MAT120

Assignment-04 Set-24

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18301297

Section : 03

1830129X

01

Date: / /

□Sat □Sun □Mon □Tue □wed □Thu □Fri

Theme:

1. $\int_0^2 \int_x^2 2y^2 \sin(xy)\, dy\, dx.$

$0 \leq x \leq 2$ so, $x \leq y \leq 2$.

so, $x \leq y \leq 2$.

$\Rightarrow 0 \leq y \leq 2$

Now, $x \leq y \leq 2$

$\Rightarrow 0 \leq x \leq y$

$\rightarrow x \leq y \leq 2$

$0 \leq x \leq y.$

Now, $\int_0^2 \int_0^y 2y^2 \sin(xy)\, dx\, dy$

$\Rightarrow \int_0^2 \left(2y^2 \int_0^y \sin(yx)\, dx\right) dy$

$\Rightarrow \int_0^2 \left(2y^2 \int_0^{y^2} \frac{\sin(u)}{y}\, du\right) dy$ [let $u = xy$]

$\Rightarrow \int_0^2 \left(2y^2 \frac{1}{y} \int_0^{y^2} \sin(u)\, du\right) dy$

$\Rightarrow \int_0^2 \left(2y^2 \frac{1}{y} \left[-\cos(u)\right]_0^{y^2}\right) dy$

$\Rightarrow \int_0^2 \left(2y \left[-\cos(u)\right]_0^{y^2}\right) dy$

$\Rightarrow \int_0^2 \left(2y\left(-\cos(y^2) - (-1)\right)\right) dy$

02

18301297

Theme: Date: / /

$\Rightarrow \int_0^2 (2y(-\cos(y^2)+1))\,dy.$

$\Rightarrow 2\int_0^2 (y(-\cos(y^2)+1))\,dy.$

$\Rightarrow 2\int_0^2 -y\cos(y^2) + y\,dy.$

$\Rightarrow 2\int_0^2 -y\cos(y^2) + \int_0^2 y\,dy.$

$= \cancel{2\int}$

$\Rightarrow 2\left(-\left[y\cos(u)\frac{1}{2y}\,du\right] + \left[\frac{y^2}{2}\right]_0^2\right)$

$\Rightarrow 2\left(-\int_0^4 \frac{\cos(u)}{2}\,du\right) + 2)$

$\Rightarrow 2\left(-\frac{1}{2}\right)\int_0^4 \cos(u)\,du + 2)$

$\Rightarrow 2\left(-\frac{1}{2}\left[\sin(u)\right]_0^4 + 2\right).$

$\Rightarrow -2\frac{1}{2}(\sin(4) - 0) + 2\cdot 2$

$\Rightarrow -\sin(4) + 4$

$\Rightarrow$ [Answer]

$u = y^2$

$= 0^2 / 2^2$

$= 0 / 4.$

$\int_0^4 \frac{\cos(u)}{2}\,du$

$= \frac{1}{2}\int_0^4 \cos(u)\,du.$

$= \frac{1}{2}[\sin(u)]_0^4$

When $u = 0 + \sin(u)$

$= 0$

$u \to 4 - \sin(u)$

$= \sin(4) - 0.$

1830292 03

Date: / /

Sat Sun Mon Tue wed Thu Fri

Theme:

2.

$$\int_{\pi/2}^{\pi}\int_{0}^{x^2} \frac{1}{x}\cos\frac{y}{x}\,dy\,dx.$$

$$\Rightarrow \int_{\pi/2}^{\pi}\left(\frac{1}{x}\int_0^{x^2}\cos\left(\frac{y}{x}\right)dy\right)dx$$

$$\Rightarrow \cancel{\int_{\pi/2}^{\pi}\frac{1}{x}\int^{x}}$$

$$\Rightarrow \int_{\pi/2}^{\pi}\left(\frac{1}{x}\int_0^{x}\cos(v)\,x\,dv\right)dx.$$

$$\Rightarrow \int_{\pi/2}^{\pi}\frac{1}{x}\cdot x\int_0^{x}\cos(v)\,dv)\,dx.$$

$$\Rightarrow \int_{\pi/2}^{\pi}\left[\sin(v)\right]_0^{x}dx.$$

$$\Rightarrow \int_{\pi/2}^{\pi}(\sin x - 0)\,dx.$$

$$\Rightarrow \int_{\pi/2}^{\pi}\sin x\,dx$$

$$\Rightarrow \left[-\cos x\right]_{\pi/2}^{\pi}$$

$$\Rightarrow -(-1-0)$$

$$\Rightarrow 1 \quad [\text{Ans}]$$

$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{1}{x}\frac{d}{dy}(y)$

$= \frac{1}{x}$

$dv = \frac{1}{x}dy$

$\Rightarrow dy = x\,dx$

$= \int \cos(v)\,x\,dv.$

$v = \frac{y}{x} = 0.$

$/\frac{x^2}{x} = x.$

$\int_0^{x}$ $\underline{x, 0}$.

