

Automata Assignment 2

1. Give a context-free grammar for each of the following languages.

- a) $L(G) = \{ 0^n 1^m 0^m 1^n \mid n, m \geq 0 \}$.
- b) $L(G) = \{ a^n b^m c^k \mid n, m, k \geq 0 \text{ and } n=2m+3k \}$.
- c) $L(G) = \{ a^n b^m \mid 0 \leq n \leq m \leq 3n \}$.
- d) $L(G) = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } j=k \}$.
- e) $L(G) = \{ a^i b^j c^k \mid j \neq i+k \}$.

2. Give context-free grammars that generate the following languages.

- a) $L(G) = \{ a^n b^m c^m d^{2n} \mid n \geq 0, m > 0 \}$.
- b) $L = \{ w \mid w \text{ contains at least three 1's} \}$.
- c) $L = \{ w \mid w \text{ starts and ends with the same symbol} \}$.
- d) $L = \{ w \mid \text{the length of } w \text{ is odd} \}$
- e) $L = \{ w \mid \text{the length of } w \text{ is odd and its middle is 0} \}$
- f) $L = \{ w \mid w \text{ contains twice as many 1s as 0s} \}$

3. Consider the following context-free grammar $\Sigma = \{0, 1\}$.

$$\begin{aligned} S &\rightarrow A \mid B \\ A &\rightarrow 0A \mid \epsilon \\ B &\rightarrow 0B \mid 1B \mid \epsilon \end{aligned}$$

Give leftmost and rightmost derivations and parse tree for the following strings

- a) 0010101
- b) 10100
- c) 00011

4. Which language generates the grammar G given by the productions.

$$\begin{aligned} S &\rightarrow aSa \mid ab \\ B &\rightarrow bB \mid b \end{aligned}$$

5. Explain why the grammar below is ambiguous.

4. Which language generates the grammar G given by the productions.

$$\begin{aligned}S &\rightarrow aSa \mid aBa \\B &\rightarrow bB \mid b\end{aligned}$$

5. Explain why the grammar below is ambiguous.

$$\begin{aligned}S &\rightarrow 0A \mid 1B \\A &\rightarrow 0AA \mid 1S \mid 1 \\B &\rightarrow 1BB \mid 0S \mid 0\end{aligned}$$

6. Given the following ambiguous context free grammar

$$\begin{aligned}S &\rightarrow Ab \mid aaB \\A &\rightarrow a \mid Aa \\B &\rightarrow b\end{aligned}$$

a) Find leftmost and rightmost derivations for $aaaaab$, $aabb$, ab .

(b) Show the parse trees for the above strings.

(c) Find an equivalent unambiguous context-free grammar.

(d) Give the unique leftmost derivation and parse tree for the above strings generated from the unambiguous grammar above.

DA ← 2

(c)

adnata / A ← A

(b)

Adnan Basad kasi

16101271

+ 2000791

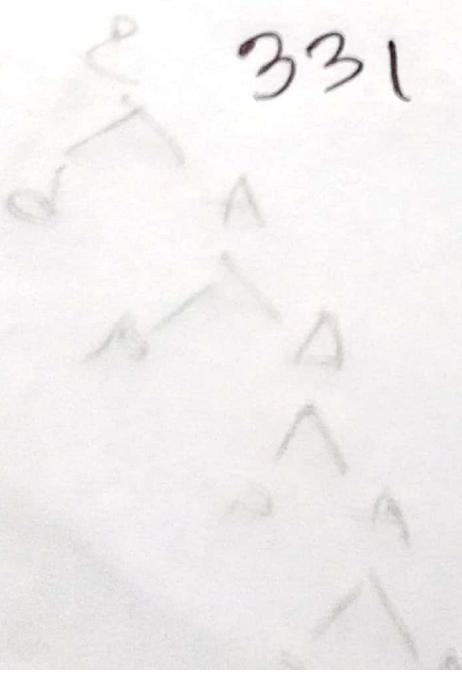
331

DA ←

daA ←

daaa ←

daaaa ←



Assignment 02

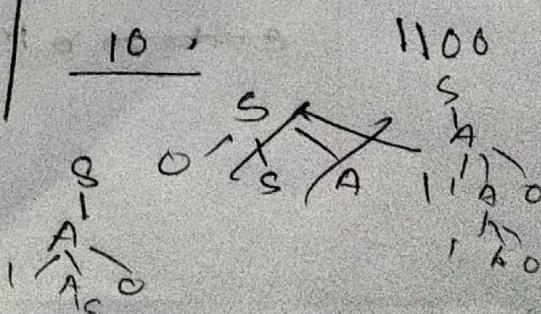
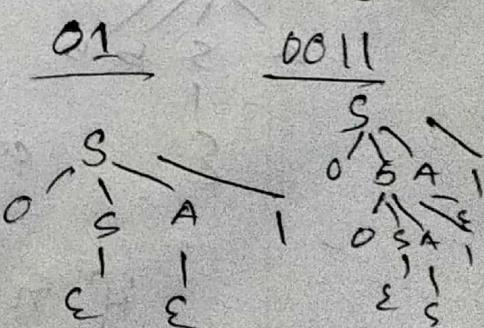
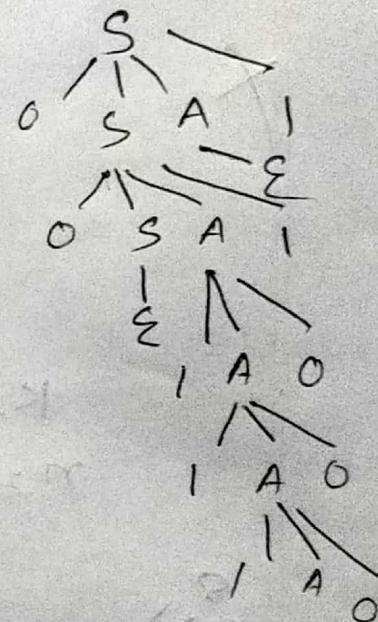
Ans to the Q-1

