

# Acknowledgement

All thanks to almighty Allah for granting me this opportunity to help the ones in need. And, to my instructor Sanjeeda Nazneen (SAN) whose efforts have perfected me in knowledge and also my special without whom this would have been impossible.

Note: Use this unofficial document at your own risk. There might be some errors and wrong answers as well, since I am not that good at Mathematics so please do consider that. I wasn't able to include few topics just because my final examination was knocking at the door.

Domain and Range

$$f(x) = \frac{1}{x-3}$$

$$11 \quad f(x) = \frac{1}{x-3}$$

$$x-3 \neq 0$$

$$y = \frac{1}{x-3}$$

$$\Rightarrow xy - 3y = 1 \quad (x \neq 0)$$

$$\Rightarrow xy = 1 + 3y$$

$$\Rightarrow x = \frac{1+3y}{y}$$

$$\therefore y \neq 0$$

$$\text{Domain: } \mathbb{R} - \{3\}$$

$$\text{Range: } \mathbb{R} - \{0\}$$

$$(0, \infty) \cup (-\infty, 0) : \text{Domain}$$

$$11 \quad f(x) = \frac{2x}{x-4}$$

$$x-4 \neq 0$$

$$y = \frac{2x}{x-4}$$

$$\Rightarrow xy - 4y = 2x$$

$$\Rightarrow x = \frac{2x+4y}{y}$$

$$\Rightarrow xy - 2x = 4y$$

$$\Rightarrow x(y-2) = 4y$$

$$\Rightarrow x = \frac{4y}{y-2} \quad (y \neq 0)$$

$$\therefore y \neq 2$$

$$12 \quad f(x) = \frac{1}{5x+7}$$

$$5x+7 \neq 0$$

$$y = \frac{1}{5x+7}$$

$$\Rightarrow 5xy + 7y = 1$$

$$\Rightarrow 5xy = 1 - 7y$$

$$\Rightarrow x = \frac{1-7y}{5y}$$

$$\therefore y \neq 0$$

$$\text{Domain: } \mathbb{R} - \{-7/5\}$$

$$\text{Range: } \mathbb{R} - \{0\}$$

$$2] f(x) = \sqrt{x^2 - 9}$$

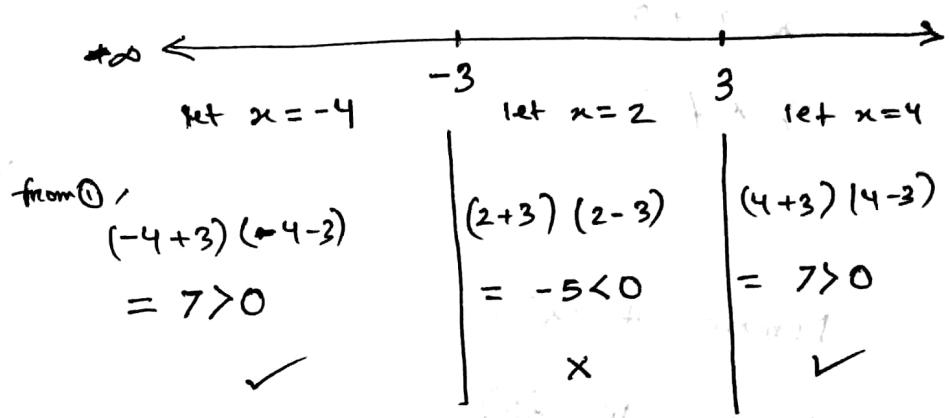
$$x^2 - 9 > 0$$

$$\Rightarrow x^2 - 3^2 > 0$$

$$\Rightarrow (x+3)(x-3) > 0 \quad \text{--- ①}$$

$$(x+3)(x-3) = 0$$

$$\Rightarrow x = -3, 3$$



$$\text{Domain: } (-\infty, -3] \cup [3, \infty)$$

$$\text{Range: } [0, \infty)$$

To find Range!

|     |           |      |     |          |
|-----|-----------|------|-----|----------|
| $x$ | $-\infty$ | $-3$ | $3$ | $\infty$ |
| $y$ | $\infty$  | 0    | 0   | $\infty$ |

$$3] f(x) = \sqrt{9-x^2}$$

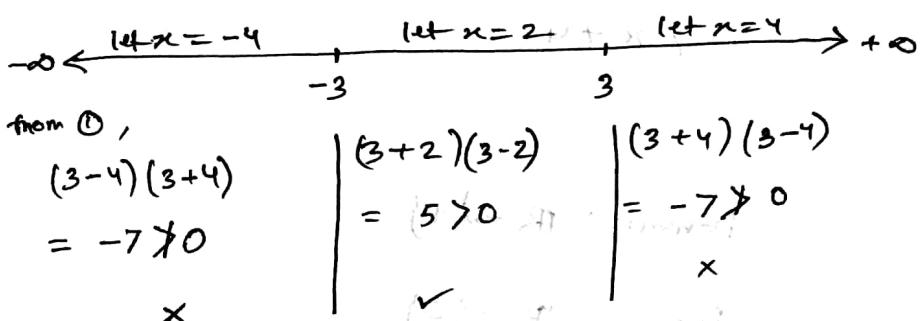
$$9-x^2 > 0$$

$$\Rightarrow 3^2 - x^2 > 0$$

$$\Rightarrow (3+x)(3-x) > 0 \quad \text{--- ①}$$

$$(3+x)(3-x) = 0$$

$$\Rightarrow x = -3, 3$$



$$\text{Domain: } [-3, 3]$$

$$\text{Range: } [0, 3]$$

To find Range!

|     |           |      |     |          |
|-----|-----------|------|-----|----------|
| $x$ | $-\infty$ | $-3$ | $3$ | $\infty$ |
| $y$ | 0         | 0    | 0   | 3        |

$$4) f(x) = \sqrt{x^2 - 5x + 6}$$

$$\text{com } x^2 - 5x + 6 \geq 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 \geq 0$$

$$\Rightarrow x(x-3) - 2(x-3) \geq 0$$

$$\Rightarrow (x-2)(x-3) \geq 0 \quad \text{--- ①}$$

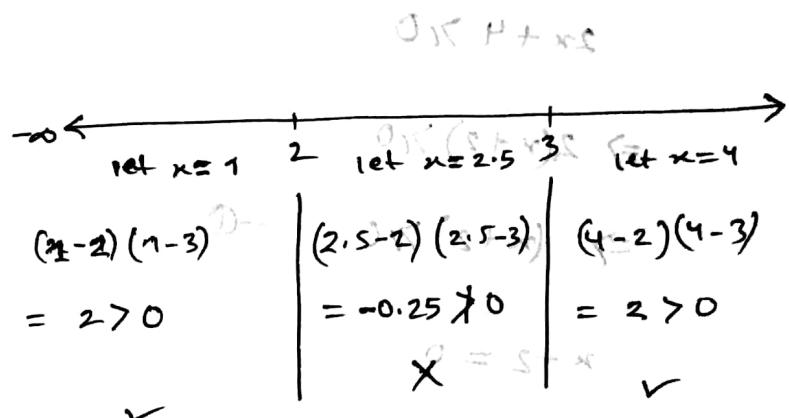
$$(x-2)(x-3) = 0$$

$$\Rightarrow x = 2, 3$$

Domain:  $(-\infty, 2] \cup [3, \infty)$

Range:  $[0, \infty)$

$$f(x) = (x)^+$$



to find range:

$$\begin{array}{c|c|c|c|c} x & -\infty & 2 & 3 & \infty \\ \hline y & \infty & 0 & 0 & \infty \end{array}$$

$$10) f(x) = -\sqrt{x^2 - 7x + 10}$$

$$0 < x \quad x^2 - 7x + 10 \geq 0$$

$$\Rightarrow x^2 - 2x - 5x + 10 \geq 0$$

$$\Rightarrow x(x-2) - 5(x-2) \geq 0$$

$$\Rightarrow (x-2)(x-5) \geq 0 \quad \text{--- ①}$$

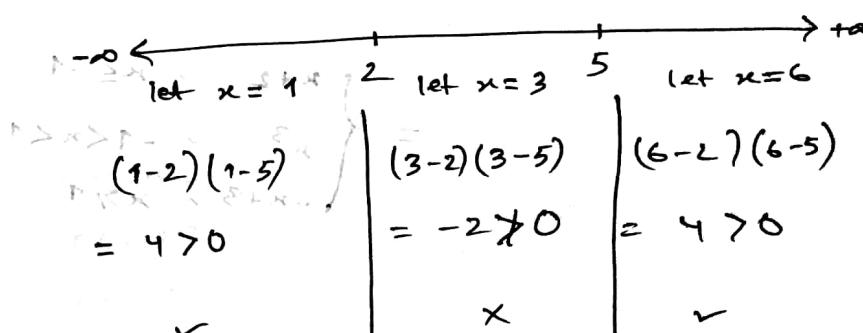
$$(x-2)(x-5) = 0$$

$$\Rightarrow x = 2, 5$$

Domain:  $(-\infty, 2] \cup [5, \infty)$

Range:  $(-\infty, 0]$

$$\begin{array}{c|c|c|c|c} x & -\infty & 2 & 3 & \infty \\ \hline y & \infty & 0 & 0 & \infty \end{array}$$

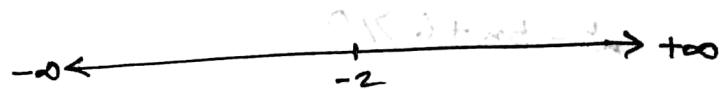


to find range:

$$\begin{array}{c|c|c|c|c} x & -\infty & 2 & 3 & \infty \\ \hline y & \infty & 0 & 0 & -\infty \end{array}$$

$$19) f(x) = \sqrt{2x+4}$$

$$2x+4 > 0$$



$$\Rightarrow 2(x+2) > 0$$

$$(x+2) > 0$$

$$\Rightarrow x = -2$$

$$\begin{aligned} \text{let } x = -3 & \quad (-3+2) \\ &= -1 > 0 \quad \checkmark \\ \text{let } x = 3 & \quad (3+2) \\ &= 5 > 0 \quad \checkmark \end{aligned}$$

to find range:  $(-\infty, \infty)$

Domain:  $[-2, \infty)$

$$\frac{x+2}{\sqrt{}} \rightarrow x+2 \geq 0 \quad \checkmark$$

Range:  $[0, \infty)$

$(\infty, \infty] \cup [5, \infty)$  : rational

$(\infty, 0]$  : irrationals

$$7) f(x) = \begin{cases} x+2 & , x \leq -1 \\ x^3 & , -1 < x < 1 \\ -x+3 & , x \geq 1 \end{cases}$$

$$f(x) = (\infty, \infty)$$

$$\begin{aligned} & (x+2)(x-1) \\ & (x^3)(x-1) \\ & (-x+3)(x-1) \end{aligned}$$

Domain:  $(-\infty, -1] \cup (-1, 1) \cup [1, \infty)$

from above

Range:  $(-\infty, 1] \cup (-1, 1) \cup [1, \infty)$

$$= (-\infty, 2]$$

$(\infty, 2] \cup [5, \infty)$  : rational

$(-\infty, 2)$  : irrationals

$$\underline{8} \quad f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 2, & x=1 \end{cases}$$

Domain:  $\mathbb{R} - \{1\} \cup \{1\}$

$\{0\} = \mathbb{R}$  : closed

$$\frac{x}{|x|} = (x) + \boxed{2}$$

$$\left\{ \begin{array}{l} 0 < x < \infty \\ 2 > x > -2 \end{array} \right\} = \text{Dom}$$

$$0 < x < \infty \Rightarrow \frac{x}{x-1} = \frac{x}{x} = (x) +$$

Range:  $\mathbb{R}$

$\{x \in \mathbb{R} : x \neq 2\}$

$$\underline{14} \quad f(x) = \begin{cases} x^2, & x < 0 \\ x, & 0 \leq x \leq 1 \\ 1/x, & x > 1 \end{cases}$$

$$x^2 + x = (x) + \boxed{1}$$

Domain:  $(-\infty, 0) \cup [0, 1] \cup (1, \infty)$

All rational

All : open

$= \mathbb{R}$

Range:  $(0, \infty) \cup [0, 1] \cup (1, 0)$

$$[\because 1/0 = \infty] \quad \text{amide} = (x) + \boxed{1}$$

$= [0, \infty)$

All : closed

$$\underline{15} \quad f(x) = \begin{cases} 2x+6, & -3 \leq x \leq 0 \\ 6, & 0 < x \leq 2 \\ 2x-6, & 2 \leq x \leq 5 \end{cases}$$

$$[x, x+2] : \text{closed}$$

Domain:  $[-3, 0] \cup (0, 2) \cup [2, 5]$

All : closed

$$\left\{ \begin{array}{l} [-3, 0] \\ (0, 2) \\ [2, 5] \end{array} \right\} = [-3, 5]$$

$$0 < x+2 < 2 \Rightarrow -3 < x < 0$$

Range:  $[0, 6] \cup \{6\} \cup [-2, 4]$

$= [-2, 6]$

$$5] f(x) = \frac{x}{|x|}$$

$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

$$|x| \neq 0$$

$$\Rightarrow x \neq 0$$

$$f(x) = \frac{x}{|x|} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$

Domain:  $\mathbb{R} - \{0\}$

Range:  $\{-1, 1\}$

$$6] f(x) = x^3 + 2$$

Domain:  $\mathbb{R}$

Range:  $\mathbb{R}$

$(-\infty, 0) \cup [1, \infty) \cup (-\infty, 1)$  Domain

$$7] f(x) = 3 \sin x$$

Domain:  $\mathbb{R}$

Range:  $[-3, 3]$

$$8] f(x) = \ln(x^2 + 1)$$

$$x^2 + 1 > 0$$

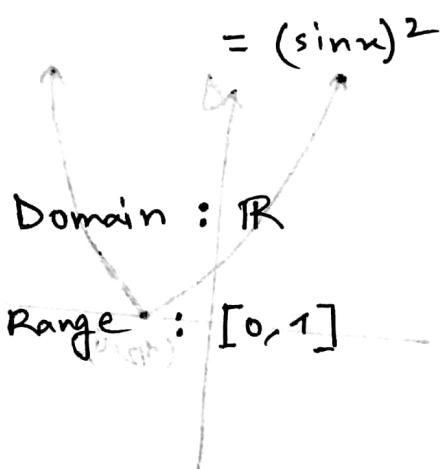
for all  $x \in \mathbb{R}, x^2 + 1 > 0$

Domain:  $\mathbb{R}$

Range:  $[0, \infty)$

| x | $-\infty$    | -1              | 0       | 1               | $\infty$     |
|---|--------------|-----------------|---------|-----------------|--------------|
| y | $\ln \infty$ | $\ln 2$         | $\ln 1$ | $\ln 2$         | $\ln \infty$ |
|   | $= \infty$   | $= \text{true}$ | $= 0$   | $= \text{true}$ | $= \infty$   |

16  $f(x) = \sin^2 x$



defn

$$\varepsilon^{-r}(t-\infty) = \beta$$

$$0 = \varepsilon^{-r}(t-\infty) + \beta = \beta$$

$$0 = t - \infty \Leftrightarrow$$

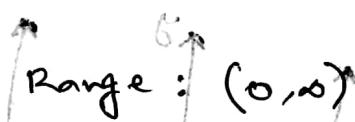
$$t = \infty \Leftrightarrow$$

$$s/t = \infty \Leftrightarrow$$

$(0, 1)$ :  $x$ -axis

17  $f(x) = e^x$

Domain:  $\mathbb{R}$



$$\varepsilon^{-r}(t-\infty) = \beta$$

$$\varepsilon^{-r}(t-\infty) = \varepsilon + \beta \Leftrightarrow$$

$$0 = \varepsilon^{-r}(t-\infty) + \beta \Leftrightarrow 0 = \varepsilon + \beta$$

$$s/t = \infty \Leftrightarrow \varepsilon^{-r} = \beta \Leftrightarrow$$

$(\varepsilon + \beta)$ :  $x$ -axis

18  $f(x) = \log x$

$$= \log e^x$$

$$= \ln x$$

Domain:  $(0, \infty)$

Range:  $\mathbb{R}$



$$\varepsilon^{-r}(t-\infty) = \beta$$

$$\varepsilon^{-r}(t-\infty) = \varepsilon + \beta \Leftrightarrow$$

$$0 = \varepsilon^{-r}(t-\infty) + \beta \Leftrightarrow 0 = \varepsilon + \beta$$

$$s/t = \infty \Leftrightarrow$$

Sketch

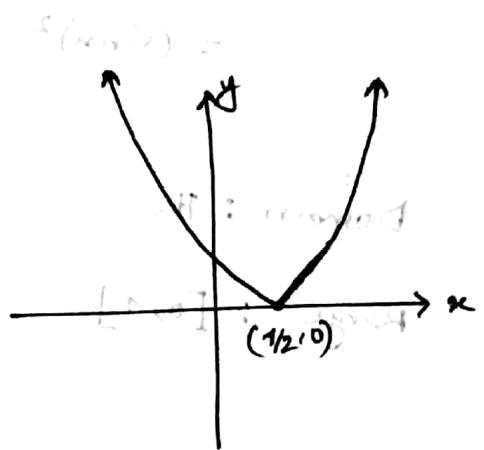
11  $y = (2x-1)^2$

$$y=0 \text{ if } (2x-1)^2=0$$

$$\Rightarrow 2x-1=0$$

$$\Rightarrow 2x=1$$

$$\Rightarrow x=1/2$$



vertex:  $(1/2, 0)$

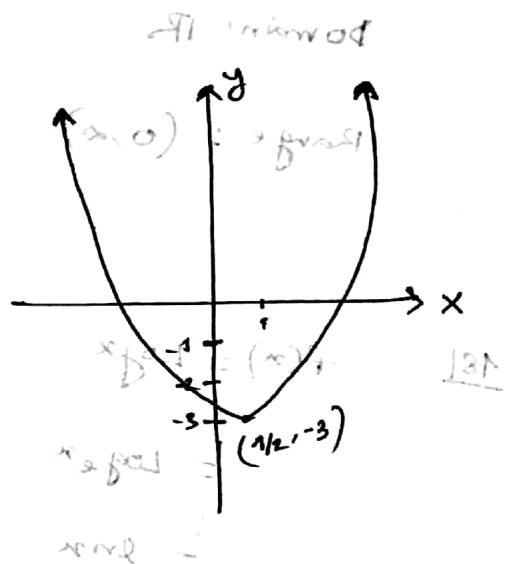
21  $y = (2x-1)^2 - 3$

$$\Rightarrow y+3 = (2x-1)^2$$

$$y+3=0 \text{ if } (2x-1)^2=0$$

$$\Rightarrow y=-3 \quad \Rightarrow x=1/2$$

vertex:  $(1/2, -3)$



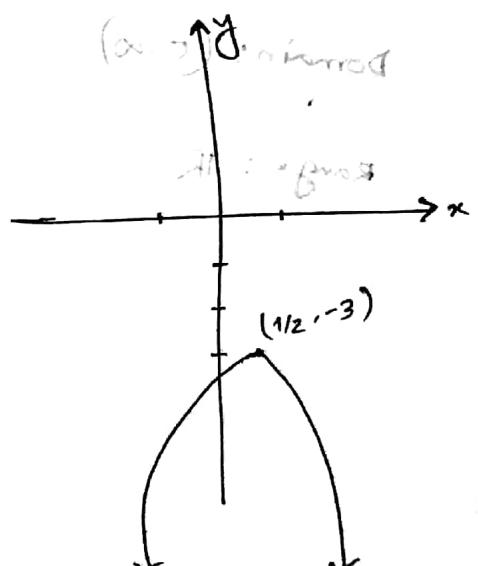
31  $y = -(2x-1)^2 - 3$

$$\Rightarrow y+3 = -(2x-1)^2$$

$$y+3=0 \text{ if } -(2x-1)^2=0$$

$$\Rightarrow y=-3 \quad \Rightarrow x=1/2$$

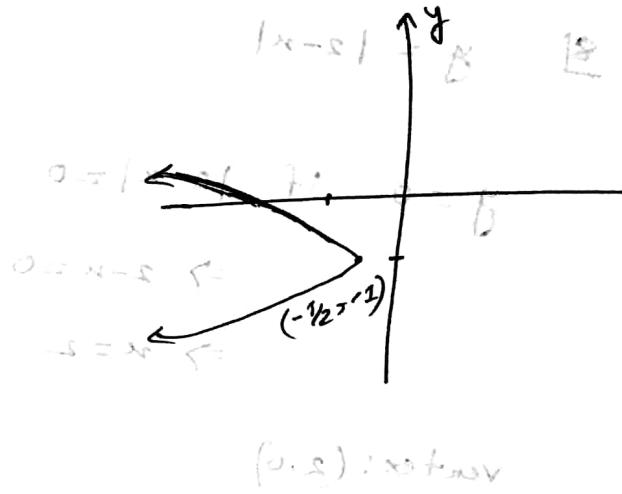
vertex  $(1/2, -3)$



$$41 \quad (y+1)^2 = -x - 1/2$$

$$(y+1)^2 = 0 \quad \text{if } -x - 1/2 = 0 \\ \Rightarrow y = -1 \quad \Rightarrow x = -1/2$$

vertex:  $(-1/2, -1)$



$$51 \quad y^2 = x$$

$$\Rightarrow y = \pm \sqrt{x}$$

$$61 \quad y = \sqrt{1+2x} - 2$$

$$\Rightarrow y + 2 = \sqrt{1+2x}$$

$$y + 2 = 0 \quad \text{if } \sqrt{1+2x} = 0$$

$$\Rightarrow y = -2 \quad \Rightarrow 1+2x=0 \\ \Rightarrow x = -1/2$$

vertex:  $(-1/2, -2)$

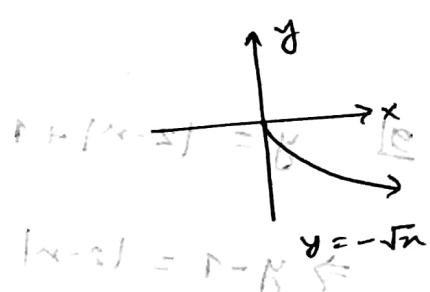
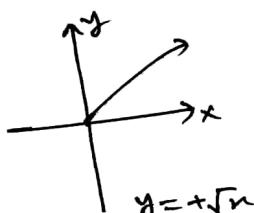
$$71 \quad y = -\sqrt{1+2x} - 2$$

$$\Rightarrow y + 2 = -\sqrt{1+2x}$$

$$y + 2 = 0 \quad \text{if } -\sqrt{1+2x} = 0$$

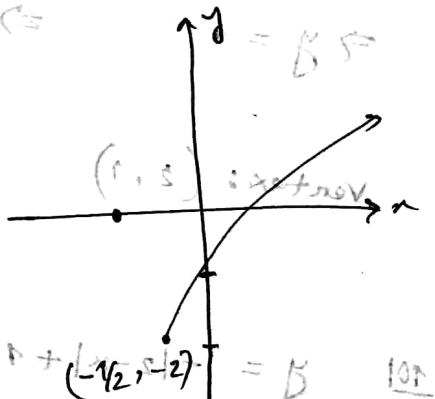
$$\Rightarrow y = -2 \quad \Rightarrow x = -1/2$$

vertex:  $(-1/2, -2)$

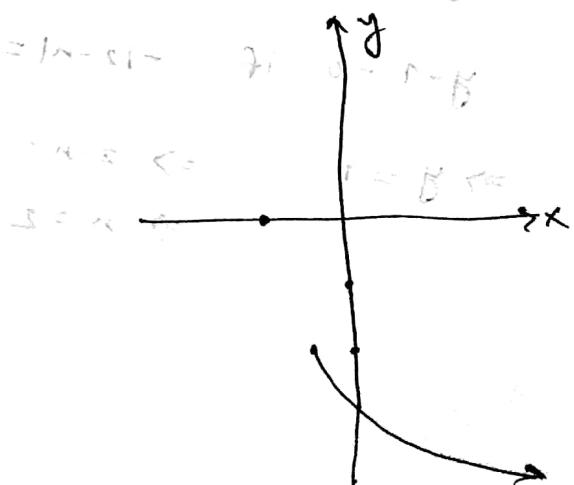


$$a = |x - s| - 2; \quad a = r - b$$

$$s = x \Leftrightarrow$$



$$s = |x - s| - 2; \quad s = r - b$$



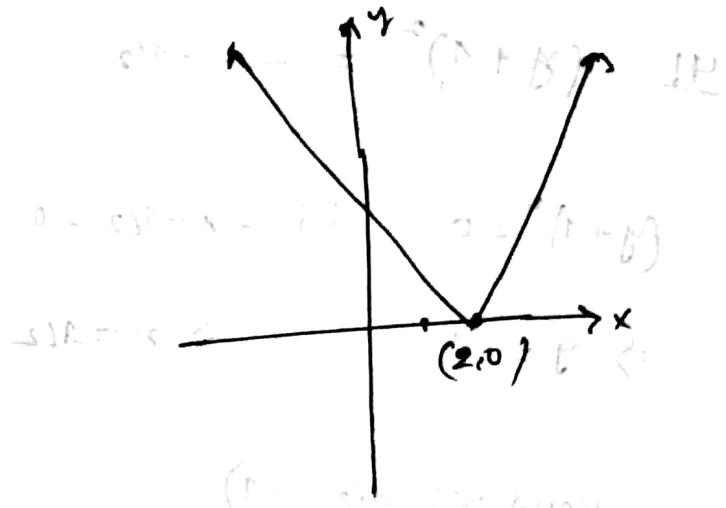
8)  $y = |2-x|$

$$y = 0 \quad \text{if} \quad |2-x| = 0$$

$$\Rightarrow 2-x = 0$$

$$\Rightarrow x = 2$$

vertex:  $(2, 0)$



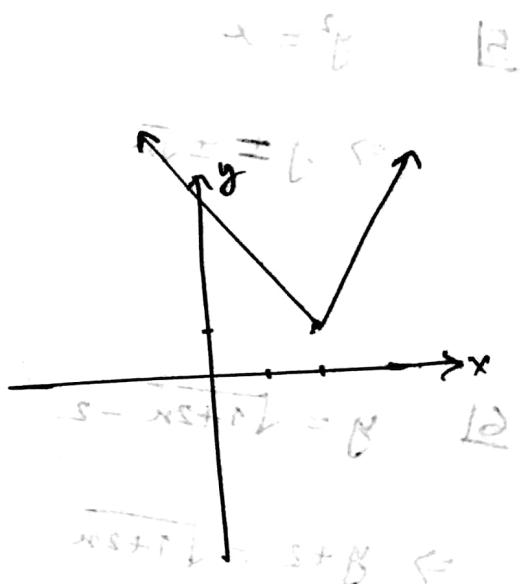
9)  $y = |2-x| + 1$

$$\Rightarrow y-1 = |2-x|$$

$$y-1 = 0 \quad \text{if} \quad |2-x| = 0$$

$$\Rightarrow y = 1 \quad \Rightarrow x = 2$$

vertex:  $(2, 1)$



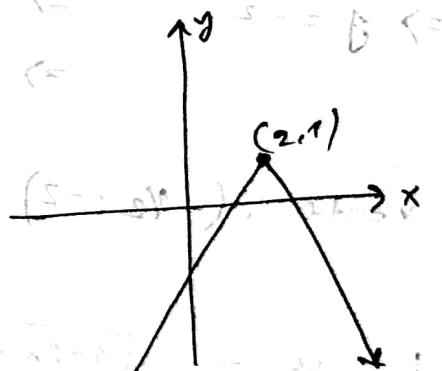
10)  $y = -|2-x| + 1$

$$y-1 = -|2-x|$$

$$y-1 = 0 \quad \text{if} \quad -|2-x| = 0$$

$$\Rightarrow y = 1 \quad \Rightarrow 2-x = 0$$

$$\Rightarrow x = 2$$



# LIMIT

1)

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1} - 1}$$

$$= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+1} + 1)}{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+1} + 1)}{(\sqrt{x+1})^2 - 1^2}$$

$$= \lim_{x \rightarrow 0} \frac{x\sqrt{x+1} + 1}{x+1-1}$$

$$= \lim_{x \rightarrow 0} \sqrt{x+1} + 1$$

$$= \sqrt{0+1} + 1$$

$$= 2$$

2)

$$\lim_{x \rightarrow 2} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$$

$$= \lim_{x \rightarrow 2} \frac{x(2x-5) + 2}{2x^2 - 4x - n + 2}$$

$$= \lim_{x \rightarrow 2} \frac{2n(n-2) - 1(n-2)}{5n(n-2) + 3(n-2)}$$

$$= \lim_{n \rightarrow 2} \frac{(n-2)(2n-1)}{(n-2)(5n+3)}$$

$$= \frac{2 \times 2 - 1}{5 \times 2 + 3}$$

$$= \frac{3}{13}$$

$$3) \lim_{x \rightarrow 0} \frac{x}{|x|}$$

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

$$\Rightarrow \frac{x}{|x|} = \begin{cases} 1 & , x > 0 \\ -1 & , x < 0 \end{cases}$$

$$L.H.L = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} (-1)$$

$$= -1$$

$$R.H.L = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} (1)$$

$$= 1$$

Doesn't exist.

$$4) f(x) = \begin{cases} 2-x & , x < 1 \\ x+1 & , x \geq 1 \end{cases}$$

$$L.H.L = \lim_{x \rightarrow 1^-} f(x)$$

$$= \lim_{x \rightarrow 1^-} (2-x)$$

$$= 2-1$$

$$= 1$$

$$R.H.L = \lim_{x \rightarrow 1^+} f(x)$$

$$= \lim_{x \rightarrow 1^+} (x+1)$$

$$= 1+1$$

$$= 2$$

Doesn't exist.

$$5) f(x) = \begin{cases} 3x-1 & , x < 1 \\ 3-x & , x \geq 1 \end{cases}$$

$$L.H.L = \lim_{x \rightarrow 1^-} f(x)$$

$$= \lim_{x \rightarrow 1^-} (3x-1)$$

$$= 3 \cdot 1 - 1$$

$$= 2$$

$$R.H.L = \lim_{x \rightarrow 1^+} f(x)$$

$$= \lim_{x \rightarrow 1^+} (3-x)$$

$$= 3-1$$

$$= 2$$

Limit exists.

6]

$$f(x) = \begin{cases} \frac{1}{x+2} & , x < -2 \\ x^2 - 5 & , -2 < x < 3 \\ \sqrt{x+13} & , x > 3 \end{cases}$$

$$\frac{3+x^2}{8-x^2} \quad \text{mid} \quad 15$$

$$L.H.L = \lim_{x \rightarrow -2^-} f(x)$$

$$= \lim_{x \rightarrow -2^-} \left( \frac{1}{x+2} \right)$$

$$= \lim_{x \rightarrow -2^-} \frac{(x+2)}{(x+2)(x-2)}$$

$$= \frac{1}{-2+2}$$

$$= \frac{1}{0}$$

$$= \infty$$

doesn't exist.

$$R.H.L = \lim_{x \rightarrow 3^-} f(x)$$

$$= \lim_{x \rightarrow 3^-} (x^2 - 5)$$

$$= (3)^2 - 5$$

$$= 4$$

$$R.H.L = \lim_{x \rightarrow 3^+} f(x)$$

$$= \lim_{x \rightarrow 3^+} (\sqrt{x+13})$$

$$= \sqrt{3+13}$$

$$= \sqrt{16}$$

$$= 4$$

Limit exists.

7)

$$\lim_{x \rightarrow \infty} \frac{3x+5}{6x-8}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{3x+5}{x}}{\frac{6x-8}{x}}$$

(x) + mit  $\frac{1}{x} \rightarrow 0$

$$= \lim_{x \rightarrow \infty} \frac{3 + \frac{5}{x}}{6 - \frac{8}{x}}$$

(x) + mit  $\frac{1}{x} \rightarrow 0$

$$= \frac{3 + \frac{5}{\infty}}{6 - \frac{8}{\infty}}$$

$$= \frac{3+0}{6-0}$$

$$= \frac{1}{2}$$

8)

$$\lim_{x \rightarrow \infty} 3 \sqrt{\frac{3x+5}{6x-8}}$$

$$= \lim_{x \rightarrow \infty} 3 \sqrt{\frac{(3x+5)/x}{(6x-8)/x}}$$

(x) + mit  $\frac{1}{x} \rightarrow 0$

$$= \lim_{x \rightarrow \infty} 3 \sqrt{\frac{3 + \frac{5}{x}}{6 - \frac{8}{x}}}$$

erneut

$$= \lim_{x \rightarrow \infty} 3 \sqrt{\frac{3+0}{6-0}}$$

$$= 3\sqrt{1/2}$$

$$\underline{9)} \lim_{x \rightarrow -\infty} \frac{4x^2 - x}{2x^3 - 5}$$

$$= \lim_{x \rightarrow -\infty} \frac{\frac{4x^2 - x}{x^3}}{\frac{2x^3 - 5}{x^3}}$$

$$= \lim_{x \rightarrow -\infty} \frac{\frac{4}{x} - \frac{1}{x^2}}{2 - \frac{5}{x^3}}$$

$$= \frac{-\infty - \frac{1}{(-\infty)^2}}{2 - \frac{5}{(-\infty)^3}}$$

$$= \frac{0 - 0}{2 - 0}$$

$$= 0$$

$$\underline{10)} \lim_{x \rightarrow \infty} (\sqrt{x^6 + 5} - x^3)$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^6 + 5} - x^3)(\sqrt{x^6 + 5} + x^3)}{\sqrt{x^6 + 5} + x^3}$$

$$= \lim_{x \rightarrow \infty} \frac{x^6 + 5 - x^6}{\sqrt{x^6(1 + \frac{5}{x^6})} + x^3}$$

$$= \lim_{x \rightarrow \infty} \frac{5}{x^3 \sqrt{1 + \frac{5}{x^6} + x^3}}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{5}{x^3}}{\frac{x^3 \sqrt{1 + \frac{5}{x^6} + x^3}}{x^3}}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{5}{x^3}}{\sqrt{1 + \frac{5}{x^6}} + 1}$$

$$= \frac{\frac{5}{(\infty)^3}}{\sqrt{1 + \frac{5}{(\infty)^6}} + 1}$$

$$= \frac{0}{\sqrt{1+0}+1}$$

$$= 0$$

111  $f(x) = \begin{cases} x^2 + 1 & , x > 0 \\ 1 & , x = 0 \\ 1+x & , x < 0 \end{cases}$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} (1+x)$$

$$= 1+0$$

$$= 1$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} (x^2 + 1)$$

$$= 0^2 + 1$$

$$= 1$$

limit exists.

$$12] \lim_{x \rightarrow \infty} \left( \sqrt{x^6 + 5x^3} - x^3 \right)$$

$$= \lim_{x \rightarrow \infty} \frac{\left( \sqrt{x^6 + 5x^3} - x^3 \right) \left( \sqrt{x^6 + 5x^3} + x^3 \right)}{\left( \sqrt{x^6 + 5x^3} + x^3 \right)}$$

$$= \lim_{x \rightarrow \infty} \frac{x^6 + 5x^3 - x^6}{\sqrt{x^6 \left( 1 + \frac{5x^3}{x^6} \right)} + x^3}$$

$\rightarrow 0 =$

$$= \lim_{x \rightarrow \infty} \frac{5x^3}{x^3 \sqrt{1 + \frac{5}{x^3}} + x^3}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{5x^3}{x^3}}{x^3 \sqrt{1 + \frac{5}{x^3}} + x^3}$$

$x^3$  का छोटा है

$$= \lim_{x \rightarrow \infty} \frac{5}{\sqrt{1 + \frac{5}{x^3}} + 1}$$

$$= \frac{5}{\sqrt{1 + 0} + 1} \underset{x \rightarrow \infty}{=} 1.414$$

$$= \frac{5}{\sqrt{1+0} + 1} \underset{x \rightarrow \infty}{=} 1.414$$

$$= \frac{5}{1+1} \underset{x \rightarrow \infty}{=} 2.5$$

$$= \frac{5}{2}$$

तो यह सही है

13)  $f(x) = \begin{cases} e^{-\frac{|x|}{2}} & , -1 < x < 0 \\ x^2 & , 0 \leq x \leq 2 \end{cases}$

L.H.L =  $\lim_{x \rightarrow 0^-} f(x)$

$$= \lim_{x \rightarrow 0^-} (e^{-\frac{|x|}{2}})$$

$$= \lim_{x \rightarrow 0^-} (e^{-\frac{-x}{2}})$$

$$= \lim_{x \rightarrow 0^-} e^{x/2}$$

$$= e^{0/2}$$

$$= e^0$$

$$= 1$$

R.H.L =  $\lim_{x \rightarrow 0^+} f(x)$

$$= \lim_{x \rightarrow 0^+} (x^2)$$

$$= \lim_{x \rightarrow 0^+} (\frac{d}{dx}(x^2))$$

$$= \lim_{x \rightarrow 0^+} (2x)$$

$$= \lim_{x \rightarrow 0^+} (\frac{2x}{x})$$

$$= \lim_{x \rightarrow 0^+} 2$$

$$= 2$$

Doesn't exist.

