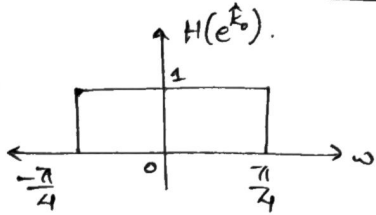


① Given:



$$\text{i.e., } H(e^{j\omega}) = \begin{cases} 1 & ; |\omega| \leq \frac{\pi}{4} \\ 0 & ; |\omega| > \frac{\pi}{4} \end{cases} = \begin{cases} 1 & ; \omega \in [-\frac{\pi}{4}, \frac{\pi}{4}] \\ 0 & ; \omega \notin [-\frac{\pi}{4}, \frac{\pi}{4}] \end{cases}$$

Step ①: Find Inverse DTFT: $h(n)$

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

$$h(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} (1) e^{j\omega n} d\omega$$

$$h(n) = \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\pi/4}^{\pi/4} = \frac{1}{2\pi} \left[\frac{e^{j\frac{\pi n}{4}}}{jn} - \frac{e^{-j\frac{\pi n}{4}}}{jn} \right]$$

$$h(n) = \frac{1}{2\pi j n} \left[e^{j\frac{\pi n}{4}} - e^{-j\frac{\pi n}{4}} \right]$$

$$h(n) = \frac{1}{2\pi j n} \left[\left(\cos\frac{\pi n}{4} + j \sin\frac{\pi n}{4} \right) - \left(\cos\frac{\pi n}{4} - j \sin\frac{\pi n}{4} \right) \right]$$

$$= \frac{1}{2\pi j n} \left[2j \sin\frac{\pi n}{4} \right] = \frac{\sin\frac{\pi n}{4}}{n\pi}$$

$$\therefore \boxed{h(n) = \frac{\sin\frac{\pi n}{4}}{n\pi}} \quad \leadsto \quad \text{approximate } \frac{\sin(\frac{\pi n}{4})}{n\pi} \text{ graph..}$$

(a) let $N=11$: (odd) (want symmetric).

If we take $N=0$ to $10 \rightarrow$ then it won't be symmetric.

So we take $N=-5$ to $5 \rightarrow$ Symmetric.

$$\Rightarrow h(-5), h(-4), \dots, h(0), \dots, h(4), \dots, h(5).$$

$$h(n) = \frac{\sin\frac{\pi n}{4}}{n\pi} \quad \text{for } |n| \leq 5.$$

We also know, that this approach suffers from lack of Causality

\therefore We multiply with a delay $\frac{-(N-1)}{2} \Rightarrow \frac{-(11-1)}{2} \Rightarrow -5$.

$$h(0) = \lim_{n \rightarrow 0} \frac{\sin n\pi/4}{n\pi} = \lim_{n \rightarrow 0} \frac{\sin n\pi/4}{n\pi/4} \cdot \frac{1}{4} = \frac{1}{4}$$

$$h(1) = h(-1) = 0.225$$

$$h(2) = h(-2) = 0.159$$

$$h(3) = h(-3) = 0.075$$

$$h(4) = h(-4) = 0$$

$$h(5) = h(-5) = -0.045$$

~~z-transform:~~

$$H(z) = \sum_{n=-5}^{n=5} h(n) \cdot z^n$$

z-transform $H(z) = \sum_{n=-5}^{n=5} h(n) \cdot z^n$

$$H(z) = h(0) + \sum_{n=1}^5 h(n)(z^n + \bar{z}^n)$$

$$H(z) = \frac{1}{4} + (0.225)(z + \bar{z}^1) + (0.159)(z^2 + \bar{z}^2) + (0.075)(z^3 + \bar{z}^3) + (0)(z^4 + \bar{z}^4) + (-0.045)(z^5 + \bar{z}^5)$$

$$H(z) = \frac{1}{4} +$$

But it's not Causal!

\therefore Multiply with delay \bar{z}^5

$$H(z) = \frac{1}{4} \bar{z}^5 + (0.225)(\bar{z}^4 + \bar{z}^6) + (0.159)(\bar{z}^3 + \bar{z}^7) + (0.075)(\bar{z}^2 + \bar{z}^8) + (-0.045)(\bar{z}^0 + \bar{z}^{10})$$

$$H(z) = -0.045 + 0.075z^{-2} + 0.225z^{-4} + 0.225z^{-6} +$$

$$H(z) = -0.045 + 0.075z^{-2} + 0.159z^{-3} + 0.225z^{-4} + 0.25z^{-5} \\ + 0.225z^{-6} + 0.159z^{-7} + 0.075z^{-8} + (-0.045)z^{-10}$$

$$\therefore h'(n) = \{-0.045, 0, 0.075, 0.159, 0.225, 0.25, 0.225, 0.159, 0.075, \\ -0, -0.045\}$$

↳ without window.

