Projection

Projection

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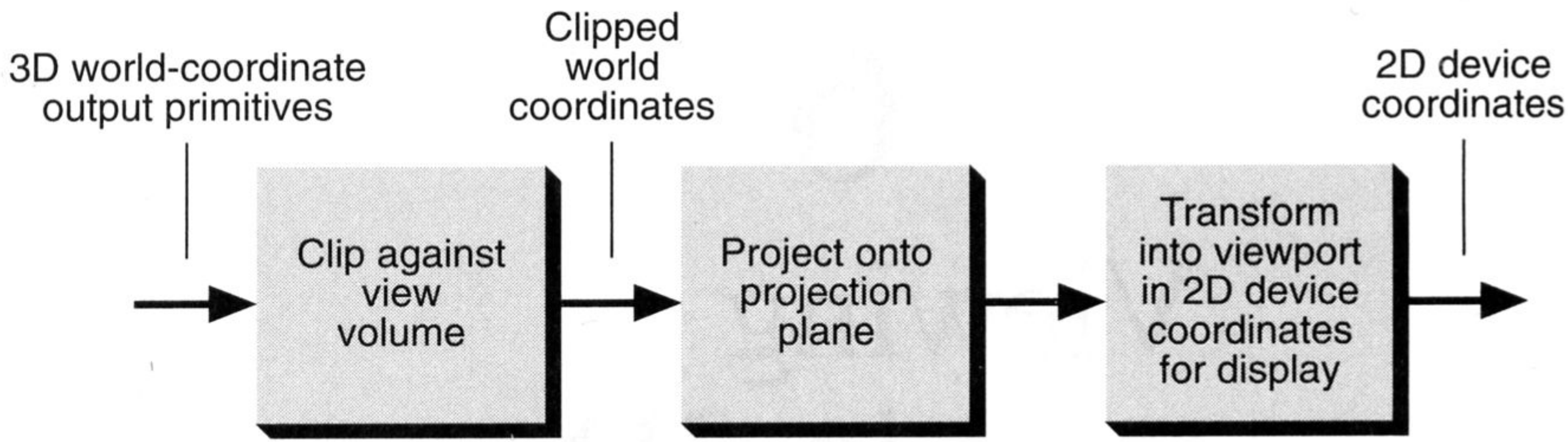
Conceptual model of 3D viewing process



Projection

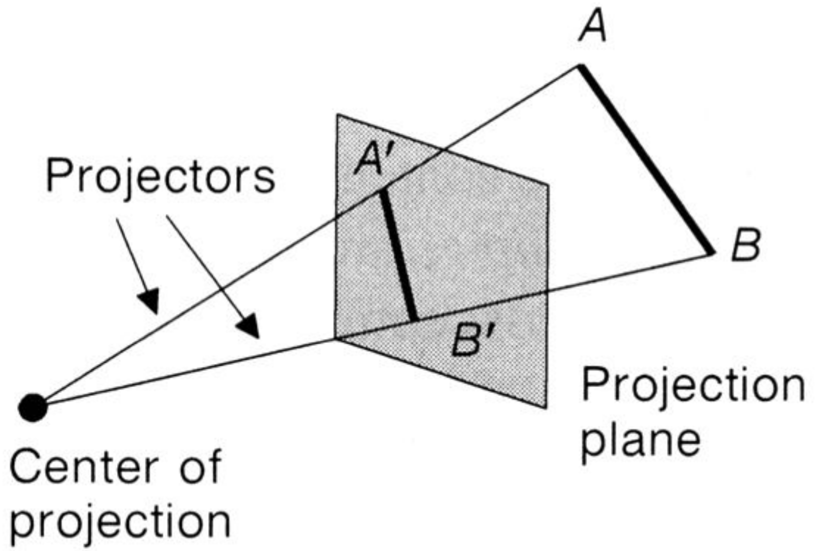
- In general, ***projections*** transform points in a coordinate system of dimension *n* into points in a coordinate system of dimension less than *n*.

- We shall limit ourselves to the projection from 3D to 2D.

- We will deal with ***planar geometric projections*** where:
  - The projection is onto a plane rather than a curved surface
  - The projectors are straight lines rather than curves

Projection

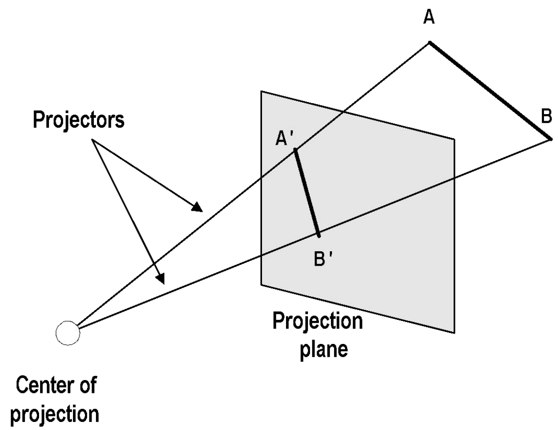
- key terms…
  - *Projection* from 3D to 2D is defined by straight *projection rays* (***projectors***) emanating from the '***center of projection***', passing through each point of the object, and intersecting the '***projection plane***' to form a projection.



