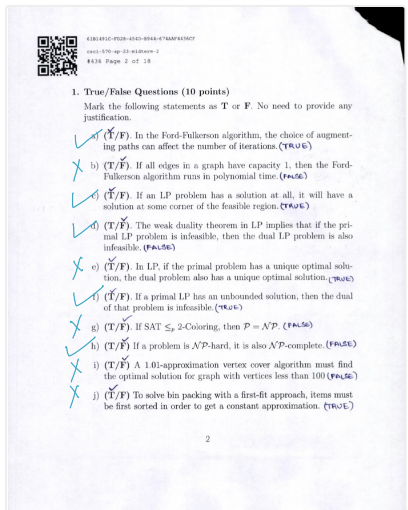
My grades for **csci-570-sp-23-midterm-2**

Q1 5



61B1491C-FD28-454D-B944-674AAF4436CF

csci-570-sp-23-midterm-2

#436 Page 2 of 18

1. True/False Questions (10 points)

Mark the following statements as **T** or **F**. No need to provide any justification.

a) **(T/F)**. In the Ford-Fulkerson algorithm, the choice of augmenting paths can affect the number of iterations. (TRUE)

b) **(T/F)**. If all edges in a graph have capacity 1, then the Ford-Fulkerson algorithm runs in polynomial time. (FALSE)

c) **(T/F)**. If an LP problem has a solution at all, it will have a solution at some corner of the feasible region. (TRUE)

d) **(T/F)**. The weak duality theorem in LP implies that if the primal LP problem is infeasible, then the dual LP problem is also infeasible. (FALSE)

e) **(T/F)**. In LP, if the primal problem has a unique optimal solution, the dual problem also has a unique optimal solution. (TRUE)

f) **(T/F)**. If a primal LP has an unbounded solution, then the dual of that problem is infeasible. (TRUE)

g) **(T/F)**. If SAT $\leq_p$ 2-Coloring, then $\mathcal{P} = \mathcal{NP}$. (FALSE)

h) **(T/F)** If a problem is $\mathcal{NP}$-hard, it is also $\mathcal{NP}$-complete. (FALSE)

i) **(T/F)** A 1.01-approximation vertex cover algorithm must find the optimal solution for graph with vertices less than 100 (FALSE)

j) **(T/F)** To solve bin packing with a first-fit approach, items must be first sorted in order to get a constant approximation. (TRUE)

2

Q2 12

2. Multiple Choice Questions (20 points)

Please select the most appropriate choice. Each multiple choice question has a single correct answer.

a) What would be the running time of the Ford-Fulkerson algorithm if we always choose the augmenting path with the least number of edges?

if FF → $O(V+V^2) \to O(V^2) \cdot O(E)$
if EK → $O(VE^2) \to O(V^5)$

a) $O(E)$
b) $O(EV)$
c) $O(E^2V)$
d) $O(EV^2)$

b) Which of the following statements is true?

a) The capacity of a cut is the sum of the capacities of all the edges that cross the cut.
b) For every node in a network, the total flow into that node equals the total flow out of that node.
c) The max flow in a network is less than or equal to the total capacity of all edges directly connected to the sink.
d) The max flow in a network is less than the total capacity of the edges in the min cut.

c) Which of the following statements is correct?

a) An edge connecting s to t is always saturated when maximum flow is reached.
b) If all capacities in a flow network are integers, then every maximum flow has integer flows on edges.
c) If in a flow network all edge capacities are distinct, then there exists a unique min-cut.
d) If f is a maximum $s-t$ flow in a flow network, then for all edges out of s are saturated.

d) Let X be a problem that belongs to the class $\mathcal{NP}$. Which one of the following is true?

