Project Proposal:

Learning Dynamical System From Time Series Data Using Neural ODE

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1. Introduction

The aim of this project is to use Neural Ordinary Differential Equations (Neural ODE) to learn the unknown dynamics of a system from time series data.

2. Problem Statement:

The problem at hand concerns discovering the unknown dynamics of a given dynamical system, represented by the equation $\dot{x} = f(x)$ with a specified initial condition $x(0) = x_0$. The data available for analysis is the time series data of $x(t)$ for the aforementioned initial condition x_0.

To address this problem, we have selected the Lotka-Volterra or Predator-Prey dynamical system, described by the equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} ax_1 - bx_1x_2 \\ cx_1x_2 - dx_2 \end{bmatrix} = f(x),$$
$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Here, the values of $a, b, c,$ and d are predefined as $1.5, 1, 1,$ and 0.5 respectively. However, the governing dynamics function $f(x)$ is unknown.

The objective of this project is to develop a neural network that can approximate $\dot{x}_1$ and $\dot{x}_2$ from the given time series data of $x_1(t)$ and $x_2(t)$, thus allowing us to discover the unknown dynamics $f(x)$.

3. Data

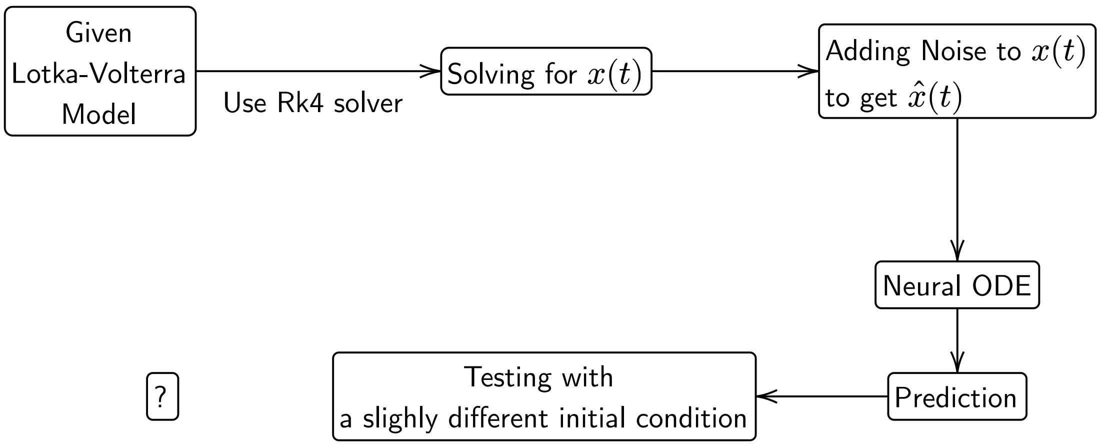
We used the ode solver `rk4` to get the time series data from the dynamics. The solution of $\dot{x} = f(x, \tau)$ with initial condition $x(0) = x_0$ can be given as:

$$x(t) = x_0 + \int_0^t f(x, \tau) d\tau.$$

In RK4 method, $\int_0^t f(x,\tau)d\tau$ is approximated as the following:

$$\begin{aligned}
\int_0^t f(x)d\tau \approx\ & \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]\Delta t, \\
k_1 =\ & f(t, x), \\
k_2 =\ & f\left(t + \frac{1}{2}\Delta t, x + \frac{\Delta t}{2} \times k_1\right), \\
k_3 =\ & f\left(t + \frac{1}{2}\Delta t, x + \frac{\Delta t}{2} \times k_2\right), \\
k_4 =\ & f(t + \Delta t, x + \Delta t \times k_2).
\end{aligned}$$

4. Methodology:



5. Literature Review:

We will examine the following three papers to provide the necessary background and context for our research:

1. "Neural Ordinary Differential Equations" by Ricky T. Q. Chen et al. (2019).