$$\therefore f(x) = \frac{4}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\right).$$

(v). দেওয়া আছে (Given that) V. V. I

$$f(x) = \begin{cases} -1, & -\pi < x < -\pi/2 \\ 0, & -\pi/2 < x < \pi/2 \\ 1, & \pi/2 < x < \pi \end{cases} \quad \ldots (1)$$

আমরা জানি, $-\pi < x < \pi$ ব্যবধিতে ফ্যুরিয়ার ধারা হইল, [We know, in the interval $-\pi < x < \pi$, the Fourier series is]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx), \ldots (2)$$

যখন $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$,

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{-\pi/2} f(x)\, dx + \int_{-\pi/2}^{\pi/2} f(x)\, dx + \int_{\pi/2}^{\pi} f(x)\, dx\right\},$$

$$= \frac{1}{\pi}\left\{-\int_{-\pi}^{-\pi/2} 1\, dx + \int_{-\pi/2}^{\pi/2} 0\, dx + \int_{\pi/2}^{\pi} 1\, dx\right\},$$

$$= \frac{1}{\pi}\left\{-\left[x\right]_{-\pi}^{-\pi/2} + 0 + \left[x\right]_{\pi/2}^{\pi}\right\},$$

$$= \frac{1}{\pi}\left\{-\left(-\frac{\pi}{2} + \pi\right) + \left(\pi - \frac{\pi}{2}\right)\right\} = \frac{1}{\pi}\left(\frac{\pi}{2} - \pi + \pi - \frac{\pi}{2}\right).$$

$$= 0.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx,$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{-\pi/2} f(x) \cos nx\, dx + \int_{-\pi/2}^{\pi/2} f(x) \cos nx\, dx + \int_{\pi/2}^{\pi} f(x) \cos nx\, dx\right\},$$

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$$=\frac{1}{\pi}\left\{-\int_{-\pi}^{-\pi/2} 1.\cos nx\, dx+\int_{-\pi/2}^{\pi/2} 0.\cos nx\, dx+\int_{\pi/2}^{\pi} 1.\cos nx\, dx\right\}$$

$$=\frac{1}{\pi}\left\{-\left[\frac{\sin nx}{n}\right]_{-\pi}^{-\pi/2}+0+\left[\frac{\sin nx}{n}\right]_{\pi/2}^{\pi}\right\},$$

$$=\frac{1}{\pi}\left\{-\frac{1}{n}(-\sin n\pi/2+\sin n\pi)+\frac{1}{n}(\sin n\pi-\sin n\pi/2)\right\},$$

$$=\frac{1}{n\pi}(\sin n\pi/2+0+0-\sin n\pi/2)=0,$$

এবং $b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx.$

বা $b_n=\frac{1}{\pi}\left\{\int_{-\pi}^{-\pi/2} f(x)\sin nx\, dx+\int_{-\pi/2}^{\pi/2} f(x)\sin nx\, dx\right.$

$$\left.+\int_{\pi/2}^{\pi} f(x)\sin nx\, dx\right\}.$$

$$=\frac{1}{\pi}\left\{-\int_{-\pi}^{-\pi/2} 1.\sin nx\, dx+\int_{-\pi/2}^{\pi/2} 0.\sin nx\, dx+\int_{\pi/2}^{\pi} 1.\sin nx\, dx\right\}$$

$$=\frac{1}{\pi}\left\{\left[\frac{\cos nx}{n}\right]_{-\pi}^{-\pi/2}+0-\left[\frac{\cos nx}{n}\right]_{\pi/2}^{\pi}\right\},$$

$$=\frac{1}{\pi}\left\{\frac{1}{n}\left(\cos\frac{n\pi}{2}-\cos n\pi\right)-\frac{1}{n}\left(\cos n\pi-\cos\frac{n\pi}{2}\right)\right\},$$

$$=\frac{1}{\pi n}\left(\cos\frac{n\pi}{2}-\cos n\pi-\cos n\pi+\cos\frac{n\pi}{2}\right).$$

$$=\frac{2}{\pi n}\left(\frac{\cos n\pi}{2}-\cos n\pi\right),$$

$$=\frac{2}{\pi n}\left\{\frac{\cos n\pi}{2}-(-1)^n\right\}.$$

$$\therefore (2)\Rightarrow f(x)=0+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\{\cos n\pi/2-(-1)^n\}\sin nx}{n},$$

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$$\text{বা } f(x) = \frac{2}{\pi}\left\{\frac{(0+1)\sin x}{1} + \frac{(-1-1)\sin 2x}{2} + \frac{(0+1)\sin 3x}{3} + \frac{(1-1)\sin 4x}{4} + \frac{(0+1)\sin 5x}{5} + \frac{(-1-1)\sin 6x}{6} + \frac{(0+1)\sin 7x}{7} + \frac{(1-1)\sin 8x}{8} + \frac{(0+1)\sin 9x}{9} + \frac{(-1-1)\sin 10x}{10} + \ldots\right\},$$

$$\text{বা } f(x) = \frac{2}{\pi}\left(\frac{\sin x}{1} - \frac{2\sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} - \frac{2\sin 6x}{6} + \frac{\sin 7x}{7} + \frac{\sin 9x}{9} - \frac{2\sin 10x}{10} + \ldots\right),$$

$$\text{বা } f(x) = \frac{2}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \ldots\right) - \frac{4}{\pi}\left(\frac{\sin 2x}{2} + \frac{\sin 6x}{6} + \frac{\sin 10x}{10} + \ldots\right) \quad \ldots (3)$$

২য় অংশ : $f(x)$ ফাংশনটি $x = \frac{\pi}{2}$ বিন্দুতে বিচ্ছিন্ন [The function $f(x)$ is discontinuous at the point $x = \pi/2$].