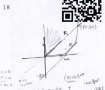
a) X cannot be solved deterministically in polynomial time.
b) If X can be solved deterministically in polynomial time, then $\mathcal{P} = \mathcal{NP}$.
c) X may be undecidable.
d) X can be solved in nondeterministic polynomial time.

e) Which of the following statements is true?

a) If X and Y are reducible to each other, then X and Y are $\mathcal{NP}$-Complete.
b) If X and Y are NP-Complete, then X and Y are reducible to each other.
c) $\mathcal{NP}$-Hard is a subset of $\mathcal{NP}$-Complete.
d) 2-SAT problem is $\mathcal{NP}$-Complete.

f) Let X, Y, Z be the problems which can be solved in time $O(\log n)$, $O(n^3)$, $O(2^n)$ respectively (where n denotes the input size). Then, we can NOT conclude that

a) Y is reducible to X.
b) Y is reducible to Z.
c) Z is reducible to Y.
d) If Z is reducible to Y, then Z is reducible to X.



g) Consider the linear program

$$\max(31x + 55y)$$

subject to:

$$2x - y \leq 80$$
$$y \geq x$$
$$x \geq 0$$
$$y \geq 0$$

$y - x \geq 0$

$x - y \leq 0 \Rightarrow$ standard form

feasible unbounded

Which of the following statement(s) are true?

a) the linear program has a single optimal solution.
b) the linear program has infinitely many optimal solutions.
c) the linear program has no optimum solution.

h) What is the feasible region in linear programming?

a) The set of all optimal solutions to the LP problem
b) The feasible region is the set of all feasible solutions that satisfy the constraints of the LP problem.
c) The set of all feasible solutions that satisfy the objective function
d) The set of all solutions that satisfy the objective function

i) Consider a linear program having an objective $max\, c^T x$, and let its dual have the objective $min\, b^T y$. Let x and y be some feasible solutions of the primal and dual respectively. Then,

a) $c^T x \geq b^T y$.
b) $c^T x \leq b^T y$.
c) $c^T x = b^T y$.
d) Any definitive comparison for the objectives is not possible.

j) A feasible solution of Linear Programming

a) Must satisfy all the constraints.
b) Does not need satisfy all the constraints, but only some of them.
c) Must be a corner point of the feasible region.
d) Must be the maximum of the objective function.

Q3 15

3. Linear Programming (15 points)

A company produces three products, Product X, Product Y, and Product Z. The company has a limited amount of resources, including production time, labor, and materials. The goal is to maximize profits subject to resource constraints. The following information is available:

- Each unit of Product X requires 1 hour of production time, 1 hour of labor, and 2 units of raw materials. Each unit of Product Y requires 2 hours of production time, 2 hours of labor, and 3 units of raw materials.
- Each unit of Product Z requires 3 hours of production time, 4 hours of labor, and 2 units of raw materials.
- The company has 200 hours of production time, 250 hours of labor, and 150 units of raw materials available.
- The profit per unit of Product X is \$50, the profit per unit of Product Y is \$60, and the profit per unit of Product Z is \$80.
- The company must produce at least 50 units of Product X, 75 units of Product Y, and 25 units of Product Z. The company can only produce a maximum of 125 units in total.

Formulate a linear programming model to help the company maximize profits. You do not have to solve the resulting LP.

a) Describe what your LP variables represent (3 points).

b) Show your objective function (3 points).

c) Show your constraints (9 points).

	Product	Time	Labor	RM	Profit	MQ
	X	1 hr	1 hr	2 u	50 \$	50u
7	Y	2 hr	2 hr	3 u	60 \$	75u
	Z	3 hr	4 hr	2u	80 \$	25u
	Total available	200 hr	250hr	150u		

max units → 125

Solution to problem 3

Formulating the given linear programming problem.

(a) Variable declaration.

Let x_1, be the number of units of Product X that will be produced.

Let x_2, be the number of units of Product Y that will be produced.

Let x_3, be the number of units of Product Z that will be produced.

(b) Objective Function

We are asked to maximize the profit earned by the company.

