Project Proposal:

Learning Dynamical System From Time Series Data Using Neural ODE

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1. Introduction

The aim of this project is to use Neural Ordinary Differential Equations (Neural ODE) to learn the unknown dynamics of a system from time series data.

2. Problem Statement:

The problem at hand concerns discovering the unknown dynamics of a given dynamical system, represented by the equation $\dot{x} = f(x)$ with a specified initial condition $x(0) = x_0$. The data available for analysis is the time series data of $x(t)$ for the aforementioned initial condition x_0.

To address this problem, we have selected the Lotka-Volterra or Predator-Prey dynamical system, described by the equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} ax_1 - bx_1x_2 \\ cx_1x_2 - dx_2 \end{bmatrix} = f(x),$$
$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Here, the values of $a, b, c,$ and d are predefined as $1.5, 1, 1,$ and 0.5 respectively. However, the governing dynamics function $f(x)$ is unknown.

The objective of this project is to develop a neural network that can approximate $\dot{x}_1$ and $\dot{x}_2$ from the given time series data of $x_1(t)$ and $x_2(t)$, thus allowing us to discover the unknown dynamics $f(x)$.

3. Data

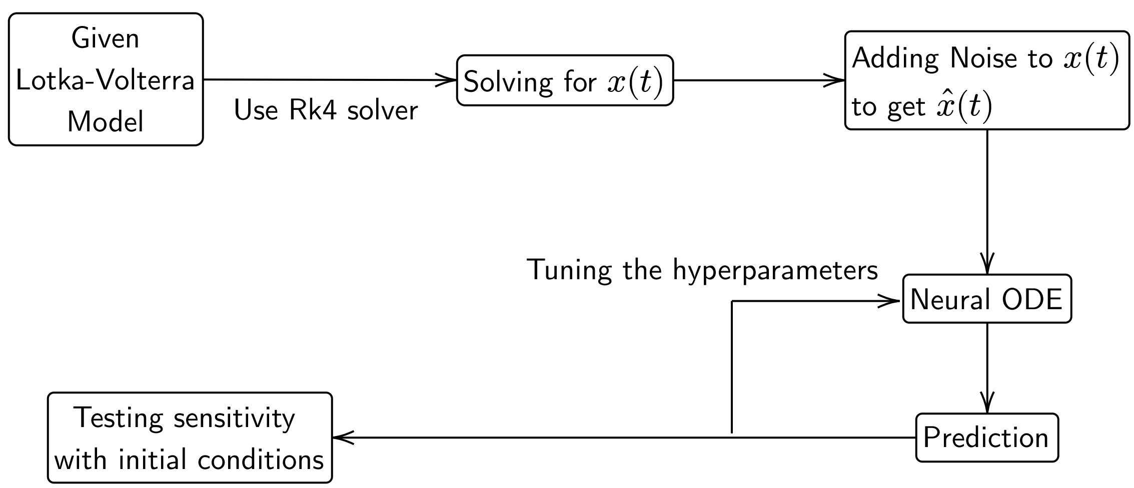
We used the ode solver `rk4` to get the time series data from the dynamics. The solution of $\dot{x} = f(x, t)$ with initial condition $x(0) = x_0$ can be given as:

$$x(t) = x_0 + \int_0^t f(x, \tau) d\tau.$$

In RK4 method, $\int_0^t f(x,\tau)d\tau$ is approximated as the following:

$$\begin{aligned}
\int_0^t f(x)d\tau \approx \ & \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]\Delta t, \\
k_1 = \ & f(t, x), \\
k_2 = \ & f\left(t + \frac{1}{2}\Delta t, x + \frac{\Delta t}{2} \times k_1\right), \\
k_3 = \ & f\left(t + \frac{1}{2}\Delta t, x + \frac{\Delta t}{2} \times k_2\right), \\
k_4 = \ & f(t + \Delta t, x + \Delta t \times k_3).
\end{aligned}$$

4. Methodology:



5. Literature Review:

We will examine the following paper to provide the necessary background and context for our research:
"Neural Ordinary Differential Equations" by Ricky T. Q. Chen et al. (2019).

6. Future goals

The outcomes of this project will be applied for further analysis in the context of the **MANE 6610 Nonlinear Control System** course instructed by **Dr. A. Agung Julius**. Specifically, we will employ Polyflow approximation to linearize the system, identify Koopman observables to obtain Koopman eigenfunctions and eigenvalues, and introduce control inputs to attain desired results. It is important to note that we have obtained permission from Dr. A. Agung Julius to use the results of this project for this purpose.

7. Conclusion

In conclusion, this project aims to leverage the power of Neural Ordinary Differential Equations (Neural ODE) to learn the unknown dynamics of a system from time series data. By implementing this approach, we anticipate gaining a deeper understanding of the underlying system dynamics, which can have broad implications for various fields of study, including control systems and machine learning. With the successful implementation of this project, we hope to contribute to the advancement of research in this area and lay the foundation for future work in the field of system identification and control.