UNIT - II

AMPLITUDE MODULATION

Modulation :- It is the process of changing the param of the carrier wave (high frequency) such as amplitude phase and frequency in accordance with the message signal is called modulation

Need for Modulation :-

The various purposes that can be served by modulation are:-

a) **Frequency Multiplexing** :- Multiplexing can be achieved by translating each one of the original signals (all of which encompass the same spectral range) to different frequency range. Suppose, say, one signal is transmitted to the freq. range f_1' to f_2' the second to the range f_1'' to f_2'' and so on. If these new freq ranges do not overlap, then the signal may be seperated at the receiving end.

b) **Practicability of Antenna** :-

For the transmission of radio signals, the antenna height must be multiple of $\lambda/4$. Here λ is wavelength, $\lambda = c/f$

So, antenna required to transmit a 10 KHz signal is

$$h = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times10\times10^3} = 7500\,m = 7.5\,km$$

for a signal 1 MHz

$$h = \lambda/4 = \frac{c}{4f} = \frac{3\times10^8}{4\times10^6} = 75\,m$$

So, modulation reduces the height of the antenna

c) **Avoids Mixing of Signals** :-

If the baseband sound signals are transmitted without using the modulation by more than one transmitter, then all the signal will be in the same frequency range i.e. 0 to 20 KHz. Therefore, all the signals get mixed together and the receiver cannot seperate them from each other.

Hence, if the signal is used to modulate a different carrier then they will occupy different slots in the freq.

nain, and hence modulation avoids mixing of signals.

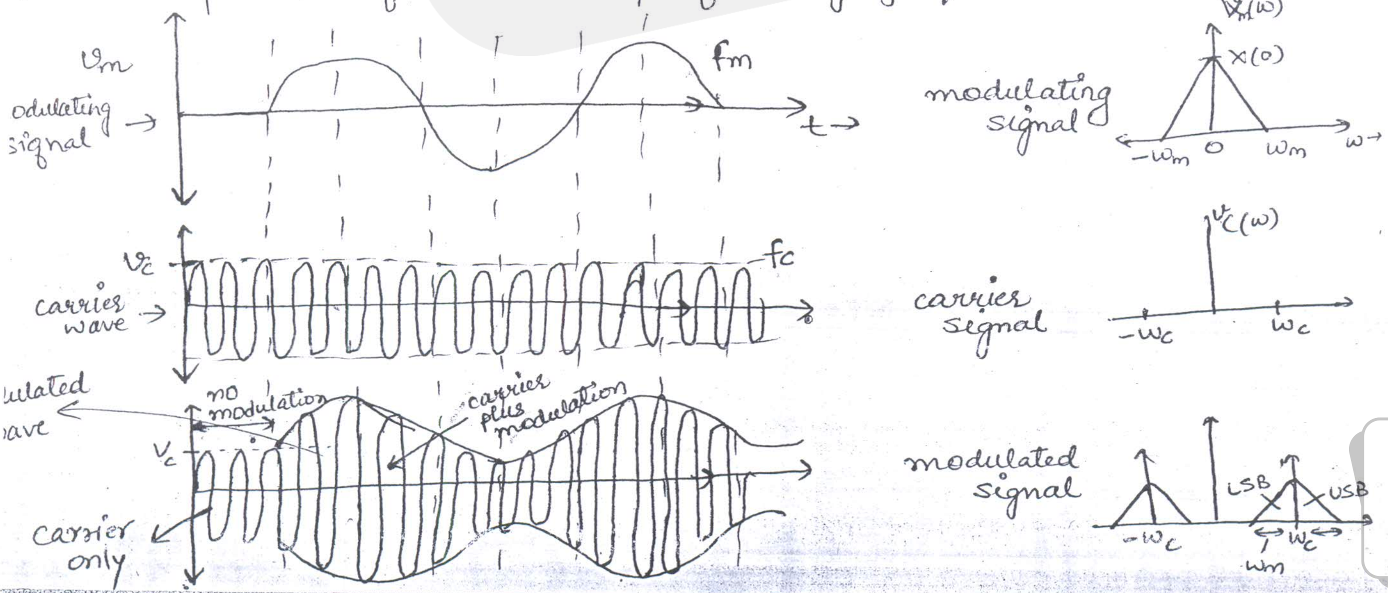
Increases the Range of Communication

frequency of baseband signal is low, and the low freq. nal can not travel a long distance when they are transmitted. attenuation reduces with the increase in frequency of the unsmitted signal and they travel longer distance.

mmon Processing :- when a no. of signals, similar in neral character but occupying different spectral ranges, need ocessing, as we go from signal to signal, we need to adjust the quency range of our processing apparatus to correspond to freq. range of the signal to be processed. So, when the ocessing apparatus is complex, it may be wiser to leave processing apparatus to operate in some freq. range & stead translate the freq. range of each signal to this ed freq. range.

MPLITUDE MODULATION :-

nplitude modulation is the process of changing the amplitude a relatively high frequency carrier signal in proportion with e instantaneous value of the modulating signal.
1 modulators are non-linear devices with two inputs and one tput. One input is a single high frequency carrier signal of nstant amplitude and the second input is comprised of relatively no-frequency information signals that may be a single frequency a complex waveform made up of many frequencies.



∴ The standard equation for AM may be expressed as

$$v_{am}(t) = [v_m(t) + v_c(t)] \cos w_c t$$

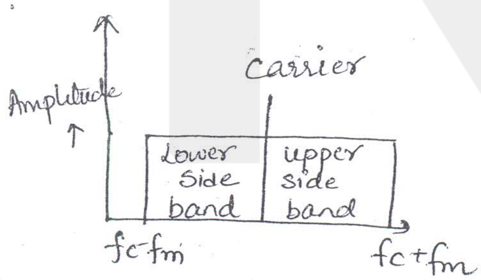
Since amplitude of carrier is constant.

$$v_{am}(t) = [v_m(t) + A] \cos w_c t$$

* Although there are several types of amplitude modulation, AM double-sideband full carrier (DSBFC) is probably the most commonly used. AM DSBFC is sometimes called conventional AM or simply AM.
* The O/P waveform contains all the frequencies that make up the AM signal and is used to transport the information through the system. Therefore, the shape of the modulated wave is called the AM envelope.

AM Frequency Spectrum and Bandwidth

AM modulator is a non linear device. Therefore, non linear mixing occurs, and the output envelope is a complex wave made up of a dc voltage, the carrier ~~function~~ frequency, a the sum (f_c+f_m) and difference (f_c-f_m) frequencies. The sum and difference frequencies are displaced from the carrier frequency by an amount equal to the modulating signal frequency.



Band of frequencies b/w f_c and $f_{...}$ is called upper sideband (USB) and the freq. within this band called an upper side frequency.

Therefore, the bandwidth (B) of an AM DSBFC wave is equal to the difference b/w the highest upper side frequency & the lowest lower side frequency so, $B = 2f_m$

Coefficient of Modulation / Modulation Index

Coefficient of modulation is a term used to describe the amo[unt] of amplitude change (modulation) present in an AM waveform. Percent modulation is simply the coefficient of modulation state[d] as a percentage

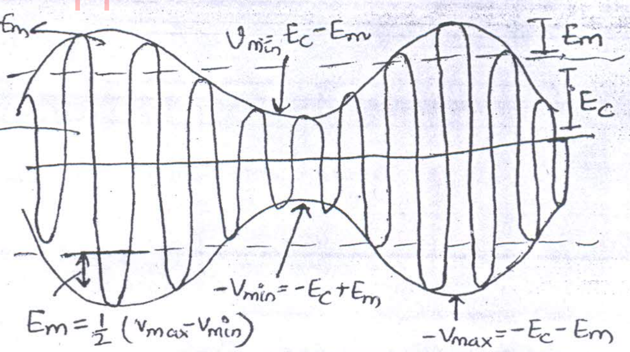
Modulation coefficient → $m = \frac{E_m}{E_c}$ → peak change in the amplitude of O/P waveform

unitless

E_c → peak amplitude of the unmodulated carrier

$n = \frac{E_m}{E_c}$, $M = \frac{E_m}{E_c} \times 100$

carrier plus upper & lower side freq.



$n = \frac{1}{2}(V_{max} - V_{min})$

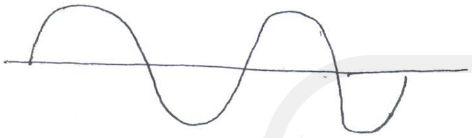
$c = \frac{1}{2}(V_{max} + V_{min})$

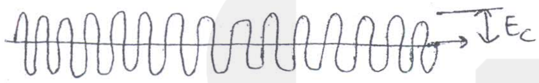
$$= \frac{\frac{1}{2}(V_{max} - V_{min})}{\frac{1}{2}(V_{max} + V_{min})} \times 100 = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} \times 100$$

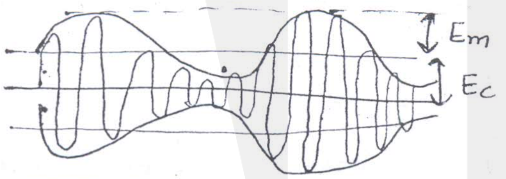
Modulation may be classified into the following two categories :-

1) Linear modulation
   ↳ $m \leq 1$

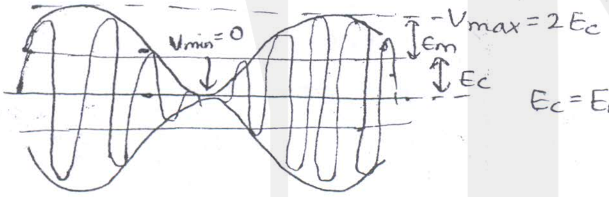
2) Over modulation
   $m > 1$, the envelope can sometimes reverse the phase. Overmodulation introduces envelope distortion. Therefore, it should be avoided.





