CSCI 570 Homework 5
Spring 2023

Due Date: Apr. 13, 2023 at 11:59 P.M.

1. There is a precious diamond that is on display in a museum at m disjoint time intervals. There are n security guards who can be deployed to protect the precious diamond. Each guard has a list of intervals for which he or she is available to be deployed. Each guard can be deployed to at most M time slots and has to be deployed to at least L time slots. Design an algorithm that decides if there is a deployment of guards to intervals such that each interval has either one or two guards deployed.

2. A company makes three products and has 4 available manufacturing plants. The production time (in minutes) per unit produced varies from plant to plant as shown below:

		Manufacturing Plant			
		1	2	3	4
Product	1	5	7	4	10
	2	6	12	8	15
	3	13	14	9	17

Similarly, the profit ($) contribution per unit varies from plant to plant as below:

		Manufacturing Plant			
		1	2	3	4
Product	1	10	8	6	9
	2	18	20	15	17
	3	15	16	13	17

If, one week, there are 35 working hours available at each manufacturing plant how much of each product should be produced given that we need at least 100 units of product 1, 150 units of product 2, and 100 units of product 3. Formulate this problem as a linear program. You do not have to solve the resulting LP.

3. Solve the following linear program using the fundamental theorem. Specifically, find all vertices of the feasible region, calculate the values of the objective function at those points, and conclude the optimal solution. (*Hint: plot the feasible region in 2D*)

$$\max(-x_1 + 4x_2)$$

subject to

$$\begin{aligned} 3x_1 + x_2 &\leq 1 \\ 3x_1 + x_2 &\geq -5 \\ x_1 - x_2 &\leq 4 \\ x_1 - x_2 &\geq -2 \\ x_2 &\leq 1 \end{aligned}$$

4. There are m basic nutritional ingredients, and Andy has to receive at least b_i units of the i-th nutrient per day to satisfy the basic minimum nutritional requirements. There are n available foods. The j-th food sells at a price c_j per unit and contains a_{ij} units of the i-th nutrient. Help Andy find the lowest cost per day to satisfy the requirement. Formulate this problem as a linear programming problem in the standard form.

5. Given an undirected graph $G = (V, E)$, a vertex cover is a subset of V so that every edge in E has at least one endpoint in the vertex cover. The problem of finding a minimum vertex cover is to find a vertex cover of the smallest possible size. Formulate this problem as an integer linear programming problem.

6. Write down the dual program of the following linear program. There is no need to provide intermediate steps.

$$\max(x_1 - 3x_2 + 4x_3 - x_4)$$

subject to

$$\begin{aligned} x_1 - x_2 - 3x_3 &\leq -1 \\ x_2 + 3x_3 &\leq 5 \\ x_3 &\leq 1 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

7. Determine whether the following linear programs are feasible bounded, feasible unbounded, or infeasible.

(a)

$$\max(x_1 + x_2)$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\leq 3 \\ 3x_1 - x_2 &\leq 2 \\ -4x_1 - x_2 &\leq 2 \end{aligned}$$

(b)

$$\max(x_1 + x_2)$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\geq 3 \\ 3x_1 - x_2 &\geq 2 \\ -4x_1 - x_2 &\geq 2 \end{aligned}$$

(c)

$$\max(x_1 + x_2)$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\geq 3 \\ 3x_1 - x_2 &\leq 2 \\ -4x_1 - x_2 &\leq 2 \end{aligned}$$

8. Show that vertex cover remains NP-Complete even if the instances are restricted to graphs with only even degree vertices.

9. Assume that you are given a polynomial time algorithm that given a 3-SAT instance decides in polynomial time if it has a satisfying assignment. Describe a polynomial time algorithm that finds a satisfying assignment (if it exists) to a given 3-SAT instance.

10. **Online Questions.** Please go to DEN (https://courses.uscden.net/) and take the online portion of your assignment.