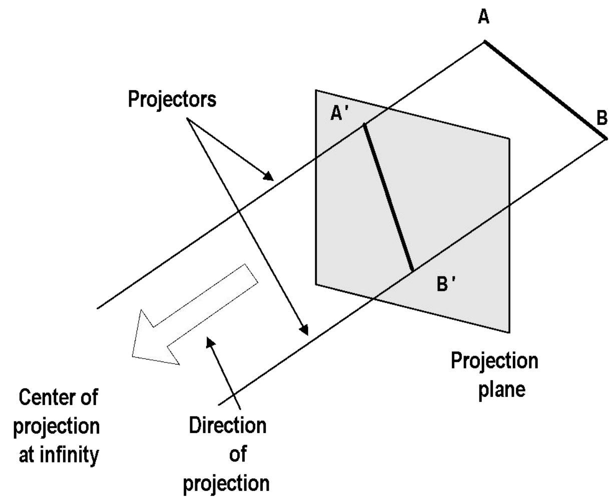
Planer Geometric Projection

- 2 types of projections
  - *perspective* and *parallel*.

- Key factor is the ***center of projection***.
  - if distance to center of projection is finite : perspective
  - if infinite : parallel



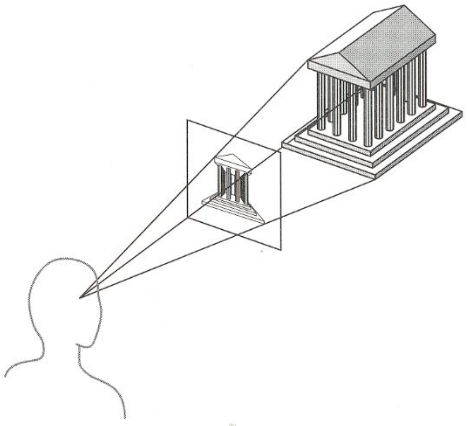
Perspective projection



Parallel projection

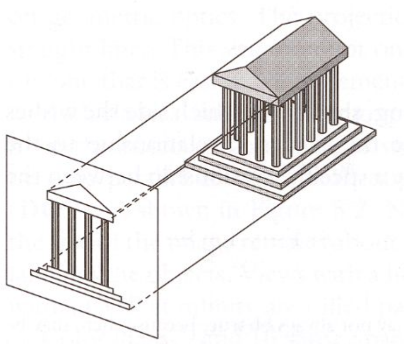
Perspective v Parallel

- Perspective:
  - visual effect is similar to human visual system...
  - has 'perspective foreshortening'
    - size of object varies inversely with distance from the center of projection.
  - Parallel lines do not in general project to parallel lines
  - angles only remain intact for faces parallel to projection plane.



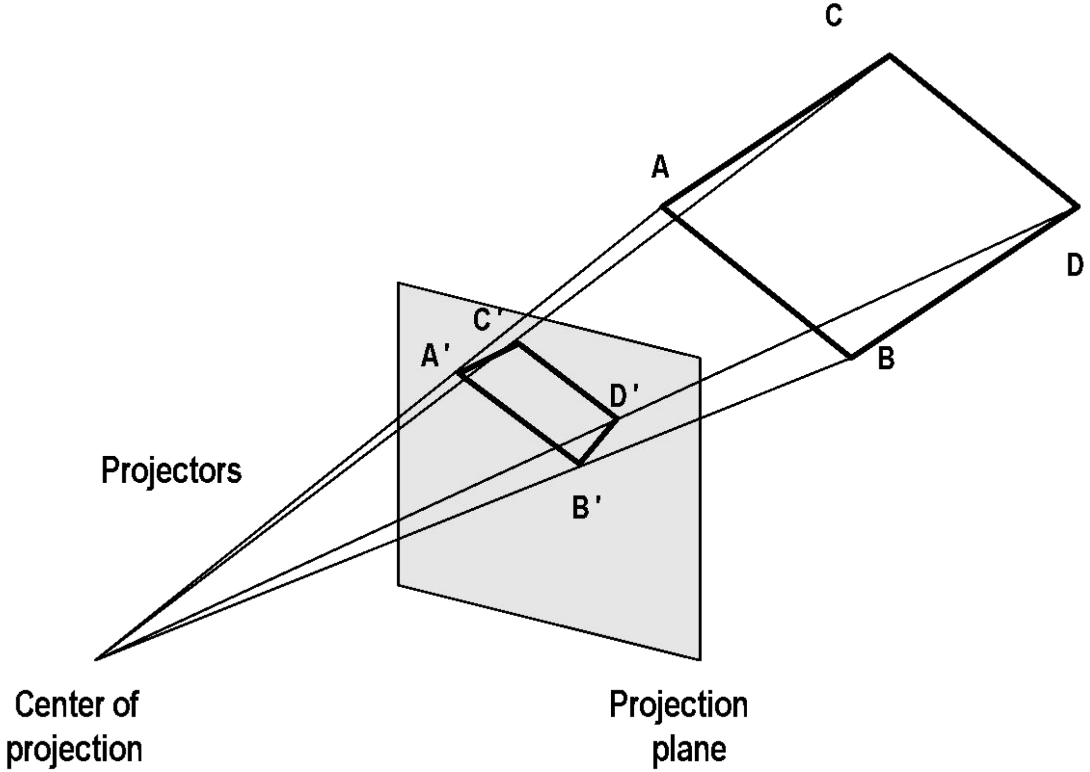
Perspective v Parallel

- Parallel:
  - less realistic view because of no foreshortening
  - however, parallel lines remain parallel.
  - angles only remain intact for faces parallel to projection plane.
  - Parallel projections are used by architects and engineers for **creating working drawing of the object**, for complete representations require two or more views of an object using different planes. Parallel Projection use to display picture in its true shape and size.



