Foundational Mathematics

Chapter 01

Orientation Programme

Mathematics Unit
Sri Lanaka Institute of Information Technology

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1. Algebraic Fractions (Introduction)

Algebraic fractions have properties which are the same as those for numerical fractions, the only difference being that the numerator (top) and denominator (bottom) are both algebraic expressions.

Example 1 Simplify each of the following fractions.

$$\textbf{(a)} \quad \frac{2b}{7b^2}, \qquad \qquad \textbf{(b)} \quad \frac{3x+x^2}{6x^2}.$$

Solution

(a) $$\frac{2b}{7b^2} = \frac{2 \times \cancel{b}}{7 \times b \times \cancel{b}} = \frac{2}{7b}$$

(b) $$\frac{3x + x^2}{6x^2} = \frac{x \times (3 + x)}{x \times 6x}$$
$$= \frac{\cancel{x} \times (3 + x)}{\cancel{x} \times 6x} = \frac{3 + x}{6x}$$

N.B. The cancellation in **(b)** is allowed since x is a common factor of the numerator and the denominator.

Sometimes a little more work is necessary before an algebraic fraction can be reduced to a simpler form.

Example 2 Simplify the algebraic fraction

$$\frac{x^2 - 2x + 1}{x^2 + 2x - 3}$$

Solution

In this case the numerator and denominator can be factored into two terms, thus $x^2 - 2x + 1 = (x - 1)^2$, and $x^2 + 2x - 3 = (x - 1)(x + 3)$.

$$\begin{aligned} \frac{x^2 - 2x + 1}{x^2 + 2x - 3} &= \frac{(x-1) \times (x-1)}{(x+3) \times (x-1)} \\ &= \frac{x-1}{x+3} \qquad \text{(Cancelling } (x-1)\text{)} \end{aligned}$$

Exercise 1. Simplify each of the following algebraic fractions.

(a) $\dfrac{8y}{2y^3}$ (b) $\dfrac{2y}{4x}$ (c) $\dfrac{7a^6b^3}{14a^5b^4}$

(d) $\dfrac{(2x)^2}{4x}$ (e) $\dfrac{5y+2y^2}{7y}$ (f) $\dfrac{5ax}{15a+10a^2}$

(g) $\dfrac{2z^2-4z}{2z^2-10z}$ (h) $\dfrac{y^2+7y+10}{y^2-25}$ (i) $\dfrac{w^2-5w-14}{w^2-4w-21}$

2. Addition of Algebraic Fractions

Addition (and subtraction) of algebraic fractions proceeds in exactly the same manner as for numerical fractions.

Example 4 Write the following sum as a single fraction in its simplest form.

$$\frac{2}{x+1}+\frac{1}{x+2}$$

Solution The *least common multiple* of the denominators is $(x + 1)(x + 2)$. Thus

$$\begin{aligned}\frac{2}{x+1} + \frac{1}{x+2} &= \frac{2 \times (x+2)}{(x+1) \times (x+2)} + \frac{1 \times (x+1)}{(x+2) \times (x+1)} \\ &= \frac{2x+4}{(x+1)(x+2)} + \frac{x+1}{(x+1)(x+2)} \\ &= \frac{(2x+4)+(x+1)}{(x+1)(x+2)} = \frac{3x+5}{(x+1)(x+2)}\end{aligned}$$

Exercise 3. Evaluate each of the following fractions. (Click on the green letters for solution.)

(a) $\dfrac{2}{y} + \dfrac{3}{z}$

(c) $\dfrac{3z+1}{3} - \dfrac{2z+1}{2}$ (b) $\dfrac{1}{3y} - \dfrac{2}{5y}$

(c) $\dfrac{x+1}{2} + \dfrac{1}{x-1}$ (d) $\dfrac{3t+1}{2} + \dfrac{1}{t}$ (f) $\dfrac{2}{w+3} \quad \dfrac{5}{w-1}$

Quiz

Which of the following values of **a** is needed if

$$\frac{\mathbf{a}}{2x+1} + \frac{1}{x+2} = \frac{4x+5}{(2x+1)(x+2)}\ ?$$

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(a) $\mathbf{a} = 3$ (b) $\mathbf{a} = -3$ (c) $\mathbf{a} = 2$ (d) $\mathbf{a} = -2$

3. Simple Partial Fractions

The last quiz was an example of *partial fractions*, i.e. the technique of decomposing a fraction as a sum of simpler fractions. This section will consider the simpler forms of this technique.

Example 5 Find the partial fraction decomposition of $4/(x^2 - 4)$.

Solution The denominator factorises as $x^2-4 = (x-2)(x+2)$. (See the package on **quadratics**.) The partial fractions will, therefore, be of the form $a/(x - 2)$ and $b/(x + 2)$. Thus

$$\frac{a}{x-2} + \frac{b}{(x+2)} = \frac{4}{(x-2)(x+2)}$$

$$\frac{a(x+2)}{(x-2)(x+2)} + \frac{b(x-2)}{(x+2)(x-2)} = \frac{4}{(x-2)(x+2)}$$

$$\frac{(a+b)x+2(a-b)}{(x-2)(x+2)} = \frac{4}{(x-2)(x+2)}$$

so that $(a + b)x + 2(a - b) = 4$

The last line is

$$(a + b)x + 2(a - b) = 4,$$

and this enables a and b to be found. For the equation to be true *for all* values of x the coefficients must match, i.e.

$$\begin{aligned} a + b &= 0 && \text{(coefficients of } x\text{)} \\ 2a - 2b &= 4 && \text{(constant terms)} \end{aligned}$$

where the first equation holds since there is no x term in $4/(x^2 - 4)$. This set of simultaneous equations may be solved to give $a = 1$ and $b = -1$. (See the package on **simultaneous equations** for a method of finding these solutions.)

Thus

$$\frac{4}{(x-2)(x+2)} = \frac{1}{x-2} - \frac{1}{(x+2)}$$

Exercise 4. For each of the following, find a and b. (Click on the green letters for solution.)

(a) $$\frac{a}{x-2}+\frac{b}{x+2}=\frac{4x}{(x-2)(x+2)}$$

(b) $$\frac{a}{z-3}+\frac{b}{z+2}=\frac{z+7}{(z-3)(z+2)}$$

(c) $$\frac{a}{w-4}+\frac{b}{w+1}=\frac{3w-2}{(w-4)(w+1)}$$

Now try this final, short quiz.

Quiz If $\frac{a}{2x-3}+\frac{b}{3x+4}=\frac{x+7}{(2x-3)(3x+4)}$, which of the following is the solution to the equation?

(a) $a = 1, b = -1$ (b) $a = -1, b = 1$

(c) $a = 1, b = 1$ (d) $a = -1, b = -1$