2+ years of teaching experience

MCA from IGDTUW

1. Time Complexity & Space Complexity
2. Asymptotic Analysis (Big O)
3. TLE
4. Big O → Meaning

Time Complexity ⇒ 2

Finding the no. of iterations

QUIZ 1

No. of times we need to divide N by 2 to reduce it to 1

$\log_2 N$

$[a, b]$

range where a, b are included.

$$\begin{array}{cc} a & b \\ [3, & 10] \end{array}$$

$\underbrace{3, 4, 5, 6, 7, 8, 9, 10}_{8 \text{ ele}}$

$b - a + 1$

$10 - 3 + 1 = 8$

(a, b)

a & b excluded

$b - a - 1$

$(7 \quad 15)$

$\underbrace{8, 9, 10, 11, 12, 13, 14}$

$a = 7 \quad b = 15$

$15 - 7 - 1 = 7$

Arithmetic Progression

$\underbrace{4, \; 7}_{3}, \underbrace{\; 10}_{3}, \underbrace{\; 13}_{3}, \underbrace{\; 16}_{3}$

$a \rightarrow$ first term

$d \rightarrow$ common difference

$N \rightarrow$ No. of terms

$a, \; a+d, \; a+2d, \; a+3d, \; \ldots \ldots \; a + (N-1)d$

sum of terms of AP $= \frac{N}{2}\left[2a + (N-1)d\right]$

Geometric Progression

$5, \; 10, \; 20, \; 40, \; 80, \; 160$

2 2 2 2 2

a → first terms

r → common ratio

N → no. of terms.

$$\underbrace{ar^0, ar^1}_{r}, \underbrace{ar^1, ar^2}_{r}, \underbrace{ar^2, ar^3}_{r} \ldots\ldots ar^{N-1}$$

$$[\overset{a}{0} \quad \overset{b}{N-1}] \rightarrow b-a+1$$

$$N-\cancel{1}-0+\cancel{1} = N$$

sum of GP
$$\begin{cases} \dfrac{a(r^N-1)}{r-1} & r>1 \\ a\left(\dfrac{1-r^N}{1-r}\right) & r<1 \end{cases}$$

$r=1$?

$$\log_2 2^{10} = 10$$

$$\log_x x^a = a$$

$$2^{\log_2 N} = N$$

Q1.

```
int func ( int N) {
    S = 0
    for( i=1 ; i<=N ; i++) {
        S = S+i
    }
    return S
}
```

Iterations
⇓
No. of times a loop will run

$[1 \quad N] \rightarrow N - \cancel{1} + \cancel{1}$

$= N$

$[a \quad b] \Rightarrow b-a+1$

```
void func( int N, int M) {
    for( i=1 ; i<=N ; i++) {
        if ( i%2 ==0) {
            print (i)
        }
    }
    for ( j=1 ; j<=M ; j++) {
        if ( j%2==0) {
            print (j)
        }
    }
}
```

$[1 \quad N] = N - \cancel{1} + \cancel{1}$

$= N$

$[1 \quad M] = M - \cancel{1} + \cancel{1}$

$= M$

$N+M$ Iterations

Q.

Q:

```
int func ( int N) {
    s = 0
    for ( i = 1 ; i <= N ; i = i+2)
        s = s+i
    }
    print (s)
}
```

i [i+2]

1
3
5
7
9
11

N

odd nos. from [1 N]

$\frac{N+1}{2}$

No. of odd nos. b/w range [1 N]

N = 7 [1 2 3 4 5 6 7] 4

N = 6 [1 2 3 4 5 6] 3

$\boxed{\frac{N}{2}}$

$\frac{7}{2} = 3.5$

$\frac{N}{2}$

$6/2 = 3$

$\frac{7}{2} \to 3.5 \to 3$

$\frac{9}{}= 2.\ x\,x\,x$

3 | 4 → 2

N = 8 [1 2 3 4 5 6 7 8]

$\frac{8}{2} = 4$

N = 9 [1 2 3 4 5 6 7 8 9]

$\frac{9}{2} = 4.5$ → 4

Formula → $\frac{N+1}{2}$ [which works for both even & odd value of N]

↓

gives the no. of odd nos. [1 N]

QUIZ

```
int func(int N){
    s = 0
    for(i=0; i<=100; i++){
        s = s + i + i^2
```

