Digital Signal Analysis.
ASSIGNMENT-1

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Question-1 : Even Odd functions :

1. Given, $x[n]$ is an odd signal if $x[-n] = -x[n] \quad \forall n$.
$x[n]$ is ~~ven~~ even signal if $x[-n] = x[n] \quad \forall n$.

(a) To show, any signal $g[n]$ can be written as sum of even or odd parts
ie., $g[n] = g_e[n] + g_o[n]$

let $g[n] = g_e[n] + g_o[n] \longrightarrow (1)$ where $g_e[n] \rightarrow$ even signal
$g_o[n] \rightarrow$ odd signal.

then, $g[-n] = g_e[-n] + g_o[-n]$

$\Rightarrow g[-n] = g_e[n] - g_o[n] \longrightarrow (2)$

Adding (1) and (2) ; $g[n] + g[-n] = 2g_e[n]$

$$\Rightarrow g_e[n] = \frac{g[n] + g[-n]}{2} \longrightarrow (3)$$

Subtracting (2) from (1); $g[n] - g[-n] = 2g_o[n]$

$$\Rightarrow g_o[n] = \frac{g[n] - g[-n]}{2} \longrightarrow (4)$$

$\therefore$ Any signal $g[n]$ can be written as sum of even and odd parts $g_e[n]$ and $g_o[n]$

where $g_e[n] = \dfrac{g[n] + g[-n]}{2}$; $g_o[n] = \dfrac{g[n] - g[-n]}{2}$ Ans.

(b) Given, $x[n]$ is even signal $\Rightarrow x[-n] = x[n] \longrightarrow (1)$
$y[n]$ is an odd signal $\Rightarrow y[-n] = -y[n] \longrightarrow (2)$

To Show; $x[n]y[n]$ is an odd signal ie., $x[-n]y[-n] = -x[n]y[n]$ $\hookrightarrow (3)$

LHS: $x[-n]y[-n] = x[n](-y[n])$ using (1) and (2)

$\Rightarrow x[-n]y[-n] = -x[n].y[n]. \Rightarrow (3)$

$\therefore$ $x[n]y[n]$ is an odd signal if $x[n]$ is even signal and $y[n]$ is odd signal.

Question-2 : Fourier Series.

1 DIRICHLET CONDITIONS :

→ these conditions provides the conditions that are necessary to be satisfied by a Periodic Signal for existence of Fourier Series.

There are four Conditions :

let $f(t)$ be the Periodic Signal of which we aim to derive the Fourier Series. ~~then,~~ and (t_1, t_2) is the range of time Period.

(1) $f(t)$ should be ~~sign~~ single valued in time Interval (t_1, t_2)

(2) $f(t)$ should have finite number of discontinous over the range of time period (t_1, t_2)

(3) $f(t)$ should have finite no. of maxima's and minima's over the range of time period (t_1, t_2).

(4) $f(t)$ should be absolutely integrable over the range of time period. (t_1, t_2)

i.e,
$$\int_{t_1}^{t_2} |f(t)|\, dt < \infty.$$

2 Trigonometric Fourier Series :- Consider a signal $f(t)$ with time Period T_0'.

$$f(t) = \underbrace{a_0}_{1} + \underbrace{\sum_{n=1}^{\infty} a_n \cos(n\omega_0 t)}_{2} + \underbrace{\sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)}_{3}.$$

where, $a_0 \rightarrow$ d.c / Avg value of $f(t)$

2 ⟶ cosine term

3 ⟶ sin term..

$$\text{where, } a_0 = \frac{1}{T_0}\int_{T_0} f(t)\,dt = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_{T_0} f(t)\cos(n\omega_0 t)\,dt = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} f(t)\sin(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T_0}\int_{T_0} f(t)\sin(n\omega_0 t)\,dt = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} f(t)\sin(n\omega_0 t)\,dt$$

where a_0, a_n, b_n are the Fourier Coefficients.

Complex Exponential Fourier Series :-

$$f(t) = \sum_{n=-\infty}^{\infty} F_n \cdot e^{jn\omega_0 t}$$

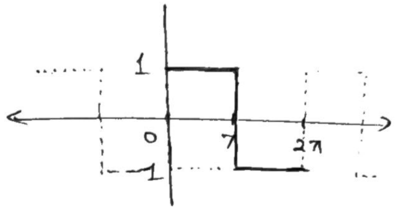
where $F_n \rightarrow$ Complex Exponential Fourier Series.

and, $F_n = \frac{1}{T_0} \int_{T_0} f(t) e^{-jn\omega_0 t} dt = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} f(t) e^{-jn\omega_0 t} dt$ {where $T_0 \rightarrow$ Time Period of $f(t)$. }.

3 (a) $f(x+2\pi) = f(x)$ (Given).

$T = 2\pi$ (Given).

$$f(x) = \begin{cases} 1 & ; x \in [0, \pi) \\ -1 & ; x \in [\pi, 2\pi). \end{cases}$$



By definition,

$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$. where $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi}$

$\boxed{\omega_0 = 1}$

clearly, $f(x)$ is an odd function (because $\cancel{f(x)} = f(-x) = f(x)$) symmetric about origin.

$a_n = 0$; $b_n \neq 0$.

And $a_0 \neq 0$.

$\therefore a_0 = \frac{1}{T} \int_{T_0} f(x)\, dx = \frac{1}{T} \int_0^T f(x) dx = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx = 0.$

$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(n\omega_0 t) dt = \frac{1}{\pi} \left[\int_0^{\pi} \cos(n\omega_0 t) dt + \int_{\pi}^{2\pi} \cos(n\omega_0 t) dt \right] = 0.$

$b_n = \frac{2}{T} \int_T f(x) \sin(n\omega_0 x) dx = \frac{2}{2\pi} \int_0^{2\pi} f(x) \cdot \sin(n\omega_0 x) dx$

$= \frac{1}{\pi} \left(\int_0^{\pi} \sin(n\omega_0 x) dx + \int_{\pi}^{2\pi} (-1) \sin(n\omega_0 x) dx \right)$

$b_n = \frac{1}{\pi} \left[\left(\frac{-\cos(nx)}{n} \right)_0^{\pi} - \left[\frac{\cos(nx)}{n} \right]_{\pi}^{2\pi} \right].$

$b_n = \frac{1}{\pi} \left[\left[\frac{-\cos n\pi + 1}{n} \right] + \left[\frac{\cancel{\cos 2\pi n}^{1} - \cos n\pi}{n} \right] \right]$

$b_n = \frac{1}{n\pi} \left[2 - 2\cos n\pi \right] = \frac{1}{n\pi} \left[2 - 2\cos(n\pi) \right]$

if $n \rightarrow$ odd ; $b_n = \frac{1}{n\pi} \left[2 - (2)(-1) \right] = \frac{4}{n\pi}$

$n \rightarrow$ even : $b_n = 0$

$$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)dx = \sum_{n=1}^{\infty} b_n \sin(nx)dx \qquad \boxed{\because \quad \omega_0 = 1.}$$

$$\cancel{f(x)=}\ b_n = \begin{cases} \frac{4}{n\pi} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even.} \end{cases}$$

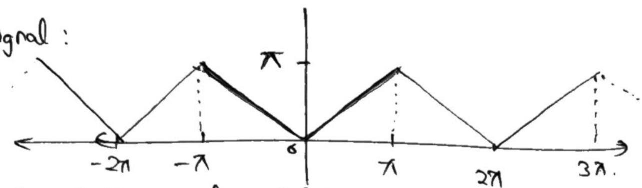
$$\therefore f(x) = \frac{4}{\pi}\left(\frac{\sin(x)}{1} + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \cdots\right) + 0\left(\frac{\sin(2x)}{2} + \frac{\sin(4x)}{4} + \frac{\sin(6x)}{6} + \cdots\right)$$

$$= \frac{4}{\pi}\left(\frac{\sin(x)}{1} + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \cdots\right)$$

(b) Given, $y = |t|$, where $-\pi \le t < \pi$ and $y(t) = y(t+2\pi)$.

