Assignment 4

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1 (a) Here,

$\Sigma f(x) = (2k \times 2) + (2k \times 4) + (2k \times 6) + (k \times 8 + 2)$

$= 34k.$

we know, $\Sigma f(x) = 1$

$\therefore 34k = 1$

$\therefore k = \frac{1}{34}$ [shown]

(b)

x	2	4	6	8
$P(X=x)$	$\frac{1}{34} \times 2 \times 2 = \frac{4}{34}$	$\frac{8}{34}$	$\frac{12}{34}$	$\frac{10}{34}$

$\therefore P(4 < x \leq 8) = \frac{12}{34} + \frac{10}{34} = \frac{22}{34}$ [Ans]

(c) $P(2 < x < 4) = 0$ [Ans]

(d) Expected value of x,

$E(X) = \left(2 \times \frac{4}{34}\right) + \left(4 \times \frac{8}{34}\right) + \left(6 \times \frac{12}{34}\right) + \left(8 \times \frac{10}{34}\right)$

$= 5.647.$ [Ans]

(e) $Var(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \left(2^2 \times \frac{4}{34}\right) + \left(4^2 \times \frac{8}{34}\right) + \left(6^2 \times \frac{12}{34}\right) + \left(8^2 \times \frac{10}{34}\right)$

$= 35.765$

$Var(x) = E(x^2) - [E(x)]^2$

$= 35.765 - (5.642)^2$

$= 3.876$ [Ans]

(4) $Var(5 - 3x) = Var(5) + Var(3x)$

$= 0 + 3^2 Var(3.876)$

$= 9 \times 3.876$

$= 34.884$ [Ans].

2. (a) $f(y \le 8) = \int_0^5 \frac{1}{25} y\,dy + \int_5^8 \frac{2}{5} - \frac{1}{25} y\,dy$

$= \frac{1}{25} \times \left[\frac{y^2}{2}\right]_0^5 + \frac{2}{5} \times [y]_5^8 - \frac{1}{25} \times \left[\frac{y^2}{2}\right]_5^8$

$= \frac{23}{25}$ [Ans]

(b) $f(y<2) + f(y>6)$

$= \int_0^2 \frac{1}{25} y\,dy + \int_6^{10} \frac{2}{5} - \frac{1}{25} y\,dy$

$= \frac{1}{25} \times \left[\frac{y^2}{2}\right]_0^2 + \frac{2}{5} [y]_6^{10} - \frac{1}{25} \times \left[\frac{y^2}{2}\right]_6^{10}$

~~$= \frac{2}{5}$ [Ans]~~

$= \frac{2}{5}$ [Ans]

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(c) $E(y) = \int_0^5 y \times \frac{1}{25} y\,dy + \int_5^{10} y \times \left(\frac{2}{5} - \frac{1}{25} y\right) dy$

$= \frac{1}{25} \times \left[\frac{y^3}{3}\right]_0^5 + \frac{2}{5} \times \left[\frac{y^2}{2}\right]_5^{10} - \frac{1}{25}\left[\frac{y^3}{3}\right]_5^{10}$

$= 5$ [Ans]

(d). $E(y^2) = \int_0^5 y^2 \times \frac{1}{25} y\,dy + \int_0^{10} y^2 \left(\frac{2}{5} - \frac{1}{25} y\right) dy$

$= \left(\frac{1}{25} \times \left[\frac{y^4}{4}\right]_0^5 + \frac{2}{5} \times \left[\frac{y^3}{3}\right]_5^{10}\right) - \left(\frac{1}{25} \times \left[\frac{y^4}{4}\right]_5^{10}\right).$

$= 29.167.$

$\therefore Var(y) = E(y^2) - [E(y)]$

$= 29.167 - 25$

$= 4.167.$

$\therefore SD(y) = \sqrt{Var(y)}$

$= \sqrt{4.167}$

$= 2.041.$

[Ans]

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3\. (a) $P(A=B) = P(0,0) + P(1,1) + P(2,2) + P(3,3)$

$= 0.08 + 0.15 + 0.1 + 0.07$

$= 0.4$ [Ans]

(b) $P(A+B=4) = P(1,3) + P(3,1) + P(2,2) + P(4,0)$

$= 0.04 + 0.03 + 0.1 + 0$

$= 0.17$.

So, $P(A+B \geq 4) = P(A+B=4) + P(A+B>4)$

$= 0.17 + P(4,1) + P(2,3) + P(4,2) + P(3,2) + P(3,3) + P(4,3)$

$= 0.28$. [Ans]

(c)

A \ B	0	1	2	3	$P_A(A)$
0	0.08	0.07	0.04	0	0.19
1	0.06	0.15	0.05	0.04	0.3
2	0.05	0.04	0.1	0.06	0.25
3	0	0.03	0.04	0.07	0.14
4	0	0.01	0.05	0.06	~~0.12~~ 0.12
	0.19	0.3	0.28	0.23	1

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$E(A) = \sum A \cdot P(A)$

$= (0 \times .19) + (1 \times .3) + (2 \times .25) + (3 \times .14) + (4 \times .12)$

$= 1.7$ [Ans]

(d) ~~P(B)~~ $P(B=2 \mid A=3) = \dfrac{P(A=3, B=2)}{P(A)}$

$= \dfrac{0.04}{0.14}$

$= 0.286$ [Ans]

(e) $P(A,B) = P_A(A) \cdot P_B(B)$

Here, $P(A,B) = P(4,0)$

$= 0.$

$P_A(A) \times P_B(B) = P_A(4) \times P_B(0)$

$= 0.12 \times 0.19$

$= 0.0228.$

So, $P(A,B) \neq P_A(A) \times P_B(B)$

So, A and B are not independent

[Ans]

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4. (a) $P(\text{drawing a red ball}) = \frac{4}{12} = \frac{1}{3}$

Let,

X = Number of trails until first success.