এখানে $f(\pi/2 - 0) = 0$ এবং $f(\pi/2 + 0) = 1$.

$$\therefore f(\pi/2) = \frac{1}{2}\{f(\pi/2 - 0) + f(\pi/2 + 0)\}$$

$$= \frac{1}{2}(0+1) = \frac{1}{2}, \quad \ldots (4)$$

এখন (3) নং এ $x = \pi/2$ বসাইয়া পাই, [Now in (3), we put $x = \pi/2$ then we get]

$$f(\pi/2) = \frac{2}{\pi}\left(\frac{\sin \pi/2}{1} + \frac{\sin 3\pi/2}{3} + \frac{\sin 5\pi/2}{5} + \frac{\sin 7\pi/2}{7} + \ldots\right) - \frac{4}{\pi}\left(\frac{\sin\pi}{2} + \frac{\sin 3\pi}{6} + \frac{\sin 5\pi}{10} + \ldots\right).$$

$$\text{বা } \frac{1}{2} = \frac{2}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\right) - \frac{4}{\pi}(0); \text{ (4) নং দ্বারা।}$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots.$$

$$=\frac{1}{3}\left\{-\left[\frac{\sin n\pi x/3}{n\pi/3}\right]_{-3}^{0}+\left[\frac{\sin n\pi x/3}{n\pi/3}\right]_{0}^{3}\right\},$$

$$=\frac{1}{3}\left\{-\frac{3}{n\pi}(0-\sin n\pi)+\frac{3}{n\pi}(\sin n\pi-0)\right\}=0,$$

এবং $b_n=\frac{1}{3}\int_{-3}^{3} f(x)\sin\frac{n\pi x}{3}dx,$

$$=\frac{1}{3}\left\{\int_{-3}^{0} f(x)\sin\frac{n\pi x}{3}dx+\int_{0}^{3} f(x)\sin\frac{n\pi x}{3}dx\right\},$$

$$=\frac{1}{3}\left\{-\int_{-3}^{0} 1.\sin\frac{n\pi x}{3}dx+\int_{0}^{3} 1.\sin\frac{n\pi x}{3}dx\right\}, \text{(1) নং দ্বারা।}$$

$$=\frac{1}{3}\left\{\left[\frac{\cos n\pi x/3}{n\pi/3}\right]_{-3}^{0}-\left[\frac{\cos n\pi x/3}{n\pi/3}\right]_{0}^{3}\right\},$$

$$=\frac{1}{3}\left\{\frac{3}{n\pi}(1-\cos n\pi)-\frac{3}{n\pi}(\cos n\pi-1)\right\},$$

$$=\frac{3.2}{3n\pi}(1-\cos n\pi)=\frac{2\{1-(-1)^n\}}{n\pi}.$$

$$\therefore (2)\Rightarrow f(x)=0+\sum_{n=1}^{\infty}\frac{2\{1-(-1)^n\}\sin n\pi x/3}{\pi n},$$

বা $f(x)=\frac{2}{\pi}\left(\frac{2\sin\pi x/3}{1}+0+\frac{2\sin 3\pi x/3}{3}+0+\frac{2\sin 5\pi x/3}{5}+\dots\right)$

$$\therefore f(x)=\frac{4}{\pi}\left(\frac{\sin\pi x/3}{1}+\frac{\sin 3\pi x/3}{3}+\frac{\sin 5\pi x/3}{5}+\dots\right).$$

5(i). দেওয়া আছে (Given that) *** (V.V.I)

$$f(x)=\begin{cases}\pi+x, & -\pi<x<0\\ \pi-x, & 0<x<\pi\end{cases}\quad\dots(1)$$

আমরা জানি, $-\pi<x<\pi$ ব্যবধিতে ফ্যুরিয়ার ধারা হইল, [We know, in he interval $-\pi<x<\pi$, the Fourier series is].

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx),\dots(2)$$

যখন $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right\}.$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} (\pi + x)\,dx + \int_{0}^{\pi} (\pi - x)\,dx\right\};$$ (1) নং দ্বারা।

$$= \frac{1}{\pi}\left\{\left[\pi x + \frac{x^2}{2}\right]_{-\pi}^{0} + \left[\pi x - \frac{x^2}{2}\right]_{0}^{\pi}\right\}.$$

$$= \frac{1}{\pi}\left\{0 - \left(-\pi^2 + \frac{\pi^2}{2}\right) + \left(\pi^2 - \frac{\pi^2}{2}\right) - 0\right\} = \frac{1}{\pi}\left(\frac{\pi^2}{2} + \frac{\pi^2}{2}\right) = \pi.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right\}.$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} (\pi + x)\cos nx\,dx + \int_{0}^{\pi} (\pi - x)\cos nx\,dx\right\}.$$

$$= \frac{1}{\pi}\left\{\left[\frac{(\pi + x)\sin nx}{n} - \frac{(1)(-\cos nx)}{n^2}\right]_{-\pi}^{0} + \left[\frac{(\pi - x)\sin nx}{n} - \frac{(-1)(-\cos nx)}{n^2}\right]_{0}^{\pi}\right\}.$$

$$= \frac{1}{\pi}\left\{\left[\frac{(\pi+x)\sin nx}{n} + \frac{\cos nx}{n^2}\right]_{-\pi}^{0} + \left[\frac{(\pi-x)\sin nx}{n} - \frac{\cos nx}{n^2}\right]_{0}^{\pi}\right\}.$$

$$= \frac{1}{\pi}\left\{\left(0 + \frac{1}{n^2}\right) - \left(0 + \frac{\cos n\pi}{n^2}\right) + \left(0 - \frac{\cos n\pi}{n^2}\right) - \left(0 - \frac{1}{n^2}\right)\right\},$$

$$= \frac{2}{\pi n^2}(1 - \cos n\pi) = \frac{2\{1 - (-1)^n\}}{\pi n^2}.$$

এবং $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx.$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right\}.$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} (\pi+x)\ \sin nx\ dx + \int_{0}^{\pi} (\pi-x)\ \sin nx\ dx\right\}; \text{ (1) নং দ্বারা ।}$$

$$= \frac{1}{\pi}\left\{\left[\frac{(\pi+x)\ (-\cos nx)}{n} - \frac{(1)\ (-\sin nx)}{n^2}\right]_{-\pi}^{0}\right.$$

$$\left. + \left[\frac{(\pi-x)\ (-\cos nx)}{n} - \frac{(-1)\ (-\sin nx)}{n^2}\right]_{0}^{\pi}\right\}.$$

$$= \frac{1}{\pi}\left[\frac{-(\pi+x)\cos nx}{n} + \frac{\sin nx}{n^2}\right]_{-\pi}^{0}$$