14)  $f(x) = \begin{cases} x^2 & , x < 1 \\ 2 \cdot 4 & , x = 1 \\ x+1 & , x > 1 \end{cases}$

L.H.L =  $\lim_{x \rightarrow 1^-} f(x)$

$$= \lim_{x \rightarrow 1^-} (x^2)$$

$$= 1^2$$

$$= 1$$

R.H.L =  $\lim_{x \rightarrow 1^+} f(x)$

$$= \lim_{x \rightarrow 1^+} (x+1)$$

$$= 1^2 + 1$$

$$= 2$$

Doesn't exist.

$$15) f(x) = \begin{cases} 2x+1 & , x < 1 \\ 3-x & , x \geq 1 \end{cases}$$

$$\text{L.H.L} = \lim_{x \rightarrow 1^-} f(x)$$

$$\begin{aligned} &= \lim_{x \rightarrow 1^-} (2x+1) \\ &= 2 \cdot 1 + 1 \\ &= 3 \end{aligned}$$

$$\text{R.H.L} = \lim_{x \rightarrow 1^+} f(x)$$

$$= \lim_{x \rightarrow 1^+} (3-x)$$

$$= 3 - 1 = 2$$

$$= 3 - \frac{1}{0.5^n}$$

$$= 2$$

doesn't exist.

$$L = "0000" = (0)^+$$

$$16) \text{ Let, } y = \left(1 + \frac{1}{n}\right)^n$$

$$\Rightarrow \ln y = \ln \left(1 + \frac{1}{n}\right)^n$$

$$\Rightarrow \ln y = n \ln \left(1 + \frac{1}{n}\right)$$

$$(x) \Rightarrow \ln y = n \left(1/n - \frac{1}{2n^2} + \frac{1}{3n^3} - \frac{1}{4n^4} + \dots\right)$$

$$(\text{using } \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots)$$

$$(\text{using } \lim_{n \rightarrow \infty} \frac{1}{n} = 0) \Rightarrow \lim_{n \rightarrow \infty} \ln y = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{2n} + \frac{1}{3n^2} - \frac{1}{4n^3} + \dots\right)$$

$$(\text{using } \lim_{n \rightarrow \infty} 0 = 0) \Rightarrow \lim_{n \rightarrow \infty} \ln y = (1 - 0 + 0 - 0 + \dots)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \ln y = 1 \Rightarrow \lim_{x \rightarrow \infty} \log_a x = 1$$

$$\Rightarrow \lim_{n \rightarrow \infty} y = e^1 = e$$

$$\begin{cases} \text{if } \log_a y = x \\ \Rightarrow y = a^x \end{cases}$$

$$\therefore \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

(proven)

### continuity

$$\begin{cases} x > 0 & f(x) = \cos x \\ 0 < x < \epsilon & f(x) = \cos x \\ x = 0 & f(x) = \cos 0 \end{cases} \Rightarrow f(x) = \cos x \quad \boxed{Q.E.D}$$

1)  $f(x) = \begin{cases} \cos x & , x > 0 \\ -\cos x & , x < 0 \end{cases}$

at  $x=0$

$$(x) \neq \text{nil} = \sin x$$

$$L.H.L = \lim_{n \rightarrow 0^-} f(n)$$

$$R.H.L = \lim_{n \rightarrow 0^+} f(n)$$

$$= \lim_{n \rightarrow 0^-} (-\cos n)$$

$$= \lim_{n \rightarrow 0^+} (\cos n)$$

$$= (-\cos 0^\circ)$$

$$= \cos 0^\circ$$

$$= -1$$

$$= 1$$

∴ same foreword

$$f(0) = \cos 0^\circ = 1$$

∴ Discontinuous.

$$f\left(\frac{1}{x} + 1\right) = B \rightarrow 2$$

$\boxed{D}$

2)  $f(x) = \begin{cases} x \cos(1/x) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

$$\begin{aligned} \left(\frac{1}{x} + 1\right) \text{ nil} &= B \rightarrow 2 \\ \left(\frac{1}{x} + 1\right) \text{ nil } x &= B \rightarrow 2 \end{aligned}$$

$$L.H.L = \lim_{x \rightarrow 0^-} f(x)$$

$$R.H.L = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^-} (x \cos 1/x)$$

$$= \lim_{x \rightarrow 0^+} (x \cos 1/x)$$

$$= \left(\lim_{x \rightarrow 0^-} x\right) \left(\lim_{x \rightarrow 0^+} \cos \frac{1}{x}\right) \rightarrow \text{nil} = B = \left(\lim_{x \rightarrow 0^+} x\right) \left(\lim_{x \rightarrow 0^-} \cos \frac{1}{x}\right)$$

$$= 0 \cdot [\text{oscillates}] = B = 0 \cdot [\text{oscillates}]$$

$$\begin{cases} x = \text{label} \\ x \rightarrow B = 0 \end{cases}$$

$$\begin{cases} x = \text{label} \\ x \rightarrow B = 0 \end{cases}$$

$$f(0) = 0$$

$$x = 1 \rightarrow B \text{ with } x =$$

∴ Continuous.

$$x = 1 \rightarrow B \text{ with } x =$$

$$3] f(x) = \begin{cases} e^{1/x}, & x \neq 0 \\ 1, & x=0 \end{cases} \text{ at } x=0, \quad \boxed{f(x)} \quad 12$$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} (e^{1/x})$$

$$= e^{1/0}$$

$$= e^{\infty}$$

$$= \infty$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} (e^{1/x})$$

$$= e^{1/0}$$

$$= e^{\infty}$$

$$= \infty$$

$$f(0) = 1$$

Discontinuous (0)

$$4] f(x) = \begin{cases} \sqrt{|x|}, & x > 0 \\ -\sqrt{|x|}, & x < 0 \end{cases} \text{ at } x=0 \quad \boxed{f(x)} \quad 12$$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} (-\sqrt{|x|})$$

$$= -\sqrt{0} + \text{mid} \rightarrow$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} (\sqrt{|x|})$$

$$\{ x = \sqrt{0} \} \text{ mid} \rightarrow$$

$$= 0$$

$$\therefore \text{at } x=0, f(x) = \sqrt{0} = 0$$

[Established]  $\Rightarrow$  continuous.

$$5) f(x) = \begin{cases} e^{-\frac{|x|}{2}}, & -1 < x < 0 \\ x^2, & 0 \leq x \leq 2 \end{cases} \quad \text{at } x=0$$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} (e^{-\frac{|x|}{2}})$$

$$= \lim_{x \rightarrow 0^-} e^{\frac{x}{2}}$$

$$= e^{0/2}$$

$$= e^0$$

$$= 1$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} x^2$$

$$= 0^2$$

$$= 0$$

$$f(0) = 0^2 = 0$$

discontinuous.

$$6) f(x) = \begin{cases} (x-a) \sin \frac{1}{x-a}, & x \neq a \\ 0, & x=a \end{cases}$$

$$\text{L.H.L} = \lim_{x \rightarrow a^-} f(x)$$

$$= \lim_{x \rightarrow a^-} \left\{ (x-a) \sin \frac{1}{x-a} \right\}$$

$$= \left( \lim_{x \rightarrow a^-} (x-a) \right) \left( \lim_{x \rightarrow a^-} \sin \frac{1}{x-a} \right)$$

$$= (a-a) \sin \frac{1}{a-a}$$

$$= 0 \cdot [\text{oscillates...}]$$

$$= 0$$

$$f(a) = 0$$

continuous.

71  $f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x=0 \end{cases}$  at  $x=0$  mid = 1.41  
 $\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x)$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x) =$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^-} (x \sin \frac{1}{x})$$

$$= \lim_{x \rightarrow 0^+} (x \sin \frac{1}{x})$$

$$= \left( \lim_{x \rightarrow 0^-} x \right) \left( \lim_{x \rightarrow 0^-} \sin \frac{1}{x} \right)$$

$$= \left( \lim_{x \rightarrow 0^+} x \right) \left( \lim_{x \rightarrow 0^+} \sin \frac{1}{x} \right)$$

$$= 0 \cdot [\text{oscillates}]$$

$$= 0 \cdot [\text{oscillates}]$$

$$= 0$$

$$= 0$$

$f(0) = 0$  even though continuous.

81  $f(x) = \begin{cases} 1, & x < 0 \\ 1 + \sin x, & 0 \leq x < \pi/2 \\ 2 + (x - \pi/2)^2, & x \geq \pi/2 \end{cases}$  at  $x=0$  and  $x=\pi/2$   $\square$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x)$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^-} (1)$$

$$= \lim_{x \rightarrow 0^+} (1 + \sin x)$$

$$= 1$$

$$= 1 + \sin 0^\circ$$

$$= 1 + 0$$

$$f(0) = 1 + \sin 0^\circ$$

$$= 1 + 0 = 1$$

hence  $f(x)$

is continuous at  $x=0$

$$\text{L.H.L} = \lim_{x \rightarrow \pi/2^-} f(x) \quad \begin{cases} 0 < x & (x) \text{ at } x \rightarrow \pi/2 \\ 0 = x & \end{cases} \quad \text{R.H.L} = \lim_{x \rightarrow \pi/2^+} f(x) \quad \begin{cases} 0 < x & (x) \neq \\ x = \pi/2 & \end{cases} \quad \boxed{15}$$

$$(x) = \lim_{x \rightarrow \pi/2^-} (1 + \sin x)$$

$$= \lim_{x \rightarrow \pi/2^+} \left\{ 2 + \left( x - \frac{\pi}{2} \right)^2 \right\}$$

$$(x) = 1 + \sin 90^\circ =$$

$$= 2 + (\frac{\pi}{2} - \frac{\pi}{2})^2$$

$$\left( \frac{1 + \sin x}{2} \right) = \frac{1 + 1}{2} =$$

$$\left( \frac{1 + \sin x}{2} \right) = \frac{2 + 0}{2} =$$

[continuous]

$$f(\frac{\pi}{2}) = 2 + (\frac{\pi}{2} - \frac{\pi}{2}) = 2$$

continuous at  $(x) = \frac{\pi}{2}$   
continuous

$$9) f(x) = |x| + |x-1| \quad \begin{cases} x > 0 & \text{at } x=0 = 0 \text{ and } n=1(x) \neq \\ x < 0 & \text{at } x=0 = 0 \text{ and } n=1(x) \neq \\ x=0 & \end{cases} \quad \boxed{18}$$

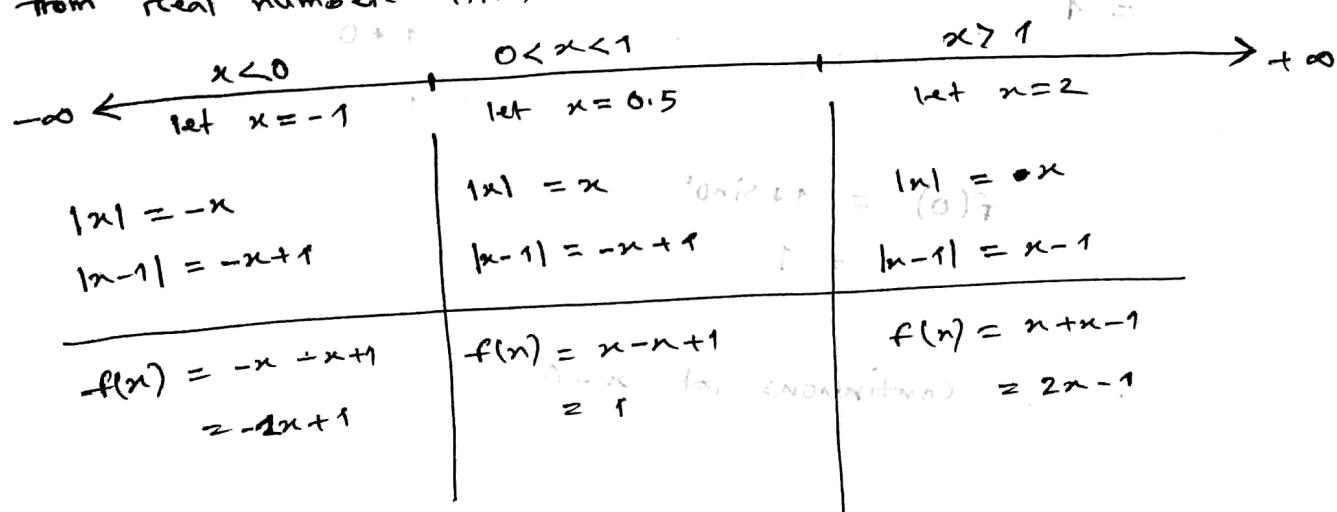
$$|x| = \begin{cases} x, & x > 0 \\ -x, & x \leq 0 \end{cases}$$

$$|x-1| = \begin{cases} x-1, & x > 1 \\ -(x-1), & x \leq 1 \end{cases}$$

$$(x) \neq \text{mid} = \text{L.H.L}$$

$$(x) \neq \text{mid} = \text{R.H.L}$$

from real number line,



$$f(x) = \begin{cases} -2x+1 & , x < 0 \\ 1 & , 0 \leq x < 1 \\ 2^{x-1} & , x > 1 \end{cases}$$

$$L.H.L = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} (-2x+1)$$

$$= 1$$

$$(x) \text{ at } x=0 = f(0) = 1$$

continuous at  $x=0$

$$(1) \quad \lim_{x \rightarrow 1^+} f(x) =$$

$$R.H.L = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} (1)$$

$$= 1$$

$$(x) \text{ at } x=1 = f(1)$$

$$(1) \quad \lim_{x \rightarrow 1^+} f(x) =$$

$$L.H.L = \lim_{x \rightarrow 1^-} f(x)$$

$$= \lim_{x \rightarrow 1^-} (1)$$

$$R.H.L = \lim_{x \rightarrow 1^+} f(x)$$

$$= \lim_{x \rightarrow 1^+} (2^{x-1})$$

$$\begin{aligned} & \text{at } x=1 \\ & 2^{x-1} \rightarrow 2^{1-1} = 2^0 = 1 \end{aligned}$$

$$\bullet \quad f(1) = 2^{1-1} = 1$$

$\therefore$  continuous at  $x=1$

$$(x) \text{ at } x=1 = f(1)$$

$$101 \quad f(x) = \begin{cases} \frac{|x-3|}{x-3}, & x \neq 3 \\ 0, & x=3 \end{cases} \quad \text{at } x=3$$

$$|x-3| = \begin{cases} x-3, & x > 3 \\ -(x-3), & x < 3 \end{cases}$$

$$f(x) = \begin{cases} 1, & x > 3 \\ -1, & x < 3 \end{cases}$$

$$\text{L.H. L} = \lim_{x \rightarrow 3^-} f(x)$$

$$= \lim_{x \rightarrow 3^-} (-1)$$

$$= -1$$

(At x=3)  $\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$ . Discontinuous.

$$\text{R.H.L} = \lim_{x \rightarrow 3^+} f(x)$$

$$= \lim_{x \rightarrow 3^+} (1)$$

$$= 1$$

$$11 \quad f(x) = \begin{cases} (1+x)^{1/n}, & x \neq 0 \\ 1, & x=0 \end{cases} \quad \text{at } x=0$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (1+x)^{1/n}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \ln f(n) = \lim_{n \rightarrow \infty} \ln (1+n)^{1/n}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \ln(f(x)) = \lim_{n \rightarrow \infty} \frac{1}{n} \ln(1+n)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \ln f(n) = \lim_{n \rightarrow \infty} \left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \dots \infty\right)$$

$$\Rightarrow \lim_{x \rightarrow 0} \ln f(x) = 1$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = e^1 = e$$

$$\therefore L.H.L = R.H.L = e$$

$$\text{But } f(0) = 1$$

Discontinuous.

$$12] \quad f(x) = \begin{cases} \frac{e^{1/x^2}}{e^{1/x^2}-1}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

$$\text{at } x=0$$

$$f(x) = \frac{e^{1/x^2}}{e^{1/x^2}-1}$$

$$= \frac{(e^{1/x^2}-1)+1}{(e^{1/x^2}-1)}$$

$$= 1 + \frac{1}{e^{1/x^2}-1}$$

$$\frac{(x)-f(x)}{0-x} \text{ mil} = (0)^{1/2}$$

$$L.H.L = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} 1 + \frac{1}{e^{1/x^2}-1}$$

$$= 1 + \frac{1}{e^{1/0}-1}$$

$$= 1 + \frac{1}{e^\infty - 1}$$

$$= 1 + \frac{1}{\infty - 1}$$

$$= 1 + 0$$

$$= 1$$

$$R.H.L = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} 1 + \frac{1}{e^{1/x^2}-1}$$

$$= 1 + \frac{1}{e^{1/0}-1}$$

$$= 1 + \frac{1}{\infty - 1}$$

$$= 0$$

$$= 1$$

$$f(0) = 1$$

$\therefore$  continuous.

23-Oct-2017

## Differentiability

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Q1 (i)  $f(x) = \begin{cases} \cos x, & x > 0 \\ -\cos x, & x < 0 \end{cases}$

at  $x=0$  left diff.  $\neq$  right diff.

$$Lf'(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0^-} \frac{-\cos x - 1}{x} \quad \text{evaluate L'Hospital's rule}$$

$$= \frac{-\cos 0 - 1}{0}$$

$$= \frac{-1 - 1}{0}$$

$$= \frac{-2}{0}$$

$$= -\infty$$

$$Rf'(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0^+} \frac{\cos x - 1}{x}$$

$$(x) \neq \lim_{x \rightarrow 0^+} \frac{\cos x - 1}{x} = 1 \cdot 4 \cdot 1$$

$$= \lim_{x \rightarrow 0^+} \frac{(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots) - 1}{x}$$

$$= \lim_{x \rightarrow 0^+} \left( -\frac{x}{2!} + \frac{x^3}{3!} - \frac{x^5}{5!} + \dots \right)$$

$$= -\frac{0}{2!} + \frac{0}{3!} - \frac{0}{5!} + \dots \infty$$

$$= 0$$

$\therefore f(x)$  is not differentiable at  $x=0$

$$(ii) f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

at  $x=0$   $\Rightarrow$  (x)  $\Rightarrow$  (vi)

$$Lf'(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0}$$

$\frac{f(x) - f(0)}{x - 0} \Rightarrow Rf'(0) = \dots$

$$= \lim_{x \rightarrow 0^-} \frac{x^2 \sin(\frac{1}{x}) - 0}{x}$$

$\frac{0 - (\frac{1}{x}) \cos x}{x} \xrightarrow[x \rightarrow 0^-]{} \frac{\infty}{0} = \infty$

$$= \lim_{x \rightarrow 0^-} x \sin(\frac{1}{x})$$

$(\frac{1}{x} \cos x) \xrightarrow[x \rightarrow 0^-]{} \infty = \infty$

$$= \left( \lim_{x \rightarrow 0^-} x \right) \left( \lim_{x \rightarrow 0^-} \frac{\sin(\frac{1}{x})}{\frac{1}{x}} + \frac{1}{x} \rightarrow 1 \right) \xrightarrow[x \rightarrow 0^-]{} \infty = \infty$$

$= 0$  [oscillates]

$$= 0$$

$\infty + \infty - \infty + \infty - \infty = 0$

Differentiable.

$$(iii) f(x) = |x| \quad \text{at } x=0$$

(v)  $\Rightarrow$  (vi)

$$f(x) = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

$$Lf'(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0^-} \frac{-x - 0}{x}$$

$$= \lim_{x \rightarrow 0^-} (-1)$$

$$= -1$$

$$Rf'(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0^+} \frac{x - 0}{x}$$

$$= \lim_{x \rightarrow 0^+} (1)$$

$$= 1$$

$\therefore$  not differentiable

$$(iv) f(x) = \begin{cases} x \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases} \quad (\frac{1}{x}) \text{ is finite at } x=0 \Rightarrow f(x) \text{ is differentiable at } x=0 \quad (ii)$$

$$L f'(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{x \cos\left(\frac{1}{x}\right) - 0}{x - 0} = \lim_{x \rightarrow 0^-} \frac{x \cos\left(\frac{1}{x}\right)}{x} = \lim_{x \rightarrow 0^-} \cos\left(\frac{1}{x}\right) = 1 \quad (0)^2 = (x)^2 \quad \text{as } x \rightarrow 0^-$$

$$= \lim_{x \rightarrow 0^-} \frac{x \cos\left(\frac{1}{x}\right) - 0}{x} = \lim_{x \rightarrow 0^-} \cos\left(\frac{1}{x}\right) = 1 \quad \text{as } x \rightarrow 0^-$$

$$= \lim_{x \rightarrow 0^-} \left( \cos \frac{1}{x} \right) = 1 \quad (0) \text{ is finite as } x \rightarrow 0^-$$

$$= \lim_{x \rightarrow 0^-} \left( 1 - \frac{1}{x^2 2!} + \frac{1}{x^4 4!} - \frac{1}{x^6 6!} + \dots \right) =$$

$$= 1 - \infty + \infty - \infty + \dots =$$

$$= -\infty \quad \text{by alternating series}$$

$$Rf'(0) = \text{Same} \quad 0 = x \rightarrow 0^+ \Rightarrow Lf' = (x)^2 = (0)^2 \quad (iii)$$

Not differentiable

$$\frac{(0)^2 - (x)^2}{0 - x} = \frac{-x^2}{-x} = x \rightarrow 0^+ \Rightarrow Rf' = (0)^2 = 0$$

$$\frac{(0)^2 - (x)^2}{0 - x} = \frac{-x^2}{-x} = x \rightarrow 0^+ \Rightarrow Rf' = (0)^2 = 0$$

$$\frac{0 - x}{x} = \frac{-x}{x} = -1 \rightarrow 0^+$$

$$\frac{0 - x}{x} = \frac{-x}{x} = -1 \rightarrow 0^+$$

$$(+) \text{ min } \pm \infty$$

$$(-) \text{ max } \pm \infty$$

$$f' =$$

$$2) f(x) = \begin{cases} x^2 - 16x & , x < 9 \\ 12\sqrt{x} & , x \geq 9 \end{cases}$$

$$L f'(9) = \lim_{x \rightarrow 9^-} \frac{f(x) - f(9)}{x - 9}$$

$$= \lim_{x \rightarrow 9^-} \frac{x^2 - 16x - 36}{x - 9}$$

$$= \lim_{x \rightarrow 9^-} \frac{x^2 - 18x + 2x - 36}{x - 9}$$

$$= \lim_{x \rightarrow 9^-} \frac{(x+2)(x-18)}{x-9}$$

$$\frac{(x+2)(x-18)}{x-9} \underset{x \rightarrow 9^-}{=} \frac{(9+2)(9-18)}{9-9}$$

$$= \infty$$

$$R f'(9) = \lim_{x \rightarrow 9^+} \frac{f(x) - f(9)}{x - 9}$$

$$= \lim_{x \rightarrow 9^+} \frac{12\sqrt{x} - 36}{x - 9}$$

$$= \lim_{x \rightarrow 9^+} \frac{12(\sqrt{x} - 3)}{(\sqrt{x} + 3)(\sqrt{x} - 3)}$$

$$= \lim_{x \rightarrow 9^+} \frac{12}{\sqrt{x} + 3}$$

$$= \frac{12}{\sqrt{9} + 3}$$

$$= 2$$

$\therefore$  not differentiable.

$$L.H.L = \lim_{x \rightarrow 9^-} f(x)$$

$$= \lim_{x \rightarrow 9^-} (x^2 - 16x)$$

$$= (9^2 - 16 \times 9)$$

$$= -63$$

$$R.H.L = \lim_{x \rightarrow 9^+} f(x)$$

$$= \lim_{x \rightarrow 9^+} (12\sqrt{x})$$

$$= 12\sqrt{9}$$

$$= 36$$

discontinuous.

$$31 \quad f(x) = \begin{cases} x^2 & , x \leq 1 \\ \sqrt{x} & , x > 1 \end{cases} \quad \text{at } x=1$$

$$L.f'(1) = \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1}$$

$$R.f'(1) = \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{x - 1}$$

$$= \lim_{x \rightarrow 1^+} \frac{\sqrt{x} - 1}{x - 1}$$

$$= \lim_{x \rightarrow 1^-} \frac{(x+1)(x-1)}{(x-1)}$$

$$= \lim_{x \rightarrow 1^+} \frac{(\sqrt{x} - 1)}{(\sqrt{x} + 1)(\sqrt{x} - 1)}$$

$$= \lim_{x \rightarrow 1^-} (x+1)$$

$$= \lim_{x \rightarrow 1^+} \frac{1}{\sqrt{x} + 1}$$

$$= 1+1$$

$$= \frac{1}{1+1}$$

$$= 2$$

$$\frac{1}{1+\sqrt{1}} = \frac{1}{2}$$

not differentiable

$$L.H. L = \lim_{x \rightarrow 1^-} f(x)$$

$$R.H. L = \lim_{\substack{x \rightarrow 1^+ \\ x \leftarrow \infty}} f(x)$$

$$(=) \lim_{x \rightarrow 1^-} (x^{\frac{1}{n}})$$

$$= 1^{\frac{1}{n}} =$$

$$\approx 1$$

$$f(1) = 1^{\frac{1}{n}} = 1$$

continuous,  $f(x)$

$$(R.H. L) = \lim_{\substack{x \rightarrow 1^+ \\ x \leftarrow \infty}} (\sqrt[n]{x})$$

$$= \sqrt[1]{1}$$

$$\approx 1$$

$$4] f(x) = \begin{cases} x^{\frac{1}{n}} + 1 & , x \leq 1 \text{ continuous at } x=1 \\ x & , x > 1 \end{cases}$$

$$L f'(1) = \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \rightarrow 1^-} \frac{x^{\frac{1}{n}} + 1 - 2}{x - 1}$$

$$= \lim_{x \rightarrow 1^-} \frac{x^{\frac{1}{n}} - 1}{x - 1}$$

$$= \lim_{x \rightarrow 1^-} \frac{(x+1)(x-1)}{(x-1)}$$

$$= \lim_{x \rightarrow 1^-} (x+1)$$

$$= 1+1$$

$$= 2$$

$$R f'(1) = \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \rightarrow 1^+} \frac{x - 2}{x - 1}$$

$$= \lim_{x \rightarrow 1^+} \frac{(x-1)-1}{(x-1)}$$

$$= \lim_{x \rightarrow 1^+} 1 - \frac{1}{x-1}$$

$$= 1 - \frac{1}{1-1}$$

$$= 1 - \infty$$

$$= \infty$$

not differentiable

$$L.H. \cdot L = \lim_{x \rightarrow 1^-} f(x) \cdot H \cdot L$$

$$R.H.C = \lim_{x \rightarrow 1^+} f(x) \cdot H \cdot L$$

$$(n) = \lim_{x \rightarrow 1^-} (n+1)$$

$$= 1 + 1$$

$$= 2$$

$$(n) = \lim_{x \rightarrow 1^+} (n)$$

$$= 1 + 1$$

$$N \in$$

$$1 = 1 = (n)$$

$$f(1) = 1 + 1 = 2$$

function is not continuous.  $\{x\}$   $\{x\}$

$$\frac{(n) - (n)}{1 - n} \quad \text{mid} = (n)'$$

$$\frac{(n) - (n)}{1 - n} \quad \text{mid} = (n)'$$

$$\frac{s - x}{t - x} \quad \text{mid} =$$

$$\frac{s - t + x}{t - x} \quad \text{mid} =$$

$$\frac{t - (t - x)}{(t - x)} \quad \text{mid} =$$

$$\frac{t - x}{t - x} \quad \text{mid} =$$

$$\frac{x}{t - x} \quad \text{mid} =$$

$$\frac{(t - x)(t + x)}{(t - x)} \quad \text{mid} =$$

$$\frac{x}{t - x} \quad \text{mid} =$$

$$(t + x) \quad \text{mid} =$$

$$B \times D =$$

$$P \times P =$$

$$P \times P =$$

Defining a limit

# Techniques of Differentiation

(v)

$$1 \quad (i) \quad y = \sin x \sin 2x \sin 3x \quad \frac{d}{dx} (f(x))^{g(x)} = \frac{df}{dx} \left( g(x) \right)^{f(x)}$$

$$\Rightarrow \frac{dy}{dx} = \sin x \sin 2x \frac{d}{dx} (\sin 3x) + \sin 3x \frac{d}{dx} (\sin x \sin 2x)$$

$$= \sin x \sin 2x \cos 3x + \sin 3x [\sin x \frac{d}{dx} \sin 2x + \sin 2x \frac{d}{dx} \sin x]$$

$$= 3 \sin x \sin 2x \cos 3x + 2 \sin x \sin 3x \cos 2x + \sin 3x \sin 2x \cos x$$

$$\left( \frac{x+1}{x-1} \right)^{1-x} f(x) = B \quad (iv)$$

$$(ii) \quad y = \frac{\cosec^3 x}{\sin(x+1)^{x-1}}$$

$$\Rightarrow y = (\cosec x)^3$$

$$\Rightarrow \frac{dy}{dx} = 3 \cosec^2 x \frac{d}{dx} (\cosec x)$$

$$= 3 \cosec x (-\cosec x \cot x)$$

$$= -3 \cosec^2 x \cosec x \cot x$$

$$(iii) \quad y = \cos 2x \cos 3x$$

$$\Rightarrow \frac{dy}{dx} = \cos 2x \frac{d}{dx} (\cos 3x) + \cos 3x \frac{d}{dx} (\cos 2x)$$

$$= -3 \cos 2x \sin 3x - 2 \cos 3x \sin 2x$$

$$(iv) \quad y = \sin^{-1}(x^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^4}} \cdot 2x$$

$$= \frac{2x}{\sqrt{1-x^4}}$$

$$(V) \quad y = \tan(\sin^{-1} x)$$

$$\Rightarrow \frac{dy}{dx} = \sec(\sin^{-1} x) \cdot \frac{1}{\sqrt{1-x^2}} \quad (i) \quad \square$$

$$\begin{aligned} & (\text{differentiate}) \frac{d}{dx} x \sin^{-1} x + (\sin^{-1} x) \frac{d}{dx} x \cos^{-1} x = \frac{1}{\sqrt{1-x^2}} \\ & = \frac{\sec(\sin^{-1} x)}{\sqrt{1-x^2}} \end{aligned}$$

$$(VI) \quad y = \cot^{-1} \left( \frac{1+x}{1-x} \right)$$

$$\Rightarrow \frac{dy}{dx} = - \frac{1}{1 + \left( \frac{1+x}{1-x} \right)^2} \cdot \frac{(1-x) \frac{d}{dx}(1+x) - (1+x) \frac{d}{dx}(1-x)}{(1-x)^2} \quad (ii) \quad \square$$

$$= - \frac{1}{1 + \frac{(1+x)^2}{(1-x)^2}} \cdot \frac{(1-x) \cdot 1 - (1+x) \cdot (-1)}{(1-x)^2} =$$

$$= - \frac{1}{\frac{(1-x)^2 + (1+x)^2}{(1-x)^2}} \cdot \frac{1-x+1+x}{(1-x)^2} =$$

$$= - \frac{2}{(1-x)^2 + (1+x)^2} = - \frac{2}{(1-2x+x^2) + (1+2x+x^2)} = - \frac{2}{2x^2+2} = - \frac{1}{x^2+1} \quad (iii)$$

$$= - \frac{2}{(1-2x+x^2) + (1+2x+x^2)} = - \frac{2}{2x^2+2} = - \frac{1}{x^2+1}$$

$$= - \frac{2}{1-2x+x^2 + 1+2x+x^2} = - \frac{2}{2x^2+2} = - \frac{1}{x^2+1} \quad (iv)$$

$$= - \frac{2}{2x^2+2} = - \frac{1}{x^2+1}$$

$$(vii) y = \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right)$$

$$\left( \frac{1-x^2}{1+x^2} \right)^{\text{1/rat}} = b \quad (x)$$

$$= \cos^{-1} \left( \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$\left( \frac{\tan \theta}{\tan^2 \theta + 1} \right)^{\text{1/rat}} \Rightarrow \theta = \tan^{-1} x$$

$$= \cos^{-1} (\cos 2\theta)$$

$$(\cos \theta)^{\text{1/rat}} =$$

$$= 2\theta$$

$$= 2\tan^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

$$\frac{2}{1+x^2} = \frac{2}{x^2+1}$$

$$(viii) y = \sin^{-1} \left( \frac{2x}{1+x^2} \right)$$

$$= \sin^{-1} \left( \frac{2\tan \theta}{1 + \tan^2 \theta} \right)$$

$$\left( \frac{2\tan \theta}{1 + \tan^2 \theta} \right)^{\text{1/rat}} = b(x)$$

$$\theta = \tan^{-1} \left( \frac{2x}{1+x^2} \right)$$

$$\text{let, } x = \tan \theta$$

$$= 2\theta$$

$$= 2\tan^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

$$\left( \frac{2\tan \theta}{1 + \tan^2 \theta} \right)^{\text{1/rat}} =$$

$$\left( \frac{2\tan \theta}{1 + \tan^2 \theta} \right)^{\text{1/rat}} =$$

$$(\theta \text{ rat})^{\text{1/rat}} =$$

$$\theta = \tan^{-1} x$$

$$(ix) \quad y = \tan^{-1} \left( \frac{2x}{1-x^2} \right)$$

$$\left( \frac{2x}{1-x^2} \right) \text{ corresponds to } (iii)$$

$$x' \text{ not } \Rightarrow \theta = \tan^{-1} \left( \frac{2\tan\theta}{1-\tan^2\theta} \right)$$

$$\left( \frac{2\tan\theta}{1-\tan^2\theta} \right) \text{ corresponds to } (ii)$$

$$= \tan^{-1} (\tan 2\theta)$$

$$(\theta \text{ s.c.}) \text{ corresponds to }$$

$$= 2\theta$$

$$\theta \text{ s.c.}$$

$$= 2\tan^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

$$\frac{s}{s^2+b^2} = \frac{\sin\theta}{\sin^2\theta + \cos^2\theta} =$$

$$(x) y = \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right)$$

$$\left( \frac{x}{\sqrt{1-x^2}} \right) \text{ corresponds to } (iii)$$

$$= \tan^{-1} \left( \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} \right)$$

$$\left( \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} \right) \text{ corresponds to } (iv)$$

$$= \tan^{-1} \left( \frac{\sin\theta}{\sqrt{\cos^2\theta}} \right)$$

$$\begin{aligned} & \text{Let } x = \sin\theta \\ & \Rightarrow \theta = \sin^{-1}x \end{aligned}$$

$$= \tan^{-1} \left( \frac{\sin\theta}{\cos\theta} \right)$$

$$\frac{s}{s^2+b^2} = \frac{\sin\theta}{\sin^2\theta + \cos^2\theta} =$$

$$= \tan^{-1} (\tan\theta)$$

$$= \theta$$

$$= \sin^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\underline{\text{2}} \quad (\text{i}) y = (\sin x)^{\ln x} \quad \frac{d}{dx} (\sin x)^{\ln x} = \frac{nb}{ab} \frac{1}{x} \quad \leftarrow$$

$$\Rightarrow \ln y = \ln(\sin x)^{\ln x} + \frac{1}{\ln x} \cdot \ln(\sin x) \quad \leftarrow$$

$$\Rightarrow \ln y = \ln x \ln(\sin x) + \cos x \ln(\sin x) \quad \leftarrow$$

$$\Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx} [\ln x \ln(\sin x)] \quad \leftarrow$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \ln x \frac{1}{x} \ln(\sin x) + \ln(\sin x) \frac{d}{dx} \ln x$$

$$\begin{aligned} &= \ln x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot \frac{1}{x} \\ &\quad (n \cos) \text{rd} = v \text{rd} \\ &= \ln x \cdot \frac{\cos x}{\sin x} + \frac{\ln(\sin x)}{x} \quad \text{vrd} \quad \leftarrow \end{aligned}$$

$$\therefore \frac{dy}{dx} = (\sin x)^{\ln x} \left[ \ln x \cot x + \frac{\ln(\sin x)}{x} \right]$$

$$(\cos x) \frac{b}{ab} (\cos x) \text{rd} + (\cos x) \text{rd} \frac{b}{ab} \text{vrd} = \frac{vb}{ab} \frac{1}{v} \quad \leftarrow$$

$$(\text{ii})^{\text{rd}} \cdot (\text{vrd}) = (\sin x)^{\cos x} + (\cos x)^{\sin x} \quad \leftarrow$$

$$(\cos x) \text{rd} \cos x + \text{vrd} \sin x \leftarrow$$

$$\text{let, } u = (\sin x)^{\cos x} \quad \text{--- ①}$$

$$[(\cos x) \text{rd} \cos x + \text{vrd} \sin x] \text{vrd} (\cos x) = \frac{vb}{ab} \quad \leftarrow$$

$$v = (\cos x)^{\sin x} \quad \text{--- ②}$$

from ①,

$$\frac{vb}{ab} + \frac{vb}{ab} = \frac{vb}{ab} \quad \leftarrow$$

$$\ln u = \ln(\sin x)^{\cos x}$$

$$[(\cos x) \text{rd} \cos x + \text{vrd} \sin x] + [(\cos x)^{\sin x} \text{rd} \sin x + (\cos x)^{\sin x} \text{vrd}] \text{vrd} (\cos x) \quad \leftarrow$$

$$\Rightarrow \ln u = \cos x \ln(\sin x)$$

$$\Rightarrow \frac{d}{dx} \ln u = \frac{d}{dx} \cos x \ln(\sin x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \cos x \frac{d}{dx} \ln(\sin x) + \ln(\sin x) \frac{d}{dx} (\cos x)$$

$$= \cos x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot (-\sin x)$$

$$= \cot x \cos x - \sin x \ln(\sin x)$$

$$\therefore \frac{du}{dx} = (\sin x)^{\cos x} [\cot x \cos x - \sin x \ln(\sin x)]$$

$$\text{and } \frac{v}{u} \frac{dv}{dx} (\text{and } v = (\sin x)^{\cos x}) \frac{dv}{dx} = \frac{v}{u} \frac{d}{dx}$$

from (ii)

$$\frac{1}{x} \cdot (\text{and } v) + \cos x \cdot \frac{1}{\sin x} \cdot \text{and } =$$

$$\ln v = \ln(\cos x)^{\sin x} \text{ and } =$$

$$\Rightarrow \frac{1}{x} \frac{dv}{dx} = \sin x \ln(\cos x) \text{ and } =$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \sin x \frac{d}{dx} \ln(\cos x) + \ln(\cos x) \frac{d}{dx} (\sin x)$$

$$= \sin x \left( \frac{1}{\cos x} \cdot (-\sin x) + \ln(\cos x) \cdot \cos x \right)$$

$$= -\sin x \tan x + \cos x \ln(\cos x)$$

$$\therefore \frac{dv}{dx} = (\cos x)^{\sin x} [-\sin x \tan x + \cos x \ln(\cos x)]$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$= (\sin x)^{\cos x} [\cot x \cos x - \sin x \ln(\sin x)] + (\cos x)^{\sin x} (-\sin x \tan x + \cos x \ln(\cos x))$$

Final answer to question 6

$$31 \quad y = \sqrt{\left(\frac{1+x}{1-x}\right)} \quad 0 = \epsilon_B^x + \epsilon_B^y - \epsilon_{ex} \quad (i)$$

$$\Rightarrow \ln y = \ln \left( \frac{1+x}{1-x} \right)^{1/2} = \left( \epsilon_B^x + \epsilon_B^y - \epsilon_{ex} \right) \frac{b}{2b} \quad \Leftarrow$$

$$\Rightarrow \ln y = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right)^{1/2} + \epsilon_B^y \frac{b}{2b} - \epsilon_{ex} \quad \Leftarrow$$

$$0 = \left[ \epsilon_B^y \frac{b}{2b} - \epsilon_{ex} \right] + \left[ \ln \left( \frac{1+x}{1-x} \right)^{1/2} - \epsilon_{ex} \right] - \epsilon_{ex} \quad \Leftarrow$$

$$\Rightarrow \ln y = \frac{1}{2} \left[ \ln (1+x) - \ln (1-x) \right] \quad \Leftarrow$$

$$0 = \frac{\epsilon_B^y}{2b} b^2 + \epsilon_B^y x^2 - \frac{\epsilon_B^y}{1b} x - \epsilon_{ex} \quad \Leftarrow$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{2} \left[ \frac{1}{1+x} \cdot 1 - \frac{1}{1-x} \cdot (-1) \right] \quad \Leftarrow$$

$$\epsilon_{ex} - \epsilon_{ex} = \frac{\epsilon_B^y}{2b} \frac{2}{x} = \frac{\epsilon_B^y}{2b} \cdot \frac{2}{x} \quad \Leftarrow$$

$$= \frac{1}{\epsilon_{ex}} \left[ \frac{1}{1+x} + \frac{1}{1-x} \right] \quad \Leftarrow$$

$$= \frac{1}{2} \left[ \frac{(1-x)+(1+x)}{(1+x)(1-x)} \right] \frac{\epsilon_B^y}{2b} \quad \Leftarrow$$

$$= \frac{1}{2} \left[ \frac{2}{1-x^2} \right]$$

$$0 = \epsilon_{ex} - \epsilon_{ex} P + \epsilon_B^y + \epsilon_x \quad (ii)$$

$$= \frac{1}{(1-x^2)} = (\epsilon_B^y + \epsilon_B^x + \epsilon_{ex}) \frac{b}{2b} \quad \Leftarrow$$

$$\therefore \frac{dy}{dx} = \sqrt{\frac{(1+x)}{(1-x)}} \cdot \frac{1}{1-x^2} = \frac{1+x}{1-x^2} \epsilon_B^y + \epsilon_x \quad \Leftarrow$$

$$0 = \left[ \frac{1+x}{1-x} \left( \epsilon_B^y + \epsilon_x \right) \right] P + \frac{1+x}{1-x} \epsilon_B^y + \epsilon_x \quad \Leftarrow$$

$$\Rightarrow \epsilon_B^y + \epsilon_x = \frac{1+x}{1-x} \epsilon_B^y + \frac{1+x}{1-x} \epsilon_x + \epsilon_x \quad \Leftarrow$$