$\Rightarrow$ Time Period, $T = 2\pi \longrightarrow (1)$

Signal :



According to Trigonometric F.S :

$$y(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t)dt + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)dt$$

$$\text{where } \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \Rightarrow \omega_0 = 1$$

$$\therefore y(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nt) + \sum_{n=1}^{\infty} b_n \sin(nt).$$

$$a_0 = \frac{1}{T}\int_{-\pi}^{\pi} y(t)dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} y(t)dt = \frac{1}{2\pi}\left[2 \times \frac{1}{2} \times \pi \times \pi\right] = \frac{\pi}{2} \Rightarrow \boxed{a_0 = \frac{\pi}{2}}$$

$$a_n = \frac{2}{T}\int_{-\pi}^{\pi} y(t)\cdot\cos(nt)dt = \frac{2}{2\pi}\int_{-\pi}^{\pi} y(t)\cdot\cos(nt)dt.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} (-t)\cos(nt)dt + \int_{0}^{\pi} (t)\cos(nt)dt\right]$$

XX

$$= \frac{1}{\pi}\left(-\left[\frac{\sin(nt)}{n}\right]_0^{-\pi} + \left[\frac{\sin(nt)}{n}\right]_0^{\pi}\right]$$

$$= \frac{1}{n\pi}\left[(\sin(-n\pi) - 0) + (\sin(n\pi) - 0)\right]$$

$$a_n = \frac{1}{n\pi}(0) = 0. \qquad \boxed{a_n = 0}$$

$$b_n = \frac{2}{T}\int_{-\pi}^{\pi} y(t)\cdot\sin(nt)dt = \frac{1}{\pi}\int_{-\pi}^{\pi} y(t)\sin(nt)dt.$$

$\cancel{b_n} = \frac{1}{\pi}\left[\int_{-\pi}^{0} f(-t) \right.$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-t)\cos(nt)\,dt + \int_{0}^{\pi}(t)\cos(nt)\,dt \right]$$

To Solve $\int t\cos(nt)\,dt = (t)\int \cos(nt)\,dt - \int\int \cos(nt)\,dt\,dt$

$$= (t)\,\frac{\sin(nt)}{n} - \int \frac{\sin(nt)}{n}\,dt$$

$$\int t\cos(nt)\,dt = \frac{t\cdot\sin(nt)}{n} + \frac{\cos(nt)}{n^2}.$$

$$\therefore a_n = \frac{1}{\pi}\left(-\left[\frac{t\cdot\sin(nt)}{n} + \frac{\cos(nt)}{n^2} \right]_{-\pi}^{0} + \left[\frac{t\sin(nt)}{n} + \frac{\cos(nt)}{n^2} \right]_{0}^{\pi} \right)$$

$$a_n = \frac{1}{\pi}\left[\cancelto{0}{\frac{-t\sin(n\pi)}{n}} + \frac{\cos(n\pi)}{n^2} + \cancelto{0}{\frac{t\sin(n\pi)}{n}} + \frac{\cos(n\pi)}{n^2} - \frac{1}{n^2} - \frac{1}{n^2} \right].$$

$$a_n = \frac{1}{\pi}\left[\frac{2\cos(n\pi)}{n^2} - \frac{2}{n^2} \right] = \frac{2}{n^2\cdot\pi}\left[\cos(n\pi) - 1\right].$$

$$a_n = \begin{cases} \frac{2}{n^2\pi}\cdot(-2) & ;\ \text{if } n \text{ is odd} \\ 0 & ;\ \text{if } n \text{ is even} \end{cases} = \begin{cases} \frac{-4}{n^2\pi} & ;\ \text{if } n \text{ is odd} \\ 0 & ;\ \text{if } n \text{ is even} \end{cases}$$

$$b_n = \frac{2}{T}\int_{-\pi}^{\pi} y(t)\sin(nt)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} y(t)\sin(nt)\,dt.$$

$$b_n = \frac{1}{\pi}\left\{ \int_{-\pi}^{0}(-t)\sin(nt)\,dt + \int_{0}^{\pi}(t)\sin(nt)\,dt \right\}$$

To Slved: $\int t\sin nt\,dt = t\int \sin(nt)\,dt + \int\int \sin(nt)\,dt\cdot dt$

$$= t\cdot\frac{-\cos nt}{n} - \int \frac{\cos nt}{n}\,dt$$

$$= \frac{-t\cos(nt)}{n} - \frac{1}{n^2}\sin(nt).$$

$$\therefore b_n = \frac{1}{\pi}\left\{ -\int_{0}^{\pi}(t)\sin(nt)\,dt + \int_{0}^{\pi}(t)\sin(nt)\,dt \right] = 0.$$

$b_n = 0.$

$$\therefore y(t) = \frac{\pi}{2} + \sum_{n=1}^{\infty} a_n\cos(nt) + 0 = \frac{\pi}{2} + \left\{ \frac{-4}{\pi}\left(\frac{\cos t}{1} + \frac{\cos 3t}{3^2} + \frac{\cos 5t}{5^2} + \cdots \right) \right\}$$

$$+ \left\{ 0\left(\cos 2t + \cos 4t + \cdots\right) \right\}$$

$y\cancel{(t) = \frac{\pi}{2}} + \cancel{\sum_n}$

$$y(t) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos t + \frac{\cos 3t}{3^2} + \frac{\cos 5t}{5^2} + \frac{\cos 7t}{7^2} + \cdots\right).$$

(c) observations :

If Signal is odd signal like $f(x) \Rightarrow$ $a_n = 0$; $\cancel{b_n \neq 0}$.
~~can be in special cases~~

If Signal is even signal like $y(t) \Rightarrow$ $\cancel{a_n \neq 0}$; $b_n = 0$.

Question 3 : Fourier Transform - (CTFT, DTFT)

① Both CTFT and DTFT are mathematical tools used to analyze signals in its frequency domain ~~used~~ in signal processing.

(i) DTFT (Discrete-time Fourier transform) :

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}.$$

where $\omega \rightarrow$ angular frequency
$j \rightarrow$ complex number.
$X(e^{j\omega}) \rightarrow$ complex valued function of frequency.

(ii) CTFT (Continous-Time Fourier Transform) :

$$\underbrace{X(\omega)}_{} = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$$

$\omega \rightarrow$ angular frequency

also sometimes denoted as $X(j\omega)$.

DIFFERENCES between DTFT and CTFT :-

① DTFT operates on discrete-time signals whereas CTFT operates on Continous-time signals.

② In CTFT, the frequency variable ω is continous and measured in radians per second whereas In DTFT, the frequency variable ω is discrete and measured in radians per sample.

③ CTFT ~~is~~ involves Integration over the entire time domain, whereas ~~DFT~~ DTFT involves summation over entire time domain.

4. Both the samples are related through sampling process.
CTFT is used for Analog signal processing whereas DTFT is used in Digital Signal Processing.
Analog Signal Processing can Include audio signals and control systems, whereas Digital Signal Processing can Include digital filtering, spectrum Analysis and signal reconstruction from samples.

2. Given, discrete time signal $x[n]$ of length N..
To show: DTFT ~~is~~ $X(e^{j\omega})$ is periodic with period 2π.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}.$$

Since, $x[n]$ is of length N, $X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n] e^{-j\omega n} + \cancel{\sum_{n=-\infty}^{0} x[n] e^{-j\omega n}}^{\,0} + \cancel{\sum_{n=N}^{\infty} x[n] e^{-j\omega n}}^{\,0}$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n] e^{-j\omega n}.$$

Put $\omega \to \omega + 2\pi$ :: $X(e^{j(\omega+2\pi)}) = \sum_{n=0}^{N-1} x[n] e^{-j(\omega+2\pi)n}$.

$$= \sum_{n=0}^{N-1} x[n] e^{-j\omega n} \cdot \cancel{e^{-2\pi n}} \to 1 \quad \because \cos(-2\pi n) - j\sin(2\pi n) = 1.$$

$$= \sum_{n=0}^{N-1} x[n] e^{-j\omega n}.$$

$\therefore X(e^{j\omega}) = X(e^{j(\omega+2\pi)})$.

$\therefore X(e^{j\omega})$ is Periodic with Period '2π'.

③ Symmetric Properties of Fourier Transform:

→ It relates the real and Imaginary part of signal.

Case ①: Signal is real. $x(t) \to$ signal.

$z = a + ib$, $a, b \in R$, $i^2 = -1$

$z = a + ib$

$z^* = a - ib$

$z - z^* = 2ib = 0$

$\Rightarrow \boxed{z = z^*}$

Similarly, $x(t) = x^*(t) \to ①$

$CTFT(x(t)) = X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$.

$\omega \to -\omega$.

$X(-\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{+j\omega t} dt = \int_t x^*(t) (e^{-j\omega t})^* dt$

$= \cancel{X^*(\omega)} \left(\int_t x(t) \cdot e^{-j\omega t} dt \right)^*$

$= X^*(\omega)$.

∴ If $x(t)$ is real, $X(-\omega) = X^*(\omega)$.

$\boxed{X(\omega) = X^*(-\omega)}$

Case ②: Signal is imaginary. ∴ $z = a + ib$; $z^* = a - ib \Rightarrow z + z^* = 2a = 0 \Rightarrow \boxed{z = -z^*}$

∴ ~~If~~ $x(t) = -x^*(t) \to ②$

$CTFT(x(t)) = X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$X(-\omega) = \int_t x(t) \cdot e^{j\omega t} dt = \int_t -x^*(t) (e^{-j\omega t})^* dt = -\left(\int_t x(t) e^{-j\omega t} dt \right)^* = -X^*(\omega)$

∴ If signal is Imaginary $\boxed{X(\omega) = -X^*(-\omega)}$

~~case 2a: x(t) is even also~~

Consider: CASE ①: $x(t)$ is real: $X(\omega) = X^*(-\omega) \to ③$

case ⓐ: $x(t)$ is even also.

$\Rightarrow x(t) = x(-t)$ and $x(t) = x^*(t)$.

$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$X(-\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-(-j\omega t)} dt = X(\omega)$.

∴ $X(\omega) = X(-\omega) \Rightarrow$ Even

$X^*(\omega) = X^*(-\omega)$.

From ③: $X(\omega) = X^*(\omega) \Rightarrow$ Real

$\Rightarrow Im(X(\omega)) = 0$

∴ $X(\omega)$ is real and even too.

case ⓑ: $x(t)$ is odd.

$x(t) = -x(-t)$, $x(t) = x^*(t)$.

$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$X(-\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-(-j\omega t)} dt$

$-t = \tau$

$d\tau = -dt$

$= -\int_{+\infty}^{-\infty} x(\tau) e^{-j\omega \tau} (-d\tau) = -X(-\omega)$

$X(\omega) = X^*(-\omega)$

$X(\omega) = -X(-\omega)$.