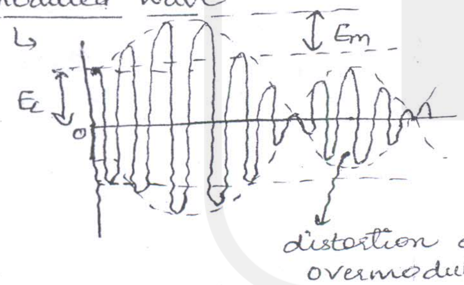


$E_m = \frac{E_c}{2}$ (50% modulated wave)



$E_c = E_m$ (100% modulated wave)

Overmoduted wave
↳



AM voltage Distribution :-

An unmodulated carrier can be described as

$$v_c(t) = E_c \sin(2\pi f_c t)$$

where $v_c(t)$ = time-varying voltage waveform for the carrier
E_c = peak carrier amplitude (volts)
f_c = carrier frequency (Hertz)

Instantaneous amplitude of modulated wave can be expressed as :-

$$v_{am}(t) = [E_c + E_m \sin(2\pi f_m t)][\sin(2\pi f_c t)]$$

where $[E_c + E_m \sin(2\pi f_m t)]$ = amplitude of the modulated wave

E_m = peak change in amplitude of envelope

f_m = frequency of modulating signal

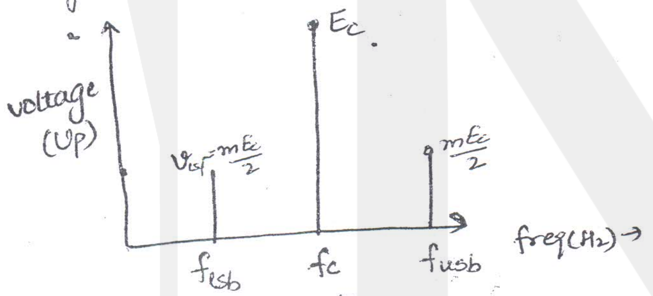
$$m = \frac{E_m}{E_c} \rightarrow E_m = m E_c$$

So, $$v_{am}(t) = [\underbrace{1}_{\text{constant}} + \underbrace{m \sin(2\pi f_m t)}_{\text{modulating signal}}][\underbrace{E_c \sin(2\pi f_c t)}_{\text{unmodulated carrier}}]$$

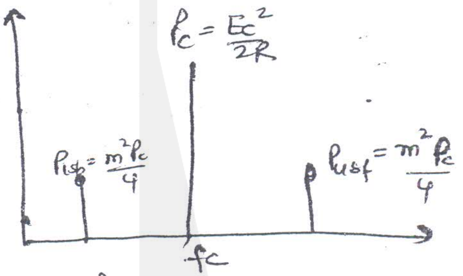
$$v_{am}(t) = E_c \sin(2\pi f_c t) + [m E_c \sin(2\pi f_m t) \cdot \sin(2\pi f_c t)]$$

$$\therefore v_{am}(t) = \underbrace{E_c \sin(2\pi f_c t)}_{\text{carrier wave}} \underbrace{- \frac{m E_c}{2} \cos[2\pi(f_c + f_m)t]}_{\text{upper side freq. signal}} + \underbrace{\frac{m E_c}{2} \cos[2\pi(f_c - f_m)t]}_{\text{lower side frequency signal}}$$

→ Amplitude of the carrier after modulation is the same as it before modulation (E_c) ∴ the amplitude of the carrier is unaffected by the modulation process.



Freq. spectrum



Power spectrum

AM Power Distribution

In any electrical circuit, the power dissipated is equal to the voltage squared divided by the resistance.

∴ The average power dissipated in a load by an unmodulated carrier is equal to the rms carrier voltage squared divided by the load resistance.

$$P_c = \frac{V_{rms}^2}{R} = \frac{(E_c/\sqrt{2})^2}{R} = \frac{E_c^2}{R}$$

The lower and upper sideband power are expressed mathematically

$$P_{usb} = P_{lsb} = \frac{(mE_c/2)^2}{2R} = \frac{m^2 E_c^2}{8R} = \frac{m^2}{4}(P_c) = \frac{m^2}{4} P_c$$

Total Power $$P_t = P_c + P_{lsb} + P_{usb}$$
$$= P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c$$

$$\boxed{P_t = \left(1 + \frac{m^2}{2}\right) P_c}$$

is important to use as high a percentage of modulation as sible while still being sure not to overmodulate. rrier power remains the same as m changes. However the sideband wer was reduced dramatically when m decreased from higher to wer. Because sideband power is proportional to the square of the odulation coefficient, a reduction in m of one-half results in a reduction of side-band power of one-forth.

AM current calculations

e relation b/w carrier current and the current of the modulated wave is

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \frac{I_t^2}{I_c^2} = 1 + \frac{m^2}{2}$$

$$I_t = \sqrt{1 + \frac{m^2}{2}} \cdot I_c$$

Modulation by a Complex Information Signal

hen several frequencies simultaneously amplitude modulate a carrier, he combined coefficient of modulation is the square root of the uadratic sum of the individual modulation indexes

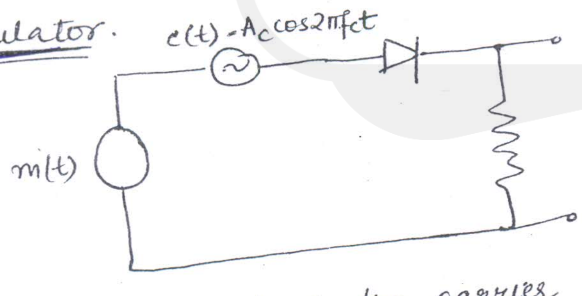
$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + m_4^2}$$

$$P_t = P_c \left(1 + \frac{m_t^2}{2}\right)$$

DSB-FC AM Modulator

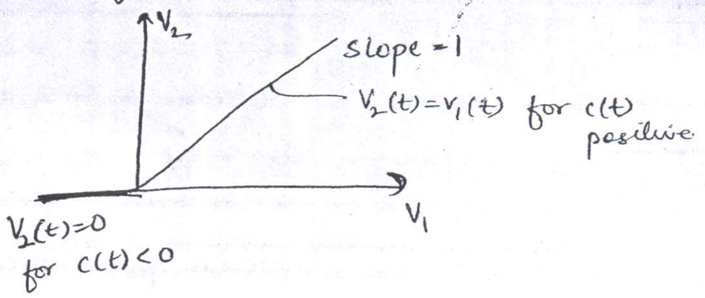
- switching modulator
- Square-law modulator * on page no 5

e generation of an AM wave may be accomplished sing various devices; one such device is a switching modulator.



t is assumed that the carrier wave c(t) applied to the diode s large in amplitude, so that it swings right across the characteristic urve of the diode

le assume that the diode acts as an ideal switch, i.e., it resents zero impedance when it is forward biased, thus approximating the transfer characteristics of the diode-load resistor combination by a piece wise linear characteristics



Accordingly for an I/P voltage $v_1(t)$ consisting of the sum of the carrier and the message signal

$m_s(t)$ — message si

$$v_1(t) = A_c \cos(2\pi f_c t) + m_s(t)$$

when $|m(t)| \ll A_c$, the resulting load voltage $v_2(t)$ is

$$v_2(t) \simeq \begin{cases} v_1(t) & c(t) > 0 \\ 0 & c(t) < 0 \end{cases}$$

i.e. the load voltage $v_2(t)$ varies periodically b/w the values $v_1(t)$ and ze at a rate equal to carrier frequency f_c. In this way, by assuming a modulating wave that is weak compared with the carrier wave, we have effeciently replaced the non-linear behaviour of the diode by an approximately equivalent piecewise linear time varying operat

$$v_2(t) = [A_c \cos(2\pi f_c t) + m_s(t)]\, g_{T_0}(t)$$

where $g_{T_0}(t)$ is a periodic pulse train of duty cycle equal to o half and period $T_0 = 1/f_c$.

Representing $g_{T_0}(t)$ by its fourier series

$$g_{T_0}(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)]$$

$$= \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \text{odd harmonic components}$$

$$v_2(t) = [A_c \cos 2\pi f_c t + m_s(t)]\left[\frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)]\right]$$

$$v_2(t) = [m_s(t) + A_c \cos 2\pi f_c t]\left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \text{odd harmonics}\right]$$

$$v_2(t) = \underbrace{\frac{1}{2}m_s(t)}_{\text{msg signal}} + \underbrace{\frac{1}{2}A_c \cos 2\pi f_c t + \frac{2}{\pi}\cos(2\pi f_c t).m_s(t)}_{\text{AM wave}} + \underbrace{\frac{2E_c}{\pi}\cos^2(2\pi f_c t)}_{\text{Second harmonics of carrier}}]$$

The component $\frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m_s(t)\right]\cos 2\pi f_c t$ is the desired AM wav with modulation index

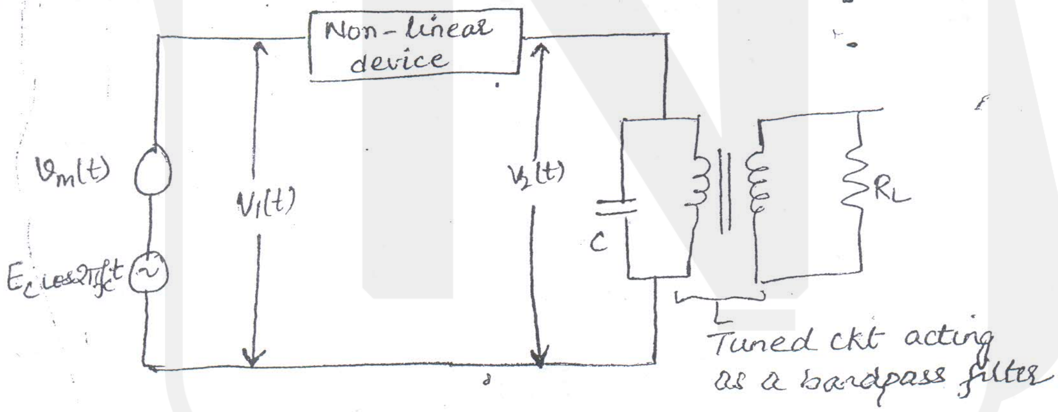
$$m = \frac{4}{\pi A_c}$$

→ The switching modulator is therefore made more sensitive by reducing the carrier amplitude A_c; however it must be maintained large enough to make the diode act as an ideal switch.

→ The unwanted terms are removed from the load voltage $v_2(t)$ by means of a band pass filter with mid-band frequency f_c and bandwidth $2W$, provided that $f_c > 2W$.
This condition ensures that the frequency seperation b/w the desired AM wave and the unwanted components are large enough for the band-pass filter to suppress the unwanted component.

2 Square law modulator

A square law modulator consists of
(a) Non-linear device
(b) A bandpass filter
(c) A carrier source and modulating signal



The modulating signal and carrier are connected in series with each other and their sum $v_1(t)$ is applied at the I/P of non-linear device, such as diode or transistor

$$v_1(t) = v_m(t) + E_c \cos 2\pi f_c t$$

I/P – O/P relationship of a non-linear device is as under

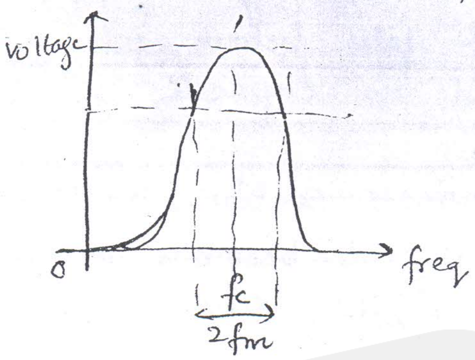
$$v_2(t) = a v_1(t) + b v_1^2(t)$$

$$v_2(t) = a[v_m(t) + E_c \cos 2\pi f_c t] + b[v_m(t) + E_c \cos 2\pi f_c t]^2$$

$$v_2(t) = \underbrace{a v_m(t)}_{\text{modulating/message signal}} + \underbrace{a E_c \cos 2\pi f_c t}_{\text{carrier signal}} + \underbrace{b v_m^2(t)}_{\text{squared msg signal}} + \underbrace{b E_c^2 \cos^2 2\pi f_c t}_{\text{squared carrier}} + \underbrace{2b v_m(t) E_c \cos 2\pi f_c t}_{\text{AM wave with only sidebands}}$$

$$v_2(t) = \underbrace{a v_m(t) + b v_m^2(t) + b E_c^2 \cos^2(2\pi f_c t)}_{\text{Unuseful}} + \underbrace{a E_c \cos(2\pi f_c t) + 2b v_m(t)\cos(2\pi f_c t)}_{\text{useful}}$$

The LC tuned ckt acts as a bandpass filter.