a) $L(G) = \{0^n 1^m 0^m 1^n \mid n, m \geq 0\}$

$$S \rightarrow 0S@1 \quad | \quad \epsilon \quad | \quad A$$

$$A \rightarrow 1A0 \quad | \quad \epsilon$$

Test
 $\underline{00} \quad \underline{11100011}$



b) $L(G) = \{ a^n b^m c^k \mid n, m, k \geq 0 \text{ and } n = 2m + 3k \}$

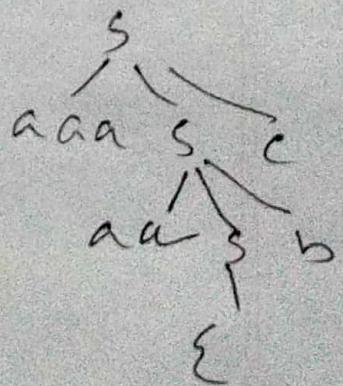
$S \rightarrow aaascc \mid aasbc \mid \epsilon$

$$\begin{array}{l} m=1 \text{ and } k=0 \\ \cancel{n=1} \quad n=2 \\ n=\cancel{6}5 \end{array}$$

$$\begin{array}{l} m=k=1 \text{ and } m=0 \\ n=3 \end{array}$$

Test
 $\overline{aaasabc}$
 \overline{aasbc}

$$2 \times 1 + 3 \times 1 = 5$$



$n=1 m=3$

$n-m$	
2	3
6	
$\frac{aaa\ mm}{aaa\ mmm}$	

c) $L(G) = \{ a^n b^m \mid 0 \leq n \leq m \leq 3n \}$

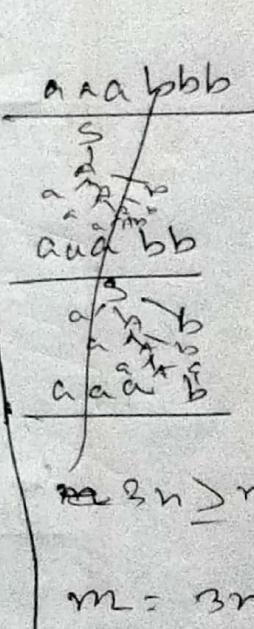
$$S \rightarrow \epsilon \mid A \mid B$$

$$A \rightarrow aA \mid ab \mid aAb \mid aBb \mid a$$

$$B \rightarrow b$$

$$A \rightarrow aAb \mid aAb \mid \epsilon \mid aA \mid a$$

$$A \rightarrow \epsilon \mid aAb \mid aAb \mid aAb \mid Ab$$



$$m = 3n$$

$$mmm \quad \star$$

$$\star \quad \star \quad \star$$

d) $L(\Sigma) = \{a^i b^j c^k \mid i, j, k \geq 0$

and $i=j$
or $j=k$

$i=j$

$S \rightarrow aAbC \mid \epsilon$

$C \rightarrow Cc \mid \epsilon$

$A \rightarrow aAb \mid \epsilon$

$j=k$

$P \rightarrow DbKc \mid \epsilon$

$D \rightarrow aD$

$K \rightarrow \cancel{bK} bKc \mid \epsilon$

Start variable = O

O $\rightarrow S \mid P$

Ans to the Q-2

(a) $L(G) = \{ a^n b^m c^m d^{2n} \mid n \geq 0, m > 0 \}$

{ e1 result }

$S \rightarrow A \underset{\cancel{B}}{P} \underset{\cancel{C}}{D}$

$A \rightarrow a \underset{\cancel{B}}{d} a \underset{\cancel{B}}{d} d \underset{\cancel{C}}{d} | c a \underset{\cancel{C}}{d} d d | B C | \cancel{E}$

$C \rightarrow b \underset{\cancel{B}}{k} c \underset{\cancel{B}}{k} c$

$K \rightarrow \epsilon$

~~$P \rightarrow \epsilon | a P c d d$~~

~~$P \rightarrow \epsilon | a P c d d$~~

$S \rightarrow A$

$A \rightarrow a P c d d | c | \cancel{A} \underset{\cancel{C}}{d} d$

$P \rightarrow \epsilon | a P c d d$

$C \rightarrow b k c$

$K \rightarrow \epsilon | b k c$

Test

bc bbcc

~~abc~~

S
|
A
|
C
|
b k c
|
 ϵ

aabbcccd
S
|
A
|
P c d d
|
b k c

$S \rightarrow \emptyset$ what can A

b) $L = \{ w | w \text{ contains at least three } 1's \}$

$S \rightarrow \emptyset \quad S \rightarrow A \mid A \mid A \mid A$
 $A \rightarrow 0 \mid 1 \mid \epsilon$

