BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences
Final Examination
Semester: Spring 2016
Course Title: Integral Calculus and Differential Equations
Course No.: MAT120

Time: 3 hours
Total Marks: 50

Date: April 13, 2016

Note: Answer any THREE from Part A and any TWO from Part B.

Part A: Integral Calculus

1. (a) Evaluate the integral $\int \cos 2x \cos 3x \, dx$. [3]

   (b) Sketch the region whose *signed area* is represented by $\int_{-1}^{4} |x-3| \, dx$ and evaluate the integral using an appropriate formula from geometry. [3]

   (c) Use x_k^* as the midpoint of each subinterval to find the *area* under the curve [4]

$$f(x) = x^2; \ -1 \le x \le 3.$$

2. (a) Evaluate the integral $\int \sec^4 x \tan^2 x \, dx$. [3]

   (b) Evaluate $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$. [3]

   (c) Sketch and find the area of the region enclosed by $y = x^2$, $y = 2 - x$, and $y = 0$. [4]

3. (a) Evaluate $\int \frac{1}{x^2 - 4x + 13} \, dx$. [3]

   (b) Find the arc length of the curve given by the following parametric equations. [3]

$$x = e^t(\cos t + \sin t), \ y = e^t(\cos t - \sin t); \ 0 \le t \le 4$$

   (c) Find the volume of solids that results when the region enclosed by the curves $y = x^2$ and $y^2 = x$ is revolved about the y-axis. [4]

4. (a) Evaluate $\int_0^1 \sin^{-1}(x) \, dx$. [3]

   (b) Evaluate $\int \frac{2x^2 + 3}{x(x-1)^2} \, dx$. [3]

Question 4 continues ...

(c) Use *cylindrical shells* to find the volume of the solid generated when the region enclosed by the curves $y^2 = x$, $y = 1$, and $x = 0$ is revolved about the x-axis. [4]

5. (a) Define *beta* and *gamma* functions. [3]

(b) Evaluate $\int_0^{\infty} e^{-x^2}\,dx$. [3]

(c) Find the *area of the surface* generated by revolving the curve given by: [4]

$$y = \sqrt{a^2 - x^2},\ a > 0,\ -a \le y \le a$$

about the x-axis.

Part B: Differential Equations

6. (a) How much of carbon-14 (^{14}C) will be left in 100 years if 50 grams are present? Assume that, the half-life of ^{14}C is 5730 years. [5]

(b) Write the standard form of the first-order linear differential equation. Solve the following differential equation: [5]

$$(x+1)\frac{dy}{dx} + (x+2)y = 2xe^{-x}.$$

7. (a) Use reduction of order to find the general solution of the differential equation [5]

$$y'' + 2y' + y = 0,$$

with one particular solution $y_1(x) = e^{-x}$ and where the prime ($'$) denotes the derivative with respect to x.

(b) Prove that the following is an exact differential equation. [5]

$$(e^x + y)dx + (2 + x + ye^y)dy = 0.$$

Also solve the equation with the initial value $y(0) = 1$.

8. (a) Solve the following differential equation: [5]

$$y'' + y = \cos x$$

by the method of undetermined coefficients. The prime ($'$) denotes the derivative with respect to x.

(b) Solve the following initial value problem: [5]

$$y'' + 6y' + 25y = 0,\quad y(0) = 1,\quad y'(0) = 1.$$

The prime ($'$) denotes the derivative with respect to x.

Inspiring Excellence

Department of Mathematics and Natural Sciences
Final Examination
Semester: Summer 2015
Course Title: Integral Calculus and Differential Equations
Course No.: MAT120

Time: 3 hours
Total Marks: 50

Date: August 16, 2015

Note: Question 1 is compulsory. Answer any THREE from Part A and any TWO from Part B.

1. Answer all of the following:

   (a) Is anything wrong with $\int_{-1}^{2} \frac{1}{x^2}\, dx = -\frac{3}{2}$? Explain. [1]

   (b) Are *gamma* and *beta* functions improper integrals? Explain. [1]

   (c) The roots of an auxiliary equation of a homogeneous differential equation with constant coefficients are -2, -2, $1+2i$, and $1-2i$. What is the general solution of that differential equation? [1]

   (d) Does $\int_{-1}^{2} x^3\, dx$ represent the *total* area under the curve $f(x) = x^3$ over $[-1, 2]$? Explain why or why not. [1]

   (e) "If two functions F and G have the same derivative then $F = G$". Provide examples to justify or negate the statement. [1]