Consider : Case (2) : $x(t)$ is Imaginary , $X(\omega) = -X^*(-\omega) \to (4)$

Case (a): Even also

$$\Rightarrow x(-t) = x(t) \ , \ x(t) = -x^*(t).$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \Rightarrow X(-\omega) = \int_{-\infty}^{\infty} x(-t) e^{-(-j\omega t)} dt = X(\omega)$$

$$\Rightarrow X^*(-\omega) = X^*(\omega) \Rightarrow \text{from (4)}, \ X(\omega) = -X^*(\omega). \Rightarrow Re(X(\omega)) = 0$$

Case (b): odd too

$$\Rightarrow x(-t) = -x(t)$$

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \Rightarrow X(-\omega) = \int_{-\infty}^{\infty} -x(-t) \, e^{j\omega t} dt$$

$$X(-\omega) = -X(\omega) \Rightarrow X^*(-\omega) = -X^*(\omega)$$

$$\text{from (4)}: X(\omega) = -(-X^*(\omega)) = X^*(\omega) \Rightarrow \boxed{Im(X(\omega)) = 0.}$$

TO PROVE: Fourier transform of real and even signal is real and even.

$\underline{x(t) = x(-t)}$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$X(-\omega) = \int_{-\infty}^{\infty} x(t) \, e^{j\omega t} dt.$$

let $t = -\tau \Rightarrow dt = -d\tau$.

$$X(-\omega) = \int_{\tau=+\infty}^{-\infty} x(-\tau) \, e^{-j\omega\tau} (-d\tau). = \int_{\tau=-\infty}^{+\infty} x(\tau) \cdot e^{-j\omega\tau} d\tau. \Rightarrow \boxed{X(-\omega) = X(\omega)} \to (5)$$

$\underline{x(t) = x^*(t)}$ $\quad X(\omega) = X^*(-\omega)$.

$\underline{\omega \Leftrightarrow -\omega}$. $\quad X(-\omega) = X^*(\omega)$.

$$\text{from (5)}. \ \boxed{X(\omega) = X^*(\omega)} \to (6)$$

From (5), (6) <u>Proved</u>

4 (a) $\boxed{\text{Linearity}}$:

$$x_1(t) \rightleftharpoons X_1(j\omega)$$
$$x_2(t) \rightleftharpoons X_2(j\omega)$$
$$\alpha x_1(t) \rightleftharpoons \alpha X_1(j\omega)$$
$$\beta x_2(t) \rightleftharpoons \beta X_2(j\omega).$$

$$\alpha x_1(t) + \beta x_2(t) \rightleftharpoons \alpha X_1(j\omega) + \beta X_2(j\omega)$$

Proof: we know $x(t) \rightleftharpoons X(j\omega)$ where $x(t) = \alpha x_1(t) + \beta x_2(t)$,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} [\alpha x_1(t) + \beta x_2(t)] \cdot e^{-j\omega t} dt$$

$$= \alpha \underbrace{\int_{-\infty}^{\infty} x_1(t) \cdot e^{-j\omega t} dt}_{X_1(j\omega)} + \beta \underbrace{\int_{-\infty}^{\infty} x_2(t) \cdot e^{-j\omega t} dt}_{X_2(j\omega)}.$$

$$X(j\omega) = \alpha X_1(j\omega) + \beta X_2(j\omega).$$

$$\therefore \alpha x_1(t) + \beta x_2(t) \rightleftharpoons \alpha X_1(j\omega) + \beta X_2(j\omega).$$

~~(b) Frequency Shifting:-~~

~~(b)~~

(c) $\boxed{\text{Time Reversal}}$:-

$$x(t) \underset{IFT}{\overset{FT}{\rightleftharpoons}} X(j\omega).$$
$$x(-t) \rightleftharpoons X(-j\omega).$$

Proof: $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$t \to -t$: $x(t) \to x(-t) \rightleftharpoons X'(j\omega) = ?$

$$X'(j\omega) = \int_{-\infty}^{\infty} x(-t) e^{-j\omega t} dt$$

let say $-t = \tau \Rightarrow t = -\tau \Rightarrow dt = -d\tau$.

$$\Rightarrow X'(j\omega) = -\int_{\infty}^{-\infty} x(\tau) \cdot e^{+j\omega\tau} d\tau.$$

$$\Rightarrow X'(j\omega) = \int_{-\infty}^{\infty} x(\tau) \cdot e^{j\omega\tau} d\tau. = \int_{-\infty}^{\infty} x(\tau) e^{-j(\omega \times -1)\tau} d\tau.$$

Substitute $t \to \tau \Rightarrow X'(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j(-\omega)t} dt \Rightarrow x(-t) \underset{IFT}{\overset{FT}{\rightleftharpoons}} X(j(-\omega))$

(b) $\boxed{\text{Frequency Shifting}}$:-

$$x(t) \overset{FT}{\rightleftharpoons} X(j\omega).$$

$$e^{\pm j\omega_0 t} \cdot x(t) \overset{FT}{\rightleftharpoons} X[j(\omega \mp \omega_0)]$$

i.e., $$e^{\pm j\omega_0 t} \cdot x(t) \overset{FT}{\rightleftharpoons} X[j(\omega \mp \omega_0)].$$

Proof: $$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$e^{+j\omega_0 t} \cdot x(t) \rightleftharpoons X'(j\omega) \quad (\text{say}).$$

$$X'(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{+j\omega_0 t} \cdot e^{-j\omega t} dt$$

$$X'(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j(\omega-\omega_0)t} dt = X[j(\omega-\omega_0)]$$

$\therefore$ $X'(j\omega) = X[j(\omega-\omega_0)]$

$\therefore$ $e^{\pm j\omega_0 t} \cdot x(t) \rightleftharpoons X[j(\omega \mp \omega_0)]$.

(f) $\boxed{\text{Convolution Property}}$:-

$$x_1(t) \rightleftharpoons X_1(j\omega) \quad (\text{Given})$$

$$x_2(t) \rightleftharpoons X_2(j\omega). \quad (\text{Given}),$$

$$x_1(t) * x_2(t) \rightleftharpoons X_1(j\omega) \cdot X_2(j\omega). \quad (\text{To prove})$$

let ~~$X_1(j\omega)$~~ $x_1(t) * x_2(t) \rightleftharpoons X'(j\omega)$.

$$X'(j\omega) = \int_{-\infty}^{\infty} (x_1(t) * x_2(t)) e^{-j\omega t} dt \quad (\text{By definition})$$

we know, $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ and $x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) d\tau$.

$$\therefore X'(j\omega) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) d\tau \right) e^{-j\omega t} dt.$$

let $t - \tau = d \Rightarrow t = \tau + d \Rightarrow dt = dd$.

$$\Rightarrow X'(j\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(d) \cdot d\tau \cdot e^{-j\omega(\tau+d)} dd.$$

$$X'(j\omega) = \int_{-\infty}^{\infty} x_1(\tau) e^{-j\omega\tau} d\tau \cdot \int_{-\infty}^{\infty} x_2(d) e^{-j\omega d} dd$$

$$X'(j\omega) = X_1(j\omega) \cdot X_2(j\omega) \quad \therefore \boxed{x_1(t) * x_2(t) \rightleftharpoons X_1(j\omega) \cdot X_2(j\omega)}$$

(9) $\boxed{\text{Parseval's Relation}}$:-

$$x(t) \rightleftharpoons X(j\omega)$$

$$E_{x(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega.$$ is the total Energy of $x(t)$.

Proof: $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t} \, d\omega.$$

$$x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega)\, e^{-j\omega t} \, d\omega.$$

$$E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$ ∵ we know: $|x(t)|^2 = x(t) \cdot x^*(t)$.
∵ By definition

$$= \int_{-\infty}^{\infty} x(t) \cdot x^*(t) \, dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot \left\{ \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega)\, e^{-j\omega t} d\omega \right\} dt$$

$$E_{x(t)} = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} X^*(j\omega) \underbrace{\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt}_{F.T} \cdot d\omega$$

$$E_{x(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega) \cdot X(j\omega)\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega.$$

(e) differentiation in time domain : if $x(t) \longleftrightarrow X(\omega)$, then

$$\frac{d(x(t))}{dt} \longleftrightarrow (j\omega)X(\omega)$$

differentiation in frequency domain : if, $x(t) \longleftrightarrow X(\omega)$, then,

$$(-jt)x(t) \longleftrightarrow \frac{d(X(\omega))}{d\omega}$$

let $\quad x(t) \xleftrightarrow{FT} X(\omega)$.

then $\quad x(t-1) \longleftrightarrow e^{-j\omega}X(\omega) \quad$ (time shift property)

$$\therefore \frac{d(x(t-1))}{dt} = \frac{d}{dt}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}(e^{-j\omega}X(\omega))\,e^{j\omega t}\,d\omega\right).$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j\omega}X(\omega)(j\omega)\,e^{j\omega t}\,d\omega.$$

$$= \cancel{\frac{d}{}\ (j\omega)\ \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j\omega}X(\omega)\,e^{j\omega t}\,d\omega}_{x(t-1)}}.$$

$$\frac{d(x(t-1))}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{e^{-j\omega}X(\omega)(j\omega)}\,e^{j\omega t}\,dt$$

$$\therefore \frac{d(x(t-1))}{dt} \xleftrightarrow{FT} (j\omega)\cdot e^{-j\omega}X(\omega).$$

$$\therefore \boxed{\frac{d^2(x(t-1))}{dt^2} \xleftrightarrow{F.T} (j\omega)^2 e^{-j\omega}X(\omega).}$$

