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Research on the pricing strategy of e-commerce agricultural products: should presale be adopted?

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Abstract
E-commerce has emerged as a promising platform for smallholder farmers to reach wider markets. To explore the feasibility of the presale strategy in the context of agricultural products, this study examines the impact of pricing and advertising on consumer behavior and optimizes sales profits for e-commerce farmers. By analyzing the results, we conclude that the dynamic pricing strategy based on presale is more appropriate for differentiated agricultural products, while the fixed pricing strategy is more suitable for homogeneous agricultural products. The findings of this study provide practical guidance for smallholder farmers to enhance their sales and profits on e-commerce platforms.

Keywords Pricing strategy · Presale · E-commerce agricultural products · Differentiated agricultural products

1 Introduction

In certain developing countries such as China and India, smallholder farmers are the primary food producers. Smallholders are farmers who own or rent small parcels of land, ranging from back gardens to several hectares of property [1]. According to statistics, there are approximately 203 million smallholder farmers in China,

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comprising 98.1% of all types of agricultural households. Additionally, around 85.2% of all households possess arable land below 1.65 acres per household [2]. The amount of arable land per household in China is merely one-third of that in South Korea, one-fortieth of that in the European Union, and one-four-hundredth of that in the United States. To put this into perspective, a typical US farm spanning 400–500 acres is equivalent to the total farmland of a village consisting of 200 households in China [3]. It is clear that a growing number of smallholder farmers are confronted with development challenges, such as being situated in isolated regions with limited access to external consumer markets, experiencing overproduction leading to sluggish sales, and facing restricted dissemination of market information. Due to their limited resources and lack of access to markets, it can be challenging for smallholder farmers to sell their crops directly to larger buyers or consumers. Consequently, for a significant period, smallholder farmers have been heavily dependent on middlemen and agents, who often take a large commission or profit margin, leaving the smallholder farmers with a small share of the final sale price. However, the rapid expansion of e-commerce has given rise to the prevalence of rural e-commerce, consequently altering the circumstances. Previous research has indicated that e-commerce, extensively adopted in emerging nations like China and India, can establish a connection between small farmers and end-consumers, and minimize information asymmetry [4]. Given these advantages, it is crucial to comprehend the behavior of smallholder farmers when utilizing e-commerce platforms.

After resolving the market link issue, new challenges can arise for smallholder farmers. In e-commerce sales, determining the price of goods is of paramount importance. In the traditional offline agricultural market, fixed pricing is common. And, due to the perishable nature of agricultural products, the change in freshness often compels retailers to implement dynamic pricing tactics to dispose of unsold goods [5–7]. In this scenario, farmers sell their agricultural products on a `cash-and-carry' basis, and the competition is limited within specific boundaries. Within a particular region, the price of a given agricultural product tends to remain stable and consistent. When farmers sell their agricultural products on e-commerce platforms, they encounter two significant changes in the market. Firstly, the competition expands beyond local farmers to countless farmers and merchants from different regions on the platform. Secondly, the e-commerce platform offers transparent price information, making it easy for consumers to compare prices and make informed purchase decisions. This makes it more important for farmers to make strategic pricing decisions. Smallholder farmers often have limited knowledge and literacy, and when it comes to complex e-commerce pricing issues, they tend to follow the crowd and emulate the practices of others. A common phenomenon in the e-commerce industry currently is the popularity of presale. Presale is a strategy used by manufacturers to transfer inventory risks to consumers [8]. This strategy is feasible for industrial products because manufacturers can adjust their production capacity accordingly. However, for agricultural products, the fixed production cycle means that the supply cannot be expanded or reduced in the short term. In this case, does dynamic pricing strategy based on presale still make sense? The existing literature on presale pricing primarily concentrates on the production and manufacturing sectors, which involve production-related decisions such as inventory management [9]. As agricultural products are perishable, it is challenging to store them for an extended period, and even with cold storage and other equipment, preservation costs can be substantial. Consequently, the current presale pricing model may not be optimal for agricultural products. Furthermore, ordinary agricultural products typically have a relatively low unit price, whereas branded agricultural products command a high premium. As such, the sale of various types of agricultural products can significantly influence pricing decisions.

This paper categorizes agricultural products into two groups: homogeneous agricultural products and differentiated agricultural products. Homogeneous agricultural products are those that are commonly found in the market and have little variation in terms of product quality. Examples include basic vegetables and rice. The latter category of agricultural products comprises those that are differentiated based on quality or branding, such as organic vegetables and Wuchang rice (a renowned rice brand in China). In the market, differentiated agricultural products are characterized by factors such as high prices, superior quality, and limited supply [10]. Larue [11] observed that wheat can be differentiated based on its intended use or country of origin. This paper highlights the fundamental contrast between the two categories, which is their capacity difference. Ordinary agricultural products are typically less technical in nature, and smallholder farmers can produce them using traditional methods. However, differentiated agricultural products rely on factors such as technology, branding, and variety to distinguish themselves. As a result, homogeneous agricultural products are often produced on a large scale, sometimes resulting in overcapacity. Mundi et al. [12] stressed the importance of categorizing non-homogeneous fruits in the agrifood industry based on their size, weight, color, and quality to meet the requirements of commercial retail formats. And, Grillo et al. [13] categorized fruit products in the supply chain based on their variety, quality, size, packaging type, and shelf-life. The categories of differentiated agricultural products differ from each other based on factors such as branding, quality standards, and more. Compared to homogenized agricultural products, differentiated agricultural products have numerous categories, and their production capacity is relatively smaller. Smallholder farmers selling their agricultural products on e-commerce platforms face strong substitutability since most of their goods lack differentiation, leading to reduced competitiveness. Consequently, price competition has emerged as the primary mode of competition for these smallholder farmers in the e-commerce industry. Given this situation, this paper aims to explore pricing strategies for these two types of agricultural products, and whether the prevalent presale model is appropriate for both. This paper explores two pricing strategies: fixed pricing strategy and dynamic pricing strategy based on presale. Fixed pricing strategy entails selling agricultural products at a constant price throughout the selling season, while dynamic pricing strategy involves setting a presale price during the presale stage, and then adjusting the spot sale price based on demand when the products are mature. The paper also addresses the implementation of pricing and advertising strategies for these two types of agricultural e-commerce products.

The primary findings of this study can be summarized as follows: (1) A combined marketing strategy of advertising and pricing for agricultural products on an e-commerce platform is effective in improving the profit level of both types of agricultural products. (2) For homogeneous agricultural products, the fixed pricing strategy is the preferable option. Studies have shown that, for homogeneous products, the quantity of products homogenizes the price [14]. It appears that the same conclusion applies to e-commerce agricultural products. (3) For differentiated agricultural products, dynamic pricing strategy based on presale can generate higher profits. (4) For both types of agricultural products, the presale price is higher than the spot sale price. To the best of our knowledge, the main contribution of this paper is its focus on the distinctive features of e-commerce transactions for agricultural products, and the resolution of pricing issues for smallholder farmers engaged in 'self-production and sales'. This differs from pricing strategies employed by large farms and agricultural enterprises, as well as from offline sales. The difference lies in the fact that large sellers wield significant power and influence in the market, while fragmented small farmers do not. Offline sales involve immediate payment and delivery, without the option of presale.

The rest of this article is as follows. Section 2 provides a literature review. Section 3 describes the model framework of the two types of agricultural products. Section 4 discusses the equilibrium results of models and comparative analysis. Section 5 focuses on numerical analysis, and Section 6 provides a summary of the research. Proofs are provided in the Appendix for illustrative purposes..

2 Literature review

Presale is primarily a pricing strategy for agricultural products, and research on pricing strategies can be divided into two categories: traditional markets (offline) and e-commerce platforms (online). In the traditional market, the long production cycle and short fresh-keeping period of agricultural products can result in different consumer perceptions during the consumption process. To adress this, Chang et al. [15] proposed a new categorized pricing strategy for agricultural products based on consumer preferences. They suggest that this strategy is particularly advantageous when customers have different preferences in the market. Recent research has explored the combination of dynamic pricing and information disclosure, concluding that a quality-based pricing strategy can lower prices, improve market demand and profit, and prevent the waste of fresh agricultural products [16]. Secondly, the traditional distribution of agricultural products involves several agents, including producers, distributors, and retailers. The pricing mechanism directly affects the profits of each agent in the supply chain, leading some studies to focus on coordinating the supply chain. For instance, Wang et al. [17] developed a dynamic pricing model that considers both decentralized and centralized decision-making, with farmer cooperatives as the core enterprise. Their findings showed that centralized decision-making leads to higher profits and lower prices. Chen et al. [18] analyzed a two-echelon supply chain of perishable food and determined the equilibrium state under both a single pricing strategy and a two-stage pricing strategy. Furthermore, sellers may choose to divide the sales period into multiple shorter periods, as suggested by Chun [19] in a multi-period pricing model for perishable or seasonal products with a limited sales period. The list price is set in advance and is non-negotiable with potential buyers. Any unsold products at the end of the sales period will be disposed of at a lower price. In addition, Jain and Hazr [20] investigated the pricing regulation and contract structure in an agricultural supply chain, with a focus on upstream decision-making. Their analysis revealed that farmers' payoffs can be higher if they strategically set the investment price. However, on the topic of pricing agricultural products through e-commerce channels, there is a lack of literature When supply and demand are uncertain and price-sensitive, Liu et al. [21] considered capacitated joint lot-sizing and pricing problems to model cases involving joint or sequential decisions. Zhou et al. [22], for instance, divided apples into high-quality and low-quality ones and discussed how to implement a price discount strategy for low-quality fruit based on consumer choice. Additionally, Rani et al. [23] developed a dynamic pricing model for organic vegetables based on four factors: supply, demand, quality, and freshness.