Part A: Integral Calculus

2. (a) Evaluate the integral $\int \sin 5x \cos 7x\, dx$. [2]

   (b) Sketch the region whose *signed area* is represented by $\int_{-1}^{4} 2x\, dx$ and evaluate the integral using the appropriate formula from geometry. [3]

   (c) Use x_k^* as the left end point to find the *area* under the curve $f(x) = 2x - x^2$ over the interval $[-1, 3]$. [4]

3. (a) Define definite integral. [2]

   (b) Evaluate $\int_{1}^{e} x^2 \ln x\, dx$. [3]

   (c) Sketch and find the area of the region enclosed by $y = x$, $y = 4x$, and $y = -x + 2$. [4]

4. (a) Evaluate $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx$. [3]
   (b) Find the arc length of the curve given by the following parametric equations.

$$x = \cos t + t\sin t,\; y = \sin t - t\cos t;\; 0 \le t \le \pi$$

[4]

   (c) Find the volume of solids that results when the region enclosed by the curves $y = x^2$ and $y = 2x$ is revolved about the x axis. [2]

5. (a) State the first fundamental theorem of calculus. [3]
   (b) Evaluate $\displaystyle\int \frac{2x-3}{x^2-3x-10}\,dx$. [4]
   (c) Use the *cylindrical shells* to find the volume of the solid generated when the region enclosed by the curves $y^2 = x$, $y = 1$, and $x = 0$ is revolved about the x axis. [2]

6. (a) Define *beta* and *gamma* functions. [3]
   (b) Evaluate $\displaystyle\int_0^1 \frac{x^2}{\sqrt{1-x}}\,dx$ using beta function. [4]
   (c) Find the *area of the surface* generated by revolving the curve given by,

$$y = \sqrt{x},\; 1 \le y \le 4$$

about the y axis.

Part B: Differential Equations

[4]

7. (a) Solve the differential equation

$$(e^y + 1)^2 e^x dx + (e^x + 1)^3 e^y dy = 0.$$

   (b) Write the standard form of the *first order linear differential equation*. Solve the following differential equation [5]

$$x\frac{dy}{dx} + (x+1)y = e^{-x}\sin 2x.$$

8. (a) Use reduction of order to find the general solution of the differential equation [4]

$$(1-x^2)y'' + 2xy' = 0,$$

with one particular solution $y_1 = 1$ and where prime ($'$) denotes the derivative with respect to x. [5]

   (b) Prove that the following differential equation is *exact* and solve.

$$(\sin y - y\sin x)dx + (\cos x + x\cos y - y)dy = 0,\; y\left(\frac{\pi}{2}\right) = 0.$$

9. (a) Solve the following differential equation by the *method of undetermined coefficients.* [4]

$$y'' + 4y' + 16y = 2x^2 + 3$$

where prime ($'$) denotes the derivative with respect to x and $y_1 = 1$..

   (b) Solve the following differential equation by variation of parameters. [5]

$$y'' + y = \sin x,$$

where prime ($'$) denotes the derivative with respect to x.

MNS Department
Final Examination
Semester: Summer 2014
Course No.: MAT 120
Course Title: Integral Calculus and Differential Equations
(Mathematics II)

Total Marks: 50 Date: August 24, 2014
Time: 3 hours

Section-A: (Integral Calculus)

Answer any **THREE** questions:

1. (a) Estimate the area of the region enclosed by the curve $f(x) = x^2$ within the interval [0, 3] by using the formula: $A = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k^*)\Delta x$ where x_k^* is the midpoint of each subinterval. [5]

   (b) Evaluate the following integrals: [2.5 x 2 = 5]
   (i) $\int(\frac{1}{x} + sec^2\pi x)$ dx, (ii) $\int_0^2 x^2 e^{-x}$ dx .

2. (a) Sketch the region whose signed area is represented by [5]
   $\int_0^4 (12 - 3x)\text{dx}$
   and evaluate the integral using the appropriate formula from geometry.

   (b) Define convergence and divergence. [1+4 = 5]
   Evaluate the integral $\int_0^5 \frac{1}{x-3}$ dx and identify whether it is converging or diverging.

3. (a) Find the area of the region which is bounded by $x = y^2$ and $y = x - 2$. [5]

   (b) Find the arc lengths of the curves given by: [5]
   $x = cos2t, \quad y = sin2t; \quad 0 \le t \le 4\pi.$

4. (a) Evaluate the following integrals: [2.5 x 2 = 5]
   (i) $\int \frac{dx}{x^2+7x+12}$, (ii) $\int_0^{\pi/6} sin^2 6x\, cos^4 3x\, dx.$

   (b) Use cylindrical shells to find the volume of a solid generated when the region in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis. [5]

[Please turn over]

5. (a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x –axis. [5]

(b) Evaluate the following integrals: [2.5 x 2 = 5]

(i) $\int_0^{\infty} \frac{1}{1+x^4}\, dx$, (ii) $\int_0^{b} y^5 \sqrt{b^2 - y^2}\, dy$.

