# MAT120

Assignment-03 Set-26

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Section : 03

Question 1:

Find the area of the surface generated by revolving the given curve

$$8xy^2 = 2y^6 \ + \ 1 \ , \ 1 \leq y \leq 2$$

$$8xy^2 = 2y^6 + 1$$

$$x = \frac{2y^6 + 1}{8y^2}$$

$$f(x') = y^3 - \frac{1}{4y^3}$$

$$f(x')^2 = y^6 - \frac{1}{2} + \frac{1}{16y^6}$$

$$S = \int_1^2 2\pi x \sqrt{1 + (y^3 - \frac{1}{4y^3})^2} dy$$

$$S = \int_1^2 2\pi x \sqrt{1 + y^6 + \frac{1}{16y^6} - 2y^3 \frac{1}{4y^3}} dy$$

$$S = \int_1^2 2\pi x \sqrt{y^6 + \frac{1}{16y^6} + \frac{1}{2}} dy$$

$$S = \int_1^2 2\pi x \sqrt{(y^3)^2 + (\frac{1}{4y^3})^2 + 2Y^3 \frac{1}{4y^3}} dy$$

$$S = \int_1^2 2\pi x \sqrt{(y^3 + \frac{1}{4y^3})^2} dy$$

$$S = 2\pi \int_1^2 (\frac{2y^6 + 1}{8y^2})(y^3 + \frac{1}{4y^3}) dy$$

$$S = 2\pi \int_1^2 (\frac{2y^6 + 1}{8y^2})(\frac{4y^6 + 1}{4y^3}) dy$$

$$S = 2\pi \int_1^2 \frac{(2y^6 + 1)(4y^6 + 1)}{32y^5} dy$$

$$S = 2\pi \int_1^2 \frac{8y^{12} 6y^6 + 1}{32y^5} dy$$

$$S = 2\pi \int_1^2 (\frac{1}{4}y^7 + \frac{3}{16}y + \frac{1}{32}y^{-5}) dy$$

$$S = 2\pi [\frac{1}{32}y^8 + \frac{3}{16}\frac{y^2}{2} + (\frac{-1}{128})y^{-4}]_1^2$$

$$S = 2\pi [\frac{1}{32}2^8 + \frac{3}{16}\frac{2^2}{2} + (\frac{-1}{128})2^{-4} - \frac{1}{32} - \frac{3}{32} + \frac{1}{128}]$$

$$= \frac{253665\pi}{15360}$$

$$= 51.8824$$

$$[Answer]$$

Question 2:

Find the area of the surface generated by revolving the given curve

$$y = \frac{1}{3}x^3 + \frac{1}{4}x^{-1} \text{ , } 1 \leq x \leq 2$$

$$\frac{dy}{dx} = \frac{d}{dx}(\frac{1}{3}x^3 + \frac{1}{4}x^{-1})$$

$$= \frac{1}{3}3x^2 + \frac{-1}{4x^2}$$

$$= x^2 - \frac{1}{4x^2}$$

$$(\frac{dy}{dx})^2 = (x^2 - \frac{1}{4x^2})^2$$

Now:

$$S = \int_b^a 2\pi y\sqrt{1 + (\frac{dy}{dx})^2}dx$$

$$= 2\pi \int_1^2 (\frac{1}{3}x^3 + \frac{1}{3}x^{-1})\sqrt{1 + (x^2 - \frac{1}{4x^2})^2}dx$$

$$= 2\pi \int_1^2 (\frac{4x^3 + 3}{12x})(x^2 + \frac{1}{4x^2})dx$$

$$=2\ \pi \int_1^2 (\tfrac{4x^3+3}{12x})(\tfrac{4x^4+1}{4x^2})dx$$

$$= 2\pi \int_1^2 (\frac{16x^8 + 4x^4 + 12x^4 + 3}{48x^3})dx$$

$$= 2\pi \int_1^2 (\frac{16x^8 + 16x^4 + 3}{48x^3})dx$$

$$= 2\pi \int_1^2 (\frac{x^5}{3} + \frac{x}{3} + \frac{x^{-3}}{16})dx$$

$$= 2\pi[\frac{x^6}{18} + \frac{x^2}{6} - \frac{1}{32x^2}]_1^2$$

$$= 2\pi(\frac{64}{18} + \frac{4}{6} - \frac{1}{128} - \frac{1}{18} - \frac{1}{6} + \frac{1}{32})$$

$$= 2\pi(\frac{515}{128})$$

$$= \pi\frac{515}{64}$$

$$= \frac{515\pi}{64}$$

$$[Answer]$$

Question 3:
Find the exact arc length of the curve

$$y = x^{\frac{2}{3}},\ 1 \leq x \leq 8$$

$$y = x^{2/3}$$

$$\frac{dy}{dx} = \frac{2}{3x^{\frac{1}{3}}}$$

$$Now:$$

$$S = \int_8^1 \sqrt{1 + (\frac{2}{3x^{\frac{1}{3}}})^2 dx}$$

$$S = \int_8^1 \sqrt{1 + (\frac{4}{9x^{\frac{2}{3}}}) dx}$$

$$= \int_8^1 \sqrt{\frac{9x^{\frac{2}{3}} + 4}{9x^{\frac{2}{3}}} dx}$$

$$= \int_8^1 \frac{\sqrt{9x^{\frac{2}{3}} + 4}}{3x^{\frac{1}{3}}} dx$$

$$= \frac{1}{3} \int_8^1 \frac{\sqrt{9x^{\frac{2}{3}} + 4}}{x^{\frac{1}{3}}} dx$$