[0 100]

a b

|
}
return s
}

$b - a + 1$

$100 - 0 + 1$

101

QUIZ

```
void func(int N){
    s = 0
    for( i=1 ; i * i <= N ; i++)
        s = s+i^2
    }
}
```

$i <= \sqrt{N}$

=

$i^2 <= N$

sqrt on both sides

$i <= \sqrt{N}$

$\underbrace{[1 \quad \sqrt{N}]}$

$\sqrt{N} \cancel{-1} \cancel{+1}$

$\sqrt{N}$

```
void func( int N){
    int i = N
    while (i > 1) {
        i = i/2
    }
}
```

i

Before	After	
N	N/2	$N/2^1$
N/2	N/4	$N/2^2$
N/4	N/8	$N/2^3$
N/8	N/16	$N/2^4$
	⋮	⋮

1 $\quad N/2^{K}$

$\boxed{\frac{N}{2^K} = 1}$ $\quad N = 2^{K}$

log on both sides $\Rightarrow$ $\log_2 N = \log_2 2^K$

$\boxed{\log_2 N = K}$

$N \longrightarrow N/2 \longrightarrow N/4 \longrightarrow N/8 \longrightarrow \cdots\cdots N/2^K$

Break → 5 min

$$\overbrace{16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1}^{K}$$

8	4	2	1
$N/2$	$N/4$	$N/8$	$N/16$
$N/2^1$	$N/2^2$	$N/2^3$	$N/2^K$

$\frac{N}{2^K} = 1$

$N = 2^K$

$\log_2 N = \log_2 (2^K)$

$\log_2 N = K$

$\log_x x^a = a$

Q: void func (N) {

```
for (i=1; i<=N; i=i*2) {
    print (i)
}
}
```

Before	After
i	
1	2
2	4
4	8
8	16
.	.
	N

$$\underbrace{N \xrightarrow{} N/2 \rightarrow N/4 \rightarrow N/8 \rightarrow \ldots 1}_{\log_2 N}$$

/2

$$\underbrace{1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \rightarrow \ldots N}$$

*2

$$\underbrace{1 \rightarrow 2^1 \rightarrow 2^2 \rightarrow 2^3 \rightarrow \ldots 2^K}$$

$N = 2^K$

$\log_2 N = \log_2 2^k$

$K = \log_2 N$

N=100 $\Rightarrow$ $\log_2 N = \log_2(100) \Rightarrow$

i $\longrightarrow$ 100

$1 \to 2 \to 4 \to 8 \to 16 \to 32 \to 64 \to 128$

$\log_2(100) = 6.6$

$\Downarrow$

6

$\log_2 N + 1$

-1

Nested Loops

```
void func( int N){
    for( i = 1 ; i <= 10; i++) {
        for (j = 1; j<=N; j++) {
            print (i*j)
```

i	j	Iterations
1	[1 N]	N +
2	[1 N]	N +
3	[1 N]	N +
4	[1 N]	N +
.		
.		

}

}

}

⋮

10 [1 N] N

$N + N + N + \cdots$ 10 times

$10 \times N$

$1 \to N$

$10 \to 10 \times N$

$N + N + \cdots$ 10 times

$[0 \quad N-1] \Rightarrow N - \cancel{1} - 0 + \cancel{1} = N$

QUIZ

```
for (i=0; i<N; i++){
    for (j=0; j<N; j++){
        ==
    }
}
```

$[0 \quad N-1] \Rightarrow N$

N^2

i	j	Iterations
0	[0 N-1]	N +
1	[0 N-1]	N +
2	[0 N-1]	N +
⋮	⋮	⋮ +
N-1	[0 N-1]	N

$N \times N$

$N \times N$

N^2

QUIZ

```
for(i=1; i<=N; i++){
    for(j=1; j<=N; j=j*2){

    }
}
```

$1 \rightarrow N$

$N \rightarrow N^2$

i	j	Iterations
1	[1 N]	$\log_2 N$ +
2	[1 N]	$\log_2 N$ +
3	[1 N]	$\log_2 N$ +
⋮		. +
N	[1 N]	$\log_2 N$

$N * \log_2 N$

Que'

```
for(i=1; i<=4; i++){
    for(j=1; j<=i; j++){
        print("Monkey")
    }
}
```

i	j	Iterations
1	[1 1]	1 +
2	[1 2]	2 +
3	[1 3]	3 +
4	[1 4]	4
		10

QUIZ

i	j	Iterations
1	[1 1]	1

```
for (i = 1 ; i <= N ; i++) {
    for (j = 1 ; j <= i ; j++) {
        print ("Monkey")
    }
}
```