Section-B: (Differential Equations)

Answer any **TWO** questions:

6. (a) Define differential equation. What is the meaning of initial value problem? Solve the initial value problem [1+1+3 = 5]

$$(e^{2y} - y)\cos x \frac{dy}{dx} = e^x \sin 2x, \quad y(0) = 0.$$

(b) Determine whether the differential equation given by [5]

$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$ is exact. If it is, solve it.

7. (a) The population of a community is known to increase at a rate proportional to the number of people present at time t. If the population triples in 8 years, how long will it take to quadruple? [5]

(b) Use reduction of order to find a second solution $y_2(x)$ of the differential equation $\frac{d^2y}{dx^2} + 9y = 0$; $y_1 = \sin 3x$. [5]

8. (a) Define complimentary function and particular solution. [1+4 = 5]
Solve it: $y^{//} + y = x\cos x - \cos x$ where $y^{//} = \frac{d^2y}{dx^2}$.

(b) Solve it: $y^{//} + y = \tan x$. [5]

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BRAC UNIVERSITY

MNS Department
Final Examinations
Semester: Spring 2014
Course No: MAT 120
Course Title: Integral Calculus and Differential Equations

Marks: 50
Time: 3 hours

Date: 26 April 2014

Section-A (Integral Calculus)

Answer any **THREE** questions:

Q1. (a) Use x_k^* as the right end point to find the area under the curve $f(x)=\frac{x^2}{2}$ over the interval $[1, 4]$. 4

(b) Define definite integral. What is the difference between definite and indefinite integrals? 3

(c) Sketch the region whose signed area is represented by $\int_{-1}^{3}(4-5x)dx$ and evaluate the integrals using the appropriate formula from geometry. 3

Q2. (a) Sketch the region enclosed by the curves $y=x^2$, $y=\sqrt{x}$ and find its area. 4

(b) Evaluate the integral $\int_0^b y^5\sqrt{b^2-y^2}\,dy$. 3

(c) Evaluate the integral $\int e^{2x}\sin x\,dx$. 3

Q3. (a) Find the volume of the solid that results when the region enclosed by the curves $x=\sqrt{1+y}$, $x=0$, $y=3$ is revolved about the $y-$axis. 4

(b) Evaluate the integral $\int \frac{dx}{x^2+3x-4}$. 3

(c) Evaluate the integral $\int_0^1 x\sqrt{1-x^2}\,dx$. 3

Q4. (a) Find the arc length of the curve $y=x^{2/3}$ from $x=1$ to $x=8$. 4

(b) Evaluate the integral $\int_0^2 \frac{1}{(x-1)^{2/3}}\,dx$. 3

(c) Evaluate the integral $\int_0^{\pi}\sin^5\theta\cos^4\theta\,d\theta$. 3

Q5. (a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis. 5

(b) Use the cylindrical shells to find the volume of the solid generated when the region enclosed by the curves $y = x^2$, $y = 0$, $x = 1$, $x = 2$ is revolved about the y-axis. 5

Section-B (Differential Equations)

Answer any **TWO** questions:

Q6. (a) Define differential equation(DE). What is the difference between the solutions of a DE and that of an algebraic equation? 2

Solve the initial-value problem: $\frac{dy}{dt} + 2y = 1, \quad y(0) = \frac{5}{2}$. 2

(b) Determine whether the differential equation given by $(x - y^3 + y^2 \sin x)\, dx = (3xy^2 + 2y \cos x)\, dy$ is exact. If it is, solve it. 3

(c) Solve the differential equation $(y^2 + yx)\, dx - x^2\, dy = 0$. 3

Q7. (a) Initially a sample contained 100 milligrams of a radioactive substance. After six hours the mass decreased by 3%. If the rate of decay is proportional to the amount of substance present at time t, find the amount remaining after 24 hours. 4

(b) Find the general solution of the differential equation $3y'' + 2y' + y = 0$. 3

(c) Use reduction of order to find a second solution $y_2(x)$ of the differential equation $9y'' - 12y' + 4y = 0; \quad y_1 = e^{2x/3}$. 3

