Neural Network
(Single Layer Perceptron) D/3 19/05/25

Healthcare Dataset (Dummy)

Age	BP	HasDisease
25	120	0
45	140	1
60	135	1
30	115	0
70	150	1

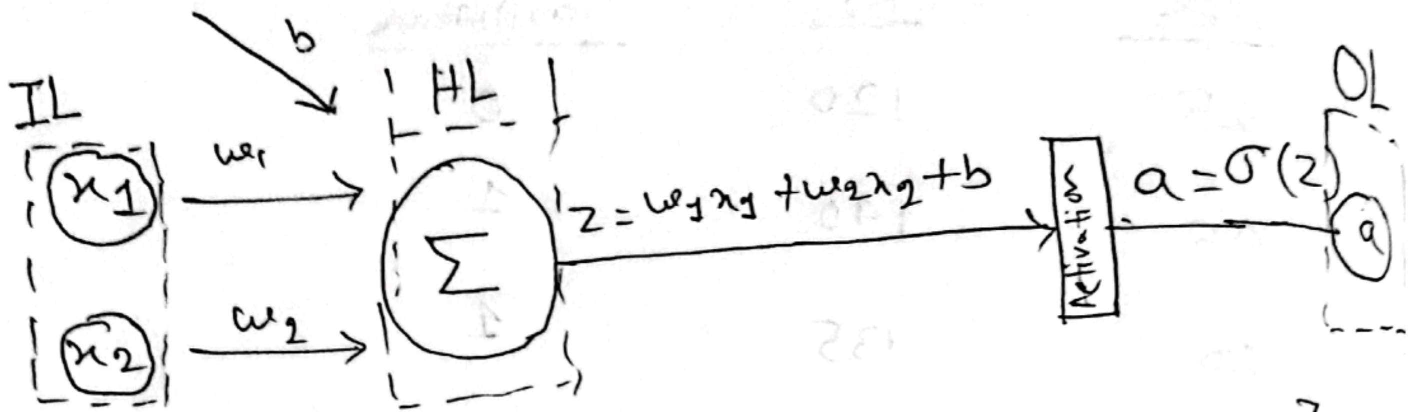
Question (a) Using the dataset simulate a back-propagation of a neural network, update w_1, w_2, b.

(b) Predict whether a patient with age = 65, and BP = 137 has disease or not.

Hint: You can use Single Neuron Perceptron.

(c) Validate whether backpropagation is optimizing weights and biases.

Single Neuron Perceptron



Here, x_1 = Age and x_2 = BP {Input features}

Binary cross entropy Loss function,

$$L = \frac{\sum_{i=1}^{n} -y_i \log a_i - (1-y_i)\log(1-a_i)}{n}$$

Sigmoid Function,

$$\sigma = \frac{1}{1+e^{-z}}$$

* IL = Input Layer
* HL = Hidden Layer
* OL = Output Layer
* ○ (round circles) are neurons.

Forward Propagation

(a)

For sample 1 ($x_1 = 25$, $x_2 = 120$)

Let, $w_1 = 0.025$, $w_2 = 0.035$, $b = 0.015$

$$z_1 = 0.025 \times \underbrace{25}_{x_1} + 0.035 \times \underbrace{120}_{x_2} + 0.015 = 4.84$$

$$a_1 = \sigma(z_1) = \frac{1}{1+e^{-4.84}} = 0.98$$

$$L_1 = -y_1 \log(a_1) - (1-y_1)\log(1-a_1)$$

$$= -0 \times \log(0.98) - (1-0)\log(1-0.98)$$

$$= 1.69$$

For sample 2 ($x_1 = 45$, $x_2 = 140$)

$$z_2 = 0.025 \times 45 + 0.035 \times 140 + 0.015 = 6.04$$

$$a_2 = \sigma(z_2) = \frac{1}{1+e^{-6.04}} = 0.99$$

$$L_2 = -y_2 \log(a_2) - (1-y_2)\log(1-a_2)$$

$$= -1 \times \log(0.99) - (1-1)\log(1-0.99)$$

$$= +0.004$$

For sample 3 $(x_1 = 60, \ x_2 = 135)$

$$z_3 = 0.025 \times 60 + 0.035 \times 135 + 0.015 = 6.24$$

$$a_3 = \frac{1}{1+e^{-6.24}} = 0.99$$

$$L_3 = -y_3 \log(a_3) - (1-y_3)\log(1-a_3)$$

$$= +0.004$$

For sample 4 $(x_1 = 30, \ x_2 = 115)$

$$z_4 = 0.025 \times 30 + 0.035 \times 115 + 0.015 = 4.79$$

$$L_4 = -y_4 \log\left(\frac{1}{1+e^{-4.79}}\right) - (1-y_4)\log\left(1 - \frac{1}{1+e^{-4.79}}\right)$$