41

$$(i) \quad 3x^4 - xy + 2y^3 = 0$$

$$\left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right) = \begin{pmatrix} 12x^3 & -1 \\ -1 & 6y^2 \end{pmatrix} = B \quad 18$$

$$\Rightarrow \frac{d}{dx} (3x^4 - xy + 2y^3) = \frac{d}{dx}(0) \text{ and } = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right) \text{ and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow 12x^3 - \frac{d}{dx} xy + \frac{d}{dx} 2y^3 = 0 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow 12x^3 - [x \frac{d}{dx} y + y \frac{d}{dx} x] + 2[3y^2 \cdot \frac{d}{dx} y] = 0 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow 12x^3 - x^2 \frac{dy}{dx} - 2xy + 6y^2 \frac{dy}{dx} = 0 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow 6y^2 \frac{dy}{dx} - x^2 \frac{dy}{dx} = 2xy - 12x^3 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow \frac{dy}{dx} (6y^2 - x^2) = 2xy - 12x^3 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\therefore \frac{dy}{dx} = \frac{2xy - 12x^3}{6y^2 - x^2} \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$(ii) \quad x^3 + y^3 + 4x^2y - 25 = 0$$

$$\Rightarrow \frac{d}{dx} (x^3 + y^3 + 4x^2y) = \frac{d}{dx}(25) \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow 3x^2 + 3y^2 \frac{dy}{dx} + 4x^2y + 4y^2x = 0 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow 3x^2 + 3y^2 \frac{dy}{dx} + 4[x^2 \frac{dy}{dx} + y \frac{d}{dx} x^2] = 0 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow 3x^2 + 3y^2 \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 0 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\Rightarrow \frac{dy}{dx} (3y^2 + 4x^2) = -8xy - 3x^2 \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$\therefore \frac{dy}{dx} = \frac{-3x^2 - 8xy}{3y^2 + 4x^2} \quad \text{and } \frac{1}{2} = B \text{ nd } \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right)$$

$$(iii) \quad x^y = y^x \quad \theta \text{ mit } x \quad (ii)$$

$$\Rightarrow \ln x^y = \ln y^x \quad \theta \text{ mit } y$$

$$\Rightarrow y \ln x = x \ln y$$

$$\theta \cos \theta \sin \theta = \frac{xy}{ab}$$

$$\Rightarrow \frac{d}{dx} (y \ln x) = \frac{d}{dx} (x \ln y)$$

$$\theta \cos \theta = \frac{xy}{ab}$$

$$\Rightarrow y \frac{d}{dx} \ln x + \ln x \frac{dy}{dx} = x \frac{d}{dx} \ln y + \ln y \frac{dx}{dx}$$

$$\theta \cos \theta = \frac{xy}{ab}$$

$$\Rightarrow y \cdot \frac{1}{x} + \ln x \frac{dy}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \ln y$$

$$\Rightarrow \ln x \frac{dy}{dx} - \frac{x}{y} \frac{dy}{dx} = \ln y - \frac{y}{x} \quad (iii)$$

$$\Rightarrow \frac{dy}{dx} \left( \ln x - \frac{x}{y} \right) = \ln y - \frac{y}{x}$$

$$\therefore \frac{dy}{dx} = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$$

51 (i)  $x = a \cos^3 \theta$

$$y = a \sin^3 \theta$$

$$\frac{dx}{d\theta} = 3 \cos^2 \theta \frac{d}{d\theta} (\cos \theta)$$

$$= -3 \cos^2 \theta \sin \theta$$

$$\frac{dy}{d\theta} = 3 \sin^2 \theta \frac{d}{d\theta} (\sin \theta)$$

$$= 3 \sin^2 \theta \cos \theta$$

$$\therefore \frac{dy}{dx} = -\frac{3 \sin^2 \theta \cos \theta}{3 \cos^2 \theta \sin \theta}$$

$$= -\frac{\sin \theta}{\cos \theta} = -\tan \theta$$

$$(ii) \quad x = \sin^2 \theta \quad \frac{dx}{d\theta} = 2 \sin \theta \cos \theta$$

$$y = \tan \theta \quad \frac{dy}{d\theta} = \sec^2 \theta$$

$$\frac{dy}{dx} = \frac{\sec^2 \theta}{2 \sin \theta \cos \theta} = \frac{\sec \theta}{2 \sin \theta}$$

$$\frac{dy}{dx} + \frac{1}{x} = \frac{\sec \theta}{2 \sin \theta} + \frac{1}{x}$$

$$(iii) \quad x = a \sec^2 \theta \quad \frac{dx}{d\theta} = \frac{ab}{\sin \theta} \frac{d\theta}{dx} = \frac{ab}{\sin \theta} \cot \theta$$

$$y = a \tan^2 \theta \quad \frac{dy}{d\theta} = \frac{ab}{\sin \theta} \frac{d\theta}{dx} = \frac{ab}{\sin \theta} \tan \theta$$

$$\frac{dy}{dx} = \frac{2 \tan \theta \sec \theta}{2 \sec^2 \theta \cdot \sec \theta \tan \theta} = \frac{2 \tan \theta \sec \theta}{2 \sec^3 \theta \tan \theta} = \frac{\tan \theta}{\sec^2 \theta} = \frac{\sin \theta}{\cos^2 \theta} = \frac{\sin \theta}{1 - \sin^2 \theta} = \frac{\sin \theta}{\cos^2 \theta}$$

$$\frac{dy}{dx} = \frac{\sin \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\sec \theta = x \quad (i)$$

$$\tan \theta = y$$

Comparing (i) & (ii)

Comparing (i) & (iii)

$\frac{dy}{dx} = \frac{1}{\cos^2 \theta}$

$$\underline{61} \quad (i) \quad \text{let, } y = \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right) \text{ then } = y \text{ stat} \quad (ii)$$

$$z = \tan^{-1} \left( \frac{2x}{1-x^2} \right) \text{ i.e. } = z$$

$$\frac{dy}{dx} = y = \cos^{-1} \left( \frac{1-\tan^2 \theta}{1+\tan^2 \theta} \right) \text{ let } x = \tan \theta$$

$$= \cos^{-1} (\cos 2\theta) \text{ i.e. } \theta = \tan^{-1} n$$

$$\frac{1}{1+x^2} = \frac{1}{1+\tan^2 \theta} = \frac{\sec^2 \theta}{\sec^2 \theta - 1} = \frac{\sec^2 \theta}{\tan^2 \theta} =$$

$$= 2 \tan^{-1} n$$

$$\frac{dy}{dx} = \frac{2}{1+n^2}$$

$$\left[ \frac{1}{1+n^2} + \frac{2}{n} \right] \text{ i.e. } = \frac{2b}{a^2}$$

$$z = \tan^{-1} \left( \frac{2\tan \theta}{1-\tan^2 \theta} \right)$$

$$= \tan^{-1} (\tan 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1} n$$

$$\frac{dz}{dn} = \frac{2}{1+n^2}$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dn}}{\frac{dz}{dn}} = 1$$

$$(ii) \text{ let, } y = x^{\sin^{-1}(x)} \stackrel{x \rightarrow 0}{\rightarrow} 1 \quad (i)$$

$$z = \sin^{-1}(x) \stackrel{x \rightarrow 0}{\rightarrow} 0 \quad (ii)$$

$$\ln y = \ln x^{\sin^{-1}(x)} \stackrel{x \rightarrow 0}{\rightarrow} \infty \quad (iii)$$

$$\Rightarrow \ln y = \sin^{-1}(x) \stackrel{x \rightarrow 0}{\rightarrow} 0 \quad (iv)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \sin^{-1}(x) \cdot \frac{1}{x} + \ln x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{\sin^{-1}(x)}{x} + \frac{\ln x}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = x^{\sin^{-1}(x)} \left[ \frac{\sin^{-1}(x)}{x} + \frac{\ln x}{\sqrt{1-x^2}} \right]$$

$$\left( \frac{\sin^{-1}(x)}{x} \right) \stackrel{x \rightarrow 0}{\rightarrow} 0 \quad (v)$$

$$\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$(0 \cdot \infty) \stackrel{x \rightarrow 0}{\rightarrow} 0$$

$$\therefore \frac{dy}{dz} = \frac{x^{\sin^{-1}(x)} \left[ \frac{\sin^{-1}(x)}{x} + \frac{\ln x}{\sqrt{1-x^2}} \right]}{\frac{1}{\sqrt{1-x^2}}}$$

$$7] \quad (i) \quad y = \ln \sqrt{\frac{1+\sin x}{1-\sin x}} \quad \text{and} \quad y = \theta \quad (i)$$

$$\Rightarrow y = \ln \left( \frac{1+\sin x}{1-\sin x} \right)^{1/2} \left( \frac{\cos -\theta}{\cos + \theta} \right) \text{ and } \frac{dy}{dx} = \frac{1}{2}$$

$$\Rightarrow y = \frac{1}{2} \ln \left( \frac{1+\sin x}{1-\sin x} \right) \left( \frac{\cos -\theta}{\cos + \theta} \right) \text{ and } \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow y = \frac{1}{2} [\ln(1+\sin x) - \ln(1-\sin x)]$$

$$[(\cos + \theta) \text{ and } (\cos - \theta)] \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \left[ \frac{1}{1+\sin x} \cdot \cos x - \frac{1}{1-\sin x} \cdot (-\cos x) \right]$$

$$[\sin x \cdot \frac{1}{\cos + \theta} + \sin x \cdot \frac{1}{\cos - \theta}] \frac{1}{2} = \frac{1}{2}$$

$$= \frac{1}{2} \left[ \frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x} \right]$$

$$[\frac{\sin x}{\cos + \theta} + \frac{\sin x}{\cos - \theta}] \frac{1}{2} =$$

$$= \frac{1}{2} \left[ \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1+\sin x)(1-\sin x)} \right]$$

$$[\frac{\cos x \sin x - \sin x + \cos x \sin x + \cos x}{(\cos + \theta)(\cos - \theta)}] \frac{1}{2} =$$

$$= \frac{1}{2} \left[ \frac{2 \cos x}{1-\sin x} \right]$$

$$= \left[ \frac{\cos x}{\frac{1-\sin x}{\cos x}} \right] \frac{1}{2} =$$

$$= \frac{1}{\cos x} \frac{\sin x}{\cos x} =$$

$$= \sec x$$

$$(ii) \quad y = \ln \sqrt{\frac{1-\cos n}{1+\cos n}} \xrightarrow{\text{multipl. by } \frac{2\pi i(2k+1)}{2\pi i(2k+1)}} n2 = B \quad (i) \quad 15$$

$$\Rightarrow y = \ln \left( \frac{1-\cos n}{1+\cos n} \right)^{1/(2\pi i(2k+1))} n2 = B \Leftarrow$$

$$\Rightarrow y = \frac{1}{2} \ln \left( \frac{1-\cos n}{1+\cos n} \right) n2 \frac{1}{2} = B \Leftarrow$$

$$[(n2k+1)n2(n2k+1)n2] \frac{1}{2} = B \Leftarrow$$

$$\Rightarrow y = \frac{1}{2} [\ln(1-\cos n) - \ln(1+\cos n)]$$

$$[(n2k+1) \xrightarrow{\text{cancel}} n2\cos \cdot \frac{1}{n2i(2k+1)}] \frac{1}{2} = \frac{1}{2} \xrightarrow{\text{cancel}} \frac{1}{2}$$

$$\frac{dy}{dn} = \frac{1}{2} \left[ \frac{1}{1-\cos n} \cdot \sin n + 1 \cdot \frac{1}{1+\cos n} \cdot \sin n \right]$$

$$\left[ \frac{n2\cos}{n2i(2k+1)} + \frac{n2\cos}{n2i(2k+1)} \right] \frac{1}{2} =$$

$$= \frac{1}{2} \left[ \frac{\sin n}{1-\cos n} + \frac{\sin n}{1+\cos n} \right]$$

$$\left[ \frac{n2\cos(n2k+1)+n2\cos(n2k+1)-n2\cos}{n2i(2k+1)(n2i(2k+1))} \right] \frac{1}{2} =$$

$$= \frac{1}{2} \left[ \frac{\sin n i \cancel{n2} \cos n + \sin n - \sin n \cos n}{\cancel{n2i(2k+1)} (1+\cos n) (1-\cos n)} \right]$$

$$= \frac{1}{2} \left[ \frac{2 \sin n}{1-\cos n} \right] =$$

$$= \frac{\sin n}{\sin n} \frac{1}{\cancel{1-\cos n}} =$$

$$= \frac{1}{\sin n} = \csc n$$

$$= \csc n$$

$$(iii) \quad y = \tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}} \quad (\text{notes}) \text{ n/c} = B \quad (v)$$

$$= \tan^{-1} \sqrt{\frac{1-\cos 2(x/2)}{1+\cos 2(x/2)}} \quad (\text{notes}) \text{ n/c} =$$

$$\pi^2 - 200 = 0 \quad \therefore = \tan^{-1} \sqrt{\frac{2\sin^2(\frac{x}{2})}{2\cos^2(\frac{x}{2})}} \quad (\text{notes}) \text{ n/c} =$$

$$= \tan^{-1}(\tan \frac{x}{2})$$

$$= \frac{x}{2} \quad (\text{notes}) \text{ n/c} =$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot (\text{notes}) \text{ n/c} =$$

$$(iv) \quad y = \tan^{-1} \left( \frac{\cos x - \sin x}{\cos x + \sin x} \right)$$

$$= \tan^{-1} \left[ \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}} \right] \quad (\text{notes}) \text{ n/c} = \frac{ab}{a+b}$$

$$= \tan^{-1} \left[ \frac{1 - \tan x}{1 + \tan x} \right]$$

$$= \tan^{-1} \left[ \frac{\tan 45^\circ - \tan x}{1 + \tan 45^\circ (\tan x)} \right]$$

$$= \tan^{-1} [\tan (45^\circ - x)]$$

$$= 45^\circ - x$$

$$\therefore \frac{dy}{dx} = -1$$

$$(v) \quad y = \sin \left( 2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) \text{ root } = \beta \quad (\text{iii})$$

$$= \sin \left( 2 \tan^{-1} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \right) \text{ let } x = \cos\theta$$

$$= \sin \left( 2 \tan^{-1} \sqrt{\frac{1-\cos 2(\theta/2)}{1+\cos 2(\theta/2)}} \right) \Rightarrow \theta = \cos^{-1} x$$

$$= \sin \left( 2 \tan^{-1} \sqrt{\frac{2\sin^2 \theta/2}{2\cos^2 \theta/2}} \right)$$

$$= \sin \left( 2 \tan^{-1} \tan \theta/2 \right) \quad \frac{\pi}{2} = \frac{\pi b}{ab}$$

$$= \sin \theta$$

$$= \sin \cdot \cos^{-1} x \quad \left( \frac{\sin(\theta/2 - \pi/2)}{\sin(\theta/2 + \pi/2)} \right) \text{ root } = b \quad (\text{vi})$$

$$\frac{dy}{dx} = \cancel{\cos \theta} \cos(\cos^{-1} x) \cdot \left( -\frac{1}{\sqrt{1-x^2}} \right) \text{ root } =$$

$$= - \frac{\cancel{x}}{\sqrt{1-x^2}} \left[ \frac{\cancel{\cos(\theta/2 - \pi/2)}}{\cancel{\cos(\theta/2 + \pi/2)}} \right] \text{ root } =$$

$$\left[ \frac{\cos(\theta/2 - \pi/2)}{\cos(\theta/2 + \pi/2)} \right] \text{ root } =$$

## Maxima and Minima

1) (i)  $f(x) = x^2 - 5x + 6$

$$f'(x) = 2x - 5$$

$$f''(x) = 2$$

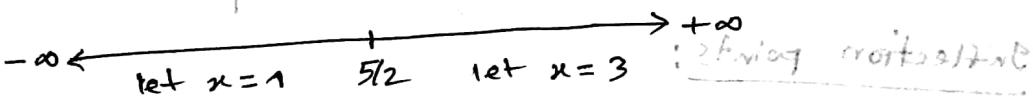
critical points:

$$f'(x) = 0$$

or  $f'(x)$  is undefined

$$0 \geq (2x - 5) \Rightarrow 2x - 5 = 0$$

$$\Rightarrow x = 5/2$$



$$f'(1) < 0$$

decreasing

$$f'(3) > 0$$

increasing

$f$  is increasing in the intervals  $(5/2, \infty)$

$f''(x) < 0$  decreasing in the intervals  $(-\infty, 5/2)$

relative maxima

inflection points:

$$f''(x) = 0$$

or  $f''(x)$  is undefined

$$\Rightarrow 2 = 0$$

$$\therefore f''(x) = 2 > 0$$

$\therefore$  function is concave up on  $\mathbb{R}$ .

$$(ii) f(x) = 5 + 12x - x^3$$

$$f'(x) = 12 - 3x^2$$

$$f''(x) = -6x$$

critical points:

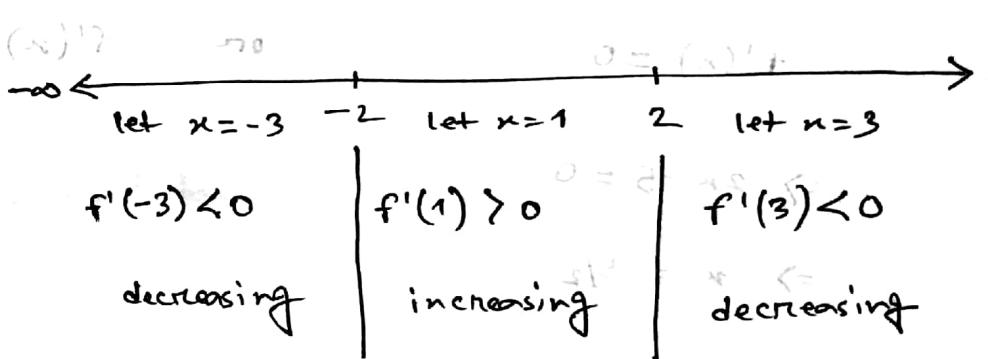
$$f'(x) = 0$$

or  $f'(x)$  is undefined

$$\Rightarrow 12 - 3x^2 = 0$$

$$\Rightarrow 3x^2 = 12$$

$$\Rightarrow x = \pm 2$$



inflection points:

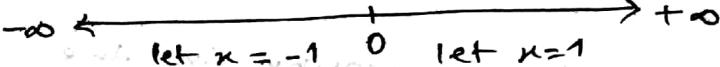
$$f''(x) = 0$$

or  $f''(x)$  is undefined

$$\Rightarrow -6x = 0$$

$$\Rightarrow x = 0$$

$$(0, \infty)$$



$$(0, \infty)$$

concave up      concave down

at  $x = 0$   $f'(0) = 12$   $f''(0) = 0$

$$0 = f''(0)$$

$$0 < x < 0$$

$$(iii) \quad f(x) = x^4 - 8x^2 + 16$$

$$\frac{f''(x)}{x^2} = (x+2)^2 = (x+2)$$

$$f'(x) = \frac{4x^3 - 16x}{x^2} = 4(x-2)$$

$$f''(x) = 12x^2 - 16$$

$$\frac{f''(x)}{x^2} = \frac{12(x-2)}{x^2} = 12(x-2)$$

critical points:

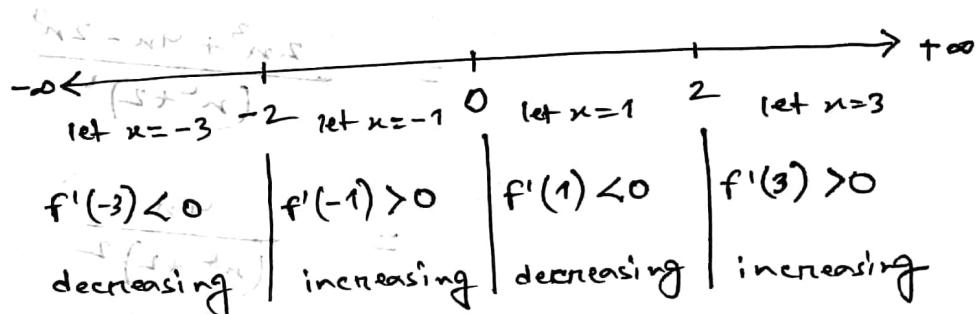
$$f'(x) = 0$$

or  $f'(x)$  is undefined

$$\Rightarrow 4x^3 - 16x = 0$$

$$\Rightarrow 4x(x^2 - 4) = 0$$

$$\Rightarrow x = 0, \pm 2$$



inflection points:

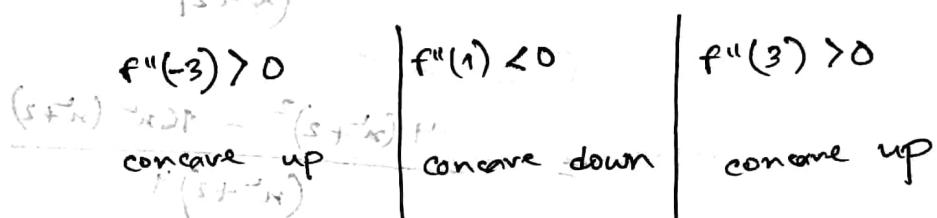
$$f''(x) = 0$$

$$\Rightarrow 12x^2 - 16 = 0$$

$$\Rightarrow 12x^2 = 16$$

$$\Rightarrow 3x^2 = 4$$

$$\Rightarrow x = \pm \frac{2}{\sqrt{3}}$$



$$(iv) \quad f(x) = \frac{x^2}{x^2+2} \quad \lim_{x \rightarrow \infty} f(x) = (x)^2 \quad (iii)$$

$$f'(x) = \frac{(x^2+2) \cdot \frac{d}{dx}(x^2) + -x^2 \cdot \frac{d}{dx}(x^2+2)}{(x^2+2)^2} = (x)^4$$

$$= \frac{(x^2+2) \cdot 2x - x^2 \cdot 2x}{(x^2+2)^2}$$

2. f'riq. bestimmen

berechnen in  $(x)^4$

$$x = \frac{(x^2+2) \cdot 2x - 2x^3}{(x^2+2)^2}$$

$$0 = (x)^4$$

$$0 = 4x^3 - 2x^2 \leftarrow$$

$$0 = (x^2 - x) \cdot x^2 \leftarrow$$

$$x^2 - x = 0 \leftarrow$$

$$\text{Faktor } x \quad \text{Faktor } (x-1) \quad 0 = \frac{2x^3 + 4x^2 - 2x^3}{(x^2+2)^2} \leftarrow$$

$$\text{O.S. } (x)^4 \quad \left| \begin{array}{l} \text{O.S. } (x)^4 \\ \text{Faktor } (x-1)^2 \end{array} \right| \quad 0 < (x-1)^2 \quad \left| \begin{array}{l} 0 < (x-1)^2 \\ \text{Faktor } (x-1)^2 \end{array} \right| \quad 0 < (x-1)^2 \quad \left| \begin{array}{l} 0 < (x-1)^2 \\ \text{Faktor } (x-1)^2 \end{array} \right| \quad 0 < (x-1)^2$$

$$f''(x) = \frac{(x^2+2)^2 \cdot \frac{d}{dx}(4x^2 - 4x \cdot \frac{d}{dx}(x^2+2)) - (x^2+2)^2 \cdot 2x^2 \cdot 2x}{(x^2+2)^4} \quad \text{2. f'riq. nachrechnen}$$

$$0 = (x)^{12}$$

$$0 = (x^2+2)^2 \cdot 4 - 4x \cdot 2(x^2+2) \cdot 2x \leftarrow$$

$$\text{O.S. } (x)^4 \quad \left| \begin{array}{l} \text{O.S. } (x)^4 \\ \text{Faktor } (x-1)^2 \end{array} \right| \quad 0 < (x-1)^2 \quad \left| \begin{array}{l} 0 < (x-1)^2 \\ \text{Faktor } (x-1)^2 \end{array} \right| \quad 0 < (x-1)^2 \quad \left| \begin{array}{l} 0 < (x-1)^2 \\ \text{Faktor } (x-1)^2 \end{array} \right| \quad 0 < (x-1)^2$$

$$= \frac{(x^2+2) \{ 4(x^2+2) - 16x^2 \}}{(x^2+2)^4}$$

$$= \frac{4x^2 + 8 - 16x^2}{(x^2+2)^3}$$

$$= \frac{8 - 12x^2}{(x^2 + 2)^3}$$

$\lim_{x \rightarrow \pm\infty} f(x) = (\infty) \neq (-\infty)$

$$\lim_{x \rightarrow \pm\infty} f'(x) =$$

critical points:

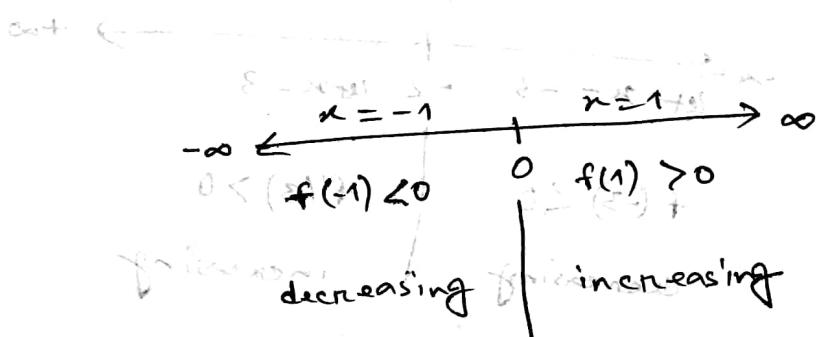
$$f'(x) = 0$$

$$\Rightarrow \frac{4x}{(x^2 + 2)^2} = 0$$

but since  $x < 0$  no solution

$$\Rightarrow 4x = 0$$

$$\Rightarrow x = 0$$



inflection points:

$f''(x) = \frac{16x^3 - 48x}{(x^2 + 2)^4}$

$$f''(x) = 0$$

$$\Rightarrow \frac{8 - 12x^2}{(x^2 + 2)^3} = 0$$

$$\Rightarrow 8 - 12x^2 = 0$$

$$\Rightarrow 12x^2 = 8$$

$$\Rightarrow 3x^2 = 2$$

$$\Rightarrow x^2 = \frac{2}{3}$$

$$\therefore x = \pm \sqrt{\frac{2}{3}}$$

$$\lim_{x \rightarrow \pm\infty} f''(x) = (-\infty) \neq$$

$\lim_{x \rightarrow \pm\infty} f''(x)$  is undefined

denominator = 0

$$\Rightarrow (x^2 + 2)^3 = 0$$

$$\Rightarrow x^2 + 2 = 0$$

$$\Rightarrow x = \pm \sqrt{-2}$$

no real solution

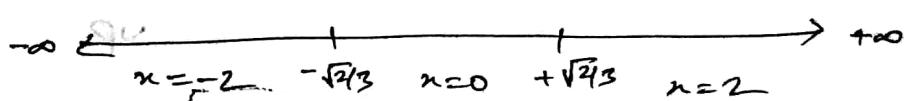
$$\lim_{x \rightarrow \pm\infty} f''(x) = \infty$$

strictly monotonic

$$(x^2 + 2)^3 = 0$$

$$\Rightarrow x = \pm \sqrt{-2}$$

no real solution



$$f''(-2) < 0$$

concave down

$$f''(0) > 0$$

concave up

$$f''(2) < 0$$

concave down

$$(v) f(x) = \sqrt[3]{x+2}$$

$$= (x+2)^{1/3}$$

$$f'(x) = \frac{1}{3}(x+2)^{-2/3} \cdot 1$$

$$f''(x) = -\frac{2}{9}(x+2)^{-5/3} \cdot 1$$

$\Rightarrow$  no minimums

critical points:  $f'(x) = 0$

$$\Rightarrow \frac{1}{3}(x+2)^{-2/3} = 0$$

no solution for  $x$  or

$$\Rightarrow x = -2$$

or  $f(x)$  is undefined

$$x = -\infty \text{ or } +\infty$$

$$x = -2$$

$$\begin{array}{c|c} x & \\ \hline -\infty & \text{let } x = -3 \\ & f'(-3) < 0 \\ & \text{decreasing} \end{array} \quad \begin{array}{c|c} x & \\ \hline -2 & \text{let } x = 3 \\ & f'(3) > 0 \\ & \text{increasing} \end{array} \quad \rightarrow +\infty$$

inflection points:

$$f''(x) = 0$$

$$\Rightarrow -\frac{2}{9}(x+2)^{-5/3} = 0$$

$$\Rightarrow x = -2$$

or  $f''(x)$  is undefined

$$x$$

$$\begin{array}{c|c} x & \\ \hline -\infty & \text{let } x = -3 \\ & f''(-3) > 0 \\ & \text{up} \end{array} \quad \begin{array}{c|c} x & \\ \hline -2 & \text{let } x = 3 \\ & f''(3) < 0 \\ & \text{down} \end{array} \quad \rightarrow +\infty$$

$$f''(-3) > 0$$

up

$$f''(3) < 0$$

down

2 (i)  $f(x) = x^3 + 3x^2 - 9x + 1$   $\therefore (x) \neq \text{d. v.}$  (ii)

$$f'(x) = 3x^2 + 6x - 9 \rightarrow \exists_{x \in \mathbb{R}} = (x)' \neq$$

critical points:

$$f'(x) = 0$$

$$\Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow x^2 - n + 3n - 3 = 0$$

$$\Rightarrow x(x+1) + 3(x+1) = 0$$

$$\therefore x = -1, -3$$

or  $f'(x)$  is undefined

$$0 = (x)' \neq$$

$$0 = x^2 - 3x \neq$$

$$\exists x \neq 0 = x \neq$$

stationary points:

$$0 = (x)' \neq$$

$$\exists x \neq 0 = x \neq$$

stationary points:

$$f'(x) = 0$$

$$\frac{x}{x+2} = (x) \neq \quad (iii)$$

$$\Rightarrow x = -3, 1$$

(ii)  $f(x) = 5 + 12x - x^3$

$$f'(x) = 12 - 3x^2 \frac{(x+2)}{(x+2)}$$

critical points:

$$\frac{x+2}{(x+2)}$$

$$(ii) f(x) = x^4 - 6x^2 - 3 \quad \text{and } f'(x) = (x)^4 \quad (i)$$

$$f'(x) = 4x^3 - 12x \quad \text{and } f''(x) = (x)^1$$

critical points:

$$f'(x) = 0$$

$$\Rightarrow 4x^3 - 12x = 0$$

$$\Rightarrow 4x(x^2 - 3) = 0$$

$$\Rightarrow x = 0, \pm\sqrt{3}$$

stationary points

X

$$0 = (x)^4$$

$$0 = \epsilon - x\delta + \bar{x}\epsilon \Leftarrow$$

$$0 = \epsilon - x\delta + \bar{x} \Leftarrow$$

$$0 = \epsilon - x\delta + x\bar{\delta} - \bar{x} \Leftarrow$$

$$0 = (1+x)\epsilon + (1-\bar{x})x \Leftarrow$$

stationary points:

$$f'(x) = 0$$

$$\epsilon, \bar{\epsilon}, x = \infty, -\infty$$

$$\Rightarrow x = 0, \pm\sqrt{3}$$

using monotone

$$(iii) f(x) = \frac{x}{x^2 + 2}$$

$$0 = (x)^4$$

$$f'(x) = \frac{(x^2 + 2) \frac{d}{dx} x - x \frac{d}{dx} (x^2 + 2)}{(x^2 + 2)^2} = x \Leftarrow$$

$$x - x\delta + \bar{x} = (x)^4 \quad (i)$$

$$= \frac{x^2 + 2 - 2x^2}{(x^2 + 2)^2} = (x)^4$$

$$= \frac{2 - x^2}{(x^2 + 2)^2}$$

using limit

to show  $(i) \Rightarrow$

$$0 = (x)^4$$

$$0 = \epsilon - x\delta + \bar{x} \Leftarrow$$

critical points:

(at  $x = \infty$ )  $f'(x)$  is undefined

$$f'(x) = 0$$

$$\Rightarrow \frac{2-x^2}{(x+2)^2} = 0$$

$$\Rightarrow 2-x^2 = 0$$

$$\Rightarrow x = \pm\sqrt{2}$$

$$\lim_{x \rightarrow -2^+} f'(x) = \infty$$

$$\Rightarrow x+2 > 0$$

$$\Rightarrow x = (\infty)^{1/2}$$

$$\Rightarrow x = \pm\sqrt{2}$$

∴ no real solution

stationary points:

$$f'(x) = 0$$

$$\Rightarrow x = \pm\sqrt{2}$$

$$\frac{P + xP}{e^{1/x} x^2} = 0$$

Let's try to fit in

(iv)  $f(x) = x^{2/3}$

$$f'(x) = \frac{2}{3} x^{-1/3}$$

$$0 = \frac{2}{3x^{1/3}}$$

critical points:

$$f'(x) = 0$$

$$\Rightarrow \frac{2}{3x^{1/3}} = 0$$

$$\Rightarrow 2 = 0$$

no solution

$$0 = (x)^{1/3}$$

$$0 = \frac{P+xP}{e^{1/x} x^2} \Leftrightarrow$$

or  $f'(x)$  is undefined

$$3x^{1/3} = 0 \Leftrightarrow$$

$$\Rightarrow x = 0$$

Let's try to find the

stationary points

stationary points:

$$f'(x) = 0$$

no stationary points.

for  $x < 0$

$$(v) f(x) = x^{1/3}(x+4)$$

string test

$$\cancel{f'(x)} (= x^{4/3} + 4x^{-1/3})$$

$0 = (x)^{1/3}$

$$f'(x) = \frac{4}{3} x^{1/3} + \frac{4}{3} x^{-2/3}$$

$$0 = \frac{4x-4}{3(x+2)}$$

$$\begin{aligned} \cancel{x^3 + x^4} &= 0 \Leftrightarrow \\ \text{notable form} &\quad = \frac{4}{3} x^{1/3} + \frac{4}{3} \frac{1}{x^{2/3}} \end{aligned}$$

$$0 = 4x - 4$$

$$= \frac{4x^{1/3} \cdot x^{2/3} + 4}{3x^{2/3}}$$

string form

$$= \frac{4x + 4}{3x^{2/3}}$$

$0 = (x)^{1/3}$

$$\cancel{x^3 + x^4} = 0$$

critical points:

or  $f'(x)$  is undefined (vi)

$$f'(x) = 0$$

$$\cancel{3x^{2/3}} = 0$$

$$\Rightarrow \frac{4x+4}{3x^{2/3}} = 0$$

$$\cancel{\frac{4}{3}} \Rightarrow x = 0$$

$$\begin{aligned} \text{notable form} &\\ \Rightarrow 4x+4 &= 0 \end{aligned}$$

string test

$$\Rightarrow x = -1$$

$0 = (-1)^{1/3}$

$$0 = x \in \mathbb{C}$$

$$0 = \frac{4}{3x^{2/3}}$$

stationary points:

$$f'(x) = 0$$

$0 = x \in \mathbb{C}$

$$\Rightarrow x = -1$$

string of points

$0 = (x)^{1/3}$

$$(vi) \quad f(x) = \cos 3x \quad \text{for } 0 < x < \pi/2$$

$$f'(x) = -3 \sin 3x \quad \text{at } x = \frac{\pi}{2} = 0$$

critical points:

$f'(x) = 0$  via, b. Tkt given condition

$$\Rightarrow -3 \sin 3x = 0$$

$$\Rightarrow \sin 3x = 0$$

$$\Rightarrow \sin 3x = \sin n\pi$$

$$\Rightarrow 3x = n\pi$$

Einsetzen in obige F. Gleichung für

$$0 < (x) \Rightarrow x = \frac{n\pi}{3} \quad [n = 0, \pm 1, \pm 2, \dots]$$

stationary points:

$$f'(x) = 0$$

$$\Rightarrow x = \frac{n\pi}{3}$$

3] (i)  $f(x) = 2x^3 - 9x^2 + 12x$   $\therefore f'(x) = \underline{}$  (iv)

$$f'(x) = 6x^2 - 18x + 12 \quad \therefore \underline{(v)}$$

$$f''(x) = 12x - 18$$

Relative extrema using 1st derivative test:

critical points:

or  $f'(x)$  is undefined

$\circlearrowleft x = 3$   $\circlearrowright$

$$f'(x) = 0$$

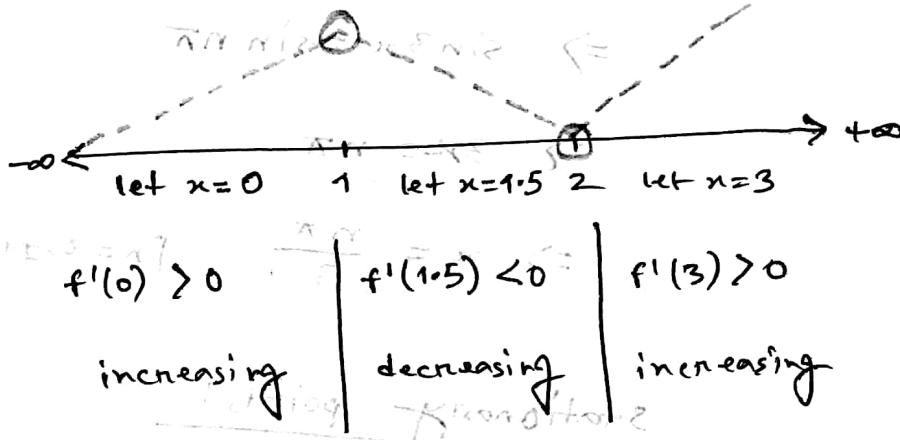
$$\Rightarrow 6x^2 - 18x + 12 = 0$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - x - 2x + 2 = 0$$

$$\Rightarrow (x-1)(x-2) = 0$$

$$\Rightarrow x = 1, 2$$



$\circlearrowleft (iv) \circlearrowright$

$\therefore$  relative maximum at  $x = 1$  and

" minimum at  $x = 2$

Relative extrema using 2nd derivative test:

| $x$ | $f''(x)$     | comment |
|-----|--------------|---------|
| 1   | $f''(1) < 0$ | maximum |
| 2   | $f''(2) > 0$ | minimum |

$$(ii) f(x) = \frac{x}{2} - \sin x \quad (x, \text{ s.t. } 0^{\circ} \leq x < 2\pi) \quad (i)$$

$$f'(x) = \frac{1}{2} - \cos x$$

$$f''(x) = \sin x$$

Critical points:

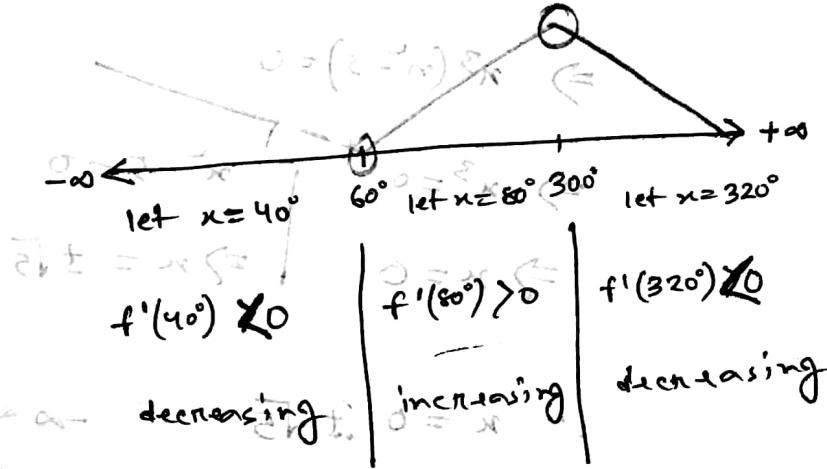
$$f'(x) = 0$$

$$\Rightarrow \frac{1}{2} - \cos x = 0$$

$$\Rightarrow \cos x = \frac{1}{2}$$

$$\Rightarrow \cos x = \cos 60^{\circ}, \cos 300^{\circ}$$

$$\begin{cases} \text{for } x = 60^{\circ}, \\ \text{for } x = 300^{\circ} \end{cases}$$



∴ relative maximum at  $x = 300^{\circ}$   
" " minimum at  $x = 60^{\circ}$

Relative extrema using 2nd derivative test:

| x             | $f''(x)$               | Comment |
|---------------|------------------------|---------|
| $60^{\circ}$  | $f''(60^{\circ}) > 0$  | minimum |
| $300^{\circ}$ | $f''(300^{\circ}) < 0$ | maximum |

extrema by 2nd D.T.

