Question - 1 : Signals :

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① (a) $x[n] = \sin^2(3n+\pi) = (\sin(\pi+3n))^2 = (-\sin(3n))^2 = \sin^2(3n)$

let us assume that $x[n]$ is periodic, then $x[n] = x[n+N]$.

where N is Period of $x[n]$.

then $\sin^2(3n) = \sin^2(3(n+N)) \Rightarrow \sin^2 3n = \sin^2(3n+3N)$.

$\Rightarrow$

we know that fundamental period of $\sin^2(\theta)$ is π.

$\therefore$ $3N = n\cdot\pi$ where n is an Integer.

$\Rightarrow$ $\boxed{N = \frac{n\pi}{3}}$

But $\nexists n \in \mathbb{Z}$ s.t $N \in$ Integers.

$\therefore$ Assumption is wrong

$x[n]$ is Aperiodic

(b) $x[n] = e^{jn\pi/8} = e^{j\left(\frac{\pi}{8}\right)n} = e^{j\omega_0 n}$.

$\omega_0 = \frac{\pi}{8} \Rightarrow \frac{\omega_0}{2\pi} = \frac{1}{16} = \frac{m}{N} \Rightarrow m = 1$ and $N = 16$.

$\Rightarrow N = 16$ for every 1 full cycle. and $\in \mathbb{Z}$.

$N \to$ fundamental period of $x[n]$

$\therefore$ $x[n]$ is periodic with fundamental period $N = 16 \in$ Integer.

(c) $x(n) = \cos(\pi n/10)\cos(\pi n/30)$.

we know : $\cos A \cos B = \frac{\cos(A-B) + \cos(A+B)}{2}$.

$\therefore x[n] = \cos\left(\frac{n\pi}{10}\right)\cos\left(\frac{n\pi}{30}\right) = \frac{\cos\left(\frac{n\pi}{10} - \frac{n\pi}{30}\right) + \cos\left(\frac{n\pi}{10} + \frac{n\pi}{30}\right)}{2}$

$= \frac{\cos\left(\frac{2n\pi}{30}\right) + \cos\left(\frac{4n\pi}{30}\right)}{2} = \frac{1}{2}\left(\cos\left(\frac{n\pi}{15}\right) + \cos\left(\frac{2n\pi}{15}\right)\right)$

$\omega_1 = \frac{\pi}{15}$; $\omega_2 = \frac{2\pi}{15}$.

$\omega_1 = 1\left(\frac{\pi}{15}\right)$; $\omega_2 = 2\left(\frac{\pi}{15}\right)$.

All of the ω_i's $i = \{1, 2\}$, are of the form $n\omega_0$, where n is an Integer and $\omega_0 = \frac{\pi}{15}$. $\therefore$ $\frac{\omega_0}{2\pi} = \frac{\pi/15}{2\pi} = \frac{1}{30} = \frac{m}{N}$

$\frac{1}{30} = \frac{m}{N} \Rightarrow m = 1$ and $N = 30 \in$ Integer.

$\therefore x[n]$ is periodic with fundamental Period $N = 30$.

(d) $x[n] = \sin(4\pi n + 3)$.

$\cancel{\omega_0 = 4\pi}$

Assume that $x[n]$ is Periodic Signal

then $x[n] = x[n+N]$ ~~for the~~ where N is a Period of Signal $x(n)$

$\Rightarrow \sin(4\pi n + 3) = \sin(4\pi(n+N) + 3)$

$\Rightarrow \sin(4\pi n + 3) = \sin(4\pi n + 3 + 4\pi N)$.

should be equal to $n \cdot 2\pi$.

$\because \sin(\theta)$ has fundamental period 2π

$4\pi N = n \cdot 2\pi \quad \forall n \in \mathbb{Z}$.

$\Rightarrow N = \frac{n}{2}$.

$n \in \mathbb{Z}$ and $N \in \mathbb{Z}$.

Smallest Possible value of $N = \frac{2}{2} = 1$ for $n = 2$

$\therefore$ fundamental Period of $x[n] \Rightarrow \boxed{N = 1}$

$\therefore x[n]$ is Periodic signal with ~~for~~ fundamental Period ~~$\$$~~ $N = 1$.

(e) $x[n] = \cancel{\cos(\pi\pi}\ \cos\left(\frac{\pi n^2}{3}\right)$.

let assume, $x[n]$ is Periodic with Period N.

