MAT215

Summer/15

1/ a/ $2\pi i$ period of e^z

b/ Euler's formula: $e^{ix} = \cos x + i\sin x$

for all integers n, De Moivere's formula: $(e^{i\theta})^n = e^{in\theta}$

If we relate the formulas
$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

c/ Cauchy Reiman equ^n: Let,
$f(z) = u(x,y) + i\,v(x,y)$ is analytic,

then,
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$$

1d

d/

e/ $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

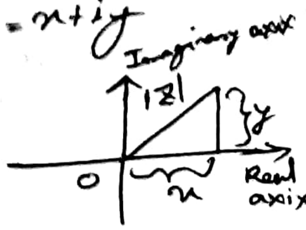
so, $\mathcal{L}\{te^{at}\} = (-1)^1 \frac{d}{ds}\left(\frac{1}{s-a}\right)$
$= -(-1) \times \frac{1}{(s-a)^2}$
$= \frac{1}{(s-a)^2}$

Scanned by CamScanner

Part A

2 a) If z is a complex number where $z = x+iy$

then modulus of z will be $\sqrt{x^2+y^2}$



argument θ is the ~~general~~ principal argument of z where

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) . \text{ Here } -180 < \theta \leq 180$$

b) polar form:

$$z = \sqrt{3} - i$$

$$r = \sqrt{(\sqrt{3})^2 + (1)^2}$$

$$= \sqrt{4} = 2$$

$$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6} + \pi$$

$$= \frac{5}{6}\pi = 150^\circ$$

Polar form so, $2 \angle 150^\circ$

c)

$$z^4 = -1$$

$$|z^4| = 1 \qquad \text{Arg}(z^4) = \tan^{-1}\left(\frac{0}{-1}\right)$$

$$= \pi$$

$$\arg(z^4) = \pi + 2n\pi$$

$$z^4 = 1\{\cos(\pi+2n\pi) + i\sin(\pi+2n\pi)\}$$

$$z = 1^{\frac{1}{4}}\left\{\cos\left(\frac{\pi+2n\pi}{4}\right) + i\sin\left(\frac{\pi+2n\pi}{4}\right)\right\}$$

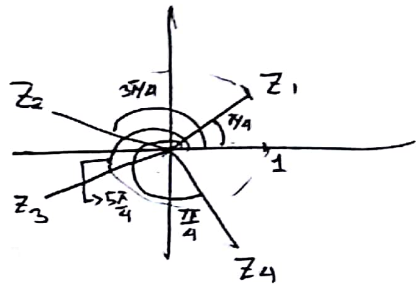
$n = 0, 1, 2, 3$

$n=0, \quad z_1 = 1 \times cis\left(\frac{\pi}{4}\right)$

$n=1, \quad z_2 = 1 \times cis\left(\frac{3\pi}{4}\right)$

$n=2; \quad z_3 = 1 \times cis\left(\frac{5\pi}{4}\right)$

$n=3, \quad z_3 = 1 \times cis\left(\frac{7\pi}{4}\right)$



3)

a) **Logarithm of a complex number**

Let, $e^{w} = z$ Let $w = u + iv$

$\Rightarrow e^{u} \cdot e^{iv} = r e^{i(\theta + 2n\pi)}$

$\Rightarrow e^{u} = r$ and $e^{iv} = e^{i(\theta+2n\pi)}$

$\Rightarrow u = \ln r$ and $v = \theta + 2n\pi$

so, $w = \ln r + i(\theta + 2n\pi)$ [n =

$\Rightarrow \log z = \ln|z| + i \arg(z)$

now, $e^{w} = z$

$\log e^{w} = \log z$

$w = \log z$

b) **Analytic function:** A single valued function $f(z)$ is said to be analytic at a point z_0 if $f(z)$ is differentiable at z_0

Example Let, $f(z) = z^2$

$\Rightarrow f'(z) = 2z$ so $f(z)$ is anlytic everx.

$f(z) = |z^2|$

at ~~as~~ $\cancel{f(z)}$ $z = x + iy$

so, $|z|^2 = x^2 + y^2$

$\cancel{f'(z) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}}$

Now,

$f(z) = u(x, y) + i v(x, y)$

we suppose

$u = x^2$ and $v = y^2$

$\boxed{3b}$

$\frac{\partial u}{\partial x} = 2x$ and $\frac{\partial v}{\partial y} = 2y$

as $\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$

so accoring to Cauchy Reiman Equation or CR equation $f(z)$ can not be analytic

Amanat

3C)

$$f(z) = \begin{cases} \dfrac{z^2+4}{z+2i} & z \neq -2i \\ -4i & z = -2i \end{cases}$$

$$\lim_{z \to -2i} \frac{z^2+4}{z+2i}$$

$$\Rightarrow \lim_{z \to -2i} \frac{z^2 - (2i)^2}{z+2i}$$

$$\Rightarrow \lim_{z \to -2i} \frac{\cancel{(z+2i)}(z-2i)}{\cancel{(z+2i)}}$$

$$\Rightarrow -4i$$

$$f(-2i) = -4i$$

L.H.L = R.HL and both of them exists so it is continous

4) b)

$$\oint_C \frac{e^{2z}}{(z+i)^4} \quad c.s \quad |z|=2$$

here,

$f(z) = e^{2z}$

~~Area kaka to~~

so,

$$\oint \frac{e^{2z}}{(z+i)^{3+1}} = \frac{2\pi i}{3!} f'''(i)$$

$f = e^{2z}$

$f' = 2e^{2z}$

$f'' = 4e^{2z}$

$f'''(z) = 8e^{2z}$

so, $f'''(i) = 8e^{2i}$

$$= \frac{2\pi i}{3!} \times 8e^{2i}$$

$$= \frac{2\pi i}{1\times 2\times 3} 8e^{2i}$$

$$= \frac{8\pi i}{3} e^{2i}$$

4) a) Cauchy Integral formula:

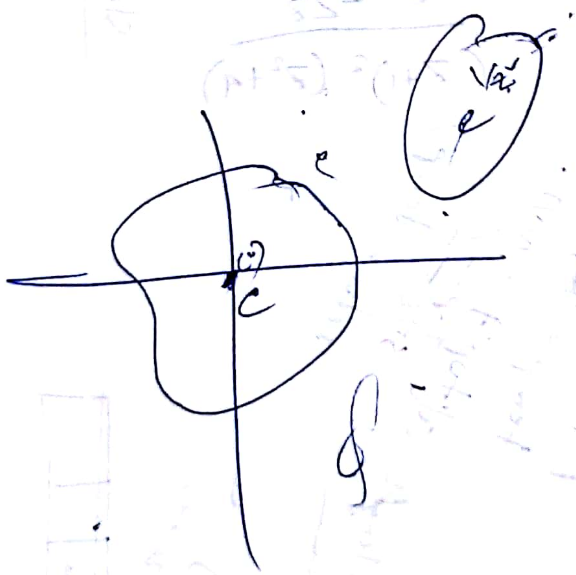
Let $f(z)$ be analytic inside and on a simple closed curve c (counterclockwise), z_0 is a point inside contour c then

$$f(z_0) = \frac{1}{2\pi i} \oint_c \frac{f(z)}{(z-z_0)} dz \quad \text{---} (1)$$

The nth derivative of $f(z)$ at $z = z_0$

$$f^{n}(z_0) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z-z_0)^{n+1}}\, dz$$

4(c)



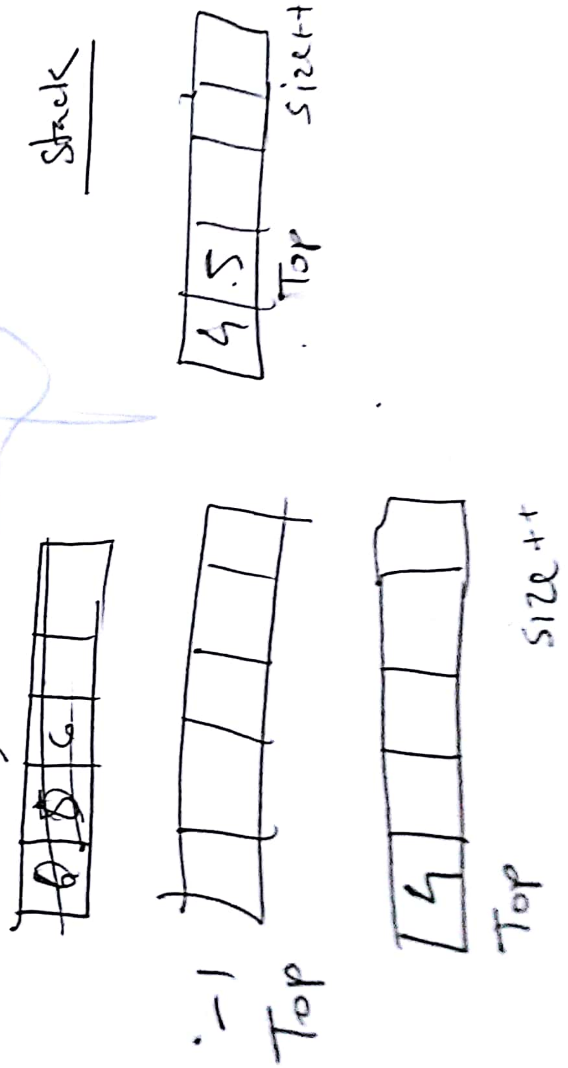
5] a] **Isolated Singular Point:** A singular point z_0 is said to be isolated if there is a neighbourhood $0 < |z - z_0| < \varepsilon$ ~~[illegible]~~ of z_0 throughout which f is analytic.

5] b]

$$\frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$$

pop()
temp = ~~top~~ a[top]
a[top] = null;
top--;
size--;
return temp;

push()
if size >= a.length
Error
top++
a[top] = e
size++;



15 c) $\oint \frac{ze^z}{(z^2-1)}$ circle $|z-i| = 2$

$$= \frac{ze^z}{(z-1)(z+1)}$$

$|z_1 - z_2| = 2$

$f(z)$ has pole of order of 1

$z = \pm 1$

$$f(z) = \frac{ze^z}{(z^2-1)}$$

$a = 1$

$m = 1$

residue at 1,

$$\oint f(z) = 2\pi i \times \lim_{z \to 1} \frac{1}{(m-1)} \frac{d^{m-1}}{dz^{m-1}} \left\{ (z-1)^m \times \frac{ze^z}{(z+1)(z-1)} \right.$$

$$= 2\pi i \lim_{z \to 1} \frac{ze^z}{z+1}$$

$$= 2\pi i \frac{e}{2}$$

$$= e \times \pi i$$

Scanned by CamScanner

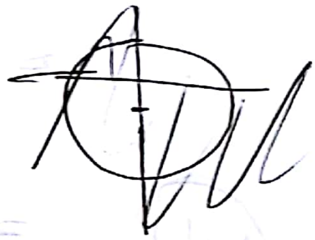
Residude
$z = -1$.

$$\oint f(z) = 2\pi i \lim_{z\to -1} \frac{1}{(1-1)!}\frac{d^0}{dz^0}(z\cancel{+1}) \times \frac{z e^z}{(z\cancel{+1})(z-1)}$$

$$= 2\pi i \lim_{z\to -1} \frac{z e^z}{(z-1)}$$

$$= 2\pi i \; \frac{-1 e^{-1}}{-2}$$

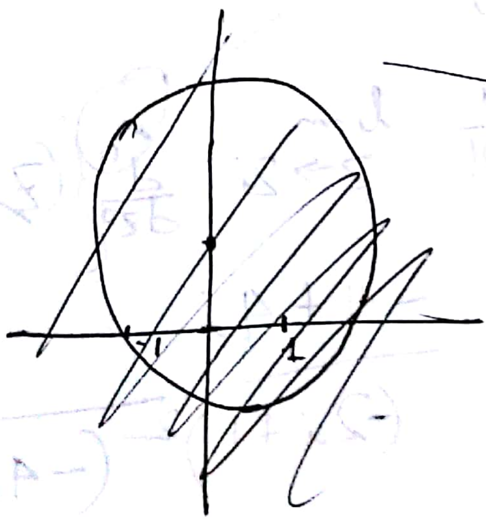
$$= \frac{\pi i}{e}$$



5) b)

$$f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$$

$$= \frac{z^2 - 2z}{(z+1)^2 (z-2i)(z+2i)}$$



Question

Residue at, $z=-1$,

$$= \frac{1}{(2-1)!} \lim_{z\to -1} \frac{d^{2-1}}{dz^{2-1}}\left(\cancel{(z+1)^2} \times \frac{z^2-2z}{\cancel{(z+1)^2}(z^2+4)}\right)$$