$$+ \left[\frac{-(\pi-x)\cos nx}{n} - \frac{\sin nx}{n^2}\right]_{0}^{\pi}\Bigg\}.$$

$$= \frac{1}{\pi}\left(\frac{-\pi}{n} + 0\right) + 0 + (0) + \left(\frac{\pi}{n} - 0\right)\Bigg\} = \frac{1}{\pi}\left(\frac{-\pi}{n} + \frac{\pi}{n}\right) = 0,$$

$$\therefore (2) \Rightarrow f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\{1-(-1)^n\}\cos nx}{n^2} + 0,$$

$$\text{বা } f(x) = \frac{\pi}{2} + \frac{2}{\pi}\left(\frac{2\cos x}{1^2} + 0 + \frac{2\cos 3x}{3^2} + 0 + \frac{2\cos 5x}{5^2} + \ldots\right).$$

$$\text{বা } f(x) = \frac{\pi}{2} + \frac{4}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\right).$$

(ii). দেওয়া আছে (given that) V.V.I

$$f(x) = \begin{cases} 1 + x/\pi, \text{ যখন } -\pi \le x < 0 \\ 1 - x/\pi, \text{ যখন } \ \ 0 \le x \le \pi \end{cases} \quad \ldots (1)$$

আমরা জানি, $-\pi \le x \le \pi$ ব্যবধিতে ফ্যুরিয়ার ধারা হইল, [We know, in the interva $-\pi \le x \le \pi$, the Fourier series is]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx), \ldots (2)$$

$$\text{যখন } a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ dx,$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\ dx + \int_{0}^{\pi} f(x)\ dx\right\}.$$

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$$=\frac{1}{\pi}\left\{\int_{-\pi}^{0}\left(1+\frac{x}{\pi}\right)dx+\int_{0}^{\pi}\left(1-\frac{x}{\pi}\right)dx\right\};\ \text{(1) নং দ্বারা।}$$

$$=\frac{1}{\pi}\left\{\left[x+\frac{x^2}{2\pi}\right]_{-\pi}^{0}+\left[x-\frac{x^2}{2\pi}\right]_{0}^{\pi}\right\}.$$

$$=\frac{1}{\pi}\left\{0-\left(-\pi+\frac{\pi^2}{2\pi}\right)+\left(\pi-\frac{\pi^2}{2\pi}\right)-0\right\}.$$

$$=\frac{2}{\pi}\left(\pi-\frac{\pi^2}{2\pi}\right)=\frac{2}{\pi}\left(\pi-\frac{\pi}{2}\right)=\frac{2}{\pi}\cdot\frac{\pi}{2}=1.$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx,$$

$$=\frac{1}{\pi}\left\{\int_{-\pi}^{0}f(x)\cos nx\,dx+\int_{0}^{\pi}f(x)\cos nx\,dx\right\},$$

$$=\frac{1}{2}\left\{\int_{-\pi}^{0}\left(1+\frac{x}{\pi}\right)\cos nx\,dx+\int_{0}^{\pi}\left(1-\frac{x}{\pi}\right)\cos nx\,dx\right\},$$

(1) নং দ্বারা।

$$=\frac{1}{\pi}\left\{\left[\left(1+\frac{x}{\pi}\right)\frac{\sin nx}{n}-\left(\frac{1}{\pi}\right)\frac{(-\cos nx)}{n^2}\right]_{-\pi}^{0}\right.$$
$$\left.+\left[\left(1-\frac{x}{\pi}\right)\frac{\sin nx}{n}-\left(-\frac{1}{\pi}\right)\frac{(-\cos nx)}{n^2}\right]_{0}^{\pi}\right\}.$$

$$=\frac{1}{\pi}\left\{\left[\left(1+\frac{x}{\pi}\right)\frac{\sin nx}{n}+\frac{\cos nx}{\pi n^2}\right]_{-\pi}^{0}\right.$$
$$\left.+\left[\left(1-\frac{x}{\pi}\right)\frac{\sin nx}{n}-\frac{\cos nx}{\pi n^2}\right]_{0}^{\pi}\right\},$$

$$=\frac{1}{\pi}\left\{\left(0+\frac{1}{\pi n^2}\right)-\left(0+\frac{\cos n\pi}{\pi n^2}\right)\right.$$
$$+\left(0-\frac{\cos n\pi}{\pi n^2}\right)$$

$$=\frac{1}{\pi}\left(\frac{2}{\pi n^2}-\frac{2\cos n\pi}{\pi n^2}\right)=\frac{2}{\pi^2 n^2}(1-\cos n\pi),$$

$$=\frac{2\{1-(-1)^n\}}{\pi^2 n^2}.$$

এবং $b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx,$

$$=\frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\sin nx\,dx+\int_{0}^{\pi} f(x)\sin nx\,dx\right\},$$

$$=\frac{1}{\pi}\left\{\int_{-\pi}^{0}\left(1+\frac{x}{\pi}\right)\sin nx\,dx+\int_{0}^{\pi}\left(1-\frac{x}{\pi}\right)\sin nx\,dx\right\},$$

$$=\frac{1}{\pi}\left\{\left[\left(1+\frac{x}{\pi}\right)\frac{(-\cos nx)}{n}-\left(\frac{1}{\pi}\right)\frac{(-\sin nx)}{n^2}\right]_{-\pi}^{0}\right.$$

$$\left.+\left[\left(1-\frac{x}{\pi}\right)\frac{(-\cos nx)}{n}-\left(-\frac{1}{\pi}\right)\frac{(-\sin nx)}{n^2}\right]_{0}^{\pi}\right\},$$

$$=\frac{1}{\pi}\left\{\left[-\left(1+\frac{x}{\pi}\right)\frac{\cos nx}{n}+\frac{\sin nx}{\pi n^2}\right]_{-\pi}^{0}\right.$$

$$\left.+\left[-\left(1-\frac{x}{\pi}\right)\frac{\cos nx}{n}-\frac{\sin nx}{\pi n^2}\right]_{0}^{\pi}\right\},$$

$$=\frac{1}{\pi}\left\{-(1+0)\frac{1}{n}+0+0+(1-0)\frac{1}{n}\right\}=\frac{1}{\pi}\left(-\frac{1}{n}+\frac{1}{n}\right)=0.$$

$$\therefore (2)\Rightarrow f(x)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{2\{1-(-1)^n\}\cos nx}{\pi^2 n^2}+0,$$

