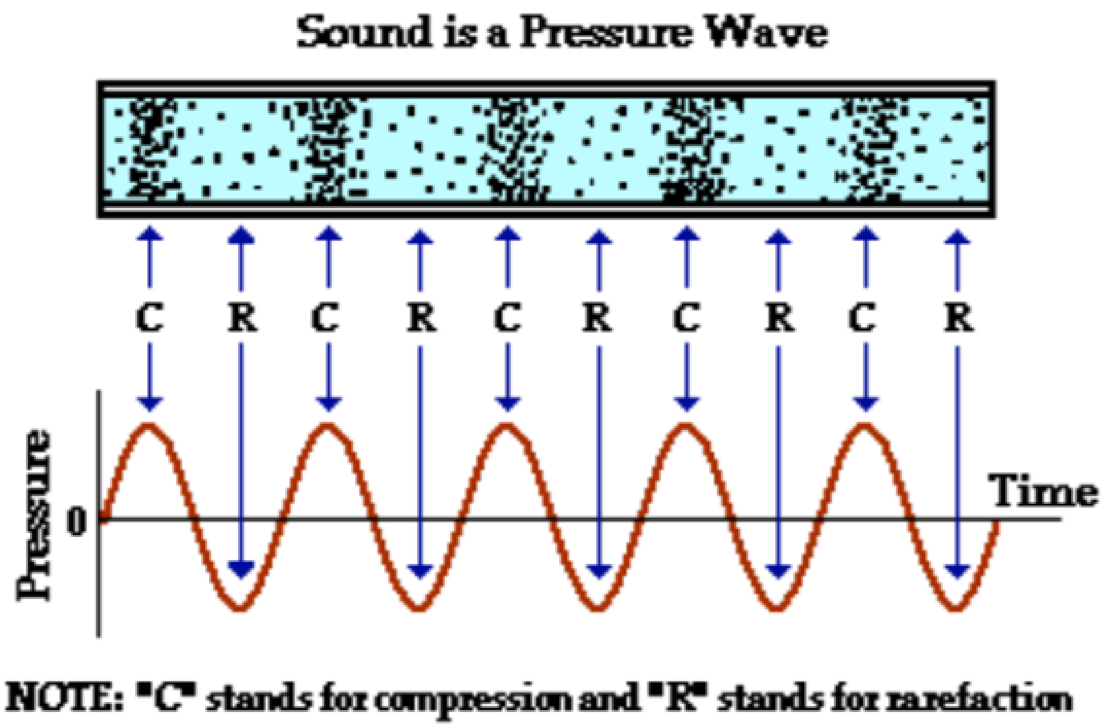
Linear Algebra and Sound

What Is Sound?

Sound is a pressure wave which is created by a vibrating object. These vibrations set particles in the surrounding medium (typical air) in vibrational motion, thus transporting energy through the medium. Since the particles are moving in parallel direction to the wave movement, the sound wave is referred to as a longitudinal wave. The result of longitudinal waves is the creation of compressions and rarefactions within the air.

The alternating configuration of C and R of particles is described by the graph of a sine wave (C=crests, R=troughs)



The speed of a sound pressure wave in air is 331.5+0.6Tc m/s , Tc temperature in Celsius The particles do not move down the way with the wave but oscillate back and forth about their individual equilibrium position.

What is meant by Wavelength, Amplitude, Frequency of a Wave?

Amplitude:

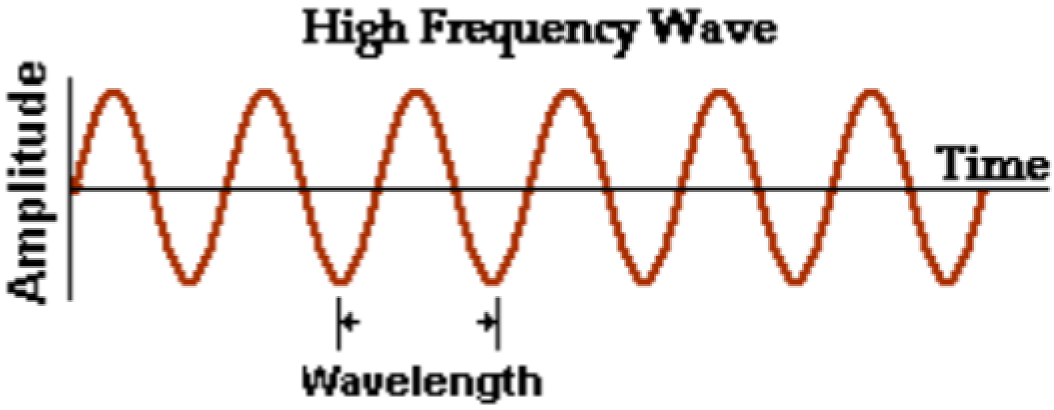
The amount of work done to generate the energy that sets the particles in motion is reflected in the degree of displacement which is measured as the amplitude of a sound.