$$= \lim_{z\to -1} \frac{d}{dz}\left(\frac{z^2-2z}{z^2+4}\right)$$

$$= \lim_{z\to -1} \frac{(z^2+4)(2z-2)-(z^2-2z)(2z)}{(z^2+4)^2}$$

Wtd

$$= \frac{5\times(-4) - 3\times(-2)}{(5)^2}$$

$$= \frac{-20+12}{25}$$

$$= -\frac{8}{25}$$

$z = 2i$,

$$= \frac{1}{0!} \lim_{z\to 2i} \frac{d^0}{dz^0}\left\{\cancel{(z+2i)} \times \frac{z^2-2z}{(z+1)^2\cancel{(z+2i)}(z-2i)}\right\}$$

$$= \frac{-4+4i}{(2i+1)^2(-4i)}$$

$z = +2i$,

$$= \frac{1}{0!} \lim_{z \to -2i} \frac{d^0}{dz^0} \left\{ \cancel{(z - 2i)} \times \frac{z^2 - 2z}{(z+1)^2 \cancel{(z-2i)} (z+2i)} \right\}$$

$$= \frac{-4 - 4i}{(2i + 1)^2 \times (+4i)}$$

$$L^{-1}\left\{\frac{2s+5}{s^2+6s+34}\right\}$$

$$L^{-1}\ \frac{2s+5}{s^2+2.3\times s+3^2+25}$$

$$L^{-1}\left\{\frac{2s+5}{(s+3)^2+5^2}\right\}$$

$$= L^{-1}\left\{\frac{2(s+3)-1}{(s+3)^2+5^2}\right\}$$

$$2L^{-1}\left(\frac{s+3}{(s+3)^2+5^2}\right) - L^{-1}\left(\frac{1}{5}\times\frac{5}{(s+3)^2+5^2}\right)$$

$$= 2\times e^{-3t}\cos 5t - \frac{1}{5}e^{-3t}\sin 5t$$

$y'' + y = t$

$y(0) = 1$

$y'(0) = -2$

$L\{y(s)\} +$

$s^2 Y(s) - s y(0) - y'(0) + s Y(s) + y(0) = \frac{1}{s^2}$

$s^2 Y(s) + s - 2 + s Y(s) + 1 = \frac{1}{s^2}$

$\Rightarrow \quad Y(s)(s^2 + s) = \frac{1}{s^2} + 1 - s$

$\Rightarrow \quad Y(s) = \frac{1 + s^2 - s^3}{s^2(s^2+s)}$

$\frac{1}{s}$

$= (-1)^n \frac{d}{ds^n}\left(\frac{1}{s}\right)$

$= -\left(-\frac{1}{s^2}\right)$

$L\{t^2 e^{at}\}$

$= (-1)^2 \times \frac{d^2}{ds^2}\left\{\frac{1}{(s-a)}\right\} \quad L\{e^{at}\} = \frac{1}{s-a}$

$=$

$s^2 Y(s)$

Scanned by CamScanner

$y'' + y = t \qquad y(0) = 1 \qquad y'(0) = -2$

$s^2 Y(s) - s y(0) - y'(0) + Y(s) = \frac{1}{s^2}$

$-s + 2 + Y(s)(1+s^2) = \frac{1}{s^2}$

$(1+s^2) Y_s = \frac{1}{s^2} - 2 - s$

$Y_s = \frac{1}{s^2(s^2+1)} - \frac{2}{(1+s^2)} - \frac{s}{(1+s^2)}$

$L^{-1}\{Y(s)\} = L^{-1}\{ \quad \}$

$L^{-1}\left\{\frac{1}{s^2(s^2+1)}\right\}$

$(B+C) = 0 \qquad \frac{1}{s^2(s^2+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{s^2+1}$

$1 = A(s^2+1) + Bs(s^2+1) + (Cs+D)s^2$

if $s = 0 \Rightarrow 1 = A$

if $s = 1 \Rightarrow 1 = 2A + 2B + C + D$

$\Rightarrow 1 = 2 + 2B + C + D$

Scanned by CamScanner

12/ b)

i) $\mathcal{L}\{e^{-3t}(3\sin 4t - 4\cos 4t)\}$

$= \mathcal{L}\{e^{-3t} \times 3\sin 4t\} - \mathcal{L}\{4 e^{-3t}\cos 4t\}$

$= 3 \times \dfrac{4}{(s+3)+4^2} + 4 \times \dfrac{(s+3)}{(s+3)^2+4^2}$

ii) $\mathcal{L}\{4t^{-1/2} - e^{-t}\}$

$4 \times \dfrac{\Gamma(-\frac{1}{2})}{s^{n+1}} - \dfrac{1}{(s+1)}$

$4t^5$

$\dfrac{5!}{s^{n+1}} = \dfrac{\sqrt{5}}{s^{n+1}}$

Scanned by CamScanner

iii)

$$L\left\{3t \sin 2t - t e^{-2t}\right\}$$

$$= 3(-1)^1 \times \frac{d}{ds} \times \left(\frac{2}{s+4}\right) - (-1)^1 \times \frac{d}{ds}\left(\frac{1}{s+2}\right)$$

$$= -6 \times (-1) \times \frac{1}{(s+4)^2} - (-1)(-1)\frac{1}{(s+2)^2}$$

$$= \frac{6}{(s+4)^2} - \frac{1}{(s+2)^2}$$

$$u(x,y) = e^{-3x} \cos 3y$$

$$\frac{\partial u}{\partial x} =$$

Scanned by CamScanner

$z^4 = 16i$

$z^4 = 16 \text{ cis } \frac{\pi}{2}$

$= 16 \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$z = 2 \text{ cis}\left(\dfrac{\frac{\pi}{2} + 2n\pi}{4}\right)$

$z^4 = 0 + 16i$

$|z^4| = \sqrt{0^2 + 16^2}$

$= 16$

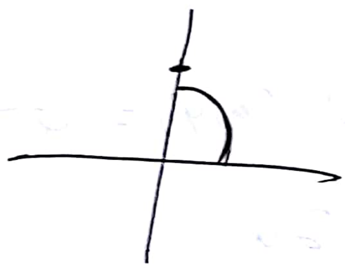
$\arg(z) =$

$\tan^{-1}\left(\frac{16}{0}\right)$

$\tan^{-1} \infty$

$\theta = \frac{\pi}{2}$

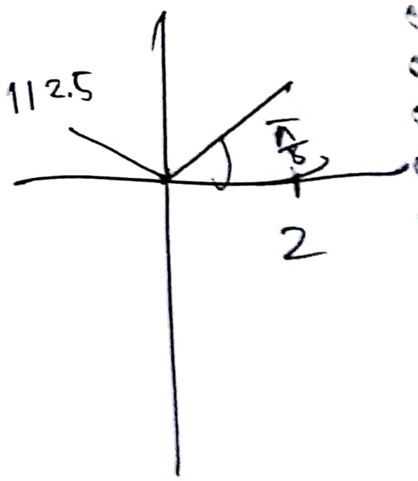
for $n=0$



$z = 2 \text{ cis } \dfrac{\frac{\pi}{2}}{4} = 2 \text{ cis } \frac{\pi}{8}$

$n=1 = 2 \text{ cis}\left(\dfrac{\frac{\pi}{2} + 2\pi}{4}\right)$

$= 2 \text{ cis } \frac{5}{8}\pi$



4)