$x[n] = x[n+N]$.

$\Rightarrow \cos\left(\frac{\pi n^2}{3}\right) = \cos\left(\frac{\pi(n+N)^2}{3}\right)$.

$\Rightarrow \cos\left(\frac{\pi n^2}{3}\right) = \cos\left(\frac{\pi n^2}{3} + \frac{\pi N^2}{3} + \frac{2\pi n N}{3}\right)$.

$\Rightarrow \frac{\pi n^2}{3} + 2d\pi = \frac{\pi(n+N)^2}{3}$ (using Trigonometric Equations)

$\Rightarrow \frac{\pi N^2}{3} + \frac{2\pi n N}{3} - 2d\pi = 0$.

Here N is dependent on the value of 'n' and hence is not a constant.

So, ~~$\cos n^2$~~ $\cos\left(\frac{\pi n^2}{3}\right)$ is not periodic $\therefore x[n]$ is Aperiodic

(f) $x(n) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(n-k) = \cdots + (-1)^{-1}\delta(n-(-1)) + (-1)^0\delta(n-0) + (-1)^1\delta(n-1) + \cdots$

$x[n] = (-1)^n \delta(0) = (-1)^n.$

$\boxed{x[n] = (-1)^n}$

Clearly $x[n]$ is periodic with fundamental Period '2'. $\Rightarrow x[n] = x[n+2]$

$\therefore$ $x[n]$ is periodic with fundamental Period '2'.

Question-1 : Signals:

(2) (a) $x[n] = \sqrt{2}\cos\left((an + 1/4)\pi\right)$.

$$x[n] = \frac{x[n]}{2} + \frac{x[n]}{2} = \cancel{x[n]+x[n]}\ \underbrace{\frac{x[n]+x[-n]}{2}}_{\text{even}} + \underbrace{\frac{x[n]-x[-n]}{2}}_{\text{odd}}$$

$\therefore$ odd part : $\dfrac{x[n] - x[-n]}{2} = \dfrac{\sqrt{2}\cos\left((an+\frac{1}{4})\pi\right) - \sqrt{2}\cos\left((-an+\frac{1}{4})\pi\right)}{2}$

$$x[n] = \sqrt{2}\cos\left(an\pi + \frac{\pi}{4}\right) = \sqrt{2}\left[\cos(an\pi)\cos\left(\frac{\pi}{4}\right) - \sin(an\pi)\sin\left(\frac{\pi}{4}\right)\right]$$

$$= \cancel{\sqrt{2}}\left[\cos(an\pi)\frac{1}{\cancel{\sqrt{2}}} - \sin(an\pi)\frac{1}{\cancel{\sqrt{2}}}\right] = \cos(an\pi) - \sin(an\pi)$$

$$\therefore \text{odd part} = \frac{\left[\cancel{\cos(an\pi)} - \sin(an\pi)\right] - \left[\cancel{\cos(-an\pi)} + \sin(+an\pi)\right]}{2}$$

$$= \cancel{\cos}\ \frac{-2\sin(an\pi)}{2} = -\sin(an\pi). \quad \boxed{\text{ODD}}$$

$$\text{Even Part} = \frac{\left[\cos(an\pi) - \sin(an\pi)\right] + \left[\cos(-an\pi) - \sin(-an\pi)\right]}{2}$$

$$= \frac{2\cos(an\pi)}{2} = \cos(an\pi) \quad \boxed{\text{EVEN}}$$

$\therefore$ odd Part : $-\sin(an\pi)$ and Even Component : $\cos(an\pi)$

(b) $x[n] = e^{jan\pi} + e^{jn\pi/b}$.

odd ~~even~~ part $= \dfrac{x[n] - x[-n]}{2}$

$$= \frac{\left(e^{jan\pi} + e^{jn\pi/b}\right) - \left(e^{-jan\pi} + e^{-jn\pi/b}\right)}{2}$$

$$= \frac{2j\sin(an\pi) + 2j\sin\left(\frac{n\pi}{b}\right)}{2} = j\left(\sin(an\pi) + \sin\left(\frac{n\pi}{b}\right)\right).$$

Even part $= \dfrac{x[n] + x[-n]}{2}$

$$= \frac{\left(e^{jan\pi} + e^{jn\pi/b}\right) + \left(e^{-jan\pi} + e^{-jn\pi/b}\right)}{2}$$

$$= \frac{2\cos(an\pi) + 2\cos\left(\frac{n\pi}{b}\right)}{2} = \cos(an\pi) + \cos\left(\frac{n\pi}{b}\right)$$

Question-1: Signals:

(3) Energy $= E = \sum_{n=-\infty}^{\infty} |x[n]|^2$ and Power; $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$.