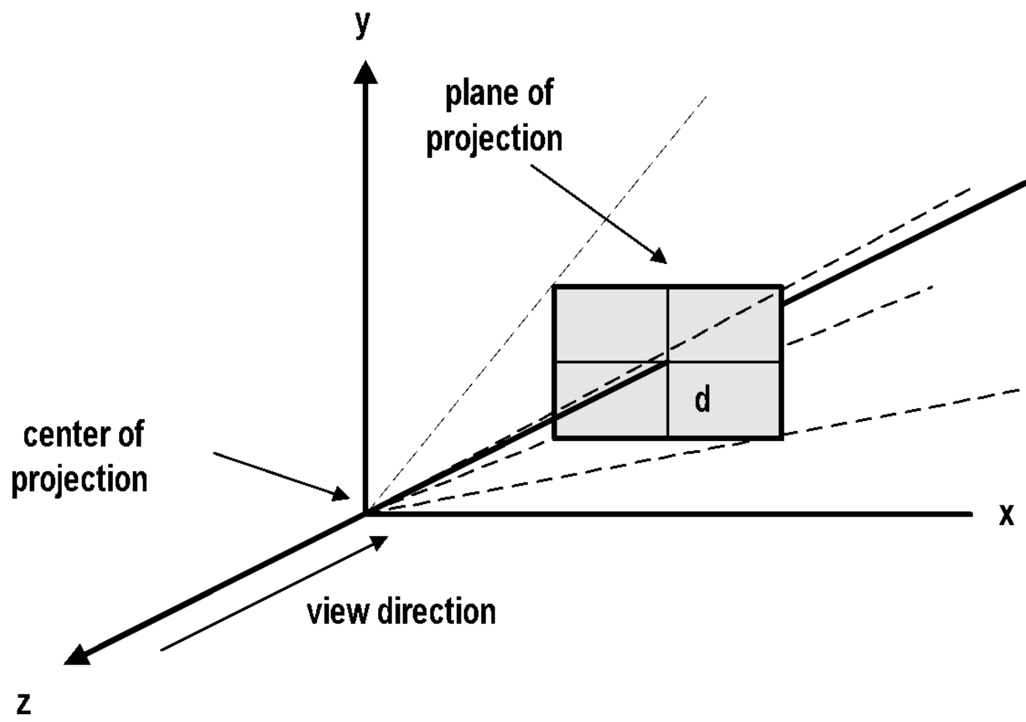
Perspective projection- anomalies

- ***Perspective foreshortening*** The farther an object is from COP the smaller it appears



Perspective foreshortening

Projective Transformation



Settings for perspective projection

Projective Transformation

$$\frac{y}{z} = \frac{y_p}{d} \Rightarrow y_p = \frac{y}{(z/d)}$$

$$z = d$$

$$(x, y, z, 1) \Rightarrow \left(\frac{x}{(z/d)}, \frac{y}{(z/d)}, d, 1 \right)$$

Projective Transformation

$$\begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \dfrac{x}{(z/d)} \\ \dfrac{y}{(z/d)} \\ d \\ 1 \end{pmatrix}$$

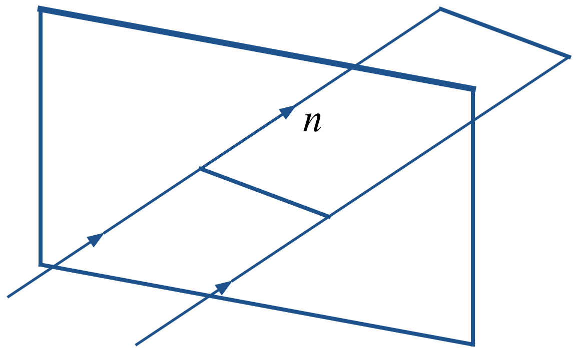
$$\begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ \dfrac{z}{d} \end{pmatrix}$$

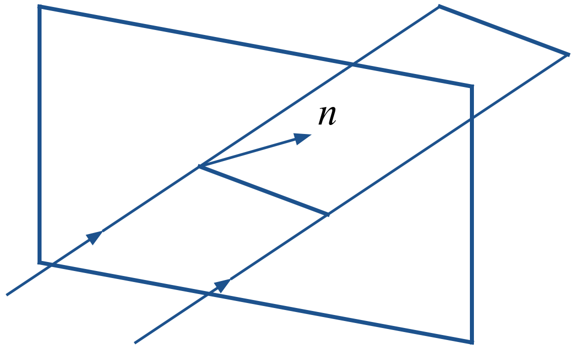
$$\begin{pmatrix} x \\ y \\ z \\ \dfrac{z}{d} \end{pmatrix} \xrightarrow[\textit{division}]{\textit{perspective}} \begin{pmatrix} \dfrac{x}{(z/d)} \\ \dfrac{y}{(z/d)} \\ d \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \dfrac{1}{d} & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ \dfrac{z}{d} \end{pmatrix}$$

Parallel projection

- 2 principle types:
  - *orthographic* and *oblique*.
- Orthographic :
  - direction of projection = normal to the projection plane.
- Oblique :
  - direction of projection != normal to the projection plane.