$\therefore$ objective function is. $\max(50x_1 + 60x_2 + 80x_3)$

good **15**
work

(c) Subject to the following constraints

(i) $x_1 + 2x_2 + 3x_3 \leq 200$ (production hours available)

(ii) $x_1 + 2x_2 + 4x_3 \leq 250$ (labour hours available)

(iii) $2x_1 + 3x_2 + 2x_3 \leq 150$ (raw material available)

(iv) $x_1 \geq 50$ (company should produce atleast 50 units of product X)

(v) $x_2 \geq 75$ (company should produce atleat 75 units of Product Y)

(vi) $x_3 \geq 25$ (company should produce atleast 25 units of product Z)

(vii) $x_1 + x_2 + x_3 \leq 125$ (company can produce a maximum of 125 units in total)

Q4 15

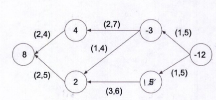


Figure 0.1: Question 4

4. Network Flow (20 points)

In the network above, the demand values are shown on vertices (supply values are negative). Lower bounds on flow and edge capacities are shown as (lower bound, capacity) for each edge. Determine if there is a feasible circulation in this graph. Please complete the following three steps below.

a) Turn the circulation with lower bounds problem into a circulation problem without lower bounds (8 points).

b) Turn the circulation with demands problem into the max-flow problem (7 points).

c) Does a feasible circulation exist? Explain your answer (5 points).

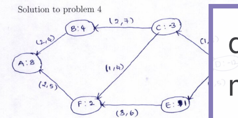
A4FBF1BA-98A6-4976-9938-74CFCC7AB811

csci-570-sp-23-midterm-2

#436 Page 10 of 18

Solution to problem 4

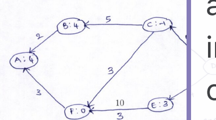
a)



to find the new demands after removing the lower bounds, we do using the below steps

$d'(A) = 8+0-2-2 = 4$
$d'(B) = 4+2-2 = 4$
$d'(C) = -3+2-1+1 = -2+1 = -1$
$d'(d) = -12+1+1 = -10$
$d'(e) = 1+3-1 = 3$
$d'(f) = 2+2-1-3 = 0$

$e(AB) = 2$
$e(BC) = 5$
$e(C,D) = 4$
$e(DE) = 4$
$e(EF) = 3$
$e(FA) = 3$
$e(CF) = 3.$



continued on page 16

didn't mention max flow, and incorrect conclusion -5

Q5 9

5. $\mathcal{NP}$ Completeness (20 points)

An organization is arranging an event spread over n days, namely $D_1, D_2, \ldots, D_n$, and needs volunteers to manage the event. On day i, at least v_i volunteers are needed. A total of m people, namely $P_1, P_2, \ldots, P_m$, have applied to volunteer with each applicant indicates the availability. An applicant, if selected, must work on all the days that he or she has indicated. The organization wants to select as few volunteers as possible. Please complete the following five questions.

a) Phrase the above optimization problem as a decision problem and show that it belongs to $\mathcal{NP}$. (5 points)

b) Show a polynomial time construction using a reduction from Vertex Cover.(6 points)

c) Write down the claim that the Vertex Cover problem is polynomially reducible to the original problem. (3 points)

d) Prove the claim in the direction from the reduced problem to the Vertex Cover problem. (3 points)

e) Prove the claim in the direction from the Vertex Cover problem to the reduced problem. (3 points)

(a) The above problem can be converted to decision problem as " Will the organisation be able to find 'v_i' volunteers on day 'i' to manage the event, such that the organisation selecte as few volunteers as possible as per their availability"

The above problem is NP, we can show it by assuming a feasible solution and verifying that solution in polynomial

a correct 5

4F7D9991-BA29-4F31-88E6-6435271B3272

csci-570-sp-23-midterm-2

#436 Page 12 of 18

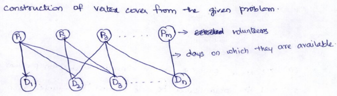
Solution to problem 5

Assuming that we are given the list of volunteers selected, then we can create an empty zero array with size 'n' (no of days) and traverse each volunteer and increment the value of the day 'D_i' on which the volunteer is available. this can be done in polynomial time.

Once this is completed, then we can traverse the array and make sure that on each day the number of volunteers is atleast 'V_i' this can also be verified in polynomial time.

$\therefore$ we can say that the given problem is in NP.