c) $L = \{ w | w \text{ starts and ends with same symbol} \}$

$S \rightarrow 0 \mid 1$
 $S \rightarrow 0A1 \mid 1A\emptyset \quad A \rightarrow 0 \mid 1$
 $A \rightarrow 0A1 \mid 1A \mid \epsilon \quad 0 \mid 1 \rightarrow 0 \mid 1$

d) $L = \{ w \mid \text{the length of } w \text{ is odd} \}$

$$\{(a+b)(a+b)\}^*$$

~~a or b~~ ~~a or b~~ ~~a or b~~

$s \rightarrow ABA|011$

$A \rightarrow 0A + 10A | \cancel{0} 0 + 1 011$

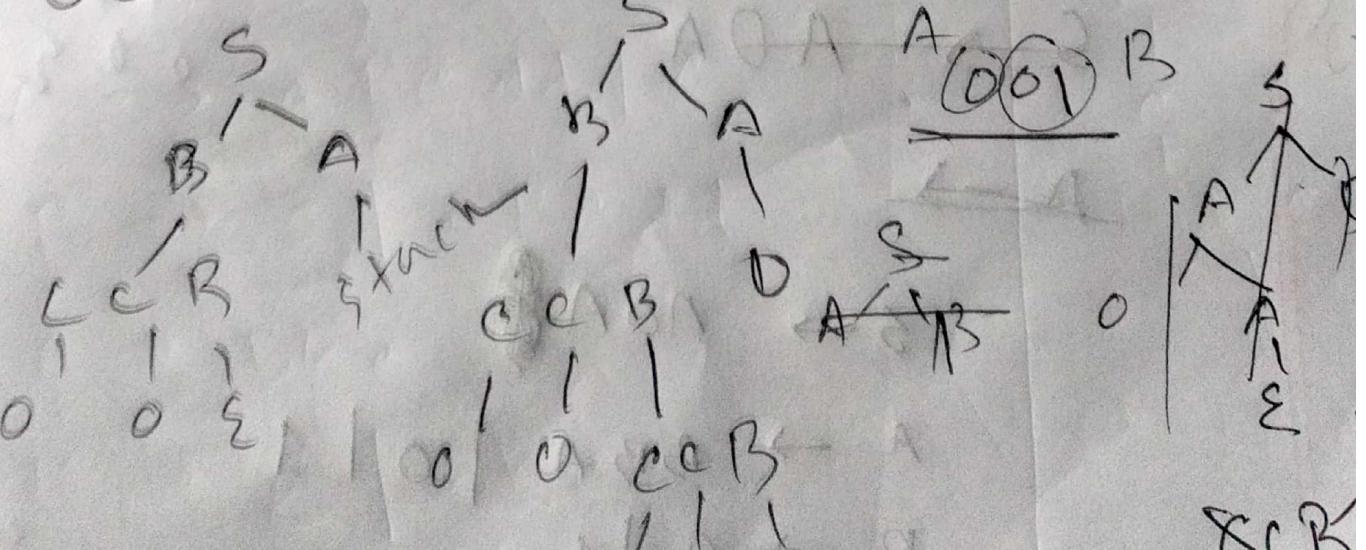
$B \rightarrow \cancel{000000}CCCB\Sigma$

$C \rightarrow a+b 011$

S / AGEI Test

Test

00



00000000

$$\begin{array}{r} \cancel{2} + \cancel{1} + \cancel{2} \\ = 5 \end{array}$$

e) L = {w | the length of w is odd
and middle is 0}

$$\begin{array}{l} \cancel{(a+b)} 0 \cancel{(a+b)} : 10 / 1000 \leftarrow S \end{array}$$

$$\left\{ \begin{array}{l} \cancel{(a+b)(a+b)} : S(a+b \text{ last}) \leftarrow A \end{array} \right.$$

$$S \rightarrow AOA | 000 | 001 | 100 | 101 \left| \begin{array}{c} 000 \\ 00 \\ \hline 0000 \end{array} \right. \begin{array}{l} 00 \\ \text{col} \\ 100 \\ 101 \end{array}$$

$$A \rightarrow BBA | \epsilon$$

$$B \rightarrow 0 | 1$$

$$S \rightarrow AOA$$

$$\begin{array}{c} S \\ | \\ A \\ | \\ BBA \\ | \\ 111 \\ | \\ 00 \end{array} \left| \begin{array}{c} A \\ \text{A} \\ \text{BBA} \\ 111 \\ 00 \\ \hline 000 \end{array} \right. \begin{array}{l} 00 \\ \text{col} \\ 100 \\ 101 \\ 101 \\ 101 \end{array}$$

~~D~~

$$S \rightarrow AOA$$

$$A \rightarrow BBA | 0 | 1$$

$$B \rightarrow 0 | 1$$

$$\begin{array}{c} 000 \\ S \rightarrow \\ | \\ B \\ | \\ 0 \\ | \\ 0 \end{array}$$

e) $L(G) = \{a^i b^j c^k \mid j \neq i+k\}$

this means $j < i+k$ or $j > i+k$

case $j > i+k$

$S_1 \rightarrow A b B A$

$A \rightarrow abA \mid \epsilon \mid bcA$

$B \rightarrow bcB \mid \epsilon$

$j < i+k$

$S_2 \rightarrow a D \mid E b C \mid a D \mid b C \mid a C$

$D \rightarrow ab \mid \epsilon$

$E \rightarrow bc \mid \epsilon$

$\xrightarrow{\text{ss}} S_3 \rightarrow S_1 \mid S_2$

8) $L\{w\}$ & w contains as many 1s as

0s



$S \rightarrow OSIS$) isosle

8

3.