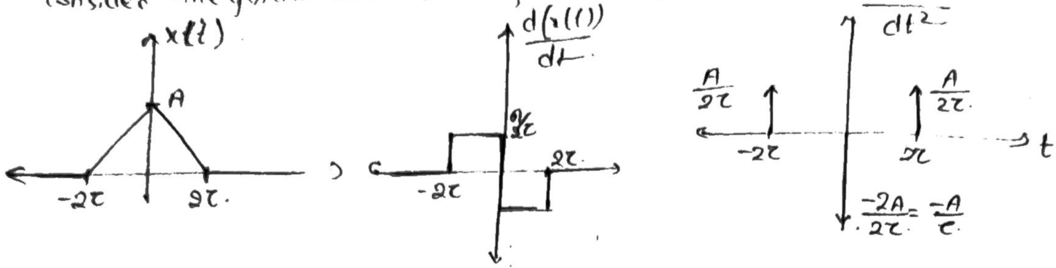
where, $X(\omega)$ is F.T of $x(t)$ and $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$

Question 3

(4) (d) $x(t) = \frac{\sin^2(kt)}{t^2} = k^2 \operatorname{sinc}^2(kt)$

$x(t) = k^2 \operatorname{sinc}^2(kt)$. To find $X(\omega) = ?$

Consider the general case of triangular function:



$$\frac{d^2(x(t))}{dt^2} = \frac{A}{2\tau}\delta(t+2\tau) - \frac{A}{\tau}\delta(t-0) + \frac{A}{2\tau}\delta(t-2\tau).$$

$$\frac{d^2(x(t))}{dt^2} = \frac{A}{2\tau}\left[\delta(t+2\tau) - 2\delta(t-0) + \delta(t-2\tau)\right].$$

$$(j\omega)^2 X(\omega) = \frac{A}{2\tau}\left[1\cdot e^{2j\omega\tau} - 2 + 1\cdot e^{-2j\omega\tau}\right]$$

$$\Rightarrow X(\omega) = \frac{-A}{2\omega^2\tau}\left[e^{2\omega\tau j} + e^{-2\omega\tau j} - 2\right] \qquad \{\because j^2\omega^2 = -\omega^2\}$$

$$X(\omega) = \frac{-A}{2\omega^2\tau}\left[2\cos(2\omega\tau) - 2\right] = \frac{-A}{\omega^2\tau}\left[\cos(2\omega\tau) - 1\right] = \frac{A}{\omega^2\tau}\left[1 - \cos(2\omega\tau)\right]$$

$$X(\omega) = \frac{A}{\omega^2\tau}\, 2\sin^2(\omega\tau) = 2A\tau \operatorname{sinc}^2(\omega\tau).$$

$\therefore$ $$A\,\mathrm{tri}\left(\frac{t}{\tau}\right) \longleftrightarrow 2A\tau \operatorname{sinc}^2(\omega\tau).$$

$$2A\tau \operatorname{sinc}^2(t\tau) \longleftrightarrow 2\pi A\,\mathrm{tri}\left(\frac{-\omega}{\tau}\right).$$

here $A = 1$; $\tau = k$.

$$\mathrm{tri}\left(\frac{t}{k}\right) \longleftrightarrow 2k \operatorname{sinc}^2(k\omega)$$

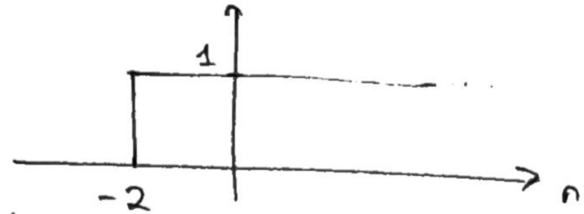
$$2k \operatorname{sinc}^2(kt) \longleftrightarrow 2\pi\,\mathrm{tri}\left(\frac{-\omega}{k}\right)$$

$$\Rightarrow k \operatorname{sinc}^2(kt) \longleftrightarrow \pi\,\mathrm{tri}\left(\frac{-\omega}{k}\right)$$

$$\Rightarrow k^2 \sin^2(kt) \longleftrightarrow k\pi\,\mathrm{tri}\left(\frac{-\omega}{k}\right).$$

$$\therefore X(\omega) = k\pi\,\mathrm{tri}\left(\frac{-\omega}{k}\right) \quad \therefore \quad x(t) = \frac{\sin^2(kt)}{t^2} \longleftrightarrow k\pi\,\mathrm{tri}\left(\frac{-\omega}{k}\right)$$

5 (a) $x[n] = \left(\frac{1}{4}\right)^n u[n+2]$ (13)



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-2}^{\infty} x[n] e^{-j\omega n} = \sum_{n=-2}^{\infty} \left(\frac{1}{4}\right)^n \cancel{u[n+2]}(1)\, e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-2}^{\infty} (4e^{j\omega})^{-n}$$

$$X(e^{j\omega}) = \frac{(4e^{+j\omega})^{-2}}{1-(4e^{j\omega})^{-1}} = \frac{(4e^{j\omega})^3}{(4e^{j\omega})-1}$$

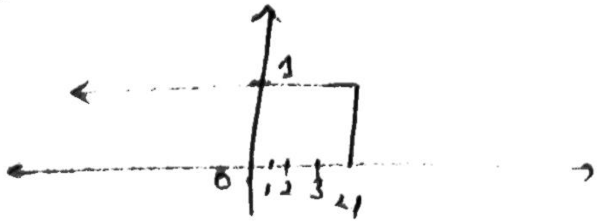
(b) $x[n] = \begin{cases} 1 & ; n \in \{0,1,2,3\} \\ 0 & ; o/w \end{cases}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=0}^{3} (1) e^{-j\omega n} = 1 + e^{-j\omega(1)} + e^{-j\omega(2)} + e^{-j\omega(3)}$$

$$\boxed{X(e^{j\omega}) = 1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega}}$$

Question - 4 Fourier Transform - (DFT).



(1) $x[n] = \frac{1}{4} u[4-n]$

$$4X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}.$$

↳ DFT.

$$4(X(k)) = \sum_{n=0}^{7} x[n]\, e^{-j\frac{2\pi}{8}k\cdot n} \qquad \because N = 8.$$

$$\Rightarrow 4(X(k)) = \sum_{n=0}^{4} (1)\, e^{-j\frac{\pi}{4}k\cdot n} + \cancel{\sum_{n=5}^{7} (0)\, e^{-j\frac{\pi}{4}kn}}\ 0.$$

$$\Rightarrow 4(X(k)) = 1\cdot e^{0} + e^{-\frac{\pi}{4}\cdot kj} + e^{-\frac{\pi}{4}2kj} + e^{-\frac{\pi}{4}3kj} + e^{-\frac{\pi}{4}4kj}$$

$$\Rightarrow 4(X(k)) = 1 + e^{-\frac{\pi}{4}kj} + e^{-\frac{\pi}{4}2kj} + e^{-\frac{\pi}{4}\cdot 3kj} + e^{-\frac{\pi}{4}4kj}$$

$$\therefore 4(X(k)) = \Big\{ (1+1+1+1+1),\ \big(1 + e^{-\frac{\pi}{4}j} + e^{-\frac{\pi}{4}2j} + e^{-\frac{\pi}{4}3j} + e^{-\frac{\pi}{4}4j}\big),$$
$$\big(1 + e^{-\frac{\pi}{4}2j} + e^{-\frac{\pi}{4}4j} + e^{-\frac{\pi}{4}6j} + e^{-\frac{\pi}{4}8j}\big),\ \big(1 + e^{-\frac{\pi}{4}3j} + e^{-\frac{\pi}{4}6j} + e^{-\frac{\pi}{4}9j} + e^{-\frac{\pi}{4}12\cdot j}\big),$$
$$\big(1 + e^{-\frac{\pi}{4}4j} + e^{-\frac{\pi}{4}8j} + e^{-3\pi j} + e^{-4\pi j}\big),\ \big(1 + e^{-\frac{\pi}{4}5j} + e^{-\frac{\pi}{4}10j} + e^{-\frac{\pi}{4}15j} + e^{-\frac{\pi}{4}20j}\big)$$
$$\big(1 + e^{-\frac{\pi}{4}6j} + e^{-\frac{\pi}{4}12j} + e^{-\frac{\pi}{4}18j} + e^{-\frac{\pi}{4}24j}\big),\ \big(1 + e^{-\frac{\pi}{4}7j} + e^{-\frac{\pi}{4}14j} + e^{-\frac{\pi}{4}21j} + e^{-\frac{\pi}{4}28j}\big) \Big\}$$

$\boxed{15}$

③ Given $X[k] = k^2$; $0 \le k \le 7$. be the 8 point DFT of a sequence $x[n]$.

To find $\sum_{n=0}^{3} x[2n+1] = ?$

Using IDFT : $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] \cdot e^{\frac{2\pi i k n}{N}} = \frac{1}{N}\sum_{k=0}^{N-1} X[k] \cdot e^{i\frac{2\pi}{N}kn}$

$\therefore \; x[n] = \frac{1}{N}\sum_{k=0}^{N-1} k^2 \cdot e^{i\frac{2\pi}{N}kn}$

$x[n] = \frac{1}{8}\sum_{k=0}^{7} k^2 \cdot e^{i\frac{2\pi}{8}\cdot kn} = \frac{1}{8}\sum_{k=0}^{7} k^2 \cdot e^{i\frac{\pi}{4}kn}$.