This paper also belongs to the field of presale research, which originated in aviation, hotel, and other industries. Fisher and Raman [24] were the first to propose the concept of presale in the retail industry. Nowadays, presale has become an effective and popular marketing strategy. Feng et al. [25] discovered that introducing an online presale channel can improve retailer profitability, and if online consumers have a high preference for presale, the retailer should reduce sales effort to save costs after introducing the presale channel. Compared to other sales models, the 'Presale and Spot-Sale' model is more beneficial to supply chain profits [26]. This model improves not only the profits of manufacturers [27], but also the profits of both retailers and manufacturers under certain conditions, such as extremely high marginal production costs [28]. Various presale strategies have been proposed based on consumers' time preferences. Wang et al. [29] presented four presale strategies, including single-stage pure presale, single-stage hybrid presale, two-stage pure presale, and two-stage hybrid presale, and found that the two-stage presale strategy is more advantageous for sellers. Zhang et al. [30] considered the effect of consumer's anticipated regret, such as action regret and inaction regret, on mixed bundle strategy in advance selling given uncertain consumer valuation. Presale is also considered a cost-effective source of financing [31] and can assist sellers in updating demand forecasts to decrease inventory risk [32], increase profits, improve consumer surplus, reduce competition [33, 34], as well as aid retailers in reducing demand uncertainty [35]. Presale can be a useful tool for retailers and manufacturers to adjust prices based on market demand fluctuations in situations where supply and demand are uncertain [36]. Due to the benefits of presale, many researchers have examined pricing strategies in this context. For instance, Xiao et al. [37]. studied the equilibrium pricing strategy for two typical presale pricing schemes: the dynamic pricing scheme and the price commitment scheme. Additionally, during presale, consumers may exhibit loss aversion, which can be considered in pricing decisions. Zhang et al. [38] developed a two-period Stackelberg game model to analyze pricing strategies in a supply chain system that included a manufacturer and an e-retailer. They also investigated pricing strategies for presale under different contracts, such as resale and agency contracts [39]. While there have been limited studies on presale in the online agricultural product market, He et al. [40] investigated the presale model of fresh agricultural produce from a competitive perspective and discovered that physical stores may not succeed if online grocery delivery costs are not high and circulation loss rates for fresh produce are high. However, for smallholder farmers engaged in 'self-production and sales' of agricultural products, can presale offer certain advantages?

Presale may not always be the optimal choice, as its effectiveness depends on market parameters and consumer behavior. Capacity is a crucial factor that affects the presale strategy. For instance, Huang et al. [41] examined a two-period pricing model where a seller offers freebies with the product during presale, and production is limited by capacity. They found that the sellers' optimal pricing strategy depends on their capacity size. Similarly, Yu et al. [42] identified that rationing capacity during the presale period is an effective tool for signaling product quality, and high-quality sellers can differentiate themselves by allocating less capacity than low-quality sellers. Wang and Zeng [43] investigated presale in a capacity-constrained model with consumer heterogeneity and demand uncertainty and revealed that the seller's optimal pricing strategy depends on capacity size. Hence, capacity plays a crucial role in the presale strategy [44]. Studies have examined limited and unlimited capacity when deciding whether to ration capacity in the presale period [45]. Based on these findings, this paper categorizes agricultural products as differentiated or homogeneous based on production capacity and explores whether these products should adopt presale strategies in the e-commerce market.

3 Model framework

In this paper, we examine a distribution channel consisting of sellers, platforms, and consumers. The online platform has the option to operate as a reseller or an online marketplace, but we consider it as an online marketplace in this study. This approach aligns with the current 'self-production and sales' model adopted by smallholder farmers. The fixed pricing strategy employs a constant price, denoted by P_i^f, throughout the entire sales season. Here, $i = h$ and d refer to homogeneous and differentiated agricultural products, respectively. The dynamic pricing strategy based on presale utilizes a price denoted by P_i^a, where a_1 and a_2 represent the presale and spot sale stages, respectively. Given that large e-commerce platforms have a monopoly on both users and data [46], advertising has become increasingly crucial, as it directly impacts product ranking when consumers search for related products [47]. Therefore, apart from considering pricing, we also factor in advertising investment. In practice, the advantages of advertising investment may not always exhibit a linear growth trend. Hence, this paper employs a quadratic function to portray the diminishing returns effect over time. When the advertising investment level is A, advertisement cost function can be expressed as $I = \frac{\mu A^2}{2}$, which $\mu > 0, A > 0, \frac{\partial I}{\partial A} > 0, \frac{\partial^2 I}{\partial A^2} > 0$. Key notations in this paper are summarized in Table 1.

The market demand is influenced by both the price and the level of advertising investment. In particular, when the price increases, the market demand decreases, whereas when the level of advertising investment increases, the market demand increases. Similar demand models can be found in the marketing literature, such as those presented by Tsay and Agrawal [48] and Bernstein and Federgruen [49]. The basic functional form is given by

$$D_i^j = C_i - \omega P_i^j + \beta A_i^j \tag{1}$$

where D_i^j, P_i^j and A_i^j refer to the realized marketing demand, the price and the advertising investment level for the e-commerce agricultural products under price strategy j, respectively. Potential market demand, denoted by C_i, is uncertain [50]. Considering the abundant market supply of agricultural products, this paper assumes that the supply surpasses the potential market demand,i.e., $S_i > C_i$. The sensitivity of market demand to price is measured by ω, while β measures the market demand's response to advertising investment, which also indicates sellers' advertising preference. Equation (1) pertaining to market demand is applicable for the fixed pricing strategy and the presale stage under the dynamic pricing strategy.

The demand function for homogeneous agricultural products and differentiated agricultural products during the spot sale stage can be expressed as follows, respectively:

$$D_h^{a_2} = (C_h - D_h^{a_1}) - \omega P_h^{a_2} + \beta A_h^a \tag{2}$$

$$D_d^{a_2} = (S_d - D_d^{a_1}) - \omega P_d^{a_2} + \beta A_d^a \tag{3}$$

Table 1 Notations for variables and parameters

$i = h, d$	types of agricultural products, $i \in \{h, d\}$, where h represents homogeneous agricultural products and d represents differentiated agricultural products
$j = f, a$	price strategy, $j \in \{f, a\}$, $a \in \{a_1, a_2\}$, where a_1 represents the presale stage and a_2 represents the spot sale stage
D_i^j	Marketing demand for i type agricultural products under price strategy j
π_i^j	Profit for i type agricultural products under price strategy j
P_i^j	Price for i type agricultural products under price strategy j
A_i^j	The level of advertising investment of i type agricultural product sunder price strategy j
I_i^j	The advertising investment cost of i type agricultural products under price strategy j
C_i	Potential market demand for i type agricultural products
S_i	Market supply of i type agricultural products
ω	The price sensitivity coefficient, $\omega > 0$
β	The reaction of market demand to advertisement investment, namely advertisement reaction coefficient, $\beta > 0$
μ	The advertisement investment cost factor of agricultural product,namely advertisement cost coefficient, $\mu > 0$

where $D_i^{a_2}$ represents the market demand during the spot sale stage when sellers adopt a presale strategy, $C_h - D_h^{a_1}$ represents the unfulfilled market demand for homogeneous agricultural products, and $S_d - D_d^{a_1}$ indicates the remaining product supply after presale. This aligns with our assumptions and is reflective of reality. Zhang et al. [51] developed a model wherein a brand sells a new product over two periods and discovered that the demand during the second period is met by the available inventory. In the case of agricultural products, the production cycle makes it challenging for sellers to adjust their production capacity to meet market demand within a short period. Therefore, the supply remains fixed for a certain period.

Within this framework, the paper examines four scenarios that involve the fixed pricing strategy and the dynamic pricing strategy based on presale for homogeneous and differentiated agricultural products. The study also includes direct comparisons of the same type of agricultural products under different pricing strategies. Section 4 presents an analysis and discussion of the results, and Sect. 5 verifies these findings using numerical simulation.

4 Results

This section aims to obtain the optimal results for the four scenarios and conduct comparative analyses. To ensure that all feasible scenarios have positive optimal results and control the range of $\frac{C_h}{S_h} = 2 - \frac{\beta^2}{\mu\omega}$, i.e., $2 - \frac{\beta^2}{\mu\omega} \in (0,\ 1)$, we assume that $\mu\omega < \beta^2 < \frac{3\mu\omega}{2}$. The complete solution is presented in the appendix.

4.1 Fixed pricing strategy for homogeneous agricultural products

We will first consider the scenario of a fixed pricing strategy for homogeneous agricultural products on an e-commerce platform. The fixed pricing strategy is quite common among sellers and ensures uniform pricing during the selling process. In this case, the market demand function and profit function of agricultural products can be expressed as follows:

$$\begin{aligned} D_h^f &= C_h - \omega P_h^f + \beta A_h^f \\ \max_{P_h^f, A_h^f} \pi_h^f &= D_h^f P_h^f - \frac{\mu (A_h^f)^2}{2} \\ \text{s.t. } S_h &> D_h^f \end{aligned} \tag{4}$$

To solve this scenario, we will use the range of decision variables and the axis of symmetry. When profit is maximized, there is one feasible situation:

Theorem 1 *The optimal decisions for sellers of homogeneous agricultural products when using a fixed pricing strategy are as follows when* $\frac{C_h}{S_h} \in \left(0, 2 - \frac{\beta^2}{\mu\omega}\right)$, *the level* *of advertising investment, price, profit, demand, consusmer surplus, and social welfare are as follows.*

$$A_h^f = \frac{C_h \beta}{2\mu\omega - \beta^2}, P_h^f = \frac{\mu C_h}{2\mu\omega - \beta^2},$$
$$\pi_h^f = \frac{\mu (C_h)^2}{2(2\mu\omega - \beta^2)}, D_h^f = \frac{\mu\omega C_h}{2\mu\omega - \beta^2},$$
$$CS_h^f = \frac{\mu^2 (C_h)^2 \omega}{2(2\mu\omega - \beta^2)^2}, SW_h^f = \frac{\mu (C_h)^2 (3\mu\omega - \beta^2)}{2(2\mu\omega - \beta^2)^2}.$$

