Mathematics

Patuakhali Science and Technology University

4^{th} semester (L-2, S-II) Final Examination of B.Sc. Engg. (CSE), July-Dec-2020, Session 2018-19

Course Code: MAT-221, Course Title: Mathematics-IV

Marks-70, Time: 3 hours, Credit: 3.00

[Figure in the right margin indicates full marks. Split answering of any question is not recommended.]

Answer any 5 of the following questions.

1. a) Define complex number. 2

b) State and prove Cauchy-Riemann Equation 6

V.V.I c) Show that the function $u = e^{-x}(x\sin y - y\cos y)$ is a harmonic function and also find the conjugate harmonic function of $u = e^{-x}(x\sin y - y\cos y)$. 6

2. (O+E) a) If $f(z)$ is analytic for all points inside of C and connected a simple closed curve C. a is any point inside C. Then $f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}dz$. 5

b) Evaluate : 9

$f(z) = \sin\pi z^2 + \cos\pi z^2$

(O+E) i. $\oint_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}dz$ where C is the circle $|z| = 3$ $\rightarrow \frac{1}{(z-1)(z-2)}$

ii. $\oint_C \frac{e^{tz}}{(z^2+1)}dz$ where C is the circle $|z| = 3$

(O+E) iii. $\oint_C \frac{ze^{tz}}{(z+1)^3}dz$ where C is the circle

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3. a) State and prove Cauchy's Residue theorem. 4

b) Evaluate $\oint_C \frac{e^{zt}}{(z^2+2z+2)}dz$, where c is the circle 5

To find the pole

(O+E) c) Show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ by using the suitable contour. 5

F 4. a) Define Fourier coefficients for a Fourier series. 3

b) Derive Fourier cosine and Fourier sine series for even and odd function. 7

c) Write down some applications of Fourier Series. 4

F 5. a) Find Fourier series of the function x^2 on the interval [-l, l]. 5

b) State and prove second translation property. 4

c) Find the Laplace transforms of following functions i) F(t)=$t\cos at$ ii) ~~F(t)=~~t^n 5

L 6. a) Write down some applications of Laplace transform. 3

b) Find the Laplace transform of $e^{3t}(5\cos 4t - 6\sin 3t)$. 6

c) Find the Laplace transform of $\int_0^t \frac{\sin t}{t}$. 5

Patuakhali Science and Technology University

4th semester (L-2, S-II) Final Examination of B.Sc. in Engg. (CSE), July-Dec-2017, Session: 2015-16,
Course Code: MAT-221, Course Title: Mathematics-IV
Marks-70, Time: 3 hours, Credit: 3.00

[Figure in the right margin indicates full marks. Split answering of any question is not recommended]

Answer any 5 of the following questions.

1. a) Write down the applications of Fourier series 3
   b) Define the half range Fourier series 4
   c) Find the Fourier series for the following function 7

$$f(x)=\begin{cases} 0 & \text{when } -\pi<x<0 \\ \dfrac{\pi x}{4} & \text{when } 0<x<\pi \end{cases}$$

and hence prove that $\dfrac{\pi}{4}=1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\ldots$

2. a) State the convolution theorem for the Fourier transformation 2
   b) Find the Fourier sine transformation of $F(x)$ where 5

$$F(x)=\begin{cases} 1, & |x|<a \\ 0, & |x|>a \end{cases}$$

   c) Solve the following boundary value problem using the Laplace transformation 7

$$\frac{\partial U}{\partial t}=\frac{\partial^2 U}{\partial x^2},\ U(x,0)=3\sin 2\pi x,\ U(0,t)=U(1,t)=0 \text{ where } 0<x<1,\ t>0$$

3. a) Find the Laplace transformation of the error function 4
   b) State the Heaviside's expansion formula and applying this formula find the value of 5

$$L^{-1}\left[\frac{3s+1}{(s-1)(s^2+1)}\right]$$

   c) Solve the following differential equation by using the Laplace transformation 5

$$\frac{d^2y}{dt^2}-3\frac{dy}{dx}+2y=e^{2t},\ y(0)=-3,\ y'(0)=5$$

4. a) State the Cauchy-Riemann equations 2
   b) Verify the Cauchy-Riemann equations for the function $f(z)=e^{z^2}$ 4
   c) Define harmonic function 2
   d) Show that the function $u=e^x(x\cos y-y\sin y)$ is a harmonic function and find the corresponding analytic function $f(z)$. From it find v. 6

5. a) State the Cauchy's integral formula for the nth derivative of an analytic function 2
   b) Show that $\oint_c \dfrac{\sin^6 z}{(z-\frac{\pi}{6})^3}dz=\dfrac{21\pi i}{16}$ where c is the circle $|z|=1$ 5
   c) Illustrate the Laurent's theorem. 3
   d) Expand $f(z)=\dfrac{z}{(z+1)(z+2)}$ in a Laurent's series for the region $1<z<2$ 4

6. a) State the Cauchy's residue theorem and evaluate $\oint_c \dfrac{e^{zt}}{(z^2+1)^2}dz$ where c is the circle $|z|=3$ 2+4
   b) Show that $\displaystyle\int_0^\infty \frac{\cos mx}{(x^2+a^2)^2}dx=\frac{\pi(1+ma)e^{-ma}}{4a^3}$ by using the suitable contour. 8

Patuakhali Science and Technology University

4th semester (L-2, S-II) Final Examination of B.Sc. in Engg. (CSE), July-Dec-2016

Session: 2014-15, Course Code: MAT-221, Course Title: Mathematics-IV

Marks-70, Time: 3 hours, Credit: 3.00

[Figure in the right margin indicates full marks. Split answering of any question is not recommended.]