(b) we know, Hamming window: $w_H(n) = \begin{cases} 0.54 - 0.46 \cos\left(2\pi \cdot \frac{n}{N-1}\right), & |n| \leq \frac{N-1}{2} \\ 0, & \text{o/w} \end{cases}$ where $N=8$

$$w_H(n) = \begin{cases} 0.54 - 0.46 \cos\left(2\pi \cdot \frac{n}{N-1}\right); & |n| \leq 4 \\ 0, & \text{o/w} \end{cases}$$

$$\therefore h(n) = h'(n) \cdot w_H(n)$$

$$\cancel{h(0) = h'(0) \cdot w_H(0) = (-0.045)(0.08) =}$$

$$h'(0) = \lim_{n \rightarrow 0} \frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi} = \lim_{n \rightarrow 0} \frac{\sin\left(\frac{n\pi}{4}\right)}{\frac{n\pi}{4}} \cdot \frac{1}{4} = \frac{1}{4} \quad (\text{L'Hospital rule})$$

$$h'(1) = h'(-1) = 0.225$$

$$h'(2) = h'(-2) = 0.159$$

$$h'(3) = h'(-3) = 0.075$$

$$h'(4) = h'(-4) = 0$$

$$h'(5) = h'(-5) = -0.045$$

$$\therefore h(0) = h'(0) \cdot w_H(0) = \frac{1}{4} \cdot 0.08 = 0.02$$

$$h(1) = h'(1) \cdot w_H(1) = (0.225) \left(0.54 - \frac{0.46}{\sqrt{2}}\right) = 0.0483 = h(-1)$$

$$h(2) = h'(2) \cdot w_H(2) = 0.0859 = h(-2)$$

$$h(3) = h'(3) \cdot w_H(3) = 0.0649 = h(-3)$$

$$h(4) = h'(4) \cdot w_H(4) = 0$$

$$\therefore h(n) = \{0, 0.0649, 0.0859, 0.0483, 0.02, 0.0483, 0.0859, 0.0649, 0\}$$

↑

(2) Given, H_d is LPF.

and Rectangular window $w(n) = \begin{cases} 1 & \text{if } n=0,1,2 \\ 0 & \text{o/w} \end{cases} \Rightarrow \underline{\text{window length} = 3 = L}$

We know, FIR filter $h(n) = w(n) h_d(n)$.

where, $h_d(n) \rightarrow$ Ideal lowpass filter impulse response

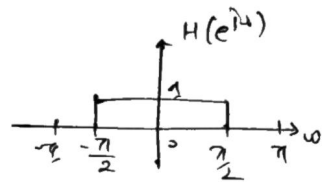
$w(n) \rightarrow$ rectangular window

$h(n) \rightarrow$ windowed impulse response

~~We know, $h_d(n) = \frac{\sin(\omega_c n)}{\pi n}$~~

1) Impulse Response for Ideal lowpass filter:

$$H_d(e^{j\omega}) = \begin{cases} 1 & -\frac{\pi}{2} \leq \omega \leq \frac{\pi}{2} \\ 0 & \text{o/w} \end{cases}$$



$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega n} d\omega$$

$$h_d(n) = \frac{1}{2\pi j n} \left[e^{j\omega n} \right]_{-\pi/2}^{\pi/2} = \frac{1}{2\pi j n} \left(2j \sin \frac{\pi n}{2} \right) = \frac{\sin\left(\frac{\pi n}{2}\right)}{\pi n}$$

$$\therefore h_d(n) = \frac{\sin\left(\frac{n\pi}{2}\right)}{\pi n} \rightarrow \text{Impulse Response of LPF.}$$

here cutoff frequency is $\frac{\pi}{2} = \omega_c$.

$$\therefore h(n) = w(n) \cdot h_d(n)$$

$$= h[0] + \dots$$

$$\boxed{h(n) = h_d[0] + h_d[1] + h_d[2]} \rightarrow \text{windowed Impulse Response}$$

$$h(n) = 0 + \frac{1}{\pi} + \frac{j}{2} (0) \Rightarrow h(n) =$$

$$h[0] = w[0] h_d[0] = (1)(0) = 0; \quad h[1] = \frac{1}{\pi}; \quad h[2] = 0$$

Now, Fourier Transform of $h[n]$ be $H(e^{j\omega})$.