4.2 Dynamic pricing strategy based on presale for homogeneous agricultural products

Price transparency is an important characteristic of e-commerce platforms. By using dynamic pricing strategies to create a series of historical prices, it can influence consumer purchasing decisions [52]. This is commonly referred to as the 'anchoring effect'. Some sellers of agricultural products adopt a presale strategy and make price decisions at both stages. The market demand function for homogeneous agricultural products in the presale stage can be expressed as follows:

$$D_h^{a_1} = C_h - \omega P_h^{a_1} + \beta A_h^a \tag{5}$$

The market demand function for homogeneous agricultural products in the spot sale stage can be expressed as:

$$D_h^{a_2} = (C_h - D_h^{a_1}) - \omega P_h^{a_2} + \beta A_h^a \tag{2}$$

Assuming sufficient supply in the homogeneous agricultural product market, the profit function resulting from the dynamic pricing strategy based on presale can be expressed as:

$$\begin{gathered} \max_{P_h^{a_1}, P_h^{a_2}, A_h^a} \pi_h^a = D_h^{a_1} P_h^{a_1} + D_h^{a_2} P_h^{a_2} - \frac{\mu (A_h^a)^2}{2} \\ \text{s.t.} \begin{cases} C_h > D_h^{a_1} \\ S_h > D_h^{a_1} + D_h^{a_2} \end{cases} \end{gathered} \tag{6}$$

Theorem 2 *The optimal decisions for sellers of homogeneous agricultural products when using the dynamic pricing strategy based on presale are as follows:*

when $\frac{C_h}{S_h} \in (0, \frac{1}{2})$, *the level of advertising investment, price, profit, demand, consusmer surplus, and social welfare are as follows:*

$$A_h^a = \frac{S_h\beta}{\mu\omega}, P_h^{a_1} = \frac{S_h\beta^2}{\mu\omega^2}, P_h^{a_2} = \frac{S_h\beta^2}{\mu\omega^2} + \frac{C_h - S_h}{\omega},$$
$$\pi_h^a = \frac{(S_h)^2\beta^2}{2\mu\omega^2} - \frac{(S_h - C_h)^2}{\omega}, D_h^a = S_h,$$
$$CS_h^a = \frac{(S_h)^2 - 2S_hC_h + 2(C_h)^2}{2\omega},$$
$$SW_h^a = \frac{(S_h)^2\beta^2}{2\mu\omega^2} + \frac{2C_hS_h - (S_h)^2}{2\omega},.$$

4.3 Fixed pricing strategy for differentiated agricultural products

In this scenario, we are considering differentiated agricultural products, which are products that have differences in quality or brand. They typically have smaller production capacity and market supply compared to homogeneous agricultural products. The market demand function and profit function for differentiated agricultural products can be expressed as follows:

$$\begin{aligned} D_d^f &= C_d - \omega P_d^f + \beta A_d^f \\ \max_{P_d^f, A_d^f} \pi_d^f &= S_d P_d^f - \frac{\mu (A_d^f)^2}{2} \\ \text{s.t. } S_d &< D_d^f \end{aligned} \tag{7}$$

Theorem 3 *The optimal decisions for sellers of differentiated agricultural products using the fixed pricing strategy on an e-commerce platform are as follows*:

$$A_d^f = \frac{S_d\beta}{\mu\omega}, P_d^f = \frac{S_d\beta^2}{\mu\omega^2} + \frac{C_d - S_d}{\omega}, D_d^f = S_d,$$
$$\pi_d^f = \frac{(S_d)^2\beta^2}{2\mu\omega^2} + \frac{S_dC_d - (S_d)^2}{\omega},$$
$$CS_d^f = \frac{(S_d)^2}{2\omega}, SW_d^f = \frac{(S_d)^2\beta^2}{2\mu\omega^2} + \frac{2S_dC_d - S_d^2}{2\omega}.$$

4.4 Dynamic pricing strategy based on presale for differentiated agricultural products

The market demand functions for presale and spot sale stages of differentiated agricultural products are similar to those of homogeneous agricultural products. The market demand and profit functions are as follows:

$$
\begin{aligned}
D_d^{a_1} &= C_d - \omega P_d^{a_1} + \beta A_d^a \\
D_d^{a_2} &= (S_d - D_d^{a_1}) - \omega P_d^{a_2} + \beta A_d^a \\
\max_{P_d^{a_1}, P_d^{a_2}, A_d^a} \pi_d^a &= D_d^{a_1} P_d^{a_1} + (S_d - D_d^{a_1}) P_d^{a_2} - \frac{\mu (A_d^a)^2}{2} \\
\text{s.t. } D_d^{a_1} &< S_d < D_d^{a_1} + D_d^{a_2}
\end{aligned}
\tag{8}
$$

Theorem 4 *The optimal decisions for sellers of differentiated agricultural products using the dynamic pricing strategy based on presale on an e-commerce platform are as follows*:

$$
\begin{aligned}
& A_d^a = \frac{S_d \beta}{\mu \omega}, P_d^{a_1} = \frac{S_d \beta^2}{\mu \omega^2} + \frac{C_d}{\omega}, P_d^{a_2} = \frac{S_d \beta^2}{\mu \omega^2}, \\
& \pi_d^a = \frac{(S_d)^2 \beta^2}{2\mu\omega^2} + \frac{(C_d)^2}{4\omega}, D_d^a = S_d, \\
& CS_d^a = \frac{2(S_d)^2 - 2S_d C_d + (C_d)^2}{4\omega}, \\
& SW_d^a = \frac{(S_d)^2 \beta^2}{2\mu\omega^2} + \frac{(S_d)^2 - S_d C_d + (C_d)^2}{2\omega}.
\end{aligned}
$$

4.5 Comparison and analysis

In this section, we analyze the relationship between variables such as profit, advertising investment level, price, consumer surplus, and social welfare, and the parameters ω or β. We compare these variables to examine the efficiencies within four scenarios.

Proposition 5 *In the model of fixed pricing strategy for homogeneous agricultural products*, *we have*:

when $\frac{C_h}{S_h} \in (0, 2 - \frac{\beta^2}{\mu\omega})$, $\frac{\partial P_h^f}{\partial \omega} < 0, \frac{\partial A_h^f}{\partial \omega} < 0, \frac{\partial \pi_h^f}{\partial \omega} < 0, \frac{\partial CS_h^f}{\partial \omega} < 0, \frac{\partial SW_h^f}{\partial \omega} < 0, \frac{\partial P_h^f}{\partial \beta} > 0,$ $\frac{\partial A_h^f}{\partial \beta} > 0, \frac{\partial \pi_h^f}{\partial \beta} > 0, \frac{\partial CS_h^f}{\partial \beta} > 0, \frac{\partial SW_h^f}{\partial \beta} > 0.$

Proposition 5 is easy to prove. According to the proposition, the optimal price, advertising investment level, profit, consumer surplus, and social welfare all decrease with an increase in price sensitivity coefficient and increase with an increase in the advertising reaction coefficient.

Proposition 6 *In the model of dynamic pricing strategy based on presale for homogeneous agricultural products*, *we have*:

when $\frac{C_h}{S_h} \in (0, \frac{1}{2})$, $\frac{\partial A_h^f}{\partial \omega} < 0$, $\frac{\partial P_h^{a1}}{\partial \omega} < 0$, $\frac{\partial P_h^{a2}}{\partial \omega} < 0$, $\frac{\partial \pi_h^a}{\partial \omega} < 0$, $\frac{\partial CS_h^a}{\partial \omega} < 0$, $\frac{\partial SW_h^a}{\partial \omega} < 0$, $\frac{\partial A_h^a}{\partial \beta} > 0$, $\frac{\partial P_h^{a1}}{\partial \beta} > 0$, $\frac{\partial P_h^{a2}}{\partial \beta} > 0$, $\frac{\partial \pi_h^a}{\partial \beta} > 0$, $\frac{\partial SW_h^a}{\partial \beta} > 0$, *and* $P_h^{a_1} > P_h^{a_2}$.

Proposition 6 is straightforward to prove. The price sensitivity coefficient and the advertising reaction coefficient jointly determine all optimal decisions, except for consumer surplus, which is only affected by the price sensitivity coefficient, leading to lower consumer surplus for more price-sensitive consumers. Notably, the presale price of homogeneous agricultural products is typically higher than the spot sale price, unlike industrial products. Agricultural products have a short shelf life, and consumers tend to prefer the first crop of some agricultural products, such as fruits. Hence, the presale price may be slightly higher. Additionally, to ensure that homogenized agricultural products sell out during the spot sale season, sellers often reduce prices to stimulate consumer demand.

Proposition 7 *After conducting a comparative analysis of the results for homogeneous agricultural products under the fixed pricing strategy and the dynamic pricing strategy based on presale*, *it is observed that*:

when $\frac{C_h}{S_h} \in (0, \frac{1}{2})$, *it implies that* S_h *is significantly larger than* C_h. *This is in line with the observation that smallholder farmers who produce undifferentiated products often face oversupply in the market. In such a scenario*, *adopting a fixed pricing strategy would result in higher profits if the value of* $\frac{S_h}{C_h}$ *falls within the range* $(0, t^*)$, *where* $t^* = \frac{(2\mu\omega-\beta^2)(2\mu\omega-\sqrt{\mu\omega(2\beta^2-\mu\omega)})}{\mu\omega(5\mu\omega-2\beta^2)}$, *i.e.*, $\pi_h^f > \pi_h^a$. *However*, *at that point*, *social welfare is always lower*, *i.e.*, $SW_h^f < SW_h^a$. *In terms of consumer surplus*, *if* $\beta^2 \in (\mu\omega, (2 - \frac{\sqrt{2}}{2})\mu\omega)$, *the dynamic pricing strategy based on presale results in higher consumer surplus*, *i.e.*, $CS_h^f < CS_h^a$.

Proposition 7 declares that a fixed pricing strategy is the optimal approach for homogeneous agricultural products. The evaluation of pricing strategies' effectiveness relies on the crucial indicators of consumer surplus and social welfare [53]. Consumer surplus measures the variance between the maximum price that consumers are willing to pay for a particular commodity and the current market price. It also represents the portion of profits allocated to consumers by sellers. In contrast, social welfare assesses the overall welfare of society, including sellers and consumers. When the value of electronic network services is uncertain, Brynjolfsson et al. [54] applied consumer surplus to gauge the value of increased product diversity at online bookstores in the digital economy. Similarly, Lence et al. [55]emphasized the critical role of social welfare in evaluating the development of differentiated products.

