MTP 290 - Problem set 6

1. Calculate the Lagrange interpolating polynomial $p_2(x)$ for the following given values of the function $y = \sin(x)$: $\sin(0) = 0$, $\sin(\pi/4) = 0.70711$ and $\sin(\pi/2) = 1.0$. From $p_2(x)$, find the approximate value of $\sin(\pi/6)$.

2. Find the unique interpolating polynomial $p_3(x)$ of degree less or equal to 3 of a function $f(x)$, that agrees with the following data: $f(0) = 0$, $f(1) = 1$, $f(2) = 8$, $f(4) = 64$.
   Then use $p_3(x)$ to find the approximate value of $f(3)$.

3. Evaluate the integral $\int_0^4 (x^2 + cosx)\ dx$ by using midpoint formula.

4. Use Trapezoidal rule with $n = 8$ to estimate

$$\int_1^5 \sqrt{1 + x^2}\ dx.$$

5. The following points were found empirically.

x	2.1	2.4	2.7	3.0	3.3	3.6
y	3.2	2.7	2.9	3.5	4.1	5.2

   Use composite Trapezoidal rule to evaluate $\int_{2.1}^{3.6} y\ dx$.

6. Approximate the integral of f(x)=x^3 on the interval [1,2] by using composite trapezoidal method
   (a) with four sub intervals,
   (b) with eight sub intervals,
   (Which approximation is much closer to the correct answer)
   (c) Compute the true error in both the cases.

7. Use Simpson rule to evaluate
   (a) $\int_0^{\pi/3} cos^2x\ dx$.
   (b) Use your answer to part (a) to deduce an approximate value of integral $\int_0^{\pi/3} sin^2x\ dx$.

8. Use Simpson's rule with 5 ordinates to find an approximate value for the integral

$$\int_4^6 \frac{1}{3 - \sqrt{x}}\ dx.$$

9. Redo Problem 6 by using Composite Simpson's rule.

10. Using Trapezoidal Rule and Simpson's rule with n=4 to approximate the value of the following integral and compute the true errors and approximation errors

$$\int_0^2 e^{x^2}\ dx.$$

11. Evaluate the following integral by using one point Gauss quadrature and compute the true error.

$$\int_0^{\pi/2} x\ sinx\ dx$$

12. Redo Problem 11 by using two point Gauss quadrature formula.