If Energy is finite, Power $= 0$, then its Energy Signal
If Power is finite, Energy $= \infty$, then its Power Signal.

(a) $x[n] = \begin{cases} 0 & ; n<0 \\ n & ; n \geq 0 \end{cases}$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \cancel{\sum_{n=-\infty}^{0} |0|^2} + \sum_{n=0}^{\infty} |n|^2.$$

$$= \sum_{n=0}^{\infty} n^2 = 0^2 + 1^2 + 2^2 + \cdots + \cdots = \infty.$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{0} |0|^2 + \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |n|^2$$

$$= \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} n^2 = \lim_{N\to\infty} \frac{1}{2N+1} (0^2 + 1^2 + \cdots + N^2).$$

$$= \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} \left(\frac{(N)(N+1)\cancel{(2N+1)}}{6} \right) = \lim_{N\to\infty} \frac{N(N+1)}{6} = \infty$$

$E = \infty$; $P = \infty \Rightarrow$ Neither Power nor Energy Signal.

(b) $x[n] = \cos(n\pi/2)$.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} \left|\cos\left(\frac{n\pi}{2}\right)\right|^2 = 2 \cdot \sum_{n=0}^{\infty} \left|\cos\left(\frac{n\pi}{2}\right)\right|^2$$

$$= 2 \left(|\cos 0|^2 + |\cos \tfrac{\pi}{2}|^2 + |\cos \tfrac{2\pi}{2}|^2 + |\cos \tfrac{3\pi}{2}|^2 + \cdots \right).$$

$$= 2 \left(1^2 + 0^2 + 1^2 + 0^2 + \cdots \right) = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\cos\left(\frac{n\pi}{2}\right)\right|^2.$$

$$= \begin{cases} \lim_{N\to\infty} \frac{1}{2N+1} \left(\cancel{2(N+1)(1)} + 1 \right)^{(N+1)} & ; \text{if } N \text{ is even} \\ \lim_{N\to\infty} \frac{1}{2N+1} (N) & ; \text{if } N \text{ is odd} \end{cases}$$

$$= \begin{cases} \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}} & ; N \text{ is even} \\ \lim_{N\to\infty} \frac{1}{2+\frac{1}{N}} & ; N \text{ is odd} \end{cases} = \begin{cases} \frac{1}{2} & \text{if } n \text{ is even} \\ \frac{1}{2} & \text{if } n \text{ is odd} \end{cases} = \frac{1}{2} \Rightarrow \text{finite}$$

Energy $= \infty$; Power $= \frac{1}{2}$ (finite) $\therefore$ $x[n]$ is a Power Signal

(c) $x[n] = \begin{cases} 3^n & , n < 0 \\ \left(\frac{1}{2}\right)^n & : n \geq 0 \end{cases}$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{-1} |3^n|^2 + \sum_{n=0}^{\infty} \left|\left(\tfrac{1}{2}\right)^n\right|^2.$$

$$= \sum_{n=-\infty}^{-1} (9)^n + \sum_{n=0}^{\infty} \left|\left(\tfrac{1}{2}\right)^{2n}\right|.$$

$$= (9^{-1} + 9^{-2} + 9^{-3} + \cdots) + \left(\left(\tfrac{1}{2}\right)^0 + \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^4 + \cdots\right)$$

$$= \frac{1/9}{1 - 1/9} + \frac{1}{1 - 1/2^2}.$$

$$= \frac{1}{8} + \frac{4}{3} \Rightarrow \text{finite}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} (3^{2n}) + \sum_{n=0}^{N} \left(\tfrac{1}{2}\right)^{2n} \right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\left(3^{-2} + 3^{-4} + \cdots + 3^{-2N}\right) + \left(\left(\tfrac{1}{2}\right)^0 + \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^4 + \cdots + \left(\tfrac{1}{2}\right)^{2N}\right) \right].$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\frac{3^{-2}\left((3^{-2})^N - 1\right)}{(3^{-2} - 1)} + \frac{1\left(\left(\tfrac{1}{2}\right)^{N+1} - 1\right)}{\tfrac{1}{2} - 1} \right]$$