Tuned to freq fc and BW = $2f_m$

This bandpass filter eliminates the unuseful terms of $v_2(t)$.

Hence the o/p voltage i.e. $v_{am}(t)$ consists of

$$v_{am}(t) = a E_c \cos(2\pi f_c t) + 2b\, v_m(t)\, E_c \cos 2\pi f_c t$$

$$v_{am}(t) = [a E_c + 2b\, v_m(t) \cdot E_c] \cos 2\pi f_c t$$

$$= a E_c \left[1 + \frac{2b}{a} v_m(t)\right] \cos 2\pi f_c t$$

So, the above expression of $v_{am}(t)$ represents an AM wave with modulating index $m = \frac{2b}{a}$

⇒ **Demodulation of AM wave**

Demodulation is the process of recovering the message signal from the received modulated signal.

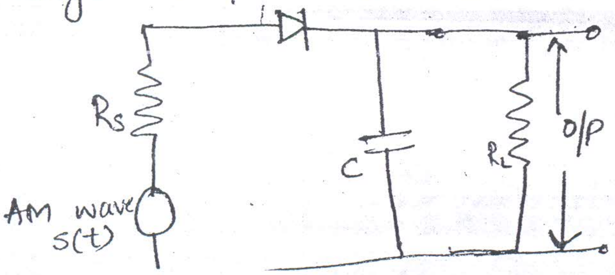
Two types of AM detector are as under :-

a) Square Law Detector

b) Envelope Detector

(a) **Envelope Detector**

- This is used in almost all commercial AM radio receivers
- For this to function properly, the AM wave has to be narrow-band, which requires that the carrier frequency be large compared to the message BW.

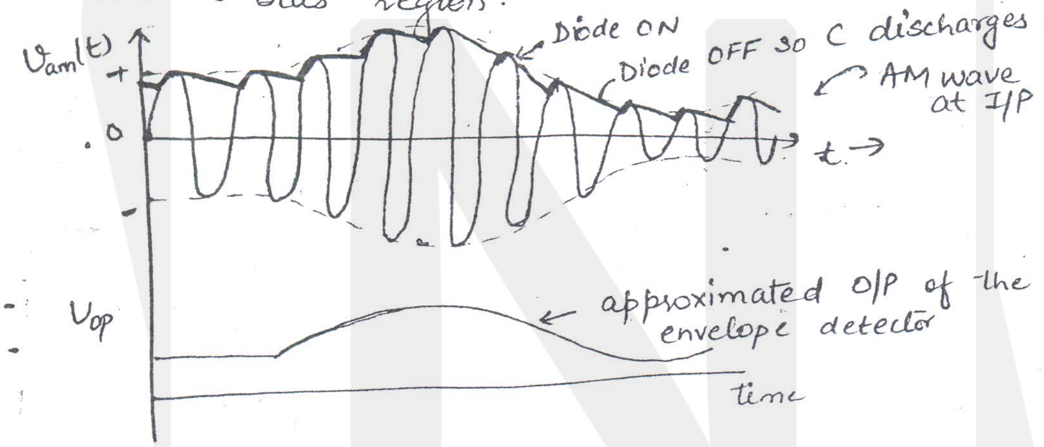


→ On a positive half cycle of the I/P signal, the diode is forward biased and the capacitor C charges up rapidly to the peak value of the I/P signal.

→ When the I/P signal falls below this value, the diode becomes reverse-biased and the capacitor C discharges slowly throug the load resistor R_L.

→ The discharging process continues until the next positive half cycle. When the I/P signal becomes greater than the voltage across the capacitor, the diode conducts again and the proc is repeated.

→ We assume the diode is ideal, presenting resistance r_f to current flow in the forward-bias region and infinite resistance in the reverse-bias region.



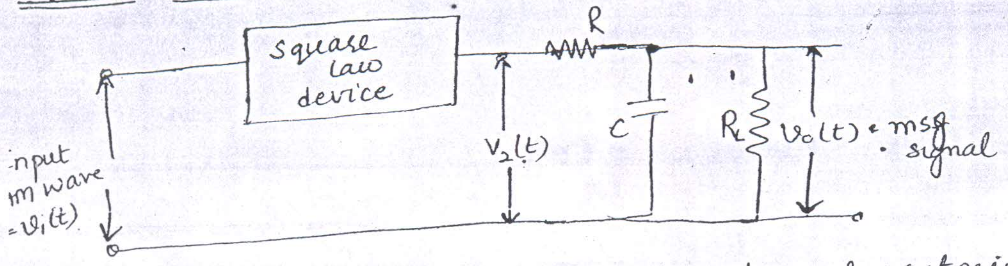
We further assume that the AM wave applied to the envelope detector is supplied by a voltage source of internal impedance R_s.
So, the charging time constant $(r_f + R_s)C$ must be short compared to carrier period $1/f_c$ i.e.

$$(r_f + R_s)C \ll \frac{1}{f_c}$$

So that the capacitor C charges rapidly and thereby follows the applied voltage upto the +ve peak when the diode is conducting.
On the other hand, discharging time constant $R_L C$ must be long to ensure that the capacitor discharges slowly through the load resistor R_L b/w +ve peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of modulating wave

$$\frac{1}{f_c} \ll R_L C \ll \frac{1}{W}$$

Square-Law Detector :-



The I/O characteristics i.e. the transfer characteristics of a square law device is non-linear and is expressed mathematically as

$$V_2(t) = aV_1 + bV_1^2(t)$$

where $V_1(t) \rightarrow$ AM wave

$$V_1(t) = E_c[1 + m\, V_m(t)]\cos 2\pi f_c t]$$

So, $V_2(t) = aE_2[1 + mV_m(t)]\cos 2\pi f_c t) + bE_c^2[1 + mV_m(t)]^2 \cos^2 2\pi f_c t$

But $\cos^2\theta = \frac{1}{2}[1 + \cos 2\theta]$

$\therefore \cos^2 2\pi f_c t = \frac{1}{2}[1 + \cos[4\pi f_c t]]$

$$V_2(t) = aE_c[1 + mV_m(t)]\cos 2\pi f_c t + \frac{bE_c^2}{2}[1 + 2mV_m(t) + m^2V_m^2(t)][1 + \cos 4\pi f_c t]$$

Out of these, the desired term is only $V_o(t) = (bE_c^2 m)V_m(t)$ which is due to the term $bV_1^2(t)$ term.

Hence the name is square law detector.

The desired term is extracted by using a LPF after the diode

So, $V_o(t) = (bE_c^2 m)V_m(t)$

$\Rightarrow$ we have recovered the message signal $V_m(t)$ at the O/P of the detector.

Que:- For an AM DSBFC envelope with $+V_{max} = 20V$ and $V_{min} = +4V$.

Determine the following

(a) Peak amplitude of the carrier

(b) Modulation coefficient and % modulation

(c) Peak amplitude of upper and lower side frequencies.

Soln:- $V_m = \frac{V_{max} - V_{min}}{2} = \frac{(20-4)}{2} = 8V$

$$V_c = \frac{V_{max} + V_{min}}{2} = \frac{20+4}{2} = \frac{24}{2} = 12V$$

Modulation coefficient

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{20-4}{20+4} = \frac{16}{24} = \frac{4}{6} = \frac{2}{3} = 0.667$$

Percentage modulation $= m \times 100\% = 66.67\%$

Peak amplitude of USB or LSB $= \frac{mV_c}{2} = \frac{0.667 \times 12}{2} = 4V$

Que :- A sinusoidal carrier has amplitude of 10V and freq. 30 KHz. It is amplitude modulated by a sinusoidal voltage of amplitude 3V and freq 1 KHz

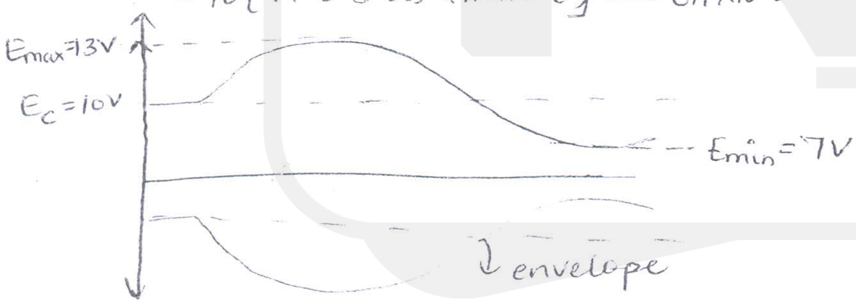
(a) Write the eq^n for modulated wave
(b) Plot the modulated wave showing maxima & minima of waveform
(c) Determine the modulation index
(d) Draw the spectrum of modulated wave.

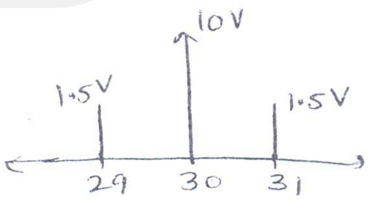
Ans: $E_c = 10V$
$E_m = 3V$
$f_c = 30 KHz$
$f_m = 1 KHz$

$m = \frac{E_m}{E_c} = \frac{3}{10} = 0.3$ → modulation index

$$v_{am}(t) = E_c[1 + m\cos\omega_m t]\cos\omega_c t$$

$$= 10[1 + 0.3\cos 2\pi \times 10^3 t]\cos 6\pi \times 10^4 t$$





$f_{USB} = f_c + f_m = 30 + 1 = 31 KHz$
$f_{LSB} = f_c - f_m = 30 - 1 = 29 KHz$

Amplitude of each sideband $= \frac{m}{2} \times E_c = \frac{.3 \times 10}{2} = 1.5V$

Que:- An AM broadcast radio transfer radiates 10K watts of power if modulation % is 60. Calculate how much of this is the carrier power.

Sol^n :- $P_t = P_c\left(1+\frac{m_a^2}{2}\right)$

$$P_c = \frac{P_t}{1+\frac{m_a^2}{2}} = \frac{10}{1+\frac{0.6^2}{2}} = \frac{10}{1.18} = 8.47\,KW$$

Que- Prove that in amplitude modulation, maximum average power transmitted by an antenna is 1.5 times the carrier power.

$$P_t = P_c\left[1+\frac{m^2}{2}\right]$$

The maximum value of m without introducing distortion in the modulated wave is $m=1$

$$P_{t(max)} = \left[1+\frac{1}{2}\right]P_c = \frac{3}{2}P_c$$

or $P_c = 66\%$ of the total Power.

