

Patuakhali Science and Technology University 4th semester (L-2, S-II) Final Examination of B.Sc. Engg. (CSE), July-Dec-2020, Session 2018-19 Course Code: MAT-221, Course Title: Mathematics-IV Marks-70, Time: 3 hours, Credit: 3.00 [Figure in the right margin indicates full marks. Split answering of any question is not recommended.] Answer any 5 of the following questions. Define complex number. State and prove Cauchy-Riemann Equation 6 Show that the function $u = e^{-x}(x \sin y - y \cos y)$ is a harmonic function and also find the conjugate harmonic function of $u = e^{-x}(x \sin y - y \cos y)$ f(z) is analytic for all points inside of C and connected a simple closed curve C. a is COTE $\int \frac{\sin \pi z^2 + \cos \pi z}{(z-1)(z-2)} dz \text{ where C is the circle } |z| = 3$ ii. $\int \frac{e^{iz}}{(z-1)(z-2)} dz \text{ where C is the circle } |z| = 3$ any point inside C. Then $f(a) = \frac{1}{2\pi i} \oint_{C} \frac{f(z)}{z - a} dz$. ii. $\oint_{C} \frac{e^{zz}}{(z^2 + 1)} dz$ where C is the circle |z| = 3 $\int \frac{ze^{iz}}{(z+1)^3} dz$ where C is the circle Evaluate $\oint \frac{e^{zz}}{(z^2+2z+2)} dz$, where c is the circle $\oint f dz$ Show that $\int_{0}^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ by using the suitable contour. 5 a) Define Fourier coefficients for a Fourier series. Derive Fourier cosine and Fourier sine series for even and odd function. Write down some applications of Fourier Series. Find Fourier series of the function x^2 on the interval [-I, I]. State and prove second translation property. Find the Laplace transforms of following functions i) F(t)=tcosat ii) E(t)=tn 5 Write down some applications of Laplace transform. 3 Find the Laplace transform of $e^{3t}(5\cos 4t - 6\sin 3t)$ 6 Find the Laplace transform of $\int_0^t \frac{\sin t}{t}$

Patuakhali Science and Technology University

4th semester (L-2, S-II) Final Examination of B.Sc. in Engg. (CSE), July-Dec-2017, Session: 2015-16, Course Code: MAT-221, Course Title: Mathematics-IV

Marks-70, Time: 3 hours, Credit: 3.00

[Figure in the right margin indicates full marks. Split answering of any question is not recommended] Answer any 5 of the following questions.



Write down the applications of Fourier series

3

b) Define the half range Fourier series

(c) Find the Fourier series for the following function

$$f(x) = \begin{cases} 0 & \text{when } -\pi < x < 0 \\ \frac{\pi x}{4} & \text{when } 0 < x < \pi \end{cases}$$

and hence prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$



State the convolution theorem for the Fourier transformation



Find the Fourier sine transformation of F(x) where

 $F(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ c) Solve the following boundary value problem using the Laplace transformation

- $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, \ U(x,0) = 3\sin 2\pi x, \ U(0,t) = U(1,t) = 0 \ where \ 0 < x < 1, \ t > 0$
- State the Heaviside's expansion formula and applying this formula find the value of $L^{-1}\left[\frac{3s+1}{(s-1)(s^2+1)}\right]$ b)

Solve the following differential equation by using the Laplace transformation

Find the Laplace transformation of the error function

 $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{2t}, \ y(0) = -3, \ y'(0) = 5$

4. a State the Cauchy-Riemann equations

b) Verify the Cauchy-Riemann equations for the function $f(z) = e^{z^2}$

c) Define harmonic function

- Show that the function $u = e^x(x\cos y y\sin y)$ is a harmonic function and find the corresponding analytic function f(z). From it find v.
- State the Cauchy's integral formula for the nth derivative of an analytic function

Show that $\oint \frac{\sin^6 z}{c(z-\pi)^3} dz = \frac{21\pi i}{16}$ where c is the circle |z|=1



- c) Illustrate the Laurent's theorem.
- d) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ in a Laurent's series for the region 1 < z < 2



- State the Cauchy's residue theorem and evaluate $\oint_C \frac{e^{iz}}{(z^2+1)^2} dz$ where c is the circle |z|=3

- Show that $\int_{0}^{\infty} \frac{\cos mx}{(x^2 + a^2)^2} dx = \frac{\pi (1 + ma)e^{-ma}}{4a^3}$ by using the suitable contour. $\frac{(-1)^{n-1}}{2n!}$



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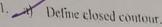
Patuakhali Science and Technology University

4th semester (L-2, S-II) Final Examination of B.Sc. in Engg. (CSE), July-Dec-2016 Session: 2014-15, Course Code: MAT-221, Course Title: Mathematics-IV

Marks-70, Time: 3 hours, Credit: 3.00

[Figure in the right margin indicates full marks. Split answering of any question is not recommended.]

Answer any 5 of the following questions.