Frequency:

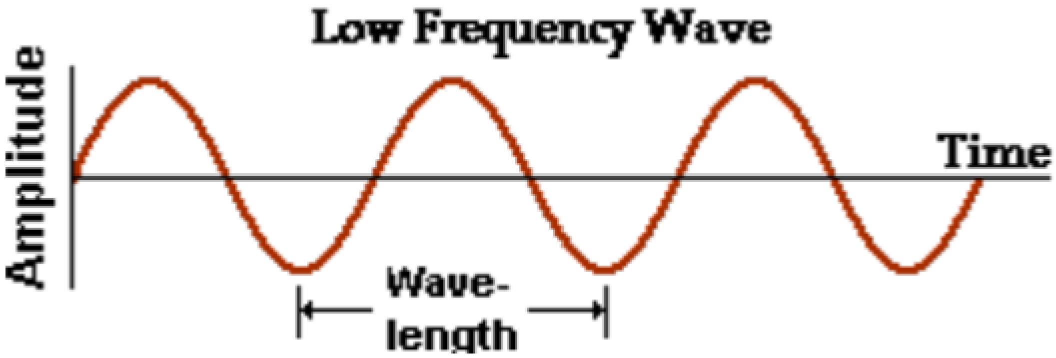
The frequency f of a wave is measured as the number of complete back-and-forth vibrations of a particle of the medium per unit of time. 1 Hertz = 1 vibration/second f = 1/Time

Wavelength:

Depending on the medium, sound travels at some speed c which defines the wavelength l: l = c/f



[Source: Internet]



[Source: Internet]

Why sound waves are Sines and Cosines only?

A basis for a linear space is a fundamental set of building blocks that can be used to make any element in the space. To analyse sound, we need to find a set of basic sounds that can be combined to make all other sounds. To do this, we first look at how sounds travel through the air and what your ear does when it receives that sound.

All sounds are produced by vibrations which cause variations in air pressure to propagate. If you hold your fingers against your throat when you speak, you can feel your larynx vibrate. When a bow is pulled across a flute string, the string vibrates. You have also probably felt these vibrations at a concert or when standing next to a loud speaker.

These variations in air pressure travel from the source of the sound to your ear where they are processed and then sent to your brain. How the ear processes sound is not completely understood, but we do know the basic story. The variations in air pressure cause your eardrums to vibrate which causes some liquid in your inner ear to slosh around. This liquid surrounds a hair-lined membrane and is enclosed in a tapered chamber. Different variations in air pressure cause differently shaped waves to propagate through the liquid. Because the chamber containing the membrane is tapered, some waves travel further than others along the membrane and stimulate different hairs. These hairs are connected to neurons that transmit the information to your brain.

A crude model of what's happening to a point on the membrane is given by the differential equation $\frac{d^2x}{dt^2} = -k\ y$ where t is time and y are the distance of that point on the

