BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences
Final Examination
Semester: Summer 2015
Course Title: Complex Variables and Laplace Transformations
Course No.: MAT215

Time: 3 hours
Total Marks: 50

Date: August 16, 2015

Note: Question 1 is compulsory. Answer any THREE from Part A and any ONE from Part B.

1. Answer all of the following:
   (a) What is the period of e^z? [2]
   (b) State Euler's formula and De Moivre's formula. How are these formulae related? [2]
   (c) Write Cauchy-Riemann equations. [2]
   (d) The function $f(z) = e^{1/z^2}$ is not analytic at $z = 0$ but $\oint_C e^{1/z^2}\,dz = 0$ around the closed contour C containing the origin. Explain why this does not violate Cauchy's theorem. [2]
   (e) $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$. Use the derivative rule to find $\mathcal{L}\{te^{at}\}$. [2]

Part A: Complex Variables

2. (a) Define *modulus*, the *principal argument*, and the *argument* of a complex number. [2]
   (b) Find the modulus, the principal argument, and the argument of the complex number $z = \sqrt{3} - i$. Also express z into polar form. [3]
   (c) Find all the complex fourth roots of -1 and show them graphically. [5]

3. (a) Define logarithm for complex number. [2]
   (b) Define *analytic* functions. Prove that $f(z) = |z|^2$ is differentiable but not analytic at $z = 0$. [3]
   (c) Consider the function: [5]

$$f(z) = \begin{cases} \dfrac{z^2+4}{z+2i} & z \neq -2i \\[2ex] -4i & z = -2i. \end{cases}$$

   Evaluate $\lim\limits_{z \to -2i} f(z)$, if it exists. Is $f(z)$ continuous at $z = -2i$?

4. (a) State *Cauchy's Integral formulae.* [2]

(b) Evaluate $\oint_C \frac{e^{2z}}{(z+i)^4}\, dz$, where C is the circle $|z| = 2$. [3]

(c) Expand $f(z) = \frac{1}{(z-1)^2(z-3)}$ in a *Laurent series* valid for:
i) $0 < |z-1| < 2$, ii) $0 < |z-3| < 2$. [5]

5. (a) Define and classify the *isolated singular points.* [2]

(b) Define *residue* of a complex function. Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles. [3]

(c) Use *Cauchy's residue theorem* to evaluate $\oint_C \frac{ze^z}{(z^2-1)}\, dz$ where C is the circle $|z - i| = 2$. [5]

6. (a) Prove that $\sin z = \sin x \cosh y + i \cos x \sinh y$. [2]

(b) Prove that, $u(x, y) = xe^x \cos 2y + ye^x \sin 2y$ is harmonic. Find the conjugate harmonic function $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is analytic. [3]

(c) Evaluate $\int_C (2\bar{z} - z)\, dz$ along the contour C defined by $x = 2t$, $y = 4t-1$, $1 \le t \le 3$. [5]

Part B: Laplace Transformations

7. (a) Define *Laplace Transformation.* Prove that, $\mathcal{L}\{\cos at\} = \frac{s}{s^2 + a^2}$; $s > 0$. [5]

(b) Find the Laplace transformations of the following functions (any two): [5]

i) $e^{-3t}(3 \sin 4t - 4 \cos 4t)$, ii) $4t^{-1/2} - e^{-t}$, iii) $3t \sin 2t - te^{-2t}$.

8. (a) Evaluate the following: [5]

$$\mathcal{L}^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\}.$$

(b) Solve the following IVP using *Laplace Transformation.* [5]

$$y'' + y = t, \ y(0) = 1, \ y'(0) = -2.$$

where $'$ denotes the derivative with respect to t.

11.44

BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences
Final Examination
Semester: Spring 2015
Course Title: Complex Variables and Laplace Transformations
Course No.: MAT215

Time: 3 hours
Total Marks: 50

Date: April 19, 2015

Note: Question No. 1 is compulsory. Answer any <u>THREE</u> from Part A and any <u>TWO</u> from Part B.

