Assignment-4-DSA

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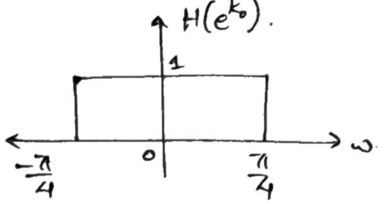
(1) Given:



i.e., $H(e^{j\omega}) = \begin{cases} 1 & ; |\omega| \le \frac{\pi}{4} \\ 0 & ; |\omega| > \frac{\pi}{4} \end{cases} = \begin{cases} 1 & ; \omega \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \\ 0 & ; \omega \notin \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]. \end{cases}$

Step (1): Find Inverse DTFT: $h(n)$

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega.$$

$$h(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} (1) e^{j\omega n} d\omega.$$

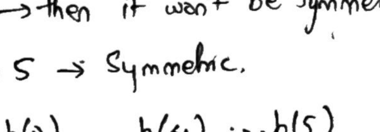
$$h(n) = \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/4}^{\pi/4} = \frac{1}{2\pi}\left[\frac{e^{j\frac{\pi n}{4}}}{jn} - \frac{e^{-j\frac{\pi n}{4}}}{jn}\right].$$

$$h(n) = \frac{1}{2\pi j n}\left[e^{j\frac{n\pi}{4}} - e^{-j\frac{n\pi}{4}}\right].$$

$$h(n) = \frac{1}{2\pi j n}\left[\left(\cos\frac{n\pi}{4} + j\sin\frac{n\pi}{4}\right) - \left(\cos\frac{n\pi}{4} - j\sin\frac{n\pi}{4}\right)\right].$$

$$= \frac{1}{\cancel{2}\pi j n}\left[\cancel{2}j \sin\frac{n\pi}{4}\right] = \frac{\sin\frac{n\pi}{4}}{n\pi}$$

$\therefore \boxed{h(n) = \frac{\sin\frac{n\pi}{4}}{n\pi}}$



approximate $\frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi}$ graph..

(a) let $N = 11$: (odd) (want Symmetric).

If we take $N = 0$ to 10 $\rightarrow$ then it won't be Symmetric.

So we take $N = -5$ to 5 $\rightarrow$ Symmetric.

$\Rightarrow$ $h(-5), h(-4) \dots h(0), \dots h(4), \dots h(5)$.

$$h(n) = \frac{\sin\frac{\pi n}{4}}{n\pi} \quad \text{for } |n| \le 5.$$

we also know, that this approach suffers from lack of Causalty

$\therefore$ we multiply with a delay $z^{-\left(\frac{N-1}{2}\right)} \Rightarrow z^{-\left(\frac{11-1}{2}\right)} \Rightarrow z^{-5}$.

$$h(0) = \lim_{n\to 0} \frac{\sin \frac{n\pi}{4}}{n\pi} = \lim_{n\to 0} \frac{\sin \frac{n\pi}{4}}{\frac{n\pi}{4}} \cdot \frac{1}{4} = \frac{1}{4}.$$

$h(1) = h(-1) = 0.225$

$h(2) = h(-2) = 0.159$

$h(3) = h(-3) = 0.075$

$h(4) = h(-4) = \cancel{0.07}\ 0.$

$h(5) = h(-5) = -0.045$.

~~DTFT~~ ~~Z-transform:~~

$$\cancel{H(z) = \sum_{n=-5}^{n=5} h(n)\cdot z^{n}}$$

Z-transform: $H(z) = \sum_{n=-5}^{n=5} h(n)\cdot z^{n}$.

