Base 2

THE HEXADECIMAL BASE

Assembly language programming
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Objectives

- Introduction to base 16.
- You will understand the special relation between base 16 and base 2.
- You will learn about the importance of base 16.
- You will recognize when would be a good time to use base 16.

Base 16

- Base 16 is called the **Hexadecimal** base, or just **hex**.
- Base 16 contains 16 symbols:
  - 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Examples:
  - $10_{16} = 0 \cdot 16^0 + 1 \cdot 16^1 = 16_{10}$
  - $2A_{16} = A \cdot 16^0 + 2 \cdot 16^1 = 10 \cdot 16^0 + 2 \cdot 16^1 = 42_{10}$
- Usually marked with a "$0x$" (Zero and x) prefix in high level programming languages.
  - $0x10, 0x2a$

Hex to Binary

- How could we convert a hexadecimal number to a binary number?
- We could use the **Remainder Evaluation** method.
  - Example: Convert $AB1C_{16}$ to base 2.

AB1C	0
558E	0
2AC7	1
1563	1

AB1	1
558	0
2AC	0
156	0

AB	1
55	1
2A	0
15	1

A	0
5	1
2	0
1	1

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$\boldsymbol{C_{16} = 1100_2}$	$\boldsymbol{1_{16} = 0001_2}$	$\boldsymbol{B_{16} = 1011_2}$	$\boldsymbol{A_{16} = 1010_2}$

Hex to Binary

- How could we convert a hexadecimal number to a binary number?
- We could use the **Remainder Evaluation** method.
  - Example: Convert $AB1C_{16}$ to base 2.

$C_{16} = 1100_2$	$1_{16} = 0001_2$	$B_{16} = 1011_2$	$A_{16} = 1010_2$

- $AB1C_{16} = (1010\ 1011\ 0001\ 1100)_2$
- Every Hex digit is represented by exactly 4 bits.

Hex to Binary (Cont.)

- Base 16 and Base 2 "Get along", because $2^4 = 16$.
  - There is a bonus lesson that explains this more rigorously.
- In order to convert hexadecimal number to a binary number, it is enough to convert every hex digit into 4 bits.
- Example:
  - $0xAD5D = ?$
    - $0xA = 10_{10} = 1010_2$
    - $0xD = 13_{10} = 1101_2$
    - $0x5 = 5_{10} = 0101_2$
  - So $0xAD5D = (1010\ 1101\ 0101\ 1101)_2$

Binary to Hex

- Example: Convert the number 101101011_2 to hexadecimal base.
  - We first divide the bits of the number into groups of 4 (beginning from the right): $(1\ 0110\ 1011)_2$.
  - The last set of bits might be of size less than 4. We could imagine that there are leading zeroes: $(0001\ 0110\ 1011)_2$.
  - We convert every quadruple of bits into one hex digit:
    - $0001_2 = 0x1$
    - $0110_2 = 0x6$
    - $1011_2 = 11_{10} = 0xB$
  - We get that $101101011_2 = 0x16B$

Bases that get along

- The same phenomenon of easy conversion happens for every two bases b, c Where $b = c^d$ for some d.
  - In base 4, every digit is represented by exactly 2 bits.
  - In base 8, every digit is represented by exactly 3 bits.
- Example:
  - Convert the number 1317_8 to binary.
    - $1_8 = 001_2$
    - $3_8 = 011_2$
    - $7_8 = 111_2$
  - So $1317_8 = (001\ 011\ 001\ 111)_2$

Why would I care about hex

- In order to talk to the processor, you have to understand binary.
- Binary numbers could sometimes be too long.
  - Many computers today work with groups of 32 bits or 64 bits.
- Hex numbers have more symbols, thus much shorter than their Binary equivalent.
  - Which one is easier to read?
    - $11011110101011011011111011101111_2$
    - $0xDEADBEEF$
- Conversion between Hex and Binary is immediate.
  - Conversion is done digit wise (unlike the general method of conversion between bases).

Exercises

- Basic conversion exercises.
  - **Use a pen and paper** to solve. Use a calculator / Computer to check your answers.
- Interesting divisibility rules in base 16.
- Some more…

Hex to Binary (Explained)

- Question: Why is every hex digit represented by 4 binary digits?
- Representation in base 16:
  - $(\ldots a_4 a_3 a_2 a_1 a_0)_{16} = a_0 \cdot 16^0 + a_1 \cdot 16^1 + a_2 \cdot 16^2 + a_3 \cdot 16^3 + \cdots$

Hex to Binary (Explained)

- Representation in base 16:
  - $(\ldots a_4 a_3 a_2 a_1 a_0)_{16} = a_0 \cdot 16^0 + a_1 \cdot 16^1 + a_2 \cdot 16^2 + a_3 \cdot 16^3 + \cdots$
- We could represent each hex digit using 4 bits:
  - $a_0 = \left(b_{3,0} b_{2,0} b_{1,0} b_{0,0}\right)_2$
  - $a_1 = \left(b_{3,1} b_{2,1} b_{1,1} b_{0,1}\right)_2$
  - $a_2 = \left(b_{3,2} b_{2,2} b_{1,2} b_{0,2}\right)_2$
  - …