Que:- The antenna current of an AM transmitter is 8A if only the carrier is sent, but it increases to 8.93A if the carrier is modulated by a single sinusoidal wave. Determine the % modulation. Also find the antenna current if the % of modulation changes to 0.8

Sol^n :- $I_t = I_c\sqrt{1+\frac{m_a^2}{2}}$

$$\frac{I_t}{I_c} = \sqrt{1+\frac{m_a^2}{2}} \Rightarrow \left(\frac{I_t}{I_c}\right)^2 = 1+\frac{m_a^2}{2}$$

$$\frac{m_a^2}{2} = \left(\frac{I_t}{I_c}\right)^2 - 1 \Rightarrow m_a = \sqrt{2\left[\left(\frac{I_t}{I_c}\right)^2-1\right]}$$

$$m_a = \sqrt{2\left[\left(\frac{8.93}{8}\right)^2-1\right]} = \sqrt{2\left[(1.16)^2-1\right]} = \sqrt{2(1.246-1)} = \sqrt{0.492} = 0.701$$
$$= 70.1\%$$

(ii) $I_t = I_c\sqrt{1+\frac{m_a^2}{2}}$

$I_c = 8A$, $m_a = 0.8$

$$I_t = 8\sqrt{1+\frac{0.64}{2}} = 8\sqrt{1.32} = 8\times1.149$$
$$= 9.19A$$

Virtues, Limitations, and Modifications of Amplitude Modulation

Virtue:-
Modulation is accomplished rather simply in the transmitter using a switching modulator or a square-law modulator. Demodulation is accomplished just as easily in the receiver using an envelope detector or a square-law detector. So, AM system is relatively cheap to build

Limitations

(1) Amplitude Modulation is wasteful of Power :- The carrier wave c(t) is completely independent of the information-bearing signal of baseband signal $v_m(t)$. The transmission of the carrier wave therefore represents a waste of power, which means that in amplitude modulation only a fraction of the total transmitted power is actually affected by m(t).

(ii) Amplitude modulation is wasteful of Bandwidth :-
The upper and lower sidebands of an AM wave are uniquely related to each other by virtue of their symmetry about the carrier frequency, hence, given the amplitude and phase spectrum of either sideband, we can uniquely determine the other. This means that insofar as the transmission of information is concerned only one sideband is necessary, and the communication channel therefore needs to provide only the same BW as the baseband signal. So, AM is wasteful of BW as it requires a transmission BW equal to twice the message BW.

Modifications :-
To overcome these limitations, we must make certain change which result in increased system complexity of AM process. In effect, we trade off system complexity for improved utilization of communication resources.
So, we can distinguish three modified forms of amplitude modulation :

1. Double - Sideband - Suppressed Carrier :- The transmitted wave consists of only the upper and lower sidebands. Transmitted power is saved here through the suppression of the carrier wave, but the channel Bandwidth requirement

is the same as before.

(2) Single sideband (SSB) modulation :-
Modulated wave consists only of the upper sideband or the lower sideband. The essential function of SSB modulation therefore to translate the spectrum of the modulating sig (with or without inversion) to a new location in the frequency domain
SSB is particularly suited for transmission of voice signa by virtue of the energy gap that exists in the spectrum of voice signal b/w zero and a few hundred hertz.
It is an optimum form of modulation in that it requires the minimum transmitted power and minimum channel BW.
Its principal disadvantage is increased cost & complexity.

(3) Vestigial Sideband (VSB) modulation :-
In this one sideband is passed almost completely and just a trace, or vestige, of the other sideband is retained.
The required channel BW is therefore in excess of the message BW by an amount equal to the width of the vestigial sideband.
This form of modulation is well suited for the transmission of wideband signals such as television signals that contain significant components at extremely low frequencies
In commercial TV broadcasting, a sizeable carrier is transmitted together with the modulated wave, which make it possible to demodulate the incoming modulated signal by an envelope detector in the receiver & thereby simplify the receiver design.

Applications
a) Radio Broadcasting
b) Picture Transmission in a TV system.

Double Sideband Suppressed Carrier (DSB-SC)

The equation of an AM wave in its simplest form is

$$v_{am}(t) = A\cos\omega_c t + A\frac{m_c}{2}\cos(\omega_c+\omega_m)t + A\cdot\frac{m_a}{2}\cos(\omega_c-\omega_m)t$$

The carrier component remain constant in amplitude & frequency. This means carrier does not convey any information. Moreover $P_t = \left(1+\frac{m^2}{2}\right)P_c$ so, for a 100% modulation about 67% power is required for transmitting the carrier which does not contain any information.

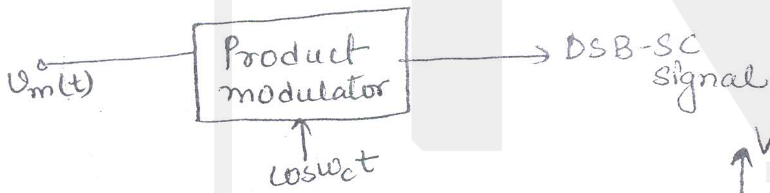
Hence, if the carrier is suppressed, only the SB remain and in this way a saving of two-third power may be achieved at 100% modulation.

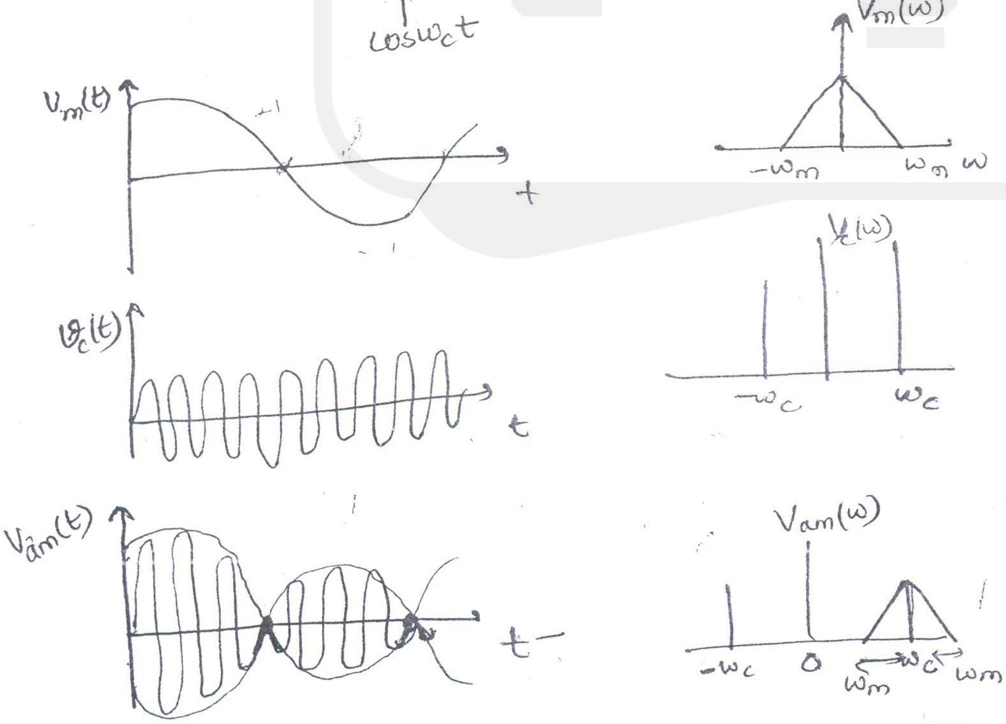
$$v_{am}(t) = E_c[1+v_m(t)]\cos\omega_c t$$

$$= E_c\cos\omega_c t + \frac{A v_m}{2}\cos(\omega_c+\omega_m)t + \frac{E_c v_m}{2}\cos(\omega_c-\omega_m)t$$

So, $v_m(t)\cos\omega_c t$ represents a DSB-SC signal.

So, DSB-SC is obtained simply multiplying modulating signal $x(t)$ with carrier signal $\cos\omega_c t$. This is achieved by a product modulator





DSB-SC exhibit ~~suppressed carrier~~ phase reversal at zero crossings i.e. whenever the baseband signal $x(t)$ crosses zero.

⇒ The envelope of the DSB-SC modulated signal is different

In a DSB-SC, the upperside band frequency is $w_c + w_m$ where the lower sideband freq is $w_c - w_m$.

$$So,\ BW = (w_c + w_m) - (w_c - w_m) = 2w_m$$

So, BW of DSB-SC is same as that of General BW.

Generation of DSB-SC signal

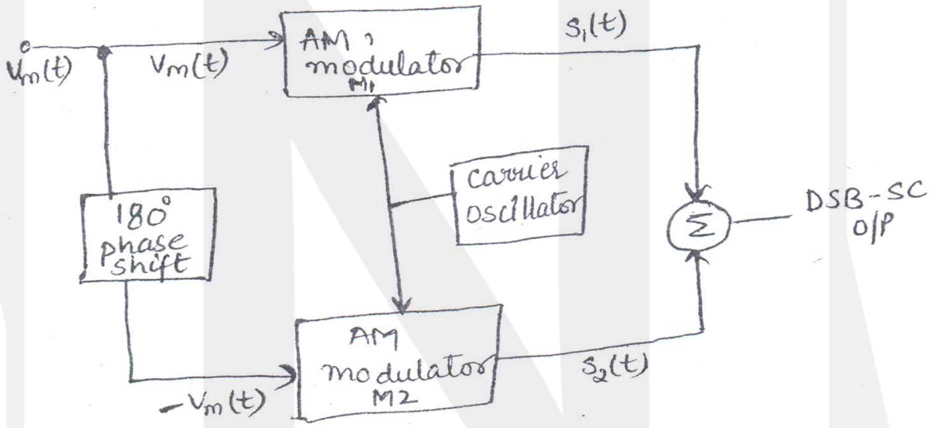
DSB-SC is generated by

a) Balanced modulator

b) Ring Modulator

Balanced Modulator

It consists of two standard modulators arranged in the balanced configuration, so as to suppress the carrier completely.