বা $f(x)=\frac{1}{2}+\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\{1-(-1)^n\}\cos nx}{n^2}.$

বা $f(x)=\frac{1}{2}+\frac{2}{\pi^2}\left(\frac{2\cos x}{1^2}+0+\frac{2\cos 3x}{3^2}+0+\frac{2\cos 5x}{5^2}+\ldots\right),$

বা $f(x)=\frac{1}{2}+\frac{4}{\pi^2}\left(\frac{\cos x}{1^2}+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\ldots\right),\ \ldots (3)$

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দ্বিতীয় অংশ ঃ যেহেতু $x = 0$ বিন্দুতে $f(x)$ ফাংশন অবিচ্ছিন্ন, কাজেই প্র শর্ত হইতে পাই, $f(0) = 1 - \frac{0}{\pi} = 1$. [Since the function f(x) is cont uous at the point $x = 0$, so from the given condition, we $f(0) = 1 - \frac{0}{\pi} = 1$].

এখন (3) নং এ $x = 0$ বসাইয়া পাই, [Now put $x = 0$ in (3), we get

$$f(0) = \frac{1}{2} + \frac{4}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right).$$

বা $1 = \frac{1}{2} + \frac{4}{\pi^2}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right)$.

বা $\frac{1}{2} = \frac{4}{\pi^2}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right)$.

$\therefore \frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

(iii) দেওয়া আছে (Given that) V.V.I

$$f(x) = \begin{cases} \pi - x, \text{ যখন } -\pi < x < 0 \\ x - \pi, \text{ যখন } \ 0 < x < \pi \end{cases} \quad \dots (1)$$

আমরা জানি, $-\pi < x < \pi$ ব্যবধিতে ফ্যুরিয়ার ধারা হইল, [We know, the interval $-\pi < x < \pi$ the Fourier series is]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx), \dots (2)$$

যখন $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right\}$.

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} (\pi - x)\,dx + \int_{0}^{\pi} (x - \pi)\,dx\right\}, \text{ (1) নং দ্বারা।}$$

$$= \frac{1}{\pi}\left\{\left[\pi x - \frac{x^2}{2}\right]_{-\pi}^{0} + \left[\frac{x^2}{2} - \pi x\right]_{0}^{\pi}\right\}.$$

$$= \frac{1}{\pi}\left\{0 - \left(-\pi^2 - \frac{\pi^2}{2}\right) + \left(\frac{\pi^2}{2} - \pi^2\right) - 0\right\}.$$

$$= \frac{1}{\pi}(\pi^2) = \pi.$$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right\},$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} (\pi-x)\cos nx\,dx + \int_{0}^{\pi} (x-\pi)\cos nx\,dx\right\},\ \text{(1) নং দ্বারা।}$$

$$= \frac{1}{\pi}\left\{\left[(\pi-x)\frac{\sin nx}{n} - (-1)\frac{(-\cos nx)}{n^2}\right]_{-\pi}^{0} + \left[(x-\pi)\frac{\sin nx}{n} - (1)\frac{(-\cos nx)}{n^2}\right]_{0}^{\pi}\right\}.$$

$$= \frac{1}{\pi}\left\{\left[(\pi-x)\frac{\sin nx}{n} - \frac{\cos nx}{n^2}\right]_{-\pi}^{0} + \left[(x-\pi)\frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right]_{0}^{\pi}\right\},$$

$$= \frac{1}{\pi}\left\{\left(0-\frac{1}{n^2}\right) - \left(0-\frac{\cos n\pi}{n^2}\right) + \left(0+\frac{\cos n\pi}{n^2}\right) - \left(0+\frac{1}{n^2}\right)\right\},$$

$$= \frac{2}{\pi n^2}(\cos n\pi - 1) = \frac{2\{(-1)^n - 1\}}{\pi n^2}.$$

এবং $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx.$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right\},$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} (\pi-x)\sin nx\,dx + \int_{0}^{\pi} (x-\pi)\sin nx\,dx\right\},\ \text{(1) নং দ্বারা।}$$

$$= \frac{1}{\pi}\left\{\left[\frac{(\pi-x)(-\cos nx)}{n} - \frac{(-1)-(-\sin nx)}{n^2}\right] + \left[\frac{(x-\pi)(-\cos nx)}{n} - \frac{(1)(-\sin nx)}{n^2}\right]\right\}$$

$$= \frac{1}{\pi}\left\{-\left[\frac{(\pi - x)\cos nx}{n} - \frac{\sin nx}{n^2}\right]_{-\pi}^{0} + \left[\frac{-(x-\pi)\cos nx}{n} + \frac{\sin nx}{n^2}\right]_{0}^{\pi}\right\}.$$

$$= \frac{1}{\pi}\left\{-\frac{\pi}{n} + \frac{2\pi\cos n\pi}{n} + 0 + \left(\frac{-\pi}{n}\right)\right\} = \frac{1}{\pi}\left(\frac{2\pi\cos n\pi}{n} - \frac{2\pi}{n}\right).$$

$$= \frac{2\pi}{\pi n}(\cos n\pi - 1) = \frac{2\{(-1)^n - 1\}}{n}.$$

$$\therefore (2) \Rightarrow f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\{(-1)^n - 1\}\cos nx}{n^2} + 2\sum_{n=1}^{\infty}\frac{\{(-1)^n - 1\}\sin nx}{n}.$$

$$\text{বা } f(x) = \frac{\pi}{2} + \frac{2}{\pi}\left(\frac{-2\cos x}{1^2} + 0 + \frac{-2\cos 3x}{3^2} + 0 + \frac{-2\cos 5x}{5^2} + \ldots\right) + 2\left(\frac{-2\sin x}{1} + 0 + \frac{-2\sin 3x}{3} + 0 + \frac{-2\sin 5x}{5} + \ldots\right).$$

$$\therefore f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\right) - 4\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\right).$$

প্রমাণ কর যে, $(-\pi, \pi)$ ব্যবধিতে $f(x) = x\cos x$ ফুরিয়ার সিরিজ

6(ii) দেওয়া আছে (Given that)