Q8. (a) Solve the differential equation $y'' - 2y' + 5y = e^x \cos 2x$ by the method of undetermined coefficients. 5

(b) Solve the differential equation $y'' + y = \sin x$ by variation of parameters. 5

THE END

BRAC University
Semester: Summer 2013
Final Examination
Course Title: Integral Calculus & Differential Equations (Mathematics II)
Course No.: MAT 120

Full Marks: 50
Time: 3 Hours

Date: Aug 24 , 2013

Section A (Integral Calculus):

Answer any **THREE** questions:

1. (a) Evaluate (i) $\int x\sqrt{1-x}\,dx$; (ii) $\int tan^5x\, sec^4x\, dx$. [4]

   (b) Find the area under the curve $y = 1 - x^3$ over the interval [-3,-1] with x_k^* as the right end point of each subinterval. [6]

2. (a) Sketch and evaluate the integral $\int_0^1 (x + 2\sqrt{1-x^2})\,dx$ using appropriate formula from geometry. [5]

   (b) Find the area and sketch the region enclosed by the curve $y^2 = 4x, y = 2x - 4$. [5]

3. (a) Sketch the curve and find the volume of the solid that results when the region enclosed by the curve $y = sinx, y = cosx, x = 0, x = \frac{\pi}{4}$ is revolved about the x-axis. [5]

   (b) Evaluate $\int_1^3 \sqrt{x}\, tan^{-1}\sqrt{x}\, dx$. [5]

4. (a) Find the arc length of the curve $24xy = y^4 + 48$ over the interval $2 \le y \le 4$. [5]

   (b) (i) Use cylindrical shell to find the volume of the solid generated when the region enclosed by the curve $y^2 = x, y = 1, x = 0$ is revolved about the x-axis. [2.5]

   (ii) Find the area of the surface generated by revolving the curve $x = 9y + 1$, $0 \le y \le 2$ about the y axis. [2.5]

5. (a) Discuss whether the limit converges or diverges using an appropriate method: [3]

$$\int_0^{\pi/2} tanx\, dx$$

   (b) Evaluate $\int \frac{1}{(1-x^2)^{3/2}}\, dx$ by trigonometric substitution. [3]

   (c) Evaluate $\int_0^1 \frac{x^3}{\sqrt{1-x^3}}\, dx$ in terms of gamma function. [4]

Section B (Differential Equations):

Answer any **TWO** questions:

6. (a) Find the general solution of the differential equation $x^2y' + x(x+2)y = e^x$. [4]

   (b) Determine whether the given differential equation is exact. If it is exact, then solve the initial value problem: [6]
   $$(x+y)^2dx + (2xy + x^2 - 1)dy = 0,\ y(1) = 1.$$

7. (a) Solve the differential equation by variation of parameter: [5]
   $$y'' + y = sec\theta\ tan\theta.$$

   (b) Initially 100 mg of a radioactive substance was present. After 6 hours, the mass decreased by 3%. If the rate of decay is proportional to the amount of the substance present at time t, find the amount remaining after 24 hours. [5]

8. (a) Solve the given differential equation by undetermined coefficient: [5]
   $$y'' - 10y' + 25y = 30x + 3.$$

   (b) Solve $xy^2\frac{dy}{dx} = y^3 - x^3$ while $y(1) = 2$. [5]

BRAC University
Semester: Fall 2013
Final Examination
Course Title: Integral Calculus & Differential Equations (Mathematics II)
Course No.: MAT 120

Full Marks: 50 **Date: 30^{th} December 2013**
Time: 3 Hours

Section A (Integral Calculus):

Answer any **THREE** questions:

1. (a) Sketch and evaluate the integral $\int_{-1}^{2} |2x-3|\, dx$ using appropriate formula from geometry. [5]

   (b) Find the area under the curve $y = 4 - \frac{1}{4}x^2$ over the interval $[0,3]$ with x_k^* as the right end point of each subinterval. [5]

2. (a) Use cylindrical shell to find the volume of the solid generated when the region enclosed by the curve $y = \frac{1}{x}, y = 0, x = 1, x = 3$ is revolved about the y-axis. Sketch the curve. [4]

   (b) Find the area and sketch the region enclosed by the curve $x = y^2,\ y = x - 2$. [6]

3. (a) Sketch the curve and find the volume of the solid that results when the region enclosed by the curve $y = \sqrt{cosx}, y = 0, x = \frac{\pi}{4}, x = \frac{\pi}{2}$ is revolved about the x-axis. [4]

   (b) Evaluate $\int_1^3 \sqrt{x}\, tan^{-1}\sqrt{x}\, dx$. [6]