2	[1 2]	+ 2 +
3	[1 3]	3 +
⋮	⋮	⋮ +
N	[1 N]	N

$$1 + 2 + 3 + 4 + \cdots N$$

$$sum = \frac{N}{2}\left[2a + (N-1)d\right]$$

$$= \frac{N}{2}\left[2(1) + (N-1)(1)\right]$$

$$= \frac{N}{2}\left[2 + N - 1\right]$$

$$= \left\| \frac{N}{2}\left[N + 1\right] \right\|$$

$$\frac{N(N+1)}{2}$$

Que.

```
for (i = 1; i <= N; i++) {
    for (j = 1 ; j <= 2^i ; j++) {
        print (i & j)
    }
```

i	j	Iteration
1	$[1 \quad 2^1]$	2^1 +
2	$[1 \quad 2^2]$	2^2 +
3	$[1 \quad 2^3]$	2^3 +
4	$[1 \quad 2^4]$	2^4 +
⋮		⋮

}

$\vdots$

$N \quad [1 \;\; 2^N] \quad 2^N$

$$\Rightarrow \underbrace{2^1 + 2^2 + 2^3 + 2^4 + \cdots 2^N}_{\text{sum of GP}}$$

$a \rightarrow 2$

$r \rightarrow 2$

no. of terms $\rightarrow N$

$$\frac{a(r^N - 1)}{r-1} \Rightarrow \frac{2(2^N - 1)}{2-1}$$

$$\Rightarrow 2(2^N - 1)$$

Goldman / Google

```
for ( i = N; i > 0; i = i/2 ) {
    for (j = 1; j <= i; j++) {
        print (i+j)
    }
}
```

Table

i	j	Iteration
N	[1 N]	N +
N/2	[1 N/2]	N/2 +
N/4	[1 N/4]	N/4 +
N/8	[1 N/8]	N/8 +
$\vdots$		$\vdots$ +
1	[1 1]	1

$$2^{\log_2 N} = N$$

$$1 = \frac{N}{N} = \frac{N}{2^{\log_2 N}}$$

$$N \xrightarrow{1} + \frac{N}{2} \xrightarrow{2} + \frac{N}{4} \xrightarrow{3} + \frac{N}{8} + \cdots \xrightarrow{\log_2 N} 1 \qquad \frac{N}{N}$$

$$\frac{N}{2^{\log_2 N}}$$

$$\frac{N}{2^0} + \frac{N}{2^1} + \frac{N}{2^2} + \frac{N}{2^3} \cdots \frac{N}{2^{\log_2 N}}$$

$$\frac{N}{2^0} + N\left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots \frac{1}{2^{\log_2 N}}\right)$$

$a = 1/2$

$r = 1/2$

no_of_terms $= \log_2 N$

$$\frac{a(1 - r^N)}{1 - r} \Rightarrow \frac{1}{2}\left(\frac{1 - (1/2)^{\log_2 N}}{1 - 1/2}\right)$$

$$\Rightarrow \cancel{\frac{1}{2}}\,\frac{\left(1 - \frac{1}{2^{\log_2 N}}\right)}{\cancel{1/2}}$$

$$\Rightarrow 1 - \frac{1}{2^{\log_2 N}}$$

$2^{\log_2 N}$ $\rightarrow 1 - \frac{1}{N}$

$$\frac{N}{2^0} + N\left(1 - \frac{1}{N}\right)$$

$$N + (N - 1) = 2N - 1$$

DOUBT

$$N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \dots 1$$

$a = N$ $\quad \log_2 N + 1$

$r = 1/2$

$$\frac{N\left(1 - \frac{1}{2^{\log_2 N+1}}\right)}{1/2}$$

$$2N\left(1 - \frac{1}{2^{\log_2 N} \cdot 2}\right)$$

$$2N\left(1 - \frac{1}{\dots}\right)$$

$$\left(\quad 2N \quad \right)$$

$$\cancel{2N}\left(\frac{2N-1}{\cancel{2N}}\right)$$

Doubt

$\log_2 N$

$$\frac{100}{2} \rightarrow \frac{50}{2} \rightarrow \frac{25}{2} \rightarrow \frac{12}{2} \rightarrow \frac{6}{2} \rightarrow \frac{3}{2} \rightarrow 1$$

$$\log_2(100) = 6$$