{2}[H_{\perp 0} - h_{ii} - h_{ei}^0] \quad (175)$$

where

$$\Delta_1^{*} = \Delta_0 - \frac{cb^2}{2I_p} h_{ii} \quad (176)$$

or

$$\frac{d}{dt} I_p(\Delta - \Delta_1) = \frac{1}{\tau_k}\frac{cb^2}{2}[H_{\perp 0} - h_i] \quad (177)$$

where

$$\Delta_1 = \Delta_0 - \frac{cb^2}{2I_p} h_i \quad (178)$$

h_i being the true additional (control) balancing field inside the casing generated by the internal and external conductors

$$h_i = h_{ii} + h_{ei} \quad (179)$$

The last form of the equation is useful in that the intensity of the control field $h_i(t)$ which goes into it can be measured directly in the experiment without triggering a discharge.

The form for the displacement of the plasma column in a thin casing can be represented usefully as

$$\Delta = \Delta_0 - \frac{cb^2}{2I_p}(h_i + h^{\sigma}) \quad (180)$$

Here the first two terms in the right-hand part represent the amount of displacement in an ideally conducting casing for a given control field h_i and the last term determines the additional displacement associated with leakage of the magnetic field through the casing walls. The additional transverse field h^{σ} occurring during this leakage is governed by the equation

$$\frac{dh^{\sigma}}{dt} = -\frac{1}{\tau_k}(H_{\perp 0} - h_i) \quad (181)$$

which is obtained on substitution of the displacement expression (180) into Eq. (177).

Consideration of the core complicates the picture somewhat. Constant C_2^V should now be determined from the condition on the core. Here, instead of Eq. (170), we obtain

$$\frac{d}{dt} I_p(\Delta - \Delta_1^{**}) = \frac{1}{\tau_k} \frac{b_4^2}{b^2 + b_4^2} \frac{cb^2}{2} (H_{\perp 0} - Q) \tag{182}$$

where field Q, when $b_5 = \infty$ and $b_5 \neq \infty$, is determined respectively by formulae

($b_5 = \infty$)

$$Q = -\frac{I_p}{cR}\left[\frac{a^2}{b_4^2}\left(\Lambda + \frac{1}{2}\right) + \frac{2R\Delta}{b_4^2}\right] + \frac{I_0}{cR}\left[\ln\frac{8R}{b_3} - \frac{1}{2}\left(1 - \frac{b_3^2}{b_4^2}\right)\right] + \frac{I_0 - I_p}{cR}\left(\ln\frac{8R}{b_4} - 1 - \frac{2R\Delta_4}{b_4^2}\right) - \frac{2\pi}{c}\sum_{e=1}^{3} i_e\left(1 + \frac{b_e^2}{b_4^2}\right) \tag{183}$$

($b_5 \neq \infty$)

$$Q = -\frac{I_p}{cR}\left[\frac{a^2}{b_4^2}\left(\Lambda + \frac{1}{2}\right) + \frac{2R\Delta}{b_4^2}\right] + \frac{I_0}{cR}\left[\ln\frac{8R}{b_3} - \frac{1}{2}\left(1 - \frac{b_3^2}{b_4^2}\right)\right] + \frac{I_0 - I_p}{cR}\left[\frac{2b_5^2}{b_5^2 - b_4^2}\ln\frac{b_5}{b_4} - \ln\frac{8R}{b_4} + 1 - \frac{2R\Delta_4}{b_4^2} + \frac{4R(\Delta_5 - \Delta_4)}{b_5^2 - b_4^2}\right] - \frac{2\pi}{c}\sum_{e=1}^{3} i_e\left(1 + \frac{b_e^2}{b_4^2}\right) \tag{184}$$

By combining a thin casing with an external maintaining field, we can solve the problem of prolonged stable maintaining of the plasma in equilibrium. The presence of the casing ensures stability of the equilibrium position in relation to rapidly changing disturbances, which would technically be difficult to balance by only one external field. As far as slow disturbances are concerned, they can be dealt with by appropriate selection of the geometry of the maintaining field, as has been discussed in Section 3.1, and by automatic regulation of the magnitude of this field with the help of feedbacks.

We shall now deal with the criterion of applicability of the approximation of a thin casing. This approximation is valid when the dipole current in the thickness of the casing wall becomes roughly homogeneous. Since the penetration of the magnetic field occurs from the internal as well as the external side owing to the presence of gaps, it is sufficient, to balance the current, if the field diffuses only into half the thickness of the casing wall. This will occur in time

$$t_0 \simeq \frac{4\pi\sigma_k}{c^2}\left(\frac{d_k}{2}\right)^2 = \frac{\pi\sigma_k d_k^2}{c^2} \tag{185}$$

Thus, the approximation of a thin casing becomes applicable in time $t \gg t_0$, counting from the beginning of the process. Figure 20, taken from Ref. [24][5] compares the results of calculating displacement in the approximation of a thin casing (broken lines) and the accurate solution of the diffusion equation for the dipole component of the magnetic field in the casing (solid lines).

