Day 1 Sakhawat

Mathmatical Assignment

1) prove that R^n is a vector space by verifying closure under Addition and closure under scalar multiplication.

Solution: let $u = (u_1, \cdots u_n)$ and $v = (v_1 \cdots v_n)$ be in R^n

and let $c \in R$

Addition:
$$u+v = (u_1+v_1, + \cdots u_n+v_n) \in R^n$$

Scalar multiplication:
$$c.v = (cv_1 + cv_2 \cdots cu_n) \in R^n$$

Since both operation remain in R^n, closure holds, Therefore, R^n is a vector space.

2) let $H = \{(x, y, z) \in R^3 \mid x+y+z = 0\}$.

Determine whether H is a subspace of R^3. Justify your Answer using the three subspace Condition.

Solution: check subspace Conditions:

• Zero vector:

$$(0,0,0) \in H \text{ since } 0+0+0=0$$

• Closure under addition:

If (x_1, y_1, z_1) and (x_2, y_2, z_2) satisfy

$$x_1+y_1+z_1=0, \quad x_2+y_2+z_2=0$$

then $(x_1+x_2)+(y_1+y_2)+(z_1+z_2)=0$

• Closure under scaler multiplication: If $c \in R$,

$$c(x+y+z) = c \cdot 0 = 0.$$

All condition hold. Hence, H is a subspace of R^3.

3) Given the vector:

$$v_1 = (1,2,3), \; v_2 = (4,5,6), \; v_3 = (2,4,6)$$

Determine whether the set $\{v_1, v_2, v_3\}$ is linerly independent. Show All neccssary steps.

Solution: observe:

$$V_3 = (2,4,6) = 2(1,2,3) = 2V_1$$

Since V_3 is scalar multiple of V_1, the vectors are linearly dependent.

4) Define a linear combination. Find the span of the vectors $(1,0,0)$ and $(0,1,0)$ in $\mathbb{R}^3$. What geometric object does this span represent?

<u>Solutions</u>:

A linear combination is

$$c_1V_1 + c_2V_2$$

for $(1,0,0)$ and $(0,1,0)$:

$$c_1(1,0,0) + c_2(0,1,0) = (c_1, c_2, 0)$$

Thus

$$\text{span}\{(1,0,0),(0,1,0)\} = \{(x,y,0)\}$$

This represent the xy plane in $\mathbb{R}^3$.

5) Determine whether the vectors.

$(1,1,0)$, $(0,1,1)$ $(1,0,1)$.

from a basis for R^3. Justify your Answer.

Solution:

From A matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

compute determinant.

$\det(A) = 1(1.1 - 1.0) - 1(0.1 - 1.1) = 2 \neq 0$.

Since determinant is non zero. vectors are linerly independent and from a basis of R^3.

6) prove that the standard basis vectors in R^n are linenly independent.

Solution: suppose: $c_1e_1 + \cdots c_ne_n = 0$

Then $(c_1 \cdots c_n) = (0, \cdots 0)$.

Thus, $c_1 = \cdots c_n = \cdots 0$, proving liner independence.

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7) Find the dimension of the subspace spanned by the vectors $(1,2,3), (2,4,6), (3,6,9)$

Justify your answer.

<u>Solution:</u>

Observe: $(2,4,6) = 2(1,2,3)$ | $(3,6,9) = 3(1,2,3)$.

All vectors are scalar multiples of $(1,2,3)$.

Thus dimension $= 1$.

8) let $V = \text{span}\{(1,0,1); (0,1,1)\} \in R^3$

Find a general vector v and determine the dimension of v.

<u>Solution:</u> General vector

$$c_1(1,0,1) + c_2(0,1,1) = (c_1; c_2, c_1 + c_2)$$

Thus

$$V = \{(x, y, x+y)\}$$

Since the two vectors are linearly independent

$\dim(V) = 2$.

9. If a set of vectors spans R^3 and contains exactly three linearly independent vectors, prove that it form a basis of R^3.

Solutions:

They satisfy Both Basis conditions:

a) Span R^3

b) Linearly independent.

Hence they form a basis.

~~10. The rank of matrix equals~~

10. Explain the relationship between the rank of a matrix And the dimension of the span of its row vector.

Solutions: The rank of matrix equals the number of linearly independent row vectors. Therefore

rank(A) = dim (span of row vectors).