18301222

04

Date: / /

□Sat □Sun □Mon □Tue □wed □Thu □Fri

Theme:

3) $\int_{1/4}^{1}\int_{x^2}^{x}\sqrt{\frac{x}{y}}\,dy\,dx.$

~~Let~~

$\Rightarrow \int_{1/4}^{1}\left(\int \sqrt{u}\left(-\frac{y^2}{x}\right)du\right)dx$

$\Rightarrow \int_{1/4}^{1}\left(\int -\frac{y^2\sqrt{u}}{x}\,du\right)dx$

$\Rightarrow \int_{1/4}^{1}\left(\int -\frac{\left(\frac{x}{u}\right)^2\sqrt{u}}{x}\,du\right)dx$

$\Rightarrow \int_{1/4}^{1}\left(\int -\frac{x}{u^{3/2}}\,du\right)dx.$

$\Rightarrow \int_{1/4}^{1} -x\int_{1/x}^{1}\frac{1}{u^{3/2}}\,du)\,dx.$

$\Rightarrow \int_{1/4}^{1}\left(-x\int_{1/x}^{1} u^{-3/2}\,du\right)dx.$

$\Rightarrow \int_{1/4}^{1}\left(-x\left[-\frac{2}{\sqrt{u}}\right]_{\frac{1}{x}}^{1}\right)dx.$

$\Rightarrow \int_{1/4}^{1}\left(-x\left(-2+2\sqrt{x}\right)\right)dx$

Side Notes.

$u = -\frac{x^2}{y^2}$

$du = -\frac{x^2}{y^2}\,dy$

$dy = \left(-\frac{y^2}{x}\right)du$

$= \int\sqrt{u}\left(-\frac{y^2}{x}\right)du.$

$\int -\sqrt{u}\,\frac{y^2}{x}\,du.$

$\Rightarrow \int -\frac{y^2\sqrt{u}}{x}\,du.$

$u = \frac{y}{x} \quad y = \frac{x}{u}.$

$\int -\frac{\left(\frac{x}{u}\right)^2\sqrt{u}}{x}\,du.$

$= \int -\frac{x}{u^{3/2}}\,du.$

$u = \frac{x}{y} = \frac{1}{x}.$

$u = \frac{x}{y} = 1.$

$= \int_{\frac{1}{x}}^{1} -\frac{u}{u^{3/2}}\,du.$

$= -x\int_{1/x}^{1} u^{-3/2}\,du$

$\lim u \to \frac{1}{x}+\left(-\frac{2}{\sqrt{u}}\right) = -2\sqrt{x}$

$u \to 1 \cdot \left(\frac{2}{\sqrt{x}}\right) = -2$

18301297

05

Theme:

Date: / /

□Sat □Sun □Mon □Tue □wed □Thu □Fri

$= \int_{1/4}^{1} -x(-2+2\sqrt{x})\,dx.$

$\Rightarrow -2\int_{1/4}^{1} x(-1+\sqrt{x})\,dx$

$\Rightarrow -2\int_{1/4}^{1} -x + x^{3/2}\,dx.$

$\Rightarrow -2\left(-\int_{\frac{1}{4}}^{1} x\,dx + \int_{1/4}^{1} x^{3/2}\,dx\right.$

$\Rightarrow -2\left(-\left(\frac{1}{2}-\frac{1}{32}\right)+\left(\frac{2}{5}-\frac{1}{80}\right)\right)$

$\Rightarrow -2\left(-\frac{15}{32} + \frac{31}{80}\right)$

$\Rightarrow -2\left(-\frac{13}{160}\right)$

$\Rightarrow \frac{13\ \cancel{26}}{\cancel{160}\ 80}$

$\Rightarrow \frac{13}{80}$

$\Rightarrow 0.1625$

[Answer]

