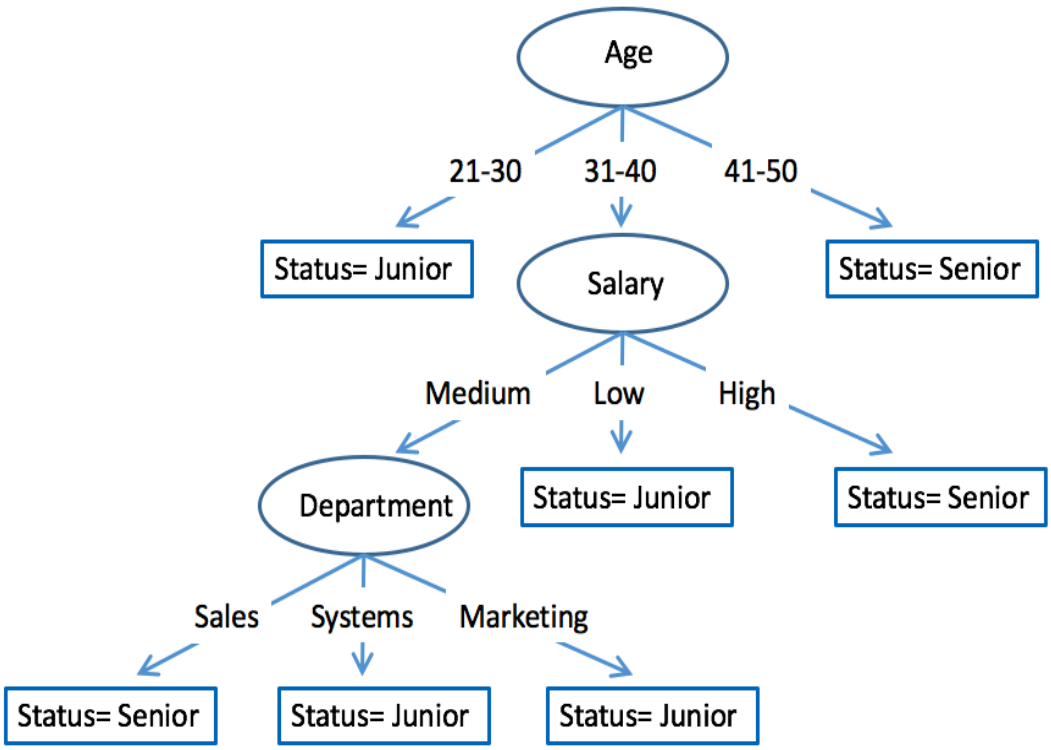
Decision Trees

Using data given in the table below as training data, answer the following questions:

1. Construct decision tree (no pruning) using Entropy.
2. Compute the following on training data:(i) individual class accuracy (ii) overall class accuracy
3. For the following test data, predict the class label for each instance using the tree

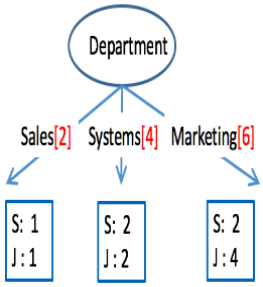
Department	Age	Salary	Status
Sales	31-40	Medium	Senior
Sales	31-40	Low	Junior
Systems	21-30	Medium	Junior
Systems	31-40	High	Senior
Systems	21-30	Medium	Junior
Systems	41-50	High	Senior
Marketing	31-40	Medium	Senior
Marketing	31-40	Medium	Junior
Marketing	41-50	High	Senior
Marketing	21-30	High	Junior
Marketing	31-40	Medium	Junior
Marketing	31-40	Medium	Junior

(a) The decision tree and the step wise procedure is as follows:



Step 1: Choosing the root node

Before splitting: $\rightarrow E(p) = -[(5/12)\log_2(5/12) + (7/12)\log_2(7/12)] = 0.9799$



$$E(D = \text{Sales}) = -[(1/2)\log_2(1/2) + (1/2)\log_2(1/2)] = 1.0000$$

$$E(D = \text{Syst.}) = -[(2/4)\log_2(2/4) + (2/4)\log_2(2/4)] = 1.0000$$

$$E(D = \text{Mark}) = -[(2/6)\log_2(2/6) + (4/6)\log_2(4/6)] = 0.9183$$

$$\rightarrow \bar{E} = (2/12)1 + (4/12)1 + (6/12)0.9183 = 0.9591$$

$$\rightarrow G(p, \text{Department}) = 0.9799 - 0.9591 = 0.0208$$

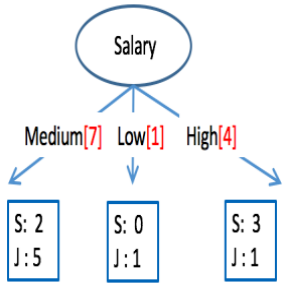
$$E(A = 31\_40) = -[(3/7)\log_2(3/7) + (4/7)\log_2(4/7)] = 0.9858$$