|              |                 |         |
|--------------|-----------------|---------|
| $\sin x < 0$ | $(x)^{1/2} < 0$ | $x < 0$ |
| $\sin x > 0$ | $(x)^{1/2} > 0$ | $x > 0$ |
| $\sin x = 0$ | $(x)^{1/2} = 0$ | $x = 0$ |
| $\sin x > 0$ | $(x)^{1/2} > 0$ | $x > 0$ |

$$\boxed{4} \quad (\text{i}) \quad f'(x) = x^3(x^2 - 5) \quad \text{where } -\frac{x}{5} = (x) \quad (\text{ii})$$

$$f''(x) = 5x^4 - 15x^2$$

$$\cos = \frac{1}{2} \pi = \left(\frac{\pi}{2}\right)^{-1}$$

or  $f'(x)$  is undefined

$$\text{series} = (x)^n$$

$$f'(x) = 0$$

$$\Rightarrow n^3(n-5) = 0$$

## String Lattices

$$\Rightarrow x^3 \geq 0$$

$$\Rightarrow x^3 = 0 \quad | \quad x - 5 = 0$$

$$\Rightarrow x = 0$$

$\zeta(-q)^7$

$$\therefore x = 0, \pm\sqrt{5}$$

W. N. & D. S. M.

Re: ~~REVIEW OF THE~~

1724 2

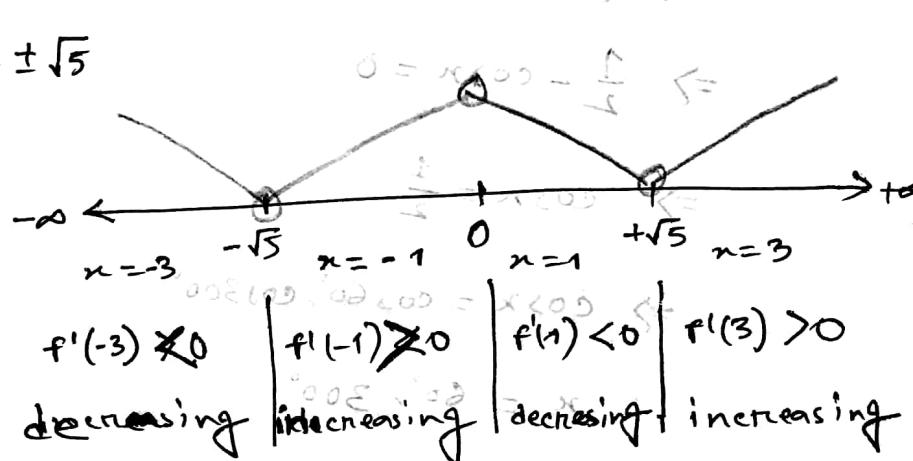
ing 2nd

o f

$$+\sqrt{3} \quad f'$$

$$= \sqrt{5} - f$$

42 | P



relative maximum at  $x=0$   
 and  
 " minimum at  $x = \pm\sqrt{5}$

Using 2nd derivative:

|                    |                                  |     |
|--------------------|----------------------------------|-----|
| <u>transient</u>   | $(\text{vol})^{\frac{1}{2}}$     | 40  |
| <u>noninvasive</u> | $0.1 (\text{vol})^{\frac{1}{2}}$ | 0.3 |
| <u>ment</u>        | $0.3 (\text{vol})^{\frac{1}{2}}$ | 0.8 |
| <u>relative</u>    |                                  |     |

| $x$         | $f''(x)$             | comment      |
|-------------|----------------------|--------------|
| 0           | $f''(0) = 0$         | inconclusive |
| $+\sqrt{3}$ | $f''(\sqrt{3}) > 0$  | minimum      |
| $-\sqrt{3}$ | $f''(-\sqrt{3}) < 0$ | minimum      |

## Leibnitz's Theorem

08-Nov-201

12]

$$x = \tan(\ln y)$$

$$\Rightarrow \ln y = \tan^{-1} x$$

$$\Rightarrow y = e^{\tan^{-1} x}$$

— ①

Differentiating ① with respect to  $x$ ,

$$y_1 = e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} = \left( e^{\tan^{-1} x} \right) \cdot \frac{1}{1+x^2} + \left( e^{\tan^{-1} x} \right) \cdot \frac{1}{(1+x^2)^2} \cdot 2x$$

$$\Rightarrow (1+x^2)y_1 = y \quad — ②$$

Differentiating ② with respect to  $x$ ,

$$(1+x^2) \frac{d}{dx}(y_1) + y_1 \frac{d}{dx}(1+x^2) = \frac{d}{dx}(y)$$

$$\Rightarrow (1+x^2)y_2 + 2x y_1 = y_1$$

$$\Rightarrow (1+x^2)y_2 + (2x-1)y_1 = 0 \quad — ③$$

Applying Leibnitz's theorem in equation ③

$$\frac{d^n}{dx^n} [(1+x^2)y_2] + \frac{d^n}{dx^n} [(2x-1)y_1] = \frac{d^n}{dx^n} [0]$$

$$\Rightarrow [y_{n+2}(1+x^2) + n c_1 y_{n+1} \cdot 2x + n c_2 y_n \cdot 2 + n c_3 y_{n-1} \cdot 0]$$

$$+ [y_{n+1}(2x-1) + n c_1 y_n \cdot 2 + n c_2 y_{n-1} \cdot 0] = 0$$

$$\Rightarrow (1+x^2)y_{n+2} + 2nx y_{n+1} + \frac{n(n-1)}{2} y_n \cdot 2 + (2x-1)y_{n+1} + 2ny_n = 0$$

$$\Rightarrow (1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0$$

Showed

$$g \rightarrow \infty \quad \log_e y = \tan^{-1} x \quad \text{as } x \rightarrow \infty$$

$$\Rightarrow \ln y = \tan^{-1} x$$

$$\Rightarrow y = e^{\tan^{-1} x}$$

$$\Rightarrow y_1 = e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$\Rightarrow y_1(1+x^2) = y$$

so by taking ratio ① & ② we get

$$\Rightarrow (1+x^2) y_1 = y$$

$$\Rightarrow (1+x^2) \frac{dy_1}{dx} + y_1 \cdot 2x = \frac{dy}{dx}$$

$$\Rightarrow (1+x^2) y_2 + 2x y_1 = y_1$$

$$\Rightarrow (1+x^2) y_2 + (2x-1) y_1 = 0$$

$$\Rightarrow [y_{n+2}^{(0)}(1+x^2) + n c_1 y_{n+1} \cdot 2x + n c_2 y_n \cdot 2 + n c_3 y_{n-1} \cdot 0]$$

$$+ [y_{n+1}(2x-1) + n c_1 y_n \cdot 2 + n c_2 y_{n-1} \cdot 0] = 0$$

$$\Rightarrow (1+x^2) y_{n+2} + y_{n+1} \cdot 2nx + \frac{n(n-1)}{2} y_n \cdot 2 + (2x-1) y_{n+1} + 2n y_n = 0$$

$$\Rightarrow [(1+x^2) y_{n+2} + (2nx+2x-1) y_{n+1} + (n^2-n+2n) y_n] = 0$$

$$\Rightarrow (1+x^2) y_{n+2} + (2nx+2x-1) y_{n+1} + n(n+1) y_n = 0$$

$$\Rightarrow [(1+x^2) y_{n+2} + (2nx+2x-1) y_{n+1} + n(n+1) y_n] = 0$$

Showed

$$y = 41^{n+1} \cdot \sin((n+1)x) + 45^n \cdot \cos((n+1)x)$$

same as g.

$$g = 41^n \cdot \sin(nx) + 45^n \cdot \cos((n+1)x) + 45^n \cdot \sin((n+1)x)$$

$$1 \quad y = \tan^{-1} x \quad x^2 + y^2 = 1$$

$$\Rightarrow y_1 = \frac{1}{1+x^2}$$

$$\Rightarrow (1+x^2)y_1 = 1 \quad 1 = \alpha b(x+\beta) \Leftarrow$$

$$\Rightarrow (1+x^2) \frac{dy_1}{dx} + y_1 \frac{d}{dx}(1+x^2) = \frac{d}{dx}(1) \quad \alpha b + (\alpha b)' \frac{b}{\alpha b} (x+\beta) \Leftarrow$$

$$\Rightarrow (1+x^2)y_2 + 2x \cdot y_1 = 0 \quad 0 = \alpha s \cdot \alpha b + \alpha b (x+\beta) \Leftarrow$$

$$\begin{aligned} & \Rightarrow [y_{n+2}(1+x^2) + n c_1 y_{n+1} \cdot 2x + n c_2 y_n \cdot 2 + n c_3 y_{n-1} \cdot 0] \\ & [0 \cdot \alpha s - \alpha b x^{3n+2} + \alpha b x^{3n+1} + \alpha s \cdot \alpha b x^{3n+1} + (x+\beta) \frac{\alpha b}{\alpha s + \alpha b}] \Leftarrow \\ & + [y_{n+1} 2x + n c_1 y_n \cdot 2 + n c_2 y_{n-1} \cdot 0] = 0 \\ 0 & = [0 \cdot \alpha s - \alpha b x^{3n+2} + \alpha b x^{3n+1} + \alpha s x^{3n+1}] + \end{aligned}$$

$$\Rightarrow (1+x^2)y_{n+2} + y_{n+1} \cdot 2nx + 2 \cdot \frac{n(n-1)}{2} y_n + 2x \cdot y_{n+1} + 2ny_n = 0$$

$$\Rightarrow (1+x^2)y_{n+2} + (2nx+2x)y_{n+1} + (n^2-n+2n)y_n = 0$$

$$0 = \alpha b(\alpha s - \alpha s - \alpha b) + \alpha s \alpha b (x s + x n s) + \alpha s \alpha b (x + \beta) \Leftarrow$$

$$\Rightarrow (1+x^2)y_{n+2} + 2(n+1)x y_{n+1} + n(n+1)y_n = 0$$

$$0 = \alpha b(1+n)s + \alpha b x(n+1)s + \alpha s \alpha b (x + \beta) \Leftarrow$$

Showed

b20263

$$2) \quad y = \cot^{-1} x$$

$$x^2 \tan^{-1} x = b - 1$$

$$\Rightarrow y_1 = -\frac{1}{1+x^2}$$

$$\overrightarrow{AB} = \overrightarrow{CB} \Leftarrow$$

$$\Rightarrow (1+x^2)y_1 = -1$$

$$1 = AB(S_i + 1) \Leftarrow$$

$$\Rightarrow (1+x^2) \frac{d}{dx}(y_1) + y_1 \frac{d}{dx}(1+x^2) = \frac{d}{dx}(-1) \frac{b}{x^2}(S_i + 1) \Leftarrow$$

$$\Rightarrow (1+x^2)y_2 + y_1 \cdot 2x = 0 \quad 0 = rB \cdot nS + sB(S_i + 1) \Leftarrow$$

$$\begin{aligned} & [0 - nb \cdot e^{2x} + s \cdot nb \cdot e^{2x} + xs \cdot ne^{2x} + (S_i + 1) \cdot ne^{2x}] \Leftarrow \\ & \Rightarrow [y_{n+2}(1+x^2) + nc_1 y_{n+1} \cdot 2x + nc_2 y_n \cdot 2 + nc_3 y_{n-1} \cdot 0] \end{aligned}$$

$$0 = [0 - nb \cdot e^{2x} + s \cdot nb \cdot e^{2x} + xs \cdot ne^{2x}] + [y_{n+1} \cdot 2x + nc_1 y_n \cdot 2 + nc_2 \cdot y_{n-1} \cdot 0] = 0$$

$$0 = nb \cdot nS + nb \cdot xs + nb \frac{(n-1)n}{2} \cdot 2 + xs \cdot n \cdot nb + s \cdot nb(S_i + 1) \Leftarrow$$

$$\Rightarrow (1+x^2)y_{n+2} + 2nx y_{n+1} + 2 \cdot \frac{n(n-1)}{2} + 2x y_{n+1} + 2n y_n = 0$$

$$0 = nb(nS + n - 1) + nb(xs + nS) + nb(S_i + 1) \Leftarrow$$

$$\Rightarrow (1+x^2)y_{n+2} + (2nx + 2n) y_{n+1} + (n^2 - n - 2n) y_n = 0$$

$$0 = nb((1+n)x^2 + nb(x(n+n))S + s \cdot nb(S_i + 1)) \Leftarrow$$

$$\Rightarrow (1+x^2)y_{n+2} + 2(n+1)x y_{n+1} + n(n+1) y_n = 0$$

hence

Showed

$$3] \quad y \sqrt{1-x^2} = \sin^{-1} x$$

$$\Rightarrow y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1 = \frac{\sqrt{1-x^2} \frac{d}{dx}(\sin^{-1}x) - \sin^{-1}x \frac{d}{dx}(\sqrt{1-x^2})}{(\sqrt{1-x^2})^2}$$

$$\Rightarrow y_1 = \frac{\sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}} - (\sin^{-1}x \cdot \frac{1}{2\sqrt{1-x^2}}) \cdot (-2x)}{1-x^2} = \frac{\sin^{-1}x + x}{\sqrt{1-x^2}} \quad <$$

$$\Rightarrow y_1 = \frac{1 + \frac{x \sin x}{\sqrt{1-x^2}}}{1-x^2} + 5t^5 \cdot 10^{10} (t-1) \text{ 不}$$

$$\Rightarrow (1-x^2) y_1 = 1 + x \star$$

$$\Rightarrow \cancel{(1-\mu)} \frac{d}{dx}(\cancel{y_1}) + y_1 + \cancel{\frac{A}{\mu x} (1-x^2)} = \cancel{\frac{d}{dx}}(1+x+y)$$

$$\Rightarrow + (1-x^2) y_1 - xy \left( \begin{matrix} = 1 \\ n+1 \end{matrix} \right)_{x+yB} x^n + (x-y)_{x+yB} \right] \in$$

$$\Rightarrow [y_{n+1}(1-x^2) + n c_1 y_n(-2x) + n c_2 y_{n-1}(-2) + n c_3 y_{n-2} \cdot 0] - [y_n \cdot x + n c_1 y_{n-1} \cdot 1 + n c_2 y_{n-2} \cdot 0] = 0$$

$$(\sin x + \cos x) = \text{ab} \cdot \frac{(x-a)\alpha}{s} = s - \sin \theta \cos(x-s) + \sin \theta (x-a)$$

$$\Rightarrow (1-x^2)y_{n+1} - 2nx y_n - n(n-1)y_{n-1} - x y_n - ny_{n-1} = 0$$

$$0 = \alpha y_n - \Rightarrow (1-\alpha^2)y_{n+1} - (2\alpha + 1) \alpha y_n + \alpha^2 y_{n-1} = 0$$

$$Q = \rho^2 \left( \frac{1}{r} + \frac{1}{n} + \frac{1}{r} e^{2\phi} \right) - r^2 n^2 e^{2\phi} \left( 1 + n^2 \right) - r^2 n^2 \left( \frac{1}{r^2} e^{-2\phi} \right)$$

$$0 = \cos((\pi n + \pi)) - e^{\pi i n B} \cos(\pi + \pi s) - e^{\pi i n B} (\pi s - \pi)^{-s}$$

5

$$y = e^{m \sin^{-1} x}$$

$$\Rightarrow y_1 = m e^{m \sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1 = \frac{my}{\sqrt{1-x^2}} = \frac{(x+1)(\frac{1}{\sqrt{1-x^2}})}{(x+1)(\frac{1}{\sqrt{1-x^2}}) - 1} = \frac{m}{x+1}$$

$$\Rightarrow (\sqrt{1-x^2}) y_1 = my - \frac{m}{x+1} = nb$$

$$\Rightarrow (1-x^2) y_1^2 = m^2 y^{2n-1}$$

$$\Rightarrow (1-x^2)^2 y_1 y_2 + y_1^2 (-2x) = m^2 y y_1 = nb$$

$$\Rightarrow (1-x^2)^2 y_1 y_2 - 2x y_1^2 - 2m^2 y y_1 = 0$$

$$\Rightarrow 2y_1 [(1-x^2)y_2 - xy_1 - my] = 0$$

$$\Rightarrow (1-x^2)y_2 - xy_1 - my = 0$$

$$\Rightarrow [y_{n+2}(1-x^2) + n c_1 y_{n+1}(-2x) + n c_2 y_n(-2) + n c_3 y_{n-1} \cdot 0]$$

$$[c_{n+2} x^{n+2} + c_{n+1} x^{n+1} + c_n x^n + c_{n-1} x^{n-1}] - my_n = 0$$

$$0 = [c_{n+2} x^{n+2} + c_{n+1} y_{n+1} + c_n y_n + c_{n-1} y_{n-1}] - my_n$$

$$\Rightarrow (1-x^2)y_{n+2} + (-2x)^n y_{n+1} - 2 \cdot \frac{n(n-1)}{2} y_n - (xy_{n+1} + ny_n)$$

$$3 = n(n-1)x^{n-1} + nbx^n - nb(1-x)x^{n-1} - nbx^{n-1} = nb(1-x) - my_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2nx+x)y_{n+1} - \cancel{(n^2-n)}y_n - \cancel{n}y_n - my_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)ny_{n+1} - (n^2-n+n+m)y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n+m)y_n = 0$$

showed

$$6) \quad y = (\sin^{-1} x)^2$$

$$\Rightarrow y_1 = 2 \sin^{-1} x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow (\sqrt{1-x^2}) y_1 = 2x y$$

$$\Rightarrow (1-x^2) y_1^2 = 4x^2 y^2$$

$$\Rightarrow (1-x^2) 2y_1 y_2 + y_1^2 (-2x) = 4y_1 \cdot 2x (-2x)$$

$$\Rightarrow (1-x^2) 2y_1 y_2 - 2x y_1^2 - 4y_1 = 0$$

$$\Rightarrow 2y_1 [(1-x^2) y_2 - x y_1 - 2] = 0$$

$$\Rightarrow (1-x^2) y_2 - x y_1 - 2 = 0$$

$$\Rightarrow (1-x^2) y_2 - n y_1 - 2 = 0$$

$$\Rightarrow [y_{n+2} (1-x^2) + n c_1 y_{n+1} (-2x) + n c_2 y_n (-2) + n c_3 y_{n-1} \cdot 0]$$

$$= - [y_{n+1} x + y_n \cdot 1 + y_{n-1} \cdot 0 \cdot n c_2] = 0$$

$$0 = ad^2 - [a^2 d + b^2 d + abd + b^2 d]$$

$$\Rightarrow (1-x^2) y_{n+2} - 2nx y_{n+1} - 2 \cdot \frac{n(n-1)}{2} y_n - xy_{n+1}$$

$$- ny_n = 0$$

$$\Rightarrow (1-x^2) y_{n+2} - (2n+1)xy_{n+1} - \frac{(n-1)n}{2} y_n - ny_n = 0$$

$$\Rightarrow (1-x^2) y_{n+2} - (2n+1)xy_{n+1} - ny_n = 0$$

shown

showed

$$7) \log_e y = a \sin^{-1} x$$

$$\Rightarrow \ln y = a \sin^{-1} x$$

$$\Rightarrow y = e^{a \sin^{-1} x}$$

$$\Rightarrow y_1 = a e^{a \sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow (\sqrt{1-x^2}) y_1' = ay: (x \sin^{-1} x)^{(1)} + ab \sin^{-1} x + ab(1-\sin^{-1} x)$$

$$\Rightarrow (1-x^2) y_1'^2 = a^2 y^2$$

$$\Rightarrow (1-x^2) 2y_1 y_2 + y_1^2 (-2x) = a^2 2y y_1$$

$$\Rightarrow (1-x^2) 2y_1 y_2 - 2x y_1^2 - a^2 2y y_1 = 0$$

$$\Rightarrow 2y_1 [(1-x^2) y_2 - xy_1 - a^2 y] = 0$$

$$\Rightarrow (1-x^2) y_2 - xy_1 - a^2 y = 0$$

$$\Rightarrow [y_{n+2}(1-x^2) + n c_1 y_{n+1}(-2x) + n c_2 y_n(-2) + n c_3 y_{n-1} 0] - [y_{n+1} x + c_1 y_n + c_2 y_{n-1} 0] - a^2 y_n = 0$$

$$\Rightarrow (1-x^2) y_{n+2} - 2nxy_{n+1} - 2 \cdot \frac{n(n+1)}{2} y_n - xy_{n+1} - y_{n-1} - a^2 y_n = 0$$

$$\Rightarrow (1-x^2) y_{n+2} - (2n+1)ny_{n+1} - (n^2-n)y_n - y_{n-1} - a^2 y_n = 0$$

$$\Rightarrow (1-x^2) y_{n+2} - (2n+1)ny_{n+1} - (n^2+n)y_n = 0$$

showed

long time

$$8] \quad y = e^{m \cos^{-1} x}$$

$$x^2 - 1 = 0$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow y_1 = m e^{m \cos^{-1} x} \cdot \left( -\frac{1}{\sqrt{1-x^2}} \right)$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow (-\sqrt{1-x^2}) y_1 = my$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow (1-x^2) y_1^2 = m^2 y^2$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow (1-x^2) 2y_1 y_2 + y_1^2 (-2x) = m^2 2y y_1$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow (1-x^2) 2y_1 y_2 - 2x y_1^2 - m^2 2y y_1 = 0$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow 2y_1 [(1-x^2)y_2 - xy_1 - m^2 y] = 0$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow (1-x^2)y_2 - xy_1 - m^2 y = 0$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow [y_{n+2}(1-x^2) + nc_1 y_{n+1}(-2x) + nc_2 y_n(-2) + nc_3 y_{n-1} \cdot 0]$$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$- [y_{n+1}x + nc_1 y_{n-1} \cdot 1 + nc_2 y_{n-1} \cdot 0] - m^2 y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - 2nx y_{n+1} - 2 \frac{n(n-1)}{n} y_n - ny_{n+1} - ny_n - m^2 y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$$

showed

9

$$\log_e y = \tan^{-1} x$$

$$\Rightarrow \ln y = \tan^{-1} x$$

$$\Rightarrow y = e^{\tan^{-1} x}$$

$\Rightarrow$  then same as 4

$$B^m = (x^2 - 1)^{1/2} + \sqrt{x^2 - 1} (x - 1)$$

10

$$y = (\cos^{-1} x)^2$$

$$\Rightarrow y_1 = 2 \cos^{-1} x \left( -\frac{1}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow (-\sqrt{1-x^2}) y_1 = 2 \cos^{-1} x$$

$$\Rightarrow (-\sqrt{1-x^2}) y_1 = 2 \sqrt{y}$$

$$\Rightarrow (1-x^2) y_1^2 = 4y$$

$$\Rightarrow y_1^2 = \frac{4y}{1-x^2} = \frac{4y}{(\cos^{-1} x)^2} = \frac{4y}{x^2 - 2x + 1}$$

$\Rightarrow$  then same as 6

11

$$\ln y = m \cos^{-1} x$$

$$\Rightarrow y^m = e^{m \cos^{-1} x}$$

$\Rightarrow$  then same as 8

## Rolle's and Mean Value Theorem

12

Q. Find for  $(\frac{1}{2}, \frac{3}{2})$  no. of points at which  $f(x)$  is zero.

1]  $f(x) = x^2 - 6x + 8 ; [2, 4]$

To :  $[\frac{1}{2}, \frac{3}{2}]$  no

Hence  $f(x)$  is differentiable on  $(2, 4)$  and continuous

on  $[2, 4]$ , if  $f(2) = 2^2 - 6 \cdot 2 + 8 = 0$

$$f(4) = 4^2 - 6 \cdot 4 + 8 = 0$$

then according to Rolle's theorem  $\exists c \in (2, 4)$  such that  $f'(c) = 0$

$$f(x) = x^2 - 6x + 8$$

$$f'(x) = 2x - 6$$

$$f'(c) = 2c - 6$$

Now,  $2c - 6 = 0$

$$\Rightarrow 2c = 6$$

$$\Rightarrow c = 3$$

$\therefore c = 3 \in (2, 4)$

(verified)

$$2] f(x) = \cos x ; [\frac{\pi}{2}, \frac{3\pi}{2}]$$

Hence  $f(x)$  is differentiable on  $(\frac{\pi}{2}, \frac{3\pi}{2})$  and continuous on  $[\frac{\pi}{2}, \frac{3\pi}{2}]$ . If

continuous b/w  $(P, S) f(\frac{\pi}{2}) = \cos \frac{\pi}{2} = 0$  &  $f(S) \neq 0$

$$\therefore f(\frac{3\pi}{2}) = \cos \frac{3\pi}{2} = 0 \quad \text{for } [\frac{\pi}{2}, \frac{3\pi}{2}] \text{ no}$$

$$0 = \sin P - \sin S = (P) +$$

since  $f(\frac{\pi}{2}) = f(\frac{3\pi}{2}) = 0$  there is at least one number

$c \in (\frac{\pi}{2}, \frac{3\pi}{2})$  such that  $f'(c) = 0$

$$f'(x) = -\sin x$$

$$-\sin c = (x) +$$

$$f'(c) = -\sin c$$

$$-\sin c = (x) +$$

Now,

$$-\sin c = 0$$

$$-\sin c = (x) +$$

$$\Rightarrow \sin c = 0$$

$$0 = 0 + 0$$

$$\Rightarrow \sin c = \sin n\pi$$

$$0 = 0 + 0$$

$$\Rightarrow c = n\pi$$

$$0 = 0 + 0$$

$$n=0 ; c=0 \notin (\frac{\pi}{2}, \frac{3\pi}{2})$$

$$n=1 ; c=\pi \in (\frac{\pi}{2}, \frac{3\pi}{2})$$

$$n=2 ; c=2\pi \notin (\frac{\pi}{2}, \frac{3\pi}{2})$$

$$\therefore c = \pi \in (\frac{\pi}{2}, \frac{3\pi}{2})$$

(verified)

$$3] f(x) = \frac{x}{2} - \sqrt{x}; [0,4]$$

$$[0,4] \subset \text{Domain}(x) \quad \square$$

Hence  $f(x)$  is differential and continuous on  $(0,4)$ . If,   
 by contradiction (iff)  $\exists c \in (0,4)$  such that  $f'(c) = 0$

$$f(0) = \frac{0}{2} - \sqrt{0} = 0$$

$$\text{and } f(4) = \frac{4}{2} - \sqrt{4} = 2 - 2 = 0$$

Since  $f(0) = f(4) = 0$ , by Rolle's theorem there is at least one number  $c \in (0,4)$  such that  $f'(c) = 0$

$$f'(x) = \frac{1}{2} - \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{2} \left(1 - \frac{1}{\sqrt{x}}\right)$$

$$f'(c) = \frac{1}{2} \left(1 - \frac{1}{\sqrt{c}}\right)$$

$$\text{Now, } \frac{1}{2} \left(1 - \frac{1}{\sqrt{c}}\right) = 0 \quad \frac{(c-a)}{(c-a)-a} = x + \varepsilon$$

$$\Rightarrow \frac{1}{2} - \frac{1}{2\sqrt{c}} = 0 \quad \frac{-\frac{1}{\sqrt{c}}}{\varepsilon} = x + \varepsilon$$

$$\Rightarrow \frac{1}{2\sqrt{c}} = 0 \quad \varepsilon = x + \varepsilon$$

$$\Rightarrow \sqrt{c} = 1 = 0$$

$$\Rightarrow \sqrt{c} = 1$$

$$\Rightarrow c = 1$$

$$\therefore c = 1 \in (0,4) \quad (\text{verified})$$

$$b. \quad [f(x)]_{x=1}^{x=2} - \frac{f(2) - f(1)}{2} = (x)^2 \quad |E$$

$$1] \quad f(x) = x^3 + x - 4 ; [-1, 2]$$

$f(x)$  are continuous b/w left endpoints of  $(x)$  & smooth  
since polynomials are everywhere ( $\mathbb{R}$ ) continuous and  
differentiable so  $f(x)$  is differentiable & continuous  
on  $(-1, 2)$ . Then by Mean value theorem there  
exists at least one number  $c \in (-1, 2)$  such that,

$$0 = (x)^2 - f(a) \quad f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{from ①} \quad \text{and} \quad f(2) =$$

$$f(x) = x^3 + x - 4$$

$$f'(x) = 3x^2 + 1$$

$$f'(c) = 3c^2 + 1$$

$$\frac{1}{2} - \frac{1}{2} = (x)^2$$

$$f(-1) = (-1)^3 + (-1) - 4 = -6$$

$$f(2) = 2^3 + 2 - 4 = 6$$

$$\left(\frac{1}{2} - 1\right) \frac{1}{2} = (x)^2$$

from ①,

$$3c^2 + 1 = \frac{6 - (-6)}{2 - (-1)} = \left(\frac{1}{2} - 1\right) \frac{1}{2} \quad \text{from}$$

$$\Rightarrow 3c^2 + 1 = \frac{12}{3} \quad 0 = \frac{1}{2} - 1 \quad \Leftarrow$$

$$\Rightarrow 3c^2 = 3 \quad 0 = \frac{1}{2} - 1 \quad \Leftarrow$$

$$\Rightarrow c^2 = 1 \quad 0 = 1 - 1 \quad \Leftarrow$$

$$\Rightarrow c = \pm 1 \quad 1 = 1 \quad \Leftarrow$$

$$\therefore c = 1 \in (-1, 2) \quad \begin{array}{l} \text{(verified)} \\ \text{(Ans)} \end{array}$$

$$2) f(x) = \sqrt{x+1} ; I [0, 3] \quad \text{Rolle's Theorem} \quad L$$

since  $f(x)$  is differentiable and continuous on  $[0, 3]$ , there is at least one number such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad c \in (0, 3) \quad \text{--- ①}$$

$$\text{--- ②} \quad \frac{(x)-f(0)}{x-0} = f'(0) +$$

$$f'(x) = \frac{1}{2\sqrt{x+1}} \cdot 1$$

$$f(0) = 1$$

$$f'(c) = \frac{1}{2\sqrt{c+1}} = (0)' +$$

$$f(3) = \frac{1}{2\sqrt{3+1}} = (3)' +$$

$$0 = (2)' +$$

from ①,

$$\frac{1}{2\sqrt{c+1}} = \frac{2-1}{3-0}$$

$$\frac{2}{3-2\sqrt{c+1}} = (2)' +$$

① mark

$$\Rightarrow 2\sqrt{c+1} = 3$$

$$\frac{c+1}{0-2} = \frac{2}{3-2\sqrt{c+1}}$$

$$\Rightarrow \sqrt{c+1} = \frac{3}{2}$$

$$\sqrt{c+1} = \frac{3}{2} \quad \text{---}$$

$$\Rightarrow c+1 = \frac{9}{4}$$

$$\sqrt{c+1} = \frac{3}{2} \quad \text{---}$$

$$\Rightarrow c = \frac{9}{4} - 1$$

$$\sqrt{c+1} = \frac{3}{2} \quad \text{---}$$

$$\Rightarrow c = \frac{5}{4}$$

$$\sqrt{c+1} = \frac{3}{2} \quad \text{---}$$

$$\therefore c = \frac{5}{4} \in (0, 3)$$

(verified).

Ans.

$$(c, 0) \rightarrow \frac{5}{4} = 0$$

$$3) f(x) = \sqrt{25-x^2} ; [0,5], f'(x) = ? \quad [E]$$

Since  $f(x)$  is differentiable and continuous on  $[0,5]$  so by Mean value theorem there is at least one number  $c \in (0,5)$  such that,

$$f'(c) = \frac{f(b)-f(a)}{b-a} \quad \text{--- ①}$$

$$\begin{aligned} f'(x) &= \frac{1}{2\sqrt{25-x^2}} \cdot (-2x) \\ &= -\frac{x}{\sqrt{25-x^2}} \end{aligned}$$

$$f'(c) = -\frac{c}{\sqrt{25-c^2}}$$

$$\begin{aligned} f(0) &= 5 \\ f(5) &= 0 \end{aligned}$$

$$\frac{f(5)-f(0)}{5-0} = -\frac{1}{\sqrt{25-0^2}}$$

from ①,

$$-\frac{c}{\sqrt{25-c^2}} = \frac{0-5}{5-0}$$

$$\Rightarrow 5c = 5\sqrt{25-c^2}$$

$$5c = 5\sqrt{25} \Leftrightarrow 5c = 25$$

$$c = \sqrt{25} \Leftrightarrow c = 5$$

$$\frac{c}{5} = 1 \Leftrightarrow c = 5$$

$$1 + \frac{c}{5} = 2 \Leftrightarrow c = 5$$

$$\frac{c}{5} = 2 \Leftrightarrow c = 10$$

$$\Rightarrow c^2 = 25 - c^2$$

$$\Rightarrow 2c^2 = 25$$

$$\Rightarrow c = \pm \frac{5}{\sqrt{2}}$$

$$(0,0) \rightarrow \frac{5}{\sqrt{2}} = 5$$

$$\therefore c = \frac{5}{\sqrt{2}} \in (0,5) \quad (\text{verified}).$$

## Partial Derivative

09-NOV-2017

1)  $f(x) = 3x^3y^2$   $\frac{\partial f}{\partial x} = 3x^2 \cdot 2y^2 + \dots$  (i)

$$f_x = 9x^2y^2$$

$$f_y = 6x^3y$$

$$\therefore f_x(x, 1) = 9x^2$$

$$\therefore f_y(1, y) = 6y$$

$$\begin{aligned} f_x(1, 2) &= 9 \cdot 4 \\ &= 36 \end{aligned}$$

$$\begin{aligned} f_y(1, 2) &= 6 \cdot 2 \\ &= 12 \end{aligned}$$

$$(62) \frac{\delta}{dx} = 12$$

2) (i)  $f(x, y) = x e^{-y} + 5y$

$$f_x = e^{-y}$$

$$f_y = -x e^{-y} + 5$$

$$\begin{aligned} \therefore f_x(3, 0) &= e^{-0} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \therefore f_y(4, 2) &= -4 \cdot 0 \cdot e^{-2} + 5 \\ &= -4e^{-2} + 5 \end{aligned}$$

(ii)  $f(x, y) = \sqrt{3x+2y}$

$$f_x = \frac{1}{2\sqrt{3x+2y}} \cdot (3+0)$$

$$= \frac{3}{2\sqrt{3x+2y}}$$

$$\begin{aligned} \therefore f_x(3, 0) &= \frac{3}{2\sqrt{3+2 \cdot 0}} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

$$f_y = \frac{1}{2\sqrt{3x+2y}} \cdot (0+2)$$

$$\begin{aligned} (62) \frac{\delta}{dy} &= \frac{1}{\sqrt{3x+2y}} \\ &= \frac{1}{\sqrt{3 \cdot 4 + 2 \cdot 2}} \end{aligned}$$

$$\begin{aligned} \therefore f_y(4, 2) &= \frac{1}{\sqrt{3 \cdot 4 + 2 \cdot 2}} \\ &= \frac{1}{\sqrt{16}} \\ &= \frac{1}{4} \end{aligned}$$

$$\text{Ques } \underline{\underline{3}} \text{ (i) } f(x, y) = 4x^2 - 2y + 7x^4y^5$$

$$f_{xx} = \frac{\partial}{\partial x} (4x^2 - 2y + 7x^4y^5) = 8x + 28x^3y^5$$

$$= 8x + 28x^3y^5$$

$$f_{xx} = \frac{\partial}{\partial x} (f_{xy}) =$$

$$= 8 + 84x^2y^5$$

$$f_{xy} = \frac{\partial}{\partial y} (f_{xx})$$

$$= 140y^4x^3$$

$$f_y = \frac{\partial}{\partial y} (4x^2 - 2y + 7x^4y^5)$$

$$= -2 + 35x^4y^4$$

$$f_{yy} = \frac{\partial}{\partial y} (f_y)$$

$$= 140x^4y^3$$

$$f_{yx} = \frac{\partial}{\partial x} (f_y)$$

$$= 140x^3y^4$$

$$(ii) f(x,y) = e^{x^2+xy+y^2} \quad (P) \frac{\partial}{\partial x} = xy^2$$

$$f_x = e^{x^2+xy+y^2} \cdot (2x+y)$$

$$f_{xx} = \frac{\partial}{\partial x} (f_x)$$

$$= e^{x^2+xy+y^2} \frac{\partial}{\partial x} (2x+y) + (2x+y) \cdot \frac{\partial}{\partial x} (e^{x^2+xy+y^2})$$

$(xP - P) \cancel{m2} = e^{x^2+xy+y^2}$

$$= e^{x^2+xy+y^2} (2+0) + (2x+y) \cdot e^{x^2+xy+y^2} \cdot (2x+y)$$

$$= 2e^{x^2+xy+y^2} + (2x+y)^2 \cdot e^{x^2+xy+y^2} (s) \frac{\partial}{\partial x} = x^2$$

$$P^2 \cdot (xP - P) \cos = (P -) \cdot (xP - P) \cos =$$

$$f_{xy} = \frac{\partial}{\partial y} (f_x) = (xP - P) \cos P =$$

$$= e^{x^2+xy+y^2} \frac{\partial}{\partial y} (2x+y) + (2x+y) \cdot \frac{\partial}{\partial y} (e^{x^2+xy+y^2})$$

