CS570
Analysis of Algorithms
Fall 2014
Exam III

Name: _____

Student ID: _____

Email:_____

Wednesday Evening Section

	Maximum	Received
Problem 1	20	
Problem 2	16	
Problem 3	16	
Problem 4	16	
Problem 5	16	
Problem 6	16	
Total	100	

Instructions:
1. This is a 2-hr exam. Closed book and notes
2. If a description to an algorithm or a proof is required please limit your description or proof to within 150 words, preferably not exceeding the space allotted for that question.
3. No space other than the pages in the exam booklet will be scanned for grading.
4. If you require an additional page for a question, you can use the extra page provided within this booklet. However please indicate clearly that you are continuing the solution on the additional page.

1) 20 pts
   Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

   [TRUE/FALSE]
   All the NP-hard problems are in NP.

   [TRUE/FALSE]
   Given a weighted graph and two nodes, it is possible to list all shortest paths between these two nodes in polynomial time.

   [TRUE/FALSE]
   In the memory efficient implementation of Bellman-Ford, the number of iterations it takes to converge can vary depending on the order of nodes updated within an iteration

   [TRUE/FALSE]
   There is a feasible circulation with demands $\{d_v\}$ if $\sum_v d_v = 0$.

   [TRUE/FALSE]
   Not every decision problem in P has a polynomial time certifier.

   [TRUE/FALSE]
   If a problem can be reduced to linear programming in polynomial time then that problem is in P.

   [TRUE/FALSE]
   If we can prove that P $\neq$ NP, then a problem A $\in$ P does not belong to NP.

   [TRUE/FALSE]
   If all capacities in a flow network are integers, then every maximum flow in the network is such that flow value on each edge is an integer.

   [TRUE/FALSE]
   In a dynamic programming formulation, the sub-problems must be mutually independent.

   [TRUE/FALSE]
   In the final residual graph constructed during the execution of the Ford–Fulkerson Algorithm, there's no path from sink to source.

2) 16 pts

In the Bipartite Directed Hamiltonian Cycle problem, we are given a bipartite directed graph $G = (V; E)$ and asked whether there is a simple cycle which visits every node exactly once. Note that this problem might potentially be easier than Directed Hamiltonian Cycle because it assumes a bipartite graph. Prove that Bipartite Directed Hamiltonian Cycle is in fact still NP-Complete.

3) 16 pts

A tourism company is providing boat tours on a river with n consecutive segments. According to previous experience, the profit they can make by providing boat tours on segment i is known as a_i. Here a_i could be positive (they earn money), negative (they lose money), or zero. Because of the administration convenience, the local community of the river requires that the tourism company should do their boat tour business on a contiguous sequence of the river segments, i.e, if the company chooses segment i as the starting segment and segment j as the ending segment, all the segments in between should also be covered by the tour service, no matter whether the company will earn or lose money. The company's goal is to determine the starting segment and ending segment of boat tours along the river, such that their total profit can be maximized. Design an efficient algorithm to achieve this goal, and analyze its run time (Note that brute-force algorithm achieves $\Theta(n^2)$, so your algorithm must do better.)

4) 16 pts

Consider the following matching problem. There are m students $s_1, s_2, \ldots, s_m$ and a set of n companies $C = \{c_1, c_2, \ldots, c_n\}$. Each student can work for only one company, whereas company c_j can hire up to b_j students. Student s_i has a preferred set of companies $\Lambda_i \subseteq C$ at which he/she is willing to work. Your task is to find an assignment of students to companies such that all of the above constraints are satisfied and each student is assigned. Formulate this as a network flow problem and describe any subsequent steps necessary to arrive at the solution. Prove correctness.

5) 16 pts

Consider a directed, weighted graph G where all edge weights are positive. You have one Star, which lets you change the weight of any one edge to zero. In other words, you may change the weight of any one edge to zero. Give an efficient algorithm using Dijkstra's algorithm to find a lowest-cost path between two vertices s and t, given that you may set one edge weight to zero. Note: you will receive 10 pts if your algorithm is efficient. You will receive full points (16 pts) if your algorithm has the same run time complexity as Dijkstra's algorithm.

6) 16 pts

   You are given n rods; they are of length $l_1, l_2, \ldots, l_n$, respectively. Our goal is to connect all the rods and form a single rod. The length after connecting two rods and the cost of connecting them are both equal to the sum of their lengths. Give an algorithm to minimize the cost of connecting them to form a single rod. State the complexity of your algorithm and prove that your algorithm is optimal.

Additional Space