c)

$$\frac{1}{(z-1)^2(z-3)}$$

$$\frac{1}{(z-1)^2 \times (z-3)} = \frac{A}{(z-1)} + \frac{B}{(z-1)^2} + \frac{C}{z-3}$$

$$\Rightarrow \quad 1 = A(z-1)(z-3) + B(z-3) + C(z-1)^2$$

when $z=1$, $1 = B(-2)$

$$\Rightarrow B = -\frac{1}{2}$$

ii)

$$\frac{1}{(z-1)^2 (z-3)}$$

$$= \frac{1}{(z-3+2)^2 (z-3)}$$

$$= \frac{1}{(u+2)^2 \times u}$$

$$= \frac{1}{u \times 4\left(1+\frac{u}{2}\right)^2}$$

From question.

$|u| < 2$

$\Rightarrow \frac{|u|}{2} < 1$

so. $\frac{1}{4u} \times \left(1 - 2\frac{u}{2} + 3\left(\frac{u}{2}\right)^2 - 4\left(\frac{u}{2}\right)^3 + \cdots\right)$

Formula

as $(1+z)^{-2} = 1 - 2z^2 + 3z^2 - 4z^2 \cdots$

1)

$f(z) \frac{1}{(z-1)^2 (z-3)}$

$0 < |z-1| < 2$

$z - 1 = u$

$\frac{1}{u^2 (z-1-2)}$

$\Rightarrow \frac{1}{u^2 (u-2)}$

$= \frac{1}{-u^2 \times 2\left(1 - \frac{u}{2}\right)}$

$= -\frac{1}{u^2 \times 2} \times \left(1 + \frac{u}{2} + \left(\frac{u}{2}\right)^2 + \left(\frac{u}{2}\right)^3 \cdots\right)$

Ans

6 a)

$\sin z = \sin(x+iy)$

$= \sin x \cos(iy) + \cos x \cdot \sin(iy)$

$= \sin x \cosh y + \cos x \, i \sinh y$

$\cos z = \cos(x+iy)$

$= \cos x \cos iy - \sin x \cdot \sin iy$

$= \cos x \cosh y - i \sin x \cdot \sinh y$.

Q. 4

$u = xe^{x}\cos 2y + ye^{x}\sin 2y$

$$\frac{\partial u}{\partial x} = \cos 2y \frac{\partial (xe^{x})}{\partial x} + y\sin y \frac{\partial (e^{x})}{\partial x}$$

$$= \cos 2y \times \left\{x e^{x} - \int 1.e^{x}\right\} + y\sin y\, e^{x}$$

$$= \cos y \left\{xe^{x} - e^{x}\right\} + y\sin y\, e^{x}$$

$$\frac{\partial^2 u}{\partial x^2} = \cos y \left\{\left[xe^{x} - \int 1.e^{x}\right] - e^{x}\right\} + y\sin y\, e^{x}$$

$$= \cos y \left[xe^{x} - 2e^{x}\right] + y\sin y\, e^{x}$$

$u = xe^{x}\cos 2y + ye^{x}\sin 2y$

$$\frac{\partial u}{\partial y} = xe^{x}\left(-\frac{\sin 2y}{2}\right) + e^{x}\left\{y\frac{\cos 2y}{2} + \int 1.\frac{\cos 2y}{2}\right\}$$

$$= xe^{x}\left(-\frac{\sin 2y}{2}\right) + e^{x}\left\{y\frac{\cos 2y}{2} - \frac{\sin 2y}{4}\right\}$$

$$\frac{\partial^2 u}{\partial y^2} =$$

$$\frac{\partial^2 u}{\partial y^2} = x e^x \left(-\frac{\cos 2y}{2} \right) + e^x \left\{ \left[y \frac{(-\sin 2y)}{4} - \int 1 \cdot \frac{-\sin 2y}{4} \right] - \frac{\cos 2y}{4} \right\}$$

$$= -\left\{ x e^x \frac{\cos 2y}{2} - e^x \frac{y \sin 2y}{4} + \frac{\cos 2y}{8} - \frac{\cos 2y}{4} \right.$$

Q1 b)

$$u = xe^x\cos 2y + ye^x\sin 2y$$

$$\frac{\partial u}{\partial x} = \cos 2y\,(xe^x + e^x) + y\sin 2y\, e^x$$

$$\frac{\partial^2 u}{\partial x^2} = \cos 2y\left\{[xe^x + e^x] + e^x\right\} + y\sin 2y\, e^x$$

$$= \cos 2y\left\{xe^x + 2e^x\right\} + y\sin 2y\, e^x$$

$$u = xe^x\cos 2y + ye^x\sin 2y$$

$$\frac{\partial u}{\partial y} = -2xe^x\sin 2y + e^x\{2y\cos 2y + \sin 2y\}$$

$$\frac{\partial^2 u}{\partial y^2} = -4xe^x\cos 2y\ \cancel{+ e^x\sin 2y}$$

$$\cancel{+ e^x\cos 2y}$$

$$+ e^x\ 2$$

$\underline{9}$ $\underline{d}$

$(2\bar{z} - z)$

$z = 2t + i(4t-1)$

$\bar{z} = 2t - i(4t-1)$

$dz = 2 + 4i$

$$\int_c \left[2\left\{ 2t - i(4t-1) \right\} - 2t - i(4t-1) \right] (2+4i)$$

$$\Rightarrow \int_c \left(\underline{4t} - \underline{8it} + 2i - \underline{2t} - \underline{4it} + i \right)(2+4i)$$

$$\Rightarrow \int_c \left(2t - 12it + 3i \right)(2+4i)$$

$$= \int_c \left\{ \underline{4t} - \underline{24it} + 6i + \underline{8it} + \underline{48t} - 12 \right\}$$

$$= \int_1^3 \left\{ 52t - 16it + 6i - 12 \right\}$$

$$= 52\left[\frac{t^2}{2}\right]_1^3 - 16\left[\frac{t^2}{2}\right]_1^3 + 6.4\left[t\right]_1^3 - 12\left[t\right]_1^3$$