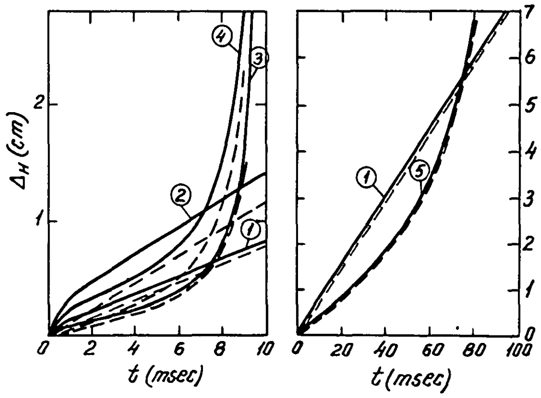

FIG. 20. Additional displacement of the plasma column caused by the damping of currents in the casing (solid lines). The broken line represents the same in the approximation of a thin casing. Calculations were carried out for the parameters of the T-5 device when $\ell_i = \frac{1}{2}$, a/b = 3/5.
(1) I_p = const., $\beta_I = 0$;
(2) I_p = const., $\beta_I = 1$;
(3) I_p = const. $\sin(2\pi/T)t$, T = 20 ms, $\beta_I = 0$;
(4) T = 20 ms, $\beta_I = 1$;
(5) T = 200 ms, $\beta_I = 0$.

Calculations were performed for the parameters of Tokamak-5 (R = 62.5 cm, b = 25 cm, d_k = 1.6 cm, $\sigma_k = 5 \times 10^{17}$ CGSE units), when $t_0 \simeq$ 5 ms. As can be seen from the figure, the relative error of the approximate solution for $t > t_0$ is less than 30%.

4.7. DETERMINATION OF THE CHARACTERISTICS OF A PLASMA COLUMN FROM MEASUREMENTS OF THE MAGNETIC FIELDS OUTSIDE THE PLASMA

Relations (5), (13) and (85) enable us, from the measurements of the magnetic fields outside the plasma, to determine such characteristics of a plasma column as the value of parameter β_I, the amount of column displacement Δ and the value of its inductance

$$L = 4\pi R\left(\ln\frac{b}{a} + \frac{\ell_i}{2}\right) \tag{186}$$

[5] In Ref. [24], where Fig. 20 was displayed, it was incorrectly stated, that it applies to T-3 instead of T-5.

Parameter β_I is determined from the pressure-balance equation (5) on the basis of measurements of the diamagnetic flux $\delta\Phi$ of the longitudinal magnetic field in the column cross-section. This equation, with allowance for column displacement Δ, is written in the form

$$\beta_I = 1 + \frac{c^2}{2\pi I_p^2} H_{\varphi e}(R)\left(1 - \frac{\Delta}{R}\right)\delta\Phi \qquad (187)$$

The combination $((L/4\pi R) + \beta_I)$ and Δ can be expressed in terms of the experimentally determined asymmetrical component of the poloidal magnetic field H_ω. In accordance with formula (13), the amplitude of the asymmetrical component $H_1(\rho)$ in region II, i.e. in the space between the internal loops and the casing, is

$$H_1^{II}(\rho) = -\frac{I_p}{cR}\ln\frac{8R}{\rho} + \frac{1}{2\pi R}\left(C_2^{II} - \frac{C_1^{II}}{\rho^2}\right) \qquad (188)$$

Substituting into this formula the values of constants C_1^{II} and C_2^{II} from formulae (113) and (114), we obtain

$$H_1^{II}(\rho) = \frac{I_p}{cR}\left[\frac{2R\Delta}{\rho^2} + \left(\Lambda + \frac{1}{2}\right)\left(1 + \frac{a^2}{\rho^2}\right) + \ln\frac{\rho}{a} - 1\right] + \frac{2\pi}{c} i_1\left(1 + \frac{b_1^2}{\rho^2}\right) \qquad (189)$$

This expression for H_1^{II} is valid in the case both of an ideal and a non-ideal casing. The difference between the two cases is contained in the expression for Δ. Quantity H_1^{II} can be conveniently determined with the help of a pair of identical magnetic probes placed in the equatorial plane of the torus at equal distances from the centre of the casing cross-section [25]. If the magnetic field generated by the control loops were purely dipolar, the difference between the signals of these probes, after integration, would give a value proportional to $H_1^{II}(\rho)$. In fact, however, the field of the control loops at the place where the probes are located differs considerably from a dipole field, since it is generated by concentrated currents, while the probes are usually located not far from these currents. This difficulty can be overcome if we exclude from the signal being measured the contribution due to the field of the control loops. Accordingly, the dipole component of this field should be subtracted from the expression for H_1^{II}. In this case, the ratio of the resulting difference signal of probes u_1 to the sum of the signals of probes u_2, according to formula (189), will be

$$\frac{u_1}{u_2} = \frac{b_m}{2R}\left[\frac{2R\Delta}{b_m^2} + \left(\Lambda + \frac{1}{2}\right)\left(1 + \frac{a^2}{b_m^2}\right) + \ln\frac{b_m}{a} - 1\right] + \frac{cb_m}{2I_p}\left[\frac{2\pi}{c} i_1\left(1 + \frac{b_1^2}{b_m^2}\right) - h_{ie}(b_m)\right] \qquad (190)$$

Here b_m is half of the distance between the probes and $h_{ie}(b_m)$ is the dipole component of the field of the control loops for $\rho = b_m$. Substituting the expression for Δ into this formula, we obtain the relationship between the combination $((L/4\pi R) + \beta_I)$ and quantity u_1/u_2. In the case of an ideal casing, for example, we have

$$\frac{L}{4\pi R} + \beta_I = \frac{1}{2} + \frac{2Rb_m}{b^2 + b_m^2}\frac{u_1}{u_2} + \frac{b_m^2}{b^2 + b_m^2}\left(1 + \ln\frac{b}{b_m}\right) - \frac{2R}{b^2}\Delta_H \qquad (191)$$