The carrier signal is connected to both the AM modulators. The message signal is applied as it is to M_1 and its inverted version at M_2. At the O/P of modulators M_1 and M2, we get the standard AM signals $s_1(t)$ & $s_2(t)$ as under:

O/P of M1 : $s_1(t) = E_c[1 + V_m(t) \cdot m] \cos 2\pi f_c t$

O/P of M2 : $s_2(t) = E_c[1 - mV_m(t)] \cos 2\pi f_c t$

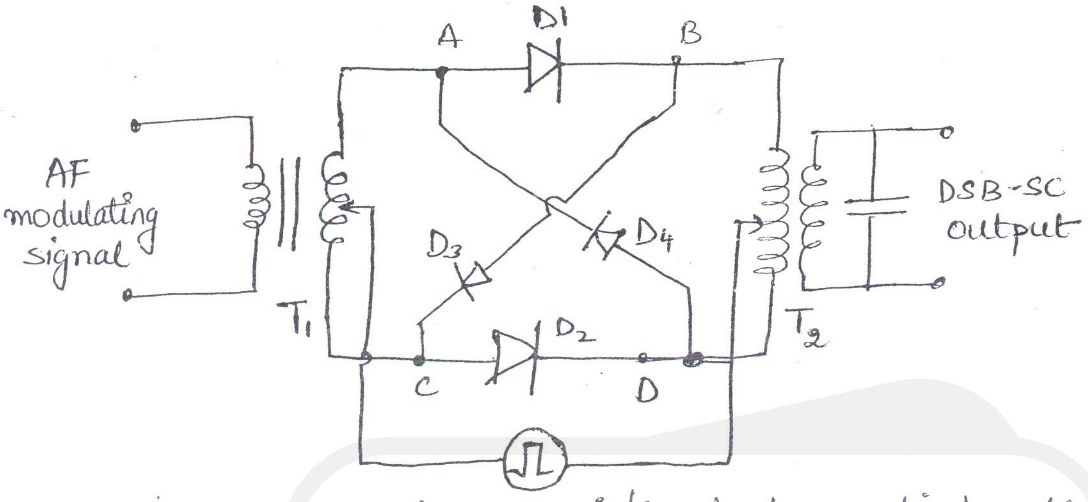
These are then applied to a subtractor and the subtractor produces the desired DSB-SC signal as

$$O/P = s_1(t) - s_2(t)$$

$$= E_c[1 + V_m(t)] \cos 2\pi f_c t - E_c[1 - mV_m(t) \cos 2\pi f_c t]$$

$$= 2m E_c \cos 2\pi f_c t$$

Ring Modulator

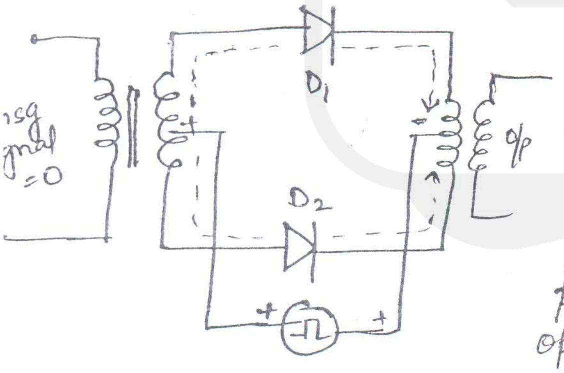


The Ring modulator consists of four diodes that form a ring in which they all point in the same way & hence the name.

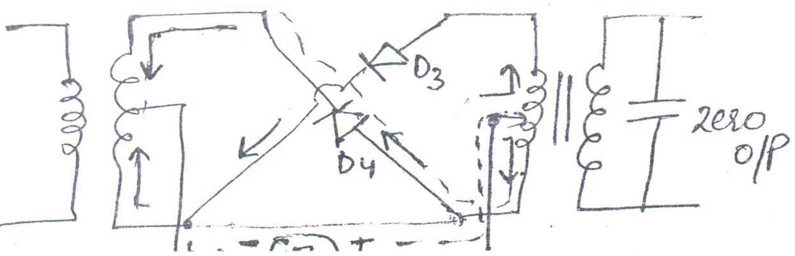
The diodes are controlled by a square wave carrier c(t) of frequency fc, which is applied longitudinally by means of two centre-tapped transformers.

The DSB-SC output is obtained at the secondary of the RF transformer T_2.

To understand how carrier suppression takes place, let us assume that the modulating signal is absent and only carrier signal is applied



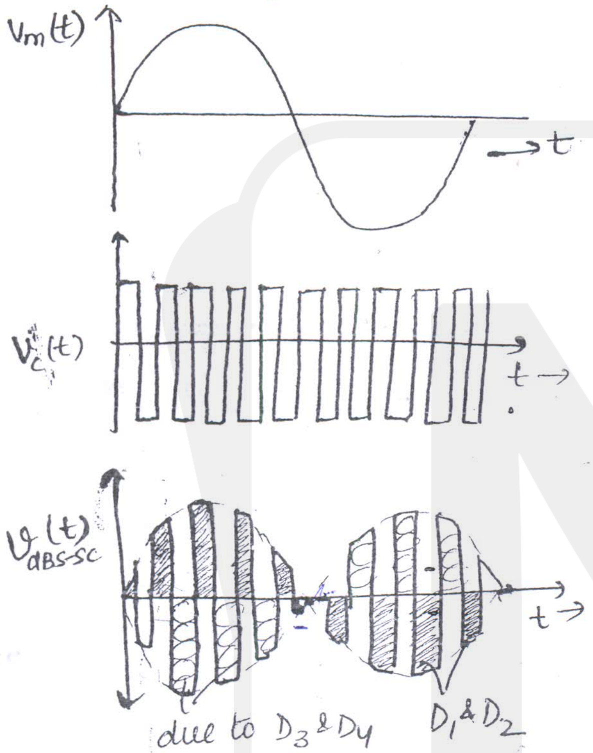
During the +ve half cycle of carrier. The diodes D_1 and D_2 are forward biased. Diodes D_3 and D_4 reverse biased. The direction of current flowing through 1° winding of transformer T_2 are equal & opposite to each other, ∴ the magnetic field produced by these currents are equal and opposite and cancel each other. Hence, the induced voltage in secondary winding is zero. Thus the carrier is suppressed in +ve half cycle of carrier



During the -ve half cycle of carrier, the current flowing in upper & lower halves of the 1° winding of T_2 are equal & opposite. Hence O/P = 0 ⇒

So when a message signal is applied through the input audio transformer T_1, diodes D_1 and D_2 are on during the +ve half cycle of the carrier and the secondary of T_1 is applied as it is across the primary of T_2.

In the negative half of the carrier, D3 and D4 are turned on & the secondary of T_1 is applied in a reversed manner across the primary of T_2. Thus the primary voltage of T_2 is negative and output voltage also becomes negative.



→ When modulating signal reverse the polarities, the operation of the ckt is same as in the positive half cycle with the only differ is that the diode pair $D_3 D_4$ will produce a positive output voltage whereas D_1 and D_2 will produce a negative o/p voltage.

So, during the +ve half cycle of the carrier, the message signal multiplied by +1 and in negative half cycle $x(t)$ is multiplied by −1. So, ring modulator

The ring modulator is sometimes referred to as a double-balanced modulator because it is balanced with respect to both the baseband signal and the square wave carrier.

Here $f_c > BW$, so as to prevent sideband overlap, which arises when sideband belonging to the adjacent harmonic frequency f_c and $3f_c$ overlap each other.

Thus provided we have $f_c > W$, we may use a band-pass filter of mid-band frequency f_c and bandwidth $2W$ to select the desired pair of sidebands around the carrier freq. f_c

Demodulation

Demodulation of a DSB-SC can be done by:

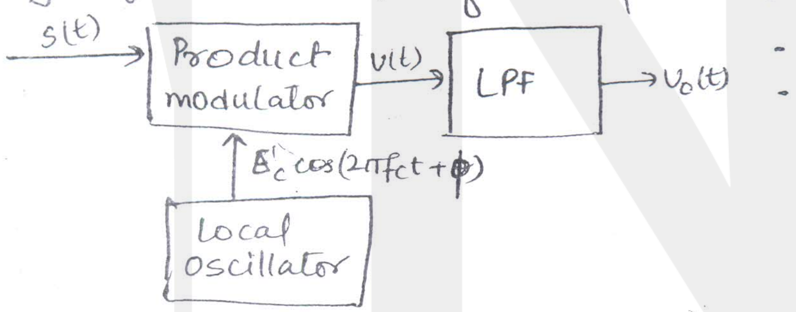
(a) Coherent Detection

(b) Costas Receiver

Coherent Detection → also known as Synchronous Detection

One problem associated with DSB-SC modulation is the phase reversal that occurs when the carrier signal waveform has a zero crossing at the half-cycle point. This causes the demodulation process to be slightly more complex to account for the phase reversal without losing message integrity.

In order to demodulate a DSB-SC signal, another product modulator acting at the original carrier frequency must be used. This is combined with a LPF to obtain the original message from the O/P of the product modulator.



It is assumed that the local oscillator signal is exactly coherent or synchronized in both phase & frequency, with the carrier wave used in the product modulator to generate s(t). This method of modulation is coherent detection or synchronous demodulation.

Here, O/P of local oscillator $= c'(t) = E_c' \cos(2\pi f_c t + \phi)$

The O/P of product modulator is given by

$$v(t) = s(t) \cdot c'(t)$$

where $s(t) = $ DSB-SC I/P $= v_m(t)\, E_c \cos 2\pi f_c t$

$$\text{or } v(t) = v_m(t) E_c \cos 2\pi f_c t \cdot E_c' \cos 2\pi f_c t + \phi$$

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\text{So, } v(t) = \frac{1}{2} v_m(t) E_c E_c' [\cos(4\pi f_c t + \phi) + \cos\phi]$$

$$= \underbrace{\frac{1}{2} E_c E_c' \cos\phi \, v_m(t)}_{\text{scaled version of msg } x(t)} + \underbrace{\frac{1}{2} v_m(t) \cos(2\pi f_c t + \phi) E_c E_c'}_{\text{Unwanted term}}$$

msg signal with $\frac{E_c \cos\phi}{2}$ amplitude

This signal is then passed through a low pass filter, which allows only the first term to pass through and reject the second term.

Therefore, Filter O/P is given by

$$V_0(t) = \frac{1}{2} E_c E_c' \cos\phi \, v_m(t)$$

The demodulated signal $v_0(t)$ is therefore proportional to m(t) wh the phase error ϕ is a constant.

→ The amplitude of this demodulated signal is maximum when $\phi = 0$ and it is minimum when $\phi = \pm\pi/2$. The zero demodula signal, which occurs for $\phi = \pm\pi/2$ represent the quadrature nul effect of the coherent detector.

→ So, the phase error ϕ in the local oscillator causes the detect O/P to be attenuated by a factor equal to $\cos\phi$. As long as t phase error ϕ is constant, the detector O/P provides an undistort version of the original baseband signal m(t).

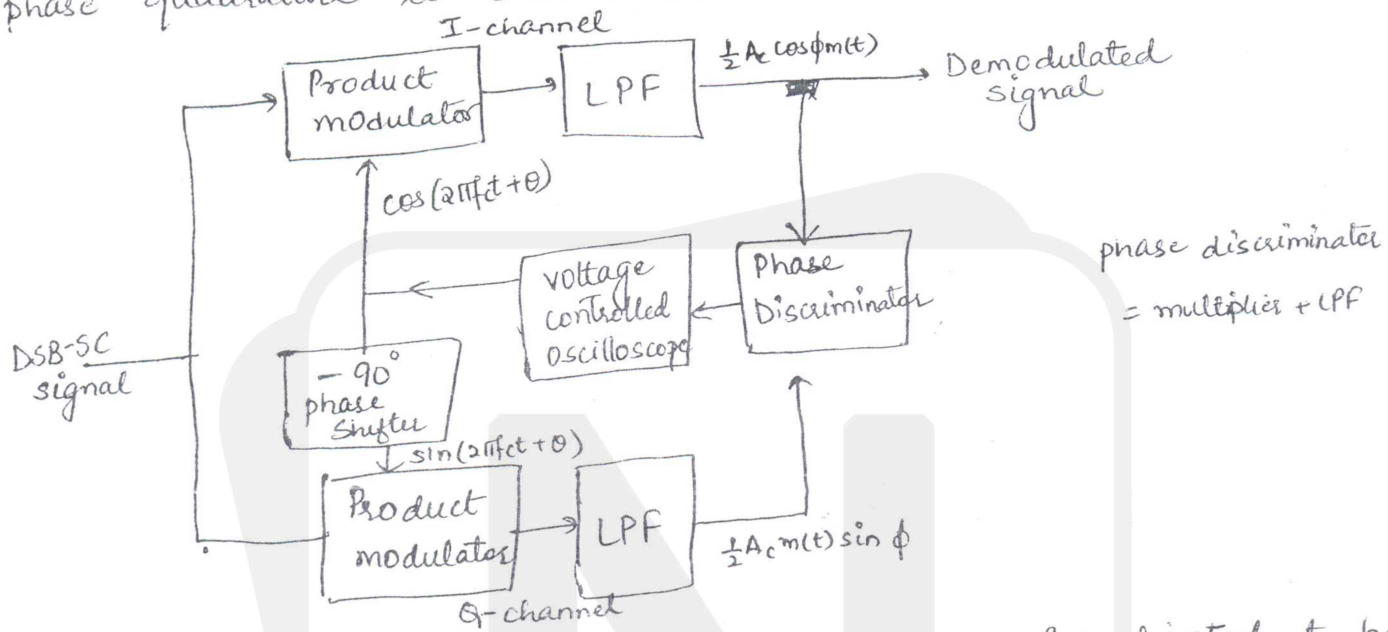
→ However, ϕ varies randomly with time resulting in varying the multiplying factor $\cos\phi$ and thus the detector O/P ~~and~~ which is undesirable

→ Thus provision must be made in the system to maintain the local oscillator in the receiver in perfect synchronism in both frequency and phase, with the carrier wave used to generate the DSB-SC modulated signal in the transmitter.