$$= \cancel{\lim_{N\to\infty} \frac{1}{2N+1} \left(\frac{1}{3^2} \cdot \frac{1 - 3^{2N}}{} \right.} = \frac{3^{-2}}{(3^{-2}-1)} \lim_{N\to\infty} \frac{3^{-2N} - 1}{2N+1} \cdot -\tfrac{1}{2} \lim_{N\to\infty} \left(\left(\tfrac{1}{2}\right)^{N+1} - 1\right)$$

$$= 0 + -\tfrac{1}{2}(-1) = \tfrac{1}{2} \Rightarrow \text{finite}.$$

E → finite and Power is finite

∴ Neither Energy nor Power Signal.

(d) $x(n) = a^n u(n) \quad , a \in R$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |a^n u(n)|^2 = \cancel{\sum_{n=-\infty}^{n=\infty}}$$

$$= \cancel{\sum_{n=-\infty}^{0} |a^n(0)|^2} + \sum_{n=0}^{\infty} |a^n|^2 = |a^0|^2 + |a^1|^2 + |a^2|^2 + \cdots$$

$$= |a^0| + |a^2| + |a^4| + |a^6| + \cdots$$

$$= a^0 + a^2 + a^4 + a^6 + \cdots$$

$$= \frac{1}{1-a^2} \Rightarrow \text{finite}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \sum_{n=0}^{N} |a^n|^2$$

$$= \lim_{N\to\infty} \left[(a^0 + a^2 + a^4 + \cdots + a^{2N}) \cdot \frac{1}{2N+1} \right]$$

$$= \lim_{N\to\infty} \frac{1(1-a^{2N})}{(2N+1)(1-a^2)} = \frac{1}{1-a^2} \lim_{N\to\infty} \frac{1-a^{2N}}{(2N+1)}$$

$$= \cancel{\frac{1}{a^2-1}} = \begin{cases} 0 & ; a \in (-1,1) \quad \left(\because \frac{1}{\infty} = 0\right) \\ \lim_{N\to\infty} \frac{1}{1-a^2} \cdot \frac{1-a^{2N}}{(2N+1)} & ; a \notin (-1,1) \end{cases}$$

$$= \begin{cases} 0 & ; a \in (-1,1) \\ \lim_{N\to\infty} \frac{a^{2N}-1}{(a^2-1)(2N+1)} & a \notin (-1,1) \end{cases} = \begin{cases} 0 & ; a \in (-1,1) \\ \lim_{N\to\infty} \frac{a^{2N}-1}{(a^2-1)(2N+1)} & a \notin (-1,1) \end{cases}$$

$$P = \begin{cases} 0 & ; |a| < 1 \\ \lim_{N\to\infty} \frac{a^{2N}-1}{(a^2-1)(2N+1)} & ; |a| \geq 1 \end{cases}$$

$\therefore$ when $|a| < 1$: Energy is finite and Power is $0 \Rightarrow$ Energy Signal

when $|a| \geq 1$: Energy is finite and Power is not $0 \Rightarrow$ Neither Energy nor Power Signal

(e) $x(n) = e^n \delta(n-4)$.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |e^n \delta(n-4)|^2 = |(e^4 \cdot (1))^2| = e^8 \Rightarrow \text{finite}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |e^n \delta(n-4)|^2.$$

if $N < 4$: $P = 0$.

if $N \geq 4$: $P = \lim_{N\to\infty} \frac{1}{2N+1} \cdot |e^4|^2 = \cancel{\lim}\ 0$.

$\therefore P = 0$.

$E =$ finite , $P = 0 \Rightarrow x(n) =$ is an Energy Signal.

Question : 2 : Systems :

① (a) $y(t) = t^2 x(t-1)$.

$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t) = t^2 x(t-1)$.

$y(t) \xrightarrow[\text{delay}]{t_0} y(t-t_0) = (t-t_0)^2 x(t-t_0-1)$. $\longrightarrow$ ①

$x(t) \xrightarrow[\text{delay}]{t_0} x(t-t_0) \longrightarrow \boxed{\text{System}} \longrightarrow t^2 x(t-t_0-1)$. $\Rightarrow$ ②

① ≠ ② ∴ $y(t)$ is Time variant System (TV).

(b) $y[n] = x[n-1] + x[n+1]$.