Proposition 6 establishes a significant relationship between price and consumer surplus As sellers' profits increase, consumers experience a significant reduction in their consumer surplus. Therefore, devising an efficient pricing strategy necessitates balancing the sellers' profit with consumer surplus. The essentiality of consumer surplus and social welfare drives this paper's emphasis on their discussion.

Proposition 8 *In the model of fixed pricing strategy for differentiated agricultural products, we have*:

$$\frac{\partial P_d^f}{\partial \omega} < 0, \frac{\partial A_d^f}{\partial \omega} < 0, \frac{\partial \pi_d^f}{\partial \omega} < 0, \frac{\partial CS_d^f}{\partial \omega} < 0, \frac{\partial SW_d^f}{\partial \omega} < 0, \frac{\partial P_d^f}{\partial \beta} > 0, \frac{\partial A_d^f}{\partial \beta} > 0, \frac{\partial \pi_d^f}{\partial \beta} > 0, \frac{\partial SW_d^f}{\partial \beta} > 0.$$

Under the fixed pricing strategy, Proposition 8 demonstrates that the impact of the price sensitivity coefficient and advertising reaction coefficient on differentiated agricultural products is similar to that of homogeneous agricultural products. In this scenario, consumer surplus is solely influenced by the price sensitivity coefficient, and a higher price sensitivity coefficient will lead to a decrease in consumer surplus. It is widely recognized that differentiated agricultural products have a clear advantage in terms of price when compared to homogeneous agricultural products. However, most consumers with high price sensitivity are more likely to refrain from making a purchase due to a low consumer surplus.

Proposition 9 *In the model of differentiated agricultural products under a dynamic pricing strategy based on presale, the following can be observed*:

$$\frac{\partial P_d^{a_1}}{\partial \omega} < 0, \frac{\partial P_d^{a_2}}{\partial \omega} < 0, \frac{\partial A_d^a}{\partial \omega} < 0, \frac{\partial \pi_d^a}{\partial \omega} < 0, \frac{\partial CS_d^a}{\partial \omega} < 0, \frac{\partial SW_d^a}{\partial \omega} < 0, \frac{\partial P_d^{a_1}}{\partial \beta} > 0, \frac{\partial P_d^{a_2}}{\partial \beta} > 0, \frac{\partial A_d^a}{\partial \beta} > 0, \frac{\partial \pi_d^a}{\partial \beta} > 0, \frac{\partial SW_d^a}{\partial \beta} > 0 \quad ,$$

and $P_d^{a_1} > P_d^{a_2}$.

Proposition 9 shows that the presale price, spot sale price, profit, advertising investment level, and social welfare of differentiated agricultural products, are negatively correlated with the price sensitivity coefficient and positively correlated with the advertising reaction coefficient. Furthermore, consumer surplus is only positively correlated with the price sensitivity coefficient. In addition, it has been observed that $P_d^{a_1} > P_d^{a_2}$. This suggests that presale price is typically higher than spot sale prices for agricultural products. According to Zhang et al. [56], there is a similar conclusion in presale studies of agricultural products that the presale price is usually higher than the spot sale price, particularly for products that have a significant driving effect.

Proposition 10 *Compares the results of the fixed pricing strategy and the dynamic pricing strategy based on presale for differentiated agricultural products and reaches the following conclusions*:

1. *The advertising investment level is the same for the dynamic pricing strategy based on presale and the fixed pricing strategy, that is*, $A_d^a = A_d^f$.

2. *The presale price, spot sale price, profit, and social welfare of the dynamic pricing strategy based on presale are higher than those of the fixed pricing strategy, specifically* $P_d^{a_1} > P_d^f, P_d^{a_2} > P_d^f, \pi_d^a > \pi_d^f, SW_d^a > SW_d^f$.
3. *The dynamic pricing strategy based on presale results in a lower consumer surplus, that is,* $CS_d^a < CS_d^f$.

Proposition 10 indicates that for differentiated agricultural products, with the same advertising investment, the presale price and spot sale price are higher under the dynamic pricing strategy than under the fixed pricing strategy. This finding is consistent with real-world examples, such as Sunshine Rose Grape, which was introduced to the Chinese market in 2015 and was initially very expensive due to limited production and consumer demand. Consistent with Proposition 7, the dynamic pricing strategy for differentiated agricultural products results in a smaller consumer surplus. Sellers can benefit from this strategy, but social welfare remains high.

5 Numerical simulation

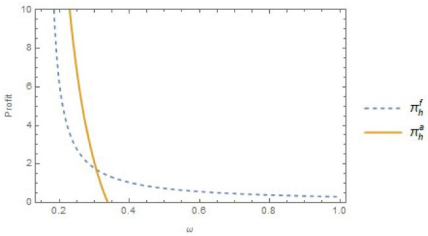
5.1 The impact of ω and β on profit, consumer surplus and social welfare for homogeneous agricultural products

The main factors that determine whether a fixed pricing strategy or a dynamic pricing strategy based on presale is adopted for homogeneous agricultural products are the price sensitivity coefficient (ω) and the advertising reaction coefficient (β). To further investigate the propositions through numerical simulation, this section focuses on a particular agricultural product with a fixed potential market demand, denoted as $C_h = 1$. To ensure that the condition of $\frac{C_h}{S_h} \in (0, \frac{1}{2})$ is met, we set $S_h = 3.2$. Moreover, we set the advertising cost coefficient at 0.3, which is a approprite level, i.e., $\mu = 0.3$. It is worth noting that the specific value of each parameter, as long as it falls within the range obtained in the above solution, will not affect the conclusion and trend observed.

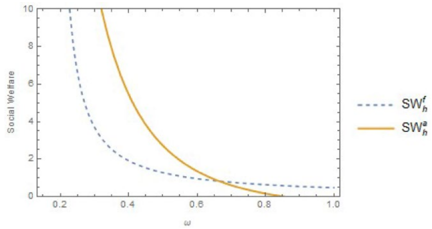
Figures 1 and 2 show that the impact of ω and β on profit,consumer surplus and social welfare for homogeneous agricultural products respectively. Except consumer surplus is only affected by the price sensitivity coefficient under the dynamic pricing strategy based on presale, other outcomes are influenced by both the advertising reaction coefficient and the price sensitivity coefficient. The results reveal that a low value of ω and a high value of β lead to higher profits.

5.2 The impact of S_h on profit,social welfare and consumer surplus for homogeneous agricultural products

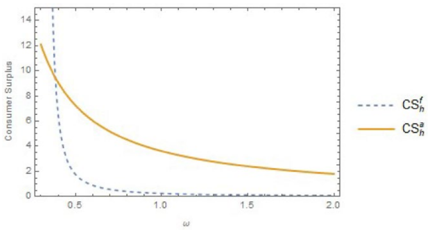
In this section, we aim to explore the impact of S_h on the profit, social welfare, and consumer surplus of homogeneous agricultural products under uniform market conditions, as discussed in Sect. 5.1. To ensure that the results are not affected by



(a):Impact of ω on profit

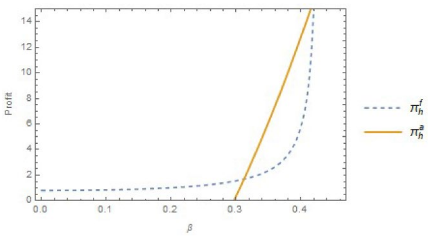


(b):Impact of ω on social welfare

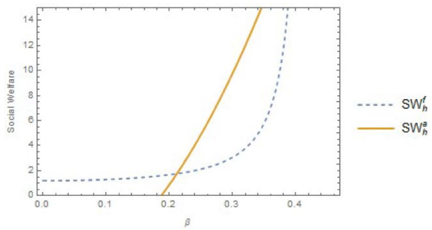


(c):Impact of ω on consumer surplus

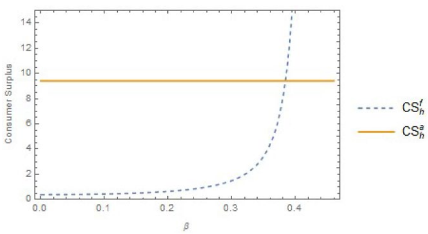
Fig. 1 When $\frac{C_h}{S_h} \in \left(0, \frac{1}{2}\right)$, the impact of ω on profit,consumer surplus and social welfare for homogeneous agricultural products



(a):Impact of β on profit



(b):Impact of β on social welfare



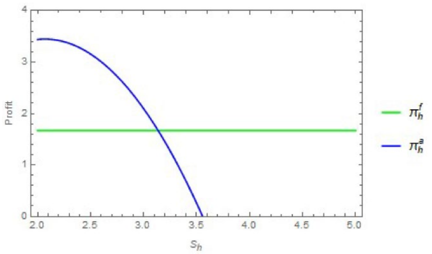
(c):Impact of β on consumer surplus

Fig. 2 When $\frac{C_h}{S_h} \in \left(0, \frac{1}{2}\right)$, the impact of β on profit,consumer surplus and social welfare for homogeneous agricultural products extreme values, we value the parameter as follows, $\mu = 0.3, C_h = 1, \beta = 0.31$ and $\omega = 0.31$. These parameter values will remain constant throughout all the numerical simulations in this study. By maintaining these constraints, we can ensure that the graphs can be compared and simultaneously displayed in Section 5.

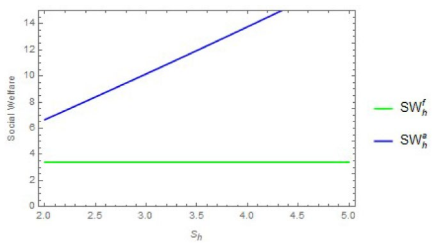
According to Fig. 3, it can be observed that, in general, the fixed pricing strategy generates significantly higher profits for homogeneous agricultural products compared to the dynamic pricing strategy based on presale when the market supply S_h is sufficiently large. Due to the low requirements on production skills and knowledge, smallholder farmers can easily produce homogeneous agricultural products, resulting in a large market supply. Fixed pricing strategy helps to stabilize prices, mitigating the impact of frequent price fluctuations, and consolidating the purchasing power of consumers who are highly sensitive to prices. Moreover, the profit, consumer surplus, and social welfare remain stable under these circumstances. However, when maximizing the sellers' profit, the consumer surplus is relatively low, leaving less profit margin for consumers.