(i)
$$\int_{0}^{\pi} \frac{1}{1 + a \sin x} dx$$
, $0 < a < 1$ (ii) $\int_{0}^{\infty} \frac{\cos x}{(x^2 + 1)(x^2 + 4)^2} dx$ (iii) $\int_{0}^{\infty} \frac{\sin mx}{x} dx$, $m > 0$

2. a) Let
$$f(z)$$
 be analytic inside and on a simple closed curve C and a is any point inside C. Then

prove that $f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz$

Find the harmonic conjugate of the function
$$u = x^3 + 6x^2y - 3xy^2 - 2y^3$$

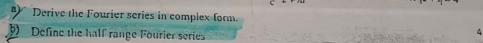
Show that $\frac{1}{2} = \frac{1}{4} \frac{ze^{4z}}{z^2} dz = (1 - \frac{1}{2})^2 z^4$

Show that
$$\frac{1}{2\pi i} \int_{C} \frac{ze^{iz}}{(z+1)^3} dz = (t-\frac{1}{2}t^2)e^{-t}$$

a) State and prove the Laurents theorem.

b) Expand
$$-\log\left(\frac{1+z}{1-z}\right)$$
 in Taylor series about $z=0$

State the Cauchy's residue theorem and evaluate
$$\int_{c}^{c} \frac{e^{3z}}{z+\pi i} dz$$
 where c is the circle $|z+1|=4$



c) Expand the Fourier series for the following function
$$f(x) = \begin{cases} 0 & when - \pi < x < 0 \end{cases}$$

$$\frac{\pi x}{4} \quad when \quad 0 < x < \pi$$

and hence prove that
$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$$

Illustrate some applications of Laplace Transformation. Also prove that if
$$L[F(t)] = f(s)$$
 5

then
$$L[F'''(t)] = s^3 f(s) - s^2 F(0) - sF'(0) - F''(0)$$

Evaluate
$$L^{-1}\left[\frac{1}{s^2(s^2+4)}\right]$$
 by use of the Convolution theorem

c) Find the solution of the following differential equation using Laplace Transformation which satisfy the given conditions

$$y'' - 9y = e'$$
, $y(0) = 1$, $y'(0) = 0$

c) Find the Fourier sine transformation of
$$F(x)$$
 where

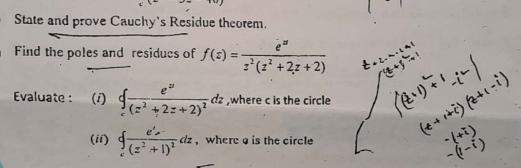
$$F(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

Patuakhali Science and Technology University 4th semester (L-2, S-II) Final Examination of B.Sc. in Engg. (CSE), July-Dec-2015 Session: 2013-14, Course Code: MAT-221, Course Title: Mathematics-IV

Marks-70, Time: 3 hours, Credit: 3.00

[Figure in the right margin indicates full marks. Split answering of any question is not recommended.] Answer any 5 of the following questions.

- 1. a Define single valued function and multiple valued functions.
 - b) State and prove Cauchy-Riemann Equation
 - c) Show that $u = e^{x}(x \cos y y \sin y)$ is harmonic. Also find the conjugate harmonic function
- State and Prove Cauchy's integral formula
 - Evaluate:
- (iii) $\oint \frac{\sin 3z}{(z+\frac{\pi}{2})} dz$, where c is the circle $\int \frac{1}{z^2-3z-40} dz$, where c is the circle $\int \frac{1}{z^2-3z-40} dz$, where c is the circle $\int \frac{1}{z^2-3z-40} dz$, where c is the circle $\int \frac{1}{z^2-3z-40} dz$
- $\int_{c}^{c} \frac{1}{(z^2-3z-40)} dz$, where c is the circle $\int_{c}^{c} \frac{1}{2} \int_{c}^{c} \frac{1}{2} dz$
- a) State and prove Cauchy's Residue theorem.

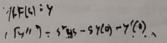


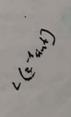
- Write down some applications of Fourier Series related to CSE b) If f(x) is an odd function then show that
 - $(i) a_0 = 0$ $(ii) a_n = 0$, and $(iii) b_n = \frac{2}{r} \int_0^x f(x) \sin nx \, dx$,
 - e) Expand the Fourier series for the following functions

$$\int_{f(x)}^{f} f(x) = \begin{cases} -1 & \text{when } -\pi < x < 0 \\ 1 & \text{when } 0 < x < \pi \end{cases}$$

Hence prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

- Illustrate some applications of Laplace Transformation. Also prove that if L[F(t)] = f(s)
- then $L[F''(t)] = s^2 f(s) sF(0) F'(0)$ Find the value of L^{-1} $\frac{1}{(s+2)^2(s-2)}$
- c) Solve the following differential equation by using Laplace Transformation V" . 78' . 58 - " ' wint V(D) - D V'(D) -1





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