$\therefore P(X=5) = (1-P)^4 \times P$

$= (1-\frac{1}{3})^4 \times \frac{1}{3}$

$= \frac{16}{243}$ [Ans]

(b) Non white ball $= 12-2 = 10$

$\therefore P(\text{drawing a non-white ball}) = \frac{10}{12} = \frac{5}{6}$

Let,

X = Number of trails until first success.

$\therefore E(X) = \frac{1}{p} = \frac{6}{5} = 1.2 \approx 2$ [Ans]

(c). $P(\text{drawing a blue ball}) = \frac{6}{12} = \frac{1}{2}$

Let, X = Number of trails until first success.

$\therefore V(X) = \frac{1-p}{p^2} = \frac{\frac{1}{2}}{\frac{1}{2}^2} = 2$ [Ans]

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5 (a) $P(\text{drawing a blue ball}) = \frac{6}{12} = \frac{1}{2}$.

Here, $n = 6$.

Let x = number of blue balls drawn

$\therefore P(x=3) = {}^6C_3 \times \left(\frac{1}{2}\right)^3 \times \left(1 - \frac{1}{2}\right)^{6-3}$

$= \frac{5}{16}$ [Ans]

(b) ~~Exam (b)~~

More than 4 blue balls after 6.

$\therefore P(x>4) = P(x=5) + P(x=6)$

$= {}^6C_5 \times \left(\frac{1}{2}\right)^5 \times \left(1-\frac{1}{2}\right)^{6-5} + {}^6C_6 \times \left(\frac{1}{2}\right)^6 \times \left(1-\frac{1}{2}\right)^{6-6}$

$= \frac{3}{32} + \frac{1}{64}$

$= \frac{7}{64}$ [Ans]

(c) $P(\text{picking red ball}) = \frac{4}{12} = \frac{1}{3}$

$n = 48$

$\therefore \mu = E(x) = np$

$= 48 \times \frac{1}{3}$

$= 16$ [Ans]

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(d) P(Picking white ball) = $\frac{2}{12} = \frac{1}{6}$

$n = 36$

$\sigma = \sqrt{np(1-p)}$

$= \sqrt{36 \times \frac{1}{6} \times \frac{5}{6}}$

$= \sqrt{5}$ [Ans]

6. (a) Average earthquakes in 12 months $\lambda_{12} = 6$.

$\therefore$ Average number of earthquake in 4 months

$\lambda_4 = \frac{6}{12} \times 4$

$= 2$ [Ans]

(b) $\lambda_{24} = \lambda_{12} \times 2 \Rightarrow 6 \times 2 \Rightarrow 12$.

let X, = Number of earthquake.

$\therefore P(X = x) = e^{-\lambda_{24}} \frac{\lambda_{24}^x}{x!}$

$= e^{-12} \frac{12^x}{x!}$

[Ans]

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(c)

$P(X \geq 9) = 1 - P(X < 9)$

$= 1 - P(X \leq 8)$

$= 1 - \left(\sum_{x=0}^{8} e^{-6} \frac{6^x}{x!} \right)$

$= 0.1528$ [Ans]

7 (a) $z = \frac{75-73}{22} = 0.09$

$P(X \geq 75) = P(Z \geq z)$

$= P(Z \geq 0.09)$

$= 1 - P(Z < 0.09)$

$= 1 - 0.5359$

$= 0.4641$ [Ans]

(b) $z = \frac{45-73}{22} = 1.27$

$P(x \leq 45) = P(Z \leq z)$

$= P(Z \leq -1.27)$

$= 0.1020$ [Ans]

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(c) $z_1 = \frac{55-73}{22}$ $\quad$ $Z_2 = \frac{90-73}{22}$

$= -0.82$ $\quad$ $= 0.77$

$P(55<x<90) = P(-0.82<Z<0.77)$

$= P(Z<0.77) - P(Z<-0.82)$

$= 0.7794 - 0.2061$

$= 0.5733$ [Ans]

(d) $P(x=65)=0$ [Ans]

(e) $P(Z<z) = 8\% = 0.08$

From ~~z~~ the Z table,

$P(Z<-1.405) \simeq 0.08$

Now, $Z = \frac{x-u}{6}$

$\therefore -1.405 = \frac{x-73}{22}$

$\therefore x \simeq 42.09$ [Ans]

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(f) Median & Mode is equal to mean.

$\therefore$ Median = ₹3

Mode = ₹3 [Ans]

8\. (a) $E(x) = 100{,}000$

$\therefore \frac{1}{\lambda} = 100000$

$\therefore \lambda = \frac{1}{100000}$

$P(x<1000) = 1 - e^{-\lambda x}$

$= 0.09516$ [Ans]

(b) $P(x > 120000) = e^{-\lambda x} = 0.30119$ [Ans]

(c) $P(60000 < x < 100000) = \lambda \int_{60000}^{100000} e^{-\lambda x} dx$

$= 0.1809$ [Ans]

(d) $P(x > a) = 10\% = 0.1$

$\therefore e^{-\lambda x} = 0.1$

$\therefore -\lambda x = -2.302585$

$\therefore x = 230258.5$ [Ans]