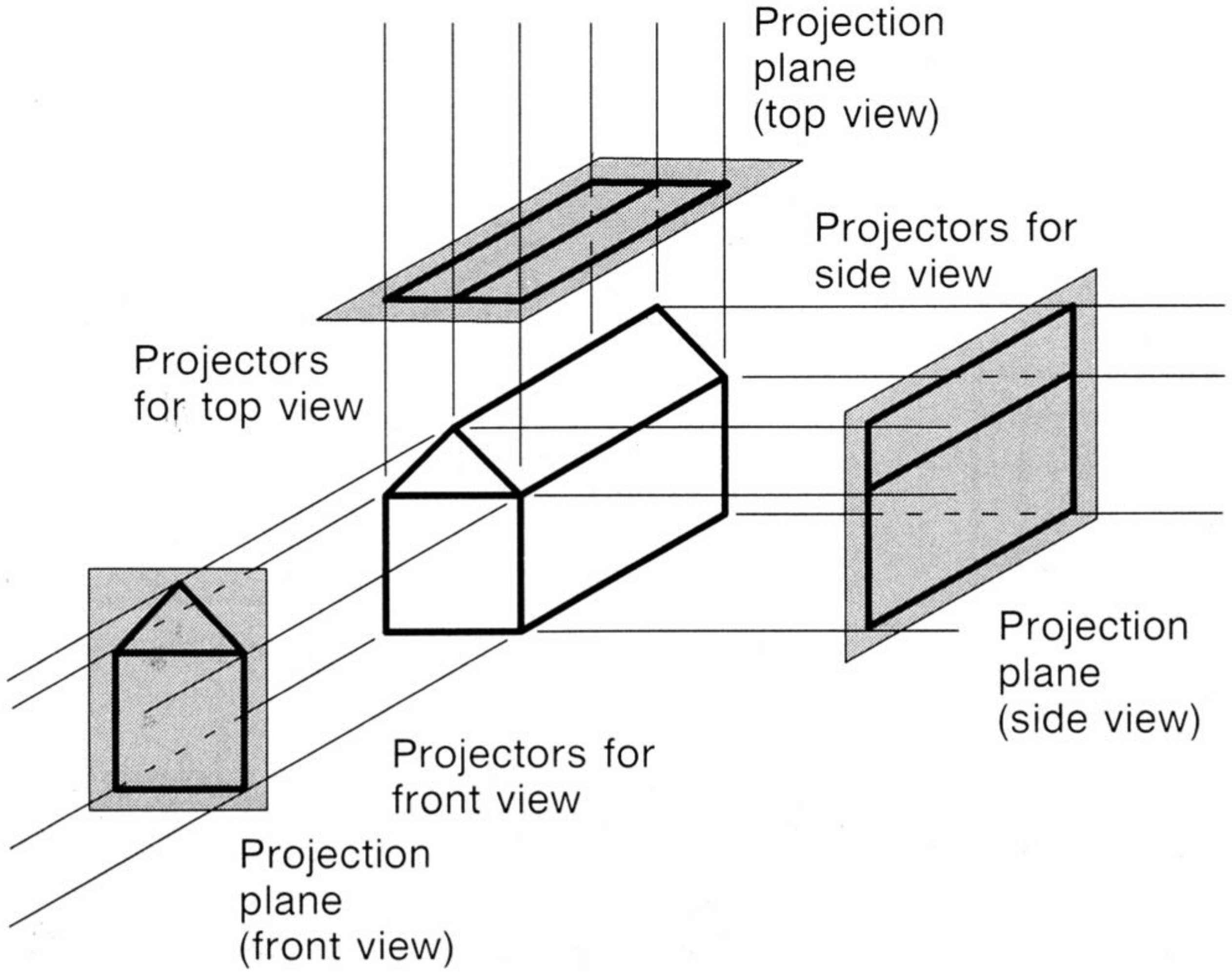




Orthographic projection

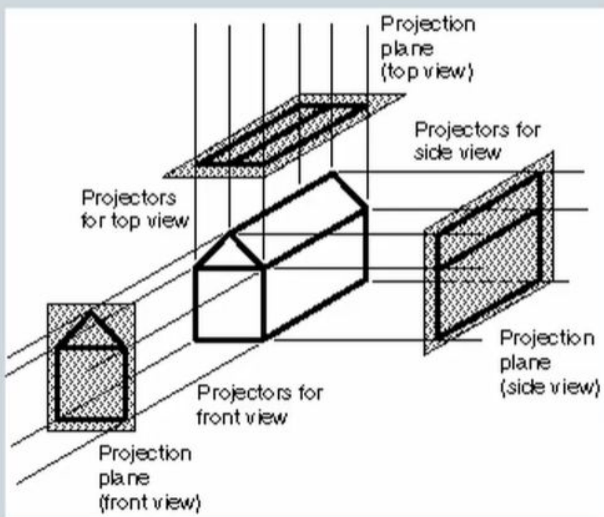
- Orthographic (or orthogonal) projections:
  - front elevation, top-elevation and side-elevation.
  - all have projection plane perpendicular to a principle axes.
- Useful because angle and distance measurements can be made...
- However, As only one face of an object is shown, it can be hard to create a mental image of the object, even when several view are available

