

Probability Assignment 1(12.13.4.11)

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Question

Two dice are thrown simultaneously, if X denotes number of sixes, find the expectation of X

For binomial,

$$X = X_1 + X_2 + \dots X_n \quad (11)$$

$$M_X = (1 - p + pz^{-1})^n \quad (12)$$

Solution

Clearly X follows a binomial distribution

$$X \sim \text{Bin}(2, 1/6) \quad (1)$$

$$E(X) = np \quad (2)$$

$$= 2 \cdot \frac{1}{6} \quad (3)$$

$$= 0.33 \quad (4)$$

$$E(X) = \frac{dM_X(z^{-1})}{dz} \Big|_{z=1} \quad (13)$$

$$= \frac{d(1 - p + pz)^n}{dz} \Big|_{z=1} \quad (14)$$

$$= (np)(1 - p + p(1))^{(n-1)} \quad (15)$$

$$= np \quad (16)$$

Proof of (2)

$$M_{X_i}(z) = E(z^{-X_i}) = \sum_{k=-\infty}^{\infty} P_{X_i}(k)z^{-k} \quad (5)$$

$$E(z^{-X_1-X_2+\dots-X_n}) = E(z^{-X_1})E(z^{-X_2})\dots E(z^{-X_n}) \quad (6)$$

$$M_X(z) = M_{X_1}(z)M_{X_2}(z)\dots M_{X_n}(z) \quad (7)$$

($\because X_1, X_2, \dots, X_n$ are independent and identically distributed)

$$\frac{dM_{X_i}(z^{-1})}{dz} = E(X_i z^{X_i-1}) \quad (8)$$

Put $z=1$ to get

$$E(X_i) = \frac{dM_{X_i}(z^{-1})}{dz} \Big|_{z=1} \quad (9)$$

For Bernoulli distribution,

$$M_{X_i}(z) = 1 - p + pz^{-1} \quad (10)$$

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