$$= 0 \times \log(0.98) - (1-0)\log(1-0.98)$$

$$= 1.69$$

$$L = \frac{L_1 + L_2 + L_3 + L_4 \cancel{+L_5}}{\cancel{5}\,4} = \frac{1.69 + 0.004\cancel{4} + 0.004 + 1.69}{4}$$

$$= 0.84\cancel{7}$$

Back Propagation

We need to update w and b. We can use the following formula:

$$w = w - \alpha * \frac{dL}{dw}$$

$$b = b - \alpha * \frac{dL}{db}$$

$$\frac{dL}{dw} = \frac{\sum_{i=1}^{n}\left(\frac{1}{1+e^{-z_i}} - y_i\right) \times x_i}{n} = \frac{\sum_{i=1}^{n}(a_i - y_i)x_i}{n}$$

$$\frac{dL}{db} = \frac{\sum_{i=1}^{n}\left(\frac{1}{1+e^{-z_i}} - y_i\right)}{n} = \frac{\sum_{i=1}^{n}(a_i - y_i)}{n}$$

* $\frac{dL}{dw}$: Derivative of the loss L with respect to the weights.

* $\frac{dL}{db}$: Derivative of the loss L with respect to bias.

* In backpropagation, we optimize weights, and biases using Gradient Descent.

$$\frac{dL}{dw} = \frac{(0.98-0)\times 25 + (0.99-1)\times 45 + (0.99-1)\times 60 + (0.98-0)\,30}{4}$$

$$= 13.21$$

So, $w_1' = w_1 - \alpha \times 13.21$

$= 0.025 - 0.001 \times 13.21$

$= 0.011$

$$\frac{dL}{dw} = \frac{(0.98-0)120 + (0.99-1)140 + (0.99-1)135 + (0.98-0)\,115}{4}$$

$$= 56.88$$

So $w_2' = w_2 - \alpha \times 56.88$

$= 0.035 - 0.001 \times 56.88$

$= -0.021$

$$\frac{dL}{db} = \frac{(0.98-0) + (0.99-1) + (0.99-1) + (0.98-0)}{4} = 0.485$$

$b' = b - \alpha \times \frac{dL}{db} = 0.015 - 0.001 \times 0.485$

$= 0.0149$

Updated: $w_1 = 0.011$, $w_2 = -0.021$, $b' = 0.014$ (Ans)

(b)

$Z_t = 65 \times 0.011 + 137 \times (-0.021) + 0.014$

$= -2.148$

$a_t = \dfrac{1}{1+e^{2.148}} = 0.10$

Since, $a_t < 0.5$, so the model will predict 0 (no disease). (Ans)

শুদ্ধি পরীক্ষা: (ঙ)

Updated $w_1 = 0.011$, $w_2 = -0.021$, $b = 0.014$

For sample 1 $(x_1 = 25, x_2 = 120)$

$z_1 = 0.011 \times 25 + (-0.021) \times 120 + 0.014 = -2.231$

$a_1 = 0.09$

$L_1 = -0 \cdot \log(0.09) - (1-0)\log(1-0.09) = 0.04$

For sample 2 $(x_1 = 45, x_2 = 140)$

$z_2 = 0.011 \times 45 + (-0.021) \times 140 + 0.014 = -2.431$

$a_2 = 0.08$

$L_2 = -1 \times \log(0.08) - (1-1)\log(1-0.08) = 1.09$

Similarly, $L_3 = 1.09$, $L_4 = 0.04$

$$L = \frac{0.04 + 1.09 + 1.09 + 0.04}{4} = 0.565$$

Previous Loss $= 0.847 > 0.565$. Hence, the backpropagation is working. (Ans)