$\cancel{x[n] =}$

$\therefore \; x[2n+1] = \frac{1}{8}\sum_{k=0}^{7} k^2 \cdot e^{i\frac{\pi}{4}k(2n+1)}$.

$\sum_{n=0}^{3} x[2n+1] = x[1] + x[3] + x[5] + x[7]$

$\cancel{= \frac{1}{N}\left(\sum_{k=0}^{N-1}\right.}$

$= \left(\frac{1}{8}\right)\left\{\left(\sum_{k=0}^{7} k^2 e^{i\frac{\pi}{4}\cdot k\cdot(1)}\right) + \left(\sum_{k=0}^{7} k^2 e^{i\frac{\pi}{4}k(3)}\right) + \left(\sum_{k=0}^{7} k^2 e^{i\frac{\pi}{4}k(5)}\right) + \left(\sum_{k=0}^{7} k^2 e^{i\frac{\pi}{4}k(7)}\right)\right\}$

$= \left(\frac{1}{8}\right)\left\{\left(e^1 + e^3 + e^5 + e^7\right)\left(\sum_{k=0}^{7} k^2 \cdot e^{i\frac{\pi}{4}k}\right)\right\}$

$= \left(\frac{1}{8}\right)\left\{\left(e^1 + e^3 + e^5 + e^7\right)\left(0 + 1^2 e^{i\frac{\pi}{4}} + 2^2 e^{i\frac{\pi}{4}\cdot 2} + 3^2 e^{i\frac{\pi}{4}3} + 4^2 \cdot e^{i\frac{\pi}{4}4}\right.\right.$
$\left.\left. + 5^2 e^{i\frac{\pi}{4}\cdot 5} + 6^2 e^{i\frac{\pi}{4}\cdot 6} + 7^2 \cdot e^{i\frac{\pi}{4}7}\right)\right\}$.

Queston-4: Fourier Transform (DFT)

(2) $X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$ → (1) (Assuming $X(k)$ is N-point DFT of $x[n]$)

$y[n] = x[n] + x[n+\frac{N}{2}]$

$\therefore Y(k) = \sum_{n=0}^{\frac{N}{2}-1} \cancel{x[n]}\, y[n] e^{-j\frac{4\pi}{N}kn}$

$Y(k) = \sum_{n=0}^{\frac{N}{2}-1} x[n] e^{-j\frac{4\pi}{N}kn} + \sum_{n=0}^{\frac{N}{2}-1} x[n+\frac{N}{2}] e^{-j\frac{4\pi}{N}kn}$

$Y(k) = \sum_{n=0}^{\frac{N}{2}-1} x[n] e^{-j\frac{4\pi}{N}kn} + \sum_{m=\frac{N}{2}}^{N-1} x[m] e^{-j\frac{4\pi}{N}k\cdot(m-\frac{N}{2})}$ $\because \{m = n+\frac{N}{2}\}$

$Y(k) = \sum_{n=0}^{\frac{N}{2}-1} x[n] e^{-j\frac{4\pi}{N}kn} + \sum_{m=\frac{N}{2}}^{N-1} x[m] e^{\frac{-4\pi}{N}km} \cdot \cancel{e^{j\frac{4\pi}{N}\frac{N}{2}k}} \to 1.$

$Y(k) = \sum_{n=0}^{\frac{N}{2}-1} x[n] e^{-j\frac{4\pi}{N}kn} + \sum_{m=\frac{N}{2}}^{N-1} x[m] e^{-j\frac{4\pi}{N}km}$

$Y(k) = \sum_{n=0}^{\frac{N}{2}-1} x[n] e^{-j\frac{4\pi}{N}kn} + \sum_{n=\frac{N}{2}}^{N-1} x[n] e^{-j\frac{4\pi}{N}kn}$

$Y(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{4\pi}{N}kn}$

$\therefore \boxed{Y(k) = X(2k)}$

(4) (a) $X(e^{j\omega}) = \cos^3\omega + \cos^2\omega$

$X(e^{j\omega}) = \frac{\cos 3\omega + 3\cos\omega}{4} + \frac{\cos 2\omega + 1}{2}$

$X(e^{j\omega}) = \frac{1}{4}\left(\cos 3\omega + 3\cos\omega + 2\cos 2\omega + 2\right)$

$X(e^{j\omega}) = \frac{1}{4}\left(\frac{e^{j3\omega} + e^{-j3\omega}}{2} + 3\cdot\frac{e^{j\omega}+e^{-j\omega}}{2} + 2\cdot\frac{e^{j2\omega}+e^{-j2\omega}}{2} + 2\right)$

$X(e^{j\omega}) = \frac{1}{8}e^{j3\omega} + \frac{1}{8}e^{-j3\omega} + \frac{3}{8}e^{j\omega} + \frac{3}{8}e^{-j\omega} + \frac{1}{4}e^{j2\omega} + \frac{1}{4}e^{-j2\omega} + \frac{1}{2}$

we know: $\delta(n-n_0) \xleftrightarrow{FT} e^{-j\omega n_0}$

IDTFT of $X(e^{j\omega})$ is

$x[n] = \frac{1}{8}\delta(n+3) + \frac{1}{8}\delta(n-3) + \frac{3}{8}\delta(n+1) + \frac{1}{4}\delta(n+2) + \frac{1}{4}\delta(n-$

$+ \frac{3}{8}\delta(n-1)$

$+ \frac{1}{2}$

where $\delta(n)$ is the kronecker delta function which equals to 1 when $n=0$ and 0 o/w.

(c) $$X(e^{j\omega}) = \frac{3e^{-j\omega}-1}{3-e^{-j\omega}}$$

we know, $a^n u[n]$, $|a|<1$ $\xleftrightarrow{F.T}$ $\frac{1}{1-ae^{-j\omega}}$ $\longrightarrow$ (1) (common Fourier transform pairs).

$$\left(\frac{1}{3}\right)^n u[n] \longleftrightarrow \frac{1}{1-\frac{1}{3}e^{-j\omega}} = \frac{3}{3-e^{-j\omega}}$$

$$\Rightarrow \left(\frac{1}{3}\right)^{n+1} u[n] \longleftrightarrow \frac{1}{3-e^{-j\omega}} \longrightarrow (2)$$

we have, $$X(e^{j\omega}) = \underbrace{\frac{3e^{-j\omega}}{3-e^{-j\omega}}}_{X_1(e^{j\omega})} - \underbrace{\frac{1}{3-e^{-j\omega}}}_{X_2(e^{j\omega})}$$

$X_1(e^{j\omega}) \xrightarrow{IDTFT} x_1(n)$, $X_2(e^{j\omega}) \xrightarrow{IDTFT} x_2[n]$ (say). (Using

$$x_2[n] = \left(\frac{1}{3}\right)^{n+1} u[n].$$

$$x_1'[n] = (3)\left[\left(\frac{1}{3}\right)^{n+1} u[n]\right] \longleftrightarrow \frac{3}{3-e^{-j\omega}}$$

$$x_1''[n] = 3\cdot\left(\frac{1}{3}\right)^{n} u[n-1] \longleftrightarrow \frac{3}{3-e^{-j\omega}}\cdot e^{-j\omega}$$

$$\therefore \cancel{x_2[n]}\ x_1[n] = x_1''[n] = \left(\frac{1}{3}\right)^{n-1} u[n-1].$$

$\therefore$ IDTFT of $X(e^{j\omega})$ is $x_1[n] - x_2[n]$

which is: $$\left(\frac{1}{3}\right)^{n-1} u[n-1] - \left(\frac{1}{3}\right)^{n+1} u[n] \Rightarrow \left(\frac{1}{3}\right)^{n-1} u[n-1] - \left(\frac{1}{3}\right)^{n+1} u[n]$$

$$\cancel{\Rightarrow \left(\frac{1}{3}\right)^n\left[u[n-1] - \left(\frac{1}{3}\right)u[n]\right]} \Rightarrow \left(\frac{1}{3}\right)^{n-1}\left[u[n-1] + \frac{1}{9}u[n]\right]$$

where $u(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n<0 \end{cases}$

(b) $X(e^{j\omega}) = \dfrac{e^{-4j\omega} + e^{-3j\omega} - e^{-j\omega} - 1}{e^{-j\omega} + 1} = \dfrac{e^{-3j\omega}(e^{-j\omega}+1) - 1\,(e^{-j\omega}+1)}{e^{-j\omega}+1}$

$= \dfrac{\cancel{(e^{-j\omega}+1)}(e^{-3j\omega}-1)}{\cancel{e^{-j\omega}+1}} = e^{-3j\omega} - 1$

$X(e^{j\omega}) = e^{-3j\omega} - 1.$

we know. $\delta[n-n_0] \longleftrightarrow e^{-j\Omega n_0}$ (or) $e^{-j\omega n_0}. \longrightarrow (1)$

where $\delta(n)$ is the kroneeker delta function which equals to ~~0 when 1 when n=0 o/w 0 when n = o/w.~~

$\delta(n) = \begin{cases} 1 ; & n=0 \\ 0 ; & o/w \end{cases}$

$\therefore \quad X(e^{j\omega}) = \underbrace{+e^{-3j\omega}}_{X_1(e^{j\omega})} \underbrace{-1}_{X_2(e^{j\omega})}$

let : $x_1[n] \longleftrightarrow X_1(e^{j\omega})$

$x_2[n] \longleftrightarrow X_2(e^{j\omega})$,

$x[n] \longleftrightarrow X(e^{j\omega})$

$= x_1[n] - x_2[n]$

~~x+t~~

Using, (1) : $x_1[n] = \delta(n-3)$.