Orthographic projection

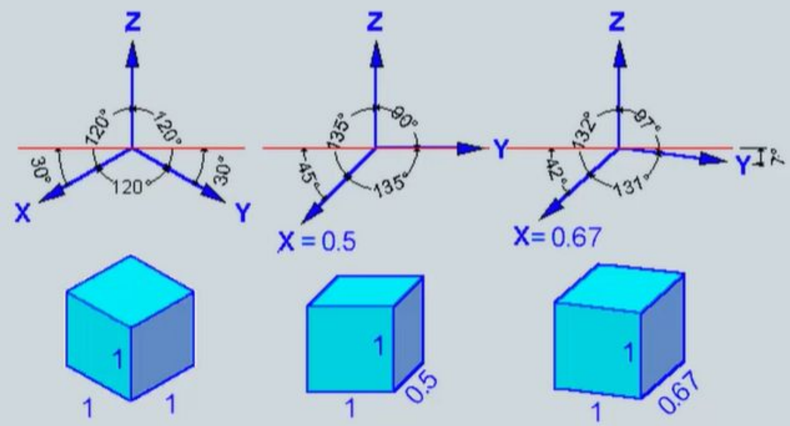


Orthographic projection

Types of Projections

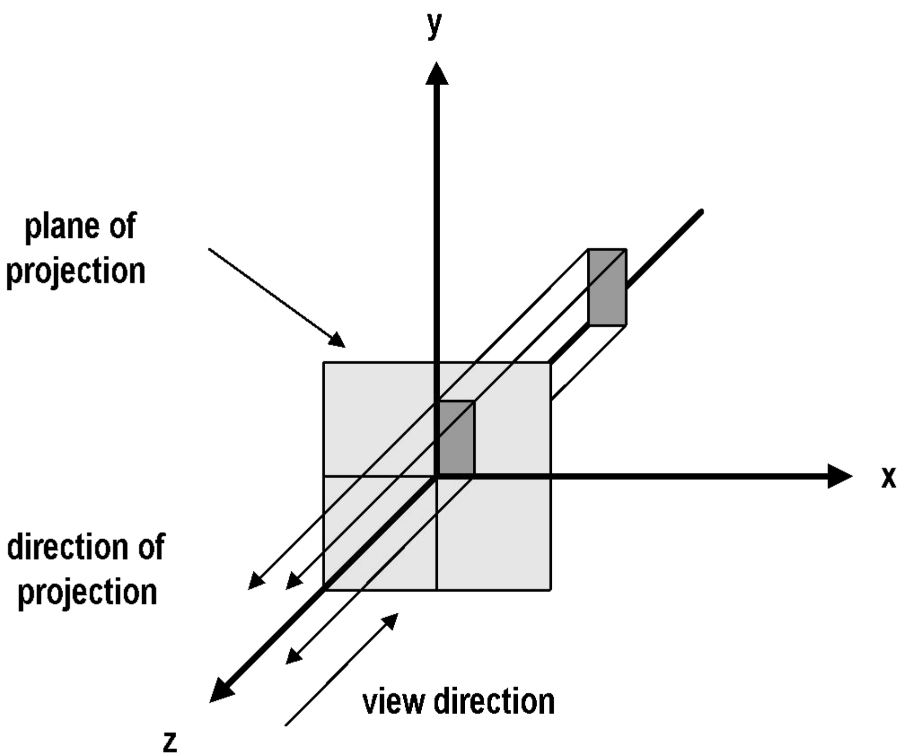


Orthographic Multiview Projection



Orthographic Axonometric Projections

Orthogonal Projection Matrix

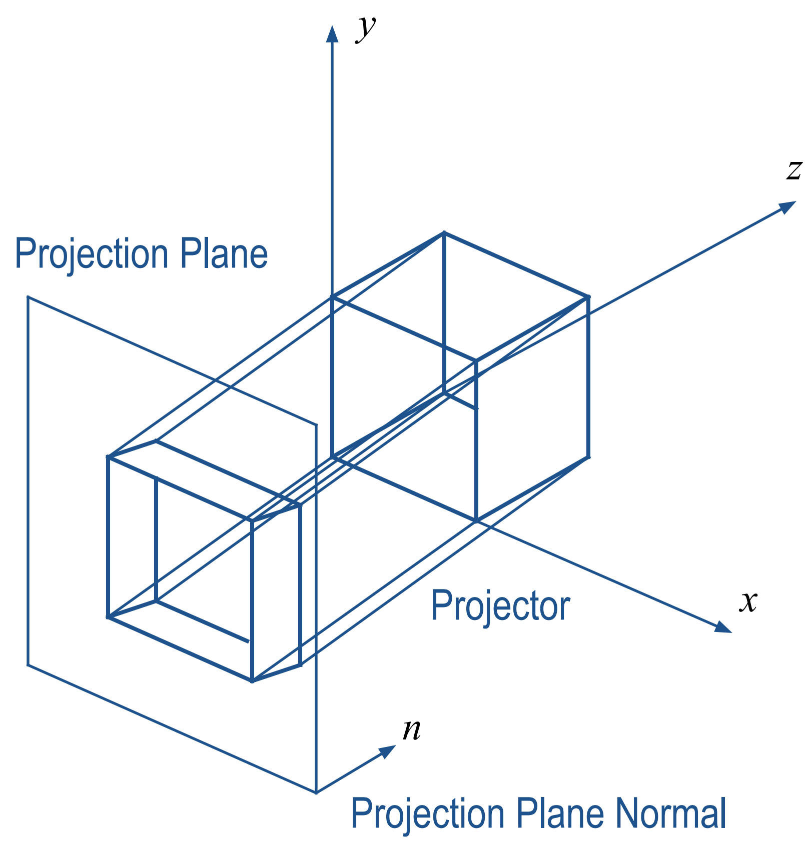


$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix}$$

Oblique parallel projection

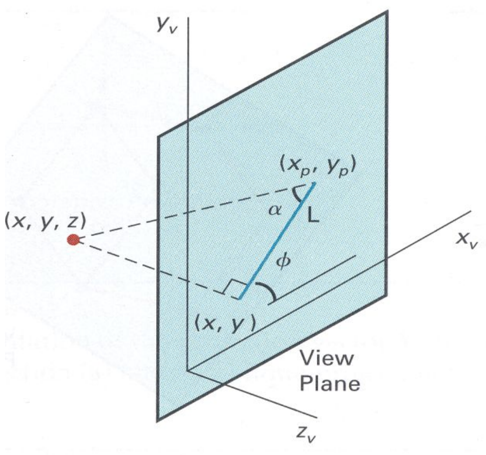
- Oblique parallel projections
  - Objects can be visualized better then with orthographic projections
  - Oblique projection is commonly used in technical drawing.
  - Can measure distances, but not angles
    - Can only measure angles for faces of objects parallel to the plane
- 2 common oblique parallel projections:
  - *Cavalier* and *Cabinet*