1. Answer all of the following:
   (a) Write the critical difference of *real valued* and *complex valued* functions. [1]
   (b) State Euler's formula. [1]
   (c) Write Cauchy-Riemann equations. [1]
   (d) "If $\oint_C f(z)\,dz = 0$, f is *analytic* on and inside the closed contour C containing the origin." Provide examples to justify or negate the statement. [1]
   (e) Relate the Laplacian to harmonic functions. [1]

Part A: Complex Variables

2. (a) State and prove *De Moivre's Theorem*. [2]
   (b) Find the *modulus*, the *principal argument*, and *argument* of the complex number $z = 1 - \sqrt{3}i$. Also express z into polar form. [3]
   (c) Find all the complex fourth roots of -1 and show them graphically. [4]

3. (a) Define branch points and branch lines of a complex function $f(z)$ with an example. [2]
   (b) Define *analytic* functions. Show that $f(z) = \bar{z}$ is non-analytic everywhere. [3]
   (c) Consider the function: [4]

$$f(x) = \begin{cases} \dfrac{z^2+3}{z-\sqrt{3}i} & z \neq \sqrt{3}i \\ 5+2\sqrt{3}i & z = \sqrt{3}i. \end{cases}$$

   i. Evaluate $\lim\limits_{z\to\sqrt{3}i} f(z)$, if it exists.

Question 3(c) continues...

ii) Is $f(z)$ continuous at $z = \sqrt{3}i$?

4. (a) State *Cauchy's Integral formulae.* [2]

   (b) Evaluate $\oint\limits_C \frac{e^{2z}}{(z+1)^4}\,dz$, where C is the circle $|z| = 2$. [3]

   (c) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a *Laurent series* valid over the regions: [4]
   i) $1 < |z| < 3$, ii) $0 < |z+1| < 2$.

5. (a) Define and classify the *singular* points. [2]

   (b) Define *residue* of a complex function. Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles. [3]

   (c) Use *Cauchy's residue theorem* to evaluate $\oint\limits_C \frac{e^{-iz}}{(z+3)(z-i)^2}\,dz$ where C is the circle $|z| = 2$. [4]

6. (a) Determine if the line integral $\int\limits_C (z^3 + 3z)\,dz$ is *independent of the path.* [2]

   (b) Evaluate $\int\limits_C (z^3 + 3z)\,dz$ along the line joining the points (2,0) to (2,2) and then from (2,2) to (0,2). [3]

   (c) Evaluate $\int\limits_C (z^3 + 3z)\,dz$ along the circle $|z| = 2$ from (2,0) to (0,2) in a counter-clockwise direction. [4]

Part B: Laplace Transformations

7. (a) Define *Laplace Transformation.* Prove that, $\mathcal{L}\{\sin at\} = \frac{s}{s^2 + a^2}$; $s > 0$. [4]

   (b) Find the Laplace transformations of the following functions (any two): [5]

   i) $e^{-3t}(3\sin 4t - 4\cos 4t)$, ii) $4t^3 - e^{-t}$, iii) $7\sin 2t - 3\cos 2t$.

8. (a) Evaluate the following: [4]

$$\mathcal{L}^{-1}\left\{\frac{2s^2 - 4}{(s+1)(s-2)(s-3)}\right\}.$$

   (b) Solve the following IVP using *Laplace Transformation.* [5]

$$y'' + y = t,\ \ y(0) = 1,\ \ y'(0) = -2.$$

   where $'$ denotes the derivative with respect to t.

9. (a) Evaluate the following: [4]

$$\mathcal{L}^{-1}\left\{\frac{4s+12}{s^2 + 8s + 16}\right\}.$$

   (b) Solve the following first order IVP using *Laplace Transformation* [5]

$$ty'' + (1-2t)y' - 2y = 0,\ \ y(0) = 1,\ \ y'(0) = 2.$$

   where $'$ denotes the derivative with respect to t.

MNS Department
Final Examination
Summer Semester 2014
Course No: MAT 215
Course Title: Complex Variables and Laplace Transformations (Mathematics III)

Time: 3 hours
Full Marks: 50 **Date: 24-08-2014**

Group A (Complex Variables): Answer any THREE questions:

1. a) Represent graphically the set of values of z for which: 5
(i) $1 < |z-2i| < 2$ and (ii) $|z-2| \le |z+2|$.

b) Prove that $|z_1 + z_2| \ge |z_1| - |z_2|$, where z_1 and z_2 are complex numbers. 4

c) Find $\lim_{z\to 0} \frac{z - \sin z}{z^3}$. 3

2. a) Compute all the roots of $(16i)^{\frac{1}{4}}$ in the form $z = a + ib$ and show them graphically. 5

b) Show that $\sin^{-1} z = -i \ln[iz \pm (1-z^2)^{1/2}]$. 4

c) If $z_1 = 1 - i$, $z_2 = -2 + 4i$, evaluate $|z_1 \overline{z_2} + z_2 \overline{z_1}|$. 3

3. (a) Define harmonic function. Is $u(x,y) = e^{-3x} \cos 3y$ harmonic? 5
If so, find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic.

b) Expand $f(z) = \frac{z}{-z^2 + 3z - 2}$ in a Laurent series valid for: 4
(i) $1 < |z| < 2$, (ii) $|z| > 2$.

c) Using the definition, find the derivative of the function: 3
$f(z) = \frac{2z}{z+i}$ at $z = i$.

4. a) State residue theorem. Hence evaluate $\oint_C \frac{z^2}{2z^2+5z+2}\,dz$ where C is the unit circle $|z|=1$. 5

b) Evaluate $\int_C \left(x^2 - iy^2\right) dz$ along the straight lines from $(1,1)$ to $(1,8)$ and then from $(1,8)$ to $(2,8)$. 4

c) Evaluate $\oint_C \frac{dz}{z-2}$ around the circle $|z-2|=4$. 3

5. a) State Cauchy integral formula. Hence evaluate $\oint_C \frac{e^z\,dz}{\left(z^2+\pi^2\right)^2}$, where C is the circle $|z|=4$. 5

b) Show that $\int_0^\infty \frac{\cos mx}{x^2+1}\,dx = \frac{\pi}{2}e^{-m}$, $m>0$. 4

c) Show that: $\ln\left(-1+\sqrt{3}\,i\right) = \ln 2 + 2\left(n+\frac{1}{3}\right)\pi i$. 3

Group B (Laplace Transformations): Answer any TWO questions:

6. a) Define Laplace transform. Show that $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$. 4

b) Evaluate $\mathcal{L}\{e^{-3t}(3\sin 4t - 4\cos 4t)\}$. 3

7. a) Solve by Laplace transform: $Y''(t)+Y(t)=t$, $Y(0)=1$, $Y'(0)=-2$. 4

b) Evaluate $\mathcal{L}^{-1}\left\{\frac{23s-15}{s^2+8}\right\}$. 3

8. a) Solve the following by Laplace transformation: 4

$t\,Y''(t)+(1-2t)Y'(t)-2Y(t)=0$, $Y(0)=1$, $Y'(0)=2$.

b) Evaluate $\mathcal{L}^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$. 3

The End

BRAC UNIVERSITY

Final Examination

Semester: Spring 2014

Course No: MAT 215

Course Title: Complex Variables and Laplace Transformations (Mathematics III)

Total Marks: 50

Time: 3 hours

Date: 26 April 2014

Group A (Complex Variables)

Answer any FOUR questions.

1. a) Prove that $|z_1 + z_2| \geq |z_1| - |z_2|$, where z_1 and z_2 are complex numbers. 4

   b) Represent graphically the set of values of z for which $|z-1| + |z+1| \leq 4$. 3

   c) Find $\lim_{z\to 0}(\cos z)^{\frac{1}{z^2}}$. 3

2. a) Compute all the roots of $(64)^{\frac{1}{6}}$ in the form $z = a + ib$ and locate them graphically. 5

   b) Show that $\ln(z-1) = \frac{1}{2}\ln[(x-1)^2 + y^2] + i\tan^{-1}\frac{y}{x-1}$. 5

3. a) Define a harmonic function. Is $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$ harmonic? If so, find a function $v(x,y)$ such that $f(z) = u + iv$ is analytic. 5

   b) Expand $f(z) = \frac{1}{z^2+4z+3}$ in a Laurent series valid for (i) $1 < |z| < 3$ and (ii) $|z| > 3$. 5

4. a) State Cauchy's Theorem. Hence verify this theorem for the integral of z^3 taken over the boundary of the rectangle with vertices at $-1, 1, 1+i, -1+i$. 5

   b) Evaluate $\oint |z|^2\, dz$ around the square with vertices at $(0,0), (1,0), (1,1), (0,1)$. 5

5. a) State Cauchy integral formula. Hence, evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{z^2 - 3z + 2} dz$. 5

   b) Evaluate $\oint_C \frac{e^{3z}}{z - \pi i} dz$, where, $C: |z-2| + |z+2| = 6$. 5

6. a) State Cauchy's residue theorem. Hence, evaluate $\int_0^\infty \frac{dx}{1+x^2}$. 5

   b) Show that $\int_0^\infty \frac{\cos mx}{x^2+a^2} dx = \frac{\pi}{2a} e^{-ma}$ $(m > 0)$. 5

Group B (Laplace Transforms): Answer any ONE question.

7. a) Define Laplace transforms. Use this definition to show that 5

   $\mathcal{L}\{\sin bt\} = \frac{b}{s^2+b^2}$.

   b) Evaluate $i)\ \mathcal{L}\{3t\sin 2t - 2t\cos 2t\}$ *and* $ii)\ \mathcal{L}^{-1}\{\frac{5s+4}{s^3} - \frac{2s-18}{s^2+9}\}$. 5

8. Solve by Laplace transformations method :

   a) $y''(t) + y(t) = t, y(0) = 1, y'(0) = -2$. 5

   b) $t\,y''(t) + (1-2t)y'(t) - 2y(t) = 0,\ y(0) = 1, y'(0) = 2$. 5

BRAC UNIVERSITY

MNS Department
Final Examination
Semester: Fall 2013
Course Title: Complex Variables and Laplace Transformations (Mathematics III)
Course No.: MAT 215

Date: December 30, 2013

Time: 3 Hours
Full Marks: 50

Answer any Five questions:

1. (a) Find all roots for the function $(-4+4i)^{1/5}$ and locate them graphically. 5

   (b) Solve $\cos hz = \frac{1}{2}$. 5

2. (a) Prove that $|z_1 + z_2| \leq |z_1| + |z_2|$. 5

   (b) Find $\lim_{z\to 2i} \frac{z^2+4}{2z^2+(3-4i)z-6i}$. 5

3. (a) Evaluate $\int (x^2 - iy^2)dz$ along the straight lines from $(1,1)$ to $(1,8)$ and then from $(1,8)$ to $(2,8)$. 5

   (b) Show that $\cos^{-1} z = -i \ln[z \pm i(1-z^2)^{1/2}]$. 5

4. (a) Determine whether the function $u = xe^x \cos y - ye^x \sin y$ is harmonic. If u is harmonic then find its conjugate harmonic function. 5

   (b) Prove that $\lim_{z\to 0} \frac{\bar{z}}{z}$ does not exist. 5

5. (a) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent series for :

   (i) $|z| > 2$, 5

   (ii) $0 < |z-2| < 1$. 5

   (b) Find $\oint \frac{e^{2z}}{(z+1)^4} dz$ around the circle $|z| = 3$.

6. (a) Show that $\ln(-1+i\sqrt{3}) = \ln 2 + 2(n+1/3)\pi i$. 5

   (b) Evaluate $\oint (\bar{z})^2 dz$ around the circle $|z-1| = 1$. 5

7. (a) Evaluate $\oint \frac{z^2}{2z^2+5z+2} dz$ using the residue at the poles around the unit circle $|z| = 1$. 5

   (b) Find all singularities for the function $f(z) = \frac{2z-3}{z^2+2z+2}$. 5

8. (a) Find $\mathcal{L}\{e^{-t}(3\sin h2t - 5\cos h2t)\}$. 5

   (b) Evaluate $\mathcal{L}^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$. 5

MNS Department
Final Examination
Semester: Fall 2014
Course No: MAT 215
Course Title: Complex Variables and Laplace Transformations

Full Marks: 50

Date: December 21, 2014

Time: 3 hours

Part A: Answer any <u>four</u> of the following problems

1. a) Find all roots of $(2+i)^{1/2}$. 2

   b) State and prove De Moivre's theorem. 3

   c) Is $u(x,y)=e^{-x}(x\sin y - y\cos y)$ harmonic? 5

   If so, find a function $v(x,y)$ such that $f(z)=u+iv$ is analytic.

2. a) Define limit and continuity of a complex function. 2

   b) Show that $|z_1 \pm z_2| \ge |z_1| - |z_2|$. 3

   c) Evaluate: (i) $\lim\limits_{z\to i}\dfrac{z^2-2iz-1}{z^4+2z^2+1}$, (ii) $\lim\limits_{z\to 0}\dfrac{z-\sin z}{z^3}$, using L'Hospital's rule. 5

3. a) Express the complex number $-1+i\sqrt{3}$ in polar form. 2

   b) Prove that $\sin^{-1} z = -i\ln\left(iz \pm \sqrt{1-z^2}\right)$. 3

   c) Define the derivative of $f(z)$. Find the derivative of $f(z)=\dfrac{2z-i}{z+2i}$ 5

   at $z=-i$ using the definition of derivative.

