Linear Transformation & matrix operations.

Day 2: mathmatics Assignment

problem 1 : Matrix composition and non commutativity.

let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

Task A: manully compute the matrix product AB.

Task B: " " " " " BA.

Task C: " " " " $AB = BA$? geomtrilly explain why stretching then rotating ~~problem~~ produces a diffrent result than rotating then stretching.

Solution:

Task A: $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

$$AB = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$$

Task B:

$$BA = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$$

Task C: $AB \neq BA$.

Geometric Explanation: order matter.

- Stretch in x by 2 then rotating 90°

  Stretch x: $(x,y) \longrightarrow (2x,y)$

  Rotate 90° $\longrightarrow (-y, 2x)$ $\quad \left[(u,v) \rightarrow (-v,u)\right]$

- rotating 90° then stretch in x by 2.

  rotate 90 : $(x,y) \longrightarrow (-y,x)$

  Stretch x : $(-2y, x)$

Both are not same.

Problem 2 Singular matrices and collapsing space.

Consider a transformers matrix C:

$$C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Task A: Calculate the determinant of matrix C,

Task B: Based on determinant, is matrix C invertible?

Task C: Geometrically, what happens to the 2D Cartesian plane when you apply transformation C? How does this relate to the concept of linearly dependent features from Day 1?

<u>Solution:</u>

Task A: $\det(C) = 1(4) - (2(2)) = 0.$

Task B: Matrix C is not invertible (it is singular).

Because the det is zero.

Task C:

Geometrically 2D plane collapses into a 1D line.

Look at the column of C:

$$C_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad C_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, = 2C_1$$

Both colum point same direction. That means transformation cant span 2D space.

C_1, C_2 are linearly Dependent.

problem 8: solving a system of Linear equations

you are tranning highly simplified linear classifier. you need to find the exact weights w_1 and w_2 tatt satisfy the following system of equations based on two data point.

$$3w_1 + 2w_2 = 7$$

$$w_1 - w_2 = -1$$

Task A: Formulate this system in $AW = b$

Task B: manually calci the inverse matrix A^{-1}.

Task C: Solve the weigh vector W by computing $A^{-1}b$

Solution: A: matrix formation:

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

Task B: $A^{-1} = -\frac{1}{5} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1/5 & 2/5 \\ 1/5 & -3/5 \end{bmatrix}$

Task C

$$W = A^{-1} b$$

$$= \begin{bmatrix} 1/5 & 2/5 \\ 1/5 & -3/5 \end{bmatrix} \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The weights are $w_1 = 1$, $w_2 = 2$.