4. (a) Find the arc length of the curve $y = x^{\frac{2}{3}}$ over the interval $1 \le x \le 8$. [5]

   (b) Evaluate $\int \frac{x^2}{\sqrt{9-x^2}} dx$ by trigonometric substitution. [5]

5. (a) Discuss whether the following integral converges or diverges using an appropriate method: [5]

$$\int_e^{\infty} \frac{1}{x(lnx)^3} dx$$

(b) Find the area of the surface generated by revolving the curve $y = \sqrt{x} - \frac{1}{3}x^{\frac{3}{2}}$, $1 \leq x \leq 3$ about the x-axis. [5]

Section B (Differential Equations):

Answer any **TWO** questions:

1. (a) Separate x and y variables of the differential equation $(e^x + e^{-x})\frac{dy}{dx} = y^2$. [4]

   (b) Determine whether the given differential equation is exact. If it is exact, solve the initial value problem: [6]

$$\left(\frac{1}{1+y^2} + cosx - 2xy\right)\frac{dy}{dx} = y(y + sinx),\ y(0) = 1.$$

2. (a) Solve the initial value problem of the linear differential equation: [6]

$$(x+1)\frac{dy}{dx} + y = lnx,\ \ y(1) = 10$$

   (b) Determine whether the given differential equation is homogeneous. If so, solve it by substitution:

$$\left(x + ye^{\frac{y}{x}}\right)dx - xe^{\frac{y}{x}}dy = 0.$$ [4]

3. (a) Let $y_1(x)$ be a solution of the differential equation given in the following. Find the second solution $y_2(x)$, so that $y_1(x)$ and $y_2(x)$ are linearly independent. [5]

$$y'' + 9y = 0,\ y_1(x) = sin3x$$

   (b) Separate t and y variables of the differential equation $\frac{dy}{dt} + 2y = 1$, and solve the initial value problem where $y(0) = \frac{5}{2}$. [5]

BRAC University
Final Examination
Semester: Spring 2013
Course Title: Integral Calculus & Differential Equations (Mathematics II)
Course No.: MAT 120

Full Marks: 50

Date: April 21 , 2013
Time: 3 Hours

Section A (Integral Calculus):

Answer any **THREE** questions:

1. (a) Find the area under the curve $y = \frac{1}{2}x$ over the interval [1,4] with x_k^* as the right end point of each subinterval. [5]

   (b) (i) Evaluate $\int \cos(lnx)\, dx$. [2]
   (ii) Find the exact arc length of the curve $24xy = y^4 + 48$, over the intervals $y = 2$, $y = 4$. [3]

2. (a) Sketch the region enclosed by the curves and find its area: [5]
   $y = 2 + |x - 1|, \qquad y = -\frac{1}{5}x + 7.$

   (b) Evaluate the integrals:
   (i) $\int_1^3 \sqrt{x}\tan^{-1}\sqrt{x}\, dx$ [3]
   (ii) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} sin\theta\sqrt{1 - 4cos^2\theta}\ d\theta$, by using an appropriate formula from geometry. [2]

3. (a) Find the volume of the solid that results when the region enclosed by the given curves $y = \sqrt{25 - x^2}$, $y = 3$, is revolved about the x-axis. [5]

   (b) Evaluate the integrals: [5]
   (i) $\int cos^5x\, dx$
   (ii) $\int t\sqrt{7t^2 + 12}\, dt$

4. (a) (i) Use cylindrical shell to find the volume of the solid generated when the region enclosed by the given curves whose equations are given below is revolved about the y-axis: $y = 2x - 1, y = -2x + 3, x = 2.$ [3]
   (ii) Evaluate $\int sin^3 2xcos^2 2x\, dx.$ [2]

(b) Find the exact area of the surface generated by revolving the curve $y = \sqrt{x} - \frac{1}{3}x^{\frac{3}{2}}$, about x-axis over $1 \leq x \leq 3$. [5]

5. (a) Evaluate the integrals:

   (i) $\int \frac{dx}{(4+x^2)^2}$, by trigonometric substitution; [3]

   (ii) $\int_e^{+\infty} \frac{1}{x ln^3 x}\, dx$, state whether the limit converges or diverges. [2]

   (b) (i) Evaluate $\int_0^1 (1 - \frac{1}{x})^{\frac{1}{3}}\, dx$ in terms of gamma function. [5]

   (ii) Evaluate $\int_0^1 x^2(1 - x^3)^{3/2}\, dx$ in terms of beta function.