$$A(t, x) = A_{max}\,\mathrm{Sin}(kx \mp \omega t + \Phi)$$

where max amplitude A, time t, and phase offset ϕ

$$\omega = \frac{2\pi}{T} = 2\pi f$$

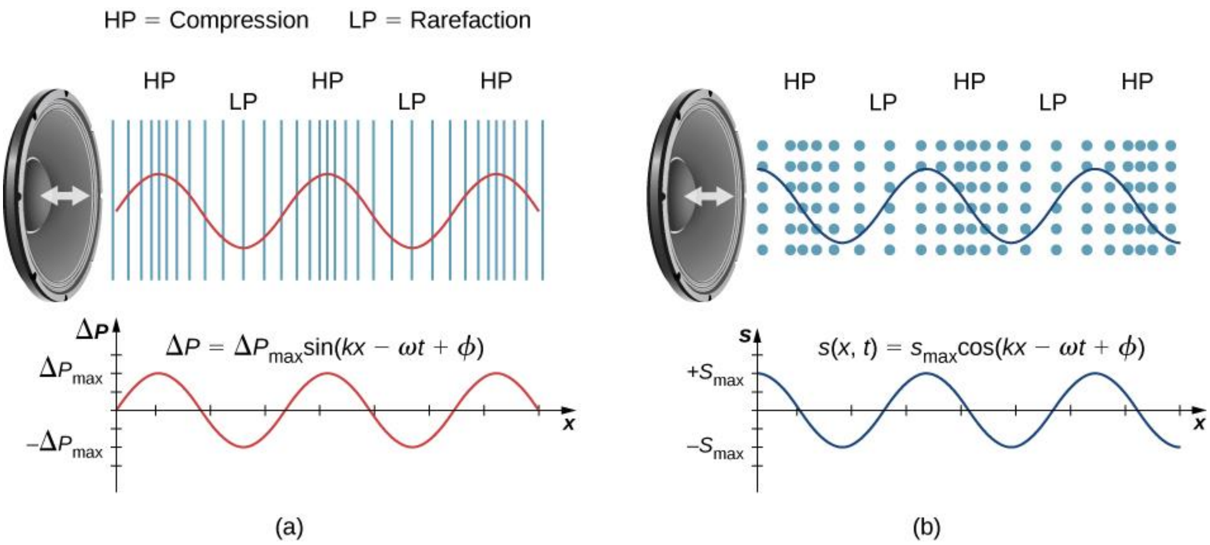
where f is frequency

$$k = \frac{2\pi}{\lambda}$$

Where k is wave number

$$v = \frac{\omega}{k} = \frac{\lambda}{T}$$

Where v is wave speed



Generation Sound with coding:

WAV file:

The WAV audio format was developed by Microsoft and has become one of the primary formats of uncompressed audio. It stores audio at about 10 MB per minute at a 44.1 kHz sample rate using stereo 16-bit samples. The WAV format is by definition, the highest quality 16-bit audio format. It is also the largest at about 10 MB per minute of CD quality audio. The quality can be sacrificed for file size by adjusting the sampling rate, data width (i.e. 8-bits), and number of channels (up to 2 for stereo).

```
# python 3.8

def create_wave_sin(freq_Hz=10000.0, duration_S=5.0, volume=0.3):
    sps = 44100  # Samples per second(sps)
    each_sample_number = np.arange(duration_S * sps)
    waveform = np.sin(2 * np.pi * each_sample_number * freq_Hz / sps)
    waveform_quiet = waveform * volume
    waveform_integers = np.int16(waveform_quiet * 30000)  # integer because
making its digital signal
    file_name = f'sine_wave(sin(x))-{freq_Hz}Hz.wav'
    write(file_name, sps, waveform_integers)  # Write the .wav file
    return file_name
```

Let say,

Frequency =1000.0 Hz

Duration = 1 second

Volume (Amplitude) = 0.3 * 30000

And we are saving as 16-bit WAV file,

Samples per Second =44.1 kHz

sine_wave(sin(x))-10000Hz.wav

Audios file for $(1000).\sin(2\pi * 10000)$

`

(just tap to play the icon)