Spring 15

7) b)

i) $$L\{e^{-3t}(3\sin 4t - 4\cos 4t)\}$$

$$= 3\times\frac{4}{(s+3)^2-4^2} - 4\times\frac{s+3}{(s+3)^2-4^2}$$

ii) $$L\{4t^3 - e^{-t}\}$$

$$= 4\,\frac{3!}{s^{3+1}} - \frac{1}{(s+1)}$$

iii) $$L\{7\sin 2t - 3\cos 2t\}$$

$$= 7\times\frac{2}{s^2-2^2} - 3\,\frac{s}{s^2-2^2}$$

$y'(0) = 2$

$y(0) = 1$

9)

b)

$$L\{ty'' + (1-2t)y' - 2y\} = 0$$

$$(-1)^1 \times \frac{d}{ds}\{s^2 Y(s) - s\,y(0) - y'(0)\}$$

$$+ L\{y'\} - L\{2t \times y'\} - 2Y(s) = 0$$

$$-2s\,Y'(s)$$

$$e^{\int P(x)\,dx}$$

$$\left(\text{Ans} \times (y)\right) = \int \text{Ans} \times \text{RHS}$$

Scanned by CamScanner

Q1 a)

$$\frac{2s^2-4}{(s-1)(s-2)(s-3)}$$

$$\frac{1}{(s-1)(s-2)(s-3)} = \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{C}{(s-3)}$$

$z^2 +$

Scanned by CamScanner

Summer 14

11 a)
$1 < |z - (0+2i)| < 2$

b)

$|x+iy-2| \leq |x+iy+2|$ $\boxed{\text{check}}$

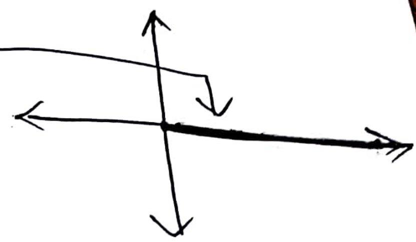
$\Rightarrow \sqrt{(x-2)^2+y^2} \leq \sqrt{(x+2)^2+y^2}$

$\Rightarrow (x-2)^2+\cancel{y^2} \leq (x+2)^2+\cancel{y^2}$

$\Rightarrow x^2-4x+\cancel{4} \leq x^2+4x+\cancel{4}$

$\Rightarrow 8x \geq 0$

$\Rightarrow x \geq 0$



11 c)

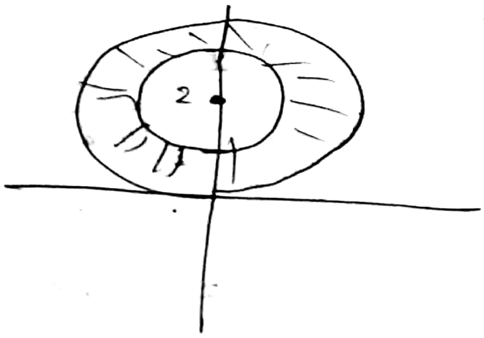
$$\lim_{z\to 0} \frac{z-\sin z}{z^3}$$

$$= \frac{1-\cos z}{3z^2}$$

$$= \frac{-\sin z}{6z} = \frac{-\cos z}{6}$$

$1 < |z - 2i| < 2$

$1 < |z - (0+2i)| < 2$

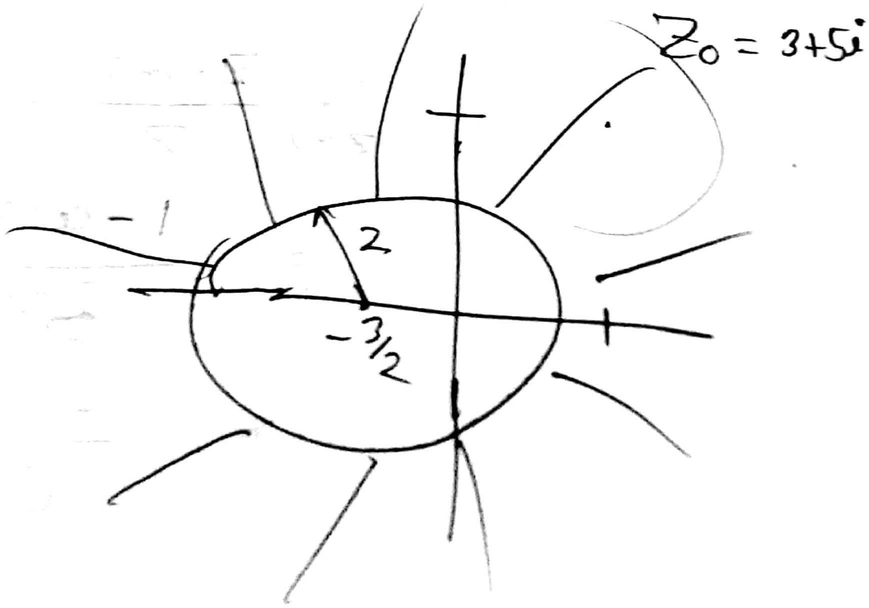


$|2z+3| > 4$

$2\,|z + \frac{3}{2}| > 4$

$|z + \frac{3}{2}| > 2$

$|z - (-\frac{3}{2} + 0i)| > 2$



11 c)

$$\lim_{z \to 0} \frac{z - \sin z}{z^3}$$

$$= \lim_{z \to 0} \frac{1 - \cos z}{3z^2}$$

$$= \lim_{z \to 0} \frac{-\sin \theta}{6z}$$

$$= \lim_{z \to 0} \frac{\cos \theta}{6}$$

$$= \frac{1}{6}$$

1(c)

Sam 14

$|z-2| + |z+2| = 6$

11 b)

$|x+iy-1| + |x+iy+1| \leq 4$

$\Rightarrow \sqrt{(x-1)^2+y^2} + \sqrt{(x+1)^2+y^2} \leq 4$

$\Rightarrow (x+1)^2 + \cancel{y^2} \leq (x-1)^2 + \cancel{y^2} + 2\sqrt{(x-1)^2+y^2}\,4 + 16$

[Sr]