$x[n] \longrightarrow \boxed{\text{System}} \longrightarrow y[n] = x[n-1] + x[n+1]$.

$y[n] \xrightarrow[\text{delay}]{n_0} y[n-n_0] = x[n-n_0-1] + x[n-n_0+1]$ $\longrightarrow$ ①

$x[n] \xrightarrow[\text{delay}]{n_0} x[n-n_0] \longrightarrow \boxed{\text{System}} \longrightarrow x[n-n_0-1] + x[n-n_0+1]$ $\longrightarrow$ ②

① = ② ∴ $y[n]$ is Time Invariant System (TIV)

(c) $y[n] = \dfrac{1}{x[n]}$

$x[n] \longrightarrow \boxed{\text{System}} \longrightarrow y[n] = \dfrac{1}{x[n]}$

$y[n] \xrightarrow[\text{delay}]{n_0} y[n-n_0] = \dfrac{1}{x[n-n_0]}$. $\longrightarrow$ ①

$x[n] \xrightarrow[\text{delay}]{n_0} x[n-n_0] \longrightarrow \boxed{\text{System}} \longrightarrow y = \dfrac{1}{x[n-n_0]}$. $\longrightarrow$ ②

① = ② ∴ $y[n]$ is Time Invariant System (TIV).

(d) $y[n] = x[n]\left(g[n] + g[n-1]\right)$.

o) if $g[n] = 1 \quad \forall n$.:

$y[n] = x[n](1+1) = 2x[n]$.

$x[n] \longrightarrow \boxed{\text{System}} \longrightarrow y[n] = 2x[n]$.

$y[n] \xrightarrow[\text{delay}]{n_0} y[n-n_0] = 2x[n-n_0]$ $\longrightarrow$ ①

$x[n] \xrightarrow[\text{delay}]{n_0} x[n-n_0] \longrightarrow \boxed{\text{System}} \longrightarrow 2x[n-n_0] \longrightarrow (2)$

$(1) = (2) \therefore y[n] = \cancel{g[n]}\, x[n](g[n] + g[n-1])$ is Time Invariant System (TIV) if $g[n] = 1, \forall n$.

b) $g[n] = n \;\forall n. \Rightarrow y[n] = x[n]\left[n + (n-1)\right] = x[n](2n-1)$.

$x[n] \longrightarrow \boxed{\text{System}} \longrightarrow y[n] = x[n](2n-1)$.

$y[n] \xrightarrow[\text{delay}]{n_0} y[n-n_0] = x[n-n_0](2(n-n_0)-1)$.

$= x[n-n_0](2n - 2n_0 - 1) \longrightarrow (1)$

$x[n] \xrightarrow[\text{delay}]{n_0} x[n-n_0] \longrightarrow \boxed{\text{System}} \longrightarrow x[n-n_0](2n-1) \longrightarrow (2)$

$(1) \neq (2) \therefore y[n] = x[n]\left[g[n] + g[n-1]\right]$ is Time Variant (TV).

if $g[n] = n \;\forall n$.

c) $g[n] = 1 + (-1)^n \;\forall n$.

$y[n] = x[n]\left[g[n] + g[n-1]\right] = x[n]\left(1 + \cancel{(-1)^n} + 1 + \cancel{(-1)^{n-1}}\right)$

$= 2x[n]$.

$x[n] \longrightarrow \boxed{\text{System}} \longrightarrow y[n] = 2x[n]$.

$y[n] \xrightarrow[\text{delay}]{n_0} y[n-n_0] = 2x[n-n_0] \longrightarrow (1)$

$x[n] \xrightarrow[\text{delay}]{n_0} x[n-n_0] \longrightarrow \boxed{\text{System}} \longrightarrow 2x[n-n_0] \longrightarrow (2)$

$(1) = (2) \therefore y[n] = x[n]\left[g[n] + g[n-1]\right]$ is Time Invariant (TIV)

if $g[n] = 1 + (-1)^n \;\forall n$.

Question : 2 : Systems

(2) a) $y(t) = x(\sin t)$. i.e., $T\{x(\sin t)\} = y$

i.e., $x(t) \longrightarrow \boxed{\text{System}}_T \; y(t) = x(\sin t)$.

$x_1(t) \longrightarrow \boxed{\text{System}}_T \longrightarrow y_1(t) = x_1(\sin t)$

$x_2(t) \longrightarrow \boxed{\text{System}}_T \longrightarrow y_2(t) = x_2(\sin t)$.