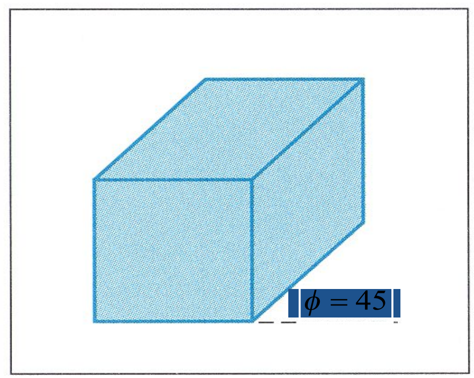
Oblique parallel projection



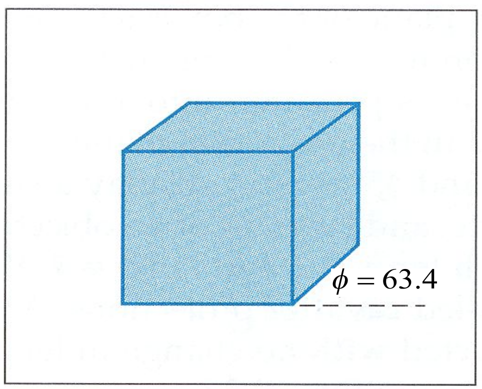
Oblique Projections

DOP not perpendicular to view plane





Cavalier
(DOP α = 45°)
tan(α) = 1



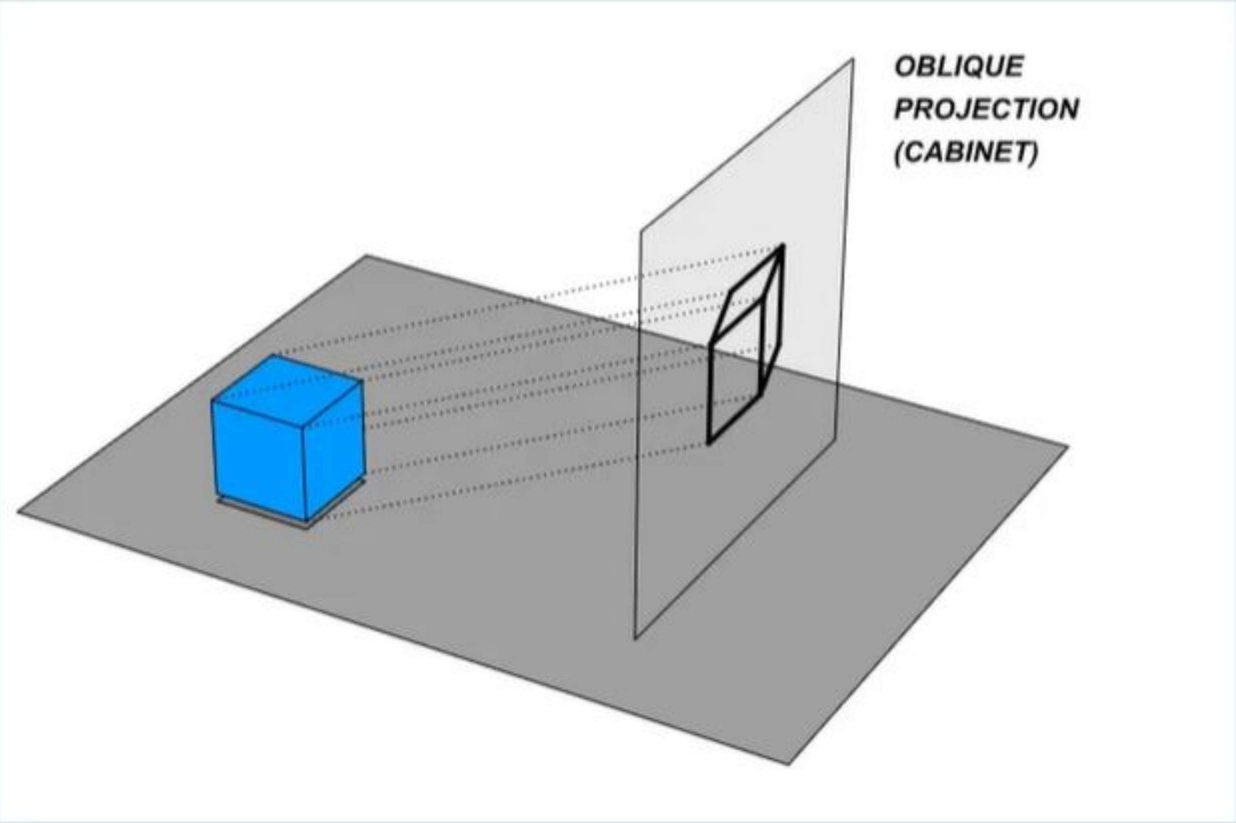
Cabinet
(DOP α = 63.4°)
tan(α) = 2

H&B

Oblique parallel projection

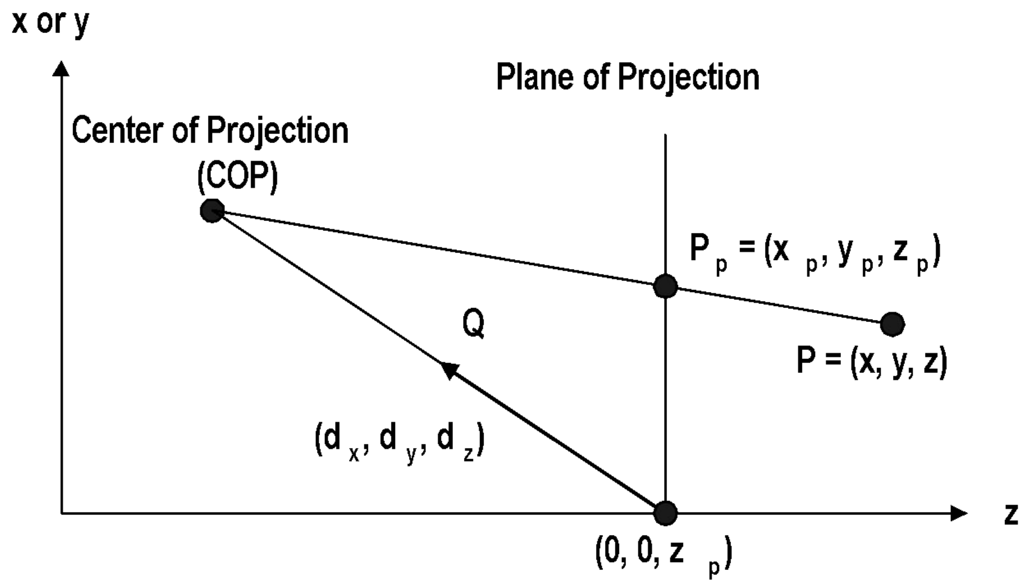
λ=1	α= 45	Cavalier projection	β= 0 - 360
λ=0.5	α = 63.4	Cabinet projection	β= 0 – 360
λ=0	α = 90	Orthogonal projection	β= 0 – 360

Oblique parallel projection



Oblique Projection (Cabinet)

Generalized Projection Matrix



$$P_p = COP + t(P - COP), \quad 0 \le t \le 1$$
$$COP = (0,0,z_p) + Q(d_x, d_y, d_z)$$

$$P' = (x', y', z')$$
$$x' = Qd_x + t(x - Qd_x)$$
$$y' = Qd_y + t(y - Qd_y)$$
$$z' = (z_p + Qd_z) + t(z - (z_p + Qd_z))$$

Generalized Projection Matrix

$$x_p = \frac{x - z\dfrac{d_x}{d_z} + z_p \dfrac{d_x}{d_z}}{\dfrac{z_p - z}{Qd_z} + 1}$$

$$z_p = \left(z_p + Qd_z\right) + t\left(z - \left(z_p + Qd_z\right)\right)$$

$$\Rightarrow t = \frac{z_p - \left(z_p + Qd_z\right)}{z - \left(z_p + Qd_z\right)}$$