The amount of displacement of the column, expressed in terms of the magnetic probe readings, takes the form

$$\Delta = \frac{b_m(b^2 - a^2)}{b^2 + b_m^2}\frac{u_1}{u_2} + \frac{a^2}{b^2}\Delta_H + \frac{a^2}{2R}\ln\frac{b}{a} + \frac{b_m^2}{2R}\frac{b^2 - a^2}{b^2 + b_m^2}\left(1 + \ln\frac{b}{b_m}\right) \qquad (192)$$

Here

$$\Delta_H = -b\frac{h_{ii}^\infty}{H_\omega(b)} = \frac{cb^2}{2I_p}\frac{2\pi}{c} i_1\left(1 - \frac{b_1^2}{b^2}\right)$$

is the additional displacement of the column due to the field of the control loops. Formulae (191) and (192) remain valid also when the plasma column volume has a constant transverse field $H_\perp$ generated before the beginning of the discharge. In this case, $\Delta_H = -b(h_{ii}^\infty + H_\perp)/H_\omega(b)$. Formulae (191) and (192) retain their form also when the probes measuring the field H_ω are located in region I, i.e. in the space between the plasma column and the control loops.

Determining the inductance of column L from expressions (187) and (191), we can calculate the effective active resistance $R_{eff.}$ of the plasma column and the rate of Joule energy release in the plasma

$$Q_I = \int\frac{j_\varphi^2}{\sigma_\parallel}d\tau = I_p^2 R_{eff.} \qquad (193)$$

with the help of the relation

$$Q_I = I_p V - \frac{d}{dt}\left(\frac{LI_p^2}{2}\right) \qquad (194)$$

where V is the voltage on the torus surface measured on the casing surface.

Relations (187) and (194) are used for calculating the energy life-time of the plasma [26]

$$\tau_E = \frac{W}{Q_I - \frac{dW}{dt}} \qquad (195)$$

Here W is the thermal energy of the plasma connected with β_I by the relation

$$W = \frac{3}{2}\pi R\frac{I_p^2}{c^2}\beta_I \qquad (196)$$

The expressions for L and Δ given above are obtained without allowance for the effect of the gaps in the casing and of the latter's finite electrical conductivity. In considering the effect of the gaps, there arises, generally speaking, the same difficulty as in the case of the field of the control loops. At the place where the probes are located the additional field due to the gaps is not equal to the dipole component of this field. Furthermore, unlike the field of the control loops, this field cannot be measured in advance so that it could be subtracted from the magnetic probe readings. In fact, however, this is not necessary if the probes are removed from the gap by a distance greater than b. At this distance the field due to the gaps is practically equal to zero, and in order to compare the results of the probe measurements with the calculated formula, it is sufficient to subtract from expression (189) the dipole component of this field determined by formula (118).

We thus obtain

$$\frac{L}{4\pi R} + \beta_I = \frac{1}{2} + \frac{1}{1+\alpha}\left[\frac{2Rb_m}{b^2+b_m^2}\frac{u_1}{u_2} + \frac{b_m^2}{b^2+b_m^2}\left(1+\ln\frac{b}{b_m}\right) - \alpha\left(\ln\frac{8R}{b}-1\right) + \frac{cR}{I_p}\left(h_{ii}^{\infty}+\alpha h_{ii}^{0}+\alpha h_{ei}^{0}\right)\right] \quad (197)$$

$$\Delta = \frac{b_m\left(b^2-\frac{a^2}{1+\alpha}\right)}{b^2+b_m^2}\frac{u_1}{u_2} - \frac{ca^2}{2I_p}\frac{1}{1+\alpha}\left(h_{ii}^{\infty}+\alpha h_{ii}^{0}+\alpha h_{ei}^{0}\right) + \frac{a^2}{2R}\ln\frac{b}{a} + \frac{\alpha}{1+\alpha}\frac{a^2}{2R}\left(\ln\frac{8R}{b}-1\right) + \frac{b_m^2}{2R}\frac{b^2-\frac{a^2}{1+\alpha}}{b^2+b_m^2}\left(1+\ln\frac{b}{b_m}\right) \quad (198)$$

Here α denotes the quantity $2\mu b/(2\pi R-\mu b)$. These formulae contain, in addition to the values of fields h_{ii}^{∞} and h_{ii}^{0} determined by the currents in the internal control loops, the value of the field h_{ei}^{0} generated by the external sources. The value of h_{ei}^{0} should be determined experimentally. It is difficult to determinate it by calculation, since it depends on the characteristics of the transformer core. In order to evaluate h_{ei}^{0}, we can use formulae (135), (140) and (145) obtained on the assumption that the core has the shape of an axisymmetrical toroidal shell and possesses an infinite magnetic permeability.

In this case of a thin casing ($t > \pi\sigma_k d_k^2/c^2$), the quantities $(L/4\pi R)+\beta_I$ and Δ are determined in terms of u_1/u_2 with the help of the usual differential equations which, without allowance for the effect of the gaps, at a = const., takes the form

$$\frac{d}{dt}I_p\left(\frac{L}{4\pi R}+\beta_I\right) + \frac{b^2}{b^2+b_m^2}\frac{I_p}{\tau_k}\left(\frac{L}{4\pi R}+\beta_I\right) = \frac{d}{dt}I_p\left[\frac{1}{2}+\frac{2Rb_m}{b^2+b_m^2}\frac{u_1}{u_2}+\frac{cR}{I_p}h_{ii}+\frac{b_m^2}{b^2+b_m^2}\left(\ln\frac{b}{b_m}+1\right)\right] - \frac{I_p}{\tau_k}\frac{b^2}{b^2+b_m^2}\left[\ln\frac{8R}{b}-\frac{3}{2}-\frac{cR}{I_p}\left(h_{ii}+h_{ei}^{0}\right)\right] \quad (199)$$