$f(x) = x\cos x, \; -\pi < x < \pi$

$\therefore \; f(-x) = -x\cos(-x) = -x\cos x = -f(x).$

$\Rightarrow f(x)$ অযুগ্ম ফাংশন। কাজেই ইহার ফ্যুরিয়ার ধারা হইল, [$\Rightarrow f(x)$ is an odd function, so its Fourier series is]

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx, \; \ldots (1)$$

$x\cos x =$

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যখন $b_n = \frac{2}{\pi}\int_0^\pi f(x) \sin nx \, dx,$

$$= \frac{2}{\pi}\int_0^\pi x \cos x \sin nx \, dx, \ \dots (2)$$

$$= \frac{1}{\pi}\int_0^\pi x\{\sin (n+1)x + \sin (n-1)x\} \, dx,$$

$$= \frac{1}{\pi}\left[x\left\{\frac{-\cos (n+1)x}{(n+1)} - \frac{\cos (n-1)x}{n-1}\right\}\right.$$

$$\left. -1\left\{\frac{-\sin(n+1)x}{(n+1)^2} - \frac{\sin(n-1)x}{(n-1)^2}\right\}\right]_0^\pi, \ n \neq 1$$

$$= \frac{1}{\pi}\left[-x\left\{\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right\}\right.$$

$$\left. + \frac{\sin(n+1)x}{(n+1)^2} + \frac{\sin(n-1)x}{(n-1)^2}\right]_0^\pi$$

$$= \frac{1}{\pi}\left[-\pi\left\{\frac{\cos (n+1)\pi}{n+1} + \frac{\cos (n-1)\pi}{n-1}\right\} + 0\right].$$

$$= -\frac{\cos (\pi + n\pi)}{n+1} - \frac{\cos (\pi - n\pi)}{n-1} = \frac{\cos n\pi}{n+1} + \frac{\cos n\pi}{n-1}.$$

$$b_n = \frac{2n \cos n\pi}{(n-1)(n+1)} = \frac{2n(-1)^n}{n^2-1}, \ n = 2, 3, 4, \dots$$

এখন (2) নং এ $n = 1$ বসাইয়া পাই, [Now we put $n = 1$ in (2) then we get]

$$b_1 = \frac{2}{\pi}\int_0^\pi x \cos x \sin x \, dx,$$

$$= \frac{1}{\pi}\int_0^\pi x \sin 2x \, dx.$$

$$= \frac{1}{\pi}\left[x\frac{(-\cos 2x)}{2} - (1).\frac{(-\sin 2x)}{2^2}\right]_0^\pi$$

$$= \frac{1}{\pi}\left[-\frac{x\cos 2x}{2} + \frac{\sin 2x}{4}\right]_0^\pi.$$

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$$= \frac{1}{\pi}\left\{\frac{-\pi\cos 2\pi}{2} + 0\right\} = -\frac{1}{2}.$$

$$\therefore (1) \Rightarrow f(x) = b_1 \sin x + \sum_{n=2}^{\infty} b_n \sin nx,$$

$$\therefore x\cos x = -\frac{1}{2}\sin x + 2\sum_{n=2}^{\infty}\frac{n(-1)^n}{n^2-1}\sin nx.$$ প্রমাণিত।

(ii) দেওয়া আছে (Given that)

$$f(x) = \begin{cases} 0, \text{ যখন } -\pi < x < 0 \\ \sin x, \text{ যখন } \quad 0 < x < \pi \end{cases} \quad \ldots (1)$$

আমরা জানি, $-\pi < x < \pi$ ব্যবধিতে ফ্যুরিয়ার ধারা হইল [We know, in the interval $-\pi < x < \pi$ the Fourier series is]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx). \ldots (2)$$

যখন $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right\}.$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi}\sin x\,dx\right\}, \text{ (1) নং দ্বারা ।}$$

$$= \frac{1}{\pi}\left\{0 - \left[\cos x\right]_0^{\pi}\right\} = \frac{1}{\pi}\{-(\cos\pi - \cos 0)\} = \frac{2}{\pi},$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx,$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right\},$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} 0\cos nx\,dx + \int_{0}^{\pi}\sin x\cos nx\,dx\right\} \ldots (3)$$

$$= \frac{1}{\pi}\left[0 + \frac{1}{2}\int_{0}^{\pi}\{\sin(n+1)x - \sin(n-1)x\}\,dx\right].$$

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$$= \frac{1}{2\pi}\left[\frac{-\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right]_0^{\pi} ; n \neq 1.$$

$$= \frac{1}{2\pi}\left\{\frac{-\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right\}.$$

$$= \frac{1}{2\pi}\left\{\frac{-\cos(\pi+n\pi)}{n+1} + \frac{\cos(\pi-n\pi)}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right\}.$$

$$= \frac{1}{2\pi}\left\{\frac{\cos n\pi}{n+1} - \frac{\cos n\pi}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right\}.$$

$$= \frac{1}{2\pi}\left\{\frac{2\cos n\pi}{n^2-1} + \frac{-2}{n^2-1}\right\} = \frac{2}{2\pi(n^2-1)}(\cos n\pi - 1).$$

$$= \frac{(-1)^n - 1}{\pi(n^2-1)} ; n = 2, 3, 4, \ldots$$

এখন (3) নং এ $n = 1$ বসাইয়া পাই, [Now we put $n = 1$ in (3) then we get]

$$a_1 = \frac{1}{\pi}\int_0^{\pi} \sin x \cos x \, dx = \frac{1}{2\pi}\int_0^{\pi} \sin 2x \, dx.$$

$$= \frac{1}{2\pi}\left[\frac{-\cos 2x}{2}\right]_0^{\pi} = -\frac{1}{4\pi}(\cos 2\pi - 1)$$

$$= 0.$$

এবং $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx.$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x) \sin nx \, dx + \int_0^{\pi} f(x) \sin nx \, dx\right\}.$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} 0 \sin nx \, dx + \int_0^{\pi} \sin x \sin nx \, dx\right\}; \ldots (4)$$

$$= \frac{1}{\pi}\left[0 + \frac{1}{2}\int_0^{\pi} \{\cos(n-1)x - \cos(n+1)x\} \, dx\right].$$

$$= \frac{1}{2\pi}\left[\frac{\sin(n-1)x}{n-1} - \frac{\sin(n+1)x}{n+1}\right]_0^{\pi}, n \neq 1.$$

$= 0$, যেহেতু $\sin(n-1)\pi = 0$, $\sin(n+1)\pi = 0$.