$$y_p = \frac{y - z\dfrac{d_y}{d_z} + z_p \dfrac{d_y}{d_z}}{\dfrac{z_p - z}{Qd_z} + 1}$$

$$z_p = z_p \frac{\dfrac{z_p - z}{Qd_z} + 1}{\dfrac{z_p - z}{Qd_z} + 1} = \frac{-z\dfrac{z_p}{Qd_z} + \dfrac{z_p^2 + z_p Qd_z}{Qd_z}}{\dfrac{z_p - z}{Qd_z} + 1}$$

Generalized Projection Matrix

$$x_p = \frac{x - z\dfrac{d_x}{d_z} + z_p \dfrac{d_x}{d_z}}{\dfrac{z_p - z}{Qd_z} + 1}$$

$$y_p = \frac{y - z\dfrac{d_y}{d_z} + z_p \dfrac{d_y}{d_z}}{\dfrac{z_p - z}{Qd_z} + 1}$$

$$z_p = \frac{-z\dfrac{z_p}{Qd_z} + \dfrac{z_p^2}{Qd_z} + z_p}{\dfrac{z_p - z}{Qd_z} + 1}$$

$$M_{gen} = \begin{pmatrix} 1 & 0 & -\dfrac{d_x}{d_z} & z_p \dfrac{d_x}{d_z} \\ 0 & 1 & -\dfrac{d_y}{d_z} & z_p \dfrac{d_y}{d_z} \\ 0 & 0 & -\dfrac{z_p}{Qd_z} & \dfrac{z_p^2}{Qd_z} + z_p \\ 0 & 0 & -\dfrac{1}{Qd_z} & \dfrac{z_p}{Qd_z} + 1 \end{pmatrix}$$

Generalized Projection Matrix

$$M_{gen} = \begin{pmatrix} 1 & 0 & -\dfrac{d_x}{d_z} & z_p \dfrac{d_x}{d_z} \\ 0 & 1 & -\dfrac{d_y}{d_z} & z_p \dfrac{d_y}{d_z} \\ 0 & 0 & -\dfrac{z_p}{Qd_z} & \dfrac{z_p^2}{Qd_z} + z_p \\ 0 & 0 & -\dfrac{1}{Qd_z} & \dfrac{z_p}{Qd_z} + 1 \end{pmatrix}$$

