MAT120

MId Set-2

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Section : 03

1. Solve the following Integrals

$$a. \int \frac{ln\,(lnx)}{x}dx$$

Let u=ln(x)

$$\frac{du}{dx} = \frac{1}{x}dx$$

dx=xdu

$$\int \frac{ln\,(u)}{x}dx$$

$$= \int ln(u)du$$

Applying Integration by parts :

$$= uln(u) - \int 1du$$

$$= uln(u) - uln(u) - u$$

u=ln(u)

$v^{'} = 1$

$$= ln(x)ln(ln(x)) - ln(x) + C$$

$$[Answer]$$

$$b. \int \frac{x + sinx}{1 + cosx} dx$$

$$= \int \frac{x}{1 + cosx} dx + \int \frac{sinx}{1 + cosx} dx$$

$$= xtan(\frac{x}{2}) + 2ln|cos(\frac{x}{2})| - ln|1 + cosx|$$

$$= xtan(\frac{x}{2}) + 2ln|cos\frac{x}{2}| - ln|1 + cosx| + C$$

$$[Answer]$$

2. Solve the following Integrals by Substitution

$$a. \int_0^3 \frac{dx}{16 + x^{2\frac{3}{2}}}$$

let

$$x = 4tan(u)$$

so:

$$\int_0^{arctan(\frac{3}{4})} \frac{1}{16sec(u)} du$$

$$= \frac{1}{16} \int_0^{arctan(\frac{3}{4})} \frac{1}{sec(u)} du$$

$$= \frac{1}{16} \int_0^{arctan(\frac{3}{4})} cos(u) du$$

$$= \frac{1}{16} [sin(u)]_0^{arctan(\frac{3}{4})}$$

$$= \frac{1}{16} \frac{3}{5}$$

$$= \frac{3}{80}$$

$$[Answer]$$

$$b. \int_1^0 \frac{y^2}{\sqrt{4-3y}} dy$$

let

$$u = \sqrt{4-3y}$$

$$du = -\frac{3}{2\sqrt{4-3y}}$$

$$= \int_2^1 -\frac{2(u^2-4)^2}{27} du$$

$$= -(\frac{-2}{27} \int_1^2 (u^2-4)^2 du)$$

$$= -(\frac{-2}{27} \int_1^2 u^4 - 8u^2 + 16du)$$

$$= -(\frac{-2}{27}(\int_1^2 u^4 du - \int_1^2 8u^2 du + \int_1^2 16du))$$

$$= -\frac{-2}{27}(\frac{31}{5} - \frac{56}{3} + 16)$$

$$= -\frac{106}{405}$$

$$= -0.26172$$

$$[Answer]$$

4. Integrate the Following using Rational Functions

$$a. \int \frac{1}{x^3 - x^2 - 9x + 9} dx$$

$$= \int -\frac{1}{8(x-1} + \frac{1}{24(x+3)} + \frac{1}{12(x-3)} dx$$

$$= -\int \frac{1}{8(x-1)} dx + \int \frac{1}{24(x+3)} dx + \int \frac{1}{12(x-3)} dx$$

$$Here \int \frac{1}{8(x-1)}$$

$$= \frac{1}{8}\int \frac{1}{x-1}dx$$

$$= \frac{1}{8}ln|x-1|$$

$$Again \int \frac{1}{24(x+3)}dx$$

$$= \frac{1}{24}ln|x+3|$$

$$Also: \int \frac{1}{12(x-3)}dx$$

$$= \frac{1}{12}ln|x-3|$$

$$Finally: -\frac{1}{8}ln|x-1| + \frac{1}{24}ln|x+3| + \frac{1}{12}ln|x-3| + C$$

$$[Answer]$$

$$b. \int \frac{3x^2-x+1}{x^3-x^2}dx$$

$$= \int \frac{3x^2-x+1}{x^2(x-1)}dx$$

Now:

$$\int \frac{3x^2-x+1}{x^3-x^2}dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$so, Ax(x-1) + B(x-1) + Cx^2 = 3x^2 - x + 1$$

if

$$x = 1$$

$$0 + 0 + C = 3$$

Similirarly if

$$x = 0$$

B=-1
or A=0

so

$$\frac{3x^2-x+1}{x^3-x^2}=\frac{0}{x}+\frac{-1}{x^2}+\frac{3}{x-1}$$
$$\frac{3x^2-x+1}{x^3-x^2}=\frac{3}{x-1}-\frac{1}{x^2}$$
$$\int\frac{3x^2-x+1}{x^3-x^2}dx=\int\frac{3}{x-1}dx-\int\frac{1}{x^2}dx$$
$$=3ln|x-1|-\frac{x^{-2+1}}{-2+1}$$
$$=3ln|x-1|+x^{-1}+C$$
$$=3ln|x-1|+\frac{1}{x}+C$$
$$[Answer]$$