Hex to Binary (Explained)

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} =$$
$$a_0 \cdot 16^0 +$$
$$a_1 \cdot 16^1 +$$
$$a_2 \cdot 16^2 +$$
$$a_3 \cdot 16^3 +$$
$$\ldots$$

$$a_0 = \left(b_{3,0} b_{2,0} b_{1,0} b_{0,0}\right)_2$$
$$a_1 = \left(b_{3,1} b_{2,1} b_{1,1} b_{0,1}\right)_2$$
$$a_2 = \left(b_{3,2} b_{2,2} b_{1,2} b_{0,2}\right)_2$$
$$a_3 = \left(b_{3,3} b_{2,3} b_{1,3} b_{0,3}\right)_2$$
$$\ldots$$

Hex to Binary (Explained)

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} = \\ a_0 \cdot 16^0 + \\ a_1 \cdot 16^1 + \\ a_2 \cdot 16^2 + \\ a_3 \cdot 16^3 + \\ \ldots$$

$$a_0 = (b_{3,0} b_{2,0} b_{1,0} b_{0,0})_2 \\ a_1 = (b_{3,1} b_{2,1} b_{1,1} b_{0,1})_2 \\ a_2 = (b_{3,2} b_{2,2} b_{1,2} b_{0,2})_2 \\ a_3 = (b_{3,3} b_{2,3} b_{1,3} b_{0,3})_2 \\ \ldots$$

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} = \\ (b_{3,0} b_{2,0} b_{1,0} b_{0,0})_2 \cdot 16^0 + \\ (b_{3,1} b_{2,1} b_{1,1} b_{0,1})_2 \cdot 16^1 + \\ (b_{3,2} b_{2,2} b_{1,2} b_{0,2})_2 \cdot 16^2 + \\ (b_{3,3} b_{2,3} b_{1,3} b_{0,3})_2 \cdot 16^3 + \\ \ldots$$

Hex to Binary (Explained)

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} =$$
$$a_0 \cdot 16^0 +$$
$$a_1 \cdot 16^1 +$$
$$a_2 \cdot 16^2 +$$
$$a_3 \cdot 16^3 +$$
$$\ldots$$

$$a_0 = \left(b_{3,0} b_{2,0} b_{1,0} b_{0,0}\right)_2$$
$$a_1 = \left(b_{3,1} b_{2,1} b_{1,1} b_{0,1}\right)_2$$
$$a_2 = \left(b_{3,2} b_{2,2} b_{1,2} b_{0,2}\right)_2$$
$$a_3 = \left(b_{3,3} b_{2,3} b_{1,3} b_{0,3}\right)_2$$
$$\ldots$$

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} =$$
$$\left(b_{3,0} b_{2,0} b_{1,0} b_{0,0}\right)_2 \cdot 16^0 +$$
$$\left(b_{3,1} b_{2,1} b_{1,1} b_{0,1}\right)_2 \cdot 16^1 +$$
$$\left(b_{3,2} b_{2,2} b_{1,2} b_{0,2}\right)_2 \cdot 16^2 +$$
$$\left(b_{3,3} b_{2,3} b_{1,3} b_{0,3}\right)_2 \cdot 16^3 +$$
$$\ldots$$

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} =$$
$$\left(b_{0,0} 2^0 + b_{1,0} \cdot 2^1 + b_{2,0} 2^2 + b_{3,0} 2^3\right) \cdot 16^0 +$$
$$\left(b_{0,1} 2^0 + b_{1,1} \cdot 2^1 + b_{2,1} 2^2 + b_{3,1} 2^3\right) \cdot 16^1 +$$
$$\left(b_{0,2} 2^0 + b_{1,2} \cdot 2^1 + b_{2,2} 2^2 + b_{3,2} 2^3\right) \cdot 16^2 +$$
$$\left(b_{0,3} 2^0 + b_{1,3} \cdot 2^1 + b_{2,3} 2^2 + b_{3,3} 2^3\right) \cdot 16^3 +$$
$$\ldots$$

Hex to Binary (Explained)

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} =$$
$$\left(b_{0,0}2^0 + b_{1,0} \cdot 2^1 + b_{2,0}2^2 + b_{3,0}2^3\right) \cdot 16^0 +$$
$$\left(b_{0,1}2^0 + b_{1,1} \cdot 2^1 + b_{2,1}2^2 + b_{3,1}2^3\right) \cdot 16^1 +$$
$$\left(b_{0,2}2^0 + b_{1,2} \cdot 2^1 + b_{2,2}2^2 + b_{3,2}2^3\right) \cdot 16^2 +$$
$$\left(b_{0,3}2^0 + b_{1,3} \cdot 2^1 + b_{2,3}2^2 + b_{3,3}2^3\right) \cdot 16^3 +$$
$$\ldots$$

- We will use the fact that $2^4 = 16$!