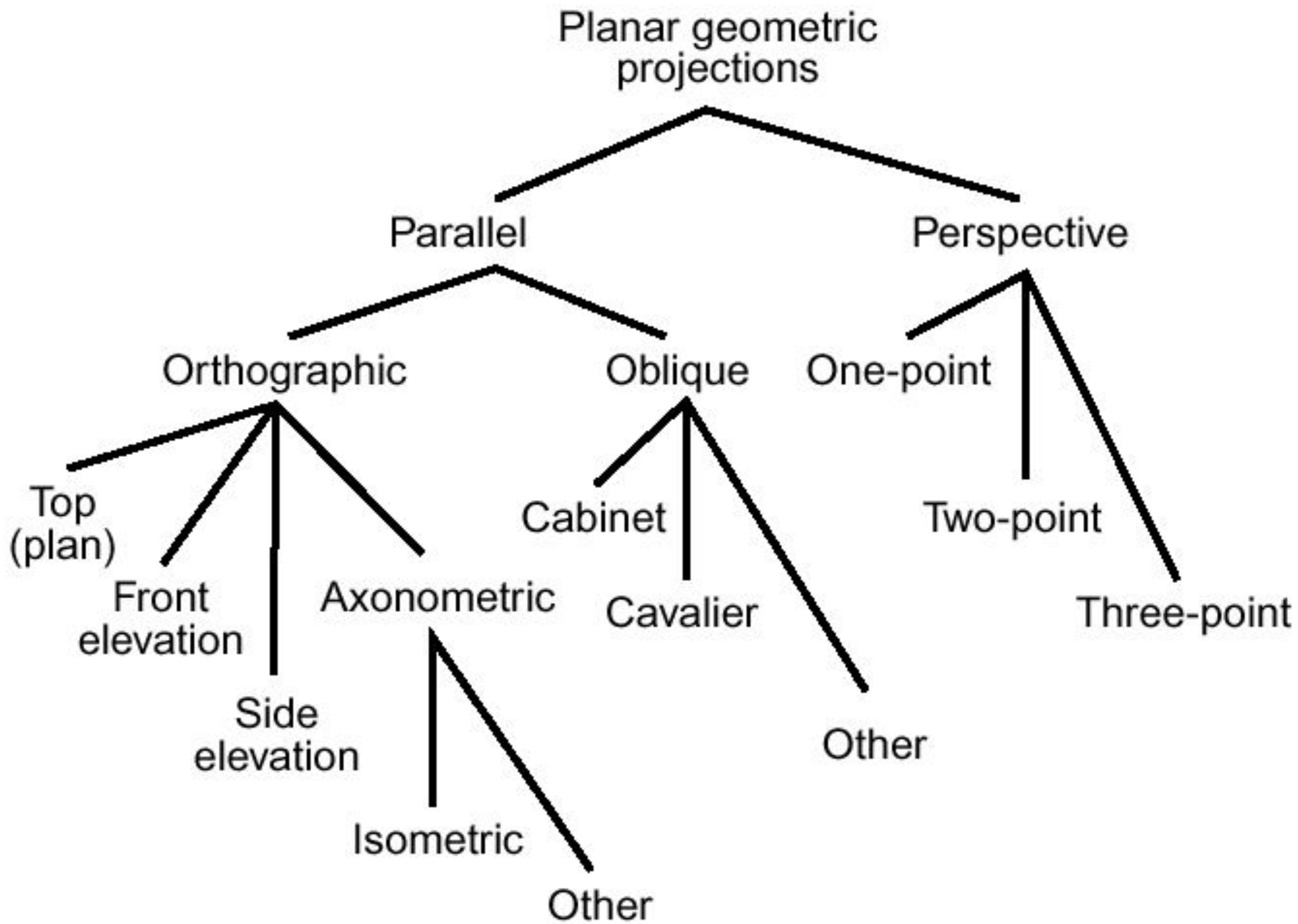
$$\xrightarrow[\;]{\substack{z_p = d,\, Q = d, \\ d_x = 0,\, d_y = 0,\, d_z = -1}} M_{per} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \dfrac{1}{d} & 0 \end{pmatrix}$$

Generalized Projection Matrix

$$M_{gen} = \begin{pmatrix} 1 & 0 & -\dfrac{d_x}{d_z} & z_p \dfrac{d_x}{d_z} \\ 0 & 1 & -\dfrac{d_y}{d_z} & z_p \dfrac{d_y}{d_z} \\ 0 & 0 & -\dfrac{z_p}{Qd_z} & \dfrac{z_p^2}{Qd_z} + z_p \\ 0 & 0 & -\dfrac{1}{Qd_z} & \dfrac{z_p}{Qd_z} + 1 \end{pmatrix}$$

$$\xrightarrow[\;]{\substack{z_p=0,\ Q=\infty, \\ d_x=0, d_y=0, d_z=1}} M_{par} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Taxonomy of Projection



Exercise

- A 3D point (10.5, -30.8, -750.0) is projected on a projection plane. Given that the center of projection plane is (0.0, 0.0, -500.0) and the coordinate of the COP is (-50.4, 40.3, 0.0). Determine the coordinate of that 3D point on the projection plane using general purpose projection matrix.