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(4) নং এ $n = 1$ বসাইয়া পাই, [We put $n = 1$ in (4) then we get]

$$b_1 = \frac{1}{\pi}\int_0^{\pi} \sin x \sin x \, dx = \frac{1}{2\pi}\int_0^{\pi}(1-\cos 2x)\, dx,$$

$$= \frac{1}{2\pi}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{1}{2\pi}(\pi - 0) = \frac{1}{2}.$$

$$\therefore (2) \Rightarrow f(x) = \frac{a_0}{2} + a_1 \cos x + \sum_{n=2}^{\infty} a_n \cos nx + b_1 \sin x + \sum_{n=2}^{\infty} b_n \sin nx,$$

$$\text{বা } f(x) = \frac{1}{2}\cdot\frac{2}{\pi} + 0 + \frac{1}{\pi}\sum_{n=2}^{\infty}\frac{\{(-1)^n - 1\}\cos nx}{(n-1)(n+1)} + \frac{1}{2}\sin x + 0.$$

$$\text{বা } f(x) = \frac{1}{\pi} + \frac{1}{\pi}\left(0 - \frac{2\cos 3x}{2.4} + 0 - \frac{2\cos 5x}{4.6} + 0 - \frac{2\cos 7x}{6.8} + \dots\right) + \frac{\sin x}{2}$$

$$\therefore f(x) = \frac{1}{\pi} - \frac{2}{\pi}\left(\frac{\cos 3x}{2.4} + \frac{\cos 5x}{4.6} + \frac{\cos 7x}{6.8} + \dots\right) + \frac{\sin x}{2}.$$

(iii)(a). দেওয়া আছে (Given that)

$\therefore f(x) = \cos \alpha x,$

$\therefore f(-x) = \cos(-\alpha x) = \cos \alpha x = f(x)$

$\Rightarrow f(x)$ যুগ্ম ফাংশন, কাজেই ইহার ফ্যুরিয়ার ধারা হইল, [$\Rightarrow f(x)$ is an even function, so its Fourier series is]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \dots (1)$$

যখন $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\, dx$ এবং $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\, dx.$

এখন $a_0 = \frac{2}{\pi}\int_0^{\pi}\cos\alpha x\, dx = \frac{2}{\pi}\left[\frac{\sin\alpha x}{\alpha}\right]_0^{\pi}$

$$= \frac{2}{\pi\alpha}(\sin\alpha\pi - 0) = \frac{2\sin\alpha\pi}{\alpha\pi}.$$

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এবং $a_n = \frac{2}{\pi}\int_0^{\pi} \cos\alpha x . \cos nx \, dx.$

$$= \frac{1}{\pi}\int_0^{\pi} \{\cos(\alpha - n)x + \cos(\alpha + n)x\} \, dx,$$

$$= \frac{1}{\pi}\left[\frac{\sin(\alpha - n)x}{\alpha - n} + \frac{\sin(\alpha + n)x}{\alpha + n}\right]_0^{\pi},$$

$$= \frac{1}{\pi}\left\{\frac{\sin(\alpha - n)\pi}{\alpha - n} + \frac{\sin(\alpha + n)\pi}{\alpha + n} - 0\right\},$$

$$= \frac{1}{\pi}\left\{\frac{\sin(\alpha\pi - n\pi)}{\alpha - n} + \frac{\sin(\alpha\pi + n\pi)}{\alpha + n}\right\},$$

$$= \frac{1}{\pi}\left(\frac{\sin\alpha\pi \cos n\pi - \cos\alpha\pi \sin n\pi}{\alpha - n} + \frac{\sin\alpha\pi \cos n\pi + \cos\alpha\pi \sin n\pi}{\alpha + n}\right),$$

$$= \frac{1}{\pi}\left(\frac{\sin\alpha\pi \cos n\pi - 0}{\alpha - n} + \frac{\sin\alpha\pi \cos n\pi}{\alpha + n}\right).$$

যেহেতু $\sin n\pi = 0$,

$$= \frac{\sin\alpha\pi . \cos n\pi}{\pi}\left(\frac{1}{\alpha - n} + \frac{1}{\alpha + n}\right) = \frac{2\alpha . \sin\alpha\pi . \cos n\pi}{\pi(\alpha^2 - n^2)},$$

$$= \frac{2(-1)^n \alpha . \sin\alpha\pi}{\pi(\alpha^2 - n^2)}.$$

$$\therefore (1) \Rightarrow f(x) = \frac{2\sin\alpha\pi}{2\alpha\pi} + \sum_{n=1}^{\infty} \frac{2(-1)^n \alpha \sin\alpha\pi \cos nx}{\pi(\alpha^2 - n^2)},$$

বা $f(x) = \frac{\sin\alpha\pi}{\alpha\pi} + \frac{2\alpha \sin\alpha\pi}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{\alpha^2 - n^2},$

বা $\cos\alpha x = \frac{\sin\alpha\pi}{\alpha\pi} + \frac{2\alpha \sin\alpha\pi}{\pi}\left(\frac{-\cos x}{\alpha^2 - 1^2} + \frac{\cos 2x}{\alpha^2 - 2^2} - \frac{\cos 3x}{\alpha^2 - 3^2} + \cdots\right).$

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$$\therefore\ \cos\alpha x = \frac{\sin\alpha\pi}{\alpha\pi} - \frac{2\alpha\sin\alpha\pi}{\pi}\cdot\left(\frac{\cos x}{\alpha^2-1^2} - \frac{\cos 2x}{\alpha^2-2^2} + \frac{\cos 3x}{\alpha^2-3^2} - \ldots\right) \quad \ldots (2)$$

(b) এখন (2) নং এ $x = \pi$ বসাইয়া পাই, [Now we put $x = \pi$ is (2) then we get]

$$\cos\alpha\pi = \frac{\sin\alpha\pi}{\alpha\pi} - \frac{2\alpha\sin\alpha\pi}{\pi}\left(\frac{-1}{\alpha^2-1^2} - \frac{1}{\alpha^2-2^2} - \frac{1}{\alpha^2-3^2} - \ldots\right).$$