$$\therefore H(e^{j\omega}) = \sum_{n=0}^{n=2} h[n] \cdot e^{-j\omega n} =$$

$$H(e^{j\omega}) = h[0] \cdot e^0 + h[1] e^{-j\omega} + h[2] e^{-2j\omega}$$

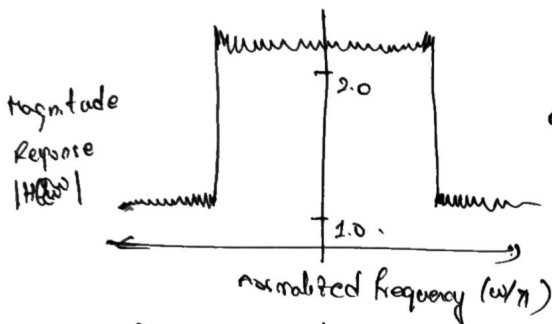
$$H(e^{j\omega}) = (0)(1) + \frac{1}{\lambda} \cdot e^{-j\omega} + (0)(e^{-2j\omega})$$

$$\boxed{H(e^{j\omega}) = \frac{1}{\lambda} e^{-j\omega}}$$

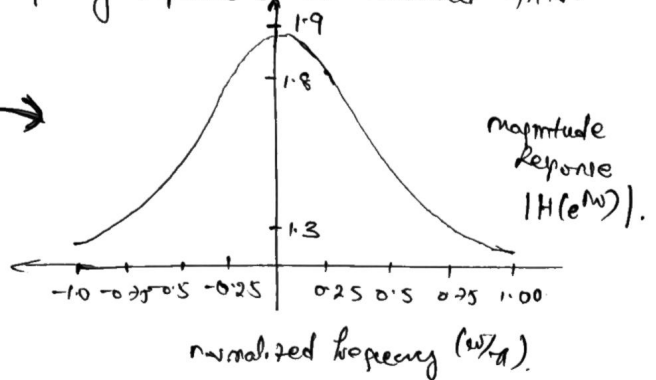
for general cut-off frequency ω_c .

$$H(e^{j\omega}) = \frac{\omega_c}{\pi} \frac{e^{-j\omega(0)}}{e^{-j\omega(0)}} + \frac{\sin(\omega_c)}{\pi} e^{-j\omega} + \frac{\sin(2\omega_c)}{2\pi} e^{-2j\omega}$$

The Plot of $H(e^{j\omega})$ will show the frequency response of the windowed filter.



On comparing with



On comparing both the figures.

OBSERVATIONS

- ① The main lobe is wider, leading to a less sharp transition from the passband to the stopband.
- ② Ripples or oscillations will appear in the stopband, known as Gibbs phenomena.
- ③ The stopband attenuation is lower than the ideal LPF.

2) We have, Rectangular window $w[n]$ is used, and the window length is $L=3$.

When the window is applied to the infinite impulse response $h_d[n]$ of an ideal low-pass filter (LPF), the windowed

Impulse response $h(n)$ becomes $h(n) = w(n) * h_d(n)$

\Rightarrow Windowed impulse response $h(n)$ has non-zero values only for $n = 0, 1, 2$, and is zero for all other values of n .

Therefore, the length of the non-zero part of $h(n)$ is 3.

which corresponds to the window length $L = 3$.