5.3 The impact of ω and β on profit,cosumer surplus and social welfare for differentiated agricultural products

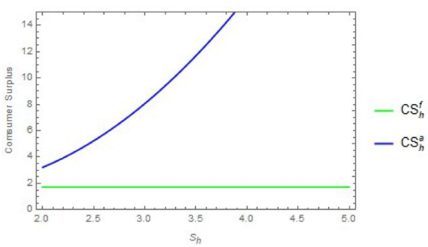
For differentiated agricultural products, we have used a consistent set of parameters for all the simulations, including $\mu = 0.3, C_d = 1, S_d = 1.2$. As in Sect. 5.1,



(a):Impact of S_h on profit



(b):Impact of S_h on social welfare

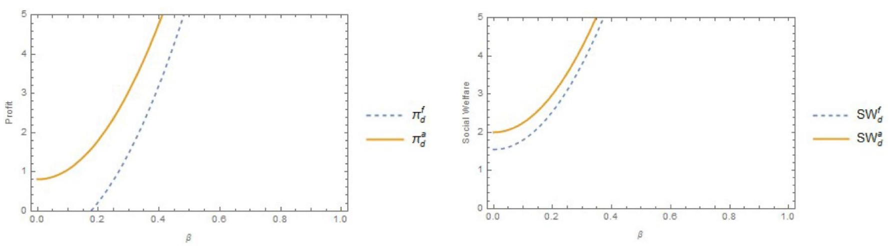


(c):Impact of S_h on consumer surplus

Fig. 3 Impact of S_h on profit,social welfare and consumer surplus for homogeneous agricultural products the potential market demand for an agricultural product remains fixed. However, to account for the limited market supply of differentiated agricultural products, we have set $S_d = 1.2$,reflecting the characteristics of such products.

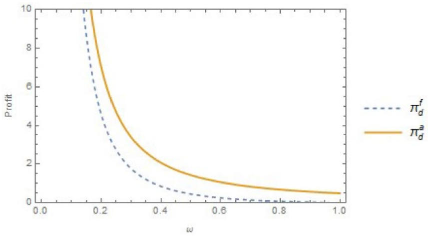
Figure 4 illustrates the impact of β on profit and social welfare for differentiated agricultural products, with a fixed value of $\omega = 0.31$. The results indicate that both profit and social welfare increase with an increase in β.Unlike Section 5.1, for differentiated agricultural products, advertising strategies can be effectively implemented to generate an appropriate profit and social welfare, even if consumers in the market have a low advertising reaction coefficient. When the consumer advertising reaction coefficient is high, it can lead to even higher profits and social welfare. In addition, the consumer surplus is only affected by the price sensitivity coefficient ω. Therefore, we set $\beta = 0.31$ to specifically observe the impact of ω on profit, consumer surplus, and social welfare. As shown in Fig. 5, all three factors decrease with an increase in the price sensitivity coefficient ω.

Based on the results shown in Figs. 4 and 5, it can be concluded that the dynamic pricing strategy based on presale generates significantly higher profit and social welfare for differentiated agricultural products compared to the fixed pricing strategy. Although consumer surplus is lower under the dynamic pricing strategy, the overall benefits to the sellers and society are higher. For smallholder farmers who produce differentiated agricultural products, the dynamic pricing strategy based on presale is more appropriate due to the higher investment in production, selection, packaging, branding, and other aspects. The dynamic pricing strategy based on presale can help reduce losses in the trading process by adjusting prices in a timely manner and making appropriate allocations, thereby avoiding losses for smallholder farmers. Additionally, it is worth mentioning that profits experience a rapid growth as consumers become more sensitive to advertisements, i.e., as β increases. To differentiate their products from homogeneous agricultural products, it is essential to promote and communicate the unique features of the product to consumers through advertising.

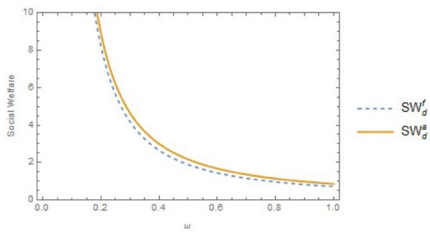


(a):Impact of β on profit (b):Impact of β on social welfare

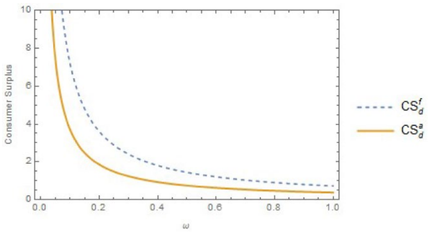
Fig. 4 Impact of β on profit and social welfare for differentiated agricultural products



(a):Impact of ω on profit



(b):Impact of ω on social welfare



(c):Impact of ω on consumer surplus

Fig. 5 Impact of ω on profit,consumer surplus and social welfare for differentiated agricultural products

6 Conclusion

With the increasing popularity of e-commerce and improved logistics services in rural areas, selling agricultural products online has become a common practice among smallholder farmers. This study examines two common pricing strategies for online sales of agricultural products: fixed pricing strategy and dynamic pricing strategy based on presale. As the majority of agricultural products sold online are homogeneous, this study focuses on the applicability of these two pricing strategies to both homogeneous and differentiated agricultural products. By analyzing the demand functions of both types of agricultural products and constructing an objective function based on profit maximization, this study presents several key findings.

Firstly, we determine the optimal pricing and advertising investment level for both types of agricultural products under the two pricing strategies. Accordingly, we get the optimal profit, consumer surplus, and social welfare in each scenario. Identifying the factors that influence these decision variables can offer valuable guidance for e-commerce agricultural sellers. Secondly, we find that for both homogeneous and differentiated agricultural products, the presale price is higher than the spot sale price, which may seem counterintuitive but is a crucial finding. Unlike industrial goods, agricultural products have a fixed supply over a period, and even if a product is found to be popular through presale, production cannot be immediately increased. This is why the cobweb model is unsolvable, despite advances in information technology. Additionally, the short shelf life of produce further reinforces the conclusion that the presale price should be higher than the spot selling price. Thirdly, we compare the optimal profits in the four scenarios and observe that the fixed pricing strategy is superior for homogeneous agricultural products, albeit with lower consumer surplus and social welfare. On the other hand, for differentiated agricultural products, the dynamic pricing strategy based on presale generates higher profits, and although consumer surplus is lower, social welfare is greater as consumer demand is better satisfied. Finally, advertising investment is particularly important for differentiated agricultural products as it helps consumers better identify the unique qualities of the products.

The findings of this paper have several implications for management. Firstly, e-commerce platforms can provide smallholder farmers with a means to sell their agricultural products, and it is important for sellers to understand their product positioning. The fixed pricing strategy is more suitable for those producing and selling homogeneous agricultural products. However, if they produce and sell unique or differentiated products, such as local specialties, handmade food or crafts, organic food, or high-quality produce, then dynamic pricing strategy based on presale can help them maximize profits. Secondly, since e-commerce platforms have borderless competition, appropriate advertising investment is necessary. Therefore, smallholder farmers, particularly those producing and selling distinctive and differentiated agricultural products, should learn to use advertising and promotion strategies. Lastly, only differentiated agricultural products can achieve higher profits, so smallholder farmers should continuously improve their knowledge, experiment with new varieties, and adopt new planting and breeding methods to differentiate themselves through competition. The government and agricultural technicians can provide support where necessary.

However, this research only considers the homogeneity and differentiation of agricultural products and has limitations. In the future, a more detailed classification of agricultural products could be considered, or further research could focus on a specific type of agricultural product, such as fresh produce.

Appendix A: Proofs

The Appendices part is started with the command. Appendix sections are then done as normal sections

Theorem 1 *When homogeneous agricultural products are sold using the fixed pricing strategy, the market demand function and profit function can be expressed as:*

$$\begin{aligned} D^f_h &= C_h - \omega P^f_h + \beta A^f_h \\ \max_{P^f_h, A^f_h} \pi^f_h &= D^f_h P^f_h - \frac{\mu (A^f_h)^2}{2} \\ \text{s.t. } S_h &> D^f_h \end{aligned} \tag{A.1}$$

According to $S_h > D_h^f$, *we get* $P_h^f > \frac{\beta}{\omega}A_h^f + \frac{(C_h - S_h)}{\omega}$. *Assuming that* $(P_h)^0 = \frac{\beta}{\omega}A_h^f + \frac{(C_h - S_h)}{\omega}$.

$$\max_{P_h^f, A_h^f} \pi_h^f = -\omega\left(P_h^f\right)^2 + \left(C_h + \beta A_h^f\right)P_h^f - \frac{\mu}{2}\left(A_h^f\right)^2 \tag{A.2}$$

The axis of symmetry of Eq. (A.2) *is* $(P_h)^* = \frac{(C_h + \beta A_h^f)}{2\omega}$. *We now have two scenarios to analyze.*

1. *If* $(P_h)^* \geq (P_h)^0$, $P_h^f = (P_h)^* = \frac{(C_h + \beta A_h^f)}{2\omega}$ *can be plug into Eq.* (A.2) *to get*

$$\begin{aligned} \max_{A_h^f} \pi_h^f &= \frac{\beta^2 - 2\mu\omega}{4\omega}\left(A_h^f\right)^2 + \frac{C_h\beta}{2\omega}A_h^f + \frac{(C_h)^2}{4\omega} \\ \text{s.t. } A_h^f &\leq \frac{2S_h - C_h}{\beta} \end{aligned} \tag{A.3}$$

When $\frac{C_h}{S_h} \in \left(0, 2 - \frac{\beta^2}{\mu\omega}\right)$,

$A_h^f = \frac{C_h\beta}{2\mu\omega - \beta^2}, P_h^f = \frac{\mu C_h}{2\mu\omega - \beta^2}, \pi_h^f = \frac{\mu(C_h)^2}{2\left(2\mu\omega - \beta^2\right)}, D_h^f = \frac{\mu C_h\omega}{2\mu\omega - \beta^2}$.