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)(z^{n} + z^{-n}).$$

$$H(z) = \frac{1}{4} + (0.225)(z + z^{-1}) + (0.159)(z^{+2} + z^{-2}) + (0.075)(z^{+3} + z^{-3})$$
$$+ (0)(z^{4} + z^{-4}) + (-0.045)(z^{5} + z^{-5}).$$

$\cancel{H(z) = \frac{1}{4} +}$

But its not Causal.:

$\therefore$ Multiply with delay z^{-5}.

$$\boxed{H(z) = \frac{1}{4} z^{-5} + (0.225)(z^{-4} + z^{-6}) + (0.159)(z^{-3} + z^{-7}) + (0.075)(z^{-2} + z^{-8}) + (-0.045)(z^{0} + z^{-10}).}$$

$\cancel{H(z) = -0.045 + 0.075z^{-2} + 0.225z^{-4} + 0.225z^{-6} +}$

$$H(z) = -0.045 + 0.075z^{-2} + 0.159z^{-3} + 0.225z^{-4} + 0.25z^{-5}$$
$$+ 0.225z^{-6} + 0.159z^{-7} + 0.075z^{-8} + (-0.045)z^{-10}.$$

$$\boxed{\therefore h'(n) = \{-0.045, 0, 0.075, 0.159, 0.225, 0.25, 0.225, 0.159, 0.075, 0, -0.045\}.}$$

↳ without window.

(b) We know, Hamming window: $w_H(n) = \begin{cases} 0.54 - 0.46\cos\left(2\pi \cdot \frac{n}{N-1}\right), & |n| \le \frac{N-1}{2} \\ 0 & ; o/w. \end{cases}$ where $N = 6$

$$w_H(n) = \begin{cases} 0.54 - 0.46\cos\left(2\pi \cdot \frac{n}{N-1}\right); & |n| \le 4 \\ 0 & ; o/w \end{cases}$$

$\therefore h(n) = h'(n) \cdot w_H(n).$

$\cancel{h(0) = h'(0)\, w_H(0) = (-0.045)(0.08)} =$

$$h'(0) = \lim_{n\to 0} \frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi} = \lim_{n\to 0} \frac{\sin\left(\frac{n\pi}{4}\right)}{\frac{n\pi}{4}} \cdot \frac{1}{4} = \frac{1}{4}$$ (L'hospital rule)

$h'(1) = h'(-1) = 0.225$

$h'(2) = h'(-2) = 0.159$

$h(3) = h'(-3) = 0.075$

$h(4) = h'(-4) = 0.0$

$h(5) = h'(-5) = -0.045$

$\therefore h(0) = h'(0) \cdot w_H(0) = \frac{1}{4} \cdot 0.08 = 0.02.$

$h(1) = h'(1) \cdot w_H(1) = (0.225)\left(0.54 - \frac{0.46}{\sqrt{2}}\right) \doteq 0.0483 = h(-1)$

$h(2) = h'(2)\, w_H(2) = 0.0859 = h(-2)$

$h(3) = h'(3)\, w_H(3) = 0.0649 = h(-3)$

$h(4) = h'(4)\, w_H(4) = 0.$

$\therefore h(n) = \{0, 0.0649, 0.0859, 0.0483, \underset{\uparrow}{0.02}, 0.0483, 0.0859, 0.0649, 0\}$

(2) Given, H_d is LPF.

and Rectangular window $w(n) = \begin{cases} 1 & ; \text{if } n = 0,1,2 \\ 0 & ; \text{o/w} \end{cases}$ $\Rightarrow$ window length $= 3 = L$.

We know, FIR filter $h(n) = w(n)\, h_d(n)$.

where, $h_d(n) \rightarrow$ Ideal lowpass filter impulse response

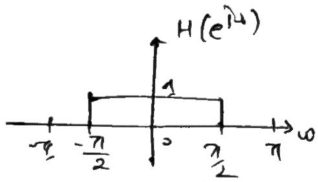
$w(n) \rightarrow$ rectangular window

$h(n) \rightarrow$ windowed impulse response

~~we know,~~ $\cancel{h_d(n) = \frac{\sin(\omega_0 n)}{\pi n}}$

1) Impulse Response for Ideal lowpass filter:

$$H_d(e^{j\omega}) = \begin{cases} 1 & ; -\frac{\pi}{2} \le \omega \le \frac{\pi}{2} \\ 0 & ; \text{o/w} \end{cases}$$



$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega n} d\omega.$$

$$h_d(n) = \frac{1}{2\pi j n} \left[e^{j\omega n}\right]_{-\pi/2}^{\pi/2} = \frac{1}{2\pi j n}\left[2j \sin\frac{\pi}{2} n\right] = \frac{\sin\left(\frac{\pi}{2} n\right)}{n\pi}$$

$$\therefore h_d(n) = \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$ $\rightarrow$ Impulse Response of LPF.

here cutoff frequency is $\frac{\pi}{2} = \omega_0$.