$$Let, u = 9x^{\frac{2}{3}} + 4$$

$$du = \frac{6}{x^{\frac{1}{3}}} dx$$

$$\frac{du}{6} = \frac{1}{x^{\frac{1}{3}}} dx$$

$$so: \frac{1}{x^{\frac{1}{3}}} dx = \frac{du}{6}$$

x=1; u=13

x=8; u=40

So :

$$= \frac{1}{3} \int_{13}^{40} \sqrt{u} \frac{1}{6} du$$

$$= \frac{1}{18} \int_{13}^{40} u^{\frac{1}{2}} du$$

$$= \frac{1}{18} [\frac{2}{3} u^{\frac{3}{2}}]_{13}^{40}$$

$$= \frac{1}{18} [(\frac{2}{3} 40^{\frac{3}{2}}) - (\frac{2}{3} 13^{\frac{3}{2}})]$$

$$= 7.634$$

$$[Answer]$$

Question 4:

Find the exact arc length of the curve

$$24xy = Y^4 + 48\ ,\ 2 \leq\ y \leq\ 4$$

$$Now : 24xy = Y^4 + 48$$

$$x = \frac{y^4}{24y} + \frac{48}{24y}$$

$$x = \frac{y^3}{24} + \frac{2}{y}$$

Derivating with respect to y :

$$\frac{d}{dy}x = \frac{d}{dy}(\frac{y^3}{24} + \frac{2}{y})$$

$$\frac{dx}{dy} = \frac{3}{24}y^2 + 2 - (1)Y^{-2}$$

$$\frac{dx}{dy} = \frac{y^2}{8} - \frac{2}{y^2}$$

$$(\frac{dx}{dy})^2 = (\frac{y^2}{8} - \frac{2}{y^2})^2$$

$$(\frac{dx}{dy})^2 = \frac{y^4}{64} - 2\frac{y^2}{8}\frac{2}{y^2} + \frac{4}{y^4}$$

$$(\frac{dx}{dy})^2 = \frac{y^4}{64} + \frac{4}{y^4} - \frac{1}{2}$$

$$Weknow : S = \int_d^c \sqrt{1 + (\frac{dx}{dy})^2 dy}$$

So

$$S = \int_4^2 \sqrt{1 + \frac{y^4}{64} + \frac{4}{y^4} - \frac{1}{2}} dy$$

$$S = \int_4^2 \sqrt{\frac{y^4}{64} + \frac{4}{y^4} + \frac{1}{2}} dy$$

$$S = \int_4^2 \sqrt{\frac{32y^2 + y^8 + 256}{64y^2}}$$

$$S = \int_4^2 \sqrt{(\frac{y^4 + 16}{8y^2})^2}$$

$$S = \int_4^2 \frac{y^4 + 16}{8y^2}$$

$$= \frac{1}{8}\int_4^2 y^2 + \frac{16}{y^2}$$

$$= \frac{1}{8}\int_4^2 y^2 dy + \frac{1}{8}\int_4^2 \frac{16}{y^2} dy$$

$$= \frac{1}{24}[4^3 - 2^3] - 2[\frac{1}{4} - \frac{1}{2}]$$

$$= \frac{1}{24}56 + \frac{1}{2}$$

$$= \frac{17}{6}$$

$$[Answer]$$

Question 5:

Determine the surface area of the solid obtained by rotating

$$y = \sqrt[3]{x}, 1 \leq \ y \leq \ 2$$

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$$

$$(\frac{dy}{dx})^2 = \frac{1}{9x^{\frac{4}{3}}}$$

y=1 ; x=1

y=2 ; x=8

Now :

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$= \sqrt{1 + \frac{1}{9x^{\frac{4}{3}}}}$$

$$= \sqrt{\frac{9x^{\frac{4}{3}} + 1}{9x^{\frac{4}{3}}}}$$

$$= \frac{\sqrt{9x^{\frac{4}{3}} + 1}}{3x^{\frac{2}{3}}}$$

$$S = \int_1^8 2\pi x \frac{\sqrt{9x^{\frac{4}{3}} + 1}}{3x^{\frac{2}{3}}}\, dx$$
$$= \frac{2\pi}{3} \int_1^8 x^{\frac{1}{3}} \sqrt{9x^{\frac{4}{3}} + 1}\, dx$$

$$Let : u = 9x^{\frac{4}{3}} + 1$$

So :

$$du = 12x^{\frac{1}{3}}\, dx$$

x=1 ; y=10

x=8 ; y=145

$$
\begin{aligned}
S &= \frac{\pi}{18}\int_{10}^{145}\sqrt{u}\,du \\
&= \frac{\pi}{27}u^{\frac{3}{2}}\Big|_{10}^{145} \\
&= \frac{\pi}{27}\left(145^{\frac{3}{2}} - 10^{\frac{3}{2}}\right)
\end{aligned}
$$

$$= 199.48$$

$$[Answer]$$