বা $$\cos\alpha\pi = \frac{\sin\alpha\pi}{\pi}\left(\frac{1}{\alpha} + \frac{2\alpha}{\alpha^2-1^2} + \frac{2\alpha}{\alpha^2-2^2} + \frac{2\alpha}{\alpha^2-3^2} + \ldots\right).$$

$$\frac{\pi\cos\alpha\pi}{\sin\alpha\pi} = \frac{1}{\alpha} + \frac{2\alpha}{\alpha^2-1^2} + \frac{2\alpha}{\alpha^2-2^2} + \frac{2\alpha}{\alpha^2-3^2} + \ldots,$$

বা $$\frac{\pi\cos\alpha\pi}{\sin\alpha\pi} - \frac{1}{\alpha} = \frac{2\alpha}{\alpha^2-1^2} + \frac{2\alpha}{\alpha^2-2^2} + \frac{2\alpha}{\alpha^2-3^2} + \ldots.$$

উভয় পক্ষকে 0 এবং x সীমার মধ্যে α–এর সম্পর্কে ইনটিগ্রেট করিয়া পাই, [Integrating both sides w. r. to α between the limits 0 and x then we get]

$$\int_0^x\left(\frac{\pi\cos\alpha\pi}{\sin\alpha\pi} - \frac{1}{\alpha}\right)d\alpha = \int_0^x\frac{2\alpha\,d\alpha}{\alpha^2-1^2} + \int_0^x\frac{2\alpha\,d\alpha}{\alpha^2-2^2} + \int_0^x\frac{2\alpha\,d\alpha}{\alpha^2-3^2} + \ldots,$$

বা $$\Big[\log\sin\alpha\pi - \log\alpha\Big]_0^x = \Big[\log(\alpha^2-1^2)\Big]_0^x + \Big[\log(\alpha^2-2^2)\Big]_0^x + \Big[\log(\alpha^2-3^2)\Big]_0^x + \ldots,$$

বা $$\left[\log\frac{\sin\alpha\pi}{\alpha}\right]_0^x = \{\log(x^2-1^2) - \log(-1^2)\} + \{\log(x^2-2^2) - \log(-2^2)\} + \{\log(x^2-3^2) - \log(-3^2)\} + \ldots \ldots (3)$$

এখন $$\left[\log\frac{\sin\alpha\pi}{\alpha}\right]_0^x = \log\frac{\sin x\pi}{x} - \log\left(\lim_{\alpha\to 0}\frac{\sin\alpha\pi}{\alpha}\right).$$

$$= \log\frac{\sin\pi x}{x} - \log\pi\left(\lim_{\alpha\to 0}\frac{\sin\alpha\pi}{\alpha\pi}\right).$$

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$$= \log\frac{\sin \pi x}{x} - \log\pi.1,$$

$$= \log\frac{\sin \pi x}{\pi x}, \dots (4)$$

(3) নং এবং (4) নং হইতে পাই, [From (3) and (4) we get]

$$\log\frac{\sin \pi x}{\pi x} = \log\left(1-\frac{x^2}{1^2}\right) + \log\left(1-\frac{x^2}{2^2}\right) + \log\left(1-\frac{x^2}{3^2}\right) + \dots,$$

বা $\log\frac{\sin \pi x}{\pi x} = \log\left\{\left(1-\frac{x^2}{1^2}\right)\left(1-\frac{x^2}{2^2}\right)\left(1-\frac{x^2}{3^2}\right)\dots\right\}$.

বা $\frac{\sin \pi x}{\pi x} = \left(1-\frac{x^2}{1^2}\right)\left(1-\frac{x^2}{2^2}\right)\left(1-\frac{x^2}{3^2}\right)\dots, \dots (5)$

এখন (5) নং এ x এর স্থলে x/π বসাইয়া পাই, [Now replacing x by x/π in (5) then we get]

$$\frac{\sin x}{x} = \left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{(2\pi)^2}\right)\left(1-\frac{x^2}{(3\pi)^2}\right)\dots,$$ প্রমাণিত।

এখন $x = \pi/2$ বসাইয়া পাই, [Now we put $x = \pi/2$ then we get]

$$\frac{\sin\pi/2}{\pi/2} = \left(1-\frac{\pi^2}{2^2\,\pi^2}\right)\left(1-\frac{\pi^2}{2^2.2^2\,\pi^2}\right)\left(1-\frac{\pi^2}{2^2.\,3^2.\,\pi^2}\right)\dots,$$

বা $\frac{2}{\pi} = \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{6^2}\right)\dots,$

বা $\frac{2}{\pi} = \left(\frac{1.3}{2.2}\right)\left(\frac{3.5}{4.4}\right)\left(\frac{5.7}{6.6}\right)\dots,$

$\therefore \frac{\pi}{2} = \frac{2.2.4.4.6.6.8.8\dots}{1.3.3.5.5.7.7.9\dots}$ প্রমাণিত।

(iv) দেওয়া আছে (Given that)

$$f(x) = \frac{\pi e^x}{2\sin h\pi}, -\pi < x < \pi.$$

আমরা জানি, $-\pi < x < \pi$ ব্যবধিতে ফ্যুরিয়ার ধারা হইল, [We know, in the interval $-\pi < x < \pi$, the Fourier series is]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx), \dots (1)$$