$$\frac{d}{dt}I_p\Delta + \frac{b^2}{b^2+b_m^2}\frac{I_p}{\tau_k}\Delta = \frac{d}{dt}I_p\left[\frac{b_m(b^2-a^2)}{b^2+b_m^2}\frac{u_1}{u_2} - \frac{ca^2}{2I_p}h_{ii} + \frac{a^2}{2R}\ln\frac{b}{a} + \frac{b_m^2}{2R}\frac{b^2-a^2}{b^2+b_m^2}\left(1+\ln\frac{b}{b_m}\right)\right] + \frac{b^2}{b^2+b_m^2}\frac{I_p}{\tau_k}\left[b_m\frac{u_1}{u_2} - \frac{c(b_m^2+a^2)}{2I_p}h_{ei}^{0} - \frac{ca^2}{2I_p}h_{ii} + \frac{a^2}{2R}\ln\frac{b}{a} + \frac{b_m^2}{2R}\left(1+\ln\frac{b}{b_m}\right) + \frac{b_m^2+a^2}{2R}\left(\ln\frac{8R}{b}-1\right)\right] \quad (200)$$

4.8. PENETRATION OF THE CORRUGATIONS OF THE LONGITUDINAL MAGNETIC FIELD INSIDE A CONDUCTING CASING

When the coils of a longitudinal magnetic field are discretely located, the strength of the field is inhomogeneous along the length of the plasma column. In an inhomogeneous field the conditions of plasma maintaining are apparently worsened. This occurs, firstly, because of the appearance of additional regions of poorly maintained trapped particles. Furthermore, the modulation of the curvature of the magnetic lines of force resulting from the inhomogeneity of the field reduces the stability of the plasma. For example, the condition necessary for the stability of flute disturbances in the vicinity of the magnetic axis in a corrugated longitudinal field $H_\varphi = H_0/(1+\kappa\sin N\varphi)^2$ has the form

$$\left(\frac{2\pi j_0 R}{cH_0}\right)^2 < 1 - \frac{3}{2}\kappa^2 N^2 \quad (201)$$

where j_0 is current density on the axis.

In order to determine experimentally how the inhomogeneity of the longitudinal magnetic field affects plasma stability and the rate of plasma escape, we can deliberately corrugate the longitudinal field. In practice, this can be conveniently done by including the corrugated field outside the plasma. Here arises the question of the time of penetration of this field inside the casing. Neglecting, for the sake of simplicity, the curvature of the torus, we find that the applied corrugated longitudinal field $H_{\varphi e}$ can be represented in the form of superposition of harmonic components

$$H_{\varphi n}^{e} = H_0 I_0\left(\frac{r}{R}n\right)\cos n\varphi \quad (202)$$

In the approximation of a thin casing, the solution for the field inside the casing takes the form (for the given harmonic component)

$$H_{\varphi n}^{i} = H_0\left(1-e^{-\frac{t}{\tau_n}}\right)I_0\left(\frac{r}{R}n\right)\cos n\varphi \quad (203)$$

The characteristic time of penetration of the n-th harmonic component is

$$\tau_n = \frac{4\pi\sigma_k d_k R}{c^2 n}\,\xi\left(\frac{b}{R}n\right) \qquad (204)$$

where

$$\xi(x) = xK_1(x)\,I_1(x)$$

For a small number of corrugations ($n \ll R/b$) $\xi = bn/2R$ and

$$\tau_n = \tau_k = \frac{2\pi\sigma_k d_k b}{c^2} \qquad (205)$$

In the opposite case of short corrugations ($(b/R)n \gg 1$) $\xi = 1/2$, so that

$$\tau_n = \frac{2\pi\sigma_k d_k R}{c^2 n} = \tau_k \frac{R}{nb} \ll \tau_k \qquad (206)$$

5. SOME NON-STANDARD CASES OF EQUILIBRIUM

5.1. EQUILIBRIUM IN A RACETRACK

In racetrack-type systems having no axial symmetry, both the amount and the direction of displacement of a plasma column inside a conducting casing change along the length of the column. The displacement vector has components along the principal normal $\vec{\nu}$ and the binormal $\vec{\beta}$ to the casing axis

$$\vec{\xi} = -\xi_\nu \vec{\nu} + \xi_\beta \vec{\beta} \qquad (207)$$

If the system has a second-order symmetry axis, displacement ξ_ν in the equatorial plane can be written in the form

$$\xi_\nu = \xi_{\nu 0} + 2\sum \xi_{\nu m}\cos\frac{4\pi}{L}ms \qquad (208)$$

and bi-normal displacement as

$$\xi_\beta = 2\sum \xi_{\beta m}\sin\frac{4\pi}{L}ms \qquad (209)$$

Here s is the length of the arc of the casing axis (the origin of the frame of reference is chosen at the intersection of the axis with the symmetry plane), and L the total length of the axis. The formulae for $\xi_{\nu m}$ and $\xi_{\beta m}$ can be obtained using the results in Ref. [27]. These formulae are

$$\xi_{\nu m} = \frac{k_m b^2}{2(1-4m^2q^2)(1-4m^2q_b^2)}\Bigg\{(1-4m^2q^2)\ln\frac{b}{a} + \Bigg[\left(\beta_I + \frac{\ell_i - 1}{2}\right)\left(1 + \frac{4m^2b^2}{a^2}q^2\right) - 4m^2q^2\left(\frac{b^2}{a^2} - 1\right)\frac{\langle\rho H_\omega\rangle}{a\,H_\omega(a)} + 4m^2q^2\left(\frac{3}{4}\frac{b^4}{a^4} - 3\frac{b^4}{a^4}m^2q^2 - \frac{b^2+a^2}{4a^2}\right)\Bigg]\left(1 - \frac{a^2}{b^2}\right)\Bigg\} \qquad (210)$$