$$S \rightarrow A \mid B$$

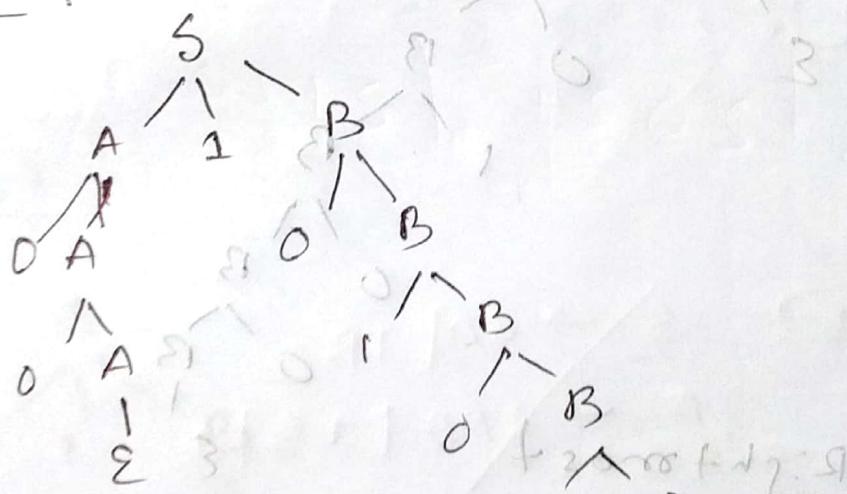
$$00101 \quad \text{Leftmost}$$

$$A \rightarrow 0A \mid \epsilon$$

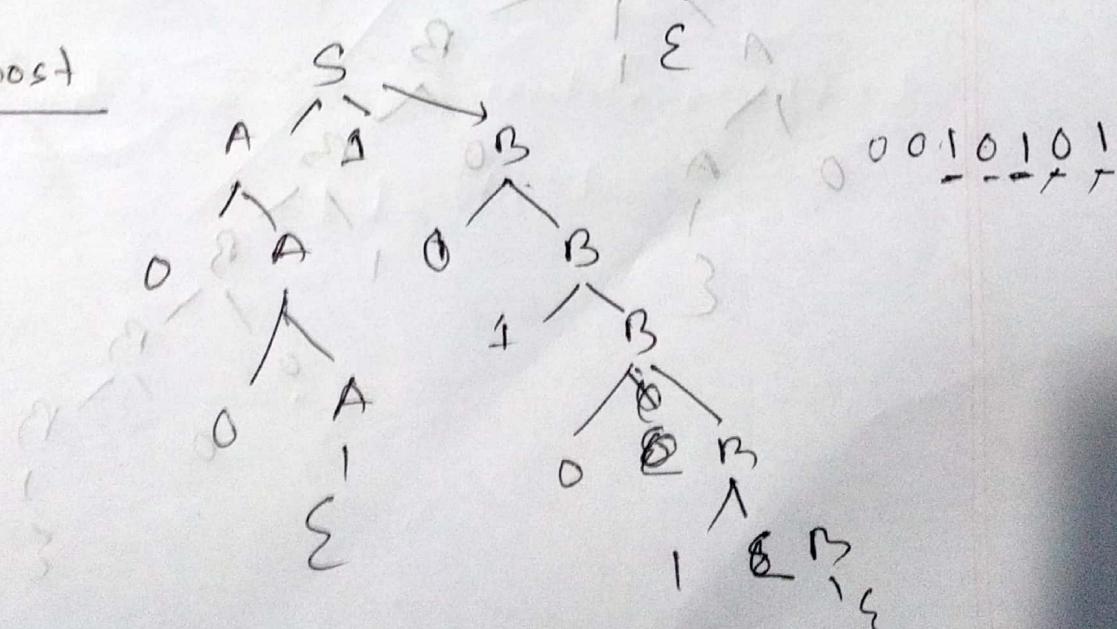
$$B \rightarrow 0B \mid 1B \mid \epsilon \quad \text{Rightmost}$$

