MAT120

Assignment-01 Set-12

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Section : 03

Question 1:

Evaluate the following integral by interpreting it as area, or otherwise:

$$GivenEquation: \int_{-1}^{5} |x-3|$$

Now
x-3=0
so, x=3

$$= \int_{-1}^{3} |x-3| + \int_{3}^{5} |x-3|$$

$$= (-\int_{-1}^{3} xdx + \int_{-1}^{3} 3dx) + (\int_{3}^{5} xdx - \int_{3}^{5} 3dx)$$

$$= (-[\frac{x^2}{2}]_{-1}^{3} + [3x]_{-1}^{3}) + ([\frac{x^2}{2}]_{3}^{5} - [3x]_{3}^{5})$$

$$= (-4+12) + (8-6)$$

$$= 8+2$$

$$= 10$$

Question 2:

Evaluate the following indefinite integral with a substitution, or otherwise:

$$GivenEquation: \int \frac{x}{\sqrt{4-x^2}} dx$$

where $4 \geq x^2$Now:
Let

$$u = 4 - x^2$$

$$du = 0 - 2xdx$$

$$So, du = -2xdx$$

$$xdx = -\frac{du}{2}$$

Now:

$$\int \frac{1}{\sqrt{4-x^2}} xdx$$

$$= -\int \frac{1}{\sqrt{u}} \frac{du}{2}$$

$$= -\frac{1}{2} \int \frac{1}{\sqrt{u}} du$$

$$= -\frac{1}{2} \int u^{\frac{-1}{2}} du$$

$$= -\frac{1}{2}2u^{\frac{1}{2}} + c$$

$$= -u^{\frac{1}{2}} + c$$

$$= -(4 - x^2)^{\frac{1}{2}} + c$$

<u>Question 3(a):</u>

If I$_n = \int x^{n-1}e^x dx$

$$Provethat : I_n = x^{n-1}e^x - (n-1)I_n - 1$$

$$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

$$u = x^{n-1}$$

$$\frac{du}{dx} = (n-1)x^{n-2}$$

$$\frac{dv}{dx} = e^x$$

$$v = e^x$$

$$Now, I_n = \int x^{n-1}e^x dx$$

$$= x^{n-1}e^x - \int e^x(n-1)x^n$$

$$= x^{n-1}e^x - (n-1)\int x^{n-2}e^x dx$$

$$= x^{n-1}e^x - (n-1)I_n - 1$$

Proved

Question 3(b):

$$I_n = \int x^{n-1} e^x dx$$

$$I_n = x^{n-1} \int e^x dx - \int \frac{d}{dx}(x^n) \int e^x dx$$

$$I_n = x^{n-1} e^x dx - \int (n-1) x^{n-2} e^x dx$$

$$I_n = x^{n-1} e^x dx - (n-1) \int e^x x^{n-2} dx$$

$$I_3 = x^2 e^x dx - 2 \int e^x x dx$$

$$= x^2 e^x dx - 2xe^x + 2 \int e^x dx$$

$$= x^2 e^x dx - 2xe^x + 2e^x$$

Question 4:

Evaluate the following integral by decomposing it into partial fractions, or otherwise:

$$GivenEquation : \int \frac{2x+7}{x^2 - 16x + 55}$$

Now:

$$\frac{2x+7}{x^2 - 16x + 55}$$

$$\frac{2x+7}{x^2 - 5x - 11x + 55}$$

$$\frac{2x+7}{x(x-5) - 11(x-5)}$$

$$\frac{2x+7}{(x-5)(x-11)}$$

so:

$$\frac{2x+7}{(x-5)(x-11)} = \frac{A}{x-5} + \frac{B}{x-11}$$

$$= \frac{A(x-11)+B(x-5)}{x^2-16x+55}$$

Now:

$$A(x-11)+B(x-5) = 2x+7$$

Let x=11,

$$A(0)+B(6) = 22+7$$

$$6B = 29$$

$$B = \frac{29}{6}$$

Again Let x=5,

$$A(-6)+B(0) = 10+7$$

$$-6A = 17$$

$$A = -\frac{17}{6}$$

$$\int \frac{2x+7}{x^2-16x+55}dx = \int \frac{\frac{-17}{6}}{x-5}dx + \int \frac{\frac{29}{6}}{x-11}dx$$

$$= \frac{-17}{6}\int \frac{1}{x-5}dx + \frac{29}{6}\int \frac{1}{x-11}dx$$

$$= \frac{-17}{6}ln|x-5| + \frac{29}{6}ln|x-11| + c$$

Question 5:

Evaluate the following improper integral with the help of Gamma functions,or otherwise:

$$GivenEquation: \int_0^\infty (3t)^7 e^{-(3t)} dt$$

Now

$$let, 3t = x$$

$$t = \frac{x}{3}$$

$$dt = \frac{1}{3}dx$$

So,

$$\int_0^\infty (x)^7 e^{-x}\frac{1}{3}dx$$

$$\frac{1}{3}\int_0^\infty (x)^7 e^{-x} dx$$

Now

$$\Gamma\left(n\right)=\int\limits_0^\infty x^{n-1}e^{-x}dx$$

$$n-1=7so,$$

n=8

Now :

$$\frac{1}{3}\Gamma\left(8\right)$$

$$=\frac{1}{3}(8-1)!$$

$$=\frac{1}{3}7!$$

=1680