When $\frac{C_h}{S_h} \in \left(2 - \frac{\beta^2}{\mu\omega}, 1\right)$,

$A_h^f = \frac{2S_h - C_h}{\beta}, P_h^f = \frac{s}{\omega}, \pi_h^f = \frac{2(S_h)^2\beta^2 - \mu\omega(2S_h - C_h)^2}{2\omega\beta^2}, D_h^f = S_h$.

2. *If* $(P_h)^* < (P_h)^0$, $P_h^f = (P_h)^0 = \frac{\beta}{\omega}A_h^f + \frac{(C_h - S_h)}{\omega}$ *can be plug into Eq.* (A.2) *to get*

$$\begin{aligned} \max_{A_h^f} \pi_h^f &= -\frac{\mu}{2}\left(A_h^f\right)^2 + \frac{S_h\beta}{\omega}A_h^f + \frac{S_hC_h - S_h^2}{\omega} \\ \text{s.t. } A_h^f &> \frac{2S_h - C_h}{\beta} \end{aligned} \tag{A.4}$$

When $\frac{C_h}{S_h} \in \left(0, 2 - \frac{\beta^2}{\mu\omega}\right)$,

$A_h^f = \frac{2S_h - C_h}{\beta}, P_h^f = \frac{S_h}{\omega}, \pi_h^f = \frac{2(S_h)^2\beta^2 - \mu\omega(2S_h - C_h)^2}{2\omega\beta^2}, D_h^f = S_h$.

When $\frac{C_h}{S_h} \in (2 - \frac{\beta^2}{\mu\omega}, 1)$,

$A_h^f = \frac{S_h\beta}{\mu\omega}, P_h^f = \frac{S_h\beta^2}{\mu\omega^2} + \frac{C_h - S_h}{\omega}, \pi_h^f = \frac{S_h^2\beta^2 + 2\mu\omega\left(S_hC_h - S_h^2\right)}{2\mu\omega^2}, D_h^f = S_h$.

In conclusion,based on the above calculation results and with profit maximization as the objective, we need compare and analyze the outcomes under the same range conditions. It is worth noting that in the real agricultural market, the supply of homogeneous agricultural products is often much larger than the potential market *demand, which is more reflective of reality. Therefore, the range of* $\frac{C_h}{S_h} \in \left(0, 2 - \frac{\beta^2}{\mu\omega}\right)$, *is more suitable for homogeneous agricultural products.*

$$\begin{aligned} &\textit{If} \frac{C_h}{S_h} \in \left(0, 2 - \frac{\beta^2}{\mu\omega}\right), \\ &\frac{2(S_h)^2\beta^2 - \mu\omega(2S_h - C_h)^2}{2\omega\beta^2} - \frac{\mu(C_h)^2}{2\left(2\mu\omega - \beta^2\right)}, \\ &= -\frac{2\left(\mu^2\omega^2(C_h)^2 - 2S_hC_h\mu\omega\beta^2 - (S_h)^2\beta^4\right)}{2\omega\beta^2\left(2\mu\omega - \beta^2\right)} \end{aligned} \tag{A.5}$$

Assuming that $\frac{C_h}{S_h} = t$, *and* $t \in (0, 2 - \frac{\beta^2}{\mu\omega})$, *we can get*

$$y_{(t)} = -2\mu^2\omega^2t^2 + 4t\mu\omega\beta^2 - 2\beta^4.$$

In this case, $y_{(t)}$ *is monotonically increasing when* $t \in (0, 2 - \frac{\beta^2}{\mu\omega})$. *And* $y_{\left(t=2-\frac{\beta^2}{\mu\omega}\right)} < 0$. *When* $t \in (0, 2 - \frac{\beta^2}{\mu\omega})$, $y_{(t)}$ *is always less than zero. To sum up, the result in this case is*

$$\begin{aligned} &A_h^f = \frac{C_h\beta}{2\mu\omega - \beta^2}, P_h^f = \frac{\mu C_h}{2\mu\omega - \beta^2}, \\ &\pi_h^f = \frac{\mu(C_h)^2}{2(2\mu\omega - \beta^2)}, D_h^f = \frac{\mu\omega C_h}{2\mu\omega - \beta^2}, \\ &CS_h^f = \frac{\mu^2(C_h)^2\omega}{2(2\mu\omega - \beta^2)^2}, SW_h^f = \frac{\mu(C_h)^2(3\mu\omega - \beta^2)}{2(2\mu\omega - \beta^2)^2}. \end{aligned} \tag{A.6}$$

Theorem 2 *When homogeneous agricultural products are sold using the dynamic pricing strategy based on presale, the market demand function and profit function can be expressed as follows.*

$$\begin{aligned} D_h^{a_1} &= C_h - \omega P_h^{a_1} + \beta A_h^a \\ D_h^{a_2} &= (C_h - D_h^{a_1}) - \omega P_h^{a_2} + \beta A_h^a \\ \max_{P_h^{a_1}, P_h^{a_2}, A_h^a} \pi_h^a &= D_h^{a_1} P_h^{a_1} + D_h^{a_2} P_h^{a_2} - \frac{\mu(A_h^a)^2}{2} \\ \text{s.t.} &\begin{cases} C_h > D_h^{a_1} \\ S_h \geq D_h^{a_1} + D_h^{a_2} \end{cases} \end{aligned} \tag{A.7}$$

According to $S_h \geq D_h^{a_1} + D_h^{a_2}$ and $C_h > D_h^{a_1}$, we get $P_h^{a_2} \geq \frac{\beta}{\omega} A_h^a + \frac{(C_h - S_h)}{\omega}$ and $P_h^{a_1} > \frac{\beta}{\omega} A_h^a$.Assuming that $(P_h^{a_1})^0 = \frac{\beta}{\omega} A_h^a$, $(P_h^{a_2})^0 = \frac{\beta}{\omega} A_h^a + \frac{(C_h - S_h)}{\omega}$.

$$
\begin{aligned}
\max_{P_h^{a_1}, P_h^{a_2}, A_h^a} \pi_h^a = & -\omega\left(P_h^{a_1}\right)^2 + \left(C_h + \beta A_h^a\right)P_h^{a_1} \\
& + \omega P_h^{a_1} P_h^{a_2} - \omega\left(P_h^{a_2}\right)^2 - \frac{\mu}{2}\left(A_h^a\right)^2 \\
\text{s.t.} & \begin{cases} P_h^{a_1} > \frac{\beta}{\omega}A_h^a \\ P_h^{a_2} \geq \frac{\beta}{\omega}A_h^a + \frac{C_h - S_h}{\omega} \end{cases}
\end{aligned}
\tag{A.8}
$$

In this case, the axis of symmetry of Eq. (A.8) is $\left(P_h^{a_1}\right)^* = \frac{c + \beta A_h^a + \omega P_h^{a2}}{2\omega}$. Now we have two situations:

1. If $(P_h^{a_1})^* \geq (P_h^{a_1})^0$, $P_h^{a_1} = (P_h^{a_1})^*$ can be plug into Eq. (A.8) to get

$$
\begin{aligned}
\max_{P_h^{a2}, A_h^a} \pi_h^a = & -\frac{3}{4}\omega\left(P_h^{a_2}\right)^2 + \frac{\beta^2 - 2\mu\omega}{4\omega}\left(A_h^a\right)^2 + \frac{C_h\beta}{2\omega}A_h^a \\
& + \frac{C_h + \beta A_h^a}{2}P_h^{a_2} + \frac{(C_h)^2}{4\omega} \\
\text{s.t.} & \begin{cases} P_h^{a_2} \geq \frac{\beta}{\omega}A_h^a - \frac{C_h}{\omega} \\ P_h^{a_2} \geq \frac{\beta}{\omega}A_h^a + \frac{C_h - S_h}{\omega} \end{cases}
\end{aligned}
\tag{A.9}
$$

In this case, the axis of symmetry of Eq. (A.9) is $\left(P_h^{a_2}\right)^* = \frac{C_h + \beta A_h^a}{3\omega}$. If there is a common range for the region of $P_h^{a_2}$, we can consider two situations for this function.

Firstly, if $S_h > 2C_h$, the common range is $P_h^{a_2} \geq \frac{\beta}{\omega}A_h^a - \frac{C_h}{\omega}$. In this case, there are two situations.

(a) If $\frac{C_h + \beta A_h^a}{3\omega} \geq \frac{\beta}{\omega}A_h^a - \frac{C_h}{\omega}$, so $\frac{C_h}{\beta} < A_h^a \leq \frac{2C_h}{\beta}$. We have

$$
\begin{aligned}
\max_{A_h^a} \pi_h^a &= \frac{2\beta^2 - 3\mu\omega}{6\omega}(A_h^a)^2 + \frac{2C_h\beta}{3\omega}A_h^a + \frac{(C_h)^2}{3\omega} \\
\text{s.t.} & \ \frac{C_h}{\beta} < A_h^a \leq \frac{2C_h}{\beta}
\end{aligned}
\tag{A.10}
$$

The axis of symmetry of Eq. (A.10) is $A_h^a = \frac{2C_h\beta}{3\mu\omega - 2\beta^2} \geq \frac{2C_h}{\omega}$. So, the optimal reault of A_h^a is $A_h^a = \frac{2C_h}{\beta}$. Combined with the previous text, we know that $2 - \frac{\beta^2}{\mu\omega} \in (0, 1)$ and all decision variables need to be meaningful, so $\mu\omega < \beta^2 < \frac{3\mu\omega}{2}$.

(b) If $\frac{C_h + \beta A_h^a}{3\omega} < \frac{\beta}{\omega}A_h^a - \frac{C_h}{\omega}$, so $A_h^a > \frac{2C_h}{\beta}$. We have

$$
\begin{aligned}
\max_{A_h^a} \pi_h^a &= -\frac{\mu}{2}(A_h^a)^2 + \frac{2C_h\beta}{\omega}A_h^a - \frac{(C_h)^2}{\omega} \\
\text{s.t.} & \ A_h^a > \frac{2C_h}{\beta}
\end{aligned}
\tag{A.11}
$$

The axis of symmetry of Eq. (A.11) is $A_h^a = \frac{2C_h\beta}{\mu\omega} > \frac{2C_h}{\omega}$. So, the optimal reault of A_h^a is $A_h^a = \frac{2C_h\beta}{\mu\omega}$.