Section B (Differential Equations):

<u>Answer any **TWO** questions:</u>

6. (a) Find the general solution of $y' + y tanx = cos^2 x$. Solve the initial value problem when $y(0) = -1$. [5]

   (b) Determine whether the differential equation given below is exact. If it is exact, then solve the following system of equation: [5]

   $$\left(\frac{1}{1+y^2} + cosx - 2xy\right)\frac{dy}{dx} = y(y + sinx), \quad y(0) = 1.$$

7. (a) Show $xdx + (y - 2x)dy = 0$, is a homogeneous differential equation. Solve the equation by using an appropriate substitution. [5]

   (b) Initially 100 ml of radioactive substance was present. After 6 hours the mass has decreased by 3%. If the rate of decay is proportional to the amount of substance present at time t, determine the half-life of the radioactive substance. [5]

8. (a) Separate x and y from $\frac{dy}{dx} = \frac{y^2-1}{x^2-1}$ by substituting the initial values $y(2) = 2$. [5]

   (b) Solve the exact equation: $(x - y^3 + y^2 sinx)dx = (3xy^2 + 2y cosx)dy$. [5]

BRAC UNIVERSITY

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Department of Mathematics and Natural Sciences
Mid-term Examination
Semester: Spring 2016
Course Title: Integral Calculus and Differential Equations
Course No.: MAT120
Sec: 01

Time: 1 hour
Total Marks: 40

Date: Feb 22, 2016

NOTE: Answer any FOUR.

1. (a) Find the following indefinite integrals (any two): [5]

i) $\int x^{1/3}(2-x)^2\,dx$, ii) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$, iii) $\int \frac{1}{4+9x^2}\,dx$.

(b) Find the *area of surface* generated by revolving the the curve $y=\sqrt{x}, 1 \le x \le 4$ about the x-axis. [5]

2. (a) Use *cylindrical shells* to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis. [5]

(b) Find the *exact arc length* of the curve $y = x^{2/3}$, $x \in [0,8]$. [5]

3. (a) Use the definition of *Area Under the Curve* with x_k^* as the right endpoint of each subinterval to find the area under the curve $f(x) = x^2$ over the interval $[0,1]$. [5]

(b) Find the volume of solids that results when the region enclosed by the curves $y = x^2$ and $y = 2x$ is revolved about the x-axis. [5]

4. (a) Sketch and find the of area of the region enclosed by the following curves: [5]

$$y = x^2,\ y = \sqrt{x},\ x = \frac{1}{4},\ x = 1.$$

(b) Evaluate $\int \sin 3x \cos 4x\,dx$. [5]

5. (a) Sketch the region whose *signed area* is represented by the definite integral: [5]

$$\int_{-3}^{3} \sqrt{9-x^2}\,dx,$$

and evaluate the integral using *geometry*.

(b) State the *Fundamental Theorem of Calculus (Part I)*. And evaluate any two of the following: [5]

i) $\int_0^2 t^2(t-3)^3\,dt$, ii) $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\,dx$, iii) $\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,dx$.

BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences
Mid-term Examination
Semester: Summer 2015
Course Title: Integral Calculus and Differential Equations
Course No.: MAT120

Time: 1 hour
Total Marks: 40

Date: June 26, 2015

Answer any FOUR:

1. (a) Find the following indefinite integrals (any two): [5]

   i) $\int x^{1/3}(2-x)^2\,dx$ ii) $\int \sin 3x \cos 4x\,dx$ iii) $\int \frac{1}{4+9x^2}\,dx.$

   (b) Find the *area of surface* generated by revolving the the curve $y=\sqrt{x}, 1 \le x \le 4$ about the x-axis. [5]

2. (a) Use *cylindrical shells* to find the volume of the solid generated when the region R in the first quadrant enclosed between $y=x$ and $y=x^2$ is revolved about the y axis. [5]

   (b) Find the *exact arc length* of the curve $y=3x^{3/2}-1$ over $x \in [0,1]$. [5]

3. (a) Use the definition of *Area Under the Curve* with x_k^* as the right endpoint of each subinterval to find the area under the curve $f(x)=9-x^2$ over the interval $[0,3]$. [5]

   (b) Find the volume of solids that results when the region enclosed by the curves $y=x^2$ and $y=2x$ is revolved about the x-axis. [5]

4. (a) Sketch the region enclosed by the following curves and find its area. [5]

$$y=x^2,\ y=\sqrt{x},\ x=\frac{1}{4},\ x=1$$

   (b) Evaluate $\int e^{2x}\cos x\,dx.$ [5]

5. (a) Sketch the region whose *signed area* is represented by the definite integral: [5]

$$\int_{-3}^{3}\sqrt{9-x^2}\,dx$$

   and evaluate the integral using *geometry*.