→ The resulting system complexity is the price that must be paid for suppressing the carrier wave to save transmitter power.

Costas Receiver

This receiver consists of two coherent detectors supplied with the same I/P signal, namely the incoming DSB-SC wave and with individual local oscillator signals that are in phase quadrature to each other



In this, the frequency of the local oscillator is adjusted to be the same as the carrier frequency f_c, which is assumed known a priori.

- The detector in the upper path is referred to as the in-phase coherent detector or I-channel and that in the lower path is referred to as quadrature-phase coherent detector or Q-channel

→ These two detectors are coupled together to form a negative feedback system designed in such a way as to maintain the local oscillator synchronous with the carrier wave.

→ Suppose that the local oscillator signal is of the same phase as the carrier signal $A_c \cos 2\pi f_c t$ used to generate the incoming DSB-SC wave. Under these conditions, we find that the I-channel output contains the desired demodulated signal $m(t)$ whereas the Q-channel O/P is zero due to the quadrature null effect of the Q-channel.

→ Now, the local ~~osc~~ oscillator phase drifts from its proper value by a small angle ϕ radians. The I channel output will remain essentially unchanged, but there will now be some signal appearing at the Q-channel output, which is proportional to

$\sin\varphi \approx \varphi$ for small φ.

→ The Q-channel o/p will have the same polarity as the I-channel o/p for one direction of local oscillator phase drift and opposite polarity polarity for the opposite direction of local oscill phase drift

→ Thus by combining the I- and Q-channel o/p channel in a phase discriminator, a dc signal is obtained that automatica corrects for local phase errors in the voltage controlled oscillosc

→ The phase control in the costas receiver ceases with the modulation and the phase lock has to be established with the reappearance of modulation. However, this is not a serious problem when receiving voice transmission, because the lock-process normally occurs so rapidly that no distortion is percep

Single-Side Band Modulation

In a band-pass channel of bandwidth 2W, we can transmit two independent signals, each of bandwidth W. However, in double-sideband modulation, we are transmitting only one such signal, where due to the symmetry of the DSB signal about the carrier frequency, the same information is transmit in the upper and lower sidebands, and only one of the sidebands need to be transmitted.

Conceptually, generation of a SSB signal is straight-forward. We first generate a double-sideband ~~system~~ signal and then apply an ideal band pass filter to the result ~~of~~ with cutoff frequencies of fc and fc+W for the upper side band for instance. But, practically the approximate construction of an ideal filter i very difficult.

Methods of Generation of SSB modulated wave

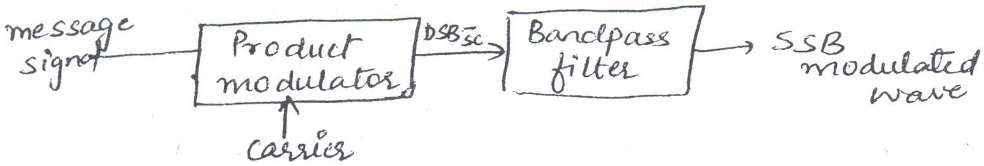
The SSB modulated wave can be generated by

a) Frequency Discrimination method

b) Phase Discrimination method

Frequency Discrimination Method

- also known as filter method.



This modulator consists of a product modulator, carrier oscillator and bandpass filter designed to pass the required sideband.

At the O/P of the product modulator, we get the DSB-SC modulated wave which consists the two sidebands only.

The bandpass filter will pass only one of these sidebands and produce the SSB modulated wave at its O/P.

→ The approximate construction of an ideal filter is very difficult.

SSB- find its greatest application in the transmission of analog voice signal.

→ The filter must satisfy the following requirements :

a) The desired sideband lies inside the passband of the filter

b) The unwanted sideband lies inside the stopband of the filter.

This indicates that the filter's transition band, seperating the passband from the stopband is twice is lowest freq. component (2f) of the message signal. This non-zero transition BW greatly simplfies the design of SSB filter.

Advantages of Frequency Discrimination

→ gives adequate sideband suppression

→ BW is sufficiently flat & wide.

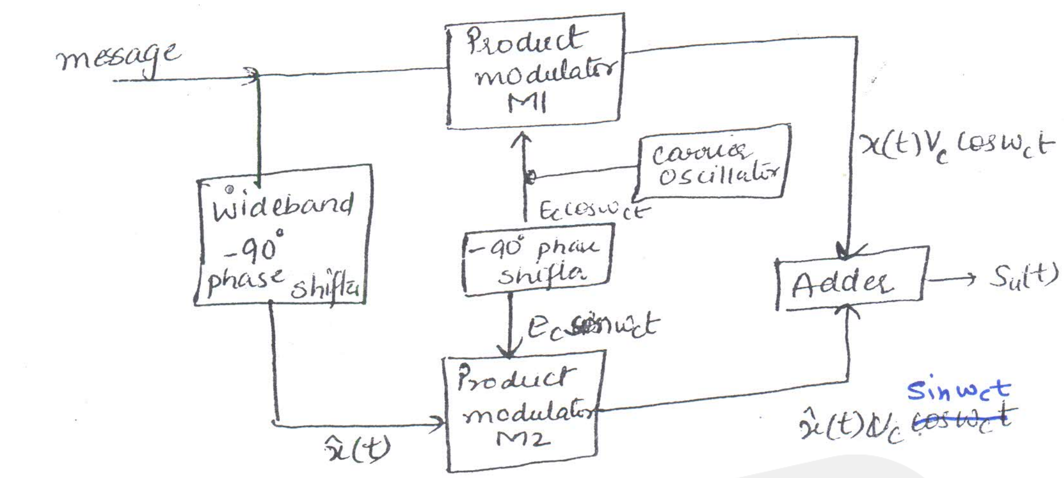
Disadvantages :-

1) Due to inability of the system to generate SSB at high radio frequencies, the frequency up conversion is necessary.

2) Low audio frequencies can not be used as the filter becomes bulky.

3. Two expensive filters are to be used one for each sideband

Phase Discrimination method



* The system is used for the suppression of lower sideband.

→ The message signal $x(t)$ is applied directly to the product modulator M_1 and through a $-90°$ phase shifter to the product modulator M2. Hence we get Hilbert transform $\hat{x}(t)$ at the O/P of wideband $-90°$ phase shifter.

$$\text{O/P of } M1 = x(t)\ V_c \cos w_c t$$

$$\text{" " } M_2 = \hat{x}(t)\ V_c \sin 2\pi f_c t$$

The O/P M_1 and M2 are applied to an adder

$$\text{Adder O/P} = x(t) V_c \cos w_c t \mp \hat{x}(t) V_c \sin w_c t$$

(-ve sign due the quadrature component)

$$\boxed{S_u(t) = V_c [x(t) \cos w_c t \mp \hat{x} V_c \sin w_c t]}$$

||ely SSB wave containing only LSB can be represented as

$$\boxed{S_l(t) = \frac{E_c}{2} [x(t) \cos 2\pi f_c t + \hat{x}(t) \sin 2\pi f_c t]}$$

where $x(t)$ = msg signal = Inphase component
$\hat{x}(t)$ = quadrature component = Hilbert transform of $x(t)$

Advantages of SSB

1. Less BW requirement as SSB requires a BW of f_m
2. Due to the transmission of one sideband, at 100% modulation the percentage power saving is 83.33%
3. Reduced interference of noise.

Disadvantages of SSB

1. Generation & Reception of SSB signal is complicated
2. The SSB transmitter and receiver need to have an excellent frequency stability. A slight change in frequency will hamper the quality of transmitted & received signal. Therefore,

SSB is not generally used for the transmission of good quality music. It is used for speech transmission

Applications:-

→ The application areas are land and air mobile communication, telemetry, military communications, navigation and amateur radio

→ Many of these applications are point to point communication

→ Most prevalent use of single sideband supressed carrier system employing FDM such as long-distance telephonic systems.

There are many advantages of SSB system over DSBFC system still it is not used widely in radio broadcasting applications. The two reasons for it are:-

(i) As the SSB transmitter and receiver require an excellent frequency stability, a small frequency shift in the system result in degradation in the quality of the transmitted signal. Thus, it is not possible to transmit good quality music

(ii) It is not possible to design a tunable receiver oscillator with very high frequency stability. Now with the advant of the freq. synthesizers, this has become possible. But such receivers are too expensive.

Demodulation of SSB

Demodulation of SSB wave can be done by

(a) Pilot carrier SSB system

(b) Coherent SSB demodulation

Pilot carrier SSB system

In a practical SSB system, the carrier is not completely suppressed. Instead a pilot carrier is transmitted along with the desired sideband. The pilot carrier is added to the SSB signal which is to be transmitted. This reinsertion of the carrier is done after the unwanted sideband is removed

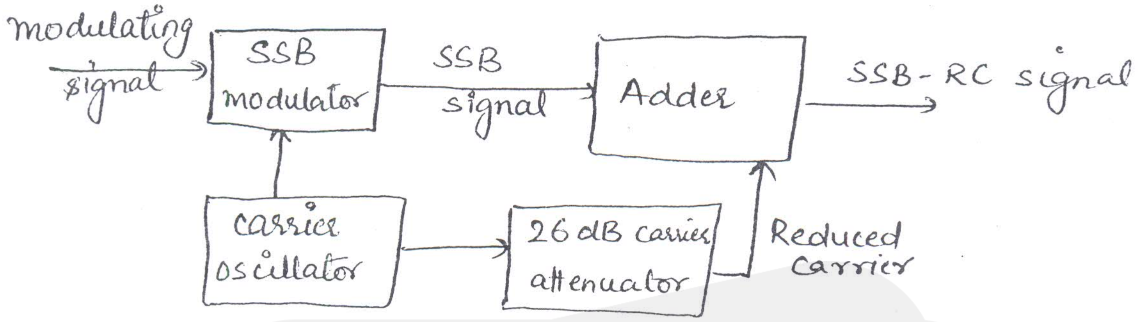
The reinserted carrier is at a very low power level, 16 to 26 dB below the normal carrier level before suppression.

