Probability Assignment 1(12.13.4.11)

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Question

Two dice are thrown simultaneously, if X denotes number of sixes, find the expectation of X

Solution

Clearly X follows a binomial distribution

$$X \sim Bin(2, 1/6) \tag{1}$$

$$E(X) = np \tag{2}$$

$$= 2.\frac{1}{6} \tag{3}$$

$$= 0.33 \tag{4}$$

Proof of (2)

$$M_{X_i}(z) = E(z^{X_i}) = \sum_{k=-\infty}^{\infty} P_{X_i}(k) z^{-k} \tag{5}$$

$$E(z^{X_1+X_2+...X_n}) = E(z^{X_1})E(z^{X_2})...E(z^{X_n}) \tag{6}$$

$$M_X(z) = M_{X_1}(z)M_{X_2}(z)...M_{X_n}(z) \tag{7}$$

($\because$ $X_1, X_2, ..., X_n$are independent and identically distributed)

$$\frac{dM_{X_i}(z)}{dz} = E(X_i z^{X_i-1}) \tag{8}$$

Put z=1 to get

$$E(X_i) = \frac{dM_{X_i}(z)}{dz}|_{z=1} \tag{9}$$

For Bernoulli distribution,

$$M_{X_i}(z) = 1 - p + pz^{-1} \tag{10}$$

For binomial,

$$X = X_1 + X_2 + ...X_n \tag{11}$$

$$M_X = (1 - p + pz^{-1})^{-n} \tag{12}$$

$$E(X) = \frac{dM_X(z)}{dz}|_{z=1} \tag{13}$$

$$= \frac{d(1 - p + pz^{-1})^{-n}}{dz}|_{z=1} \tag{14}$$

$$= (-n)(1 - p + p(1))^{-(n+1)} \frac{-p}{1^2} \tag{15}$$

$$= np \tag{16}$$

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