   (b) State the *first fundamental theorem of calculus*. And evaluate any two of the following: [5]

   i) $\int_0^2 t^2(t-3)^3\,dt$ ii) $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\,dx$ iii) $\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,dx.$

MNS Department
Midterm Examination
Semester: Spring 2015
Course No: MAT 120 (Section-6)
Course Title: Integral Calculus and Differential Equations

Time: 1 hour **Date: 14 March 2015**
Marks: 40

Answer any <u>four</u> from the following questions:

1. (*i*) Evaluate $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$. 5

   (*ii*) Sketch the region enclosed by the curves $y = x^2$, $y = x + 2$ and find its area. 5

2. (*i*) Evaluate $\int_2^5 e^{2x-1} dx$. 5

   (*ii*) Use x_k^* as the midpoint of each subinterval to find the area under the curve $f(x) = 4 - x^2$ over the interval $[0, 2]$. 5

3. (*i*) Sketch the region whose area is represented by $\int_{x=0}^{1} (3x+1)\,dx$ and evaluate the integral using an appropriate formula from geometry. 5

   (*ii*) Find the volume of the solid generated when the region enclosed by $y = \sqrt{3-x}$, $y = 0$ and $x = -1$ is revolved about the x- axis. 5

4. (*i*) Evaluate $\int_{x=0}^{1} \sqrt{1+x^2}\, dx$. 5

   (*ii*) Use cylindrical shells to find the volume of the solid generated when the region enclosed by the curves $y = x^2, x = 0, x = 2$ is revolved about the y- axis. 5

5. (*i*) Find the arc length of the curve $y = x^{3/2}$ from $x = 1$ to $x = 2$. 5

   (*ii*) Evaluate $\int x \cos x \, dx$. 5

MNS DEPARTMENT

Midterm Examination

Semester: Summer 2014

Course Title: Integral Calculus and Differential Equations

Course No: MAT 120 (Section-1)

Date: 02 July 2013

Time: 1 hour

Marks: 40

Answer any four of the following questions:

1. **(a)** Evaluate $\int \sqrt{x}\,\tan^{-1}\sqrt{x}\,dx$ using integration by parts. [3.5]

   (b) (i) Integrate by trigonometric substitution: $\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx$. [3.5]

   (ii) Integrate: $\int sin^4x\,cos^5x\,dx$. [3]

2. **(a)** Use cylindrical shells to find the volume of a solid that is generated when the region enclosed by the given curves is revolved about the y-axis: $y = 2x - x^2, y = 0$. [5]

   (b) Find the area of the surface generated by revolving the given curve about the y-axis: $x = 9y + 1, 0 \le x \le 2$. [5]

3. **(a)** Evaluate: **(i)** $\int[\ln(e^x) + \ln(e^{-x})]\,dx$, **(ii)** $\int_0^{\ln 5} e^x(3 - 4e^x)dx$. [3+3.5]

   (b) Find the exact arc length of the curve $x = \frac{1}{3}t^3, y = \frac{1}{2}t^2$ over the interval from $t = 0$ to $t = 1$. [3.5]

4. **(a)** Sketch the region enclosed by the curves $x = y^2, y = x - 2$ and also find the area of the bounded region. [4]

   (b) Sketch the function & evaluate the integral using formula from geometry [4]

   $$\int_0^3 \sqrt{6x - x^2}\,dx.$$

   (c) Indicate u substitution: $\int(3 - \tan x)\,sec^2x\,dx = -\int u\,du$ if $u =$ ________ and $du =$ ________ . [2]

5. **(a)** Use signed area theorem with x_k^* as the left end point of each subinterval to find the area under the curve $f(x) = \frac{x}{2}$ over the interval $[1,4]$. [5]

   (b) State whether the limit converges or diverges: $\int_0^{\pi/6} \frac{\cos x}{\sqrt{1-2\sin x}}$ [5]

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BRAC UNIVERSITY

Department of Mathematics and Natural Sciences

Mid-term Examination

Semester: Fall 2014

Course Title : Integral Calculus and Differential Equations (Math II)

Course No.: MAT120

Section: 09

Time : 1 Hour

Total marks : 40

Date : Oct 29, 2014

Answer any FOUR:

1. Find the following indefinite integrals (any two). [5×2]

i) $\int \tan^{-1} x \, dx$ ii) $\int e^{\sqrt{x}} dx$ iii) $\int x^2\sqrt{1+x}dx$