The freq. of the pilot carrier is same as that of the original carrier

The pilot carrier acts as a reference signal to help the demodulation process in the receiver. The receiver can then use the automatic frequency control technique (AFC)

The pilot carrier SSB systems are identical to other SSB systems. They are widely used in transmarine point to point radio telep and mobile communications.

- This system is also called Reduced carrier SSB or SSB-RC sys

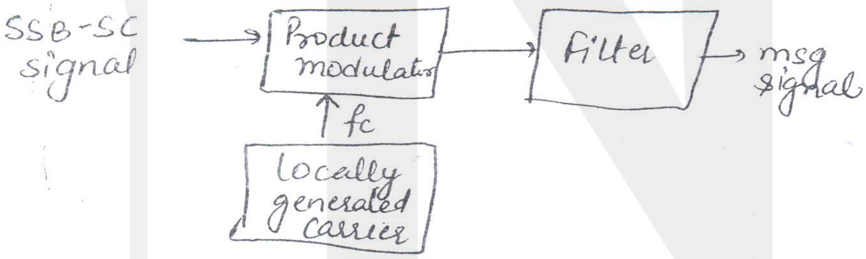


Demodulation of SSB wave

The special requirements of SSB receiver are as follows:

1) High Reliability
2) Excellent suppression of adjacent signals
3) High signal to noise ratio
4) Ability to modulate SSB.

Coherent SSB demodulation



The received SSB signal is first multiplied by a locally generated carrier signal. This locally generated carrier signal should have exactly the same frequency as that of the suppressed carrier. The product modulator multiplies the two signals at its input and the product signal is passed through a LPF with bandwidth equal to

Let the SSB wave at the I/P be given by

$$s(t) = \frac{1}{2} V_c [x(t) \cos 2\pi f_c t \pm \hat{x}(t) \sin 2\pi f_c t]$$

Locally generated carrier is $\cos 2\pi f_c t$

∴ O/P of product modulator is given by

$$v(t) = s(t) \cos 2\pi f_c t$$

$$v(t) = \frac{1}{2} V_c [x(t) \cos 2\pi f_c t \pm x(t) \sin(2\pi f_c t)] \cos 2\pi f_c t$$

$$V(t) = \frac{1}{2} V_c x(t) \cos 2\pi f_c t \cdot \cos 2\pi f_c t \pm \frac{V_c}{2} \hat{x}(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\therefore v(t) = \frac{1}{4} V_c\, x(t)\left[\cos 4\pi f_c t + \cos 0\right] \pm \frac{1}{4} V_c\, \hat{x}(t)\left[\sin 4\pi f_c t - \sin 0\right]$$

$$v(t) = \underbrace{\frac{1}{4} V_c\, x(t)}_{\text{scaled message signal}} + \underbrace{\frac{1}{4} V_c\left[x(t)\cos 4\pi f_c t \pm \hat{x}(t)\sin 4\pi f_c t\right]}_{\text{unwanted}}$$

when this signal is passed through a filter, we get the scaled message signal.

$$V_0(t) = \frac{1}{2} V_c\, x(t)$$

* In practice, the locally generated ~~phase~~ carrier is not in perfect synchronization with the carrier signal, and hence a phase error ϕ may arise in locally generated wave.

The detector O/P will get modified due to phase error

$$V_0(t) = \frac{1}{4} V_c\, x(t)\cos\phi \pm \frac{1}{4} V_c\, \hat{x}(t)\sin\phi$$

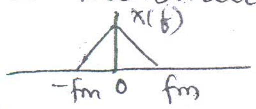
+ sign → SSB with USB
− sign → SSB with LSB

Due to the presence of hilbert transform $\hat{x}(t)$ in the O/P, the detector o/p will suffer from phase distortion. Such a phase distortion does not have serious effects with the voice communication. But in the transmission of music & video, it will have untolerable effects.

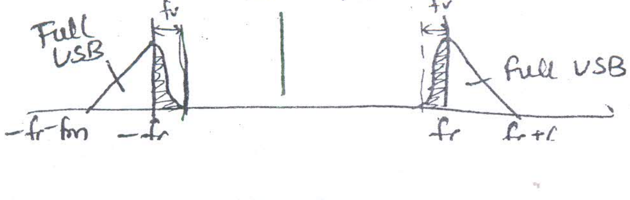
Vestigial Sideband Transmission

In this form of amplitude modulation, the carrier and one complete sideband are transmitted, but only part of the second sideband is transmitted.

The carrier is transmitted at full power.



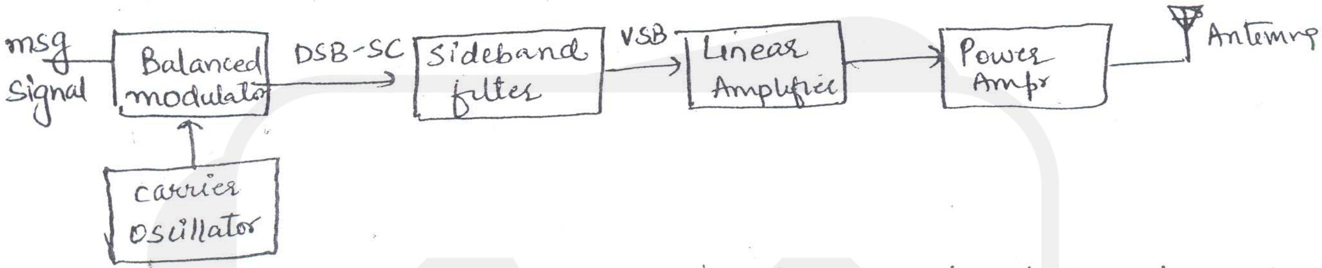
← spectrum of msg signal



Spectrum of VSB

In a VSB, the streingent frequency-response requirement on the sideband filter in SSB-SC system can be relaxed by allowing a part of the signal (vestige) to appear in the O/P of the modulator. Due to this, the design of the sideband filter is designed to a great extend. But the BW of the signal is increased slightly

Generation of VSB modulated wave

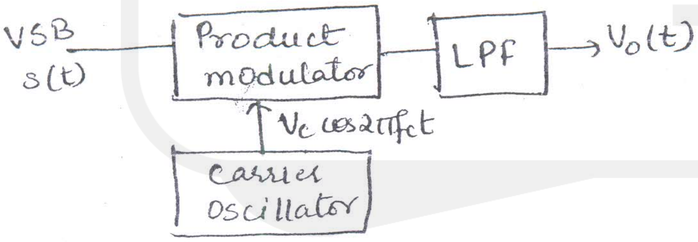


Let $H(f)$ denote the transfer function of the filter following the product modulator. The spectrum of the modulated signal $s(t)$ produced by passing the freq-shifted signal $u(t)$ through the filter $H(f)$ is given by

$$S(f) = U(f)H(f)$$
$$= \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]H(f)$$

where $M(f)$ is the fourier transform of baseband signal $m(t)$

Detection of VSB



The O/P of the product modulator is given by

$$m(t) = s(t) \cdot c(t)$$
$$= s(t)\, V_c \cos 2\pi f_c t$$

Taking the fourier transform of both sides

$$M(f) = S(f) \otimes \left[\tfrac{1}{2}\delta(f_c + f) + \tfrac{1}{2}\delta(f - f_c)\right]$$
$$= \tfrac{1}{2}S(f+f_c) + \tfrac{1}{2}S(f-f_c)$$

But $S(f) = \frac{V_c}{2}\left[X(f-f_c) + X(f+f_c)\right] H(f)$

$$M(f) = \frac{V_c}{2}\left[X(f-2f_c)H(f-f_c) + X(f+2f_c)H(f+f_c)\right] + \frac{V_c}{4} X(f)\left[H(f-f_c)H(f+f_c)\right]$$

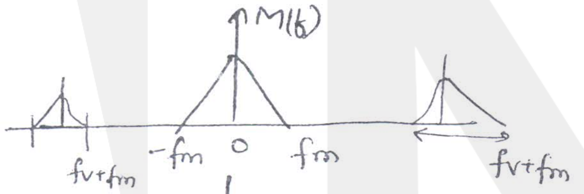
The high freq. components are removed by LPF

So O/P $= \frac{V_c}{4} X(f)\left[H(f-f_c) + H(f+f_c)\right]$

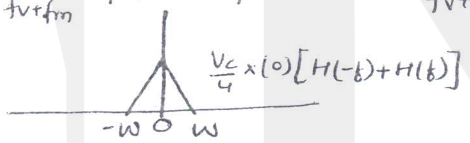
If we want to obtain the undistorted message signal $x(t)$ at the O/P of the demodulator, then $V_o(f)$ should be scaled version of $X(f)$

∴ For this the transfer function $H(f)$ should satisfy the condition

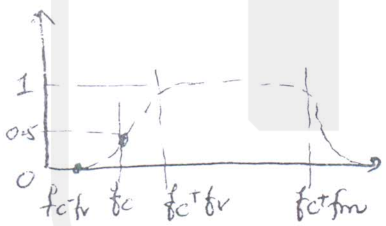
$$H(f-f_c) + H(f+f_c) = 2H(f+f_c)$$



→ spectrum of product modulator O/P



→ Spectrum of VSB demodulator



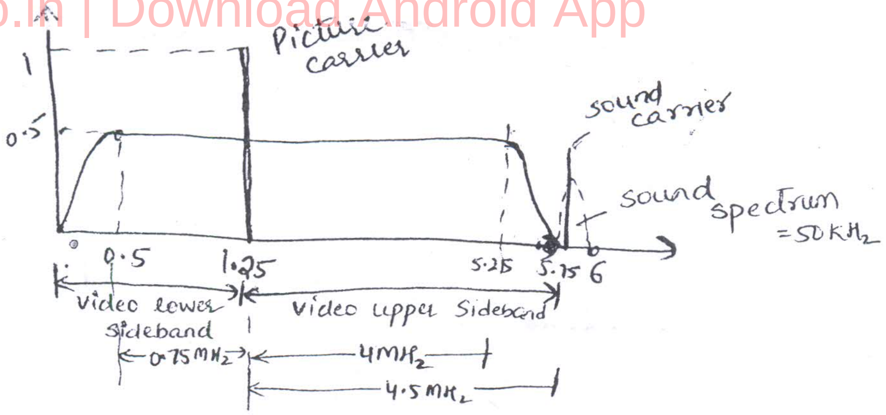
→ Freq. response of the sideband shaping filter for obtaining the VSB modulated wave with vestige of LSB.

Application of VSB :-

→ mainly used for the TV transmission all over the world. In the TV transmission, it is necessary to transmit the video information and audio information simultaneously. The bandwidth of the video signal is about 4.2 MHz.

Hence if we use DSB-FC i.e. AM transmission for TV signal, then the system BW will be $4.2 \times 2 = 8.4$ MHz.