$$\xi_{\beta m} = \frac{k_m b^2}{2(1-4m^2q^2)(1-4m^2q_b^2)}\,2mq\Bigg\{\frac{b^2}{a^2}(1-4m^2q^2)\times\ln\frac{b}{a} + \Bigg[\left(\beta_I + \frac{\ell_i}{2}\right)\left(1 + \frac{b^2}{a^2}\right) - \left(4m^2q^2\frac{b^2}{a^2} - 1\right)\times\frac{\langle\rho H_\omega\rangle}{a\,H_\omega(a)} - \frac{3}{4} + \frac{b^2}{4a^2} - 4m^2q^2\frac{b^2}{a^2}\Bigg]\left(1 - \frac{a^2}{b^2}\right)\Bigg\} \qquad (211)$$

Here k_m represents the coefficients of the Fourier expansion of the curvature of axis k(s)

$$k(s) = k_0 + 2\sum_{m=1}^{\infty} k_m \cos\frac{4\pi}{L}ms \qquad (212)$$

In the case of a racetrack with bends having a radius of curvature R

$$k_m = \frac{1}{\pi m R}\sin\frac{2\pi^2 mR}{L} \qquad (213)$$

Quantities k_0, q and q_b are determined by formulae

$$k_0 = \frac{2\pi}{L} \qquad (214)$$

$$q = \frac{2\pi a H_s}{LH_\omega(a)}, \quad q_b = \frac{2\pi b H_s}{LH_\omega(b)} = q\frac{b^2}{a^2} \qquad (215)$$

The average amount of displacement is described by a formula similar to that for displacement in a torus with the radius of curvature replaced by $L/2\pi$:

$$\xi_{\nu 0} = \frac{k_0 b^2}{2}\left[\ln\frac{b}{a} + \left(\beta_I + \frac{\ell_i - 1}{2}\right)\left(1 - \frac{a^2}{b^2}\right)\right] \qquad (216)$$

We also introduce the formulae describing the amount of displacement of the magnetic axis in relation to the centre of the magnetic surface with radius ρ in the simplest case of a homogeneous current

$$\xi_0 = \frac{k_0\rho^2}{2}\left(\beta_I + \frac{1}{4}\right) \qquad (217)$$

$$\xi_{\nu m} = \frac{k_m\rho^2}{2}\left[\beta_I\frac{1+4m^2q^2}{(1-4m^2q^2)^2} + \frac{1}{4}\,\frac{1+12m^2q^2}{1-4m^2q^2}\right] \qquad (218)$$

$$\xi_{\beta m} = \frac{k_m\rho^2}{2}\left[4\beta_I\frac{mq}{(1-4m^2q^2)^2} + \frac{2mq}{1-4m^2q^2}\right] \qquad (219)$$

From formulae (210) and (211) it is seen that the distortion of the shape of the plasma column is large only when $q<1$. However, in the practically important case where $q \gtrsim 1$, we can neglect terms of type $1/4m^2q^2$ in comparison with unity. Here the series in the displacement formulae are summed, and we obtain the simple result

$$\xi_\nu = \xi_{\nu 0} - \frac{3}{8}[k(s) - k_0](b^2 - a^2) \tag{220}$$

$$\xi_\beta = -\frac{aH_\omega(a)}{2H_s}\left[\ln\frac{b}{a} + \left(1 - \frac{a^2}{b^2}\right)\left(1 + \frac{\langle\rho H_\omega\rangle}{aH_\omega(a)}\right)\right] \times \int_0^s [k(s) - k_0]\,ds \tag{221}$$

As can be seen, displacement ξ_ν is smaller where the curvature is greater (the axis of the plasma column tends to assume the shape of a circle). It follows from the formulae for vertical displacement that the column axis extends outside the plane. It rises slightly at two opposite places and drops at two other places where the straight sections of the racetrack join the curved ones, so that its lateral projection takes the shape of a figure 8.

5.2. A PLASMA COLUMN WITH RUN-AWAY ELECTRONS

The case of anisotropy of plasma pressure $p_\parallel \gg p_\perp$ can actually occur if a small group of 'run-away' electrons attains velocities close to that of light. Under this condition, the whole longitudinal current may be transported by the accelerated electrons. The number of electrons needed for this purpose is comparatively small. Their linear density is determined by formula

$$N = \frac{I_p}{ev} = \frac{10^9 I_a}{4.8}\frac{c}{v} \tag{222}$$

where I_a is current expressed in amperes. For example, when $I_a = 5\times 10^4$, $v = c$, we have $N \simeq 10^{13}$ cm^{-1}, whereas the linear density of plasma in Tokamaks is usually about 10^{15} - 10^{16} cm^{-1}. Apparently, most of the current is carried by run-away electrons in the experiments described in Refs [17, 28]. The average longitudinal pressure of the run-away electrons moving with velocity v is

$$\langle p_\parallel \rangle = \frac{m\langle n\rangle v^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{223}$$

so that

$$\beta_\parallel = \frac{2c^2\langle p_\parallel\rangle \pi a^2}{I_p^2} = \frac{2I_c}{I_p}\frac{E}{mc^2}\frac{2 + \frac{E}{mc^2}}{1 + \frac{E}{mc^2}} \tag{224}$$

where E is the kinetic energy of the run-away electrons, $I_c = mc^3/e \approx 1.7\times 10^4$ A. If the strength of the current in the plasma I_p is not too high in comparison with I_c, quantity $\beta_\parallel$ can attain values $\gtrsim 1$ at electron energy $E \sim mc^2$. It is possible that the anomalously large displacement of the column at a low value of $\beta_\perp$ described in Ref. [29] is due to the above-mentioned effect (see also Ref. [30]).