$x_2[n] = \delta(n)$.

$\therefore \quad x_1[n] - x_2[n] \longleftrightarrow e^{-3j\omega} - 1$

$\delta(n-3) - \delta(n) \longleftrightarrow e^{-3j\omega} - 1$

$\therefore$ IDTFT of $X(e^{j\omega})$ is $\underline{\delta(n-3) - \delta(n)}$

(5) (a) $x[n] = \{1, 2, 3, 6\} \xrightarrow{N=6} x[n] = \{1, 2, 3, 6, 0, 0\}$

$X(k) = \sum_{n=0}^{N-1} x[n] e^{-\frac{2\pi}{N} knj}$

Here $N = 6 \Rightarrow X(k) = \sum_{n=0}^{5} x[n] e^{-\frac{2\pi}{6} knj}$

$X(k) = (1) e^{-\frac{\pi}{3}k(0)j} + (2)(e^{-\frac{\pi}{3}k(1)j}) + (3)(e^{-\frac{\pi}{3}k(2)j}) + (6)(e^{-\frac{\pi}{3}(k)(3)j})$
$+ 0 + 0$

$X(k) = 1 + 2e^{-\frac{k\pi}{3}j} + 3e^{-\frac{2\pi}{3}kj} + 6e^{-\pi kj} \longrightarrow (1)$

$X(0) = 1 + 2 + 3 + 6 = \cancel{\ } \; 12.$

$X(1) = 1 + 2e^{-\frac{\pi}{3}j} + 3e^{-\frac{2\pi}{3}j} + 6e^{-\pi j}.$

~~X(2) =~~ $= 1 + 2\left(\cos\frac{\pi}{3} - j\sin\frac{\pi}{3}\right) + 3\left(\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3}\right) + 6\left(\cos\pi - j\sin\pi\right).$

$=$

$$x(1) = 1+2\left(\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)+3\left(\frac{-1}{2}-j\frac{\sqrt{3}}{2}\right)+6(-1).$$

$$= \cancel{1+2(\tfrac{-1}{}}\quad 1+1-\sqrt{3}j-\frac{3}{2}-\frac{3\sqrt{3}}{2}j-6.$$

$$= \frac{-11}{2}-\frac{5\sqrt{3}}{2}j.$$

$$x(2) = 1+2e^{\frac{-2\pi}{3}j}+3e^{-\frac{4\pi}{3}j}+6e^{-2\pi j}$$

$$= 1+2\left(\cos\frac{2\pi}{3}-j\sin\frac{2\pi}{3}\right)+3\left(\cos\frac{4\pi}{3}-j\sin\frac{4\pi}{3}\right)+6\left(\cos 2\pi-j\sin 2\pi\right)$$

$$= 1+2\left(\frac{-1}{2}-j\frac{\sqrt{3}}{2}\right)+3\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}j\right)+6\left(1-0j\right)$$

$$= 1-1-\sqrt{3}j+-\frac{3}{2}+\frac{3\sqrt{3}}{2}j+6$$

$$= \frac{9}{2}+\frac{\sqrt{3}}{2}j$$

$$x(3) = 1+2e^{-\pi j}+3e^{-2\pi j}+6e^{-3\pi j}$$

$$= 1+2(\cos\pi-\sin\pi j)+3(\cos 2\pi-j\sin 2\pi)+6(\cos 3\pi-j\sin 3\pi)$$

$$= 1+(-2)+3(1)+6(-1)$$

$$= -4$$

$$x(4) = 1+2e^{-\frac{4\pi}{3}j}+3e^{\frac{-8\pi}{3}j}+6e^{-4\pi j}$$

$$= 1+2\left(\cos\frac{4\pi}{3}+-j\sin\frac{4\pi}{3}\right)+3\left(\cos\frac{8\pi}{3}-j\sin\frac{8\pi}{3}\right)+6(1)$$

$$= 1+2\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}j\right)+3\left(\frac{-1}{2}-j\frac{\sqrt{3}}{2}\right)+6$$

$$= 1-1+\sqrt{3}j+-\frac{3}{2}-\frac{3\sqrt{3}}{2}+6$$

$$= \frac{9}{2}-\frac{\sqrt{3}}{2}j$$

$$x(5) = 1+2e^{\frac{-5\pi}{3}j}+3e^{-\frac{10\pi}{3}j}+6e^{-5\pi j}$$

$$= 1+2\left(\cos\frac{5\pi}{3}-j\sin\frac{5\pi}{3}\right)+3\left(\cos\frac{10\pi}{3}-j\sin\frac{10\pi}{3}\right)+6\left(\cos 5\pi-j\sin 5\pi\right)$$

$$= 1+2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)+3\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}j\right)+6(-1)$$

$$= \frac{-11}{2}+\frac{5\sqrt{3}}{2}j$$

Clearly, $X(1)$ and $X(5)$ are conjugate to each other
$X(2)$ and $X(4)$ are conjugate to each other

(b) we know, $x[n] \longleftrightarrow X(k)$

$$\cancel{x[n]} \quad x[n-n_0] \longleftrightarrow e^{-j\frac{2\pi}{N}kn_0} X(k).$$

$\therefore$ $x[n] = \{1, 2, 3, 4\}$

$x[n-10] = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 3, 4\}$

$y[n] = x[n-10]$

$$\therefore \quad Y(k) = e^{-j\frac{2\pi}{N}k(10)} X(k)$$

$$Y(k) = e^{-j\frac{2\pi}{6}(k)(10)} X(k) = e^{-\frac{10k}{3}\pi j} X(k)$$

$$\boxed{Y(k) = e^{-\frac{10k\pi}{3}j} X(k)}$$

(c) $Y(k) = e^{-\frac{10\pi}{3}kj} X(k)$.

$x[n] \longleftrightarrow X(k)$.

$y[n] \longleftrightarrow Y(k)$.

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\cdot e^{\frac{\pi}{3}knj}$$

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} e^{-\frac{10k\pi}{3}j} X(k)\cdot e^{\frac{\pi}{3}knj}$$

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot e^{\frac{k\pi}{3}(n-10)j} \cdot \frac{1}{N}\sum_{k=0}^{N-1}\left(X(k)\cdot e^{\frac{2\pi k}{N}(n-10)j}\right).$$

~~$y[n] \longleftrightarrow X(k-10) = Y(k)$.~~

$y[n] = x[n-10]$.

$$\boxed{\cancel{Y(k) = X(k-10)}}$$

$x[n] \longleftrightarrow X(k)$

$x[n-10] \longleftrightarrow e^{-\frac{2\pi k}{N}(10)}\cdot X(k)$

$$\therefore \boxed{y[n] = x[n-10]}$$

(d) To find: DFT of $x[n] \cdot \cos\left(\frac{2\pi}{N} k_0 n\right)$.

we know: $$x_1[n] \cdot x_2[n] \longleftrightarrow \frac{1}{N} X_1(k) \circledast X_2(k), = Y(k) \text{ (say)}$$

where, $$X_1(k) \circledast X_2(k) = \sum_{i=0}^{N-1} X_1(i) X_2(k-i).$$

$$x_1[n] = x[n] \longleftrightarrow X_1(k)$$

$$x_2[n] = \cos\left(\frac{2\pi}{N} k_0 n\right) \longleftrightarrow X_2(k)$$

$$\cos\left(\frac{2\pi k_0 n}{N}\right) = \frac{e^{j\frac{2\pi}{N}k_0 n} + e^{-j\frac{2\pi}{N}k_0 n}}{2}.$$

$$\therefore X(k) = \sum_{n=0}^{N-1}\left\{\frac{1}{2}\left(e^{j\frac{2\pi}{N}k_0 n} + e^{-j\frac{2\pi}{N}k_0 n}\right)\cdot e^{-j\frac{2\pi}{N}kn}\right\}$$

$$= \sum_{n=0}^{N-1}\left\{\frac{1}{2}\left[e^{j\frac{2\pi}{N}n(k_0-k)} + e^{-j\frac{2\pi}{N}n(k_0+k)}\right\}\right.$$

$$= \sum_{n=0}^{N-1}\left\{\frac{1}{2}\left[e^{-j\frac{2\pi}{N}(k-k_0)n} + e^{-j\frac{2\pi}{N}n(k+k_0)}\right]\right\}$$

$$X_2(k) = \cancel{\sum_{n=0}^{N-1}\frac{1}{2}}\,\cancel{N}\ \frac{1}{2}\left[N\,\delta(k-k_0) + N\,\delta(k+k_0)\right]$$