a) 0010101

Leftmost :-



Rightmost

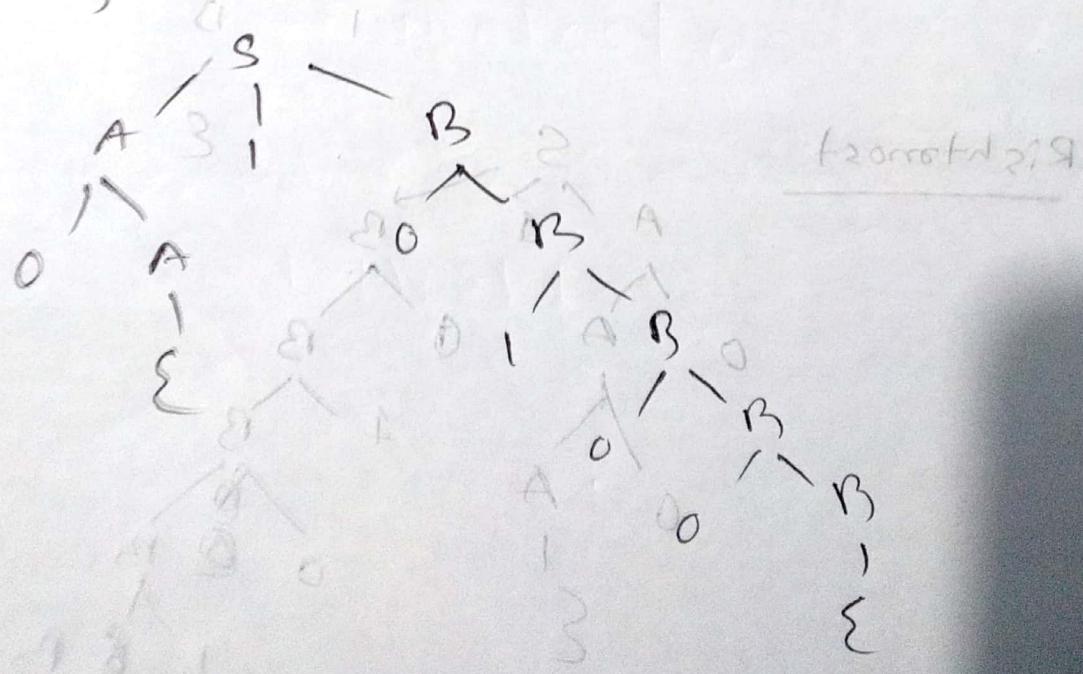
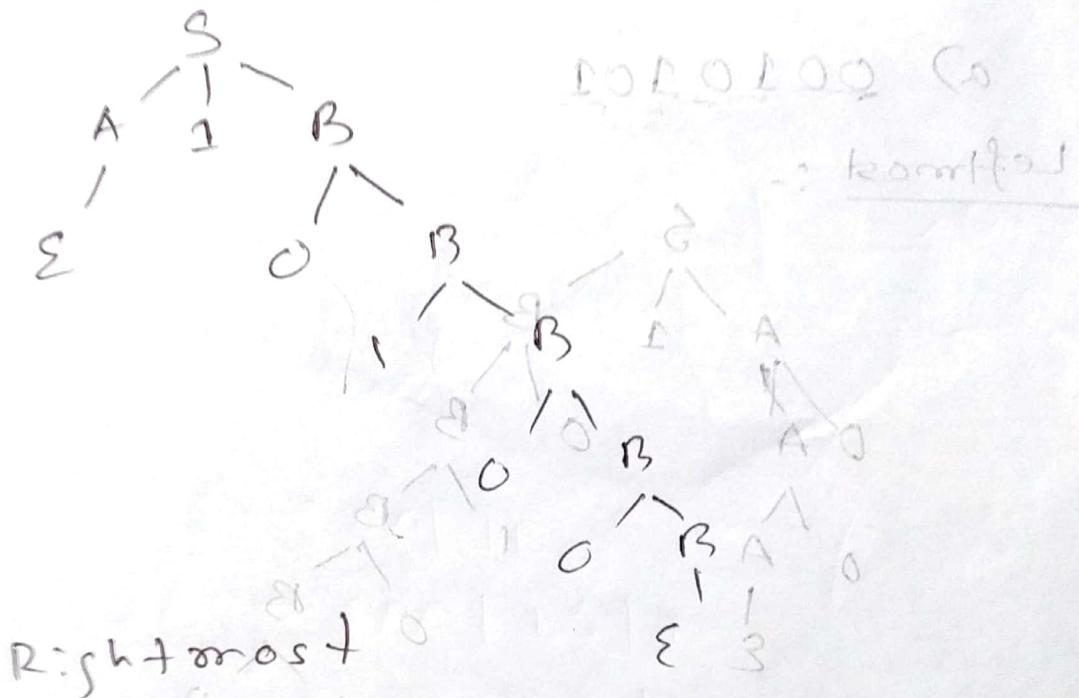