The existence of runaway electrons transporting the most part of longitudinal current was observed in experiments on T-5, T-6 Tokamak devices.

5.3. EQUILIBRIUM AT HIGH PLASMA PRESSURE

It was assumed in the foregoing that the asymmetry of distribution of the magnetic field in the cross-section of a plasma column was small, i.e. according to formula (26), $(a/R)\Lambda \ll 1$, or

$$\beta_I + \frac{\ell_i}{2} \ll \frac{R}{a} \tag{225}$$

Under this condition, the magnitude of the maintaining transverse field is small in comparison with the field of the current on the plasma column surface $H_{\perp 0} \sim (a/R)H_I$. With increasing distance from the column surface the current field diminishes and may become equal to the maintaining field. Since these fields on the inner side of the torus have opposite directions, here occurs the zero point of the total poloidal field which determines the position of the separatrix of the magnetic surfaces (see Fig. 4). With increasing plasma pressure the field $H_{\perp 0}$ needed for maintaining increases and the zero point approaches the plasma column. The value of $\beta_I = \beta_I^c$ at which the separatrix coincides with the plasma column boundary will be called critical. The critical value β_I^c depends on current distribution in the configuration. We shall consider a very simple model of a plasma column with surface current. In this case, the plasma pressure p = const. right up to the column boundary, where it abruptly drops to zero. At the boundary the following pressure-balance equation should be satisfied

$$H_\omega^2 = 8\pi p - (H_{\varphi e}^2 - H_{\varphi i}^2)\frac{r_0^2}{r^2} \tag{226}$$

Here $H_{\varphi e}$ and $H_{\varphi i}$ are respectively the intensity of the external and internal longitudinal fields for $r = r_0$. The maximum pressure is determined from the condition that the current field H_ω becomes zero on the internal surface of the torus when $r = r_{min.}$. This condition leads to the relationship between $p_{max.}$ and the pressure of the longitudinal magnetic field

$$H_{\varphi e}^2 - H_{\varphi i}^2 = 8\pi p_{max.}\frac{r_{min.}^2}{r_0^2} \tag{227}$$

Thus, when $p = p_{max.}$, the poloidal field on the column surface is distributed in accordance with the law

$$H_\omega = \sqrt{8\pi p_{max.}}\left(1 - \frac{r_0^2}{r^2}\right)^{\frac{1}{2}} \tag{228}$$

In the case of a column with circular cross-section, assuming $r_0 = R$ and $r = R + a\cos\omega$ and neglecting the value of the ratio a/R in comparison with unity, we obtain

$$H_\omega = 2\sqrt{8\pi p_{max.}\frac{a}{R}}\left|\cos\frac{\omega}{2}\right| \tag{229}$$

From the condition $\oint H_\omega a d\omega = (4\pi/c)I_p$, we find the relationship between $p_{max.}$ and the magnitude of the current, i.e. the critical value β_I^c

$$\beta_I^c = \frac{\pi^2}{16}\frac{R}{a} \tag{230}$$

To investigate how β_I^c depends on the shape of the plasma column cross-section, we apply formula (228) to a column of rectangular cross-section with sides h and d (the first-named being parallel to the symmetry axis). Calculating the circulation of the magnetic field along the perimeter of the cross-section on the assumption that $h/R \ll 1$ and $d/R \ll 1$, we obtain

$$\left(\frac{4\pi}{c}I_p\right)^2 = 8\pi p_{max.}\frac{2d}{R}\left(h + \frac{4}{3}d\right)^2 \tag{231}$$

Hence

$$\beta_I^c = \frac{2c^2 p_{max} hd}{I_p^2} = 2\pi\frac{hR}{\left(h + \frac{4}{3}d\right)^2} \tag{232}$$

For fixed thickness d of the plasma column, β_I^c has the maximum value $\beta_I^c = (3\pi/8)(R/d)$ when $h = (4/3)d$. For a fixed cross-sectional area $S = hd$, the maximum value $\beta_I^c = (9\pi/32)(R/d)$ is obtained for the form of the cross-section elongated along the symmetry axis, with $h = 4d$.

It is to be noted, that in the case of $2(h+d) \gtrsim 2\pi R$ even for a rotation number $q > 1$ the poloidal magnetic field may be comparable with the toroidal field. In these conditions one may hope to achieve stable confinement in Tokamaks for $\beta = 8\pi p/(H_\omega^2 + H_\varphi^2) \sim 1$ [31, 32].

