Short Questions: Rapid conceptual checks: matrices & transformations.

Question 1: Dimensionality and matrix multiplication.

Let matrix X (your dataset) have dimension 1000X50 (1000 samples, 50 features).

Let Matrix W (your weight matrix) have dimension 50X10.

A) What are the dimensions of the resulting matrix product XW?

B) Can you compate the product WX? If not, mathmatically explain why?

Solutions:

A) Dimensim of product XW is 1000, 10

B) NO. Dimensim are (50X10) multi (1000, 50). not possible.

Question 2: Affine vs Linear Transformations.

The one equation of a dense lager in a NNs is:

$$Z = Wx + b$$

Where W is the weight matrix and b is bias vector (assume $b \neq 0$).

In the strict mathmatical sense. define in linear algebra, is this a linear ~~transformation~~?

Solution:

No. It is not strict linear ~~transformation~~. Because of the bias beut b,

$$T(0) = W.0 + b = b.$$

Since origin move, the strict properties of linear algbra are violated. This is properly called an affine transformation

Question 3: Geometric Intuition

You are given a 2X2 transformation matrix T:

$$T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

If you meltiply a vector $v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ by T, What is the geometric effect on the vector in the 2D cartesian plane?

Solution

multiply $T.V = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

The matrix flip the x condinates sign and keep the y coordinate the same.

So it is a reflection accross the y axis in the 2D cartesian plane.

- onginal vector: (3,4)
- Transformed ~~v~~: (-3,4)

Question 4: Commutativity

True or False: For any ~~two~~ nxn matrix A and B;

$AB = BA$.

If False provide simple sentence geometric explin?

Solution: order matter in geometric.

Streting then rotation or rationing the streting is not same. result ditrent.

Question 5: Systems of Equations and Inverses.

You are trying to solve the system $Ax=b$ using python `np.linalg.inv(A) @ b`. However, python throws a singular matrix error.

A) What must be the exact value of the determinant of A, $\det(A) = ?$

B) Geometrically, what has the transformation A done to the vector space that makes it impossible to find a single, unique solution of x?

Solution

A): $\det(A) = 0$.

B): The matrix has collapsed the multidimensional space into a lower dimensional. Bcz the space collaped into a line or point, you can't invert the operation to find a unique x.