5. Use the Reduction Formula to evaluate the following integrals:

$$a.\int cos^6xdx$$

Now let

$$z=\int cos^6xdx$$
$$=\int cos^5xcosxdx$$
$$=\int cos^5xdxsinx$$
$$=cos^5xsinx+5\int cos^4xsin^2xdx$$
$$=cos^5xsinx+5\int cos^4x(1-cos^2x)dx$$
$$=cos^5xsinx+5\int cos^4x-5z$$

So

$$6z=cos^5xsinx+5\int cos^4xdx$$
$$z=\frac{5}{6}\int cos^4xdx+\frac{cos^5xsinx}{6}$$

Now:

$$\int cos^4xdx$$

$$= \int cos^3xdxsinx$$

$$= cos^3xsinx + 4\int cos^2xsin^2xdx$$

$$= cos^3xsinx + 4\int cos^2xdx - \int cos^4xdx$$

$$= \frac{sinxcos^3x}{5} + \frac{4}{5}\int cos^2xdx$$

Also

$$\int cos^2xdx$$

$$= \int cosxsinxdx$$

$$= cosxsinx + \int sin^2xdx$$

$$= cosxsinx + \int dx - \int cos^2xdx$$

$$= \frac{1}{2}x + \frac{sinxcosx}{2} + C$$

Finally:

$$= \frac{sinxcos^5x}{6} + \frac{sinxcos^3x}{6} + \frac{2sinxcosx}{5} + \frac{2x}{5} + c$$

$$[Answer]$$

$$b. \int sec^5xdx$$

let

$$z = \int sec^5 x dx$$

$$= \int sec^3 x sec^2 x dx$$

$$= sec^3 x \int sec^2 x dx - \int (\frac{d}{dx} sec^3 x \int sec^2 dx) dx$$

$$= sec^3 x tan x - \int 3 sec^3 x tan^2 x dx$$

$$= sec^3 x tan x - 3 \int sec^3 x (sec^2 x - 1) dx$$

$$= sec^3 x tan x - 3 \int sec^5 x dx + 3 \int sec^3 x dx$$

$$= sec^3 x tan x - 3z + 3 \int sec^3 x dx$$

So:

$$4z = sec^3 x tan x + 3 \int sec^3 x dx$$

$$z = \frac{1}{4} = sec^3 x tan x + \frac{3}{4} \int sec^3 x dx$$

$$\int sec^3 x dx = \frac{1}{2} (sec x tan x + log|sec x + tan x| + C)$$

$$\int sec^5 x dx = \frac{1}{4} sec^3 x tan x + \frac{3}{8} sec x tan x + log|sec x + tan x| + C$$

$$[Answer]$$

6. Use Gamma function or Beta function to evaluate the following

$$a. \int_{\infty}^{0} \sqrt{x} e^{-3\sqrt{x}}$$

let

$$e^{-3\sqrt{x}}$$

$$x = z^3$$

$$dx = 3z^2 dz$$

Now:

$$\int_{\infty}^{0} \sqrt{z}^3 e^{-z} 3z^2 dz$$

$$= 3\int_{\infty}^{0} z^{\frac{3}{2}} z^2 e^{-z} dz$$

$$= 3\int_{\infty}^{0} z^{\frac{7}{2}} e^{-z} dz$$

$$= 3\int_{\infty}^{0} z^{\frac{9}{2}-1} e^{-z} dz$$

$$= 3\Gamma\frac{9}{2}$$

$$[Answer]$$

$$b. \int_{1}^{o} (1-x^3)^{\frac{-1}{2}} dx$$

let

$$x^3 = y$$

$$x = y^{\frac{1}{3}}$$

$$dx = y^{\frac{1}{3}} dy$$

when x=0; y=0
x=1; y=1
so

$$\int_{1}^{0} y^{\frac{1}{3}} (1-y)^{\frac{-1}{2}} dy$$

$$= \int_{1}^{0} y^{\frac{4}{3}-1} (1-y)^{\frac{1}{2}-1}$$

$$= \beta(\frac{4}{3}, \frac{1}{2})$$

$$[Answer]$$