$P - = (P, S) \cancel{m2} \therefore$

$$= e^{x^2+xy+y^2} \cdot 1 + (2x+y) \cdot e^{x^2+xy+y^2} \cdot (x+2y)$$

$$= e^{x^2+xy+y^2} + (2x+y)(x+2y) \frac{\partial}{\partial x} = x^2 \quad (ii)$$

$$f_y = e^{x^2+xy+y^2} \cdot (x+2y) \quad (s) \frac{\partial}{\partial x} = x^2$$

$$f_{yy} = \frac{\partial}{\partial y} (f_y)$$

$$= e^{x^2+xy+y^2} \frac{\partial}{\partial y} (x+2y) + (x+2y) \frac{\partial}{\partial y} (e^{x^2+xy+y^2})$$

$$= (P + S) \cancel{m2} = (P, S -) \cancel{m2}$$

$$= e^{x^2+xy+y^2} \cdot 2 + (x+2y) \cdot e^{x^2+xy+y^2} \cdot (x+2y)$$

$$= 2e^{x^2+xy+y^2} + (x+2y)^2 \cdot e^{x^2+xy+y^2}$$

$$f_{xy} = \frac{\partial}{\partial x} (f_y) \quad (ii)$$

$$= e^{x^2+xy+y^2} \cdot 1 + (x+2y) \cdot e^{x^2+xy+y^2} (2x+y)$$

$$= e^{x^2+xy+y^2} + (x+2y)(2x+y) \cdot e^{x^2+xy+y^2}$$

$$\text{4) } (i) z = \sin(y^2 - 4x)$$

$$(i) z = \sin(y^2 - 4x) \quad (ii) f_x = \frac{\partial}{\partial x} (z) =$$

$$f_x = \frac{\partial}{\partial x} (z) = \cos(y^2 - 4x) \cdot (-4) \quad f_y = \frac{\partial}{\partial y} (z) =$$

$$= \cos(y^2 - 4x) \cdot (-4) \quad = \cos(y^2 - 4x) \cdot 2y$$

$$= -4 \cos(y^2 - 4x) \quad = 2y \cos(y^2 - 4x)$$

$$\therefore f_x(2,1) = -4 \quad \therefore f_y(-2,4) = 2 \cdot 4 \cos(16 + 8)$$

$$(iii) z = (x+y)^{-4} \cdot (6+x^2) + (x+y)^{-2} = 7.30$$

$$(iii) z = (x+y)^{-4} \cdot (6+x^2) + (x+y)^{-2} =$$

$$f_x = \frac{\partial}{\partial x} (z)$$

$$= -1 \cdot (x+y)^{-2} \cdot 1$$

$$= -\frac{1}{(x+y)^2}$$

$$f_y = \frac{\partial}{\partial y} (z)$$

$$= -1 \cdot (x+y)^{-2} \cdot 1$$

$$= -\frac{1}{(x+y)^2} = 66$$

$$\therefore f_x(2,1) = -\frac{1}{(2+1)^2} \quad \therefore f_y(-2,4) = -\frac{1}{(-2+4)^2}$$

$$= -\frac{1}{9} \quad = -\frac{1}{4}$$

$$51 \quad f(x, y, z) = x^3 y^5 z^7 + x y^2 + y^3 z^3 \quad \frac{\partial}{\partial x} = 1857$$

$$f_x = 3x^2 y^5 z^7 + y^2 + 0 \quad \frac{\partial}{\partial x} = 1857$$

$$f_y = 5x^3 y^4 z^7 + 2xy + 3y^2 z \quad \frac{\partial}{\partial y} = 1857$$

$$f_z = 7x^3 y^5 z^6 + 0 + y^3 \quad \frac{\partial}{\partial z} = 1857$$

$$f_{xy} = \frac{\partial}{\partial x} (3x^2 y^5 z^7 + y^2) \quad \frac{\partial}{\partial y} = 1857$$

$$= 15x^2 y^4 z^7 + 2y \quad \frac{\partial}{\partial y} = 1857$$

$$f_{yz} = \frac{\partial}{\partial z} (5x^3 y^4 z^7 + 2xy + 3y^2 z) \quad \frac{\partial}{\partial z} = 1857$$

$$= 35x^3 y^4 z^6 + 3y^2$$

$$f_{xz} = \frac{\partial}{\partial z} (3x^2 y^5 z^7 + y^2) \quad \frac{\partial}{\partial z} = 1857$$

$$= 21x^2 y^5 z^6$$

$$f_{zz} = \frac{\partial}{\partial z} (7x^3 y^5 z^6 + y^3) \quad \frac{\partial}{\partial z} = 1857$$

$$= 42x^3 y^5 z^5$$

$$f_{zyy} = ((f_z)_y)_y$$

$$\therefore f_{zy} = \frac{\partial}{\partial y} (7x^3 y^5 z^6 + y^3)$$

$$= 35x^3 y^4 z^6 + 3y^2$$

$$\therefore f_{zyy} = \frac{\partial}{\partial y} (35x^3y^4z^6 + 3y^2) \\ = 140x^3y^3z^6 + 6y$$

$$f_{zxy} = ((f_z)_x)_y$$

$$\therefore f_{zx} = \frac{\partial}{\partial x} (7x^3y^5z^6 + y^3)$$

$$= 21x^2y^5z^6 + 0$$

$$\therefore f_{zxy} = \frac{\partial}{\partial y} (21x^2y^5z^6)$$

$$= 105x^2y^4z^6 + 0$$

$$f_{zyx} = ((f_z)_y)_x$$

$$f_{zy} = \frac{\partial}{\partial y} (7x^3y^5z^6 + y^3) \\ = 35x^3y^4z^6 + 3y^2$$

$$\therefore f_{zyx} = \frac{\partial}{\partial x} (35x^3y^4z^6 + 3y^2) \\ = 105x^2y^4z^6 + 0$$

$$= 105x^2y^4z^6$$

$$f_{xyz} = ((f_x)_x)y = z \cdot 3x^2y^5z^7 + 0 = 3x^2y^5z^8$$

$$f_{xx} = \frac{\partial}{\partial x} (3x^2y^5z^7 + y^2)$$

$$= 6xy^5z^7$$

$$f_{xxy} = \frac{\partial}{\partial y} (6xy^5z^7)$$

$$= 30xy^4z^7$$

$$f_{xxyz} = \frac{\partial}{\partial z} (30xy^4z^7)$$

$$= 210xy^4z^6$$

$$= 210xy^4z^6$$

15

6)

$$f(x,y,z) = \sqrt{xy} + \ln(x^2z^3) - x\tan z$$

$$= x^{1/2}y^{1/2} + \ln(x^2z^3) - x\tan z$$

$$f_x = y^{1/2} \cdot \frac{1}{2}x^{-1/2} + \frac{1}{x^2z^3} \cdot 2xz^3 - \tan z$$

$$= y^{1/2} \cdot \frac{1}{2}x^{-1/2} + \frac{2}{x} - \tan z$$

$$= y^{1/2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}} + \frac{2}{x} - \tan z$$

$$f_{xy} = \frac{1}{2} \frac{1}{\sqrt{x}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{y}} + 0 - 0 = 0$$

$$= \frac{1}{4\sqrt{xy}}$$

$$0 = 0$$

$$0 = 0$$

$$f_z = 0 + \frac{1}{x^2 z^3} \cdot x^2 z^2 - x \sec^2 z (\sec(z)) = -\sec^2 z$$

$$= \frac{3}{z} - x \sec z \quad (\sec^2 z) \frac{d}{dz} = \sec z$$

$$(\sec^2 z) \frac{d}{dz} =$$

$$f_{xyz} = 0$$

$$(\sec^2 z) \frac{d}{dz} = \sec z$$

$$(\sec^2 z) \frac{d}{dz} =$$

$$(\sec^2 z) \frac{d}{dz} = -\sec z$$

7]  $f(x, y, z) = (x^2 - y^2) \cos x + x^5 z^2 - 30z$   
 $= x^2 \cos x - y^2 \cos x + x^5 z^2 - 30z$

$$f_x = 2x \cos x - \sin x \cdot x^2 + y^2 \sin x + 5x^4 z^2$$

$$= 2x \cos x - (\sin x) x^2 + y^2 \sin x = (2x \cos x) +$$

$$f_{xy} = 0 - 0 + 2y \sin x + 0$$

$$= 2y \sin x$$

$$f_{xz} = -2x \sin x + 2 \cos x - 2x \sin x - x \cos x + y \cos x + 20x^3 z^2$$

$$f_{xxz} = -2 \sin x + 2 \cos x - x \sin x - x \cos x =$$

$$f_{xxz} = -2 + 0 - 0 + 0 + 0 + 40x^3 z$$

$$= 40x^3 z$$

$$f_z = 0 + 2x^5 z - 30$$

$$f_{zy} = 0$$

$$f_{zyz} = 0$$

$$8| \quad f(x,y) = x^3 \ln(xy) + x^4 y - e^{3x} x^2$$

$$f_{xx} = x^3 \cdot \frac{1}{xy} \cdot 2xy + \ln(xy) \cdot 3x^2 + 4x^3 y - [e^{3x} \cdot 2x + x^2 e^{3x} \cdot 3]$$

$$= 2x^2 + 3x^2 \ln(xy) + 4x^3 y - 2xe^{3x} - 3x^2 e^{3x}$$

$$f_{xx} = 4x + 3 \left[ x^2 \frac{1}{xy} \cdot 2xy + \ln(xy) \cdot 2x \right] + 12x^2 y - 2[x e^{3x} \cdot 3 + e^{3x} \cdot 1] \\ - 3 [x^2 e^{3x} \cdot 3 + e^{3x} \cdot 2x]$$

$$= 4x + 6x + 6x \ln(xy) + 12xy - 6xe^{3x} - 2e^{3x} - 9x^2 e^{3x} - 6xe^{3x}$$

$$= 10x + 6x \ln(xy) + 12xy - 12xe^{3x} - 9x^2 e^{3x} - 2e^{3x}$$

$$f_{xy} = 0 + 3x^2 \frac{1}{xy} \cdot x^2 + 4x^3 - 0 - 0$$

$$= \frac{3x^2}{y} + 4x^3$$

$$f_y = x^3 \frac{1}{xy} \cdot x + x^4 - 0$$

$$= \frac{x^3}{y} + x^4$$

$$= x^3 \cdot y^{-1} + x^4$$

$$f_{xy} = x^3 \left[ (-1)y^{-1-1} \right] + 4x^3$$

$$= -x^3 \frac{1}{y^2} + 4x^3$$

$$f_{yy} = -x^3 \frac{1}{y^2} + 0$$

$$= -x^3 \frac{1}{y^2}$$

## Successive Differentiation

17-Nov-20

a.

$$\underline{1} \quad y = x^n$$

$$x \cdot \frac{1}{(d+x)} = x^6$$

Differentiating  $y$  with respect to  $x$  successively.

$$y_1 = n x^{n-1} \quad \frac{d}{dx} \cdot \frac{1}{(d+x)} (1-) = x^5$$

$$y_2 = n(n-1) x^{n-2} \quad \frac{d}{dx} \cdot \frac{1}{(d+x)} (2-) = x^4$$

$$y_3 = n(n-1)(n-2) x^{n-3} \quad \frac{d}{dx} \cdot \frac{1}{(d+x)} [((n-1)-) \dots (2-)] (3-) = x^3$$

$$\therefore y_n = n! x^{n-n} \quad \frac{d}{dx} \cdot \frac{1}{(d+x)} [((n-1)-) \dots (2-)] (n-) =$$

$$\underline{2} \quad y = (ax+b)^n$$

$$\frac{1}{(d+x)} = x^6$$

Differentiating  $y$  with respect to  $x$  successively.

$$y_1 = n (ax+b)^{n-1} \cdot a \quad \frac{d}{dx} \cdot \frac{1}{(d+x)} (1-) = x^5 \quad [\text{chain rule}]$$

$$y_2 = n(n-1) (ax+b)^{n-2} \cdot a^2 \quad \frac{d}{dx} \cdot \frac{1}{(d+x)} (2-) (1-) = x^4$$

$$y_3 = n(n-1)(n-2) (ax+b)^{n-3} \cdot a^3 \quad \frac{d}{dx} \cdot \frac{1}{(d+x)} (3-) (2-) (1-) = x^3$$

$$\therefore y_n = n! (ax+b)^{n-n} \cdot a^n$$

$$= a^n n!$$

$$3) f = \ln(ax+b)$$

$$y_1 = \frac{1}{ax+b} \cdot a$$

$$x = B \quad \square$$

$$= a(ax+b)^{-1}$$

Erklärung: Es ist folgendes zu beachten bei Differenzierbarkeit

$$y_2 = (-1)(ax+b)^{-2} \cdot a^2$$

$$x = B$$

$$y_3 = (-1)(-2)(ax+b)^{-3} \cdot a^3 \cdot (1-n) \alpha = ab$$

$$\therefore y_n = (-1)(-2) \dots [-(n-1)] \cdot a^n \cdot (ax+b)^{-n} \cdot a^n$$

$$= (-1)^{n-1} [1 \cdot 2 \dots (n-1)] \cdot (ax+b)^{-n} \cdot a^n$$

$$= (-1)^{n-1} (n-1)! \cdot (ax+b)^{-n} \cdot a^n$$

$$x = B \quad \square$$

$$4) f = \frac{1}{x+a}$$

Erklärung: Es ist folgendes zu beachten bei Differenzierbarkeit

$$= (x+a)^{-1}$$

$$y_1 = (-1)(x+a)^{-2} \cdot 1$$

$$= (-1)(-2)(x+a)^{-3} \cdot 1$$

$$y_2 = (-1)(-2)(-3)(x+a)^{-4} \cdot 1$$

$$y_3 = (-1)(-2)(-3)(-4)(x+a)^{-5} \cdot 1$$

$$\therefore y_n = (-1)^n \cdot (1 \cdot 2 \cdot 3 \dots n) \cdot (x+a)^{-(n+1)}$$

$$= (-1)^n \cdot n! \cdot (x+a)^{-(n+1)}$$

5)

$$y = e^{ax}$$

$$(d+e\cos)x + b \stackrel{!}{=} 5$$

$$y_1 = a e^{ax} \quad (d+e\cos)x + b \stackrel{!}{=} 5$$

$$[ \text{ansatz: } y_2 = a^2 e^{ax}] \quad (d+e\cos + \frac{\pi}{2}) \sin x = 5$$

$$y_3 = a^3 e^{ax}$$

$$\therefore y_n = a^n e^{ax} \quad (d+e\cos + \frac{\pi}{2}) \sin^n x = 5$$

$$(d+e\cos + \frac{\pi}{2} + \frac{\pi}{2}) \cos x =$$

$$6) \quad y = \sin(ax+b) \quad (d+e\cos + \frac{\pi}{2} - 2) \cos x =$$

$$y_1 = \cos(ax+b) \cdot a$$

$$y_2 = -a \sin(ax+b) \quad | \quad \begin{array}{l} \text{det. } \theta = ax+b \\ \sin(\frac{\pi}{2} + \theta) = \cos \theta \end{array}$$

$$= a \sin(\frac{\pi}{2} + ax + b) \quad | \quad \sin(\frac{\pi}{2} + \theta) = \cos \theta$$

$$y_2 = a^2 \cos(\frac{\pi}{2} + ax + b)$$

$$= a^2 \sin(\frac{\pi}{2} + \frac{\pi}{2} + ax + b) \quad | \quad \sin(\pi + \theta) = -\sin \theta$$

$$= a^2 \sin(2 \cdot \frac{\pi}{2} + ax + b)$$

$$\therefore y_n = a^n \sin(n \cdot \frac{\pi}{2} + ax + b) \quad | \quad (n \in \mathbb{N})$$

$$y_n = (\sin n \cdot \frac{\pi}{2} + \sin ax + \sin b) \quad | \quad \sin x = 5$$

$$5a + (5x - 5) \stackrel{!}{=} 5$$

$$+ 5^2 + 5^2 - 1, 5^2 + 5^2 - 2$$

$$5a + 5^2 - 1, 5^2 + 5^2 - 2$$

$$7] \quad y = \cos(ax+b)$$

$$y_1 = -a \sin(ax+b)$$

$$= a \cos\left(\frac{\pi}{2} + ax + b\right) \quad [\because \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta]$$

$$y_2 = -a^2 \sin\left(\frac{\pi}{2} + ax + b\right)$$

$$= a^2 \cos\left(\frac{\pi}{2} + \frac{\pi}{2} + ax + b\right)$$

$$= a^2 \cos\left(2 \cdot \frac{\pi}{2} + ax + b\right)$$

$$\therefore y_n = a^n \cos\left(n \cdot \frac{\pi}{2} + ax + b\right)$$

$$\Theta \cos\left(3 \cdot \frac{\pi}{2}\right) \sin x =$$

$$b) \quad y = e^{ax} \sin bx$$

$$\begin{aligned} y_1 &= e^{ax} \cdot b (\cos bx + \sin bx \cdot a \cdot e^{ax}) \\ &= b e^{ax} \cos bx + a e^{ax} \sin bx \\ &= b e^{ax} \cos bx + ay \end{aligned}$$

$$y_2 = b [e^{ax} \cdot \left( -b \sin bx + \cos bx (a e^{ax}) \right)] + a y_1$$

$$= -b^2 e^{ax} \sin bx + ab e^{ax} \cos bx + a y_1$$

$$= -b^2 y + a(y_1 - ay) + a y_1$$

$$= -b^2 y + a y_1 - a^2 y + a y_1$$

$$= -(a^2 + b^2) y + 2 a y_1$$

Showed.

Q

$$y = e^x \sin x$$

$$y_1 = e^x \cos x + \sin x e^x$$

$$\begin{aligned}y_2 &= -e^x \sin x + \cos x e^x + e^x \cos x + \sin x \cdot e^x \\&= 2e^x \cos x\end{aligned}$$

$$y_3 = 2[-e^x \sin x + \cos x e^x]$$

$$= 2[e^x \cos x - e^x \sin x]$$

$$y_4 = 2[\cos x e^x - e^x \sin x - e^x \cos x - e^x \sin x]$$

$$= 2[-2e^x \sin x]$$

$$= -4e^x \sin x$$

$$\therefore y_4 + 4y = -4e^x \sin x + 4e^x \sin x$$

$$= 0$$

## Maclaurin and Taylor Series

(i) Expand  $y = \sin x$  in the power of  $(x - \frac{\pi}{2})$  or

find the Taylor series for  $y = \sin x$  at  $x_0 = \frac{\pi}{2}$

$$f(x) = \sin x = (s)^{n+1} \cdot$$

$$f'(x) = \cos x = (s)^n \cdot$$

$$f''(x) = -\sin x = (s)^{n+2} \cdot$$

$$f'''(x) = -\cos x$$

$$f^4(x) = \sin x = \frac{(s-x)}{P} + \frac{(s-x)}{1!} \cdot$$

$$f^5(x) = \cos x = \frac{(s-x)}{1!} +$$

$$f(\pi/2) = 1 = e^{i\pi/2} = i$$

$$f'(\pi/2) = 0 = e^{i\pi/2} = i$$

$$f''(\pi/2) = -1 = e^{i\pi/2} = i$$

$$f'''(\pi/2) = 0$$

$$f^4(\pi/2) = 1 \text{ and } \sin x = \text{odd} \therefore$$

$$\therefore \sin x = 1 + (x - \frac{\pi}{2}) \cdot 0 + \frac{(x - \frac{\pi}{2})^2}{2!} \cdot (-1) + \frac{(x - \frac{\pi}{2})^3}{3!} \cdot i$$

$$+ \frac{(x - \frac{\pi}{2})^4}{4!} \cdot 1 + \frac{(x - \frac{\pi}{2})^5}{5!} \cdot 0 + \dots \infty$$

$$= 1 + \frac{(x - \frac{\pi}{2})^2}{2!} + \frac{(x - \frac{\pi}{2})^4}{4!} + \dots \infty$$

$$+ \frac{(x - \frac{\pi}{2})^6}{6!} + \dots \infty$$

$$+ \frac{(x - \frac{\pi}{2})^8}{8!} + \dots \infty$$

$$+ \frac{(x - \frac{\pi}{2})^{10}}{10!} + \dots \infty$$

$$+ \frac{(x - \frac{\pi}{2})^{12}}{12!} + \dots \infty$$

$$(ii) \quad y = \ln x \quad \text{at } x_0=2 \quad \text{or} \quad x-2=0$$

$$(\frac{x}{2}-x)y_1 = \frac{1}{x} \Rightarrow y_1 = x^{-1}$$

$$f'(2) = 2^{-1} = \frac{1}{2}$$

$$\frac{x}{2} = x \Rightarrow x-2 = 0$$

$$f''(2) = -\frac{1}{2^2} = -\frac{1}{4}$$

$$y_3 = 2x^{-3} = (\ln x)''$$

$$f'''(2) = 2 \cdot \frac{-3}{2^3} = \frac{2}{8} = \frac{1}{4}$$

$$y_4 = -6x^{-4} = (\ln x)'''$$

$$f^4(2) = -6 \cdot \frac{1}{2^4} = -\frac{6}{16} = -\frac{3}{8}$$

$$y_5 = 24x^{-5} = (\ln x)''''$$

$$f^5(2) = 24 \cdot \frac{-5}{2^5} = \frac{24}{32} = \frac{3}{4}$$

$$0 = (\ln x)''''$$

$$0 = (x)''''$$

$$\therefore \ln x = \ln 2 + \frac{1}{2}(x-2) - \frac{1}{4} \cdot \frac{(x-2)^2}{2!} + \frac{1}{4} \cdot \frac{(x-2)^3}{3!} - \frac{3}{8} \cdot \frac{(x-2)^4}{4!}$$

$$0 = (\ln x)'''' + \frac{3}{4} \cdot \frac{(x-2)^5}{5!} + \dots \infty (x)''''$$

$$\exists f(x) = e^{ax} + (r) \cdot \frac{(\frac{x}{2}-x)}{1!} + 0 \cdot \frac{(\frac{x}{2}-x)}{2!} + \dots \text{ power of } x-1.$$

$$\therefore f(x) = e^{ax} + r \cdot \frac{(\frac{x}{2}-x)}{1!}$$

$$f(1) = e^a$$

$$f'(x) = ae^{ax}$$

$$f'(1) = ae^a$$

$$f''(x) = a^2 e^{ax}$$

$$f''(1) = a^2 e^a$$

$$f'''(x) = a^3 e^{ax} + \frac{r(\frac{x}{2}-x)}{1!}$$

$$f'''(1) = a^3 e^a =$$

$$f^4(x) = a^4 e^{ax}$$

$$f^4(1) = a^4 e^a$$

$$f^5(x) = a^5 e^{ax}$$

$$f^5(1) = a^5 e^a$$

$$\therefore e^{ax} = e^a + (x-1) \cdot ae^a + \frac{(x-1)^2}{2!} a^2 e^a + \frac{(x-1)^3}{3!} a^3 e^a + \frac{(x-1)^4}{4!} a^4 \cdot e^a$$

$$+ \frac{(x-1)^5}{5!} a^5 \cdot e^a + \dots \infty$$

3]  $y = e^{ax}$  in the power of  $x$ . 14

$$f(x) = e^{ax}$$

$$f(0) = 1$$

$$f'(x) = ae^{ax}$$

$$f'(0) = a$$

$$f''(x) = a^2 e^{ax}$$

$$f''(0) = a^2$$

$$f'''(x) = a^3 e^{ax}$$

$$f'''(0) = a^3$$

$$f^4(x) = a^4 e^{ax}$$

$$f^4(0) = a^4$$

$$f^5(x) = a^5 e^{ax}$$

$$f^5(0) = a^5$$

$$\therefore e^{ax} = 1 + ax + \frac{a^2 x^2}{2!} + \frac{a^3 x^3}{3!} + \frac{a^4 x^4}{4!} + \frac{a^5 x^5}{5!} + \dots \infty$$

4]  $y = \cos x$  in the power of  $x$ .

$$f(x) = \cos x$$

$$f(0) = 1$$

$$f'(x) = -\sin x$$

$$f'(0) = 0$$

$$f''(x) = -\cos x$$

$$f''(0) = -1$$

$$f'''(x) = \sin x$$

$$f'''(0) = 0$$

$$f^4(x) = \cos x$$

$$f^4(0) = 1$$

$$f^5(x) = -\sin x$$

$$f^5(0) = 0$$

$$\therefore \cos x = 1 + 0 - \frac{x^2}{2!} + 0 + \frac{x^4}{4!} + 0 \dots \infty$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \infty$$

$$4) \quad y = e^x \cos x$$

$$f(x) = e^x \cos x$$

$$f'(x) = e^x(-\sin x) + \cos x e^x$$

$$= -e^x \sin x + \cos x e^x$$

$$f''(x) = -e^x \cos x - \sin x e^x + \cos x e^x - e^x \sin x$$

$$= -2e^x \sin x$$

$$f'''(x) = -2[e^x \cos x + \sin x e^x]$$

$$= -2e^x \cos x + 2e^x \sin x$$

$$= -2e^x(\cos x + \sin x)$$

$$f''(x) = -2[e^x(-\sin x + \cos x) + (\cos x + \sin x) \cdot e^x]$$

$$= -2[-e^x \sin x + e^x \cos x + e^x \sin x + e^x \cos x]$$

$$= -2[2e^x \cos x]$$

$$= -4e^x \cos x$$

$$f(0) = e^0 \cdot \cos 0^\circ = 1$$

$$(r+x)\alpha = 0 \quad | \times$$

$$f'(0) = 0 - 0 + 1 = 1(0) +$$

$$(r+x)\alpha = (x) +$$

$$f''(0) = -2e^0 \sin 0^\circ = 0(r)$$

$$r^-(r+x) = \frac{r}{r+x} = (x)' +$$

$$f'''(0) = -2e^0 (\sin 0^\circ + \cos 0^\circ) = -2 \cdot 1 \cdot (0) = (x)''' +$$

$$f^4(0) = -4e^0 \cdot \cos 0^\circ = 4$$

$$r^-(r+x) \cdot 1 = (x)^{IV} +$$

$$r^2 = (x)^2 +$$

$$\therefore P_n(x) = 1 + 1 \cdot x + 0 + (-2) \cdot \frac{x^3}{3!} + (-4) \cdot \frac{x^4}{4!} (x)^2 +$$

$$\dots + \frac{P_n}{n!} r^2 = 1 + x + 0 - \frac{2x^3}{3!} - \frac{4x^4}{4!} + 0 = (r+x)\alpha \quad \therefore$$

$$\therefore P_0 = 1$$

$$P_1 = 1 + x$$

$$P_2 = 1 + x + 0 = 1 + x$$

$$P_3 = 1 + x + 0 - \frac{2x^3}{3!}$$

$$P_4 = 1 + x - \frac{2x^3}{3!} - \frac{4x^4}{4!}$$

$$5] \quad y = \ln(x+1)$$

$$r = 0.02 \cdot 3 = (0)^3 r$$

$$f(x) = \ln(x+1)$$

$$f(0) = \ln 1 = 0 = (0)^1 r$$

$$f'(x) = \frac{1}{x+1} = (x+1)^{-1}$$

$$f'(0) = 1 = (0)^0 r$$

$$f''(x) = -(x+1)^{-2}$$

$$f''(0) = -1 = (0)^1 r$$

$$f'''(x) = -2(x+1)^{-3}$$

$$f'''(0) = 2 = (0)^2 r$$

$$f^4(x) = -6(x+1)^{-4}$$

$$f^4(0) = -6$$

$$f^5(x) = 24(x+1)^{-5}$$

$$f^5(0) = 24$$

$$\therefore \ln(x+1) = 0 + \frac{x^1}{1!} - \frac{x^2}{2!} + 2 \frac{x^3}{3!} - 6 \frac{x^4}{4!} + 24 \frac{x^5}{5!} + \dots \infty$$

$$r = 0^9$$

$$x+r = 1^9$$

$$x+r = 0+x+r = 1^9$$

$$-\frac{x^2}{2!} - 0 + x+r = 1^9$$

$$\frac{x^1}{1!} - \frac{x^2}{2!} - x+r = 1^9$$

## Indeterminate Forms

18-Nov-2017

Ans

Ans

$x \rightarrow \infty$

$x \leftarrow \infty$

$$1] \lim_{x \rightarrow 1} \frac{\ln x}{x-1}$$

$$\left[ \frac{0}{1-1} = \frac{0}{0} \text{ form} \right]$$

By L' Hospital's rule:

$$\lim_{x \rightarrow 1} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{1}$$

$$= 1$$

$$2] \lim_{x \rightarrow 3} \frac{x-3}{3x^2 - 13x + 12}$$

By L' Hospital's rule:

$$\lim_{x \rightarrow 3} \frac{\frac{d}{dx}(x-3)}{\frac{d}{dx}(3x^2 - 13x + 12)}$$

$$= \lim_{x \rightarrow 3} \frac{1}{6x - 13}$$

$$= \frac{1}{6 \cdot 3 - 13}$$

$$= \frac{1}{5}$$

$$\frac{(x+12) \frac{b}{ab}}{(x-x) \frac{b}{ab}} \text{ with } x \leftarrow x$$

$$\frac{26.205}{1} \text{ with } x \leftarrow x$$

$$\frac{1-}{1-} =$$

$$1- =$$

$$\frac{1- \cancel{x}-x}{\cancel{x}x} \text{ with } 1$$

$$\frac{(1-\cancel{x}-x) \frac{b}{ab}}{(\cancel{x}x) \frac{b}{ab}} \text{ with } 0 \leftarrow x$$

$$\frac{1-3x+x}{6x+1} \text{ with } 0 \leftarrow x$$

$$\frac{1-3x+x}{6x+1} \text{ with } 0 \leftarrow x$$

$$\frac{1}{6x+1} \times \frac{6x}{6x+1} \text{ with } 0 \leftarrow x$$

$$\frac{1}{6+1} =$$

$$\text{Ex 41 - 8} \lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi}$$

By L'Hospital's rule:  $\frac{0}{0} = \frac{0}{0}$

$$\frac{\cos x}{1} \text{ mit } x \leftarrow \pi$$

$$\lim_{x \rightarrow \pi} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x - \pi)}$$

: stetigkeitsw. L'H

$$= \lim_{x \rightarrow \pi} \frac{\cos x}{1}$$

$$\frac{(\cos \pi) \frac{b}{ab}}{(\pi - \pi) \frac{b}{ab}} \text{ mit } b \leftarrow 0$$

$$= \frac{-1}{1}$$

$$\frac{-1}{0} \text{ mit } b \leftarrow 0$$

$$= -1$$

$$\pi =$$

$$51 \quad \lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3}$$

$$\frac{0-0}{0+0-0} \text{ mit } b \leftarrow 0$$

By L'Hospital's rule:

: stetigkeitsw. L'H

$$\lim_{x \rightarrow 0} \frac{\frac{d}{dx}(x - \tan^{-1} x)}{\frac{d}{dx}(x^3)}$$

$$\frac{(\pi - x) \frac{b}{ab}}{(0+0-0) \frac{b}{ab}} \text{ mit } b \leftarrow 0$$

$$= \lim_{x \rightarrow 0} \frac{1 - \frac{1}{1+x^2}}{3x^2}$$

$$\frac{1}{3} \text{ mit } b \leftarrow 0$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1+x^2-1}{1+x^2}}{3x^2}$$

$$\frac{1}{3} \text{ mit } b \leftarrow 0$$

$$= \lim_{x \rightarrow 0} \frac{x^2}{1+x^2} \times \frac{1}{3x^2}$$

$$\frac{1}{3} \text{ mit } b \leftarrow 0$$

$$= \frac{1}{3+0}$$

$$= 1/3$$

$$6] \lim_{x \rightarrow +\infty} \frac{e^{3x}}{x^2}$$

(positive)  $\infty$       (negative)  $\infty$

By L' Hospital's rule:

$$\lim_{x \rightarrow +\infty} \frac{3e^{3x}}{2x}$$

$$[\frac{\infty}{\infty} \text{ form}]$$

(positive)  $\infty$       (negative)  $\infty$

$$= \lim_{x \rightarrow +\infty} \frac{9e^{3x}}{2}$$

$$\frac{x+3}{x^2} \quad \text{mild} \quad 0 < x$$

$$= \frac{9e^{3x}}{2}$$

$$\frac{1}{x^2} \quad \text{mild} \quad 0 < x$$

$$= \frac{\infty}{2}$$

$$\frac{1}{2} \quad \text{mild}$$

$$7] \lim_{x \rightarrow 0} \frac{a^x - 1 - x \ln a}{x^2}$$

By L' Hospital's rule:

$$\lim_{x \rightarrow 0} \frac{a^x \ln a - 0 - \ln a \cdot 1}{2x}$$

$$\frac{\infty}{\infty} \quad \text{mild} \quad 0 < x$$

$$= \lim_{x \rightarrow 0} \frac{a^x \ln a - \ln a}{2x}$$

$$[\frac{0}{0} \text{ form}] \quad \text{mild} \quad 0 < x$$

$$= \lim_{x \rightarrow 0} \frac{\ln a \cdot a^x \cdot \ln a - 0}{2}$$

$$\frac{1}{2} \quad \text{mild} \quad 0 < x$$

$$= \frac{\ln a \cdot a^0 \cdot \ln a}{2}$$

$$\frac{1}{2} \quad \text{mild}$$

$$= \frac{(\ln a)^2}{2}$$

$$\frac{1}{2}$$

$$\underline{11} \lim_{x \rightarrow 0} \frac{\ln(\sin x)}{\ln(\tan x)}$$

By L' Hospital's rule:

$$\lim_{x \rightarrow 0} \frac{\frac{1}{\sin x} \cdot \cos x}{\frac{1}{\tan x} \cdot \sec^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{\cot x}{\csc x \sec^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sec^2 x}$$

$$= \frac{1}{\sec^2 0^\circ}$$

$$= 1$$

$$\underline{13} \lim_{x \rightarrow 0} \frac{\sin 2x}{x}$$

By L' Hospital's rule:

$$\lim_{x \rightarrow 0} \frac{2 \cos 2x}{1}$$

$$= \lim_{x \rightarrow 0} 2 \cos 2x$$

$$= 2 \times 1$$

$$= 2$$

$$\begin{aligned} & \text{mid} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

l'Hospital's rule

$$\begin{aligned} & \frac{x^2 - 2x}{x^2} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

$$\begin{aligned} & \frac{x^2 - 2x}{x} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

$$\begin{aligned} & \frac{x^2 - 2x}{1} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

$$\begin{aligned} & \frac{\sin(x) - x}{x} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

$$\begin{aligned} & \frac{\cos x - 1 - x \sin x}{x^2} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

$$\begin{aligned} & \frac{\cos x - 1 - x \sin x}{x^2} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

$$\begin{aligned} & \frac{0 - 0 - 0}{x^2} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

$$\begin{aligned} & \frac{0}{x^2} \\ & x \leftarrow 0 \\ & x + \epsilon \rightarrow \infty \end{aligned}$$

$$\underline{14} \quad \lim_{x \rightarrow 0} \frac{\sin x}{x^2}$$

By L' Hospital's rule :

$$\lim_{x \rightarrow 0} \frac{\cos x}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x}{2}$$

$$= \frac{0}{2}$$

$$= 0$$

$$\underline{15} \quad \lim_{x \rightarrow \infty} \frac{x}{e^x}$$

By L' Hospital's rule :

$$\lim_{x \rightarrow \infty} \frac{1}{e^x}$$

$$= \frac{1}{e^\infty}$$

$$= \frac{1}{\infty}$$

$$= 0$$

171       $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x}$

By L'Hospital's rule :

$$\lim_{x \rightarrow 0} \frac{2 \cos 2x}{5 \cos 5x}$$

$$= \frac{2 \times 1}{5 \times 1}$$

$$= \frac{2}{5}$$

## Transformation of Co-ordinates

$$O = \alpha + \beta x + \gamma y + \delta x^2 + \epsilon xy + \zeta y^2 + \ldots$$

Formulas:

Relation between cartesian and Polar co-ordinates:

$$1. \quad r = \sqrt{x^2 + y^2}$$

$$O = \alpha + \beta x + \gamma y + \delta x^2 + \epsilon xy + \zeta y^2 + \ldots$$

Polar  $\rightarrow (r, \theta)$

$$2. \quad \theta = \tan^{-1} \left( \frac{y}{x} \right)$$

Rectangular/Cartesian  $\rightarrow (x, y)$   
coordinates

$$3. \quad x = r \cos \theta$$

$$4. \quad y = r \sin \theta$$

Shifting Origin Formula:

$$5. \quad x = x' + h$$

$$6. \quad y = y' + k$$

Rotation of axes:

$$7. \quad x = x' \cos \theta - y' \sin \theta$$

$$8. \quad y = x' \sin \theta + y' \cos \theta$$

General equation of 2nd degree,

$$9. ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Transformed equation,

$$10. ax^2 + 2hxy + by^2 + c = 0$$

$$\left(\frac{y}{x}\right)^2 + \frac{2h}{a} \cdot \frac{y}{x} + \frac{b}{a} = 0$$

$$2h \cdot \frac{y}{x} = -a \cdot \frac{y^2}{x^2}$$

$$2h \cdot \frac{y}{x} = k$$

Let  $m = \frac{y}{x}$  put this

$$a + b m^2 + c = 0$$

$$A + Bm^2 + C = 0$$

Comparing with standard form

$$Bm^2 + (A-C)m + (C-A) = 0$$

Discriminant  $\Delta = p^2 - 4q$

1

- (i) Find the polar co-ordinates of the point  $\frac{(x, y)}{(2\sqrt{3}, -2)}$
- $$\left(-\frac{\pi}{6}\right)^{\text{r rad}} = \theta$$
- $$\sqrt{x^2 + y^2} = r$$

Hence Given,

$$\left(-\frac{\pi}{6}\right)^{\text{r rad}} =$$

$$x = 2\sqrt{3}$$

$$y = -2$$

$$\sqrt{x^2 + y^2} =$$

$$\sqrt{(2\sqrt{3})^2 + (-2)^2} =$$

We know,

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(2\sqrt{3})^2 + (-2)^2}$$

$$= \sqrt{12 + 4}$$

$$= 4$$

$$\left(-\frac{\pi}{6}\right)^{\text{r rad}} = \theta$$

$$\sqrt{x^2 + y^2} = r$$

$$\sqrt{12 + 4} = r$$

$$4 = r$$

And,

$$\theta = \tan^{-1} \left( \frac{y}{x} \right)$$

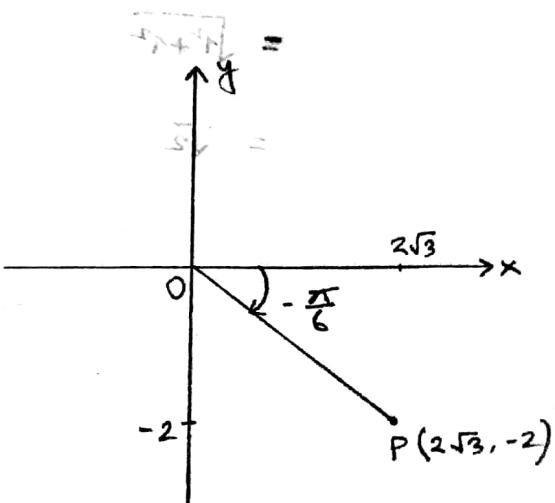
$$= \tan^{-1} \left( \frac{-2}{2\sqrt{3}} \right)$$

$$= \tan^{-1} \left( -\frac{1}{\sqrt{3}} \right)$$

$$= -\tan^{-1} \tan \left( -\frac{\pi}{6} \right)$$

$$= 2\pi - \frac{\pi}{6}$$

$$= \frac{11\pi}{6}$$



$$\therefore \text{Polar co-ordinates: } (r, \theta) = \left(4, \frac{11\pi}{6}\right)$$

$$(ii) \quad (0, -2)$$

(ii) Transforming into rectangular using add. unit (i)

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{0^2 + (-2)^2}$$

$$= 2$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \tan^{-1}\left(\frac{-2}{0}\right)$$

$$= \tan^{-1}(\infty)$$

$$= \tan^{-1} \tan\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2}$$

$$\boxed{r^2} = 4$$

$$\boxed{r(\sin\theta + i \cos\theta)} =$$

$$(iii) \quad (1, 1)$$

$$\boxed{r^2} =$$

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \tan^{-1}\left(\frac{1}{1}\right)$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1} \tan\left(\frac{\pi}{4}\right)$$

$$\left(\frac{\pi}{4}\right) \text{ rad} = ^{\circ} \text{ rad} =$$

$$\left(\frac{\pi}{4}\right) \text{ rad} = ^{\circ} \text{ rad} =$$

$$\therefore \text{Polar co-ordinates : } (r, \theta) = \left(\sqrt{2}, \frac{\pi}{4}\right)$$

$$\left(\frac{\pi}{4}\right) \text{ rad} = ^{\circ} \text{ rad} =$$

2]

(i) Find the rectangular co-ordinates of the point  $\left(7, \frac{2\pi}{3}\right)$

We know,

$$\theta = \tan^{-1} \frac{y}{x}$$

Want SW

$$\theta_{\text{SW}} = \sqrt{b^2 + c^2}$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x = 7 \cos \frac{2\pi}{3}$$

$$r \cos \theta = 7 \sin \frac{2\pi}{3}$$

$$r = \sqrt{b^2 + c^2}$$

LIA

(ii)  $(8, \frac{9\pi}{4})$

$$(\frac{8}{\sqrt{2}}) \cos \frac{\pi}{4} = \sqrt{2} \quad (ii)$$

$$x = r \cos \theta$$

$$y = r \sin \theta \quad \cos \theta = \sqrt{2} \quad \therefore$$

$$\begin{bmatrix} x = 8 \cos \frac{9\pi}{4} \\ y = 8 \sin \frac{9\pi}{4} \end{bmatrix}$$

$$\left[ \frac{8 \sin \frac{9\pi}{4}}{\sqrt{2}} \right] \theta = \pi \quad \therefore$$

$$(\theta + \pi) \frac{\pi}{2} = \pi \quad \therefore$$

(iii)  $(0, \pi)$

$$[\cos \theta = 0, \sin \theta = -1] \quad \frac{\pi}{2} = 0 \quad \text{why}$$

$$x = r \cos \theta$$

$$y = r \sin \theta \quad \frac{\pi}{2} = \pi \quad \therefore$$

$$= 0 \cos \pi$$

$$= 0 \sin \pi$$

$$= 0$$

$$\left( \frac{x+y}{\sqrt{2}} \right) \frac{\pi}{2} = \pi \quad \therefore$$

$$(x+y) \theta = \pi \quad \therefore$$

$\therefore$  Rectangular (co-ordinates)  $(x, y) = (0, 0)$

QED

3) using (i) if  $r = a \sin \theta$  interpret form of part (i)

$$\Rightarrow \sqrt{x^2 + y^2} = a \sin \theta$$

We know,

$$y = r \sin \theta$$

$$0 < \theta < \pi = a \sin \theta \cdot \sin \theta \quad [r = a \sin \theta]$$

$$\Rightarrow x^2 + y^2 = a^2 \sin^2 \theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = \sin^2 \theta$$

$$\Rightarrow x^2 + y^2 = a^2$$

Ans.