$x(-1+\sqrt{x})$

$= -x + x\sqrt{x}$

$= -x + x^{3/2}$

$\int_{\frac{1}{4}}^{1} x\,dx$

$\left[\frac{x^2}{2}\right]_{\frac{1}{4}}^{1}$

$\lim x \to \frac{1}{4}^+ \left(\frac{x^2}{2}\right)$

$= \frac{(\frac{1}{4})^2}{2} = \frac{1}{32}$

$x \to 1^- \left(\frac{x^2}{2}\right)$

$= \frac{1^2}{2} = \frac{1}{2}$

$\int_{\frac{1}{4}}^{1} x^{3/2}\,dx$

$= \left[\frac{x^{3/2+1}}{\frac{3}{2}+1}\right]_{\frac{1}{4}}^{1}$

$= \left|\frac{x^{\frac{5}{2}}\cdot 2}{5}\right|_{\frac{1}{4}}^{1}$

$= \frac{2}{5}x^{5/2}\Big|_{\frac{1}{4}}^{1}$

$\lim x \to \frac{1}{4}^+ \left(\frac{2}{5}x^{5/2}\right)$

$= \frac{2}{5}\left(\frac{1}{4}\right)^{5/2}$

$= \frac{1}{80}$

$x \to 1^- \left(\frac{2}{5}x^{5/2}\right)$

$= \frac{2}{5}\cdot 1^{5/2} \Rightarrow \frac{2}{5}$

06

18301292

Date: / /

□Sat □Sun □Mon □Tue □wed □Thu □Fri

Theme:

4 $\int_0^5 \int_0^\pi x(1-\sin\theta)\,d\theta\,dx$

$\Rightarrow \int_0^5 x \int_0^\pi (1-\sin\theta)\,d\theta\,dx$

$\Rightarrow \int_0^5 \left(x \int_0^\pi d\theta - \int_0^\pi \sin(\theta)\,d\theta\right) dx.$

$\Rightarrow \int_0^5 \left(x\ \pi - [-\cos[\theta]_0^\pi\right) dx$

$\Rightarrow \int_0^5 (x(\pi-2))\,dx.$

$\Rightarrow (\pi-2)\int_0^5 x\,dx$

$\Rightarrow (\pi-2)\left|\frac{x^2}{2}\right|_0^5$

$\Rightarrow (\pi-2)\left(\frac{25}{2}-0\right)$

$\Rightarrow (\pi-2)\frac{25}{2}$

$\Rightarrow \frac{25\pi}{2} - 25$

$\Rightarrow 14.26990$

[Ans]

lim.

$\theta \to 0 + (-\cos(\theta))$

$= -\cos(0)$

$= -1$

$\theta \to 0 + (-\cos(\theta))$

$= -\cos(\theta)$

$= -1$

lim

$x \to 0 + \left(\frac{x^2}{2}\right)$

$= 0$

$x \to 5 \cdot \left(\frac{x^2}{2}\right)$

$= \frac{25}{2}.$

08

1830127.

Date: / /

Sat Sun Mon Tue wed Thu Fri

Theme:

1.5

$$\int_0^{\pi/4}\int_0^2 \frac{1}{1+r^2} r\,dr\,d\theta$$

$$\Rightarrow \int_0^{\pi/4}\int_0^2 \frac{r}{1+r^2}\,dr\,d\theta.$$

$$= \int_0^{\pi/4}\left(\int_0^2 \frac{r}{1+r^2}\,dr\right)d\theta$$

$$\Rightarrow \int_0^{\pi/4}\left(\int_1^5 \frac{1}{2u}\,du\right)d\theta$$

$$\Rightarrow \int_0^{\pi/4}\left(\frac{1}{2}\left[\ln(u)\right]_0^5\right)d\theta.$$

$$\Rightarrow \int_0^{\pi/4} \frac{1}{2}(\ln(5)-0)\,d\theta.$$

$$\Rightarrow \int_0^{\pi/4} \frac{1}{2}\ln(5)\,d\theta.$$

$$\Rightarrow \left[\frac{1}{2}\ln(5)\theta\right]_0^{\pi/4}$$

$$\Rightarrow \frac{\pi}{8}\ln(5)-0$$

$$\Rightarrow \frac{\pi}{8}\ln(5)$$

$\rightarrow$ [Ans.]

$u = 1+r^2$.

$$\frac{d}{dr}(1+r^2)$$

$$\Rightarrow \frac{d}{dr}\cdot + \frac{d}{dr}(r^2)$$

$$= 0+2r$$

$$= 2r.$$

$du = 2r\,dr.$

$dr = \frac{1}{2r}\,du.$

$$\int \frac{r}{u}\frac{1}{2r}\,du$$

$$\Rightarrow \int \frac{1}{2u}\,du$$

$(0,2)$

again; $u = 1+r^2$

$= 1+0^2$

$= 1$

or, $u = 1+r^2$

$= 1+2^2$

$= 5.$

lim. $u \to 1$ & $\ln(u)$

$= \ln(1)$

$= 0$

$u \to 5$

$\Rightarrow \ln(5)$

lim $\theta \to \pi/4$

$\rightarrow \frac{1}{2}\ln(5)\frac{\pi}{4}$

$= \frac{\pi}{8}\ln(5)$

lim $\theta \to 0$.

$\rightarrow \frac{1}{2}\ln(5)\cdot 0$

$= 0.$