$$E(A = 21\_30) = -[(0/3)\log_2(0/3) + (3/3)\log_2(3/3)] = 0$$

$$E(A = 41\_50) = -[(2/2)\log_2(2/2) + (0/2)\log_2(0/2)] = 0$$

$$\rightarrow \bar{E} = (7/12)0.9858 + (3/12)0 + (2/12)0 = 0.5747$$

$$\rightarrow G(p, \text{Age}) = 0.9799 - 0.5747 = 0.4052$$



$$E(S = \text{Med}) = -[(2/7)\log_2(2/7) + (5/7)\log_2(5/7)] = 0.8631$$

$$E(S = \text{Low}) = -[(0/1)\log_2(0/1) + (1/1)\log_2(1/1)] = 0$$

$$E(S = \text{High}) = -[(3/4)\log_2(3/4) + (1/4)\log_2(1/4)] = 0.8113$$

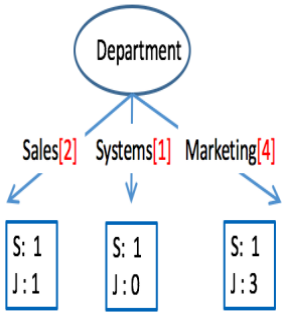
$$\rightarrow \bar{E} = (7/12)0.8631 + (1/12)0 + (4/12)0.8113 = 0.7739$$

$$\rightarrow G(p, \text{Salary}) = 0.9799 - 0.7739 = 0.2060$$

Winner → Age

Step 2: Choosing the next test attribute

$$\rightarrow E(A = 31-40) = 0.9858$$



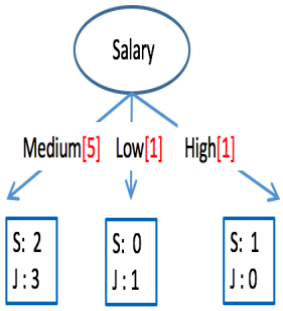
$$E(D = \text{Sales}) = -[(1/2)\log_2(1/2) + (1/2)\log_2(1/2)] = 1.0000$$

$$E(D = \text{Syst.}) = -[(1/1)\log_2(1/1) + (0/1)\log_2(0/1)] = 0$$

$$E(D = \text{Mark}) = -[(1/4)\log_2(1/4) + (3/4)\log_2(3/4)] = 0.8113$$

$$\rightarrow \bar{E} = (2/7)1 + (1/7)0 + (4/7)0.8113 = 0.7493$$

$$\rightarrow G(p, \text{Department}) = 0.9858 - 0.7493 = 0.2365$$



$$E(S = \text{Med}) = -[(2/5)\log_2(2/5) + (3/5)\log_2(3/5)] = 0.9710$$

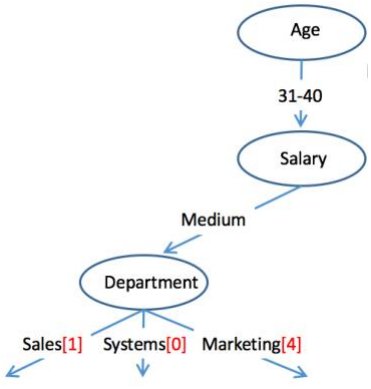
$$E(S = \text{Low}) = -[(0/1)\log_2(0/1) + (1/1)\log_2(1/1)] = 0$$

$$E(S = \text{High}) = -[(1/1)\log_2(1/1) + (0/1)\log_2(0/1)] = 0$$

$$\rightarrow \bar{E} = (5/7)0.9710 + (1/7)0 + (1/7)0 = 0.6935$$

$$\rightarrow G(p, \text{Salary}) = 0.9858 - 0.6935 = 0.2923$$

Winner → Salary



Now, for this branch, there appear to be 2 issues in the end which are handled as follows (as per the textbook):

1. There are no training examples associated with D=Systems, so it is declared as a leaf node with the same class as the majority class of the training examples associated with its Parent node. (S=2, J=3→ so we choose Junior).

2. Marketing node is still impure (S=1, J=3), so it is assigned the label of its majority class.

(b) Based on the decision tree constructed above, the following confusion matrix can be plotted:

	Predicted: Senior	**Predicted:** Junior
Actual: Senior	4 (TP)	1 (FN)
Actual: Junior	0 (FP)	7 (TN)

$\rightarrow$ Accuracy_Senior $= 4/5 = 0.8$ $\rightarrow$ Accuracy_Junior $= 7/7 = 1$ $\rightarrow$ Accuracy $= 11/12$

(c) Based on the decision tree constructed in (a):

Department	Age	Salary	Status
Sales	31-40	Low	Junior
Systems	31-40	Medium	Junior
Marketing	31-40	High	Senior
Marketing	21-30	Low	Junior