$\therefore$ DFT of $\left\{x(n)\cdot\cos\left(\frac{2\pi}{N}k_0 n\right)\right\}$

$$\text{is } \cancel{\frac{1}{N}\sum_{n=0}^{N-1}\{X_1(k}$$

$$= \frac{1}{N}\sum_{i=0}^{N-1}\left(X_1(i)\cdot X_2(k-i)\right).$$

$$\therefore Y(k) = \frac{1}{N}\sum_{i=0}^{N-1}\left(X_2(i)\cdot X_1(k-i)\right)$$

$$= \frac{1}{\cancel{N}}\sum_{i=0}^{N-1}\left[\frac{\cancel{N}}{2}\left(\delta(i-k_0) + \delta(i+k_0)\right)\cdot X_1(k-i)\right].$$

$$\Rightarrow 2(Y(k)) = \sum_{i=0}^{N-1}\left[\delta(i-k_0) + \delta(i+k_0)\right]X_1(k-i)$$

$$\Rightarrow 2(Y(k)) = \sum_{i=0}^{N-1}\left(\delta(i-k_0) + \delta(i+k_0)\right]X(k-i),$$

$$\boxed{Y(k) = \begin{cases} \dfrac{X(k-k_0)}{2} ; & \text{if } k_0 \in (0, N-1]. \\ \dfrac{X(k+k_0)}{2} ; & \text{if } k_0 \in [-N+1, 0) \ \cancel{i = k_0 (-1) = -k_0}. \\ X(k) ; & \cancel{\text{else}} \text{ if } k_0 = 0 \\ 0 & ; \text{o/w} \end{cases}}$$

By definition of kronecker delta function:

$$\therefore \text{DFT of } x(n)\cdot\cos\left(\frac{2\pi}{N}k_0 n\right) \text{ is } \begin{cases} \dfrac{X(k-k_0)}{2} ; & \text{if } k_0 \in (0, N-1] \\ \dfrac{X(k+k_0)}{2} ; & \text{if } k_0 \in [-(N-1), 0) \\ X(k) & : \text{if } k_0 = 0 \\ 0. & : \text{o/w} \end{cases}$$

Question-5: Convolution:

① (a) $f[n] = 2\delta[n+10] + 2\delta[n-10]$, $g[n] = 3\delta[n+5] + 2\delta[n-5]$.

let $x[n] = f[n] * g[n]$.: To find, $x[n]$

$$(f*g)[n] = \sum_{k=-\infty}^{\infty} f[k]\cdot g[n-k] ,$$

$$= \sum_{k=-\infty}^{\infty} (2\delta[k+10] + 2\delta[k-10])(3\delta[n-k+5] + 2\delta[n-k-5])$$

$$= 2(3\delta[n+15] + 2\delta[n+5]) + 2(3\delta[n-5] + 2\delta[n-15]).$$

$$= 6\delta[n+15] + 4\delta[n+5] + 6\delta[n-5] + 4\delta[n-15]$$

$$\therefore \boxed{x[n] = 6\delta[n+15] + 4\delta[n+5] + 6\delta[n-5] + 4\delta[n-15]}$$

(b) $f[n] = (-1)^n$, $g[n] = \delta[n] + \delta[n-1]$.

$x[n] = f[n] * g[n] = ?$

$$f[n] * g[n] = \cancel{(-1)}^n \sum_{k=-\infty}^{\infty} f[k]\, g[n-k]$$

$$f[n] * g[n] = \sum_{k=-\infty}^{\infty} (-1)^k (\delta[n-k] + \delta[n-k-1]).$$

$$f[n] * g[n] = (-1)^n + (-1)^{n-1} = (-1)^n + (-1)^n/(-1) . = 0.$$

$$\therefore \boxed{x[n] = 0}$$

Question 6: Sampling:

(1) Aliasing:

-> is the overlapping of frequency components resulting from a sample rate below the Nyquist rate.

-> This overlap results in distortion or artifacts when the signal is reconstructed from samples which causes the reconstructed signal to differ from the original continuous signal.

Reduction of Aliases:-

-> Increase the Sampling rate according to Nyquist sampling theorem which states that $\omega_s \geq 2\omega_m$

where ω_s -> Sampling angular frequency

ω_m -> Message angular frequency

-> low pass filter: this removes the high-frequency components that could cause aliasing. Ensures only frequencies below the Nyquist frequency are sampled.

-> Anti-Aliasing Filters:- removes frequencies above Nyquist ~~filters~~ frequency before sampling.

$$x(t) = \frac{1}{2\pi} \cdot \underbrace{\cos(4000\pi t)}_{m_1(t)} \underbrace{\cos(1000\pi t)}_{m_2(t)}.$$

$m_1(t)$ has Nyquist rate f_{s_1}

$m_2(t)$ has Nyquist rate f_{s_2}.

then Nyquist rate of $x(t)$ is $f_{s_1} + f_{s_2}$

$$\therefore f_{s_1} = 2\left(\frac{4000\pi}{2\pi}\right) = 4000\ Hz$$

$$f_{s_2} = 2\left(\frac{1000\pi}{2\pi}\right) = 1000\ Hz.$$

$$f_{s_1} + f_{s_2} = \underline{5000\ Hz}$$

$$\therefore \text{Nyquist Interval of } x(t) = \frac{1}{f_{s_1} + f_{s_2}} = \frac{1}{5000} \text{ sec.}$$

$\therefore$ Nyquist Rate of $x(t) = 5000\ Hz$

Nyquist Interval of $x(t) = \frac{1}{5000}$ sec. $= \underline{0.0002 \text{ sec}}$

(2) $x(t) = \underbrace{10\cos\left(1000t + \frac{\pi}{3}\right)}_{m_1(t)} + \underbrace{20\cos\left(2000t + \frac{\pi}{6}\right)}_{m_2(t).}$ (say)

Angulara frequencys

~~$\omega_1 =$~~

ω_1 = Angular frequency of $m_1(t)$ = 1000

ω_2 = Angular frequency $m_2(t)$ = 2000.

Nyquist rate of $x(t)$ = 2x (max Angular Freq)

$= 2 \times 2000$

$= 4000 \frac{\text{radians}}{\text{sec}} = \omega_s$ (Nyquist Rate).

To find, Maximum Allowable time Interval b/w sample values that will ensure perfect signal reproduction.

$T_s = \frac{1}{f_s} = \cancel{\frac{\omega}{2\pi}} = \frac{2\pi}{\omega} = \frac{2\pi}{\cancel{4000}_{2000}} = \frac{\cancel{3.14}^{1.57}}{\cancel{2000}} = \cancel{0.0001}\ \underline{0.00157\,\text{sec}}$

∴ Maximum Allowable time Interval $= \frac{2\pi}{4000}$ sec

No. of Sample values to reproduce in 1 hour.

$= \dfrac{1 \times 60 \times 60}{\frac{2\pi}{4000}} = \dfrac{60 \times 60 \times \cancel{4000}^{2000}}{\cancel{2\pi}} = \dfrac{60 \times 60 \times 2000}{\pi}.$

$\approx$ 2292993 samples $\approx$ 2300000 samples

$\frac{2\pi}{4000}$ sec & $\frac{60 \times 60 \times 2000}{\pi}$ samples Ans

(3) three signals: $x_1(t)$, $x_2(t)$, $x_3(t)$.

$X_1(\Omega) = 0$, $|\Omega| \geq 120$.

$X_2(\Omega) = 0$; $|\Omega| \leq 60$, $|\Omega| \geq 100$.

(a) $x(t) = x_1(t) + x_2(t)$.

~~let ω_{s_1} be~~ ~~Nyquist rate of~~ $x_1(t)$ ~~is ω_{s_1}, $x_2(t)$ is~~ ω_{s_2} (Angular written above)

~~then Nyquist~~

let Angular frequency of $x_1(t)$ is ω_1

$x_2(t)$ is ω_2.

then Nyquist rate of $x(t) = 2 \times (\max\{\omega_1, \omega_2\})$.

~~(a) $x(t) = x_1(t) + x_2(t)$~~

(c) $x(t) = \cos(3.6\pi t + 9.23)$.

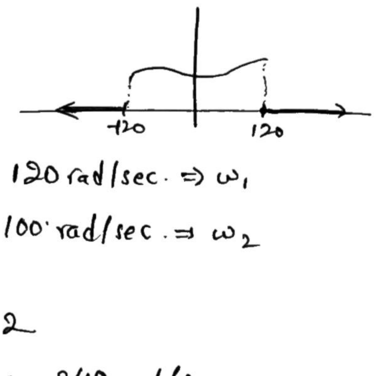
$\omega_m = 3.6\pi$ rad/s

$\omega_s \geq 2 \cdot \omega_m \Rightarrow \omega_s \geq 2 \times 3.6\pi$ rad/sec.

$\therefore \omega_s|_{min} = 2 \times 3.6\pi$ rad/s. $\Rightarrow f_s|_{min} = \frac{2 \times 3.6\pi}{2\pi} = 3.6$ Hz

$\boxed{f_s|_{min} = 3.6 \text{ Hz}}$

(a) $x(t) = x_1(t) + x_2(t)$.



Max Angular frequency in $x_1(t) \Rightarrow 120$ rad/sec. $\Rightarrow \omega_1$

Max Angular frequency in $x_2(t) \Rightarrow 100$ rad/sec. $\Rightarrow \omega_2$

$\therefore \omega_s$ of $x(t) \geq \max\{\omega_1, \omega_2\} \times 2$

$= 120 \times 2$ rad/s $= 240$ rad/sec.

$\therefore f_s \geq \frac{240}{2\pi}$. $\Rightarrow \boxed{f_{s\,minimum} = \frac{120}{\pi} \text{ Hz}}$.