$\Rightarrow 4x-16 \leq +8\sqrt{(x-1)^2+y^2}$

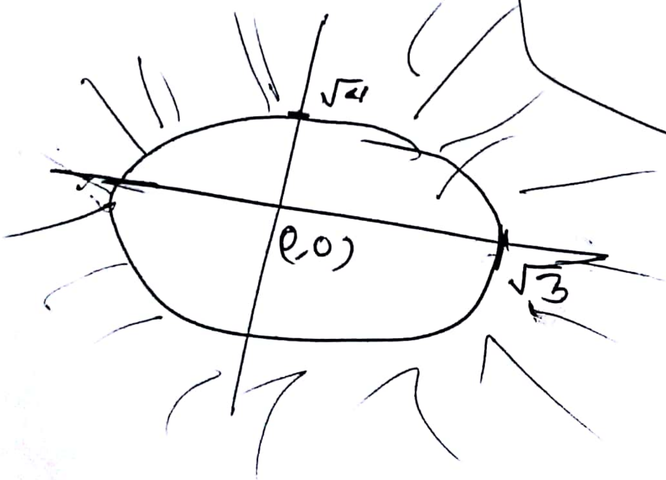
$\Rightarrow \{(x-4)\}^2 \leq \{2\sqrt{(x-1)^2+y^2}\}^2$

$\Rightarrow x^2-8x+16 \leq 4(x-1)^2+4y^2$

$\Rightarrow x^2-\cancel{8x}+16 \leq 4x^2-\cancel{8x}+4+4y^2$

$\Rightarrow 12 \leq 3x^2+4y^2$

$\Rightarrow \boxed{1 \leq \frac{x^2}{(\sqrt{4})^2} + \frac{y^2}{(\sqrt{3})^2}}$



|c|

$$\lim_{z\to 0} (\cos z)^{\frac{1}{z^2}}$$

$$z = (16i)^{\frac{1}{4}}$$

$$z^4 = (16i)$$

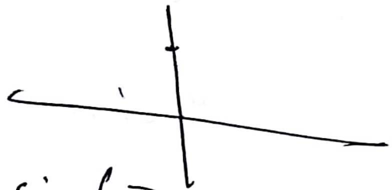
$$|z^4| = 16$$

$$\tan^{-1} \left(\frac{16}{0}\right)$$

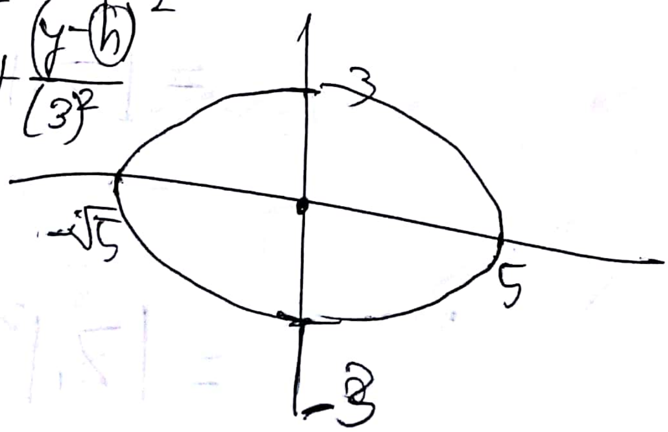
$$= \frac{\pi}{2} + 2k\pi \qquad \cancel{\theta} \neq \infty$$

$$z^4 = 16\left\{\cos\left(\frac{\pi}{2}+2k\pi\right) + i\sin\left(\frac{\pi}{2}+2k\pi\right)\right\}$$

$$z = 2\left\{\cos\left(\frac{\frac{\pi}{2}+2k\pi}{4}\right) + i\sin\left(\frac{\frac{\pi}{2}+2k\pi}{4}\right)\right.$$



$$\frac{(x-h)^2}{(\sqrt{5})^2} + \frac{(y-k)^2}{(3)^2}$$



$|z|^2 = z\cdot\bar{z}$

$z = z_1 + z_2$

$\bar{z} = \overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2$

$$|z_1+z_2|^2 = (z_1+z)\,(\overline{z_1+z_2})$$
$$= (z_1+z_2)\,(\bar{z}_1+\bar{z}_2)$$

$$\cancel{2\mathrm{Re}(z_1 z_2) \leq |z_1 z_2|} = |z_1|^2$$
$$+ z_1\bar{z}_2$$
$$+ z_2\bar{z}_1 + |z_2|^2$$

$$= |z_1|^2 + z_1\bar{z}_2 + \overline{z_1\bar{z}_2} + |z_2|^2$$

$$= |z_1|^2 + 2\,\mathrm{Real}(z_1\bar{z}_2) + |z_2|^2$$

$$2\,\mathrm{Real}(z_1\bar{z}_2) \leq 2|z_1 z_2|$$

$$|z_1|^2 + |z_2|^2 + 2\,\mathrm{Real}|z_1\bar{z}_2| \leq |z_1|^2 + |z_2|^2 + 2|z_1 z_2|$$

$$|z_1 + z_2|^2 \leq \{|z_1| + |z_2|\}^2$$

$|z_1 z_2|$

$= |z_1||z_2|$

$$\boxed{|z_1 - z_2| \leq |z_1| + |z_2|}$$

$$|z_1| = \left|\cancel{(z_1 - z_2)} + z_2\right| \leq |z_1 - z_2| + |z_2|$$

$$\Rightarrow |z_1| - |z_1| \leq |z_1 - z_2|$$

$\Rightarrow$

3) b) $f(z) = \dfrac{z}{-z^2+3z-2}$

$= \boxed{\dfrac{z}{(z-2)(z-1)}}$

$\dfrac{z}{(z-2)}$

i) $1 < |z| < 2$

$|z| < 2$

$\frac{z}{2} < 1$

$\dfrac{z}{2\left(\frac{z}{2}-1\right)(z-1)}$

$\dfrac{1}{(1-z)} = 1 + z + z^2 + z^3 \ldots$

$z < 1$

$z \geq 1$

$1 \geq \frac{1}{z}$

$\Rightarrow \dfrac{z}{-2\left(1-\frac{z}{2}\right)(z-1)}$

$\dfrac{z}{2\left(1-\frac{z}{2}\right) z\left(1-\frac{1}{z}\right)}$

$\dfrac{1}{(1+z)} = 1 - z + z^2 - z^3 \ldots$

$|z| < 1$

$=$

$$\frac{z}{(z-2)(z-1)}$$

$$\frac{z}{(z-2)(z-1)} = \frac{A}{z-2} + \frac{B}{z-1}$$

$$z = A(z-1) + B(z-2)$$

$z=1$

$1 = -B$

$B = -1$

1) $1 \geq z \geq 2$

$z=2,$

$2 = A$

$1 < |z| \leq 2$

$\frac{|z|}{2} < 1$ $\quad z \geq 1$

$$= \frac{2}{z-2} - \frac{1}{z-1}$$

$$= \frac{2}{\left(\frac{z}{2}-1\right)} - \frac{1}{z}$$

Scanned by CamScanner

$$\frac{2}{z-2} - \frac{1}{z-1}$$

$$\frac{z}{-\cancel{2}\left(1-\frac{z}{2}\right)} - \frac{1}{z\left(1-\frac{1}{z}\right)}$$