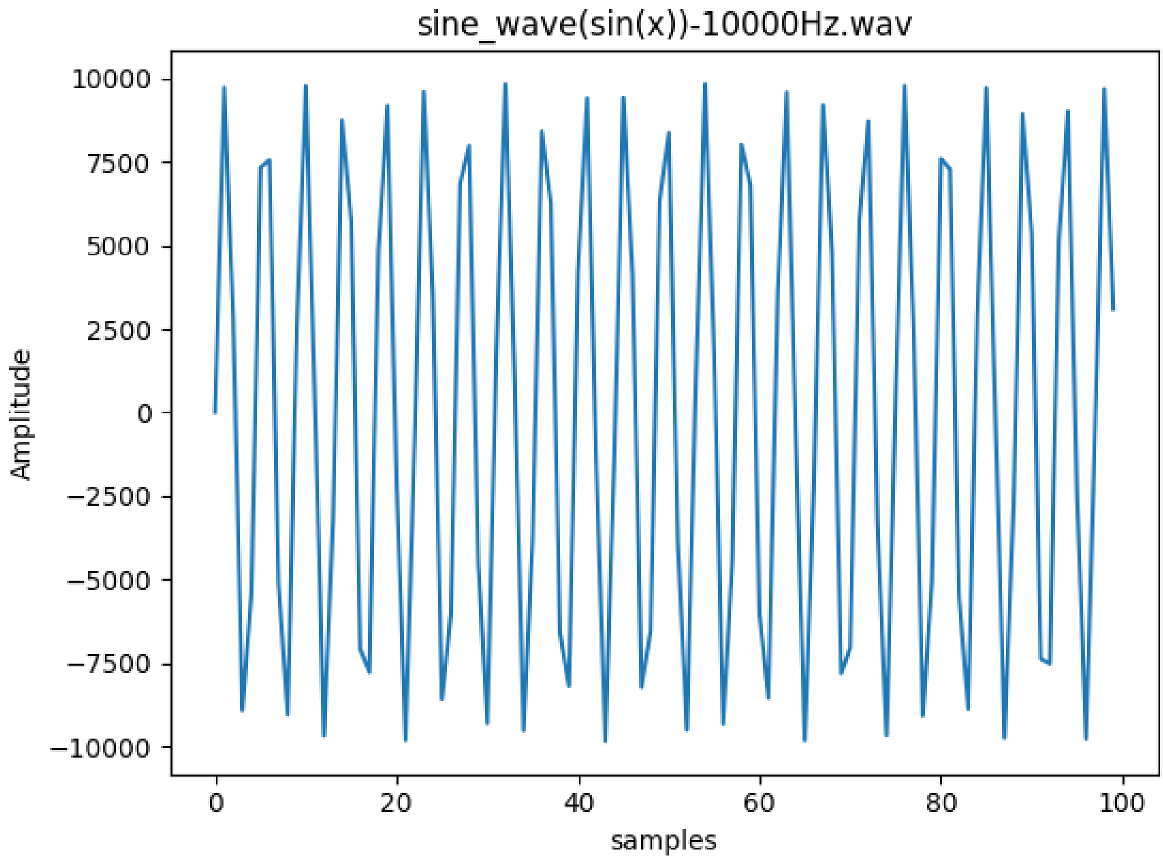


Fig: First 100 samples of the sound

Now we try Sa, Re, Ga, Ma, Pa, Dha, Ni notes

We now we have the frequencies of the following notes:

- Sa 240 Hz,
- Re 270 Hz,
- Ga 300 Hz,
- Ma 320 Hz
- Dha 400 Hz
- Pa 360 Hz,
- and Ni 450 Hz [Sourer : link]

Sa:

Frequency =240 Hz

Duration = 1 second

Volume (Amplitude) = 0.3 * 30000

And we are saving as 16-bit WAV file,

Samples per Second =44.1 kHz

sine_wave(sin(x))-240 Hz.wav

Audios file for Sa: (just tap to play the icon)

Re:

Frequency =270 Hz

Duration = 1 second

Volume (Amplitude) = 0.3 * 30000

And we are saving as 16-bit WAV file,

Samples per Second =44.1 kHz

sine_wave(sin(x))-270 Hz.wav

Audios file for Re: (just tap to play the icon)

Ga:

Frequency =320 Hz

Duration = 1 second

Volume (Amplitude) = 0.3 * 30000

And we are saving as 16-bit WAV file,

Samples per Second =44.1 kHz

sine_wave(sin(x))-300 Hz.wav

Audios file for Ga: (just tap to play the icon)

Ma:

Frequency =320 Hz

Duration = 1 second

Volume (Amplitude) = 0.3 * 30000

And we are saving as 16-bit WAV file,

Samples per Second =44.1 kHz

sine_wave(sin(x))-320Hz.wav

Audios file for Ra: (just tap to play the icon)

Dha:

Frequency =400 Hz

Duration = 1 second

Volume (Amplitude) = 0.3 * 30000

And we are saving as 16-bit WAV file,

Samples per Second =44.1 kHz

sine_wave(sin(x))-400 Hz.wav

Audios file for Ra: (just tap to play the icon)

Pa:

Frequency =360 Hz

Duration = 1 second

Volume (Amplitude) = 0.3 * 30000

And we are saving as 16-bit WAV file,

Samples per Second =44.1 kHz

sine_wave(sin(x))-360 Hz.wav

Audios file for Ra: (just tap to play the icon)

Ni:

Frequency =450 Hz

Duration = 1 second

Volume (Amplitude) = 0.3 * 30000

And we are saving as 16-bit WAV file,

Samples per Second =44.1 kHz

sine_wave(sin(x))-450 Hz.wav

Audios file for Ra: (just tap to play the icon)