b) 10100

$S \rightarrow A \rightarrow \epsilon$

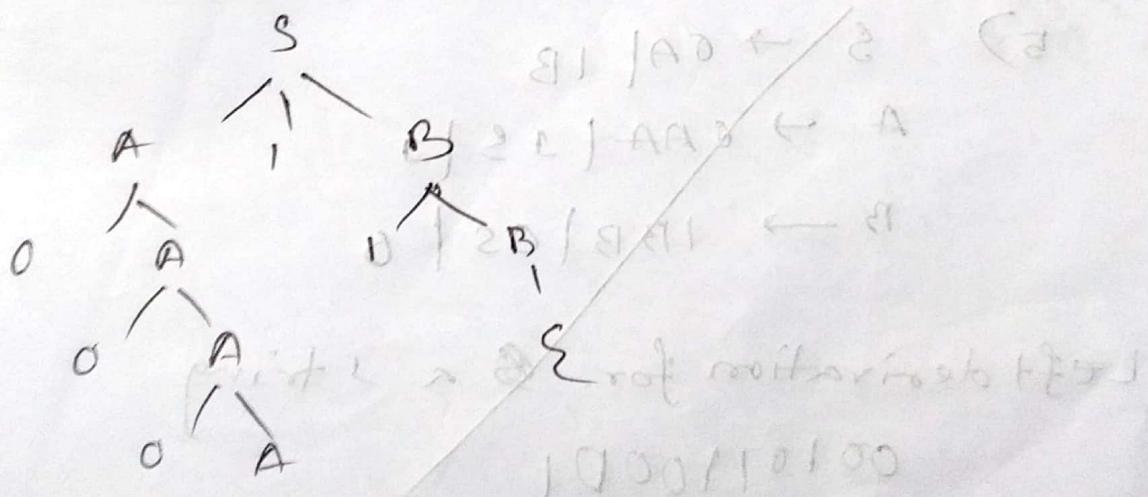
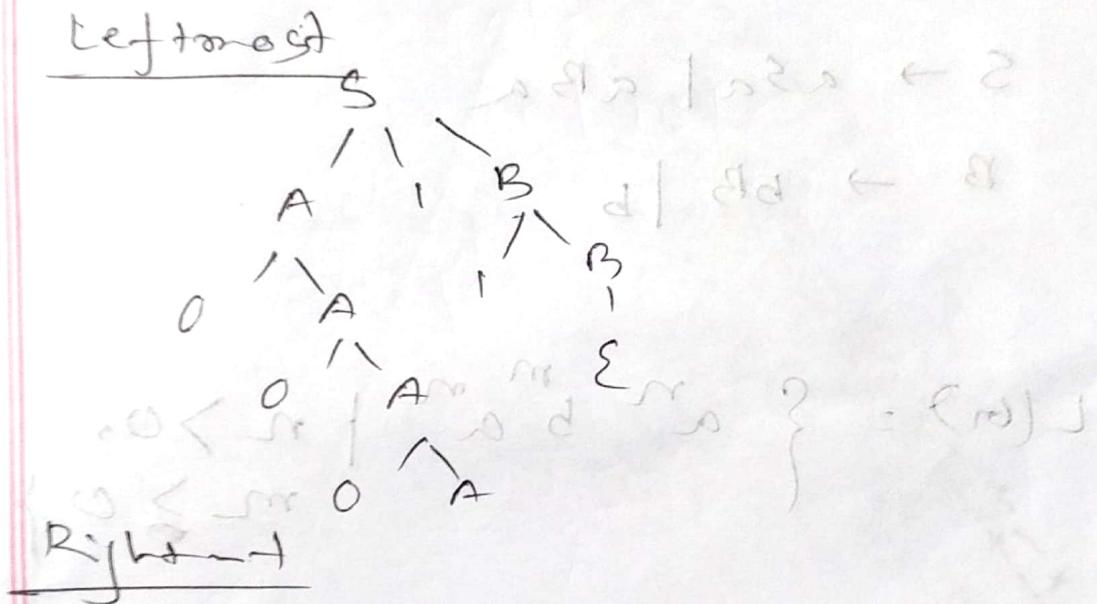
$S \rightarrow A \rightarrow A$

leftmost $\rightarrow S \rightarrow A \rightarrow B$



$\Rightarrow 00011$

(A)



4)

1 1 0 0 0 0

$S \rightarrow aSa \mid aBa$

$B \rightarrow bB \mid b$

Rejected

$L(G) = \{ a^n b^m a^n \mid n > 0, m \geq 0 \}$

5)

$S \rightarrow aA \mid 1B$

$A \rightarrow aAA \mid 1S \mid \epsilon$

$B \rightarrow 1BB \mid 0S \mid \epsilon$

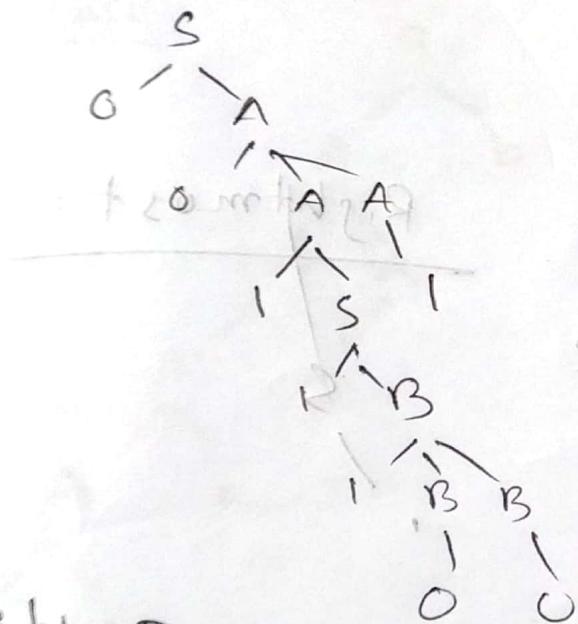
Left derivation for ~~00101100D1~~ a string

00101100D1

5.