$$-1 \times \left\{ 1+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3+\cdots \right\}$$

$$-\frac{1}{z} \times \left\{ 1+\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3 \cdots \right\}$$

$$|z|>2 \Rightarrow |z-1|>1$$

$$\frac{2}{z-2} - \frac{1}{z-1}$$

$$\frac{2}{(z-1+1)} -$$

$$\frac{z}{(z-2)(z-1)}$$

$$\boxed{z>2}$$

$$1>\frac{2}{z}$$

$$u=z-1$$

$$\frac{z}{(z-1-1)(z-1)}$$

$$\boxed{\begin{array}{l} z>2 \\ z-1>1 \\ u>1 \end{array}}$$

$$\frac{z}{(u-1)u}$$

$$\Rightarrow 1>\frac{1}{u}$$

$$\Rightarrow \frac{1}{u}<1$$

$$\frac{z}{u}\times\frac{1}{u\left(1-\frac{1}{u}\right)}$$

$$=\frac{z}{u^2}\times\frac{1}{\left(1-\frac{1}{u}\right)}$$

$$=\frac{z}{u^2}\times\left\{1+\left(\frac{1}{u}\right)^2+\left(\frac{1}{u}\right)^3\ldots\right.$$

$$\boxed{\text{where } u=z-1}$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sinh y = \frac{e^{y} - e^{-y}}{2}$$

$$\cosh y = \frac{e^{y} + e^{-y}}{2}$$

$$\sin(iy) = i\sinh y$$

$$\cos iy = \cosh y$$

$\cosh z = \frac{1}{2}$

$\frac{e^{z} + e^{-z}}{2} = \frac{1}{2}$

$e^{y} + e^{-y} = 1$

$(e^{y})^2 + 1 = (e^{y})$

$(e^{y})^2 - (e^{y}) + 1 = 0$

$e^{z} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4.1.1}}{2.1}$

$= \frac{1+\sqrt{3}\,i}{2}, \quad \frac{1-\sqrt{3}\,i}{2}$

$\boxed{e^{z} = e^{x+iy} \quad e^{z} = e^{x}.e^{iy}}$

$e^{z} = \frac{1+\sqrt{3}i}{2}$

$e^{x}.e^{iy} = \frac{1}{2} + \frac{\sqrt{3}}{2} i \qquad re^{i\theta}$

$= 1 r^{i(\frac{\pi}{3}+2n\pi)}$

Scanned by CamScanner

$e^{x} = 1$ $\quad y = \frac{\pi}{3} + 2\pi$

$x = 0$

$z = 0 + i\left(\frac{\pi}{3} + 2\pi\right)$

$$u = e^{-x}\left(x \sin y - y \cos y\right)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x} = e^{-x} x \sin y - y \cos y e^{-x}$$

$$= \sin y \left(e^{-x} - x e^{-x}\right) + y \cos y e$$

$$\frac{\partial^2 u}{\partial x^2} = \sin y\left(-e^{-x} - \left(x(-e^{-x}) + e^{-x}.1\right)\right)$$

$$= \sin y\left(-e^{-x} + x e^{-x} - e^{-x}\right)$$

$$= -2e^{-x} \sin y + x e^{-x} . \sin$$

$$\frac{\partial u}{\partial y} = e^{-x} \cdot x \cos y - e^{-x}\left(y(-\sin y) + \cos y\right)$$

$$\frac{\partial^2 u}{\partial y^2} = -e^{-x} x \sin y - e^{-x}\left(\left(-y \cos y - \sin y\right) + \cos y\right)$$

$$=$$

$$u = e^{-x}\left(x \sin y - y \cos y\right)$$

$$\frac{\partial u}{\partial y} = e^{-x}\left(x \cos y - \left(y(-\sin y) + \cos y\right)\right)$$

$$\frac{\partial u}{\partial y^{2}} = e^{-x}\left(-x \sin y + y \sin y + \cos y\right)$$

Scanned by CamScanner

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = \sin y \, e^{-x} - \sin y \, x e^{-x} + y\cos y \, e^{-x}$$

LIATE

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = -e^{-x} x\cos y - e^{-x} y\sin y + e^{-x}\cos y$$

$$V = e^{-x}(-\cos y) + x e^{-x}(\cos y) + e^{-x}\left(y(-\sin y) - \int(-\sin y)\right)$$

$$\downarrow$$

$$e^{-x}\left(y(-\sin x) + \cos y\right) + f(x)$$

$$\frac{\partial v}{\partial x} = (+\cos y) \times e^{-x} + \left(x(-e^{-x}) + e^{-x}\right)\cos y$$

$$- e^{-y}(-y\sin x + \cos y) + f'(x)$$

$$- xe^{-x}\cos y + e^{-x}\cos y - e^{-x} y\sin y + f'(x) = - x e^{-x}\cos y - e^{-x} y\sin y + e^{-x}\cos y$$

$$f'(x) = 0$$

$$f(x) = C$$

$$L^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$$

$$= L^{-1}\left\{\frac{4s+12}{(s+4)^2}\right\}$$

$$= L^{-1}\left\{\frac{4(s+4)-4}{(s+4)^2}\right\}$$

$$= \frac{4(s+4)}{(s+4)^2} - \frac{4}{(s+4)^2}$$

$$= L^{-1}\left\{\frac{4}{(s+4)}\right\} - L^{-1}\left\{\frac{4}{(s+4)^2}\right\}$$

$$= 4e^{-4t} + L^{-1}\left\{4\times\frac{1!}{(s+4)^{1+1}}\right\}$$

$t e^{4}$ at

$$\frac{1}{s-(-4)}$$

$$4e^{-4t} + 4\times t\,e^{4t}$$

Scanned by CamScanner

$L\{1\} = \frac{1}{s}$

$L\{e^{at}\sin kt\} = \frac{k}{(s-a)^2+k^2}$

$L\{\sin kt\} = \frac{\cancel{k}}{s^2+\cancel{k}^2}$

$L\{\sinh at\} = \frac{a}{s^2-a^2}$

$L\{\cos at\} = \frac{s}{s^2+a^2}$

$L\{\cosh at\} = \frac{s}{s^2-a^2}$

$L\{e^{at}.1\} = \frac{1}{(s-a)}$

$L\{t^n 1\} = (-1)^n \frac{d^n}{ds^n}\left(\frac{1}{s}\right)$

$L\{t.1\} = (-1)^1 \frac{d}{ds}\frac{1}{s}$

$= \frac{1}{s^2}$

s^{-1}

$= -1 s^{-2}$

$$L^{-1}\left\{\frac{1}{(s+5)^3}\right\}$$

$$L\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\frac{1}{2!} \times L^{-1}\left\{\frac{2!}{(s+5)^{2+1}}\right\}$$