In the case of a plasma column of circular cross-section, on the assumption that current density is homogeneous along the symmetry axis, the function of the magnetic surfaces inside the plasma column is [33-35]

$$\psi_i = -\frac{2\pi R I_p}{c}\left(1 - \frac{\rho^2}{a^2}\right)\left[1 - \frac{\rho}{R}\left(\beta_I + \frac{1}{4}\right)\cos\omega\right] \tag{233}$$

From formula (7) we obtain the expression for field H_ω on the plasma column surface

$$H_\omega = H_I(1 + \lambda\cos\omega) \tag{234}$$

where

$$\lambda = \frac{a}{R}\left(\beta_I - \frac{3}{4}\right) \tag{235}$$

The magnetic field becomes zero on the internal surface when $\lambda = 1$. Hence $\beta_I^c = (R/a) + 0.75$. A more accurate calculation with allowance for terms of the order of (ρ^2/R^2) in the expression for ψ_i gives the formula

$$\beta_I^c = \frac{R}{a} + 0.5 \tag{236}$$

As numerical calculations show, this formula is valid down to $a/R = 0.7$. Thus, for the chosen form of current distribution the simplest dependence of field H_ω on angle ω (26) remains unchanged without the assumption that coefficient $\lambda = (a/R)\Lambda$ is small. Consequently, expression (32) for flux function outside the plasma column also remains valid

$$\psi_e = -\frac{4\pi R}{c}I_p\left(\ln\frac{8R}{\rho} - 2\right) + \frac{2\pi I_p}{c}\left[\ln\frac{\rho}{a} + \left(\beta_I - \frac{1}{4}\right)\left(1 - \frac{a^2}{\rho^2}\right)\right]\rho\cos\omega \tag{237}$$

From conditions $\partial\psi/\partial\rho = 0$, $\partial\psi/\partial\omega = 0$ we determine the points at which the poloidal field is equal to zero, i.e. the position of the magnetic axis and that of the zero point of the separatrix. The magnetic axis is displaced outwards in relation to the centre of the plasma column cross-section by distance [34, 35]

$$\Delta = R\frac{\sqrt{1 + 3\left(\beta_I + \frac{1}{4}\right)^2\frac{a^2}{R^2}} - 1}{3\left(\beta_I + \frac{1}{4}\right)} \tag{238}$$

This formula coincides with the approximate formula (217) only if $(\beta_I + 1/4)(a/R) \ll 1$. Displacement of the magnetic axis at the critical pressure is

$$\Delta_{max.} \simeq \frac{a}{3} \tag{239}$$

The approximate formula (217) gives a higher value of $\Delta_{max.} \simeq a/2$.

The zero point of the separatrix is situated on the internal side of the column at distance

$$\rho_s = R\frac{1 + \sqrt{1 - \frac{a^2}{R^2}\beta_I^2}}{\beta_I} \tag{240}$$

from the column axis. This formula is written on the assumption that $\rho_s/R \ll 1$.

It should be borne in mind that the presence of critical β_I, generally speaking, does not mean that the value of the permissible plasma pressure in the Tokamak has a limit. In fact, let a plasma column be generated inside an ideal casing. If the plasma is heated sufficiently rapidly so that the condition of 'freezing' of the magnetic field in the plasma is satisfied, then the topology of the magnetic configuration cannot be disturbed, i.e. even on unlimited increase of pressure the topology of the enclosed toroidal magnetic surfaces with one magnetic axis theoretically remains unchanged. Equilibrium in this case is ensured by the compression of the magnetic flux and, consequently, by the increased intensity of the magnetic field between the expanding heated plasma and the casing wall [37] (see Fig.10). The examples of equilibria for arbitrary β_I are given in Refs [38, 39].

5.4. TOKAMAK WITH MAXIMUM PERMISSIBLE PLASMA PRESSURE – 'MAXIMAK'

It is of interest to create a Tokamak system in which during the whole duration of the discharge the thermal energy of the plasma (proportional to the product of the plasma pressure and the cross-sectional area of the column) is at maximum for a given value of current in the plasma, i.e. parameter β_I is equal to its critical value. For the sake of brevity, we shall term a system with maximum β_I a 'Maximak'.

The fundamental difference between a Maximak and an ordinary Tokamak is the presence of a separatrix, which constitutes the true boundary of the plasma. The separatrix acts as a natural diverter [40] which does not disturb the symmetry of the system. The plasma lying outside the separatrix spreads along the lines of force of the magnetic field and can, in principle, be absorbed.

Since the rotational transform angle ι on the separatrix is equal to zero, parameter $q = 2\pi/\iota$ which characterizes the stability of the plasma with current will theoretically be equal to infinity at the plasma-column boundary (in practice the maximum value of parameter q will be finite because of the spread of the region near the separatrix owing to its high sensitivity to disturbance). Thus, in this system the maximum value will be attained not only by parameter β_I but also by parameter q and, consequently, by the shear of the magnetic lines of force.

The actual construction of a Maximak system is a complex experimental problem. One of the difficulties lies in the need for matching the strength of the current I_p in the plasma and the confining magnetic field $H_\perp$ with the values of plasma pressure p and plasma radius a in accordance with formulae (241) and (242)

$$p \simeq \frac{RI_p^2}{2\pi c^2 a^3} \tag{241}$$

$$H_\perp \simeq \frac{I_p}{ca}\left[1 + \frac{a}{R}\left(\ln\frac{8R}{a} + \frac{\ell_i}{2} - \frac{3}{2}\right)\right] \tag{242}$$

Here the problem of stability of a plasma column with a free boundary in relation to screw disturbances arises afresh, since the topology of the system differs fundamentally from that of an ordinary Tokamak (the external magnetic surfaces are open).

5.5. DETERMINATION OF PLASMA COLUMN PARAMETERS FROM MEASUREMENTS OF THE MAGNETIC FIELDS OUTSIDE THE PLASMA FOR AN ARBITRARY SHAPE OF COLUMN CROSS-SECTION

Formula (6), which is used for determining parameter β_I from measurements of the diamagnetic effect of a plasma was obtained in a 'quasi-cylindrical' approximation for low asymmetry of field distribution in the plasma column cross-section. In this approximation, formulae (191), (197) and (199), which are used for determining the self-induction of the plasma column from the asymmetry of the poloidal magnetic field, are valid.