(ii)  $r = \sqrt{a} \cos\left(\frac{\theta}{2}\right)$

$\left(\frac{\text{MC}}{P}, 8\right)$  (ii)

$$\Rightarrow r = a \cos^2\left(\frac{\theta}{2}\right)$$

$\theta < 0 < \pi = \alpha$

$$\Rightarrow r = a \left[ \frac{1 + \cos^2\left(\frac{\theta}{2}\right)}{2} \right]$$

$$\left[ \because 2 \cos^2\theta = 1 + \cos 2\theta \right]$$

$$\therefore \cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\Rightarrow r = \frac{a}{2} (1 + \cos \theta)$$

Now,  $\cos \theta = \frac{x}{r} \quad [\because x = r \cos \theta]$

$(\text{TR}, 0)$  (iii)

$$\therefore r = \frac{a}{2} \left( 1 + \frac{x}{r} \right)$$

$\theta < 0 < \pi = \alpha$

$$\Rightarrow r = \frac{a}{2} \left( \frac{r+x}{r} \right)$$

$\Gamma_{\text{cos } \theta \geq 0}$

$$\Rightarrow 2r^2 = a(r+x)$$

$\theta =$

$$\Rightarrow 2(x^2 + y^2) = a(\sqrt{x^2 + y^2} + x) \quad \text{using result from (i)}$$

Ans.

$$41 \quad (i) \quad 9x^2 + 4y^2 = 36 \quad \text{--- } ①$$

We know,

$$x = r \cos \theta \quad \text{and} \quad y = r \sin \theta$$

$$y = r \sin \theta$$

putting the values of  $x$  and  $y$  in eqn ① we get,

$$\Rightarrow (r \cos \theta)^2 + 4(r \sin \theta)^2 = 36$$

$$\Rightarrow 9r^2 \cos^2 \theta + 4r^2 \sin^2 \theta = 36$$

$$\Rightarrow r^2 (9 \cos^2 \theta + 4 \sin^2 \theta) = 36$$

$$\Rightarrow (r^2 \cos^2 \theta + r^2 \sin^2 \theta) = \frac{36}{r^2}$$

$$(ii) \quad x^3 = y^2 (2a - x) \leftarrow \text{let } x = r \cos \theta \quad y = r \sin \theta \quad \text{then } 2a - x = 2a - r \cos \theta \leftarrow$$

$$\Rightarrow (r \cos \theta)^3 = (r \sin \theta)^2 (2a - r \cos \theta)$$

$$\Rightarrow r^3 \cos^3 \theta = r^2 \sin^2 \theta (2a - r \cos \theta)$$

$$\Rightarrow r \cos^3 \theta = 2a \sin^2 \theta - r \cos \theta \cdot \sin^2 \theta$$

$$\Rightarrow r \cos^3 \theta + r \cos \theta \cdot \sin^2 \theta = 2a \sin^2 \theta \quad \text{after division}$$

$$\Rightarrow r \cos \theta (\cos^2 \theta + \sin^2 \theta) = 2a \sin^2 \theta$$

$$\Rightarrow r \cos \theta = 2a \sin^2 \theta$$

Ans.

5]  $2x^2 + y^2 - 4x + 4y = 0 \quad \text{--- } ①$

Given,

New origin,  $O'(h, k) = (1, -2)$

We know,

$$x = x' + h \Rightarrow x = x' + 1$$

$$y = y' + k \Rightarrow y = y' - 2$$

substituting the values of  $x$  and  $y$  in eqn ①,

$$2(x' + 1)^2 + (y' - 2)^2 - 4(x' + 1) + 4(y' - 2) = 0$$

$$\Rightarrow 2(x'^2 + 2x' + 1) + (y'^2 - 4y' + 4) - 4x' - 4 + 4y' - 8 = 0$$

$$\Rightarrow 2x'^2 + 4x' + 2 + y'^2 - 4y' + 4 - 4x' - 4 + 4y' - 8 = 0$$

$$\Rightarrow 2x'^2 + y'^2 - 6 = 0$$

Removing suffixes,  $\Rightarrow 2x^2 + y^2 - 6 = 0$

$$2x^2 + y^2 - 6 = 0 = (2x^2 + y^2) - 6$$

Ans.

6)

$$\textcircled{1} \quad x^2 + y^2 - 8x + 14y + 5 = 0 \quad \xrightarrow{\text{New origin } (h, k) = (4, -7)} \textcircled{1}$$

Given : New origin  $(h, k) = (4, -7)$

We know,

$$x = x' + h \Rightarrow x = x' + 4$$

$$y = y' + k \Rightarrow y = y' - 7$$

Substituting the values of  $x$  and  $y$  in equation  $\textcircled{1}$

$$(x' + 4)^2 + (y' - 7)^2 - 8(x' + 4) + 14(y' - 7) + 5 = 0$$

$$\Rightarrow x'^2 + 8x' + 16 + y'^2 - 14y' + 49 - 8x' - 32 + 14y' - 98 + 5 = 0$$

$$\Rightarrow x'^2 + y'^2 - 60 = 0$$

Removing suffixes,

$$x^2 + y^2 - 60 = 0$$

$$x^2 + y^2 = 60$$

Excluding answer

$$x^2 + y^2 = 60$$

$$11) \quad 2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0 \quad \text{--- } ①$$

12

Given, New Origin  $O'(h, k) = (-2, 3)$

We know,

$$x = x' + h \Rightarrow x = x' - 2$$

$$y = y' + k \Rightarrow y = y' + 3$$

Substituting the values of  $x$  and  $y$  in eqn ①,

$$2(x'-2)^2 + 4(x'-2)(y'+3) + 5(y'+3)^2 - 4(x'-2) - 22(y'+3) + 7 = 0$$

$$\Rightarrow 2(x'^2 - 4x' + 4) + 4(x'y' + 3x' - 2y' - 6) + 5(y'^2 + 6y' + 9) - (4x' - 8) - 22(y' + 3) + 7 = 0$$

$$\Rightarrow 2x'^2 - 8x' + 8 + 4x'y' + 12x' - 8y' - 24 + 5y'^2 + 30y'$$

$$+ 45 - 4x' + 8 - 22y' - 66 + 7 = 0$$

$$\Rightarrow 2x'^2 + 5y'^2 + 4x'y' + 4x' - 22 = 0$$

Removing suffixes,

$$2x^2 + 5y^2 + 4xy + 4x - 22 = 0$$

Ans.

$$7x^2 - 2xy + y^2 + 1 = 0 \quad \text{--- (1)} \quad [2]$$

Given,  $\theta = \tan^{-1}(1/2)$   
 Now  $(x, y) = (1, 1)^\circ$  implies  $\sin \theta = 1/2$   
 $= 26.56^\circ$

We know,

$$x = x' \cos \theta - y' \sin \theta \quad \text{and} \quad y = x' \sin \theta + y' \cos \theta$$

$$\Rightarrow x = 0.9x' - 0.4y' \quad \Rightarrow y = 0.4x' + 0.9y'$$

$$\begin{cases} \cos 26.56^\circ = 0.8 \\ \sin 26.56^\circ = 0.6 \end{cases}$$

① Now put all the terms in eqn ① to get rid of suffixes

substituting the values of  $x$  and  $y$  in the eqn ①,

$$0 = 2 - (1-1)^2 P - (s-1)s Q - ((1-1)P + (s-1)s) Qs + (s+1)s M$$

$$7(0.9x' - 0.4y')^2 - 2(0.9x' - 0.4y')(0.4x' + 0.9y') + (0.4x' + 0.9y')^2 + 1 = 0$$

$$(1+1)s^2 P + (s-1)(s+1)s Qs + (s+1)s P + (s+1)s M \Leftarrow$$

$$\Rightarrow 7(0.81x'^2 - 0.72x'y' + 0.16y'^2) - 2(0.36x'^2 - 0.16x'y' + 0.81x'y' - 0.36y'^2) + (0.16x'^2 + 0.72x'y' + 0.81y'^2) + 1 = 0$$

$$P + Qs^2 + P^2 + 8P - 8Qs - P^2 P + P^2 Qs + P P + 5s^2 P + 5s^2 M \Leftarrow$$

$$\Rightarrow 5.67x'^2 - 5.04x'y' + 1.12y'^2 - 0.72x'^2 - 0.32x'y' + 1.62x'y' - 0.72y'^2 + 0.16x'^2 + 0.72x'y' + 0.81y'^2 + 1 = 0$$

$$0 = 5.11x'^2 - 3.02x'y' + 1.21y'^2 + 1 \quad \text{--- (2)}$$

2nd part removed

Removing suffixes,

$$5.11x^2 - 3.02xy + 1.21y^2 + 1 = 0$$

Ans.

$$8] \quad 11x^2 + 24xy + 4y^2 - 20x - 40y - 5 = 0 \quad \text{--- } ①$$

Given,  
New origin  $O'(h, k) = (2, -1)$

We know,

$$x = x' + h \quad \text{and} \quad y = y' + k$$

$$\Rightarrow x = x' + 2 \quad \Rightarrow y = y' - 1$$

$$y = y' - 1$$

$$x = x' + 2$$

substituting the values of  $x$  and  $y$  in the eqn ①

$$① \quad 11(x' + 2)^2 + 24(x' + 2)(y' - 1) + 4(y' - 1)^2 - 20(x' + 2) - 40(y' - 1) - 5 = 0$$

$$\Rightarrow 11(x'^2 + 4x' + 4) + 24(x'y' + 2y' - x' - 2) + 4(y'^2 - 2y' + 1)$$

$$- 20x' - 40 - 40y' + 40 - 5 = 0$$

$$\Rightarrow 11x'^2 + 44x' + 44 + 24x'y' + 48y' - 24x' - 48 + 4y'^2 - 8y' + 4$$

$$\Rightarrow 11x'^2 + 4y'^2 + 24x'y' - 20x' - 40y' + 40 - 5 = 0$$

$$\Rightarrow 11x'^2 + 4y'^2 + 24x'y' - 5 = 0$$

Removing suffixes,

$$11x^2 + 4y^2 + 24xy - 5 = 0 \quad \text{--- } ⑪$$

Also Given,  $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$$= 53.1^\circ$$

From book to refraction angle  $\theta$  of prism

We know,

$$x = x' \cos \theta - y' \sin \theta \quad \text{and} \quad y = x' \sin \theta + y' \cos \theta$$

$$= 0.6x' - 0.8y'$$

$$= 0.8x' + 0.6y'$$

$$\begin{cases} \cos 53.1^\circ = 0.6 \\ \sin 53.1^\circ = 0.8 \end{cases}$$

$$\begin{aligned} z &= x' \left| \begin{array}{l} = \frac{6x'}{10} - \frac{8y'}{10} \\ \therefore z = \frac{6x' - 8y'}{10} \end{array} \right| \quad r = d \left| \begin{array}{l} = \frac{8x'}{10} + \frac{6y'}{10} \\ \therefore r = \frac{8x' + 6y'}{10} \end{array} \right| \quad R = R \\ &= \frac{3x' - 4y'}{5} \end{aligned}$$

$$O = E - BDR - xR + \sqrt{B^2 + R^2} + BxR = (B+R) + \dots$$

Substituting the values of  $x$  and  $y$  in the eqn ⑪

$$11\left(\frac{3x' - 4y'}{5}\right)^2 + 4\left(\frac{4x' + 3y'}{5}\right)^2 + 24\left(\frac{3x' - 4y'}{5}\right)\left(\frac{4x' + 3y'}{5}\right) - 5 = 0$$

$$\Rightarrow 11(9x'^2 - 24x'y' + 16y'^2) + 4(16x'^2 + 24x'y' + 9y'^2)$$

$$\text{⑪} \quad + 24(12x'^2 - 16x'y' + 9x'y' - 12y'^2) - 125 = 0$$

$$\begin{aligned} \Rightarrow 99x'^2 - 264x'y' + 176y'^2 + 64x'^2 + 96x'y' + 36y'^2 + 288x'^2 - 384x'y' \\ + 216x'y' - 288y'^2 - 125 = 0 \end{aligned}$$

$$\Rightarrow 451x'^2 - 76y'^2 - 336x'y' - 125 = 0 \quad \text{Ans.}$$

Removing suffixes,

$$451x^2 - 76y^2 - 336xy - 125 = 0$$

$\therefore$  Ans.

$R = x - y$

$$91 \quad 9x^2 + 15xy + y^2 + 12x - 11y - 5 = 0 \quad \text{--- (1)}$$

Comparing eqn (1) with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$9x^2 + 15xy + y^2 + 12x - 11y - 5 = 0$$

Here,

$$\begin{array}{l|l|l|l|l} a = 9 & 2h = 15 & b = 1 & 2g = 12 & 2f = -11 \\ & \Rightarrow h = \frac{15}{2} & & \Rightarrow g = 6 & \Rightarrow f = -\frac{11}{2} \\ & & & & c = -5 \end{array}$$

$$\text{Let } F(x, y) = 9x^2 + 15xy + y^2 + 12x - 11y - 5 = 0$$

$$(i) \text{ To find } a_i \text{ in } F \text{ w.r.t } x \text{ to convert into first standard form}$$

$$\therefore \frac{\partial F}{\partial x} = 18x + 15y + 12 = 0$$

$$0 = 3 - \left( \frac{15}{2} + 12 \right) \Rightarrow 6x + 5y + 4 = 0 \quad \text{--- (II)} + \left( \frac{15}{2} + 12 \right) \text{ m}$$

$$\therefore \frac{\partial F}{\partial y} = 15x + 2y - 11 = 0 \quad \text{--- (III)}$$

$$0 = 3 - \left( \frac{15}{2} + 12 \right) \Rightarrow 15x + 2y - 11 = 0 \quad \text{--- (III)}$$

Solving equations (II) and (III)

$$(II) \times 2 - (III) \times 5,$$

$$\begin{array}{r} 12x + 10y + 8 = 0 \\ 75x + 10y - 55 = 0 \\ \hline -63x + 63 = 0 \end{array}$$

$$\Rightarrow -63(x-1) = 0$$

$$\Rightarrow x-1 = 0$$

$$\therefore x = 1$$

$$x=1 \Rightarrow ⑪ - 0 = AP + PC + AB - f^2 + PAf^2 + f^2C \quad ⑩$$

$$6 \cdot 1 + 5y + 4 = 0$$

$$\Rightarrow 5y = -10 \quad \text{from subtraction } ⑪ - ⑩ \quad \text{from } ⑩$$

$$\therefore y = -2$$

$$0 = AP + PC + AB - f^2 + PAf^2 + f^2C \quad ⑩$$

$\therefore$  New origin  $O'(\alpha, \beta) = (1, -2)$

$$AP = 3 \quad | \quad \alpha = 3 \quad | \quad \beta = B^2 \quad | \quad \Delta = d \quad | \quad PA = \alpha \quad | \quad B = R$$

$$PC = 2 \quad | \quad \alpha = 2 \quad | \quad \beta = C^2 \quad | \quad \Delta = d \quad | \quad CP = \alpha \quad | \quad R = C$$

We know,

Transformed eqn,  $ax^2 + 2hxy + by^2 + c = 0 \quad ⑭$

$$0 = AP + PC + AB - f^2 + PAf^2 + f^2C = (PA)^2 + (PC)^2$$

where,

$$c = g\alpha^2 + f\beta^2 + C^2 + A^2 = \frac{-76}{25} \quad \therefore$$

$$⑬ \quad = (6 \times 1) + \left(-\frac{11}{2}\right)(-2) + (-5)$$

$$= 6 + 11 - 5$$

$$⑭ \quad = \frac{0 = \alpha^2 + \beta^2 + \Delta^2}{12} \leftarrow \frac{-76}{25} \quad \therefore$$

$\therefore$  from eqn ⑭,

⑦ box ⑬ into eqn ⑭ for  $\alpha, \beta$

$$9x^2 + 15xy + y^2 + 12 = 0$$

Ans.

$$x^2 + \frac{15}{9}xy + y^2 = -12$$

$$x^2 + \frac{5}{3}xy + y^2 = -12$$

$$(x + \frac{5}{6}y)^2 = -12$$

$$x + \frac{5}{6}y = \pm i\sqrt{12}$$

$$x = \pm i\sqrt{12} - \frac{5}{6}y$$

$$141 \quad 9x^2 + 24xy + 2y^2 - 6x + 20y + 41 = 0 \quad \text{--- (i)}$$

Comparing eqn (i) with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Hence,

$$\begin{array}{l|l|l|l|l|l} a = 9 & 2h = 24 & b = 2 & 2g = -6 & 2f = 20 & c = 41 \\ \hline & \Rightarrow h = 12 & & \Rightarrow g = -3 & \Rightarrow f = 10 & \end{array}$$

(i)  $\rightarrow$  Q = 0 represents a point. Ans. inconsistent

$$\text{Let } F(x, y) = 9x^2 + 24xy + 2y^2 - 6x + 20y + 41 = 0$$

$$\therefore \frac{\partial F}{\partial x} = 18x + 24y - 6 = 0 \quad \text{--- (ii)}$$

$$(\Rightarrow 3x + 4y - 1 = 0) \quad \text{--- (ii)}$$

$$\therefore \frac{\partial F}{\partial y} \Rightarrow 24x + 4y + 20 = 0 \quad \text{--- (iii)}$$

Solving equations (ii) and (iii)

$$\begin{aligned} \text{(iii)} - \text{(ii)} , \quad & 24x + 4y + 20 = 0 \\ & \underline{-} \quad \underline{-} \quad \underline{+} \\ & \underline{3x + 4y - 1 = 0} \\ & \underline{21x + 19 = 0} \end{aligned}$$

$$\Rightarrow 21(x+1) = 0$$

$$\Rightarrow x+1 = 0$$

$$\therefore x = -1$$

$$x = -1 \Rightarrow \textcircled{11} \quad 0 = px^2 + qy^2 + rx + sy + t \quad \text{LHS}$$

$$3 \cdot (-1) + 4y - 1 = 0$$

$\Rightarrow 4y - 4 = 0$   $\Rightarrow$   $4y = 4$

$$\therefore y = 1$$

$$\therefore \text{New origin } O'(\alpha, \beta) = (-1, 1)$$

we know,

$$\text{Transformed eqn, } ax^2 + 2hxy + by^2 + c = 0 \quad \text{--- \textcircled{14}}$$

Now find \textcircled{11} term now must be constant at both ends

where,

$$\left( \frac{\partial}{\partial x} \right)^2 c = g\alpha + f\beta + c$$

no different between 2nd

$$= (-3)(-1) + 10 \cdot 1 + 41$$

$$\left( \frac{\partial}{\partial y} \right)^2 c = 54$$

$$c =$$

$\therefore$  from eqn \textcircled{14},

$$9x^2 + 24xy + 2y^2 + 54 = 0$$

$$\frac{P}{a} = 54 \text{ & } 20.0 \text{ --- } \boxed{}$$

$$\frac{Q}{b} = 24 \text{ & } 20.0 \text{ --- } \boxed{}$$

$$\text{Ans. } \frac{P}{a} =$$

$$\frac{12x+16y}{3} =$$

$$3x+4y + 20.0 = 0$$

$$12x+16y =$$

$$121 \quad 9x^2 + 24xy + 2y^2 + 54 = 0 \quad \text{--- } ①$$

(i)  $\theta = \tan^{-1} \frac{h}{a-b}$

comparing eqn ① with the following eqn

$$ax^2 + 2hxy + by^2 + c = 0$$

$a = A$        $b = B$        $c = C$

Hence,

$$\begin{array}{|c|c|c|c|} \hline a = 9 & 2h = 24 & b = 2 & c = 54 \\ \hline & \Rightarrow h = 12 & & \end{array}$$

want to

(vi)  $\theta = \tan^{-1} \frac{h}{a-b}$  for removing xy term

Now, to remove xy term from eqn ① both axes

are rotated through an angle  $\theta = \frac{1}{2} \tan^{-1} \left( \frac{2h}{a-b} \right)$

$$\begin{aligned} \theta &= \frac{1}{2} \tan^{-1} \left( \frac{24}{9-2} \right) \\ &= 36.9^\circ \end{aligned}$$

We know,

$$\begin{aligned} x &= x' \cos \theta - y' \sin \theta \\ &= \frac{4x'}{5} - \frac{3y'}{5} \\ &= \frac{4x' - 3y'}{5} \end{aligned}$$

(vi)  $\theta = 36.9^\circ$

$$\left[ \begin{array}{l} \because \cos 36.9^\circ = \frac{4}{5} \\ \sin 36.9^\circ = \frac{3}{5} \end{array} \right]$$

$$y = x' \sin \theta + y' \cos \theta$$

$$= \frac{3x' + 4y'}{5}$$

Substituting the values of  $x$  and  $y$  in the eqn ①

$$9 \left( \frac{4x' - 3y'}{5} \right)^2 + 24 \left( \frac{4x' - 3y'}{5} \right) \left( \frac{3x' + 4y'}{5} \right) + 2 \left( \frac{3x' + 4y'}{5} \right)^2 + 54 = 0$$

$$\Rightarrow 9(16x'^2 - 24x'y' + 9y'^2) + 24(12x'^2 - 9x'y' + 16y'^2) + 2(9x'^2 + 24x'y' + 16y'^2) + 1350 = 0$$

$$\Rightarrow 144x'^2 - 216x'y' + 81y'^2 + 288x'^2 - 216x'y' + 384x'y' - 288y'^2 + 18x'^2 + 48x'y' + 32y'^2 + 1350 = 0$$

$$\Rightarrow 450x'^2 - 175y'^2 + 1350 = 0$$

$$\Rightarrow 18x'^2 - 7y'^2 + 54 = 0$$

Removing suffixes,

$$18x^2 - 7y^2 + 54 = 0$$

Ans.

Ans. = first 200

Ans. = 100000

$$\text{Ques} \quad 10) \quad 11x^2 + 3xy + 7y^2 + 19 = 0 \quad \text{--- (1)}$$

$\alpha = \mu_2 + \left( \frac{\mu_1 + \mu_2}{2} \right) s + \left( \frac{\mu_1 - \mu_2}{2} \right) h \cos \theta + \left( \frac{\mu_1 + \mu_2}{2} \right) c \sin \theta$   
 comparing eqn (1) with the following eqn

$$(ax^2 + 2hxy + by^2 + c) = 0$$

$$\text{Here, } \alpha = \mu_2 + \left( \frac{\mu_1 + \mu_2}{2} \right) s +$$

$$a = 11 \quad | \quad 2h = 3 \quad | \quad b = 7 \quad | \quad c = 19$$

$$\Rightarrow h = \frac{3}{2} + \frac{\mu_1 + \mu_2 - 2\mu_2}{2}$$

$$\alpha = \mu_2 + \mu_2 + \mu_1 + \mu_2 +$$

Now, to remove  $xy$  term from eqn (1) both axes are rotated

$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{2h}{a-b} \right)$$

$$= \frac{1}{2} \tan^{-1} \left( \frac{3}{11-7} \right) \quad \text{Rearranging}$$

$$\theta = \frac{1}{2} \times 22.5^\circ$$

$$= 18.4^\circ$$

we know,

$$\begin{aligned} x &= x' \cos \theta - y' \sin \theta \\ &= 0.95x' - 0.32y' \\ &= \frac{95x'}{100} - \frac{32y'}{100} \\ &= \frac{95x' - 32y'}{100} \end{aligned}$$

$$\left[ \begin{array}{l} \because \cos 18.4^\circ = 0.95 \\ \sin 18.4^\circ = 0.32 \end{array} \right]$$

$$\begin{aligned} y &= x' \sin \theta + y' \cos \theta \\ &= 0.32x' + 0.95y' \\ &= \frac{32x' + 95y'}{100} \end{aligned}$$

substituting the values of  $x$  and  $y$  in the eqn ①

$$11(0.9x' - 0.32y')^2 + 3(0.9x' - 0.32y')(0.32x' + 0.95y') \\ + 7(0.32x' + 0.95y')^2 + 19 = 0$$

$$\Rightarrow 11(0.9x'^2 - 0.6xy' + 0.1y'^2) + 3(0.3x'^2 + 0.9xy' - 0.1xy' - 0.3y'^2) \\ + 7(0.1x'^2 + 0.6xy' + 0.9y'^2) + 19 = 0$$

$$\Rightarrow 9.9x'^2 - 6.6xy' + 1.1y'^2 + 0.9x'^2 + 2.7x'y' - 0.3xy' - 0.9y'^2 \\ + 0.7x'^2 + 4.2xy' + 6.3y'^2 + 19 = 0$$

$$\Rightarrow 11.5x'^2 + 6.5y'^2 + 19 = 0$$

$$x' = (\frac{v}{k} + \frac{v}{k} \cos \theta + \frac{v}{k} \sin \theta) \cos \theta + (\frac{v}{k} + \frac{v}{k} \cos \theta + \frac{v}{k} \sin \theta) \sin \theta$$

Removing suffixes,

$$11.5(x^2 + 6.34y^2 + 19) = 0 \Rightarrow x^2 + 6.34y^2 - 19 = 0$$

$$x^2 = (\frac{v^2}{k^2} - \frac{v^2}{k^2} \cos^2 \theta + \frac{v^2}{k^2} \sin^2 \theta) - \frac{v^2}{k^2}$$

$$x^2 = \frac{v^2}{k^2} - \frac{v^2}{k^2} \cos^2 \theta$$

$$x^2 = \frac{v^2}{k^2} - \frac{v^2}{k^2}$$

similarly for  $y$

$$y^2 = \frac{v^2}{k^2} - \frac{v^2}{k^2}$$

$\therefore$

$$\textcircled{1} \quad 13) \quad x^2 + 2\sqrt{3}xy - y^2 = 2a^2 \quad \text{--- } \textcircled{1}$$

We know,  $(x\cos\theta + y\sin\theta)(x\sin\theta - y\cos\theta) = (x^2 - y^2)\cos 2\theta$

$$x = x\cos\theta + y\sin\theta \quad y = x\sin\theta - y\cos\theta$$

$$= x'\cos 30^\circ - y'\sin 30^\circ \quad = x'\sin 30^\circ + y'\cos 30^\circ$$

$$\left[ \begin{array}{l} \because \cos 30^\circ = \frac{\sqrt{3}}{2} \\ \sin 30^\circ = \frac{1}{2} \end{array} \right] \quad = x'\left(\frac{\sqrt{3}}{2}\right) - y'\left(\frac{1}{2}\right) + \left( x'\left(\frac{1}{2}\right) + y'\left(\frac{\sqrt{3}}{2}\right) \right)$$

$$x = x\cos\theta + \left(\frac{\sqrt{3}x' - y'}{2}\right) \quad y = \frac{x' + \sqrt{3}y'}{2}$$

substituting the values of  $x$  and  $y$  in the eqn  $\textcircled{1}$ ,

$$\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 + 2\sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2}\right)\left(\frac{x' + \sqrt{3}y'}{2}\right) - \left(\frac{x' + \sqrt{3}y'}{2}\right)^2 = 2a^2$$

$$\Rightarrow (3x'^2 - 2\sqrt{3}x'y' + y'^2) + 2\sqrt{3}(3x'^2 + 3x'y' - x'y' - \sqrt{3}y'^2) - (x'^2 + 2\sqrt{3}x'y' + 3y'^2) = 8a^2$$

$$\Rightarrow 3x'^2 - 2\sqrt{3}x'y' + y'^2 + 6x'^2 + 6\sqrt{3}x'y' - 2\sqrt{3}x'y' - 6y'^2 - x'^2 - 2\sqrt{3}x'y' - 3y'^2 = 8a^2$$

$$\Rightarrow 8x'^2 - 6\sqrt{3}x'y' + 6\sqrt{3}x'y' - 8y'^2 = 8a^2$$

$$\Rightarrow 8x'^2 - 8y'^2 = 8a^2$$

$$\Rightarrow x'^2 - y'^2 = a^2$$

Removing suffixes,

$$x^2 - y^2 = a^2$$

Ans.

# Pair of Straight Lines

29<sup>th</sup> Nov 2017

$$11 \quad (i) \quad 2y^2 - xy - x^2 + y + 2x - 1 = 0 \quad \text{--- (1)}$$

Comparing eqn (1) with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{array}{c|c|c|c|c|c} a = -1 & 2h = -1 & b = 2 & 2g = 2 & 2f = 1 & c = -1 \\ & \Rightarrow h = -\frac{1}{2} & & \Rightarrow g = 1 & \Rightarrow f = \frac{1}{2} & \\ & & & & & \end{array}$$

eqn (1) will represent a pair of straight lines if,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\begin{aligned} (0,1) \cdot (1,0) &= (-1) \cdot 2 \cdot 1 + 2 \cdot \frac{1}{2} \cdot 1 \cdot \left(-\frac{1}{2}\right) + 1 \cdot \left(\frac{1}{2}\right)^2 - 2 \cdot 1^2 + 1 \cdot \left(-\frac{1}{2}\right)^2 \\ &= 2 - \frac{1}{2} + \frac{1}{4} - 2 + \frac{1}{4} \end{aligned}$$

$$= \frac{0}{\Delta} \quad \text{work out}$$

Hence eqn (1) represents a pair of straight lines.

Now,

$$\text{Let } F(x,y) = 2y^2 - xy - x^2 + y + 2x - 1 = 0$$

$$(F)_x =$$

$$\therefore \frac{\partial F}{\partial x} = -y - 2x + 2 = 0$$

$$\Rightarrow 2x + y - 2 = 0 \quad \text{--- (1)}$$

$$\therefore \frac{\partial F}{\partial y} = 4y - x + 1 = 0$$

$$\Rightarrow x - 4y - 1 = 0 \quad \text{--- (2)}$$

solving equations ⑪ and ⑬,

$$\text{⑪} = \alpha + \beta + \gamma + \delta - \text{pd} - \text{pf} - \text{sd} - \text{sf} \quad (i) \quad (1)$$

⑪ - ⑬ × 2,

$$2x + y - 2 = 0$$

$$\begin{aligned} \text{⑪} - 2\text{⑬} &= 2x + y - 2 = 0 \\ &\underline{\quad (-1) \quad (+) \quad (+)} \\ &9y = 0 \end{aligned}$$

$$\begin{array}{c|c|c|c|c|c} \text{P} = 1.5 & \text{P} = 7.5 & \Rightarrow y = 0 & \text{sd} & \text{L} = 11.5 & \text{L} = 28 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \frac{1}{8} = 7.5 & \text{P} = 11.5 & & & \frac{1}{8} = 11.5 & \end{array}$$

$y = 0 \Rightarrow \text{⑪},$

$$2x - 2 = 0$$

∴  $2x = 2 \Rightarrow x = 1$  (using ⑪)  $\Rightarrow \text{⑪} = 1$  (using ⑪)

$$\Rightarrow x = 1$$

$\therefore (\frac{1}{8}, 0)$  is the point (of) intersection.  $(x, y) = (1, 0)$

$$\frac{1}{p} + s - \frac{1}{p} + \frac{1}{s} - s =$$

Then,

we know,

$$\text{Angle, } \theta = \tan^{-1} \left( \frac{2\sqrt{h^2 - ab}}{a+b} \right)$$

$$= \tan^{-1} \left( \frac{2\sqrt{\frac{1}{4} + 2}}{1} \right)$$

$$\alpha + \beta + \gamma + \delta - \text{pd} - \text{pf} - \text{sd} - \text{sf} = (1.5) \pi + 90^\circ$$

$$= \tan^{-1}(3)$$

$$= 71.6^\circ \quad \text{Ans.}$$

⑫ = 11.5  $\Rightarrow$  11.5  $\times$  100%  $\approx 37.5\%$   $\text{Ans.}$

$$\theta = \pi + \alpha - \tan^{-1} \frac{ab}{p+q}$$

$$\text{⑬} = 11.5 - 11.5 = 0$$

$$(ii) 2x^2 - 2xy + x + 2y - 3 = 0 \quad \text{---} \quad \text{eqn } ①$$

Comparing eqn ① with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{array}{l|l|l|l|l|l} a = 2 & 2h = -2 & b = 0 & 2g = 1 & 2f = 2 & c = -3 \\ \Rightarrow h = -1 & & & \Rightarrow g = \frac{1}{2} & \Rightarrow f = 1 & \end{array}$$

eqn ① will represent a pair of straight lines if,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\begin{aligned} \Delta &= (0 + 2 \cdot 1 \cdot \frac{1}{2} \cdot (-1) - 2 \cdot 1^2 - 0 - (-3) \cdot (-1)^2) \\ &= 0 - 1 - 2 + 3 \\ &= 0 \end{aligned}$$

Hence eqn ① represents a pair of straight lines

Now,

$$\text{Let } F(x, y) = 2x^2 - 2xy + x + 2y - 3 = 0$$

$$\therefore \frac{\partial F}{\partial x} = 4x - 2y + 1 = 0 \quad \text{---} \quad \text{②}$$

$$\frac{\partial F}{\partial y} = -2x + 2 = 0 \quad \text{---} \quad \text{③}$$

P.T.O

Solving equations ⑪ and ⑬  $\Rightarrow$  ⑪ + ⑬  $\times 2$

$$⑪ + ⑬ \times 2, \quad 4x - 2y + 1 = 0$$

$$0 = \frac{-4x^2 + 4 + 4x^2 + 4x + 2y^2 + 5y}{-2y + 5} = 0$$

$$\Rightarrow y = \frac{5}{2}$$

$$\begin{array}{c|c|c|c|c|c} S = 0 & S = 72 & R = 18 & O = 8 & S = 144 & A = 10 \\ \hline y = \frac{5}{2} \Rightarrow ⑪ & R = 75 & \frac{1}{2} = 6.5 & & R = 15 & \end{array}$$

$$4x - 2 \cdot \frac{5}{2} + 1 = 0$$

$$4x - 5 + 1 = 0 \Rightarrow 4x - 4 = 0 \Rightarrow x = 1$$

$$0 = \Rightarrow x = 1 \quad L = 72 - 18 - 18 - 8 = 4$$

so the point of intersection  $(x, y) = (\alpha, \beta) = (1, \frac{5}{2})$

$$S + R + O = 0 =$$

We know,

$$\text{Angle, } \theta = \tan^{-1} \left( \frac{2\sqrt{h^2 - ab}}{a+b} \right)$$

$$= \tan^{-1} \left( \frac{2\sqrt{(-1)^2 - 0}}{2+0} \right)$$

$$⑪ \Rightarrow \theta = \tan^{-1} \left( \frac{2}{2} \right) = \frac{\pi}{4}$$

$$= \tan^{-1}(1)$$

$$⑬ \Rightarrow \theta = 45^\circ$$

Ans.

$$(iii) \quad x^2 + 3xy + 2y^2 + \frac{1}{8}x - \frac{1}{32} = 0 \quad \text{eqn } ① \text{ finite}$$

Comparing eqn ① with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{array}{l|l|l|l|l|l} a = 1 & 2h = 3 & b = 2 & 2g = \frac{1}{8} & f = 0 & c = -\frac{1}{32} \\ \Rightarrow h = \frac{3}{2} & & & \Rightarrow g = \frac{1}{16} & & \end{array}$$

eqn ① will represent a pair of straight lines if,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$(1 \cdot 2 \cdot -\frac{1}{32}) - 1 \cdot 0 - 2 \cdot (\frac{1}{16})^2 - (-\frac{1}{32})(\frac{3}{2})^2 = 0$$

$$= -\frac{1}{16} - \frac{1}{128} + \frac{9}{128}$$

$$= \frac{-8 - 1 + 9}{128} = 0 \quad \text{Ans}$$

$$= \left( \frac{0}{128} \right) \quad \text{Ans} \\ = 0 \quad [\because \frac{0}{\text{something}} = 0]$$

Hence eqn ① represents a pair of straight lines.

$$\text{Let } F(x, y) = x^2 + 3xy + 2y^2 + \frac{1}{8}x - \frac{1}{32} = 0$$

$$\therefore \frac{\partial F}{\partial x} = 2x + 3y + \frac{1}{8} = 0 \quad \text{--- ⑪}$$

$$\frac{\partial F}{\partial y} = 3x + 4y = 0 \quad \text{--- ⑫}$$

solving equations ⑪ and ⑫ we get point of intersection (iii)

$$⑪ \times 3 - ⑫ \times 2,$$

$$6x + 9y + \frac{3}{8} - 6x - 8y = 0$$

$$\Rightarrow y = -\frac{3}{8}$$

$$\Rightarrow y = -\frac{3}{8}$$

$$y = -\frac{3}{8} \Rightarrow ⑫, \quad \left| \begin{array}{l} \frac{3}{2}x - \frac{3}{8} \\ 3x - 4 \cdot \frac{3}{8} = 0 \end{array} \right| \quad \left| \begin{array}{l} \frac{3}{2}x = \frac{3}{8} \\ 3x = \frac{3}{2} \end{array} \right| \quad \left| \begin{array}{l} x = \frac{1}{2} \\ x = \frac{1}{2} \end{array} \right|$$

so we have  $x = \frac{1}{2}$  &  $y = -\frac{3}{8}$  as the solution this D op's

$$\Rightarrow x = \frac{1}{2}$$

so the point of intersection  $(x, y) = (\alpha, \beta) = \left(\frac{1}{2}, -\frac{3}{8}\right)$

$$\frac{\partial}{\partial x} + \frac{\partial}{\partial y} = \frac{\partial}{\partial x} = 2$$

we know,

$$\text{Angle, } \theta = \tan^{-1} \left( \frac{2\sqrt{h^2-ab}}{a+b} \right)$$

$$= \tan^{-1} \left( \frac{2\sqrt{(3/2)^2 - 1 \cdot 2}}{1+2} \right)$$

$$= \tan^{-1} \left( \frac{2\sqrt{9/4 - 2}}{3} \right)$$

$$= \tan^{-1} \left( \frac{2\sqrt{2/4}}{3} \right)$$

$$= \tan^{-1} \left( \frac{1}{3} \right)$$

$$\theta = 18.4^\circ$$

Ans.

$$(iv) \quad 21x^2 + 40xy - 21y^2 + 44x + 122y - 17 = 0 \quad \text{for } \begin{matrix} x=1 \\ y=1 \end{matrix} \quad \text{Eqn } ①$$

Comparing eqn ① with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{Eqn } ② \quad \begin{matrix} x=1 \\ y=1 \end{matrix}$$

Hence,

$$\begin{array}{l|l|l|l|l|l|l} a = 21 & 2h = 40 & b = -21 & 2g = 44 & 2f = 122 & c = -17 \\ (1)(2)-2(3) = (2)(3)-4(1) & \Rightarrow h = 20 & & \Rightarrow g = 22 & \Rightarrow f = 61 & & \end{array}$$

Eqn ① will represent a pair of straight lines if -

$$\Delta = ab(c + 2fg - af^2 - bg^2 - ch^2) \neq 0$$

$$\begin{aligned} \Delta &= 21 \cdot (-21) \cdot (-17) + 2 \cdot 61 \cdot 22 \cdot 20 - 21 \cdot (61)^2 + 21 \cdot (22)^2 + 17 \cdot (20)^2 \\ &= 7497 + 53680 - 78141 + 10164 + 6800 \\ &= 0 \end{aligned}$$

Hence eqn ① represents a pair of straight lines.

$$\text{Let } F(x, y) = 21x^2 + 40xy - 21y^2 + 44x + 122y - 17 = 0$$

$$\therefore \frac{\partial F}{\partial x} = 42x + 40y + 44 = 0 \quad \text{--- } ②$$

$$\frac{\partial F}{\partial y} = 40x - 42y + 122 = 0 \quad \text{--- } ③$$

From ①,

$$42x + 40y + 44 = 0$$

$$\Rightarrow 42x = -44 - 40y$$

$$\Rightarrow x = \frac{-22 - 20y}{21} \quad \text{--- } ④$$

Putting value of  $x$  in eqn (III) we get  $y = 1$

$$y = 1$$

Substituting the value of  $y$  in eqn (IV) we get  $x = -2$

$$y = 1 \Rightarrow (IV)$$

$$x = -2$$

So the point of intersection,  $(x, y) = (a, b) = (-2, 1)$

We know, angle forming a triangle will be

$$\text{Angle}, \theta = \tan^{-1} \left( \frac{2\sqrt{h^2 - ab}}{a + b} \right)$$

$$= \tan^{-1} \frac{2\sqrt{(20)^2 - (21)(-21)}}{21 - 21}$$

$$= \tan^{-1} \frac{2\sqrt{841}}{0}$$

$$= \tan^{-1} \frac{2\sqrt{841}}{0} =$$

Now multiply the value to find the angle in degrees  $\theta$  aps shown

$$= 90^\circ + 180^\circ + 180^\circ + 180^\circ = (4 \times 90^\circ) = 360^\circ$$

$$\therefore \theta = 360^\circ + 180^\circ + 180^\circ + 180^\circ = 1080^\circ$$

$$= 1080^\circ + 180^\circ + 180^\circ + 180^\circ = 1080^\circ$$

$$= 1080^\circ + 180^\circ + 180^\circ + 180^\circ = 1080^\circ$$

$$= 1080^\circ + 180^\circ + 180^\circ + 180^\circ = 1080^\circ$$

$$= 1080^\circ + 180^\circ + 180^\circ + 180^\circ = 1080^\circ$$

$$2] \text{ (i) } 2\lambda xy - y^2 + 4x + 2y + 8 = 0 \quad \text{---} \textcircled{1} \quad \text{(ii)}$$