To ~~Prove~~ verify: $T\{a\, x_1(t) + b x_2(t)\} = a \cdot T\{x_1(t)\} + bT\{x_2(t)\}$

LHS: $T\{a x_1(t) + b x_2(t)\} = a\, x_1(\sin t) + b x_2(\sin t) \rightarrow ①$

RHS: $a \cdot T\{x_1(t)\} + bT\{x_2(t)\} = a \cdot x_1(\sin(t)) + b\, x_2(\sin t) \rightarrow ②$

① = ② ∴ the system follows law of Superposition [in simpler terms it follows Law of Addition and law of Homogeneity].

∴ The System is linear.

b) $y(t) = \begin{cases} 0 & ; \text{ if } t < 0 \\ x(t) + x(t-2) & ; \text{ if } t \geq 0 \end{cases}$

case ①: $t < 0$.

$y(t) = 0$. for any input ∴ the system satisfy the equation

$T\{a(x_1(t)) + b x_2(t)\} = a T\{x_1(t)\} + bT\{x_2(t)\}$.

∴ $y(t)$ will be linear for $t < 0$.

Case ②: $t \geq 0$.

$x(t) \longrightarrow \boxed{\text{System}}_T \longrightarrow x(t) + x(t-2) = y(t)$

$y(t) = x(t) + x(t-2)$.

to ~~prove~~ verify: $T\{a x_1(t) + b x_2(t)\} = aT\{x_1(t)\} + bT\{x_2(t)\}$.

LHS: $T\{a x_1(t) + b x_2(t)\} = (a x_1(t) + b x_2(t)) + (a x_1(t-2) + b x_2(t-2))$

$= a(x_1(t) + x_2(t-2)) + b(x_2(t) + x_2(t-2))$. → ①

RHS: $aT\{x_1(t)\} + bT\{x_2(t)\}$

$= a\{x_1(t) + x_1(t-2) + b(x_2(t) + x_2(t-2))\} \rightarrow ②$

① = ② : they system follows law of Superposition

∴ System is linear

(c) $y(t) = \dfrac{d(x(t))}{dt}$: $x(t) \longrightarrow \boxed{\text{System}}^{T} \longrightarrow \dfrac{d(x(t))}{dt} = y(t)$.

To ~~prove~~ verify: $T\{ax_1(t) + bx_2(t)\} = aT\{x_1(t)\} + bT\{x_2(t)\}$

$$LHS = T\{ax_1(t) + bx_2(t)\}$$
$$= \frac{d(ax_1(t) + bx_2(t))}{dt} = a\frac{d(x_1(t))}{dt} + b\frac{d(x_2(t))}{dt} \quad \text{--- (1)}$$

$$RHS = aT\{x_1(t)\} + bT\{x_2(t)\}.$$
$$= a.\frac{d(x_1(t))}{dt} + b\frac{d(x_2(t))}{dt} \longrightarrow (2)$$

(1) = (2) $\therefore$ law of superposition is valid

the system is linear

(d) $y[n] = \sum_{m=0}^{M} a x[n-m] + \sum_{m=1}^{N} bx[n-m]$.

$x(n) \longrightarrow \boxed{\text{System } T} \longrightarrow \sum_{m=0}^{M} ax[n-m] + \sum_{m=1}^{N} bx[n-m]$.

To ~~prove~~ verify: $T\{Ax_1(t) + Bx_2[n]\} = AT\{x_1(n)\} + BT\{b x_2[n]\}$.

LHS: $T\{Ax_1[n] + Bx_2[n]\} = T\{A(x_1[n]) + Bx_2[n]\}$

$$= \sum_{m=0}^{M} a.(Ax_1(n-m) + Bx_2(n-m]) + \sum_{m=1}^{N} b(Ax_1[n-m] + Bx_2[n-m]).$$

$$= \sum_{m=0}^{M} aAx_1(n-m] + \sum_{m=0}^{M} aBx_2[n-m] + \sum_{m=1}^{N} bAx_1[n-m] + \sum_{m=1}^{N} bBx_2[n-m]. \quad \text{(1)}$$

RHS: $AT\{x_1[n]\} + BT\{x_2[n]\}$.