It is of interest to generalize these formulae for the case of a plasma column cross-section of arbitrary shape, and in particular for the case, considered in the preceding section, of the configuration of a Maximak, the external magnetic surfaces of which are not generally represented by a torus. It appears that such a generalization can be performed, using only the condition of small toroidality $\rho/R \ll 1$. Let us assume that the magnetic probes are located around the plasma column on a circle of radius ρ (Fig.21), which in the general case does not, of course, coincide with the magnetic surface. The expressions for the azimuthal H_ω and radial H_ρ fields on this circle, under the condition of symmetry in relation to the equitorial plane, can be written in the form

$$H_\omega = H_I(1 + \lambda_1 \cos\omega + \lambda_2 \cos 2\omega + \lambda_3 \cos 3\omega + \ldots) \tag{243}$$

$$H_\rho = H_I(\mu_1 \sin\omega + \mu_2 \sin 2\omega + \mu_3 \sin 3\omega + \ldots) \tag{244}$$

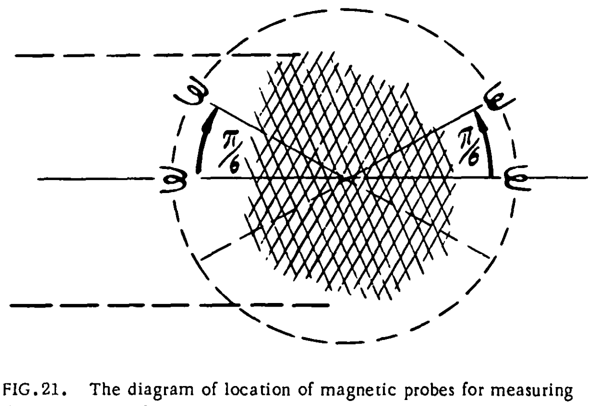

FIG.21. The diagram of location of magnetic probes for measuring the amplitudes of three harmonic components of the poloidal magnetic field.

We shall assume that the circle of radius ρ is not far from the plasma; in this case, coefficients $\lambda_1, \lambda_2, \ldots$ are of the order of unity (or smaller). Then, neglecting the terms of the order of ρ/R in comparison with unity, we can obtain [41]

$$\beta_I = 1 + \frac{c^2 H_{e0}}{2\pi I^2}\delta\phi \tag{245}$$

$$\beta_I + \frac{\ell_i}{2} - 1 = \frac{R}{\rho}\left\{\lambda_1 + \mu_1 + \frac{1}{2}\sum_{k=1}^{\infty}\left[(\lambda_k - \mu_k)\mu_{k+1} + (\lambda_k - \mu_k)\lambda_{k+1}\right]\right\} \tag{246}$$

i.e. the pressure-balance equation (6) remains valid even when $\beta_I \sim R/a$, while the second relation becomes more complicated. In the simplest case, where the probes are located on the surface of an ideal casing, $\mu_1 = \mu_2 = \ldots = 0$, and formula (246) takes the form

$$\beta_I + \frac{\ell_i}{2} - 1 = \frac{R}{a}\left(\lambda_1 + \frac{1}{2}\sum_{k=1}^{\infty}\lambda_k\lambda_{k+1}\right) \tag{247}$$

When the positions of the probes are properly chosen (near the magnetic surface), coefficients λ_k will rapidly decrease with increasing k, and in order to determine the combination $\beta_I + (\ell_i/2)$, it will be sufficient to confine ourselves to a finite number of terms in the sum over k. We assume, for example, that only coefficients λ_1, λ_2 and λ_3 are substantial, whereas coefficients λ_k for $k > 3$ and coefficients μ_k are negligibly small. Placing the probes at $\omega = 0$, π, $\pi/6$ and $5\pi/6$, we obtain

$$\lambda_1 = \frac{H_\omega\left(\frac{\pi}{6}\right) - H_\omega\left(\frac{5\pi}{6}\right)}{\sqrt{3}\,H_I} \tag{248}$$

$$\lambda_2 = \frac{H_\omega(0) + H_\omega(\pi) - H_\omega\left(\frac{\pi}{6}\right) - H_\omega\left(\frac{5\pi}{6}\right)}{H_I} \tag{249}$$

$$\lambda_3 = \frac{H_\omega(0) - H_\omega(\pi)}{2H_I} - \frac{H_\omega\left(\frac{\pi}{6}\right) - H_\omega\left(\frac{5\pi}{6}\right)}{\sqrt{3}\,H_I} \tag{250}$$

$$H_I = H_\omega\left(\frac{\pi}{6}\right) + H_\omega\left(\frac{5\pi}{6}\right) - \frac{H_\omega(0) + H_\omega(\pi)}{2} \tag{251}$$

i.e. to determine the combination $\beta_I + (\ell_i/2)$ in this case it is sufficient to measure field H_ω at four points.

Equation (194), which gives the effective active resistance of the plasma column, retains its form in the case under consideration if L denotes 'internal' inductance $L = 2\pi R\ell_i$ determined by formulae (246) and (247) and V is replaced by $\overline{V}$, where

$$\overline{V} = \frac{c\rho}{4\pi I_p}\int_0^{2\pi} V(\rho,\omega)H_\omega\,d\omega \tag{252}$$

If the circle on which the probes measuring ℓ_i are located coincides with the surface of the ideally conducting casing — a magnetic surface — voltage $V(\rho,\omega) = \text{const.} = V_0$ and formula (252) gives $\overline{V} = V_0$. In the general case, $\overline{V}$ is determined by measuring the distribution of voltage $V(\rho,\omega)$ and of magnetic field $H_\omega(\rho,\omega)$ around the plasma and calculating integral (252).

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