Comparing eqn ① with the general eqn of 2nd degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Hence,

$$\begin{array}{l|l|l|l|l|l} a=0 & 2h=2\lambda & b=-1 & 2g=4 & 2f=2 & c=8 \\ \hline h=0 & \Rightarrow h=\lambda & b=-1 & \Rightarrow g=2 & \Rightarrow f=1 & \end{array}$$

∴ eqn ① will represent a pair of straight lines if,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow 0 + 4\lambda - 0 + 4 - 8\lambda^2 = 0$$

$$\Delta = 4(\lambda - 1)(\lambda + 1) + 4(\lambda - 1)^2 - \frac{1}{2}(1 - 4\lambda^2) \left( 4\lambda^2 - 4\lambda - 4 \right) \Leftarrow$$

$$\Rightarrow 8\lambda^2 - 4\lambda - 4 = 0$$

$$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$$

$$\Rightarrow 2\lambda^2 - 2\lambda + \lambda - 1 = 0 \quad \Leftarrow$$

$$\Rightarrow 2\lambda(\lambda - 1) + 1(\lambda - 1) = 0 \quad \Leftarrow$$

$$\Rightarrow (\lambda - 1)(2\lambda + 1) = 0 \quad \Leftarrow$$

$$\Rightarrow \lambda = 1, -\frac{1}{2} \quad \Leftarrow$$

Ans.

$$(ii) \quad 2x^2 + xy - y^2 - 2x - 5y + k = 0 \quad \text{--- } ① \quad \text{KSE } (ii) \quad 12$$

Comparing eqn ① with the general eqn of 2nd degree

$$0 = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Hence,

$$\begin{cases} a = 2 \\ h = 1 \\ r = 1 \end{cases} \Rightarrow \begin{cases} 2h = 1 \\ \Rightarrow h = \frac{1}{2} \end{cases} \quad \begin{cases} b = -1 \\ \Rightarrow g = -1 \end{cases} \quad \begin{cases} 2g = -2 \\ \Rightarrow g = -1 \end{cases} \quad \begin{cases} 2f = -5 \\ \Rightarrow f = -\frac{5}{2} \end{cases} \quad \begin{cases} c = k \end{cases}$$

The eqn ① will represent a pair of straight lines if,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Delta = P^2 - 4Q^2 + 0 - 4P^2 + 0 \Leftarrow$$

$$\Rightarrow 2(-1)k + 2(-\frac{5}{2})(-1)\frac{1}{2} - 2(-\frac{5}{2})^2 + 1 \cdot (-1)^2 - k(\frac{1}{2})^2 = 0$$

$$\Delta = P^2 - 4Q^2 + 8 \Leftarrow$$

$$\Rightarrow -2k + \frac{5}{2} - \frac{25}{4} + 1 - \frac{k}{4} = 0$$

$$\Rightarrow \frac{-8k + 10 - 50 + 4 - k}{4} = 0$$

$$\Rightarrow -9k - 36 = 0$$

$$\Rightarrow -9k = 36$$

$$\Rightarrow k = -4$$

Ans.

$$(iii) \quad x^2 - 2xy + 2y^2 + 3x - 5y + 2 = 0 \quad \text{Eqn ①} \quad \text{Ans. (vi)}$$

Comparing eqn ① with the general equation of 2nd degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{aligned} a &= 1 & 2h &= -2 & b &= 2 & 2g &= 3 & 2f &= -5 & c &= 2 \\ &\Rightarrow h = -\frac{\lambda}{2} &&& && \Rightarrow g = \frac{3}{2} && \Rightarrow f = -\frac{5}{2} && \end{aligned}$$

eqn ① will represent a pair of straight lines if,

1. discriminant of eqn ① is zero i.e.  $\Delta = 0$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow 1 \cdot 2 \cdot 2 + 2 \cdot (-\frac{5}{2}) \cdot \frac{3}{2} \cdot (-\frac{\lambda}{2}) - 1 \cdot (-\frac{5}{2})^2 - 2 \cdot (\frac{3}{2})^2 - 2 \cdot (-\frac{\lambda}{2})^2 = 0$$

$$\Rightarrow 4 + \frac{15\lambda}{4} - \frac{25}{4} - \frac{9}{2} - \frac{25}{2} = 0$$

$$\Rightarrow \frac{16 + 15\lambda - 25 - 18 - 25}{4} = 0$$

$$\Rightarrow -2\lambda^2 + 15\lambda - 27 = 0$$

$$\Rightarrow 2\lambda^2 - 15\lambda + 27 = 0$$

$$\Rightarrow 2\lambda^2 - 6\lambda - 9\lambda + 27 = 0$$

$$\Rightarrow 2\lambda(\lambda - 3) - 9(\lambda - 3) = 0$$

$$\Rightarrow (\lambda - 3)(2\lambda - 9) = 0$$

$$\Rightarrow \lambda = 3, \frac{9}{2}$$

Ans.

$$(iv) \quad 12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0 \quad \text{--- } ① \quad (iii)$$

Comparing eqn ① with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{array}{l|l|l|l|l|l} a = 12 & 2h = -10 & b = 2 & 2g = 11 & 2f = -5 & c = \lambda \\ \hline & \Rightarrow h = -5 & & \Rightarrow g = \frac{11}{2} & \Rightarrow f = -\frac{5}{2} & \end{array}$$

To find triplets following the condition how ① i.e. eqn ① will represent a pair of straight lines if,

$$\begin{aligned} D &= abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \\ D &= (12)(2)(\lambda) + 2(-5)(\frac{11}{2})(-\frac{5}{2}) - 12(-\frac{5}{2})^2 - 2(\frac{11}{2})^2 - \lambda \cdot (-5)^2 = 0 \\ &\Rightarrow 12 \cdot 2 \cdot \lambda + 2 \cdot (-\frac{5}{2}) \cdot \frac{11}{2} \cdot (-5) - 12 \cdot (-\frac{5}{2})^2 - 2 \cdot (\frac{11}{2})^2 - \lambda \cdot (-5)^2 = 0 \\ &\Rightarrow 24\lambda + \frac{275}{2} - 75 - \frac{121}{2} - 25\lambda = 0 \\ &\Rightarrow \frac{48\lambda + 275 - 150 - 121 - 50\lambda}{2} = 0 \\ &\Rightarrow 4 - 2\lambda = 0 \\ &\Rightarrow -2\lambda = -4 \\ &\Rightarrow \lambda = 2 \end{aligned}$$

*Ans. 2 or -2*

$$3] \quad (i) \quad x^2 + xy - 6y^2 - x - 8y - 2 = 0 \quad \text{Comparing with } (x-h)^2 + (y-k)^2 = r^2$$

comparing eqn ① with the general eqn of 2nd degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{array}{l|l|l|l|l} a = 1 & 2h = 1 & b = -6 & 2g = -1 & 2f = -8 \\ \Rightarrow h = \frac{1}{2} & & & \Rightarrow g = -\frac{1}{2} & \Rightarrow f = -4 \\ \hline \frac{(x-h)(y-k)}{r^2} & = & \frac{(x-h)^2 + (y-k)^2}{r^2} & = & \end{array} \quad | \quad c = -2$$

$$\text{Let } F(x, y) \equiv x^2 + xy - 6y^2 - x - 8y - 2 = 0$$

$$\left( \frac{\partial F}{\partial x} - \frac{\partial F}{\partial y} \right) = +2x + y - 1 = 0 \quad \text{--- } ⑪ \quad \left( \frac{\partial F}{\partial x} \right)$$

$$\left( \frac{\partial F}{\partial x} - \frac{\partial F}{\partial y} + \frac{2F}{\partial y} \right) = x - 12y - 8 = 0 \quad \text{--- } ⑫ \quad \left( \frac{\partial F}{\partial y} \right)$$

(solving equations ⑪ and ⑫,  $\frac{\partial F}{\partial x} = 2x + y - 1$ ,  $\frac{\partial F}{\partial y} = -12y - 8$ )

$$⑪ - ⑫ \times 2,$$

$$\begin{aligned} 8x - 24y - 16 &= 2x + y - 1 = 0 \\ 2x - 24y - 16 &= 0 \\ \cancel{2x} \cancel{- 24y} \cancel{- 16} &= 0 \\ 0 &= 25y + 15 = 0 \quad \Rightarrow \quad y = -\frac{3}{5} \end{aligned}$$

Now

$$y = -\frac{3}{5} \Rightarrow ⑪,$$

$$2x - \frac{3}{5} - 1 = 0$$

$$\Rightarrow 2x = \frac{3}{5} + 1$$

$$\Rightarrow x = \frac{4}{5}$$

$$\therefore (x, y) \stackrel{\text{①}}{=} (\alpha, \beta) = \left( \frac{4}{5}, -\frac{3}{5} \right)$$

support point to represent the center of the circle. A point of intersection

We know,

$$x^2 + 2xy + y^2 + 2gx + 2fy + c = 0$$

$$\Rightarrow \frac{(x-\alpha)^2 - (y-\beta)^2}{a-b} = \frac{(x-\alpha)(y-\beta)}{1}$$

$$\Rightarrow \frac{\left(x - \frac{4}{5}\right)^2 - \left(y + \frac{3}{5}\right)^2}{1} = \frac{\left(x - \frac{4}{5}\right)\left(y + \frac{3}{5}\right)}{\frac{1}{2}}$$

$$\Rightarrow \frac{\left(\frac{5x-4}{5}\right)^2 - \left(\frac{5y+3}{5}\right)^2}{7} = -2\left(xy + \frac{3x}{5} - \frac{4y}{5} - \frac{12}{25}\right)$$

$$\Rightarrow \frac{25x^2 - 40x + 16}{25} - \frac{25y^2 + 30y + 9}{25} = 14\left(\frac{25xy + 15x - 20y - 12}{25}\right)$$

$$\Rightarrow \frac{25x^2 - 40x + 16 - 25y^2 - 30y - 9}{25} = 14\left(\frac{25xy + 15x - 20y - 12}{25}\right)$$

$$\Rightarrow 25x^2 - 40x + 16 - 25y^2 - 30y - 9 = 350xy + 210x - 280y - 168$$

$$\Rightarrow 25x^2 - 350xy - 25y^2 - 250x + 310y + 193 = 0$$

$$\frac{x}{5} = k, \frac{y}{5} = l$$

Ans.

$$\textcircled{1} \quad \frac{x}{5} = k \quad \frac{y}{5} = l$$

$$x = 5k, y = 5l$$

$$x^2 + y^2 = 25k^2 + 25l^2 = 25$$

$$x^2 + y^2 = 25$$

$$(ii) \quad 8x^2 - 14xy + 6y^2 + 2x - y - 1 = 0 \quad \text{--- } \textcircled{1}$$

comparing eqn ① with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$a = b \quad \leftarrow$$

Here,

$$\begin{array}{l} a = 8 \\ \quad \quad \quad | \quad 2h = -14 \\ \quad \quad \quad | \quad \Rightarrow h = -7 \\ \quad \quad \quad | \quad b = 6 \end{array} \quad \text{--- } \textcircled{2} \quad \leftarrow$$

$$\text{Let } F(x, y) \equiv 8x^2 - 14xy + 6y^2 + 2x - y - 1 = 0$$

$$\therefore \frac{\partial F}{\partial x} = 16x - 14y + 2 = 0 \quad (\text{eqn } \textcircled{3})$$

$$\Rightarrow 8x - 7y + 1 = 0 \quad \text{--- } \textcircled{4}$$

$$\frac{\partial F}{\partial y} = 14x - 12y - 1 = 0 \quad \text{--- } \textcircled{5}$$

solving equations ④ and ⑤,

from ④,  $8x - 7y + 1 = 0$

$$\Rightarrow 8x = 7y - 1$$

$$\Rightarrow x = \frac{7y - 1}{8} \quad \text{--- } \textcircled{6}$$

$$8x - 7y + 1 = 0 \Rightarrow 8\left(\frac{7y - 1}{8}\right) - 7y + 1 = 0 \Rightarrow 7y - 1 - 7y + 1 = 0$$

$$\Rightarrow -14\left(\frac{7y - 1}{8}\right) + 12y - 1 = 0$$

$$\Rightarrow -\frac{98y + 14}{8} + 12y - 1 = 0$$

$$\Rightarrow \frac{14 - 98y + 96y}{8} = 1 \quad \text{(iii)}$$

$$\Rightarrow 14 - 2y = 8$$

$$\Rightarrow y = 3$$

$$y = 3 \Rightarrow \textcircled{W} \quad d = 3 \quad | \quad M = 18 \quad | \quad x = 18$$

$$x = \frac{7 \cdot 3 - 1}{8} \quad | \quad 18 = 18$$

$$3 + 1 + p = 8 \cdot \frac{5}{2} \text{ feet per min} - \text{the rest (base) } 7 \text{ feet}$$

$$\therefore (x,y) = (\alpha, \beta) = \left(\frac{5}{2}, 3\right)$$

We know,

$$\frac{(x-\alpha)^L - (y-\beta)^L}{a-b} = \frac{(x-\alpha)(y-\beta)}{h}$$

$$\Rightarrow \frac{(x - \frac{5}{2})^2 - (y-3)^2}{8-6} = \frac{(x - \frac{5}{2})(y-3)}{-7}$$

$$\Rightarrow \frac{x^2 - 5x + \frac{25}{4} - y^2 + 6y - 9}{2} = \frac{xy - 3x - \frac{5y}{2} + \frac{15}{2}}{2}$$

$$\Rightarrow -7x^2 + 35x - \frac{175}{4} + 7y^2 - 42y + 63 = 2xy - 6x - 5y + 15$$

$$\Rightarrow 28n^2 - 140n + 175 - 28y^2 + 168y - 252 + 8xy - 24x - 20y + 60 = 0$$

$$\Rightarrow 28x^2 + 8xy - 28y^2 - 164x + 148y - 17 = 0$$

6-18-1980 - 10:00 AM

Ans.

$$(iii) \quad 2x^2 + xy - y^2 - 3x + 6y - 9 = 0 \quad \text{--- } \textcircled{1}$$

comparing eqn  $\textcircled{1}$  with the general eqn of 2nd degree.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Hence,

$$a = 2 \quad | \quad 2h = 1 \quad | \quad b = -1 \\ \Rightarrow h = \frac{1}{2}$$

$$\text{Let } F(x, y) \equiv 2x^2 + xy - y^2 - 3x + 6y - 9 = 0 \quad \text{--- } \textcircled{ii}$$

$$\therefore \frac{\partial F}{\partial x} = 4x + y - 3 = 0 \quad \text{--- } \textcircled{III}$$

$$\frac{\partial F}{\partial y} = x - 2y + 6 = 0 \quad \text{--- } \textcircled{IV}$$

solving equations  $\textcircled{III}$  and  $\textcircled{IV}$ ,

$$\textcircled{III} - \textcircled{IV} \times 4,$$

$$4x + y - 3 - 4x + 8y - 24 = 0$$

$$\Rightarrow 9y = 27$$

$$\Rightarrow y = 3$$

$$y = 3 \Rightarrow \textcircled{III},$$

$$x = 0$$

$$\therefore (x, y) = (\alpha, \beta) = (0, 3)$$

We know,

$$\frac{(x-\alpha)^L - (y-\beta)^L}{a-b} = \frac{(x-\alpha)(y-\beta)}{h}$$

$$\Rightarrow \frac{(x-0)^L - (y-3)^L}{2+1} = \frac{(x-0)(y-3)}{\frac{1}{2}}$$

$$\Rightarrow \frac{x^2 + 0 + 0 - y^2 + 6y - 9}{3} = \frac{xy - 3x - 0 - 0}{1/2} \quad (iii)$$

$$\Rightarrow \frac{x^2 - y^2 + 6y - 9}{3} = 2xy - 6x$$

$$\Rightarrow x^2 - 6xy - y^2 + 18x + 6y - 9 = 0 \quad \text{Ans.}$$

$$(iv) \quad 2x^2 + 7xy + 6y^2 + 13x + 22y + 20 = 0 \quad \text{--- ①}$$

comparing eqn ① with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Hence,

$$\begin{array}{l|l|l} a = 2 & 2h = 7 & b = 6 \\ & \Rightarrow h = \frac{7}{2} & \end{array}$$

$$\text{Let } F(x, y) \equiv 2x^2 + 7xy + 6y^2 + 13x + 22y + 20 = 0$$

$$\therefore \frac{\partial F}{\partial x} = 4x + 7y + 13 = 0 \quad \text{--- ②}$$

$$\frac{\partial F}{\partial y} = 7x + 12y + 22 = 0 \quad \text{--- ③}$$

solving equations ② and ③,

$$\text{②} \times 7 - \text{③} \times 4,$$

$$28x + 49y + 91 - 28x - 48y - 88 = 0$$

$$\Rightarrow y = -3$$

$$t = -3 \Rightarrow \textcircled{1},$$

$$4x - 21 + 13 = 0$$

$$\Rightarrow 4x = 8$$

$$\Rightarrow x = 2$$

$$\therefore (x, y) = (\alpha, \beta) = (2, -3)$$

we know,

$$\frac{(x-\alpha)^L - (y-\beta)^L}{a-b} = \frac{(x-\alpha)(y-\beta)}{h}$$

$$\Rightarrow \frac{(x-2)^L - (y+3)^L}{2-b} = \frac{(x-2)(y+3)}{\frac{7}{2}}$$

$$\Rightarrow \frac{x^L - 4x + 4 - y^L - 6y - 9}{-4} = \frac{2(ny + 3n - 2y - 6)}{7}$$

$$\Rightarrow 7x^L - 28x + 28 - 7y^L - 42y - 63 = -8xy - 24x + 16y + 48$$

$$\Rightarrow 7x^L + 8xy - 7y^L - 4x - 58y - 83 = 0$$

Ans.

circle

11 - Dec - 2017

- 1] (i) Hence, (iii)  
 centre  $(-2, -1)$  radius  
 radius 4

the equation of the circle,

$$\begin{aligned}
 (x-h)^2 + (y-k)^2 &= r^2 \\
 \Rightarrow (x+2)^2 + (y+1)^2 &= 4^2 \\
 \Rightarrow x^2 + 4x + 4 + y^2 + 2y + 1 &= 16 \\
 \Rightarrow x^2 + y^2 + 4x + 2y - 11 &= 0
 \end{aligned}$$

Ans.

- (ii) Hence,  
 centre  $(9, 0)$   
 radius 1

the equation of the circle,

$$\begin{aligned}
 (x-h)^2 + (y-k)^2 &= r^2 \\
 \Rightarrow (x-9)^2 + (y-0)^2 &= 1^2 \\
 \Rightarrow x^2 - 18x + 81 + y^2 &= 1 \\
 \Rightarrow x^2 + y^2 - 18x + 80 &= 0
 \end{aligned}$$

Ans.

(iii) Here,

centre  $(0, 0)$

radius 5

the equation of the circle,

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\Rightarrow (x-0)^2 + (y-0)^2 = 5^2$$

$$\Rightarrow x^2 + y^2 = 25$$

$$\Rightarrow x^2 + y^2 - 25 = 0$$

Ans.

(b.c) answer

A student

solving came to nothing

$$r^2 = (x-h)^2 + (y-k)^2$$

$$r^2 = (x-0)^2 + (y-0)^2$$

$$r^2 = y^2 + x^2 + 25$$

$$x^2 + y^2 - 25 = 0$$

A student

2

$$(i) \quad 5x^2 + 5y^2 - 11x - 9y - 12 = 0 \quad (ii)$$

$$\Rightarrow x^2 + y^2 - \frac{11}{5}x - \frac{9}{5}y - \frac{12}{5} = 0 \quad \text{--- (1)}$$

Comparing eqn (1) with the general eqn of a circle,

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{array}{c|c|c} g = -\frac{11}{5} & f = -\frac{9}{5} & c = -\frac{12}{5} \\ 2g = -\frac{11}{5} & 2f = -\frac{9}{5} & c = -\frac{12}{5} \\ \Rightarrow g = -\frac{11}{10} & (\Rightarrow f = -\frac{9}{10}) & \end{array}$$

(i - g - f - c) = (r - s -) entered ..

$\therefore$  centre  $(-g, -f) = \left(-\frac{11}{10}, -\frac{9}{10}\right)$ , radius

$$\text{radius, } r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{\left(-\frac{11}{10}\right)^2 + \left(-\frac{9}{10}\right)^2 + \frac{12}{5}}$$

$$= \sqrt{\frac{121 + 81 + 240}{100}}$$

$$= \frac{\sqrt{442}}{10}$$

Ans.

$$(ii) \quad x^2 + y^2 + 2x + 2y + 1 = 0 \quad \text{--- } ①$$

Comparing eqn ① with the general eqn of a circle,

we find  $x^2 + y^2 + 2gx + 2fy + c = 0$  npo prissapnoo

$$0 = x^2 + y^2 + 2gx + 2fy + c$$

Hence,

$$\begin{array}{l|l|l} 2g = 2 & 2f = 2 & c = 1 \\ \Rightarrow g = 1 & \Rightarrow f = 1 & \frac{c}{r^2} = \frac{1}{r^2} \\ \therefore \text{centre } (-g, -f) = (-1, -1) & & \end{array}$$

radius,  $\sqrt{\frac{c}{r^2}} = \sqrt{g^2 + f^2 - c^2}$  entnes i.

$$= \sqrt{1^2 + 1^2 - 1} \quad \text{or. 2nd way}$$

$$\left. \frac{1}{r^2} + \left( \frac{c}{r^2} - 1 \right) = \left( \frac{1}{r^2} - 1 \right) \right\} =$$

Ans.

$$\frac{0.01 + 1.01 - 1.01}{0.01} =$$

$$\frac{0.01}{0.01} = 1$$

Ans

$$(iii) \quad x^2 + y^2 + 2x - 4y - 8 = 0 \quad (F.F.) \quad (F.S.) \quad (E.T.) \quad (I) \quad 18$$

Comparing eqn ① with the general eqn of a circle

$$① - x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{So}$$

Here,

: solving with respect to finding ① obtain

$$\begin{array}{l|l|l} 2g = 2 & 2f = -4 & c = -8 \\ \text{②} & & \\ \Rightarrow g = 1 & \Rightarrow f = -2 & \text{③} \leftarrow (\text{E.T}) \\ \text{④} & \text{⑤} & \text{⑥} \leftarrow (\text{F.S}) \end{array}$$

$$\therefore \text{centre } (-g, -f) = (-1, 2) \quad . \quad \text{⑦} \leftarrow (\text{F.T})$$

$$\text{radius, } r = \sqrt{g^2 + f^2 - c}$$

$$\text{⑧} \quad = \sqrt{(-1)^2 + 2^2 + 8} \quad \leftarrow \text{④} + \text{⑤}$$

$$= \sqrt{13} \quad \leftarrow \text{⑧} + \text{⑥}$$

$$\text{⑨} \quad \text{Ans.}$$

$$\therefore \text{Ans.} \quad \text{⑩} \leftarrow \text{⑦}$$

$$D = 2\pi r - \pi R -$$

$$\frac{\pi D}{2} = D \quad \leftarrow$$

$$\frac{\pi D}{2} = D - R$$

$$31 \quad (i) \quad (1, 3), (2, -1), (-1, 1) \text{ & } \text{P.M. } 10:00 \text{ A.M.}$$

Let the equation of the circle be,

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{--- (1)}$$

circle (1) passing through the points:

$$(1, 3) \rightarrow (1), \quad 1 + 9 + 2g + 6f + c = 0 \quad \text{--- (2)}$$

$$(2, -1) \rightarrow (2), \quad 4 + 1 + 4g - 2f + c = 0 \quad \text{--- (3)}$$

$$(-1, 1) \rightarrow (1), \quad 1 + 1 + 2g + 2f + c = 0 \quad \text{--- (4)}$$

Now,

$$(2) + (3) \Rightarrow 12 + 8f + 2c = 0 \quad \text{--- (5)}$$

$$(4) \times 2 \Rightarrow 2 + 4g + 4f + 2c = 0 \quad \text{--- (6)}$$

$$(5) - (6) \times 4,$$

$$-24 - 10c = 0$$

$$\Rightarrow c = -\frac{24}{10}$$

$$\Rightarrow c = -\frac{12}{5}$$

$$c = -\frac{12}{5} \rightarrow \textcircled{vi},$$

$$9 + 2f + 3 \times \left(-\frac{12}{5}\right) = 0$$

$$\Rightarrow 2f = \frac{36}{5} - 9$$

$$\Rightarrow 2f = \frac{-9}{5}$$

$$\Rightarrow f = -\frac{9}{10}$$

$$f = -\frac{9}{10} \rightarrow \textcircled{iv},$$

$$2 - 2g + 2f + c = 0$$

$$\Rightarrow 2g = 2 + 2f + c$$

$$\Rightarrow 2g = 2 - 2 \cdot \frac{9}{10} - \frac{12}{5}$$

$$\Rightarrow 2g = \frac{10 - 21}{5}$$

$$\Rightarrow 2g = -\frac{11}{5}$$

$$\Rightarrow g = -\frac{11}{10}$$

from ①,

$$x^2 + y^2 - \frac{11}{5}x - \frac{9}{5}y - \frac{12}{5} = 0$$

$$\Rightarrow 5x^2 + 5y^2 - 11x - 9y - 12 = 0 \quad \text{Ans.}$$

$$(iii) \quad (3,1), (4,-3), (1,-1)$$

Let the equation of the circle be,

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{--- (1)}$$

circle (1) passing through the points:

$$(3,1) \rightarrow (1),$$

$$3^2 + 1^2 + 2g \cdot 3 + 2f \cdot 1 + c = 0$$

$$\Rightarrow 6g + 2f + c = -10 \quad \text{--- (2)}$$

$$(4,-3) \rightarrow (1),$$

$$4^2 + (-3)^2 + 2g \cdot 4 + 2f \cdot (-3) + c = 0$$

$$\Rightarrow 8g - 6f + c = -25 \quad \text{--- (3)}$$

$$(1,-1) \rightarrow (1),$$

$$1^2 + (-1)^2 + 2g \cdot 1 + 2f \cdot (-1) + c = 0$$

$$\Rightarrow 2g - 2f + c = -2 \quad \text{--- (4)}$$

Now,

$$(2) - (4) \times 3 \Rightarrow$$

$$6g + 2f + c + 10 - (6g - 6f - 3c - 6) = 0$$

$$\Rightarrow 8f - 2c + 4 = 0 \quad \text{--- (5)}$$

$$(3) - (4) \times 4 \Rightarrow$$

$$8g - 6f + c + 25 - (8g + 8f - 4c - 8) = 0$$

$$\Rightarrow 2f - 3c + 17 = 0 \quad \text{--- (6)}$$

$$\textcircled{V} - \textcircled{VI} \times 4 \Rightarrow$$

$$8f - 2c + 4 - 8f + 12c - 68 = 0$$

$$\Rightarrow 10c - 64 = 0$$

$$\Rightarrow c = \frac{64}{10}$$

$$\Rightarrow c = \frac{32}{5}$$

$$c = \frac{32}{5} \rightarrow \textcircled{VI},$$

$$2f - 3 \cdot \frac{32}{5} + 17 = 0$$

$$\Rightarrow 2f = \frac{96}{5} - 17$$

$$\Rightarrow f = \frac{11}{10}$$

from \textcircled{IV},

$$2g - 2 \cdot \frac{11}{10} + \frac{32}{5} = -2$$

$$\Rightarrow 2g = \frac{22}{10} - \frac{32}{5} - 2$$

$$\Rightarrow g = -\frac{31}{10}$$

from \textcircled{I},

$$x^2 + y^2 - 2 \cdot \frac{31}{10}x + 2 \cdot \frac{11}{10}y + \frac{32}{5} = 0$$

$$\Rightarrow x^2 + y^2 - \frac{31}{5}x + \frac{11}{5}y + \frac{32}{5} = 0$$

$$\Rightarrow 5x^2 + 5y^2 - 31x + 11y + 32 = 0$$

Ans.

# Tangent and Normal of circle

12<sup>th</sup> Dec 2017

$$① \quad x^2 + y^2 - 2x + 4y + 3 = 0 \quad — ①$$

$$x^2 + y^2 - 8x - 2y + 9 = 0 \quad \text{---} \quad \textcircled{11}$$

comparing circles ⑩ and ⑪ with the general eqn of a circle,

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Here,

$$\textcircled{1} \Rightarrow [m = 2f - 2c] \quad ; \quad 2g = -2 \quad ; \quad 2f = 4 \quad ; \quad c = 3$$

$$\Rightarrow g = -1 \quad \Rightarrow f = 2$$

$$\therefore \text{centre}_g(-g, -f) = (1, -2)$$

$$\text{radius, } r_1 = \sqrt{g^2 + f^2 - c} \\ = \sqrt{2}$$

$$\textcircled{11} \Rightarrow 2g = -8 ; \quad 2f = -2 ; \quad c = 9$$

$$\Rightarrow g = -4 \quad \Rightarrow f = -1$$

$$\therefore \text{centre, } c_2 = (4, 1)$$

$$\text{Radius, } r_2 = \sqrt{8}$$

$$= 2\sqrt{2}$$

$$\frac{(x - (-2 + \sqrt{5})) \cdot (x - (4 + \sqrt{5}))}{(-2 - \sqrt{5}) \cdot (4 - \sqrt{5})}$$

$$\therefore c_1c_2 = \sqrt{(1-4)^2 + (-2-1)^2} \quad \left[ \because PQ = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} \right]$$

$$= \sqrt{18}$$

$$= \sqrt{32}$$

( $\mu_1, \mu_2$ ) die Stufen der denkt- und schreibt-

Now,

$$\text{Circles } C_1 \text{ and } C_2 \text{ touch each other at } A(x, y) \text{ where } x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad (1)$$

and as the angle between the lines (i) and (ii) is  $90^\circ$  from (ii) we have

$$\therefore x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} + \alpha r^2 + \beta = S_0$$

where

$$= \frac{\sqrt{2} \cdot 4 + 2\sqrt{2} \cdot 1}{\sqrt{2} + 2\sqrt{2}} \quad [\text{Hence, } m = n] \quad (1)$$

$$\Leftrightarrow S_0 = 7 \Leftrightarrow P = 7 \Leftrightarrow S = B$$

$$\therefore x = \frac{4\sqrt{2} + 2\sqrt{2}}{3\sqrt{2}} \quad S = 7 \Leftrightarrow P = B \Leftrightarrow$$

$$(P, B) = 80^\circ \text{ and } S =$$

$$= 2$$

$$\overline{AB} = S^2 \times \text{constant}$$

$$(S, P) = (7 - B) \text{ and } S = B$$

$$\overline{AB} = P^2 \times \text{constant}$$

$$\overline{AB} = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\overline{AB} =$$

$$= \frac{\sqrt{2} \cdot 1 + 2\sqrt{2} \cdot (-2)}{\sqrt{2} + 2\sqrt{2}} \quad (S, P) =$$

$$= \frac{\sqrt{2} - 4\sqrt{2}}{3\sqrt{2}} \quad [(P, S) + (B, S)] = 80^\circ \text{ and } \\ = -1$$

$$\therefore A(x, y) = (2, -1)$$

Hence the circles touch one another at  $(2, -1)$

Ans.

2)

$$x^2 + y^2 - 9x + 14y - 7 = 0 \quad \text{--- (i) } \quad 18$$

$$x^2 + y^2 + 15x + 14 = 0 \quad \text{--- (ii) } \quad x^2 + y^2 + 2ax + 2bx + c = 0$$

$$\text{--- (iii) } \quad 0 = a + bx + c$$

Let the equation of the circle be,

$$x^2 + y^2 - 9x + 14y - 7 + k(x^2 + y^2 + 15x + 14) = 0 \quad \text{--- (iv)}$$

$$0 = (1 + kp + ka + kb + kc)x + (p - b + kc)$$

equation (iv) passing through the point (2, 5),

$$0 = a + bp + ka + kb + kc + p - b + kc \Leftarrow$$

$$(2^2 + 5^2 - 9 \cdot 2 + 14 \cdot 5 - 7) + k(2^2 + 5^2 + 15 \cdot 2 + 14) = 0$$

$$0 = a + kp + ka + kb + kc + p - b + kc \Leftarrow$$

$$\Rightarrow (4 + 25 - 18 + 70 - 7) + k(4 + 25 + 30 + 14) = 0$$

$$0 = (1 + k) + kp + ka + kb + kc + p - b + kc \Leftarrow$$

$$\Rightarrow k = -\frac{74}{73}$$

$$\text{--- (v) } \quad 0 = \left(\frac{1+k}{k+1}\right) + B\left(\frac{kp}{k+1}\right) + C\left(\frac{ka}{k+1}\right) + D\left(\frac{kb}{k+1}\right) + E\left(\frac{kc}{k+1}\right) + F + G \Leftarrow$$

from (v), Imp. (independent term)  $\Rightarrow$  (vi)  $\Rightarrow$   $B = 0$

$$x^2 + y^2 - 9x + 14y - 7 + \left(-\frac{74}{73}\right)(x^2 + y^2 + 15x + 14) = 0$$

$$\Rightarrow 73x^2 + 73y^2 - 657x + 1022y - 511 - 74x^2 - 74y^2 - 1110x - 1036 = 0$$

$$\Rightarrow -x^2 - y^2 - 1767x + 1022y + \frac{1547}{525} = 0$$

$$\Rightarrow x^2 + y^2 + 1767x - 1022y - \frac{1547}{525} = 0 \quad \Leftarrow$$

Ans.

$$\left(\frac{-1547}{525} - \frac{1022}{525} \cdot \frac{1767}{525} \cdot 1\right) = (7 - 6 - 1) \text{ or } 0$$

$$31. \quad x^2 + y^2 - 1 = 0 \quad \text{--- (i)}$$

$$x^2 + y^2 + 2x + 4y + 1 = 0 \quad \text{--- (ii)}$$

$$x + 2y + 5 = 0 \quad \text{--- (iii)}$$

Now eliminate  $x$  from eqns (i) & (ii)

(iv) Let the equation of the circle be, be

$$(x^2 + y^2 - 1) + k(x^2 + y^2 + 2x + 4y + 1) = 0$$

(i.e.,  $x$ ) taking both sides of eqn (iii) multiply

$$\Rightarrow x^2 + y^2 - 1 + x^2 k + y^2 k + 2xk + 4yk + k = 0$$

$$0 = (1+k)x^2 + (1+k)y^2 + 2xk + 4yk + k - 1$$

$$\Rightarrow x^2 + y^2 + \frac{2k}{1+k}x + \frac{4k}{1+k}y + \frac{k-1}{1+k} = 0$$

$$0 = (1+k)x^2 + (1+k)y^2 + 2xk + 4yk + (k-1)$$

$$\Rightarrow x^2 + y^2 + \left(\frac{2k}{1+k}\right)x + \left(\frac{4k}{1+k}\right)y + \left(\frac{k-1}{1+k}\right) = 0 \quad \text{--- (iv)}$$

Comparing eqn (iv) with the general eqn of a circle,

$$0 = (x^2 + y^2) + 2gx + 2fy + c = 0$$

$\therefore$  comparing eqn (iv) with the general eqn of a circle,

$$2g = \frac{2k}{1+k} \quad | \quad 2f = \frac{4k}{1+k} \quad | \quad c = \frac{k-1}{1+k}$$

$$\Rightarrow g = \frac{k}{1+k} \quad | \quad \Rightarrow f = \frac{2k}{1+k} \quad | \quad c = \frac{k-1}{1+k}$$

$$\therefore \text{centre } (-g, -f) = \left(-\frac{k}{1+k}, -\frac{2k}{1+k}\right)$$

$$\text{radius } r = \sqrt{g^2 + f^2} \leftarrow (\text{Euler}) = (1+1) \leftarrow$$

$$= \sqrt{\left(\frac{k}{1+k}\right)^2 + \left(\frac{-2k}{1+k}\right)^2 - \left(\frac{k-1}{1+k}\right)^2} \leftarrow$$

$$= \sqrt{\frac{k^2 + 4k^2 - (k-1)(1+k)}{(1+k)^2}} \leftarrow \begin{matrix} \text{H.P} \\ \text{P} = 1 \end{matrix} \leftarrow$$

$$= \sqrt{\frac{5k^2 - k^2 + 1}{(1+k)^2}} \leftarrow \begin{matrix} \text{P.H.C.P} \\ \text{P} = 1 \end{matrix} \leftarrow$$

$$= \frac{\sqrt{4k^2 + 1}}{\sqrt{(1+k)^2}} \leftarrow \begin{matrix} \text{P.H.C.P} \\ \text{P} = 1 \end{matrix} \leftarrow$$

$$= \frac{\sqrt{4k^2 + 1}}{1+k} \quad \text{VI most}$$

we know,  $\left(\frac{1-k}{1+k}\right) + \left(\frac{2k}{1+k}\right) + \infty\left(\frac{1-k}{1+k}\right) + 5 + \infty$

$$OP = r$$

$$\Rightarrow \left| \frac{\left(\frac{-k}{1+k}\right) + 2\left(\frac{-2k}{1+k}\right) + 5}{\sqrt{1^2 + 2^2}} \right| \stackrel{\text{E.S}}{=} \frac{\sqrt{4k^2 + 1} + \infty}{1+k} \quad \left[ \because OP = \sqrt{\frac{ah+bk+e}{a^2+b^2}} \right]$$

$$\Rightarrow \left| \frac{\left(\frac{-5k + 5(1+k)}{1+k}\right)}{\sqrt{5}} \right| = \frac{\sqrt{4k^2 + 1}}{1+k}$$

$$\Rightarrow \left| \frac{(\sqrt{5})^2}{\sqrt{5}(1+k)} \right| = \frac{\sqrt{4k^2 + 1}}{1+k}$$

$$\Rightarrow |\sqrt{5}| = \sqrt{4k^2 + 1}$$

$$\Rightarrow (1\sqrt{5}1)^r = (\sqrt{4k^2+1})^r$$

$$\Rightarrow 5^{r+1} = 4k^2 + 1$$

$$\Rightarrow 4k^2 = 4(1)(1-4) - 4k^2 + 1$$

$$\Rightarrow k^2 = 1$$

$$\Rightarrow k = \pm 1$$

$$\therefore k = 1$$

$$[\because k \neq -1]$$

from ④,

$$x^2 + y^2 + \left(\frac{2-1}{1+1}\right)x + \left(\frac{4-1}{1+1}\right)y + \left(\frac{1-1}{1+1}\right) = 0$$

$$\pi = 90^\circ$$

$$x^2 + y^2 + x + 2y = 0 \quad \left( \frac{(1-1)}{1+1}x + \frac{(4-1)}{1+1}y \right)$$

Ans.

$$\frac{x^2 + y^2}{1+1} = \left( \frac{(1-1)x + (4-1)y}{1+1} \right)$$

$$\frac{x^2 + y^2}{2} = \left( \frac{(4-1)y}{(1+1)x} \right)$$

$$1) \quad x^2 + y^2 - 4x + 6y - 3 = 0 \quad \text{--- (1)}$$

Comparing eqn (1) with the general eqn of a circle,  
 $x^2 + y^2 + 2gx + 2fy + c = 0$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

From the two equations, we get  
 Here,

$$\begin{array}{l|l|l} 2g = -4 & 2f = 6 & c = -3 \\ \Rightarrow g = -2 & \Rightarrow f = 3 & \end{array}$$

$$\therefore \text{centre } (-g, -f) = (2, -3)$$

$$\text{radius } r = \sqrt{g^2 + f^2 - c} = 4$$

$$r = R$$

$$R = 4$$

$$O = 4$$

Let the equation of the tangent line be,

$$(A)x + (B)y + C = 0, \text{ such that } A^2 + B^2 \neq 0$$

$$3x - 4y + k = 0 \quad \text{--- (2)}$$

$$\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$$

We know,

$$OP = r$$

$$\Rightarrow \left| \frac{3 \times 2 - 4 \times (-3) + k}{\sqrt{3^2 + (-4)^2}} \right| = 4 \quad \text{work sw}$$

$$|57 + k| = 20$$

$$\Rightarrow \left| \frac{18+k}{5} \right| = 4 \quad \text{--- (3)}$$

$$\Rightarrow \frac{18+k}{5} = \pm 4$$

$$18 + k = \pm 20 \quad \text{--- (4)}$$

from (3),

$$(+) k = 2 \Rightarrow 3x - 4y + 2 = 0 \quad \text{--- (5)}$$

$$(-) k = -38 \Rightarrow 3x - 4y - 38 = 0$$

Ans.

$$5] \quad x^2 + y^2 + 2gx + 2fy = 0 \quad \text{--- (i)}$$

$$x^2 + y^2 + 2g_1x + 2f_1y = 0 \quad \text{--- (ii)}$$

$$0 = g + f_1^2 + g^2 + f_1^2 - 2g_1x - 2f_1y$$

comparing circles (i) and (ii) with the general eqn of a circle

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad h = 0$$

$$\text{Hence,} \quad | \quad g = g \quad | \quad h = 0$$

$$(i) \Rightarrow$$

$$g = g$$

$$(x - c) = \left| \begin{array}{l} (i) \Rightarrow \\ g - g = g_1 \text{ subtract } . \end{array} \right.$$

$$f = f$$

$$P = \sqrt{g^2 + f^2} = \left| \begin{array}{l} f = f_1 \text{ subtract } \\ c = 0 \end{array} \right.$$

$$c = 0$$

on axis transform ent

$$\therefore \text{centre, } c_1 = (-g, -f)$$

$$\text{radius, } r_1 = \sqrt{g^2 + f^2}$$

$$(ii) \Rightarrow$$

$$g - g = g_1$$

$$f = f_1$$

$$c = 0$$

to no longer ent

$$\therefore \text{centre, } c_2 = (-g_1, -f_1)$$

$$O = A + P = 0$$

$$\text{radius, } r_2 = \sqrt{g_1^2 + f_1^2}$$

we know

we know,

$$P = \left| \begin{array}{l} x + (x - c) \times P = x \\ f_1 - f_1 = f_1 \end{array} \right| \leftarrow$$

$$\Rightarrow \left( \sqrt{(-g + g_1)^2 + (-f + f_1)^2} \right)^2 = \left( \sqrt{g^2 + f^2} \pm \sqrt{g_1^2 + f_1^2} \right)^2$$

$$\Rightarrow g^2 - 2gg_1 + g_1^2 + f^2 - 2ff_1 + f_1^2 = g^2 + f^2 \pm 2\sqrt{g^2 + f^2} \cdot \sqrt{g_1^2 + f_1^2}$$

$$\Rightarrow -2(gg_1 + ff_1) = \pm 2\sqrt{(g^2 + f^2)(g_1^2 + f_1^2)} \quad [\because \sqrt{ab} = \sqrt{a} \cdot \sqrt{b}]$$

$$0 = gg_1 + ff_1 \quad \text{as } g \neq 0 \quad \text{and } f \neq 0$$

$$\Rightarrow [-2(gg_1 + ff_1)]^2 = [\pm 2\sqrt{(g^2 + f^2)(g_1^2 + f_1^2)}]^2$$

$$\Rightarrow gg_1^2 + 2gg_1ff_1 + f_1^2 = g^2g_1^2 + g_1^2f^2 + gf_1^2 + f^2f_1^2$$

$$\Rightarrow g_1^2f^2 - 2gg_1ff_1 + g^2f_1^2 = 0 \quad | \quad \alpha = \beta$$

$$\Rightarrow (gf)^2 - 2(gf)(gf_1) + (gf_1)^2 = 0 \quad | \quad \alpha = \beta$$

$$\Rightarrow (g_1f - gf_1)^2 = 0 \quad | \quad (\alpha, \beta) = (\alpha, \beta) \text{ or } \alpha = \beta$$

$$\Rightarrow g_1f - gf_1 = 0$$

$$\Rightarrow f_1g = fg_1$$

Showed. word sw  
m = 90

$$z = \left| \frac{s + \alpha - \beta i}{s - \alpha - \beta i} \right| <$$

$$z = \left| \left( 1 - \frac{\alpha - \beta i}{s + \alpha - \beta i} \right) \right| <$$

$$(s + \alpha - \beta i) \in s = (s - \alpha + \beta i) <$$

$$s + \alpha - \beta i = s + \alpha - \beta i <$$

$$s = s + \alpha - \beta i + \alpha - \beta i <$$

$$0 = (\alpha - \beta i) \in p + (\alpha - \beta) \in s <$$

$$\frac{1}{s} \in s \text{ and } 0 <$$

QED

6]

$$x^2 + y^2 - 6x + 4y - 12 = 0 \quad \text{--- (1)}$$

Comparing eqn (1) with the general eqn of a circle,

Here,

$$\begin{aligned} 2g &= -6 & 2f &= 6 & c &= -12 \\ \Rightarrow g &= -3 & \Rightarrow f &= 3 & & \end{aligned}$$

$$\therefore \text{centre } (-g, -f) = (3, -2)$$

$$\text{radius, } r = \sqrt{(-3)^2 + 2^2 + 12} \\ = \sqrt{9 + 4 + 12} \\ = 5$$

We know,

$$OP = r \quad \text{[distance from origin]}$$

$$\Rightarrow \left| \frac{12 - 2k + 7}{\sqrt{9^2 + k^2}} \right| = 5$$

$$\Rightarrow \left( \left| \frac{19 - 2k}{\sqrt{9^2 + k^2}} \right| \right)^2 = 5^2$$

$$\Rightarrow (19 - 2k)^2 = 25(16 + k^2)$$

$$\Rightarrow 361 - 76k + 4k^2 = 400 + 25k^2$$

$$\Rightarrow -21k^2 - 76k - 39 = 0$$

$$\Rightarrow 21k^2 + 63k + 13k + 39 = 0$$

$$\Rightarrow 21k(k+3) + 13(k+3) = 0$$

$$\Rightarrow k = -3, -\frac{13}{21}$$

Ans.