$$= A\left(\sum_{m=0}^{M} ax_1(n-m] + \sum_{m=1}^{N} bx_1(n-m]\right) + B\left(\sum_{m=0}^{M} ax_2[n-m] + \sum_{m=1}^{N} bx_2[n-m]\right)$$

$$= \sum_{m=0}^{M} aAx_1[n-m] + \sum_{m=1}^{N} bAx_1[n-m] + \cancel{aB\Sigma} \sum_{m=0}^{M} aBx_2(n-m]$$
$$+ \sum_{m=1}^{N} bBx_2[n-m]. \longrightarrow (2).$$

(1) $\neq$ (2) $\because$ law of superposition is invalid $\therefore$ System is Not-Linear

(e) $y[n] = ax[n] + \frac{b}{x[n-1]}$

$x[n] \longrightarrow \boxed{\text{System}(T)} \longrightarrow ax[n] + \frac{b}{x[n-1]} = y[n]$.

To Prove or Disprove:

To verify: $T\{Ax_1[n] + Bx_2[n]\} = AT\{x_1[n]\} + BT\{x_2[n]\}$

$LHS = T\{Ax_1[n] + Bx_2[n]\} = T\{Ax_1[n] + Bx_2[n]\}$

$= a(Ax_1[n] + Bx_2[n]) + \frac{b}{(Ax_1[n-1] + Bx_2[n])} \longrightarrow (1)$

$RHS = A \cdot T\{x_1[n]\} + BT\{x_2[n]\}$

$= A\left(ax_1[n] + \frac{b}{x_1[n-1]}\right) + B\left(ax_2[n] + \frac{b}{x_2[n-1]}\right)$. $\to (2)$

=

(1) $\neq$ (2) $\therefore$ law superposition is not valid

$\therefore$ System is not linear.

Question-2: Systems:

(3) (a) $y(t) = x(t-2) + x(2-t)$.

we know: Causal Systems are those systems where output of the system is independent of future values of input

And Non-Causal system, output of System can depends on future values of i/p at any instance of time.

Critical Points: ~~{2}~~ {1}

case (1): $t > 1$:

$y(t) = x(t-2) + x(2-t)$.

~~$2-t<0$; $t-2>0$; $t-2<t$~~ ; $2-t \leq 1$; $t-2<t$

$\therefore$ $y(t)$ is causal for $t > 1$

case (2): $t = 1$

$y(1) = \cancel{x(0) + x(0)} \Rightarrow y(t)$ is causal at ~~$t=2$~~ $t = 1$.

$= x(-1) + x(1)$.

case ③: ~~t < 2~~, $t < 1$

$y(t) = x(t-2) + x(2-t)$

$t-2 < -1$ and $\underbrace{2-t > 1}$.

$x(2-t)$ is dependent future Input.

$\therefore$ $y(t)$ is Non-Causal in range $t < 1$.

$\therefore$ Overall system $y(t)$ is Non-causal.

(b) $y(t) = x(t)\underbrace{\cos(3t)}$.

$\cos(3t)$ is just a coefficient and doesn't reflect any input Signal

$\therefore$ $y(t)$ depends only on current Input $x(t)$. $\forall t$

$\therefore$ system $y(t)$ is Causal System

(c) $y(t) = \int_{-\infty}^{2t} x(k)\,dk$.

Case ①: when $t > 0$.

$y(t) = \int_{-\infty}^{0} x(k)dk + \int_{0}^{2t} x(k)dk$ ~~⇒ Present output~~

$= \int_{-\infty}^{t} x(k)dk + \underbrace{\int_{t}^{2t} x(k)dk}$

Present output depends on future ~~output~~ input

Case ②: when $t \leq 0$.

$y(t) = \int_{-\infty}^{2t} x(k)dk$ => Present output depends only on past or present input.

$\therefore$ overall system $y(t)$ ~~$y(t)$~~ is Non-Causal since for $t > 0$,

Present output depends on future input

(d) $y[n] = \sum_{k=0}^{\infty} x[n+k]$.

On expanding,

$y[n] = x[n] + x[n+1] + x[n+2] + \cdots$

Clearly, present output depends on future inputs $\forall n$.

$\therefore$ System ~~$y[n]$~~ is Non-Causal.

(e) $y[n] = \sum_{k=0}^{\infty} x[n-k]$.

On expanding,

$y[n] = x[n] + x[n-1] + x[n-2] + \cdots + \cdots$

Clearly, present output $y[n]$ depends only on present and past inputs.

$\therefore$ the system is Causal.