$\therefore h(n) = w(n) \cdot h_d(n)$

$\cancel{= h(0) +}$

$\boxed{h(n) = h_d(0) + \cancel{h_d(1) + h_d(2)}}$ $\rightarrow$ ~~windowed Impulse Response~~

$\cancel{h(n) = 0 + \frac{1}{\pi} + \frac{?}{\pi}(0) \Rightarrow h(n) =}$

$h(0) = w(0)\, h_d(0) = (1)(0) = 0$; $h(1) = \frac{1}{\pi}$; $h(2) = 0$.

Now, Fourier Transform of $h[n]$ be $H(e^{j\omega})$.

$$\therefore\ H(e^{j\omega}) = \sum_{n=0}^{n=2} h[n]\cdot e^{-j\omega n} =$$

$$H(e^{j\omega}) = h[0]\cdot e^{0} + h[1] e^{-j\omega} + h[2] e^{-2j\omega}$$

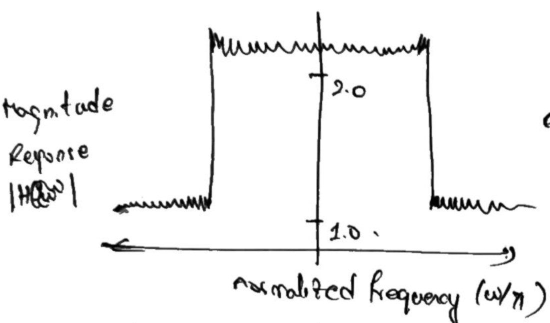
$$H(e^{j\omega}) = (0)(1) + \frac{1}{\pi}\cdot e^{-j\omega} + (0)(e^{-2j\omega})$$

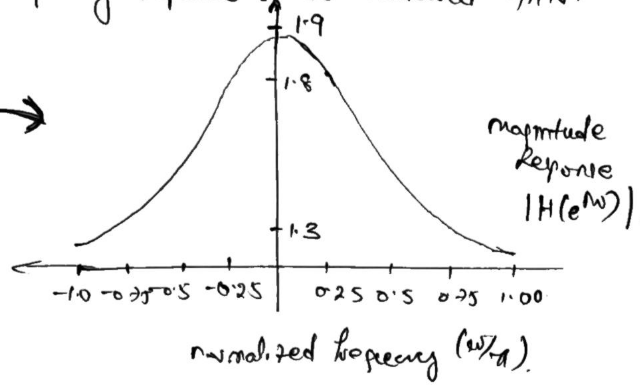
$$\boxed{H(e^{j\omega}) = \frac{1}{\pi} e^{-j\omega}}$$

for general cut-off frequency Ω_c.

$$H(e^{j\omega}) = \frac{\Omega_c}{\pi} e^{-j\omega(0)} + \frac{\sin(\Omega_c)}{\pi} e^{-j\omega} + \frac{\sin(2\Omega_c)}{2\pi} e^{-2j\omega}.$$

The Plot of $H(e^{j\omega})$ will show the frequency response of the windowed filter.





~~On Comparing with~~

On Comparing both the figures.

OBSERVATION

① The main lobe is wider, leading to a less sharp transition from the passband to the stopband.

② Ripples or oscillations will appears in the stopband, known as Gibbs phenomena

③ The Stopband attenuation is lower than the Ideal LPF.

2) We have, Rectangular window $w[n]$ is used, and the window length is $L = 3$,

When the window is applied to the Infinite impulse Response $h_d[n]$ of an ideal low-pass filter (LPF), the windowed

Impulse response $h[n]$ becomes $h[n] = w[n] * h_d[n]$

$\Rightarrow$ Windowed impulse response $h[n]$ has non-zero values only for $n = 0, 1, 2.,$ and is zero for all other values of n.

Therefore, the length of the non-zero part of $h[n]$ is 3. which corresponds to the window length $L = 3$.

Since, the windowing method produces a finite impulse response (FIR) filter with the same length as the window, the resulting filter length M, is also equal to 3

$\boxed{L = M = 3}$