Answer any 5 of the following questions.

1. a) Define closed contour. 2

   b) Evaluate the following integrals by using the method of contour 4×3

   (i) $\int_0^{2\pi} \frac{1}{1+a\sin x}\,dx,\ 0<a<1$ (ii) $\int_0^{\infty} \frac{\cos x}{(x^2+1)(x^2+4)^2}\,dx$ (iii) $\int_0^{\infty} \frac{\sin mx}{x}\,dx,\ m>0$

2. a) Let $f(z)$ be analytic inside and on a simple closed curve C and a is any point inside C. Then prove that $f'(a) = \frac{1}{2\pi i}\oint_c \frac{f(z)}{(z-a)^2}\,dz$ 5

   b) Find the harmonic conjugate of the function $u = x^3 + 6x^2y - 3xy^2 - 2y^3$ 4

   c) Show that $\frac{1}{2\pi i}\oint_c \frac{ze^{tz}}{(z+1)^3}\,dz = (t - \frac{1}{2}t^2)e^{-t}$ 4

3. a) State and prove the Laurents theorem. 6

   b) Expand $\log\left(\frac{1+z}{1-z}\right)$ in Taylor series about $z=0$ 3

   c) State the Cauchy's residue theorem and evaluate $\oint_c \frac{e^{3z}}{z+\pi i}\,dz$ where c is the circle $|z+1|=4$ 2+3

4. a) Derive the Fourier series in complex form. 4

   b) Define the half range Fourier series 2

   c) Expand the Fourier series for the following function 8

$$f(x) = \begin{cases} 0 & when\ -\pi < x < 0 \\ \frac{\pi x}{4} & when\ 0 < x < \pi \end{cases}$$

   and hence prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

5. a) Illustrate some applications of Laplace Transformation. Also prove that if $L[F(t)] = f(s)$ then $L[F'''(t)] = s^3 f(s) - s^2F(0) - sF'(0) - F''(0)$ 5

   b) Evaluate $L^{-1}\left[\frac{1}{s^2(s^2+4)}\right]$ by use of the Convolution theorem 4

   c) Find the solution of the following differential equation using Laplace Transformation which satisfy the given conditions 5

   $y'' - 9y = e^t$, $y(0) = 1, y'(0) = 0$

6. a) Explain Fourier Transformation with its applications $e^{-st}\,dt$ 4

   b) Find the relation between Fourier and Laplace Transformations 4

   c) Find the Fourier sine transformation of $F(x)$ where 6

$$F(x) = \begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$$

Patuakhali Science and Technology University

4^{th} semester (L-2, S-II) Final Examination of B.Sc. in Engg. (CSE), July-Dec-2015

Session: 2013-14, Course Code: MAT-221, Course Title: Mathematics-IV

Marks-70, Time: 3 hours, Credit: 3.00

[Figure in the right margin indicates full marks. Split answering of any question is not recommended.]

Answer any 5 of the following questions.

1. a) Define single valued function and multiple valued functions. 3

   b) State and prove Cauchy-Riemann Equation 7

   c) Show that $u = e^{x}(x\cos y - y\sin y)$ is harmonic. Also find the conjugate harmonic function of u 4

2. a) State and Prove Cauchy's integral formula 6

   b) Evaluate: (i) $\oint_c \frac{e^{iz}}{(z-4)^3}dz$, where c is the circle

   (ii) $\oint_c \frac{\sin 3z}{(z+\frac{\pi}{2})}dz$, where c is the circle $|z|$

   (iii) $\oint_c \frac{1}{(z^2-3z-40)}dz$, where c is the circle $|z|$

3. a) State and prove Cauchy's Residue theorem. 4

   b) Find the poles and residues of $f(z) = \frac{e^{zt}}{z^2(z^2+2z+2)}$ 4

   c) Evaluate: (i) $\oint_c \frac{e^{zt}}{(z^2+2z+2)^2}dz$, where c is the circle 6

   (ii) $\oint_c \frac{e^{z}}{(z^2+1)^2}dz$, where c is the circle

4. a) Write down some applications of Fourier Series related to CSE 3

   b) If $f(x)$ is an odd function then show that 4

   (i) $a_0 = 0$ (ii) $a_n = 0$, and (iii) $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$,

   c) Expand the Fourier series for the following functions 7

   $$f(x) = \begin{cases} -1 & when\ -\pi < x < 0 \\ 1 & when\ 0 < x < \pi \end{cases}$$

   Hence prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

5. a) Illustrate some applications of Laplace Transformation. Also prove that if $L[F(t)] = f(s)$ then $L[F''(t)] = s^2 f(s) - sF(0) - F'(0)$ 5

   b) Find the value of $L^{-1}\left[\frac{1}{(s+2)^2(s-2)}\right]$ 4

   c) Solve the following differential equation by using Laplace Transformation 5

   $y'' + 2y' + 5y = e^{-t}\sin t,\ y(0) = 0,\ y'(0) = 1$