Question-3: Convolution

(1) $x(n) = \{-1, 1, 0, 1\}$ and $h(n) = \{1, 2, 3, 4, 5\}$.

Linear Convolution:

	1	2	3	4	5
-1	-1	-2	-3	-4	-5
1	+1	2	3	4	5
0	0	0	0	0	0
1	1	2	3	4	5

$\Rightarrow \{-1, -1, -1, 0, 1, 8, 4, 5\}$

Circular Convolution: $x(n) = \{-1, 1, 0, 1, 0\}$ and $h(n) = \{1, 2, 3, 4, 5\}$

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} (-1+3+5) \\ (1-2+4) \\ (2-3+5) \\ (1+3-4) \\ (2+4-5) \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 4 \\ 0 \\ 1 \end{bmatrix} = \{7, 3, 4, 0, 1\}$$

Linear Convolution: $\{-1, -1, -1, 0, 1, 8, 4, 5\}$

Circular Convolution: $\{7, 3, 4, 0, 1\}$.

(2) $x_1(n) = \{2, 1, 2, 1\}$; $x_2(n) = \{1, 2, 3, 4\}$

we know the property of Circular Convolution : $y(n) = x_1(n) \circledast x_2(n) \xleftrightarrow{F.T} X_1(k) \cdot X_2(k)$.

ie, convolution in time domain is multiplication in frequency domain.

Here., $N = 4$.

we need to find $X_1(k)$ and $X_2(k)$.

DFT of $x_1(n)$:

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} (2+1+2+1) \\ (\cancel{2}\cancel{-j}\cancel{-2}+\cancel{j}) \\ (2-1+2-1) \\ (2+\cancel{j}-2-\cancel{j}) \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$X_1(k) = \{6, 0, 2, 0\}$

DFT of $x_2[n]$:

$$\begin{pmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{bmatrix} (1+2+3+4) \\ (1-2j-3+4j) \\ (1-2+3-4) \\ (1+2j-3-4j) \end{bmatrix} = \begin{pmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{pmatrix}$$

$$X_2(k) = \{10, -2+2j, -2, -2-2j\}$$

$$X_1(k) \cdot X_2(k) = \cancel{[6\ 0\ 2\ 0] \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}} = [60\ \ 0\ \ -4\ \ 0]$$

$$= [6\ 0\ 2\ 0] \cdot [10\ \ -2+2j\ \ -2\ \ -2-2j] = [60\ \ 0\ \ -4\ \ 0]$$

$$= \cancel{\{(60-8+8j-4+0)\ (6} = [60\ \ \cancel{-4+8j}\ 0\ \ -4\ \ 0]$$

$$\therefore X_1(k) \cdot X_2(k) = \cancel{\{60, -4+8j, -4, 0\}}\ \{60, 0, -4, 0\}$$

Now, IDFT of $X_1(k) \cdot X_2(k) = x'[n]$ (say).

$$x'[n] = \frac{1}{N}[W_N^*] X_N(k)$$

$$\begin{bmatrix} x'(0) \\ x'(1) \\ x'(2) \\ x'(3) \end{bmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix} \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix} = \frac{1}{4} \cancel{\begin{bmatrix} (60-4+8j-4) \\ (60-4j-8+4) \\ (60+4-8j-4) \\ (60+4j+8+4) \end{bmatrix}} = \frac{1}{4} \begin{bmatrix} 52+8j \\ 56-4j \\ 60-8j \\ 72+4j \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 13+2j \\ 14-1j \\ 15-2j \\ 18+j \end{bmatrix}} = \frac{1}{4} \begin{pmatrix} (60-4) \\ (60+4) \\ (60-4) \\ (60+4) \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 56 \\ 64 \\ 56 \\ 64 \end{pmatrix} = \begin{pmatrix} 14 \\ 16 \\ 14 \\ 16 \end{pmatrix}$$

$\therefore$ IDFT $(X_1(k) \cdot X_2(k)) = \cancel{\{13+2j, 14-j, 15-2j, 18+j\}} = x_1[n] \circledast x_2[n]$

$= \{14, 16, 14, 16\} = x_1[n] \circledast x_2[n]$.

$\therefore$ Circular Convolution of $x_1(n)$ and $x_2(n)$ is $\cancel{\{13+2j, 14-j, 15-2j, 18+j\}}$

$\{14, 16, 14, 16\}$

Ans