(b) construction of vertex cover from the given problem.



here $|VC|$, all the edges should

possible if the total number of

is atleast V_i $\therefore$ for all the d

be ΣV_i $\therefore |VC| = k = \Sigma V_i$

12

partly 1

corrected

b For each node v in V, we construct an applicant Pv

7C9C96D2-3F9A-4269-AB8A-1151BA27D056

csci-570-sp-23-midterm-2

Solution to problem 5

correct 3

(c) The ~~vertex cover the~~ graph G has a vertex cover vc of size 'k' if and only if we are able to find volunteers such that on each day there are more than or equal to 'Vi' (d). Proving the above claim.

incorrect **0**

Here we need to assume that we have the solution to given ... are able to solve the VC with size 'k'

if we are given ... on which each volunteer is available then in the construction we have made the total vertex cover of the graph shall be $\underline{\underline{\Sigma V_i = 'k'}}$.

incorrect **0**

2.) Here we need to assume that we have a vertex cover of size 'k' ... that our problem also has a solution.

if the vertex cover as per our construction is $'k' = \Sigma V_i$, then it means than we are able to satisfy the constraint of having at least 'V_i' number of volunteers on each day 'i'.

Q6b 1

Q6a 4

Q6c 1

6. Approximation Algorithm (15 points)

There are N products with infinite supply available and the price of each one of product i is p_i, $1 \leq p_i \leq 100$. You have a \$100 gift card, and to better utilize it, you try to buy a bunch of products to maximize the total value, subject to the limit \$100. For example, if there are 3 products with prices \$49.99, \$89.99, \$50, then the optimal solution is to buy two product 3, and the total value is \$100.

a) Suppose every price is more than 50, that is, for every i we have $50 < p_i \leq 100$. Prove that you can find the optimal solution in $O(N)$ time. (4 points)

b) Suppose there exists a price no more than 50, that is, there exists an i so that $1 \leq p_i \leq 50$. Prove that buying $\lfloor 100/p_i \rfloor$ product i alone is a $\frac{2}{3}$-approximation algorithm. Here $\lfloor x \rfloor$ is the greatest integer less than or equal to x, for example, when $p_i = 30$, you buy $\lfloor 100/30 \rfloor = 3$ product i. (6 points)

c) Without the assumptions in a) and b), design a $\frac{2}{3}$-approximation algorithm that runs in $O(N)$. Prove your results. (5 points)

(a). As mentioned we need to best utilize the gift card value. since every item is between $50 < pi \leq 100$ (strictly greater than 50) So, there is no way we can buy two ~~same~~ products (same/different).
we need to initialise $i=0$ & $max=0$
Here, we need to traverse each object 'i' and update 'i' and 'max' variable values if $P_i > max$

So, basically we shall be picking the item with max price.

Since we are traversing entire list only once, it can be done in O(N) time.

Solution to problem 6

(b). Here, it is mentioned that the price of object is between $1 \leq P_i \leq 50$ (both 1 & 50 inclusive).

we can assume that as mentioned in the question, the optimal algorithm will try to maximize the number of objects & also try to use the gift card in the best way possible.

So it can choose 3-obj

$P_i = 33.8333...$ $\lfloor 100/P_i \rfloor$

But our greedy algorithm

whoose $P_i = 50$ $\lfloor 100/P_i$

∴ now we can find

b) incorrect solution 1

(c) (Here we need to sort the items in ascending order) → optional step ~~and~~ first chec if we have an item in such a way that buying 'n' items of it will completely use the gift card capacity. ~~For example~~ and ~~such that~~ For example if we have an item with $P_i = 50$ then buying 2 products of 50 each will use the available limit.

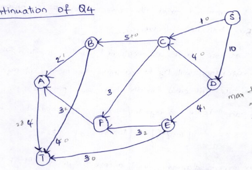
c) first, sorting takes 1

more than O(N) time; second if you don't sort, your algorithm is also incorrect.

Additional space

continuation of Q4

(b)



max flow
= 4+1+3+1
= 9

adding a new vertex 'S' source and connecting it to nodes with supply (-ve weights)
and adding a new vertex 'T' sink and connecting it to all nods with demand (+ve weight)

max flow in above graph is : 9

(c) The feasibility can be checked using the formula $\Sigma d(v) = 0$

(i) in the original graph $= -12 - 3 + 8 + 4 + 2 + 1 = 0$
(ii) in reduced graph $= -10 - 1 + 4 + 4 + 3 = 0$

$\therefore$ circulation is feasible.

Additional space

Additional space