Secondly, if $S_h < 2C_h$, the common range is $P_h^{a_2} \geq \frac{\beta}{\omega}A_h^a + \frac{C_h - S_h}{\omega}$. In this case, there are two situaitions.

(c) If $\frac{C_h+\beta A_h^a}{3\omega} \geq \frac{\beta}{\omega}A_h^a + \frac{C_h-S_h}{\omega}$, so $\frac{S_h-C_h}{\beta} < A_h^a \leq \frac{3S_h-2C_h}{2\beta}$. We have

$$\begin{aligned} \max_{A_h^a} \pi_h^a &= \frac{2\beta^2-3\mu\omega}{6\omega}(A_h^a)^2 + \frac{2C_h\beta}{3\omega}A_h^a + \frac{(C_h)^2}{3\omega} \\ \text{s.t.}\ \frac{S_h-C_h}{\beta} &< A_h^a \leq \frac{3S_h-2C_h}{2\beta} \end{aligned} \tag{A.12}$$

The axis of symmetry of Eq. (A.12) is $A_h^a = \frac{2C_h\beta}{3\mu\omega-2\beta^2} > \frac{3S_h-2C_h}{2\omega}$. So, the optimal reault of A_h^a is $A_h^a = \frac{3S_h-2C_h}{2\omega}$.

(d) If $\frac{C_h+\beta A_h^a}{3\omega} < \frac{\beta}{\omega}A_h^a + \frac{C_h-S_h}{\omega}$, so $A_h^a > \frac{3S_h-2C_h}{2\beta}$. We have

$$\begin{aligned} \max_{A_h^a} \pi_h^a &= -\frac{\mu}{2}(A_h^a)^2 + \frac{S_h\beta}{\omega}A_h^a - \frac{4C_hS_h-3(S_h)^2}{4\omega} \\ \text{s.t.}\ A_h^a &> \frac{3S_h-2C_h}{2\beta} \end{aligned} \tag{A.13}$$

The axis of symmetry of Eq. (A.13) is $A_h^a = \frac{S_h\beta}{\mu\omega} > \frac{3S_h-2C_h}{2\omega}$. So, the optimal reault of A_h^a is $A_h^a = \frac{S_h\beta}{\mu\omega}$.

2. If $(P_h^{a_1})^* < (P_h^{a_1})^0$, $P_h^{a_1} = (P_h^{a_1})^0$ can be plug into Eq. (A.8) to get

$$\begin{aligned} \max_{P_h^{a_2}, A_h^a} \pi_h^a &= -\omega\left(P_h^{a_2}\right)^2 - \frac{\mu}{2}\left(A_h^a\right)^2 + \frac{C_h\beta}{\omega}A_h^a + \beta A_h^a P_h^{a_2} \\ \text{s.t.} &\begin{cases} P_h^{a_2} < \frac{\beta}{\omega}A_h^a - \frac{C_h}{\omega} \\ P_h^{a_2} > \frac{\beta}{\omega}A_h^a + \frac{C_h-S_h}{\omega} \end{cases} \end{aligned} \tag{A.14}$$

In this case, the axis of symmetry of Eq. (A.14) is $\left(P_h^{a_2}\right)^* = \frac{\beta}{2\omega}A_h^a$. In order to get a common range for the range region about $P_h^{a_2}$, we have $S_h > 2C_h$. In this case, we have three situations.

(e) If $\frac{\beta}{2\omega}A_h^a > \frac{\beta}{\omega}A_h^a - \frac{C_h}{\omega}$, so $P_h^{a_2} = \frac{\beta}{\omega}A_h^a - \frac{C_h}{\omega}$ should be pluged into the Eq. (A.14).

$$\begin{aligned} \max_{A_h^a} \pi_h^a &= -\frac{\mu}{2}(A_h^a)^2 + \frac{2C_h\beta}{\omega}A_h^a - \frac{(C_h)^2}{\omega} \\ \text{s.t.}\ A_h^a &< \frac{2C_h}{\beta} \end{aligned} \tag{A.15}$$

The axis of symmetry of Eq. (A.15) is $A_h^a = \frac{2C_h\beta}{\mu\omega} > \frac{2C_h}{\beta}$. So, the optimal reault of A_h^a is $A_h^a = \frac{2C_h}{\beta}$.

(f)If $\frac{\beta}{2\omega}A_h^a < \frac{\beta}{\omega}A_h^a + \frac{C_h-S_h}{\omega}$, so $P_h^{a_2} = \frac{\beta}{\omega}A_h^a + \frac{C_h-S_h}{\omega}$ should be pluged into the Eq. (A.14).

$$\begin{aligned} \max_{A_h^a} \pi_h^a &= -\frac{\mu}{2}(A_h^a)^2 + \frac{S_h\beta}{\omega}A_h^a - \frac{(C_h - S_h)^2}{\omega} \\ \text{s.t. } A_h^a &> \frac{2(S_h - C_h)}{\beta} \end{aligned} \tag{A.16}$$

The axis of symmetry of Eq. (A.16) is $A_h^a = \frac{S_h\beta}{\mu\omega} > \frac{2(S_h-C_h)}{\beta}$. So, the optimal reault of A_h^a is $A_h^a = \frac{S_h\beta}{\mu\omega}$.

(g)If $\frac{\beta}{\omega}A_h^a + \frac{C_h-S_h}{\omega} < \frac{\beta}{2\omega}A_h^a < \frac{\beta}{\omega}A_h^a - \frac{C_h}{\omega}$, so $P_h^{a_2} = \frac{\beta}{2\omega}A_h^a$ should be pluged into the Eq. (A.14).

$$\begin{aligned} \max_{A_h^a} \pi_h^a &= -\frac{\beta^2 - 2\mu\omega}{4\omega}(A_h^a)^2 + \frac{C_h\beta}{\omega}A_h^a \\ \text{s.t. } \frac{2C_h}{\beta} &< A_h^a < \frac{2(S_h - C_h)}{\beta} \end{aligned} \tag{A.17}$$

The axis of symmetry of Eq. (A.17) is $A_h^a = \frac{2C_h\beta}{2\mu\omega-\beta^2}$, and $\frac{2C_h}{\beta} < \frac{2C_h\beta}{2\mu\omega-\beta^2} < \frac{2(S_h-C_h)}{\beta}$. So, the optimal reault of A_h^a is $A_h^a = \frac{2C_h\beta}{2\mu\omega-\beta^2}$.

Like the **Theorem** 1, the range, that is $\frac{C_h}{S_h} \in (0, \frac{1}{2})$, is more suitable for homogeneous agricultural products. So, when $S_h > 2C_h$,

(a)and(e)$A_h^a = \frac{2C_h}{\beta}, P_h^{a_1} = \frac{2C_h}{\omega}, P_h^{a_2} = \frac{C_h}{\omega}, \pi_h^a = \frac{(C_h)^2(3\beta^2--2\mu\omega)}{\omega\beta^2}, CS_h^a = \frac{(C_h)^2}{\omega}, SW_h^a = \frac{2(C_h)^2(2\beta^2-\mu\omega)}{\omega\beta^2}$;

(b)$A_h^a = \frac{2C_h\beta}{\mu\omega}, P_h^{a1} = \frac{2C_h\beta^2}{\mu\omega^2}, P_h^{a2} = \frac{2C_h\beta^2}{\mu\omega^2} - \frac{C_h}{\omega}, \pi_h^a = \frac{2(C_h)^2\beta^2}{\mu\omega^2} - \frac{(C_h)^2}{\omega}, CS_h^a = \frac{(C_h)^2}{\omega}, SW_h^a = \frac{2(C_h)^2\beta^2}{\mu\omega^2}$;

(f)$A_h^a = \frac{S_h\beta}{\mu\omega}, P_h^{a_1} = \frac{S_h\beta^2}{\mu\omega^2}, P_h^{a_2} = \frac{S_h\beta^2}{\mu\omega^2} + \frac{C_h-S_h}{\omega}, \pi_h^a = \frac{(S_h)^2\beta^2}{2\mu\omega^2} - \frac{(S_h-C_h)^2}{\omega}, CS_h^a = \frac{(S_h)^2--2S_hC_h+2(C_h)^2}{2\omega}, SW_h^a = \frac{(S_h)^2\beta^2}{2\mu\omega^2} + \frac{2C_hS_h-(S_h)^2}{2\omega}$;

(g)$A_h^a = \frac{2C_h\beta}{2\mu\omega-\beta^2}, P_h^{a_1} = \frac{2C_h\beta^2}{\omega(2\mu\omega-\beta^2)}, P_h^{a_2} = \frac{C_h\beta^2}{\omega(2\mu\omega-\beta^2)}, CS_h^a = \frac{(C_h)^2}{\omega}, \pi_h^a = \frac{(C_h)^2\beta^2(4\mu\omega-3\beta^2)}{\omega(2\mu\omega-\beta^2)^2}, SW_h^a = \frac{(C_h)^2\beta^2(4\mu\omega-3\beta^2)}{\omega(2\mu\omega-\beta^2)^2} + \frac{(C_h)^2}{\omega}$.

In order to make the sellers' profit maximized, we get the final optimal results by conparing. When $\frac{C_h}{S_h} \in (0, \frac{1}{2})$, the level of advertising investment, price, profit, demand, consusmer surplus, and social welfare are as follows.

$$\begin{aligned} A_h^a &= \frac{S_h\beta}{\mu\omega}, P_h^{a_1} = \frac{S_h\beta^2}{\mu\omega^2}, P_h^{a_2} = \frac{S_h\beta^2}{\mu\omega^2} + \frac{C_h - S_h}{\omega}, \\ \pi_h^a &= \frac{(S_h)^2\beta^2}{2\mu\omega^2} - \frac{(S_h - C_h)^2}{\omega}, D_h^a = S_h, \\ CS_h^a &= \frac{(S_h)^2 - 2S_hC_h + 2(C_h)^2}{2\omega}, SW_h^a = \frac{(S_h)^2\beta^2}{2\mu\omega^2} + \frac{2C_hS_h - (S_h)^2}{2\omega},. \end{aligned}$$

In the same way, we get the **Theorem** 3 and **Theorem** 4.