Hex to Binary (Explained)

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} =$$
$$(b_{0,0}2^0 + b_{1,0} \cdot 2^1 + b_{2,0}2^2 + b_{3,0}2^3) \cdot 16^0 +$$
$$(b_{0,1}2^0 + b_{1,1} \cdot 2^1 + b_{2,1}2^2 + b_{3,1}2^3) \cdot 16^1 +$$
$$(b_{0,2}2^0 + b_{1,2} \cdot 2^1 + b_{2,2}2^2 + b_{3,2}2^3) \cdot 16^2 +$$
$$(b_{0,3}2^0 + b_{1,3} \cdot 2^1 + b_{2,3}2^2 + b_{3,3}2^3) \cdot 16^3 +$$
$$\ldots$$

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} =$$
$$(b_{0,0}2^0 + b_{1,0} \cdot 2^1 + b_{2,0}2^2 + b_{3,0}2^3) \cdot 2^0 +$$
$$(b_{0,1}2^0 + b_{1,1} \cdot 2^1 + b_{2,1}2^2 + b_{3,1}2^3) \cdot 2^4 +$$
$$(b_{0,2}2^0 + b_{1,2} \cdot 2^1 + b_{2,2}2^2 + b_{3,2}2^3) \cdot 2^8 +$$
$$(b_{0,3}2^0 + b_{1,3} \cdot 2^1 + b_{2,3}2^2 + b_{3,3}2^3) \cdot 2^{12} +$$
$$\ldots$$

Hex to Binary (Explained)

$$(\ldots a_4a_3a_2a_1a_0)_{16} =$$
$$\left(b_{0,0}2^0 + b_{1,0}\cdot 2^1 + b_{2,0}2^2 + b_{3,0}2^3\right)\cdot 16^0 +$$
$$\left(b_{0,1}2^0 + b_{1,1}\cdot 2^1 + b_{2,1}2^2 + b_{3,1}2^3\right)\cdot 16^1 +$$
$$\left(b_{0,2}2^0 + b_{1,2}\cdot 2^1 + b_{2,2}2^2 + b_{3,2}2^3\right)\cdot 16^2 +$$
$$\left(b_{0,3}2^0 + b_{1,3}\cdot 2^1 + b_{2,3}2^2 + b_{3,3}2^3\right)\cdot 16^3 +$$
$$\ldots$$

$$(\ldots a_4a_3a_2a_1a_0)_{16} =$$
$$\left(b_{0,0}2^0 + b_{1,0}\cdot 2^1 + b_{2,0}2^2 + b_{3,0}2^3\right)\cdot 2^0 +$$
$$\left(b_{0,1}2^0 + b_{1,1}\cdot 2^1 + b_{2,1}2^2 + b_{3,1}2^3\right)\cdot 2^4 +$$
$$\left(b_{0,2}2^0 + b_{1,2}\cdot 2^1 + b_{2,2}2^2 + b_{3,2}2^3\right)\cdot 2^8 +$$
$$\left(b_{0,3}2^0 + b_{1,3}\cdot 2^1 + b_{2,3}2^2 + b_{3,3}2^3\right)\cdot 2^{12} +$$
$$\ldots$$

$$(\ldots a_4a_3a_2a_1a_0)_{16} =$$
$$b_{0,0}2^0 + b_{1,0}\cdot 2^1 + b_{2,0}2^2 + b_{3,0}2^3 +$$
$$b_{0,1}2^4 + b_{1,1}\cdot 2^5 + b_{2,1}2^6 + b_{3,1}2^7 +$$
$$b_{0,2}2^8 + b_{1,2}\cdot 2^9 + b_{2,2}2^{10} + b_{3,2}2^{11} +$$
$$b_{0,3}2^{12} + b_{1,3}\cdot 2^{13} + b_{2,3}2^{14} + b_{3,3}2^{15} +$$
$$\ldots$$

Hex to Binary (Explained)

$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} =$
$b_{0,0}2^0 + b_{1,0} \cdot 2^1 + b_{2,0}2^2 + b_{3,0}2^3 +$
$b_{0,1}2^4 + b_{1,1} \cdot 2^5 + b_{2,1}2^6 + b_{3,1}2^7 +$
$b_{0,2}2^8 + b_{1,2} \cdot 2^9 + b_{2,2}2^{10} + b_{3,2}2^{11} +$
$b_{0,3}2^{12} + b_{1,3} \cdot 2^{13} + b_{2,3}2^{14} + b_{3,3}2^{15} +$
$\ldots$

$$(\ldots a_4 a_3 a_2 a_1 a_0)_{16} = \left(\ldots\ \ b_{3,3}b_{2,3}b_{1,3}b_{0,3}\ \ b_{3,2}b_{2,2}b_{1,2}b_{0,2}\ \ b_{3,1}b_{2,1}b_{1,1}b_{0,1}\ \ b_{3,0}b_{2,0}b_{1,0}b_{0,0}\right)_2$$

- Every hex digit is represented by exactly 4 bits.