(b) $x(t) = x_1(t) \cdot x_2(t)$.

$\omega_1 = 120$ rad/s

$\omega_2 = 100$ rad/s.

$\left.\begin{array}{l}\omega_{s_1} = 2 \times 120 = 240 \text{ rad/s} \\ \omega_{s_2} = 2 \times 100 = 200 \text{ rad/s.}\end{array}\right\}$ $\omega_s = \omega_{s_1} + \omega_{s_2}$

$= 240 + 200$

$= 440$ rad/s.

$f_s \geq \frac{440}{2\pi}$ Hz $\Rightarrow f_s \geq \frac{220}{\pi}$ Hz $\Rightarrow \boxed{f_s|_{min} = \frac{220}{\pi} \text{ Hz}}$

Question-7

① ~~(a)~~ $x(t) = \begin{cases} -2\sin(\pi x/4); & 0 \le x < 4 \\ x-4 & ; 4 \le x < 5 \\ 1 & ; 5 < x < 7 \\ 8-x & ; 7 \le x \le 10. \end{cases} = \begin{cases} -2\sin\left(\frac{\pi t}{4}\right) & ; 0 \le x < 4 \\ x-4 & ; 4 \le x < 5 \\ 1 & ; 5 < x < 7 \\ 8-x & ; 7 \le x \le 10 \end{cases}$

Given: $f_s = 1000\,Hz \Rightarrow T_s = 1\,ms.$

(a) Sample points: ~~0~~, 1ms, 2ms, ..., 10000ms.

(b) Quantisation Intervals and Corresponding digital words.

$$\text{Step size} = \frac{V_{max} - V_{min}}{2^N} = \frac{2-(-2)}{2^2} = 1 \Rightarrow \boxed{\text{Stepsize} = 1}$$

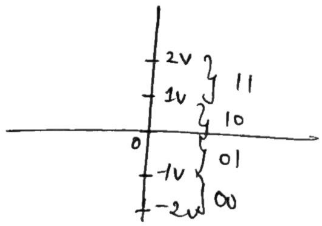
$\therefore$ Quantisation Intervals and Digital word

(−2V to −1V) = 00

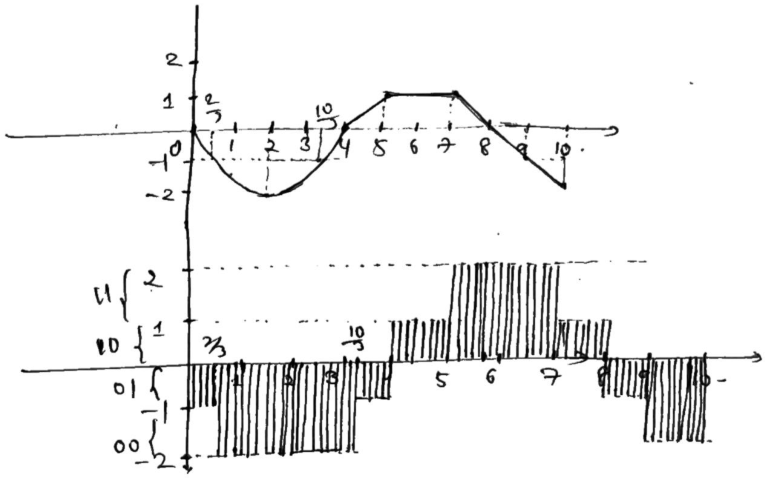
(−1V to 0V) = 01

(0V to 1V) = 10

(1V to 2V) = 11



(c)



(d) $\underbrace{\overline{01}\;\overline{01}\;\overline{01}\;\overline{01}\cdots\overline{01}}_{\frac{2}{3}\times 10000\text{ times}} \Big| \underbrace{\overline{00}\;\overline{00}\;\overline{00}\cdots\overline{00}}_{\frac{8}{3}\times 1000\text{ times}} \Big| \underbrace{\overline{01}\cdots\overline{01}}_{\frac{2}{3}\times 10000\text{ time}} \Big| \underbrace{\overline{10}\cdots\overline{10}}_{10000\text{ times}} \Big| \underbrace{\overline{11}\cdots\overline{11}}_{2\times 1000\text{ times}}$

(int) {

~~$\frac{8}{3}$ x 10000 times~~

$\cdots \Big| \underbrace{\overline{10}\;\overline{10}\cdots\overline{10}}_{1000\text{ times}} \Big| \underbrace{\overline{01}\cdots\overline{01}}_{1000\text{ times}} \Big| \underbrace{\overline{00}\;\overline{00}\cdots\overline{00}}_{2\times 1000\text{ time}}$ }

40

(e) Bitrate = $\frac{\text{Number of bits transmitted}}{\text{time taken to transmit the bits.}} = \frac{2\times(10{,}000)}{10 \text{ (sec)}} = 2000 \text{ bits/sec}$

(f) Quantization error = $\frac{\text{stepsize}}{2} = \frac{1}{2} = 0.5$

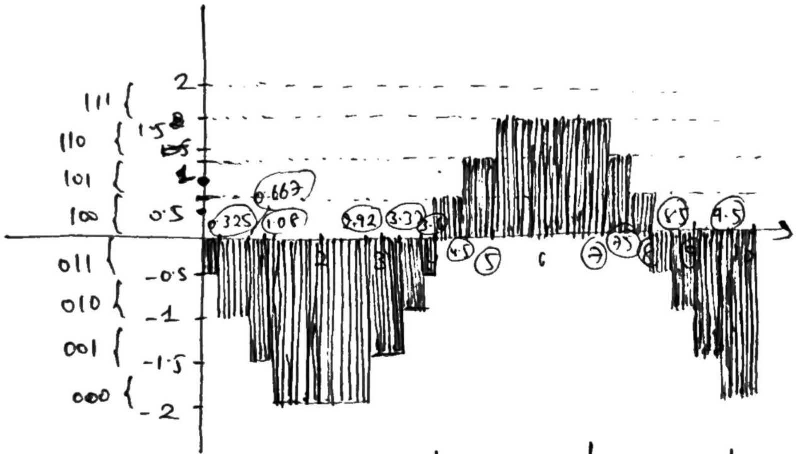
(2) (a) Sample points: 1ms, 2ms, ... 10000 ms.

(b) Stepsize = $\frac{V_{max}-V_{min}}{2^N} = \frac{2-(-2)}{2^3} = \frac{4}{8} = \frac{1}{2} = 0.5$

−2 – −1.5	000
−1.5 – −1	001
−1 – −0.5	010
−0.5 – 0	011
0 – +0.5	100
+0.5 – 1	101
1 – 1.5	110
1.5 – 2	111

$[-2,-1.5) \rightarrow 000$
$[-1.5,-1) \rightarrow 001$
$[-1,-0.5) \rightarrow 010$
$[-0.5,0) \rightarrow 011$
$[0,0.5) \rightarrow 100$
$[0.5,1) \rightarrow 101$
$[1,1.5) \rightarrow 110$
$[1.5,2] \rightarrow 111$.

(c)



(d) $\underbrace{\overline{011}\ \overline{011}\cdots\overline{011}}_{0.325\times1000 \text{ times}}$ | $\underbrace{\overline{010}\ \overline{010}\cdots\overline{010}}_{0.342\times1000 \text{ times}}$ | $\underbrace{\overline{001}\ \overline{001}\cdots\overline{001}}_{0.413\times1000 \text{ times}}$ | $\underbrace{\overline{000}\cdots\overline{000}}_{1.84\times1000 \text{ times}}$ | $\underbrace{\overline{001}\cdots\overline{001}}_{0.41\times1000 \text{ times}}$ | $\underbrace{\overline{010}\cdots\overline{010}}_{0.27\times1000 \text{ times}}$ | $\underbrace{\overline{011}\cdots\overline{011}}_{0.4\times1000 \text{ times}}$

$\underbrace{\overline{100}\ \overline{100}\cdots\overline{100}}_{0.5\times1000 \text{ times}}$ | $\underbrace{\overline{101}\ \overline{101}\cdots\overline{101}}_{0.5\times1000 \text{ times}}$ | $\underbrace{\overline{110}\ \overline{110}\cdots\overline{110}}_{2\times1000 \text{ times}}$ | $\underbrace{\overline{101}\cdots\overline{101}}_{0.5\times1000 \text{ times}}$ | $\underbrace{\overline{100}\cdots\overline{100}}_{0.5\times1000 \text{ times}}$ | ..

$\underbrace{\overline{011}\cdots\overline{011}}_{0.5\times1000 \text{ times}}$ | $\underbrace{\overline{010}\ \overline{010}\cdots\overline{010}}_{0.5\times1000 \text{ times}}$ | $\underbrace{\overline{001}\cdots\overline{001}}_{0.5\times1000 \text{ times}}$ | $\underbrace{\overline{000}\cdots\overline{000}}_{0.5\times1000 \text{ times}}$

(e) Resulting bitrate $= \frac{3 \times 10000}{10} = 3000$ bits/sec

(f) Quantization - error $= \frac{\text{Step-size}}{2} = \underline{0.25}$.

ADVANTAGES of INCREASING QUANTIZATION BITS

1. Precision is High
2. Low Quantization Error
3. Quality Increases
4. Increases the Dynamic Range of values that can be accurately represented

DISADVANTAGES

1. Higher memory Usage
2. High Computational Complexity
3. High Power Consumption
4. Increased Quantization Noise