2. a) Use *cylindrical shells* to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis. [5]

b) Find the *area* of the surface generated by revolving the given curve about the x-axis. [5]

$$y = \sqrt{x}, \qquad 4 \leq x \leq 9$$

3. a) Find the *volume* of the solid that results when the region enclosed by the given curves is revolved about the y-axis. [5]

$$y^2 = x, \qquad y = x - 2$$

b) Find the *arc length* of the plane curve over the interval (any one). Leave your answer in exact form. [5]

i) $24xy = y^4 + 48;\ y \in [2,4]$ ii) $x = (1+t)^2, y = (1+t)^3; t \in [0,1]$

4. a) Use the definition of *Area Under the Curve* with x_k^* as the *right* endpoint of each subinterval to find the area under the curve $f(x) = x^2$over the interval $[0, 3]$. [7]

b) Sketch the region whose *signed* area is represented by the definite integral: [3]

$$\int_{-1}^{2} |2x - 3| \, dx$$

and evaluate the integral using an appropriate formula from geometry.

5. a) Sketch the region enclosed by the curves and find its area. [5]

$$y = x^2, \qquad y = \sqrt{x}, \qquad x = \frac{1}{4}, \qquad x = 1$$

b) Use *Fundamental Theorem of Calculus* to obtain the following definite integrals (any two). [2.5×2]

i) $\displaystyle\int_1^3 \frac{x+2}{(x^2+4x+7)} dx$ ii) $\displaystyle\int_1^e x^2 \ln x \, dx$ ii) $\displaystyle\int_{4\pi^2}^{9\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

BRAC UNIVERSITY

MNS Department

Midterm Examination

Semester: Spring 2014

Course No: MAT 120 (Sec 04)

Course Title: Integral Calculus and Differential Equations

(Mathematics II)

Time: 1 hour

Total Marks: 20

Date: 12-03-2014

Answer any four of the following questions:

Q1. Estimate the area of the region enclosed by the curve $f(x) = x^2 + 3$ within the interval $[0, 3]$ by using the formula: [5]

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

where x_k^* is the right endpoint of each subinterval.

Q2. Evaluate the following integrals: [2.5 x2 = 5]

(a) $\int_0^{\pi/8} 2x\, cos2x\, dx$ (b) $\int \frac{1}{(1-3x)^2} dx$

Q3. (a) Find the area of the region which is bounded by $y = 2x + 6$, $y = x^2$, $x = 0$ and $x = 2$. [2.5]

(b) Find the arc length of the curves [2.5]

$$x = cos2t,\ y = sin2t;\quad 0 \le t \le 2\pi.$$

Q4. (a) Find the volume of the solid generated when the region enclosed by the curve $y = x^2$ over the interval [0, 2] is revolved about the line $y = -1$. [2.5]

(b) Use cylindrical shells to find the volume of a solid generated when the region in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis. [2.5]

Q5. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis. [5]

BRAC UNIVERSITY

MNS Department

Midterm Examination

Semester: Fall 2013

Course Title: Integral Calculus and Differential Equations (Mathematics II)

Course No: MAT 120 (Section-7)

Time: 1 Hour

Marks: 20

Date: 31 October, 2013

Answer any <u>four</u> of the following questions:

1. (a) Evaluate: $\int \sin^3 x\cos^2 x \, dx$. 2

   (b) Sketch the region and find the area of the region bounded by the curves $y^2 = 4x$, $y = 2x - 4$. 3

2. Use x_k^* as the **midpoint** of each subinterval to find the area under the curve $f(x) = 4 - \frac{x^2}{4}$ over the interval $[0,3]$. 5

3. (a) Find the area of the surface that is generated by revolving the curve $y = \sqrt{4 - x^2}$ about the x-axis over the interval $-1 \leq x \leq 1$. 2

   (b) Use **cylindrical shells** to find the volume of the solid generated when the region enclosed by the curves $xy = 4, x + y = 5$ is revolved about the **x-axis**. 3

4. (a) Sketch the region and use appropriate formula from **geometry** to evaluate the definite integral: $\int_{-3}^{0}(2 + \sqrt{9 - x^2})dx$. 2

   (b) Find the exact length of the plain curve $y = 3x^{\frac{3}{2}} - 1$ from $x = 0$ to $x = 1$. 3

5. (a) Evaluate $\int \sin^{-1} x \, dx$. 2

   (b) Use trigonometric substitution to evaluate: $\int \frac{\sqrt{x^2-9}}{x} dx$. 3