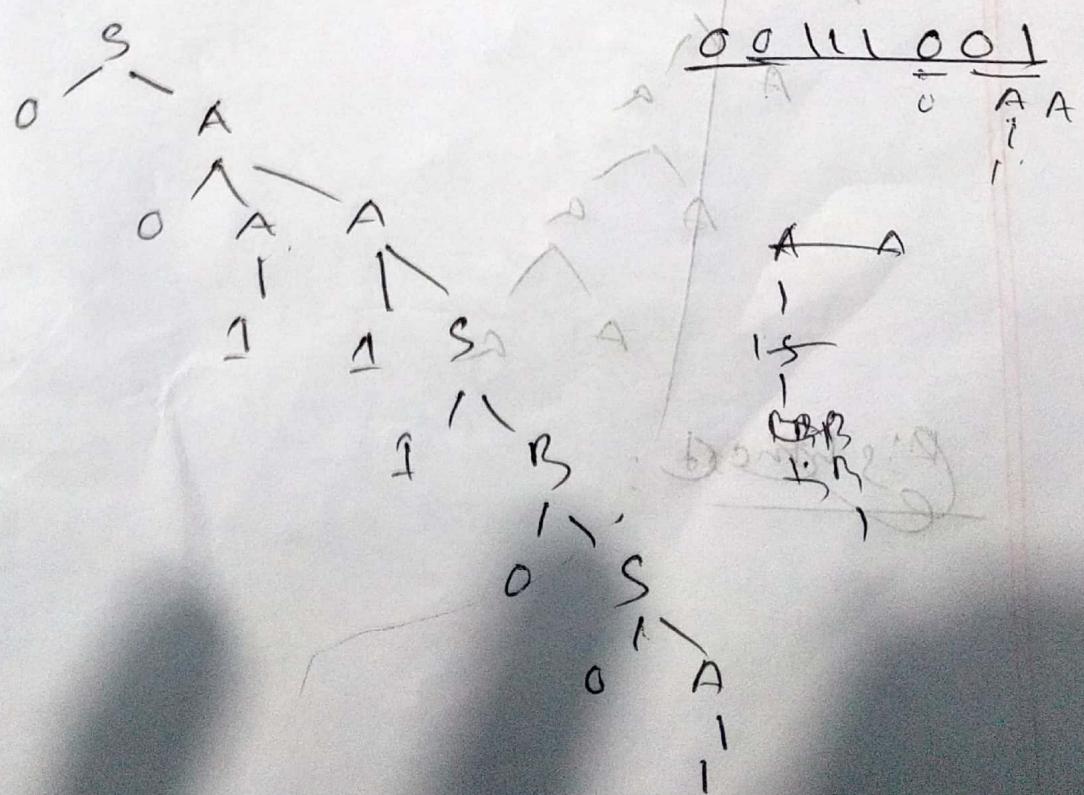
String : 00111001

1st Left D.



2nd Left D.

~~BBst~~



6

Left

$$a) \quad S \rightarrow Ab$$

$$\rightarrow Aab$$

$$\rightarrow Aaab$$

$$\rightarrow Aaaaab$$

$$\rightarrow Aaaaab$$

$$\rightarrow aaaaab$$

Right

$$S \rightarrow Ab$$

$$\rightarrow Aab$$

$$\rightarrow Aaab$$

$$\rightarrow Aaaaab$$

$$\rightarrow Aaaaab$$

$$\rightarrow aaaaab$$

do do

do do

↓A ← 2

do ←

do ←

do ←

do do

do do

do ←

aabb

1/29/1

dA ← S (d)

Left:

dssA ←

S → Ab dssA ←

→ Aab dssA ←

→ Aaab dssA ←

→ stuck dssA ←

Right

1/29/2

S → aAB dA ← S

→ aab dssA ←

stuck dssA ←

dssA ←

ab dssA ←

dssA ←

Left

S → Ab

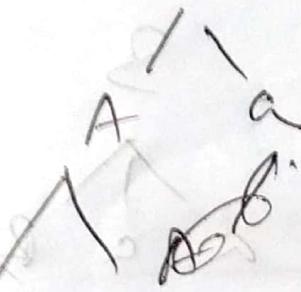
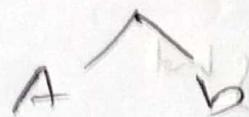
→ ab

Right

S → Ab

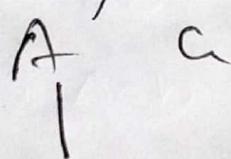
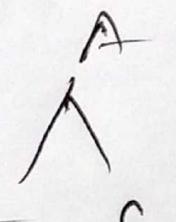
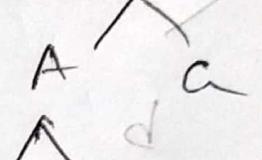
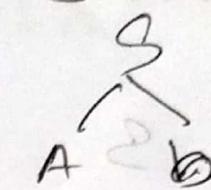
→ ab