If we add the guard band inserted b/w the sound & picture carriers and the interchannel guard band, then the BW will be about 9 MHz. This is a very large BW and should be



So, using a VSB, the total BW of a TV channel reduces to 6 MHz from 9 MHz.

Advantages of VSB

1) Reduced BW
2) Practical filters used due to partial suppression of LSB.

Performance Comparison of AM

Parameter of comparison	Double Sideband Full carrier DSB-FC	Double sideband suppressed carrier DSB-SC	Single Side Band SSB	Vestigial sideband VSB
Carrier suppression	No	Yes	Yes	No
Sideband suppression	No	No	Yes (one sideband)	Yes (one sideband is partially suppressed)
Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
Transmission Efficiency	Minimum	Moderate	Maximum	Moderate
Application	Radio Broadcasting	Radio Broadcasting	Point to point mobile communication	TV

Representation of Bandpass Signals and Systems

Bandpass or Narrowband signal

It is a signal $x(t)$ whose frequency domain representation $X(f)$ is non-zero for frequencies in a usually small neighborhood of some high frequency f_0
i.e. $X(f) \equiv 0$ for $|f - f_0| \geq W$, where $W < f_0$

Bandpass System:

It is a system which passes signal with frequency component in the neighborhood of some high frequency f_0 i.e.

$$H(f) = 1 \quad \text{for } |f - f_0| \leq W$$

and highly attenuates frequency components outside of this frequency band.

Alternatively, we may say that a bandpass system is one whose impulse response is a bandpass signal.

In the above defination, f_0 need not be the center of the signal BW, or be located in the signal BW at all.

f_0 is usually referred to as the central frequency of the bandpass signal.

A monochromatic signal is a bandpass signal for which $W = 0$

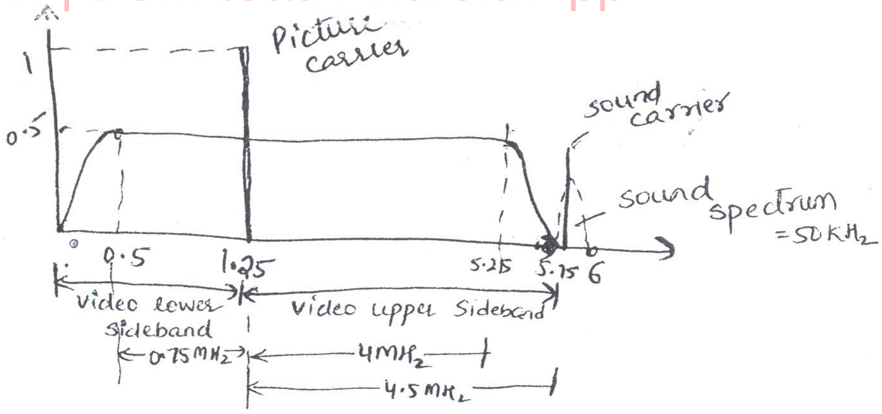
Representation of Bandpass Signal (using Hilbert Transform)

Let $x(t) = A\cos(2\pi f_0 t + \theta)$ be a monochromatic signal

Phasor component of signal $= X = Ae^{j\theta}$ → contains information about the amplitude & phase of the signal but does not have any information concerning the frequency of it

To find the O/P of a LTI system driven by this sinusoidal signal, it is enough to multiply the phasor of the excitation signal by the value of the freq. response of the system computed at the I/P frequency to obtain the phasor corresponding to the output.

O/P phasor = Freq. response of system × I/P phasor



So, using a VSB, the total BW of a TV channel reduces to 6 MHz from 9 MHz.

Advantages of VSB

1) Reduced BW
2) Practical filters used due to partial suppression of LSB.

Performance Comparison of AM

Parameter of comparison	Double Sideband Full carrier DSB-FC	~~Single~~ Double Sideband Suppressed Carrier DSB-SC	Single Side Band SSB	Vestigial Sideband VSB
Carrier suppression	None	Yes	Yes	No
Sideband suppression	No	No	Yes (one sideband)	Yes (one sideband is partially suppressed)
Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
Transmission Efficiency	Minimum	Moderate	Maximum	Moderate
Application	Radio Broadcasting	Radio Broadcasting	Point to point mobile communication	TV

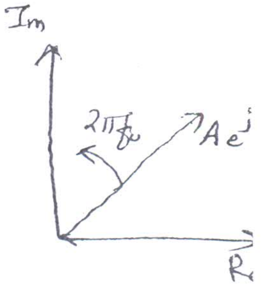
To obtain the phasor corresponding to the input, we first introduce the signal $z(t)$ as

$$z(t) = A e^{j(2\pi f_0 t + \theta)}$$
$$= A\cos(2\pi f_0 t + \theta) + jA\sin(2\pi f_0 t + \theta)$$
$$= x(t) + j x_q(t)$$

where $x_q(t) \to 90°$ phase shifted version of the original signal

So, $z(t)$ represents a vector rotating at an angular frequency equal to $2\pi f_0$.

So, X is obtained from $z(t)$ by deleting the rotation at the angular frequency $2\pi f_0$ or by equivalently rotating the vector corresponding to $z(t)$ at an angular frequency equal to $2\pi f_0$ in the opposite direction which is equivalent to multiplying by $e^{-j2\pi f_0 t}$ or



$$X = z(t) \cdot e^{-j2\pi f_0 t}$$

In the freq. domain it is equivalent to shifting $Z(f)$ to the left by f_0.

The frequency domain representation of $Z(f)$ is obtained by deleting the negative frequencies from $X(f)$ and multiplying the positive frequencies by two.

$$Z(f) = 2u_{-1}(f)X(f)$$

The signal $z(t)$ ~~is~~ called the analytic signal corresponding to $x(t)$ or pre-envelope of $x(t)$

To obtain the time domain representation of $z(t)$, we first start with finding a signal whose fourier transform is $u_{-1}(f)$

$$\mathcal{F}[u_{-1}(f)] = \frac{1}{2}\delta(t) + \frac{1}{j2\pi f}$$

Applying duality theorem

$$\mathcal{F}\left[\frac{1}{2}\delta(t) + \frac{j}{2\pi t}\right] = u_{-1}(f)$$

Using convolution

$$z(t) = \left(\delta(t) + \frac{j}{\pi t}\right) \circledast x(t)$$
$$= x(t) + j\frac{1}{\pi t} \circledast x(t)$$
$$= x(t) + j\hat{x}(t)$$

where $\hat{x}(t) = \frac{1}{\pi t} \circledast x(t)$

Comparing

$$z(t) = \underbrace{A\cos(2\pi f_0 t + \theta)}_{x(t)} + j\,\underbrace{A\sin(2\pi f_0 t + \theta)}_{\hat{x}(t)}$$

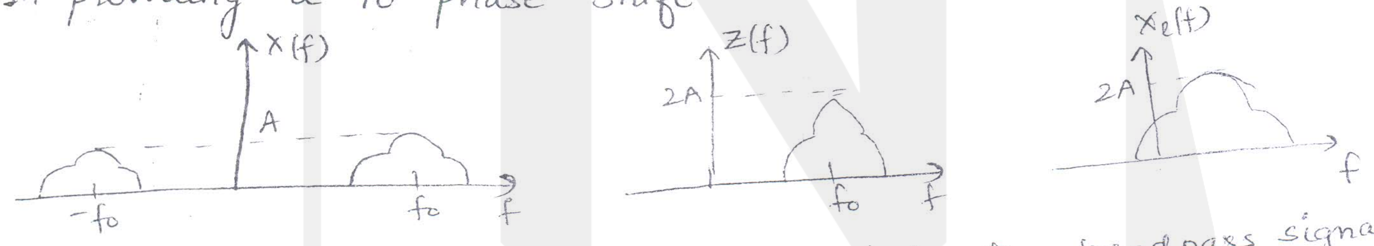
Here $\hat{x}(t)$ plays the same role as $A\sin(2\pi f_0 t + \theta)$
It is called Hilbert transform of $x(t)$
Here no change of domain is involved.
Infact, Hilbert transform is a simple filter

Hilbert's transform work in freq. domain

$$\mathcal{F}\left[\frac{1}{\pi t}\right] = -j\,\mathrm{sgn}(f)$$

$$= \begin{cases} -j & f>0 \\ 0 & f=0 \\ +j & f<0 \end{cases} = \begin{cases} e^{-j\pi/2} & f>0 \\ 0 & f=0 \\ e^{j\pi/2} & f<0 \end{cases} = e^{-j\pi/2\,\mathrm{sgn}(f)}$$

So, Hilbert transform is equivalent to a $-\frac{\pi}{2}$ phase shift for positive frequencies and $+\frac{\pi}{2}$ phase shift for negative frequencies and can be represented by a filter with transfer function $H(f) = -j\,\mathrm{sgn}(f)$
This filter is called a quadrature filter, emphasizing its role in providing a 90° phase shift



To obtain the equivalent of a 'phasor' for the bandpass signal we have to shift the spectrum of $z(t)$ i.e. $Z(f)$ to the left by f_0 to obtain a signal denoted by $x_l(t)$, which is low pass representation of the bandpass signal $x(t)$

$$X_l(f) = Z(f+f_0) = 2u_{-1}(f+f_0)\,X(f+f_0)$$

$$x_l(t) = z(t)\,e^{-j2\pi f_0 t}$$

$$z(t) = x(t) + j\,\hat{x}(t)$$

$$= x_l(t)\,e^{j2\pi f_0 t}$$

$$= (x_c(t) + j\,x_s(t))\,e^{j2\pi f_0 t}$$

$$= (x_c(t)\cos 2\pi f_0 t - x_s(t)\sin 2\pi f_0 t) + j\,[x_c(t)\sin 2\pi f_0 t + x_s(t)\cos 2\pi f_0 t]$$

So, equating the real & imaginary parts

$$x(t) = x_c(t)\cos 2\pi f_0 t - x_s(t)\sin 2\pi f_0 t$$

$$\hat{x}(t) = x_c(t)\sin 2\pi f_0 t + x_s(t)\cos 2\pi f_0 t$$

We define $V(t)$, the envelope of $x(t)$ as

$$V(t) = \sqrt{x_c^2(t) + x_s^2(t)}$$

and $\theta(t)$ = phase of $x(t)$ = $\arctan \frac{x_s(t)}{x_c(t)}$

So, $x_l(t) = V(t) \cdot e^{j\theta(t)}$.

which looks more like the familiar phase relation $X = Ae^{j\theta}$. The only difference is that in this case the envelope $V(t)$ and phase $(\theta(t))$ are both slowly time-varying functions

So, $x_l(t) = V(t) \cdot e^{j\theta(t)}$

$$z(t) = x(t) + j\hat{x}(t)$$
$$= x_l(t) \cdot e^{j2\pi f_0 t}$$
$$= V(t) \cdot e^{j\theta(t)} \cdot e^{j2\pi f_0 t}$$
$$= V(t)\cos[2\pi f_0 t + \theta(t)] + jV(t)\sin[2\pi f_0 t + \theta(t)]$$

So, $x(t) = V(t)\cos[2\pi f_0 t + \theta(t)]$

$\hat{x}(t) = V(t)\sin[2\pi f_0 t + \theta(t)]$