# Simultaneous Equations

$2x - y = 1$

$3x + y = 9$

- Simply 2 equations
  - With 2 unknowns
  - Usually x and y
- To SOLVE the equations means we find values of x and y that
  - Satisfy BOTH equations
  - At same time [simultaneously]

# Elimination Method

We have the same number of y's in each

| | |
|---|---|
| A | $2x - y = 1$ |
| B | $3x + y = 9$ |

+

If we ADD the equations, the y's disappear

$$5x = 10$$

Divide both sides by 5

$$x = 2$$

$$2 \times 2 - y = 1$$

Substitute x = 2 in equation A

$$4 - y = 1$$

$$y = 3$$

Answer

$x = 2, y = 3$

## Elimination Method

We have the same number of y's in each

$$
\begin{array}{lrl}
A & 5x + y = 17 & \\
B & 3x + y = 11 & - \\
\hline
 & 2x \quad = 6 & \\
 & x \quad = 3 &
\end{array}
$$

If we SUBTRACT the equations, the y's disappear

Divide both sides by 2

$$5 \times 3 + y = 17$$

Substitute x = 3 in equation A

$$15 + y = 17$$
$$y = 2$$

Answer

$x = 3, y = 2$

## What if NOT same number of x's or y's?

A $3x + y = 10$

B $5x + 2y = 17$

If we multiply A by 2 we get 2y in each

A $6x + 2y = 20$

B $5x + 2y = 17$ −

$x = 3$

In B $5 \times 3 + 2y = 17$

$15 + 2y = 17$

$y = 1$

Answer

$x = 3, y = 1$

## …if multiplying 1 equation doesn't help?

$$\begin{array}{ll} A & 3x + 7y = 26 \\ B & 5x + 2y = 24 \\ A & 15x + 35y = 130 \\ B & 15x + 6y = 72 \\ \hline & 29y = 58 \\ & y = 2 \end{array}$$

**Multiply A by 5 & B by 3, we get 15x in each**

−

**Could multiply A by 2 & B by 7 to get 14y in each**

$$\text{In B} \quad 5x + 2 \times 2 = 24$$

$$5x = 20$$

$$x = 4$$

Answer

$x = 4, y = 2$

# Quadratic equations

The general form of a quadratic equation is the following:

$$ax^2 + bx + c = 0$$

- The $a$ represents the numerical coefficient of $x^2$ , $b$ represents the numerical coefficient of $x$, and $c$ represents the constant numerical term.
- One or both of the last two numerical coefficients maybe zero. The numerical coefficient $a$ cannot be zero.

Some examples of quadratic equations include:

$$3x^2 + 9x - 2 = 0$$
$$6x^2 + 11x = 7$$
$$4x^2 = 13$$

# Quadratic equations

- A quadratic equation has two roots, both of which satisfy the equation.
- The two roots of the quadratic equation $x^2 + 5x + 6 = 0$ are
  - $x = 2$
  - $x = 3.$
- Substituting either of these values for x in the equation makes it true.

# Solving quadratic equations:

- <u>Taking the square root</u>

To determine which technique can be used, the equation must be written in general form:

$$ax^2 + bx + c = 0$$

- If the equation is a *pure* quadratic equation $(b = 0)$ it can be solved by taking the square root.
- Ex. $4x^2 - 1 = 0$, $4x^2 = +1$, $x^2 = 1/4$, taking the square root of ¼ we get the two solutions

  $x = +1/2$ and $x = -1/2$

## Factoring

If the numerical constant *c* is zero, the equation can be solved by factoring.

Ex. $4x^2 - 3x = 0$ ,

$$x(4x - 3) = 0,$$

for the zero – factor property

$$x = 0 , 4x - 3 = 0,$$

so the two solutions are $x = 0$ and $x = +3/4$

## Factoring

Certain other equations can also be solved by factoring and applying the zero – factor property.

Ex. $x^2 + 5x + 6 = 0$,

if we factor we have

$$(x + 3)(x + 2) = 0$$

then $x + 3 = 0$ , $x + 2 = 0$

so the two solutions are

$x = -3$ and $x = -2$

- The solution(s) to a quadratic equation can always be calculated using the **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The "±" means you need to do a plus AND a minus, and therefore there are normally TWO solutions ! You can try to solve any quadratic equation by using the quadratic formula.

## Solving a Quadratic Equation by the Quadratic Formula

- Solve $2x^2 + x - 3 = 0$.

Solution:

$$a = 2, b = 1, c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1 + 24}}{4}$$

$$x = \frac{-1 \pm \sqrt{25}}{4}$$

$$x = \frac{-1 + 5}{4} \quad \text{or} \quad x = \frac{-1 - 5}{4}$$

$$x = \frac{4}{4} \quad \text{or} \quad x = \frac{-6}{4} = -\frac{3}{2}$$

# Rewriting a Quadratic Equation before Solving

- Solve $-x^2 = 8x + 1.$

Solution:

$$-x^2 - 8x - 1 = 0$$

$$a = -1, b = -8, c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(-1)(-1)}}{2(-1)}$$

$$x = \frac{8 \pm \sqrt{64 - 4}}{-2}$$

$$x = \frac{8 \pm \sqrt{60}}{-2}$$

$$x = \frac{8 \pm \sqrt{4 \cdot 15}}{-2}$$

$$x = \frac{8 + 2\sqrt{15}}{-2} \quad \text{or} \quad x = \frac{8 - 2\sqrt{15}}{-2}$$

$$x = -4 + \sqrt{15} \quad \text{or} \quad x = -4 - \sqrt{15}$$