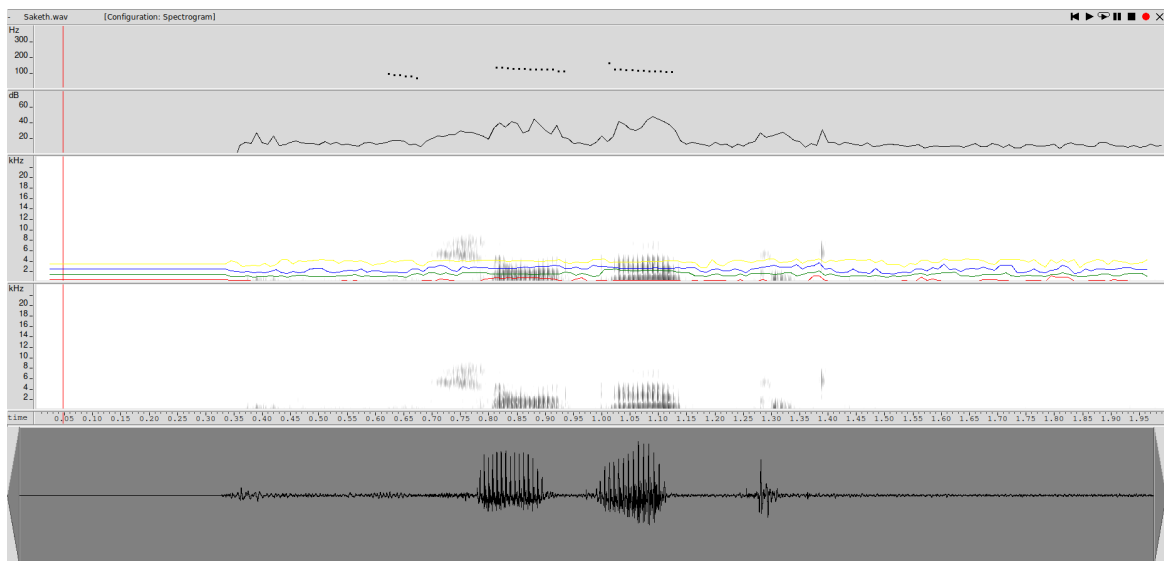
Since, the windowing method produces a finite impulse response (FIR) filter with the same length as the window, the resulting filter length M , is also equal to 3

$$L = M = 3$$

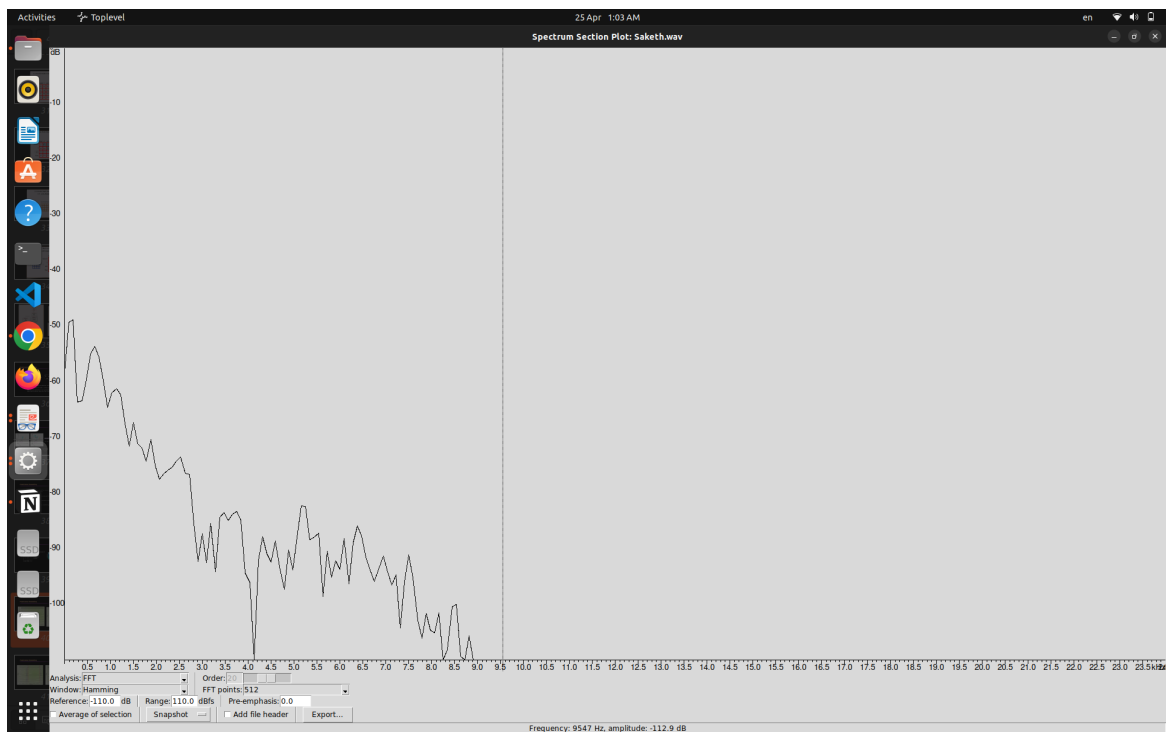
Wavesurfer - Saketh

Observations:

- Phonemes vs Timestamps:
"sa" : 0.725 - 0.94 s
"ke" : 0.96 - 1.16 s
"th" : 1.23 - 1.40 s
- Different plots on Wave surfer:
order:
i) pitch plot
ii) power plot
iii) Formants
iv) spectrogram



FFT spectrum:



LPC spectrum:

