MAT120

Assignment-02 Set-12

Tasnim Sakib Apon

18301297

Section : 03

1. Evaluate the following indefinite integral by using a trigonometric substitution or otherwise

$$\int \frac{8}{4+9x^2}dx$$

$$Weknow: \int \frac{1}{\sqrt{1+u^2}}du = tan^{-1}(u) + C$$

$$Let, x = \frac{2}{3}u$$

$$dx = \frac{2}{3}du$$

$$SO: u = \frac{3}{2}x$$

$$Now, \int \frac{8}{\sqrt{4+9(\frac{2}{3}u)^2}}\frac{2}{3}du$$

$$= \frac{16}{3}\int \frac{1}{\sqrt{4+9*\frac{4}{9}u^2}}du$$

$$= \frac{16}{3}\int \frac{1}{\sqrt{4+4u^2}}du$$

$$= \frac{16}{3}\int \frac{1}{2\sqrt{1+u^2}}du$$

$$= \frac{8}{3}\int \frac{1}{\sqrt{1+u^2}}du$$

$$= \frac{8}{3}tan^{-1}u + C$$

$$= \frac{8}{3}tan^{-1}\frac{3}{2}x + C$$

$$Answer$$

2. Evaluate the following indefinite integral by using a trigonometric substitution or otherwise

$$\int \frac{1}{\sqrt{4-9x^2}}dx$$

We know :

$$\int \frac{1}{\sqrt{1-u^2}}du = sin^{-1}(u) + C$$

Let :

$$x = \frac{2}{3}u$$

$$dx = \frac{2}{3}du$$

$$SO : u = \frac{3}{2}x$$

Now:

$$\int \frac{1}{\sqrt{4-9(\frac{2}{3}u)^2}}\frac{2}{3}du$$

$$= \frac{2}{3}\int \frac{1}{\sqrt{4-9*\frac{4}{9}u^2}}du$$

$$= \frac{2}{3}\int \frac{1}{\sqrt{4-4u^2}}du$$

$$= \frac{2}{3}\int \frac{1}{2\sqrt{1-u^2}}du$$

$$= \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}du$$

$$= \frac{1}{3}sin^{-1}(u) + C$$

$$= \frac{1}{3}sin^{-1}\frac{3}{2}x + C$$

$$Answer$$

3. Integrate the following with the help of Gamma functions or otherwise

$$\int_0^\infty x^5 e^{\frac{-x^2}{5}} dx$$

let,

$$u = \frac{x^2}{5}$$

$$du = \frac{1}{5} 2x dx$$

$$dx = \frac{5}{2x} du$$

$$x = \sqrt{5u}$$

Now :

$$\int_0^\infty x^5 e^{-u} \frac{5}{2x} du$$

$$= \frac{5}{2} \int_0^\infty (\sqrt{5u})^4 e^{-u} du$$

$$= 25 \frac{5}{2} \int_0^\infty u^2 e^{-u} du$$

$$= \frac{125}{2} \int_0^\infty u^{3-1} e^{-u} du$$

$$= \frac{125}{2} \Gamma 3$$

$$= \frac{125}{2} 2$$

$$= 125$$

$$Answer$$

4. Evaluate the following with the help of trigonometric form of Beta functions or otherwise

$$\int_0^{\frac{3\pi}{2}} sin^6(\frac{x}{3})cos^4(\frac{x}{3})dx$$

$$= \int_0^{\frac{3\pi}{2}} (sin2x\frac{x}{3})^6 cos^4(\frac{x}{3})dx$$

$$= \int_0^{\frac{3\pi}{2}} 2^6 sin^6\frac{x}{3}cos^6(\frac{x}{3})dx$$

$$= 2^6\int_0^{\frac{2\pi}{2}} sin^6\frac{x}{3}cos^{10}(\frac{x}{3})dx$$

$$Let, \frac{x}{3} = z$$

$$dx = 3dz$$

So :

$$2^6\int_0^{\frac{\pi}{2}} sin^6 zcos^{10}z3dz$$

$$= 3 * 2^6\int_0^{\frac{\pi}{2}} sin^6 zcos^{10}zdz$$

$$= 3 * 2^5\int_0^{\frac{\pi}{2}} 2sin^6 zcos^{10}zdz$$

$$2x - 1 = 6$$

so

$$x = \frac{7}{2}$$

$$2y - 1 = 10$$

so

$$y = \frac{11}{2}$$

so

x 0 $3\pi/2$

y 0 $\pi/2$

$$= 3 * 2^5\beta(\frac{7}{2}, \frac{11}{2})$$

$$= 96\frac{\frac{5}{2}!\frac{9}{2}!}{8!}$$

$$Answer$$

5. Evaluate the following with the help of Beta functions or otherwise

$$\int_0^{3^{\frac{2}{3}}} x^{\frac{7}{2}}(3 - x^{\frac{3}{2}})^4 dx$$

$$= \int_0^{2} x^{\frac{7}{2}}(3 - x^{\frac{3}{2}})^4 dx$$

$$= \int_0^{2} x^{\frac{7}{2}} 3^4 (1 - \frac{x^{\frac{3}{2}}}{3})^4 dx$$

$$= 81 \int_0^{2} x^{\frac{7}{2}} (1 - \frac{x^{\frac{3}{2}}}{3})^4 dx$$

Let

$$u = \frac{x\frac{3}{2}}{3}$$

$$du = \frac{xdx}{2}$$

$$x = 3u^{\frac{-1}{2}}$$

$$dx = \frac{3du}{u^{\frac{1}{2}}}$$

when x=0; u=0

when x=2; u=1

So :

$$= 81 \int_0^{1} (3u)^3 (1-u)^4 \frac{3du}{u^{\frac{1}{2}}}$$

$$= 6561 \int_0^{1} u^3 (1-u)^4 \frac{du}{u^{\frac{1}{2}}}$$