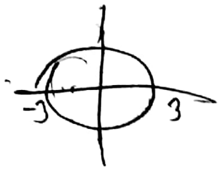
$$\frac{1}{2!} \times e^{-5t} \cdot t^2$$

$$L\{t^{1/2}\} = \frac{\sqrt{\left(\frac{1}{2}+1\right)}}{s^{\frac{1}{2}+1}}$$

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n}(f(s))$$

$$f^{n}(z) = \frac{n!\, f(z)}{2\pi i (z-z_0)^{n+1}} \oint$$

$$\oint \frac{e^{2z}}{(z+1)^4} dz \qquad |z| = 3$$



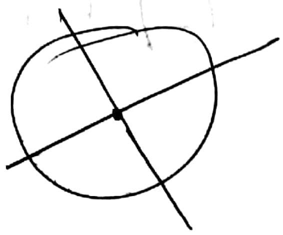
$$f(z) = e^{2z}$$

$$\oint \frac{e^{2z}}{(z+1)^{3+1}} \qquad z_0 = -1$$

$$= \frac{2\pi i}{3!} f'''(-1)$$

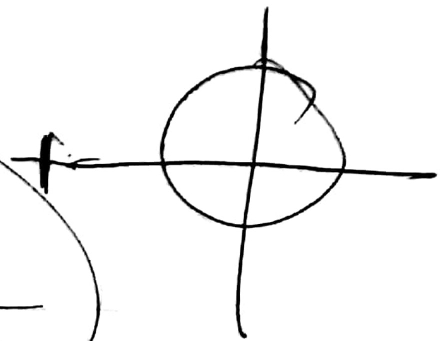
$$|z-0| = 3$$

$$f^{0}(z) = e^{2z}$$



$$\oint \frac{z^2}{2z^2+5z+2}$$

$$\frac{z^2}{\left(z+\frac{1}{2}\right)^2(z+2)}$$



Residue at $z_0 = -2$ (a) $\qquad z=-2,\ z=-\frac{1}{2}$

$$\lim_{z\to -2} \frac{1}{(m-1)!} \frac{d^{(m-1)}}{dz^{m-1}}\left[(z+2)\times\frac{z^2}{\left(z+\frac{1}{2}\right)^2(z+2)}\right]$$

$z - z_0$

$$\lim_{z\to a} \frac{1}{(m-1)!} \times \frac{d^{(m-1)}}{dz^{(m-1)}}\left[(z-a)\times\frac{z^2}{\left(z+\frac{1}{2}\right)^2(z+}\right.$$

$$\int_0^\infty \frac{dx}{1+x^2}$$

$i^2 = -1$

$$\int_0^\infty \frac{1}{x^2 - i^2}\, dx$$

$$\int_0^\infty \frac{1}{(x-i)(x+i)}$$

Ans $2\pi i$ $\left\{ \text{Sum of all Reside} \right\}$

Scanned by CamScanner

$$t y''(t) + (1-2t)\, y'(t) - 2y(t) = 0$$

$$(-1)\frac{d}{ds}\left\{ s^2 Y(s) - s\,y(0) - y'(0) \right\}$$

$$+ \left(s \cdot Y(s) - y(0) \right) - 2(-1)\frac{d}{ds}\left\{ sY(s) - y(0) \right\}$$

$$-2Y(s) = 0$$

$$\Rightarrow (-1)\left\{ s^2 Y'(s) + Y(s)\times 2s - 1 \right\}$$

$$+ s\,Y(s) - 1 + 2\left\{ s\,Y'(s) + Y(s) - 1 \right\}$$

$$-2Y(s) = 0$$

$$\Rightarrow \cancel{-Y'(s)\,s^2 + Y'(s)\times 2s}$$

$$Y'(s)\left\{ -s^2 + 2s \right\} + Y(s)\left(s - 2s + 1 - 2\right)$$

$$-2 =$$

$$Y'(s) + \frac{-(s+1)}{-s^2+2s} Y(s) = \frac{2}{-s^2+2s}$$

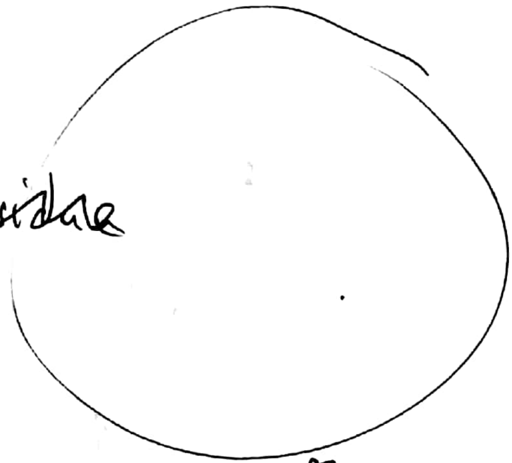
$$IF = e^{\int \frac{(s+1)}{-s^2+2s}}$$

$$\frac{d}{ds}\left(IF \times Y(s)\right) = \int \frac{2}{-s^2+2s} \times IF$$

Scanned by CamScanner

$$\oint \frac{e^{3z}}{z-\pi i}$$

at πi inside



$$\cancel{2\pi i \times} \int e^{\cancel{3(\pi i)}}$$

$-3.1416\, i$

$$f(z) = e^{3z}$$

$$z_0 = \pi i$$

Scanned by CamScanner

4| b|

$$\int_c (x^2 - iy^2)$$

straight joining $(1,1)$ $(1,8)$

$x = 1$

$\cancel{\frac{x-1}{1-1}} = \cancel{\frac{y-1}{1-8}}$

$dx = 0$

$x + iy = z$

$dx + idy = dz$

$idy = dz$

$$\int_1^8 (1 - iy^2)\, \cancel{i} dy$$

$$= \int_1^8 (i + y^2)\, dy$$

$$iy + \frac{y^3}{3}$$