4. a) Define a harmonic function. 2

   b) State Cauchy's Integral formula, hence evaluate $\oint_C \dfrac{e^{2z}}{(z+1)^4}dz$ 3

   over the region $|z|=3$.

   c) Show that $\dfrac{1}{2\pi i}\int_C \dfrac{e^{zt}}{z^2+1}dz=\sin t$ around the circle $|z|=3$ using Residue theorem. 5

5. a) Define Laurent Series. 2

b) Expand $\frac{4z+4}{z(z-3)(z+2)}$ in Laurent Series within $2<|z|<3$. 3

c) Show that $\int_0^\infty \frac{1}{x^4+x^2+1}dx = \frac{\pi\sqrt{3}}{6}$. 5

6. a) State Cauchy's Residue theorem. 2

b) Evaluate $\oint_C |z|^2 dz$ around the square with vertices at (0,0), (1,0), (1,1), (0,1). 3

c) Evaluate $\oint_C \frac{3z^3+2}{(z-1)(z^2+9)}dz$ around the circle $|z|=4$ using Cauchy's Residue theorem. 5

Part B: Answer any <u>one</u> of the following problems

7. a) Define Laplace transformation. 2

b) Find the Laplace transformation of: 4

(i) $e^{-t}(3\sinh 2t - 5\cosh 2t)$, (ii) t^2e^{-3t}.

c) Find the Laplace transformation of: 4

(i) $\left\{\frac{2s-5}{s^2-9}\right\}$, (ii) $\left\{\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\}$.

8. a) Define inverse Laplace transformation 2

b) Solve the following IVPs using Laplace Transformation: 4+4

(i) $Y''+9Y=\cos 2t,\quad Y(0)=1, Y\left(\frac{\pi}{2}\right)=-1.$

(ii) $Y''+2Y'+5Y=e^{-t}\sin t,\quad Y(0)=0, Y'(0)=1.$

BRAC UNIVERSITY

MNS Department
Semester: Spring 2015
Mid Term Examination
Course No.: MAT 215 (Section: 07)
Course Title: Complex Analysis and Laplace Transformations (Mathematics III)

Full Marks: 40 **Date: March 14, 2015**
Time: 1 Hour

Answer any **FOUR** questions:

1. (i) Find all the roots of the equation $z^5 = -4 + 4i$ and locate them graphically. 5
   (ii) Find all the points of discontinuity for the function 5
   $$f(z) = \frac{3z^2+4}{z^4-16}.$$

2. (i) Prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. 5
   (ii) Evaluate: $\lim_{z\to 0}\frac{z-\sin z}{z^3}$. 5

3. (i) Prove that: $|z_1 + z_2| \le |z_1| + |z_2|$. 5
   (ii) Show that: $\ln(-1 + i\sqrt{3}) = \ln 2 + i.2\left(n + \frac{1}{3}\right)\pi$. 5

4. (i) Use definition of derivative to find the derivative of the function: 5
   $f(z) = \frac{3}{z^2}$ at $z = 1 + i$.
   (ii) Show that: $\cos^{-1} z = -i\ln\left(z \pm i\sqrt{1 - z^2}\right)$. 5

5. (i) Solve: $\cos hz = \frac{1}{2}$ 5
   (ii) Find all values of z such that $e^{2z-1} = 1$. 5

MNS Department
Midterm Examination
Semester: Fall 2014
Course Title: Complex Variables and Laplace Transformations
Course No: MAT 215 (Section-3)

Date: 28 October 2014

Time: 1 hour
Marks: 40

Answer any four questions:

1. (a) Find all the roots of the complex number $(1+\sqrt{3}\,i)^{1/5}$. 4

(b) Express in polar form and illustrate graphically: $2+2\sqrt{3}\,i$. 3

(c) If $z_1 = 1-i$, $z_2 = -2+4i$ & $z_3 = \sqrt{3}-2i$, evaluate: $\overline{(z_2+z_3)(z_1-z_3)}$. 3

2. (a) Prove that: $|z_1 - z_2| \geq |z_1| - |z_2|$. 5

(b) Using the definition, find the derivative of $f(z) = \dfrac{2z-i}{z+2i}$ at $z = -i$. 5

