DSA-Assignment-1

Deadline: 20th March 2024

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1. Solve all the question and submit a handwritten document
2. Plagiarism will be penalised
3. Submit a pdf of the form **<roll_no>_dsa1.pdf**

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1 Even Odd Functions

1. A signal $x[n]$ is defined to be odd if $x[-n] = -x[n]$ for all n. The signal $x[n]$ is defined to be even if $x[-n] = x[n]$ for all n.

(a) Any signal $g[n]$ can be written as the sum of even and odd parts $g_e[n] + g_o[n]$. Find these parts in terms of $g[n]$.

(b) If $x[n]$ is an even signal and $y[n]$ is an odd signal, then show that $x[n]y[n]$ is an odd signal.

2 Fourier Series

1. What are Dirichlet Conditions? State them:
2. Give Formulas for Trigonometric Fourier Series and Exponential Fourier Series:
3. Find the Trigonometric Fourier Series of the following functions:

(a) Square wave function with period 2π: $f(x + 2\pi) = f(x)$

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x < \pi \\ -1 & \text{if } \pi \leq x < 2\pi \end{cases}$$

(b) $y = |t|$, where $-\pi \leq t < \pi$ and $y(t) = y(t + 2\pi)$.

(c) Give your observations about a_n and b_n in the above two questions

3 Fourier Transform - (CTFT,DTFT)

1. What are the formulas for DTFT and CTFT, and how do they differ in signal processing?

2. Consider a discrete-time signal $x[n]$ of length N. The DTFT of the signal is given by $X(e^{j\omega})$. Show that DTFT is periodic with period 2π.

3. What are symmetry properties of Fourier Transform ? Prove that Fourier transform of a real and even signal is real and even.

4. Prove below properties of CTFT
   (a) Linearity
   (b) Frequency Shifting
   (c) Time Reversal
   (d) Find Fourier Transform of the given signal (Hint: use duality property)

$$x(t) = \frac{\sin^2 kt}{t^2}$$

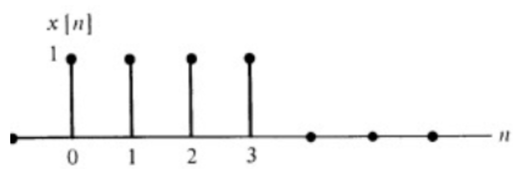
   (e) Use property of differentiation in time domain and frequency domain to find

$$\mathcal{CTFT}\left\{\frac{d^2}{dt^2}x(t-1)\right\}$$

   (f) Convolution Property
   (g) Parseval's Relation

5. Compute the DTFT for the following signals:

$$(a) \quad x[n] = \left(\frac{1}{4}\right)^n u[n+2]$$



(b)

4 Fourier Transform - (DFT)

1. Compute 8-point DFT for the signal $x(n) = \frac{1}{4}u(4-n)$, where $u(.)$ represents the unit step function.

2. Let $x[n]$ values be non-zero for $0 \leq n \leq N-1$, else zero. Let $y[n] = x[n] + x\left[n + \frac{N}{2}\right]$, $0 \leq n \leq N-1$ else zero and $Y(k)$ is the $\frac{N}{2}$-point DFT of $y[n]$. Then, what is the relation between $Y(k)$ and $X(k)$?

3. Given $X[k] = k^2$, $0 \leq k \leq 7$ be 8-point DFT of a sequence x[n],find the value of $\sum_{n=0}^{3} x[2n+1]$

4. Determine the Inverse Fourier transform of the following:
   (a)

$$X(e^{jw}) = cos^3 w + cos^2 w$$

   (b)

$$X(e^{jw}) = \frac{e^{-4jw} + e^{-3jw} - e^{-jw} - 1}{e^{-jw} + 1}$$

   (c)

$$X(e^{jw}) = \frac{3e^{-jw} - 1}{3 - e^{-jw}}$$

5. Let $x[n] = \{1, 2, 3, 6\}$ then

(a) Compute 6-point DFT of $x[n]$ and is represented as $X(k)$. Comment on the relation between:
- $X(1)$ and $X(5)$
- $X(2)$ and $X(4)$

(b) Compute 6-point DFT of $x[n-10]$ and is represented as $Y(k)$; What is relation between $Y(k)$ and $X(k)$;

(c) Obtain $y[n]$ by computing IDFT of $Y(k)$; What is the relation between $y[n]$ and $x[n]$?

(d) Find DFT of $x[n]cos(\frac{2\pi}{N}k_0 n)$ in terms of $X(k)$; here k_o is an integer constant.

5 Convolution

1. Find the Convolution of the following functions:-

(a)
$$f[n] = 2\delta[n+10] + 2\delta[n-10], \quad g[n] = 3\delta[n+5] + 2\delta[n-5]$$

(b)
$$f[n] = (-1)^n, \quad g[n] = \delta[n] + \delta[n-1]$$

6 Sampling

1. What is aliasing? What can be done to reduce aliasing?
   Let $x(t) = \frac{1}{2\pi}cos(4000\pi t)cos(1000\pi t)$ be a continuous-time signal. Find the Nyquist rate and Nyquist interval for this signal.
2. A waveform, $x(t) = 10cos(1000t + \pi/3) + 20cos(2000t + \pi/6)$ is to be uniformly sampled for digital transmission. What is the maximum allowable time interval between sample values that will ensure perfect signal reproduction? If we want to reproduce 1 hour of this waveform, how many sample values need to be stored?
3. Consider three signals $x_1(t)$ and $x_2(t)$ and $x_3(t)$ with Fourier transforms satisfying:

$$X_1(\Omega) = 0, |\Omega| \geq 120$$
$$X_2(\Omega) = 0, |\Omega| \leq 60, |\Omega| \geq 100$$

   Determine the minimum frequency f_s, at which we must sample the following signals to prevent aliasing.

   (a) $x(t) = x_1(t) + x_2(t)$

   (b) $x(t) = x_1(t)x_2(t)$

   (c) $x(t) = \cos(3.6\pi t + 9.23)$

7 Quantization

1. Consider the analog waveform $x(t)$ and answer the following questions.

$$x(t) = \begin{cases} -2\sin(\pi x/4) & 0 \leq x < 4 \\ x - 4 & 4 \leq x < 5 \\ 1 & 5 < x < 7 \\ 8 - x & 7 \leq x \leq 10 \end{cases}$$

It is sampled at 1000 Hz and quantized with a 2-bit quantizer with input range -2V to 2V.

(a) Indicate the sample points.

(b) State the quantization intervals and the corresponding digital words.

(c) Sketch the digital word assigned to each sample point.

(d) Indicate the stream of bits generated after the quantization is complete.

(e) What is the resulting bit rate?

(f) What is the quantization error?

Answer all of the above questions for a 3-bit quantizer as well.

2. Mention advantages/disadvantages of increasing quantization bits.