b) left S



a

Right S



a

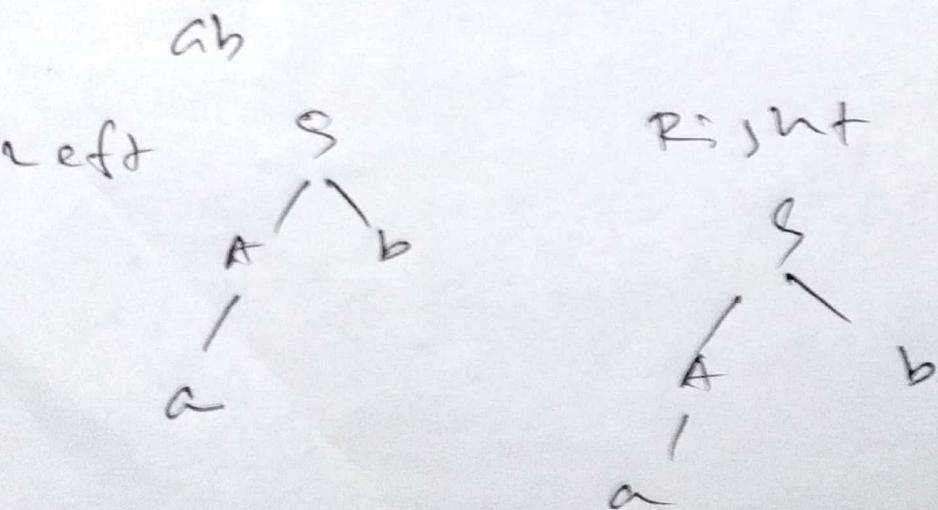
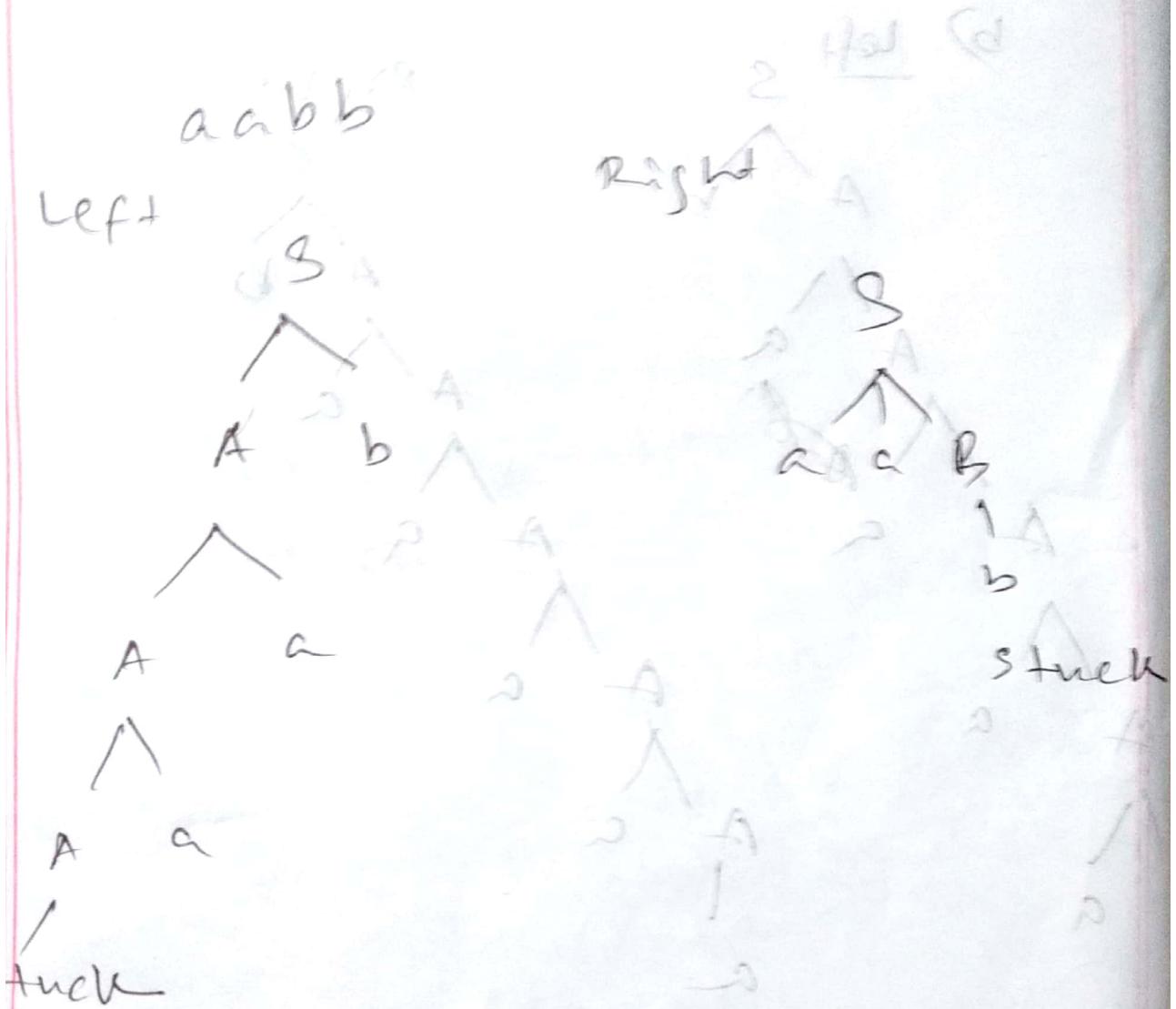
12

2

A

1

S



c)

$$S \rightarrow Ab$$

$$A \rightarrow a \mid Aa$$

d)

aaaaab

Leftmost

$$S \rightarrow Ab$$

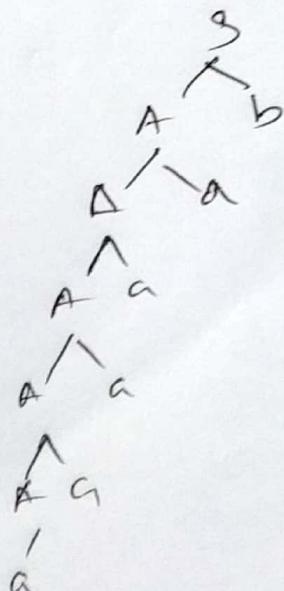
$$\rightarrow Aab$$

$$\rightarrow Aaab$$

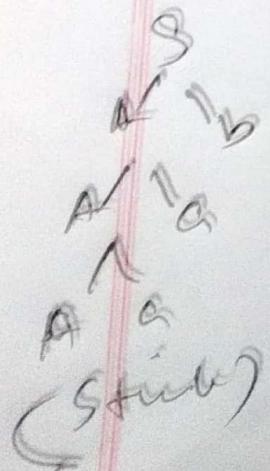
$$\rightarrow Aaaab$$

$$\rightarrow Aaaaab$$

$$\rightarrow aaaaab$$



aabb



$$S \rightarrow Ab$$

$$\rightarrow Aab$$

$$\rightarrow Aaab$$

(Stuck)

$$ab$$
$$S \rightarrow Ab$$
$$\rightarrow ab$$