3. (a) Evaluate $\lim_{z \to i} \dfrac{z^2 - 2iz - 1}{z^4 + 2z^2 + 1}$; using L'Hospital's rule. 3

(b) Define continuity and differentiability. 3

(c) Describe geometrically the set of points z satisfying the conditions:

(i) $\operatorname{Re}(z) \geq 0$, (ii) $1 < |z-2i| < 2$. 4

4. (a) Find all values of z such that $e^z = 1+\sqrt{3}\,i$. 5

(b) Prove that $\sin^{-1} z = -i \ln[iz \pm (1-z^2)^{1/2}]$. 5

5. (a) Show that (i) $\exp(2+3\pi i) = -e^2$, (ii) $\sin z = \sin x \cosh y + i \cos x \sinh y$. 5

(b) State and prove the De Moiver's theorem. 5

BRAC UNIVERSITY

MNS Department

Midterm Examination

Semester: Summer 2014

Course No: MAT 215 (Sec 02)

Course Title: Complex Variables and Laplace Transformations

(Mathematics III)

Time: 1 hour

Total Marks: 20

Date: 01-07- 2014

Answer any four of the following questions:

Q1. (a) Write down Euler's formula. 1

(b) If $Z_1 = 1 - i,\ Z_2 = -2 + 4i$ and $Z_3 = 3 - 2i$, evaluate $\text{Im}\{\ 2Z_1 + 3Z_2^2 - Z_3\}$. 2

(c) Prove that $\sin(x + iy) = \ \sin x \cos hy + i \cos x \sin hy$. 2

Q2. (a) State De Moivre's Theorem. 2

(b) Find six distinct sixth roots of $-1 + i\sqrt{3}$ and plot them in a complex plane. 3

Q3. (a) Prove that $\lim_{z\to 0} \frac{\overline{z}}{z}$ does not exist. 2

(b) Find the derivative of $f(z) = \frac{2z-i}{z+2i}$ at the point z = i using the definition of derivative. 3

Q4. (a) Find $\lim_{z\to 0} \frac{z - \sin z}{z^3}$. 2

(b) Define harmonic function. Check whether the function $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic or not. If it is, find the conjugate harmonic function of u. 3

Q5. (a) Write down Cauchy-Riemann equations. 1

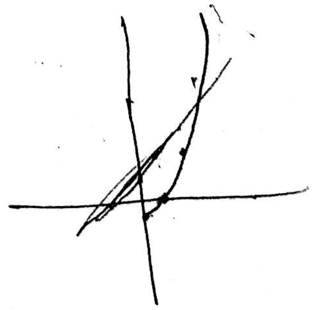
(b) Find all values of z such that $e^z = 1 + \sqrt{3}i$. 2

(c) Prove that $\cos^{-1} z = -i \ln[z \pm i\sqrt{1 - z^2}]$. 2

$y = \frac{x^2-1}{x-1}$

$= \frac{(x+1)(x-1)}{x-1}$

$= x+1$



MNS Department

Fall 2013

Mid Term Examination

Course No.: MAT 215 (Section: 04)

Course Title: Complex Analysis and Laplace Transformations (Mathematics III)

Full Marks: 40 | **Date: November 02, 2013**

Time: 1 Hour

Answer any **FOUR** questions:$(10 \times 4 = 40)$

1. (i) Find all roots of the equation $z^5 = -4 + 4i$ and locate them graphically. [5]

   (ii) Find all points of discontinuity for the function [5]

$$f(z) = \frac{3z^2+4}{z^4-16}.$$

2. (i) Find the roots of the equation $z^4 + z^2 + 1 = 0$. [5]

   (ii) Evaluate $\lim_{z\to(1+i\sqrt{3})} \frac{z^3+8}{z^4+4z^2+16}$. [5]

3. (i) Prove that $|z_1 + z_2| \leq |z_1| + |z_2|$. [5]

   (ii) Show that $\ln(-1 + i\sqrt{3}) = \ln 2 + i.2\left(n + \frac{1}{3}\right)\pi$. [5]

4. (i) Verify that the Cauchy-Riemann equations are satisfied for: [5]

$$f(z) = e^x(\cos y + i \sin y).$$

   (ii) Show that $\sin^{-1} z = -i \ln[iz \pm (1 - z^2)^{1/2}]$. [5]

5. (i) Find the conjugate of u for $u = -x^3 + 3xy^2 + 2y + 1$. [5]

   (ii) Find the values of z for $e^{2z-1} = 1$. [5]