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যখন $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx,$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\pi\, e^x\, dx}{2\, \sinh\pi} = \frac{1}{2\sinh\pi}\left[e^x\right]_{-\pi}^{\pi}.$$

$$= \frac{1}{2\sinh\pi}(e^{\pi} - e^{-\pi}) = \frac{\sinh\pi}{\sinh\pi} = 1.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx,$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\pi\, e^x}{2\, \sinh\pi}\cos nx\, dx = \frac{1}{2\sinh\pi}\int_{-\pi}^{\pi} e^x \cos nx\, dx,$$

$$= \frac{1}{2\, \sinh\pi}\left[\frac{e^x(\cos nx + n\sin nx)}{n^2 + 1^2}\right]_{-\pi}^{\pi},$$

$$= \frac{1}{2\sinh\pi}\left\{\frac{e^{\pi}(\cos n\pi + 0)}{n^2 + 1^2} - \frac{e^{-\pi}(\cos n\pi + 0)}{n^2 + 1^2}\right\};$$

$$= \frac{\cos n\pi}{2(n^2 + 1^2)\sinh\pi}(e^{\pi} - e^{-\pi}) = \frac{\cos n\pi .\, \sinh\pi}{(n^2 + 1^2)\sinh\pi},$$

$$= \frac{(-1)^n}{(n^2 + 1)},$$

এবং $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx,$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\pi e^x .\, \sin nx\, dx}{2\sinh\pi} = \frac{1}{2\sinh\pi}\int_{-\pi}^{\pi} e^x \sin nx\, dx,$$

$$= \frac{1}{2\sinh\pi}\left[\frac{e^x(\sin nx - n\cos nx)}{n^2 + 1}\right]_{-\pi}^{\pi},$$

$$= \frac{1}{2\sinh\pi}\left\{\frac{e^{\pi}(0 - n\cos n\pi)}{n^2 + 1} - \frac{e^{-\pi}(0 - n\cos n\pi)}{n^2 + 1}\right\},$$

$$= \frac{-n\cos n\pi}{2(n^2 + 1)\sinh\pi}(e^{\pi} - e^{-\pi}) = \frac{-n\cos n\pi .\, \sinh\pi}{(n^2 + 1)\sinh\pi}$$

$$= \frac{-n(-1)^n}{n^2 + 1}.$$

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$$\therefore (1) \Rightarrow f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2+1} - \sum_{n=1}^{\infty} (-1)^n \frac{n \sin nx}{n^2+1},$$

$$\text{বা } f(x) = \frac{1}{2} + \left(\frac{-\cos x}{1^2+1} + \frac{\cos 2x}{2^2+1} - \frac{\cos 3x}{3^2+1} + \ldots\right)$$
$$- \left(\frac{-\sin x}{1^2+1} + \frac{2\sin 2x}{2^2+1} - \frac{3\sin 3x}{3^2+1} + \ldots\right).$$

$$\text{বা } f(x) = \frac{1}{2} - \left(\frac{\cos x}{1^2+1} - \frac{\cos 2x}{2^2+1} + \frac{\cos 3x}{3^2+1} - \ldots\right)$$
$$+ \left(\frac{\sin x}{1^2+1} - \frac{2\sin 2x}{2^2+1} + \frac{3\sin 3x}{3^2+1} - \ldots\right).$$

দ্বিতীয় অংশে : [When $x = \pm\pi$ then the sum of the series $= \frac{1}{2} \{f(-\pi+0) + f(\pi-0)\}$

যখন $x = \pm\pi$ তখন ধারাটির যোগফল $= \frac{1}{2}\{f(-\pi+0) + f(\pi-0)\}$.

$$\text{বা ধারাটির যোগফল} = \frac{1}{2}\left(\frac{\pi e^{-\pi}}{2\sinh\pi} + \frac{\pi e^{\pi}}{2\sinh\pi}\right)$$
$$= \frac{\pi}{2\sinh\pi}\left(\frac{e^{\pi}+e^{-\pi}}{2}\right).$$
$$= \frac{\pi\cosh\pi}{2\sinh\pi} = \frac{\pi}{2}\coth\pi.$$

(v) মনে করি [Let] $f(x) = \frac{x(\pi^2 - x^2)}{12}, \ldots (1)$

$$\therefore f(-x) = \frac{-x\{\pi^2 - (-x)^2\}}{12} = \frac{-x(\pi^2 - x^2)}{12} = -f(x).$$

$\Rightarrow f(x)$ অযুগ্ম ফাংশন, কাজেই ইহার ফ্যুরিয়ার ধারা হইল, [$\Rightarrow f(x)$ is an odd function, so its Fourier series is]

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx, \ldots (2)$$

যখন $b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx \, dx,$

বা $b_n = \frac{2}{\pi}\int_0^{\pi} \frac{x(\pi^2 - x^2)}{12} \sin nx \, dx$; (1) নং দ্বারা

$$= \frac{1}{6\pi}\int_0^{\pi} (\pi^2 x - x^3) \sin nx \, dx.$$

$$= \frac{1}{6\pi}\left[(\pi^2 x - x^3)\frac{(-\cos nx)}{n} - (\pi^2 - 3x^2)\frac{(-\sin nx)}{n^2} + (-6x)\frac{\cos nx}{n^3} - (-6)\frac{\sin nx}{n^4}\right]_0^{\pi}$$

$$= \frac{1}{6\pi}\left[\frac{-(\pi^2 x - x^3)\cos nx}{n} + \frac{(\pi^2 - 3x^2)\sin nx}{n^2} - \frac{6x\cos nx}{n^3} + \frac{6\sin nx}{n^4}\right]_0^{\pi}.$$

$$= \frac{1}{6\pi}\left\{\left(0 + 0 - \frac{6\pi\cos n\pi}{n^3} + 0\right) + 0 - 0 + 0\right\}.$$

$$= -\frac{\cos n\pi}{n^3} = \frac{-(-1)^n}{n^3}.$$

$$\therefore (2) \Rightarrow f(x) = -\sum_{n=1}^{\infty} \frac{(-1)^n \sin nx}{n^3}$$

বা $f(x) = -\left(-\frac{\sin x}{1^3} + \frac{\sin 2x}{2^3} - \frac{\sin 3x}{3^3} + \dots\right)$

$\therefore \frac{x(\pi^2 - x^2)}{12} = \frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \dots$, (1) নং দ্বারা।

7. মনে করি [Let] $f(x) = x, \dots (1)$

$\therefore f(-x) = -x = -f(x).$

$\Rightarrow f(x)$ অযুগ্ম ফাংশন, কাজেই ইহার ফ্যুরিয়ার ধারা হইল, [$\Rightarrow f(x)$ is an odd function, so its Fourier series is]

$-\pi < x < \pi$

$$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin nx, \dots (2)$$

$x = 2\left(\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3}\right)$

যখন $b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx \, dx,$

$x^2 = \frac{\pi^2}{3} - 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2}\right)$

$\frac{x(\pi^2 - x^2)}{12} = \frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} + \dots$