Proposition 6 From the **Theorem** 2, we can obtain that

When $\frac{C_h}{S_h} \in (0, \frac{1}{2})$, $P_h^{a_1} - P_h^{a_2} = \frac{S_h-C_h}{\omega} > 0$.

Thus, $P_h^{a_2} < P_h^{a_1}$.

Proposition 7 From the **Theorem** 1 and **Theorem** 2, we can obtain that

When $\frac{C_h}{S_h} \in (0, \frac{1}{2})$,assuming that $\frac{C_h}{S_h} = t$ and $\beta^2 = x$, and $t \in (0, \frac{1}{2})$,

For the profit,

$\pi_h^f - \pi_h^a == \frac{\mu^2\omega^2(C_h)^2-(S_h)^2\beta^2(2\mu\omega-\beta^2)+2\mu\omega(S_h-C_h)^2(2\mu\omega-\beta^2)}{2\mu\omega^2(2\mu\omega-\beta^2)}$

$y_{(t)} = \mu\omega(5\mu\omega - 2\beta^2)t^2 - 4\mu\omega(2\mu\omega - \beta^2)t + (\beta^2 - 2\mu\omega)^2$

The axis of $y_{(t)}$ is $t^0 = \frac{4\mu\omega-2\beta^2}{5\mu\omega-2\beta^2}$, and $t^0 \in (\frac{1}{2}, \frac{2}{3})$. So, $y_{(t)}$ is monotonically decreasing when $t \in (0, \frac{1}{2})$.

$y_{(t=0)} = (\beta^2 - 2\mu\omega)^2 > 0$

$y_{(t=\frac{1}{2})} = \beta^4 - \frac{5}{2}\mu\omega\beta^2 + \frac{5}{4}\mu^2\omega^2$, and the axis of $y_{(t=\frac{1}{2})}$ is $(\beta^2)^* = \frac{5}{4}\mu\omega$, $\frac{5}{4}\mu\omega \in (\mu\omega, \frac{3}{2}\mu\omega)$, we can get:

When $\beta^2 = \frac{5}{4}\mu\omega$, $y_{(t=\frac{1}{2})} = -\frac{5}{16}\mu^2\omega^2 < 0$; when $\beta^2 = \mu\omega$, $y_{(t=\frac{1}{2})} = -\frac{1}{4}\mu^2\omega^2 < 0$; when $\beta^2 = \frac{3}{2}\mu\omega$, $y_{(t=\frac{1}{2})} = -\frac{1}{4}\mu^2\omega^2 < 0$. So, $y_{(t=\frac{1}{2})} < 0$, $y_{(t=0)} > 0$. There exists t^* to make $y_{(t=t^*)} = 0$,and $t^* = \frac{(2\mu\omega-\beta^2)(2\mu\omega-\sqrt{\mu\omega(2\beta^2-\mu\omega)})}{\mu\omega(5\mu\omega-2\beta^2)}$.

To sum up, when $\frac{S_h}{C_h} \in (0, t^*)$, i.e., $\pi_h^f > \pi_h^a$; when $\frac{S_h}{C_h} \in (t^*, \frac{1}{2})$, i.e., $\pi_h^f < \pi_h^a$.

For the consumer surplus,

$CS_h^f - CS_h^a = \frac{\mu^2\omega^2(C_h)^2-((S_h)^2-2S_hC_h+2(C_h)^2)(2\mu\omega-\beta^2)}{2\omega(2\mu\omega-\beta^2)^2}$

$y_{(t)} = (-2\beta^4 + 8\mu\omega\beta^2 - 7\mu^2\omega^2)t^2 + 2(2\mu\omega - \beta^2)^2 t - (2\mu\omega - \beta^2)^2$

The axis of $y_{(t)}$ is $t^0 = \frac{\beta^4-4\mu\omega\beta^2+4\mu^2\omega^2}{2\beta^4-8\mu\omega\beta^2+7\mu^2\omega^2}$, and $t^0 > 1$. So, $y_{(t)}$ is monotonically increasing when $t \in (0, \frac{1}{2})$.

$y_{(t=0)} = -(2\mu\omega - \beta^2)^2 < 0$

$y_{(t=\frac{1}{2})} = -\beta^4 + 4\mu\omega\beta^2 - \frac{7}{2}\mu^2\omega^2$

The axis of $y_{(t=\frac{1}{2})}$ is $(\beta^2)^0 = 2\mu\omega$, $2\mu\omega \notin (\mu\omega, \frac{3}{2}\mu\omega)$, we can get:

When $\beta^2 = \mu\omega$, $y_{(t=\frac{1}{2})} = -\frac{1}{2}\mu^2\omega^2 < 0$; when $\beta^2 = \frac{3}{2}\mu\omega$, $y_{(t=\frac{1}{2})} = \frac{1}{4}\mu^2\omega^2 > 0$. There exists $(\beta^2)^* = (2 - \frac{\sqrt{2}}{2})\mu\omega$ to make $y_{(t=\frac{1}{2})} = 0$.

To sum up, when $\beta^2 \in (\mu\omega, (2 - \frac{\sqrt{2}}{2})\mu\omega)$, $CS_h^f < CS_h^a$; when $\beta^2 \in ((2 - \frac{\sqrt{2}}{2})\mu\omega\frac{3}{2}\mu\omega)$, $CS_h^f > CS_h^a$.

For the social welfare,

$SW_h^f - SW_h^a = \frac{\mu^2\omega^2(C_h)^2(3\mu\omega-\beta^2)-(S_h)^2\beta^2(2\mu\omega-\beta^2)^2-\mu\omega(2S_hC_h-(S_h)^2)(2\mu\omega-\beta^2)^2}{2\mu\omega^2(2\mu\omega-\beta^2)^2}$.

$y_{(t)} = \mu^2\omega^2(3\mu\omega - \beta^2)t^2 - -2\mu\omega(2\mu\omega - \beta^2)^2 t + (2\mu\omega - \beta^2)^2(\mu\omega - \beta^2)$

The axis of $y_{(t)}$ is $t^0 = \frac{(2\mu\omega-\beta^2)^2}{\mu\omega(3\mu\omega-\beta^2)}$, and $t^0 \in (\frac{1}{6}, \frac{1}{2})$. So, $y_{(t)}$ is monotonically decreasing when $t \in (0, t^0)$ and increasing when $t \in (t^0, \frac{1}{2})$.

$y_{(t=0)} = (2\mu\omega - \beta^2)^2(\mu\omega - \beta^2) < 0$

$y_{(t=\frac{1}{2})} = -\beta^6 + 4\mu\omega\beta^4 - \frac{17}{4}\mu^2\omega^2\beta^2 + \frac{3}{4}\mu^3\omega^3$

$y_{(t=\frac{1}{2})} = -x^3 + 4\mu\omega x^2 - \frac{17}{4}\mu^2\omega^2 x + \frac{3}{4}\mu^3\omega^3$.

$y'_{(t=\frac{1}{2})} = -3x^2 + 8\mu\omega x - \frac{17}{4}\mu^2\omega^2$

The axis of $y'_{(t=\frac{1}{2})}$ is $x^* = \frac{4}{3}\mu\omega$, $\frac{4}{3}\mu\omega \in (\mu\omega, \frac{3}{2}\mu\omega)$. When $\beta^2 = \mu\omega$, $y'_{(t=\frac{1}{2})} = \frac{3}{4}\mu^2\omega^2 > 0$; when $\beta^2 = \frac{3}{2}\mu\omega$, $y_{(t=\frac{1}{2})} = \mu^2\omega^2 > 0$; when $\beta^2 = \frac{4}{3}\mu\omega$, $y_{(t=\frac{1}{2})} = \frac{13}{12}\mu^2\omega^2 > 0$. Thus, $y_{(t=\frac{1}{2})}$ is monotonically increasing when $\beta^2 \in (\mu\omega, \frac{3}{2}\mu\omega)$. When $\beta^2 = \frac{3}{2}\mu\omega$, $y_{(t=\frac{1}{2})} = 0$,so $y_{(t=\frac{1}{2})}$ is always less than 0.

To sum up, $y_{(t=\frac{1}{2})} < 0$ and $y_{(t=0)} < 0$,so $SW_h^f < SW_h^a$.

Proposition 9 From the Theorem 4, we can obtain that
$P_d^{a_1} - P_d^{a_2} = \frac{C_d}{\omega} > 0$, thus $P_d^{a_1} > P_d^{a_2}$.

Proposition 10 From the Theorems 3 and 4, we can obtain that
$A_d^f = A_d^a = \frac{S_d\beta}{\mu\omega}$;
$P_d^{a_1} - P_d^f = \frac{2S_d - C_d}{2\omega} > 0$, thus $P_d^{a_1} > P_d^f$;
$P_d^{a_2} - P_d^f = \frac{S_d - C_d}{\omega} > 0$, thus $P_d^{a_2} > P_d^f$;
$\pi_d^a - \pi_d^f = \frac{(C_d)^2 - 4S_dC_d + 4S_d^2}{4\omega} > 0$, thus $\pi_d^a > \pi_d^f$;
$CS_d^a - CS_d^f = \frac{C_d(C_d - 2S_d)}{4\omega} < 0$, thus $CS_d^a < CS_d^f$;
$SW_d^a - SW_d^f = \frac{2(S_d)^2 - 3S_dC_d + (C_d)^2}{2\omega}$.

Assuming that $\frac{C_d}{S_d} = t$,and $t \in (0, 1)$, $y_{(t)} = 2 - 3t + t^2 = (t-1)(t-2)$, $y_{(t)}$ is monotonically decreasing function when $t \in (0, 1)$, and $y_{(t=1)} = 0$. We can get $y_{(t)} > 0$, thus $SW_d^a > SW_d^f$.

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Declarations

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