## Assignment

Semester: Fall 2017

Course ID: MAT 110 (Section: 02)

Course Title: Differential Calculus and Coordinate Geometry

### **Answer all the questions:**

- Q1. Determine the equation  $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$  after rotating of axes through [5]  
 $30^\circ$ .
- Q2. Use Lagrange multipliers to find the maximum and minimum values of [5]  
 $f(x, y) = xy$  subject to the constraint  $4x^2 + 8y^2 = 16$ . Also find the points at which these extreme values occur.
- Q3. Locate all relative maxima, relative minima and saddle points, if any of [5]  
 $f(x, y) = y^2 + xy + 3y + 2x + 3$ .
- Q4. Find the value of k so that the following equation may represent a pair of straight [5]  
lines:  
 $2x^2 + xy - y^2 - 2x - 5y + k = 0$ .
- Q5. Find the equation of the circle passing through the points  $(3, 1), (4, -3), (1, -1)$ . [5]

$$Q1. \quad x^2 + 2\sqrt{3}xy - y^2 = 2a^2 \quad \text{--- } ①$$

We know,

$$\begin{aligned}
 x &= x'\cos\theta - y'\sin\theta \quad \text{and} \quad y = x'\sin\theta + y'\cos\theta \\
 &= x'\cos 30^\circ - y'\sin 30^\circ && = x'\sin 30^\circ + y'\cos 30^\circ \\
 &= x' \frac{\sqrt{3}}{2} - y' \frac{1}{2} && = x' \frac{1}{2} + y' \frac{\sqrt{3}}{2} \\
 &= \frac{\sqrt{3}x' - y'}{2} && = \frac{x' + \sqrt{3}y'}{2}
 \end{aligned}$$

$\left[ \begin{array}{l} \because \cos 30^\circ = \frac{\sqrt{3}}{2} \\ \sin 30^\circ = \frac{1}{2} \end{array} \right]$

substituting the values of  $x$  and  $y$  in the equation ①,

$$\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 + 2\sqrt{3} \left(\frac{\sqrt{3}x' - y'}{2}\right) \left(\frac{x' + \sqrt{3}y'}{2}\right) - \left(\frac{x' + \sqrt{3}y'}{2}\right)^2 = 2a^2$$

$$\Rightarrow (3x'^2 - 2\sqrt{3}x'y' + y'^2) + 2\sqrt{3}(\sqrt{3}x'^2 + 3x'y' - x'y' - \sqrt{3}y'^2) - (x'^2 + 2\sqrt{3}x'y' + 3y'^2) = 8a^2$$

$$\Rightarrow 3x'^2 - 2\sqrt{3}x'y' + y'^2 + 6x'^2 + 6\sqrt{3}x'y' - 2\sqrt{3}x'y' - 6y'^2 - x'^2 - 2\sqrt{3}x'y' - 3y'^2 = 8a^2$$

$$\Rightarrow 8x'^2 - 6\sqrt{3}x'y' + 6\sqrt{3}x'y' - 8y'^2 = 8a^2$$

$$\Rightarrow 8x'^2 - 8y'^2 = 8a^2$$

$$\Rightarrow x'^2 - y'^2 = a^2$$

Removing suffixes,

$$x^2 - y^2 = a^2$$

Answer.

$$Q2. \quad f(x, y) = xy$$

$$g(x, y) = 0$$

$$\Rightarrow 4x^2 + 8y^2 - 16 = 0 \quad \text{--- } \textcircled{1}$$

We know,

$$\bar{\nabla}f = \lambda \bar{\nabla}g$$

$$\Rightarrow \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} = \lambda \left( \frac{\partial g}{\partial x} \hat{i} + \frac{\partial g}{\partial y} \hat{j} \right)$$

$$\Rightarrow y \hat{i} + x \hat{j} = \lambda (8x \hat{i} + 16y \hat{j})$$

$$\because 8\lambda x = y \quad \text{and} \quad 16\lambda y = x$$

$$\Rightarrow \lambda = \frac{y}{8x} \quad \text{--- } \textcircled{II}$$

$$\Rightarrow \lambda = \frac{x}{16y} \quad \text{--- } \textcircled{III}$$

from  $\textcircled{II}$  and  $\textcircled{III}$

$$\frac{y}{8x} = \frac{x}{16y}$$

$$\Rightarrow 16y^2 = 8x^2$$

$$\Rightarrow 8y^2 = 4x^2 \quad \text{--- } \textcircled{IV}$$

from  $\textcircled{1}$ ,

$$4x^2 + 8y^2 - 16 = 0$$

$$\Rightarrow 4x^2 + 4x^2 = 16$$

$$\Rightarrow 8x^2 = 16$$

$$\Rightarrow x^2 = 2$$

$$\Rightarrow x = \pm \sqrt{2}$$

$$x = \pm\sqrt{2} \Rightarrow \text{IV},$$

$$y = \pm 1$$

$$x = -\sqrt{2} \Rightarrow \text{IV},$$

$$y = \pm 1$$

|                     |                 |                  |                  |                   |
|---------------------|-----------------|------------------|------------------|-------------------|
| $(x, y)$            | $(\sqrt{2}, 1)$ | $(\sqrt{2}, -1)$ | $(-\sqrt{2}, 1)$ | $(-\sqrt{2}, -1)$ |
| $f(x, y)$<br>$= xy$ | $\sqrt{2}$      | $-\sqrt{2}$      | $-\sqrt{2}$      | $\sqrt{2}$        |

$\therefore$  Maximum value =  $\sqrt{2}$

Minimum value =  $-\sqrt{2}$

Thus, the function  $f(x, y)$  has an absolute maximum of  $\sqrt{2}$  at  $(\sqrt{2}, 1)$  and  $(-\sqrt{2}, -1)$ , also an absolute minimum of  $-\sqrt{2}$  at  $(\sqrt{2}, -1)$  and  $(-\sqrt{2}, 1)$ .

Answer.

$$Q3. \quad f(x,y) = y^2 + xy + 3y + 2x + 3$$

to find critical points:

|                     |                        |                    |                    |
|---------------------|------------------------|--------------------|--------------------|
| $f_x = 0$           | $f_y = 0$              | $f_x$ is undefined | $f_y$ is undefined |
| $\Rightarrow y+2=0$ | $\Rightarrow 2y+x+3=0$ | $f_x = y+2$        | $f_y = 2y+x+3$     |
| $\Rightarrow y=-2$  | $\Rightarrow -4+x+3=0$ | $\times$           | $\times$           |
|                     | $\Rightarrow x=1$      |                    |                    |

$$\text{critical point } (x,y) = (1, -2)$$

second partial test:

$$f_{xx} = \frac{\partial}{\partial x} (f_x)$$

$$= 0$$

$$f_{yy} = \frac{\partial}{\partial y} (f_y)$$

$$= 2$$

$$f_{xy} = \frac{\partial}{\partial y} (f_x)$$

$$= 1$$

We know,

$$D = f_{xx} \cdot f_{yy} - f_{xy}^2$$

$$= -1$$

| $(x_0, y_0)$ | $f_{xx} = 0$ | $f_{yy} = 2$ | $f_{xy} = 1$ | $D$      | comment   |
|--------------|--------------|--------------|--------------|----------|---|
| $(1, -2)$    | $0 = 0$      | $2 > 0$      | $1 > 0$      | $-1 < 0$ | $\because D < 0 \text{ so}$<br>it has a<br>saddle point |

Answer.

$$Q4. \quad 2x^2 + xy - y^2 - 2x - 5y + k = 0 \quad \text{--- } ①$$

comparing equation ① with the general equation of 2nd degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{array}{c|c|c|c|c|c} a = 2 & 2h = 1 & b = -1 & 2g = -2 & 2f = -5 & c = k \\ \hline \Rightarrow h = \frac{1}{2} & & & \Rightarrow g = -1 & \Rightarrow f = -1 & \end{array}$$

equation ① will represent a pair of straight lines if,

$$\Delta = abc + 2fg - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow 2 \cdot (-1) \cdot k + 2 \cdot \left(-\frac{5}{2}\right) \cdot (-1) \cdot \frac{1}{2} - 2 \cdot \left(-\frac{5}{2}\right)^2 + 1 \cdot (-1)^2 - k \cdot \left(\frac{1}{2}\right)^2 = 0$$

$$\Rightarrow -2k + \frac{5}{2} - \frac{25}{2} + 1 - \frac{k}{4} = 0$$

$$\Rightarrow \frac{-8k + 10 - 50 + 4 - k}{4} = 0$$

$$\Rightarrow -9k - 36 = 0$$

$$\Rightarrow -9k = 36$$

$$\Rightarrow k = -4$$

Answer.

Q5. Let the equation of the circle be,

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{--- } \textcircled{1}$$

circle  $\textcircled{1}$  passing through the points  $(3, 1)$ ,  $(4, -3)$  and  $(1, -1)$

$(3, 1) \rightarrow \textcircled{1}$ ,

$$3^2 + 1^2 + 2g \cdot 3 + 2f \cdot 1 + c = 0$$

$$\Rightarrow 6g + 2f + c = -10 \quad \text{--- } \textcircled{2}$$

$(4, -3) \rightarrow \textcircled{1}$ ,

$$4^2 + (-3)^2 + 2g \cdot 4 + 2f \cdot (-3) + c = 0$$

$$\Rightarrow 8g - 6f + c = -25 \quad \text{--- } \textcircled{3}$$

$(1, -1) \rightarrow \textcircled{1}$ ,

$$1^2 + (-1)^2 + 2g \cdot 1 + 2f \cdot (-1) + c = 0$$

$$\Rightarrow 2g - 2f + c = -2 \quad \text{--- } \textcircled{4}$$

$\textcircled{2} - \textcircled{4} \times 3 \rightarrow$

$$6g + 2f + c + 10 - 6g + 6f - 3c - 6 = 0$$

$$\Rightarrow 8f - 2c + 4 = 0 \quad \text{--- } \textcircled{5}$$

$\textcircled{3} - \textcircled{4} \times 4 \rightarrow$

$$8g - 6f + c + 25 - 8g + 8f - 4c - 8 = 0$$

$$\Rightarrow 2f - 3c + 17 = 0 \quad \text{--- } \textcircled{6}$$

$$\textcircled{v} - \textcircled{vi} \times 4 \rightarrow$$

$$8f - 2c + 4 - 8f + 12c - 68 = 0$$

$$\Rightarrow 10c - 64 = 0$$

$$\Rightarrow c = \frac{64}{10}$$

$$\Rightarrow c = \frac{32}{5}$$

$$c = \frac{32}{5} \rightarrow \textcircled{vi},$$

$$2f - 3 \cdot \frac{32}{5} + 17 = 0$$

$$\Rightarrow 2f = \frac{96}{5} - 17$$

$$\Rightarrow f = \frac{11}{10}$$

from  $\textcircled{iv}$ ,

$$2g - 2 \cdot \frac{11}{10} + \frac{32}{5} = -2$$

$$\Rightarrow 2g = \frac{22}{10} - \frac{32}{5} - 2$$

$$\Rightarrow g = -\frac{31}{10}$$

from  $\textcircled{1}$ ,

$$x^2 + y^2 - 2 \cdot \frac{31}{10}x + 2 \cdot \frac{11}{10}y + \frac{32}{5} = 0$$

$$\Rightarrow x^2 + y^2 - \frac{31}{5}x + \frac{11}{5}y + \frac{32}{5} = 0$$

$$\Rightarrow 5x^2 + 5y^2 - 31x + 11y + 32 = 0$$

Answer.



Semester: Fall 2017  
Course ID: MAT 110  
Course Title: Mathematics I  
Section: 02  
Quiz: 01

Name: [REDACTED]  
ID: [REDACTED]  
Date: Oct 3, 2017  
Time: 20 Minutes  
Marks obtained: 25

**Answer the following questions: (Total Marks: 25)**

**Q1.** Find domain and range of the following functions:

$$6+6=12$$

$$(i) \quad f(x) = \frac{1}{x-3}$$

(ii)  $g(x) = \sqrt{2x + 4}$

**Q2.** Sketch the graph of the functions below:

$$3+3=6$$

(i)  $y = 1 - |2x + 4|$

(ii)  $y = 2 + (x - 3)^2$

Q3. Find  $\lim_{x \rightarrow 1} f(x)$  for  $f(x) = \begin{cases} 2x + 1, & x < 1 \\ 3 - x, & x > 1. \end{cases}$

7

Q1 (i)  $f(x) = \frac{1}{x-3}$

$x-3 \neq 0$

$\Rightarrow x \neq 3$

domain:  $\mathbb{R} - \{3\}$

range:  $\mathbb{R} - \{0\}$

Let  $y = \frac{1}{x-3}$

$\begin{aligned} & \Rightarrow xy - 3y = 1 \\ & \Rightarrow ny = 1 + 3y \\ & \Rightarrow x = \frac{1+3y}{y} \end{aligned}$

$\therefore y \neq 0$

$$(ii) \quad g(n) = \sqrt{2n+4}$$

$$2n+4 \geq 0 \quad \text{--- ①}$$

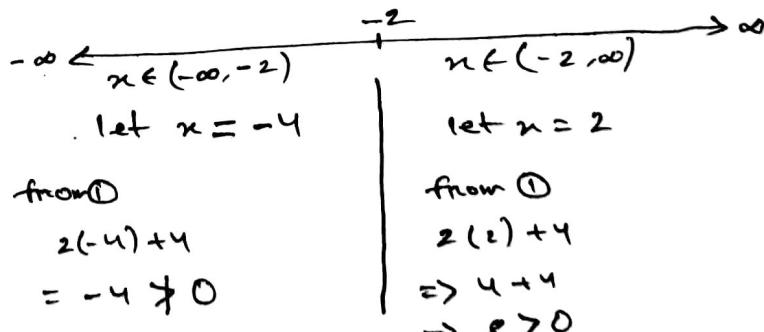
$$\Rightarrow 2n + 4 = 0$$

$$\Rightarrow 2n = -4$$

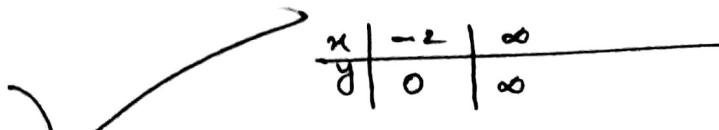
$$\Rightarrow n = -2$$

Domain:  $[-2, \infty)$

Range :  $[0, \infty)$



### To find Range



Q2

(i)  $y = 1 - |2x+4|$

$\Rightarrow y - 1 = -(2x+4)$

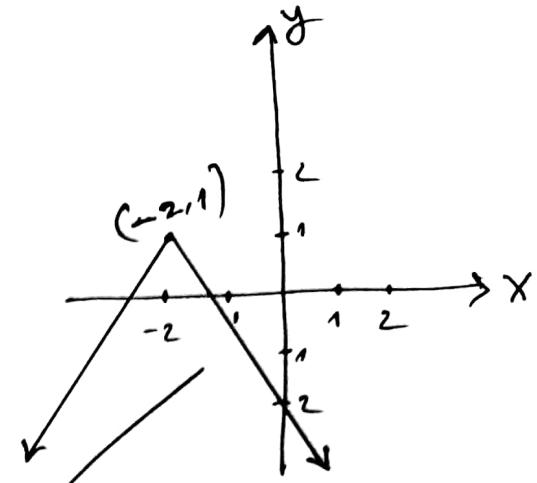
$y-1=0 \quad \text{if} \quad |2x+4|=0$

$\Rightarrow y = 1$

$\Rightarrow 2x = -4$

$\Rightarrow x = -2$

vertex: ~~(-1/2, 1)~~ (-2, 1)



(ii)  $y = 2 + (x-3)^2$

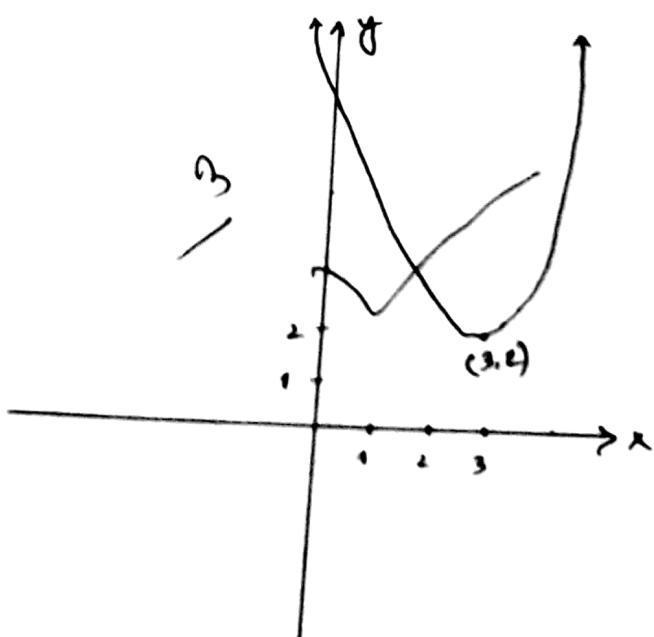
$\Rightarrow y - 2 = (x-3)^2$

$y-2=0 \quad \text{if} \quad (x-3)^2=0$

$\Rightarrow y = 2$

$x = 3$

vertex: (3, 2)



$$\underline{93} \quad f(n) = \begin{cases} 2n+1 & n < 1 \\ 3-n & n \geq 1 \end{cases} \quad \lim_{n \rightarrow 1} f(n)$$

$$L.H.L = \lim_{n \rightarrow 1^-} f(n)$$

$$= \lim_{n \rightarrow 1^-} (2n+1)$$

$$= 2 \cdot 1 + 1$$

$$= 3$$

$$R.H.L = \lim_{n \rightarrow 1^+} f(n)$$

$$= \lim_{n \rightarrow 1^+} (3-n)$$

$$= 3 - 1$$

$$= 2$$

$$\therefore L.H.L \neq R.H.L$$

$\therefore$  limit does not exist .



Semester: Fall 2017  
 Course ID: MAT 110  
 Course Title: Mathematics I  
 Section: 02  
 Quiz: 02

Name: [REDACTED]  
 ID: [REDACTED]  
 Date: Oct 15, 2017  
 Time: 20 Minutes  
 Marks obtained: 25

**Answer the following questions: (Total Marks: 25)**

Q1. Test the continuity and differentiability of the following function at  $x = 0$ : 12

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Q2. Find the differential coefficients of the following functions: 13

$$(i) \quad y = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$(ii) \quad y = (\sin x)^{\tan x}.$$

$$Q1. \quad f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases} \quad \text{at } x=0$$

$$L-f'(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0^-} \frac{x^2 \sin(1/x) - 0}{x - 0}$$

$$= \lim_{x \rightarrow 0^-} x \sin(1/x)$$

$$= \left( \lim_{x \rightarrow 0^-} x \right) \left( \lim_{x \rightarrow 0^-} \sin \frac{1}{x} \right)$$

$$= (0) \cdot \left( \sin \frac{1}{0} \right)$$

$$= 0 \cdot \cancel{\sin \frac{1}{0}}$$

= 0. [it oscillates between -1 and 1 without approaching a limit]

$$= 0$$



R.T.O

$$Rf'(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0^+} \frac{x \sin(1/x) - 0}{x - 0}$$

$$= \lim_{x \rightarrow 0^+} x \sin 1/x$$

$$= \left( \lim_{x \rightarrow 0^+} x \right) \left( \lim_{x \rightarrow 0^+} \sin \frac{1}{x} \right)$$

$\Rightarrow 0 \cdot [$  oscillates between -1 and 1 without approaching a limit]

$$\therefore Lf'(0) = Rf'(0)$$

$\therefore$  Differentiation at  $x=0$

And a differentiable function is always

continuous. So the function is both continuous and differentiable at  $x=0$ .

Ans

Q2 (i)  $y = \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right)$

$$= \tan^{-1} \left( \frac{\sin \theta}{\sqrt{1-\sin^2 \theta}} \right)$$

$$\text{let, } x = \sin \theta$$

$$\Rightarrow \theta = \sin^{-1} x$$

$$= \tan^{-1} \left( \frac{\sin \theta}{\sqrt{(\cos \theta)^2}} \right)$$

$$= \tan^{-1} \left( \frac{\sin \theta}{\cos \theta} \right)$$

$$= \tan^{-1} (\tan \theta)$$

$$= \theta$$

$$= \sin^{-1} x$$

6.5

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Ans.

(ii)  $y = (\sin x)^{\tan x}$

$$\Rightarrow \ln y = \ln(\sin x)^{\tan x}$$

$$\Rightarrow \ln y = \tan x \ln(\sin x)$$

$$\Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx} (\tan x \ln \sin x)$$

[Applying uv method]

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \tan x \frac{d}{dx} \ln \sin x + \ln \sin x \frac{d}{dx} \tan x$$

$$= \tan x \cancel{\frac{1}{\sin x}} \cdot \cos x + \ln \sin x \cdot \sec^2 x$$

$$= \tan x \frac{\cos x}{\sin x} + \sec^2 x \ln \sin x$$

$$= \tan x \cdot \cot x + \sec^2 x \ln \sin x$$

$$= \tan x \cdot \frac{1}{\tan x} + \sec^2 x \ln \sin x$$

$$= 1 + \sec^2 x \ln \sin x$$

$$\Rightarrow \frac{dy}{dx} = y (1 + \sec^2 x \ln \sin x)$$

$$= (\sin x)^{\tan x} [1 + \sec^2 x \ln \sin x]$$

6.5



Ans:

Semester: Fall 2017  
 Course ID: MAT 110  
 Course Title: Mathematics I  
 Section: 02  
 Quiz: 03

Name: \_\_\_\_\_  
 ID: \_\_\_\_\_  
 Date: Nov 12, 2017  
 Time: 20 Minutes  
 Marks obtained: 25

**Answer the following questions: (Total Marks: 25)**

Q1. If  $y = e^{m \sin^{-1} x}$ , then show that

10

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0.$$

Q2. Verify the statement of Mean Value Theorem for the function

7

$$f(x) = \sqrt{x + 1}; \quad [0, 3].$$

Q3. Find  $f_{xy}$  for the function  $f(x, y) = e^{x^2 + xy + y^2}$ .

8

911

$$y = e^{m \sin^{-1} x} \quad \text{--- } \textcircled{1}$$

Differentiating  $\textcircled{1}$  with respect to  $x$ ,

$$y_1 = m e^{m \sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow (\sqrt{1-x^2}) y_1 = m e^{m \sin^{-1} x}$$

$$\Rightarrow (\sqrt{1-x^2}) y_1 = my$$

$$\Rightarrow (1-x^2) y_1' = my^2 \quad \text{--- } \textcircled{2}$$

10

Differentiating  $\textcircled{2}$  with respect to  $x$ ,

$$(1-x^2) 2y_1 y_2 + y_1^2 (-2x) = m^2 2yy_1$$

$$\Rightarrow 2y_1 y_2 (1-x^2) - 2xy_1^2 = m^2 2yy_1$$

$$\Rightarrow 2y_1 y_2 (1-x^2) - 2xy_1^2 - m^2 2yy_1 = 0$$

$$\Rightarrow 2y_1 [(1-x^2)y_2 - xy_1 - my] = 0$$

$$\Rightarrow (1-x^2)y_2 - xy_1 - my = 0 \quad \text{--- } \textcircled{3}$$

Applying Leibnitz theorem in equation (11)

$$[y_{n+2}(1-x^2) + n c_1 y_{n+1}(-2x) + n c_2 y_n(-2) + n c_3 y_{n-1}(0)] \\ - [y_{n+1}x + n c_1 y_n \cdot 1 + n c_2 y_{n-1} \cdot 0] - m^2 y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - 2nx y_{n+1} - 2 \cdot \frac{n(n-1)}{2} \cdot y_n - ny_{n+1} - ny_n - m^2 y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - 2nx y_{n+1} - ny_{n+1} - (n^2-n)y_n - ny_n - m^2 y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)ny_{n+1} - (n^2-n+n+m^2)y_n = 0$$

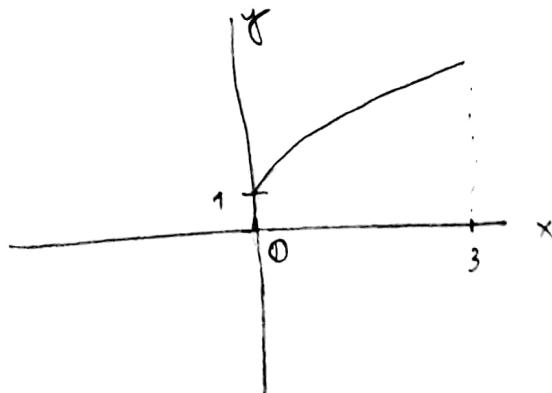
$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)ny_{n+1} - (n^2+m^2)y_n = 0$$

(showed)

Q4  $f(x) = \sqrt{x+1} \quad ; [0, 3]$

Hence,  $y = \sqrt{x+1}$

~~graph~~



Here, the function is differentiable on  $(0, 3)$  and continuous on  $[0, 3]$ . So by Mean value theorem there is at least one number  $c \in (0, 3)$  such

that,

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{--- (1)}$$

$$f(0) = \sqrt{0+1} = 1$$

$$f(3) = \sqrt{3+1} = 2$$

$$f'(x) = \frac{1}{2\sqrt{x+1}} \cdot 1$$

$$= \frac{1}{2\sqrt{x+1}}$$

$$\therefore f'(c) = \frac{1}{2\sqrt{c+1}}$$

from (1),

$$\frac{1}{2\sqrt{c+1}} = \frac{2-1}{3-0}$$

f

$$\Rightarrow 2\sqrt{c+1} = 3$$

$$\Rightarrow \sqrt{c+1} = \frac{3}{2}$$

$$\Rightarrow c+1 = \frac{9}{4}$$

$$\Rightarrow c = \frac{9}{4} - 1$$

$$= \frac{5}{4}$$

$$= \frac{5}{4}$$

$$\therefore c = \frac{5}{4} \in (0, 3) \quad \text{verified}$$

$$93] f(x,y) = e^{x^2+xy+y^2}$$

$$f_x = \frac{\partial}{\partial x} \cancel{e^{x^2+xy+y^2}} (e^{x^2+xy+y^2})$$

$$= e^{x^2+xy+y^2} \cdot (2x+y+0)$$

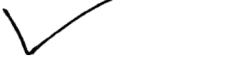
$$= (2x+y) e^{x^2+xy+y^2}$$

$$f_{xy} = \frac{\partial}{\partial y} [(2x+y) e^{x^2+xy+y^2}]$$

$$= (2x+y) \cdot e^{x^2+xy+y^2} \cdot (0+x+2y) + e^{x^2+xy+y^2} \cdot (0+1)$$

$$= (x+2y)(2x+y) e^{x^2+xy+y^2} + e^{x^2+xy+y^2}$$

8



Ans:

Semester: Fall 2017

Course ID: MAT 110

Course Title: Mathematics I

Section: 02

Quiz: 04

Name: [REDACTED]

ID: [REDACTED]

Date: Dec 03, 2017

Time: 40 Minutes

Marks obtained: 25

**Answer the following questions: (Total Marks: 25)**

- Q1. Determine whether the following equation represents a pair of straight lines. Find 12  
their equations, point of intersection and the angle between them.

$$x^2 + 3xy + 2y^2 + \left(\frac{1}{8}\right)x - \left(\frac{1}{32}\right) = 0.$$

- Q2. Remove the terms in  $x$ ,  $y$  and  $xy$  from the equation  $9x^2 + 24xy + 2y^2 - 6x + 13$   
 $20y + 41 = 0$  and write the transformed equation.

Q1  $x^2 + 3xy + 2y^2 + \left(\frac{1}{8}\right)x - \frac{1}{32} = 0 \quad \dots \quad ①$

~~1. Transform the given equation into the general form of 2nd degree.~~

Comparing eqn ① with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Here,

$$\begin{array}{l|l|l|l|l|l} a = 1 & 2h = 3 & b = 2 & 2g = \frac{1}{8} & 2f = 0 & c = -\frac{1}{32} \\ \hline \Rightarrow h = \frac{3}{2} & & & \Rightarrow g = \frac{1}{16} & \Rightarrow f = 0 & \end{array}$$

eqn ① will represent a pair of straight lines if ,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$= 1 \cdot 2 \cdot \left(-\frac{1}{32}\right) + 0 - 0 - 2 \cdot \left(\frac{1}{16}\right)^2 - \left(-\frac{1}{32}\right) \cdot \left(\frac{3}{2}\right)^2$$

$$= -\frac{1}{16} - \frac{1}{256} + \frac{9}{128}$$

$$= \frac{-16 - 1 + 18}{256}$$

$$= \frac{0}{256} \quad [\because \frac{0}{\text{something}} = 0]$$

$$= 0$$

$\therefore$  eqn ① represents a pair of straight lines .

For the point of intersection,

$$\text{Let } F(x, y) \equiv x^2 + 3xy + 2y^2 + \frac{1}{8}x - \frac{1}{32} = 0$$

$$\therefore \frac{\partial F}{\partial x} = 2x + 3y + \frac{1}{8} = 0 \quad \text{--- (I)}$$

$$\frac{\partial F}{\partial y} = 3x + 4y = 0 \quad \text{--- (II)}$$

$$(I) \times 3 - (II) \times 2 \Rightarrow$$

$$6x + 9y + \frac{3}{8} - 6x - 8y = 0$$

$$\Rightarrow y = -\frac{3}{8}$$

from (II),

$$3x + 4\left(-\frac{3}{8}\right) = 0$$

$$\Rightarrow 3x = \frac{12}{8}$$

$$\Rightarrow x = \frac{3}{2}$$

$$\Rightarrow x = \frac{1}{2}$$

$$\therefore (x, y) = (\alpha, \beta) = \left(\frac{1}{2}, -\frac{3}{8}\right)$$

To Find Angle:

$$\theta = \tan^{-1} \left( \frac{2\sqrt{h^2 - ab}}{a+b} \right)$$

$$= \tan^{-1} \left( \frac{2\sqrt{\left(\frac{3}{2}\right)^2 - 2}}{3} \right)$$

$$= \tan^{-1} \left( \frac{2\sqrt{\frac{9}{4} - 2}}{3} \right)$$

$$= \tan^{-1} \left( \frac{2 \cdot \frac{3}{2} - 2}{3} \right)$$

$$= \tan^{-1} \left( \frac{1}{3} \right)$$

$$= 18.43^\circ$$

Ans.

$$9x^2 + 24xy + 2y^2 - 6x + 20y + 41 = 0 \quad \text{--- (1)}$$

comparing eqn (1) with the general eqn of 2nd degree,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Hence,

$$\begin{array}{l|l|l|l|l|l} a=9 & 2h=24 & b=2 & 2g=-6 & 2f=20 & c=41 \\ & \Rightarrow h=12 & & \Rightarrow g=-3 & \Rightarrow f=10 & \end{array}$$

Remove in the terms  $x, y$ :

$$\text{Let } F(x, y) = 9x^2 + 24xy + 2y^2 - 6x + 20y + 41 = 0$$

$$\begin{aligned} \frac{\partial F}{\partial x} &= 18x + 24y - 6 = 0 \\ \Rightarrow 3x + 4y - 1 &= 0 \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \frac{\partial F}{\partial y} &= 24x + 4y + 20 = 0 \\ \Rightarrow 12x + 2y + 10 &= 0 \\ \Rightarrow 6x + y + 5 &= 0 \quad \text{--- (11)} \end{aligned}$$

$$(1) \times 2 - (11)$$

$$6x + 8y - 2 - 6x - 2y - 10 = 0$$

$$\Rightarrow 7y - 12 = 0$$

$$\Rightarrow y = 2$$

$$\Rightarrow y = 1$$

$$\text{from (1), } 3x + 4 \cdot 1 - 1 = 0$$

$$\Rightarrow 3x = -3$$

$$\Rightarrow x = -1 \quad \therefore (x, y) = (\alpha, \beta) = (-1, 1)$$

We know

Transformed equation,

$$ax^2 + 2hxy + by^2 + c = 0 \quad \text{--- (iv)}$$

where

$$c = gh + f\beta + c$$

$$= (-3)(-1) + 10 \cdot 1 + 41$$

$$= 3 + 10 + 41$$

$$= \cancel{-} 54$$

from (iv),

$$gx^2 + 2hxy + by^2 + \cancel{54} = 0 \quad \text{--- (v)}$$

To remove  $xy$  term,

$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{2h}{a-b} \right)$$

$$= \frac{1}{2} \tan^{-1} \left( \frac{2 \times 12}{9-2} \right)$$

$$= \frac{1}{2} \tan^{-1} \left( \frac{24}{7} \right)$$

$$= 36.87^\circ$$

$$\cos 36.87^\circ = 0.8$$

$$\sin 36.87^\circ = 0.6$$

we know,

$$\begin{aligned}x &= x' \cos \theta - y' \sin \theta \\&= 0.8x' - 0.6y' \\&= \frac{8x' - 6y'}{10}\end{aligned}$$

$$\begin{aligned}y &= x' \sin \theta + y' \cos \theta \\&= 0.6x' + 0.8y' \\&= \frac{6x' + 8y'}{10}\end{aligned}$$

substituting the values of  $x$  and  $y$  in the eqn ①

$$\begin{aligned}&9\left(\frac{8x' - 6y'}{10}\right)^2 + 24\left(\frac{8x' - 6y'}{10}\right) \cdot \left(\frac{6x' + 8y'}{10}\right) + 2\left(\frac{6x' + 8y'}{10}\right)^2 + 54 = 0 \\&\Rightarrow \frac{9(64x'^2 - 96x'y' + 36y'^2)}{100} + \frac{24(48x'^2 + 64x'y' - 36x'y' - 48y'^2)}{100} + \frac{2(36x'^2 + 26x'y' + 64y'^2)}{100} + 54 = 0 \\&\Rightarrow \frac{9(64x'^2 - 96x'y' + 36y'^2)}{100} + \frac{1152x'^2 + 1536x'y' - 864x'y' - 1152y'^2}{100} \\&\quad + \frac{2(36x'^2 + 26x'y' + 64y'^2)}{100} + 54 = 0 \\&\Rightarrow \frac{576x'^2 - 864x'y' + 324y'^2}{100} + \frac{1152x'^2 + 672x'y' - 1152y'^2}{100} + \frac{72x'^2 + 192x'y' + 128y'^2}{100} + 54 = 0 \\&\Rightarrow 576x'^2 - 864x'y' + 324y'^2 + 1152x'^2 + 672x'y' - 1152y'^2 + 72x'^2 + 192x'y' + 128y'^2 + 5400 = 0 \\&\Rightarrow 1800x'^2 - 700y'^2 + 5400 = 0 \\&\Rightarrow 18x'^2 - 7y'^2 + 54 = 0\end{aligned}$$

removing suffixes,

$$18x^2 - 7y^2 + 54 = 0$$

Ans.