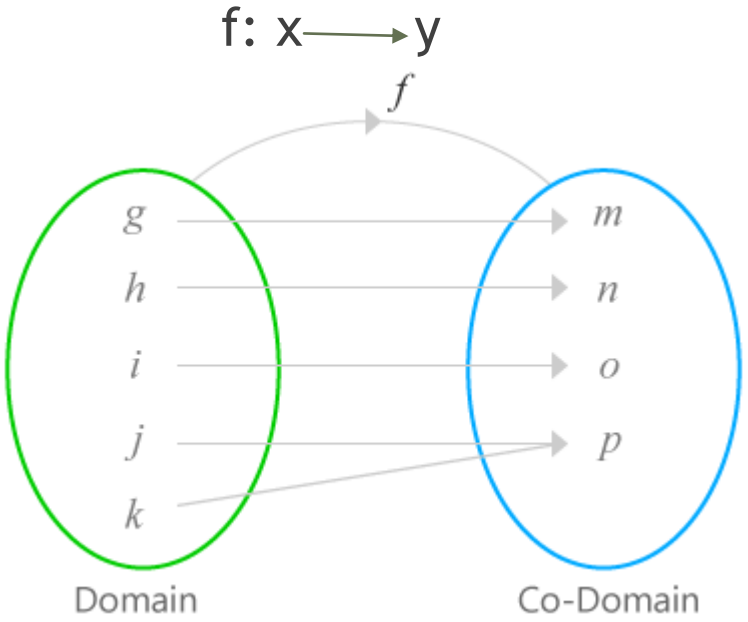
Functions

Mathematics for Computing (IT 1030)

Definition

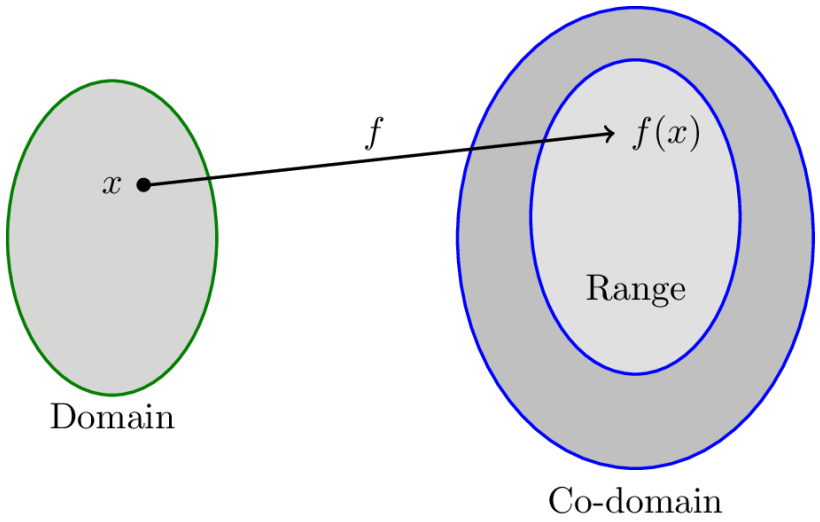
- A **function *f* from a set *X* to a set *Y*** is a relationship between elements of X and elements of Y with the property that <u>each element</u> of X is related to a <u>unique</u> element of Y.



Range/Image

- The unique element *y* to which *f* sends *x* is denoted by *f(x)* and is called ***f* of *x***, or the **value of *f* at *x***, or the **image of *x* under *f***.
- The set of all values of *f* taken together is called the **range of *f*** or the **image of *X* under *f***.

range of f = {y ϵ Y | y=f(x), for some x in X }



Examples

- Which of the following are functions ?

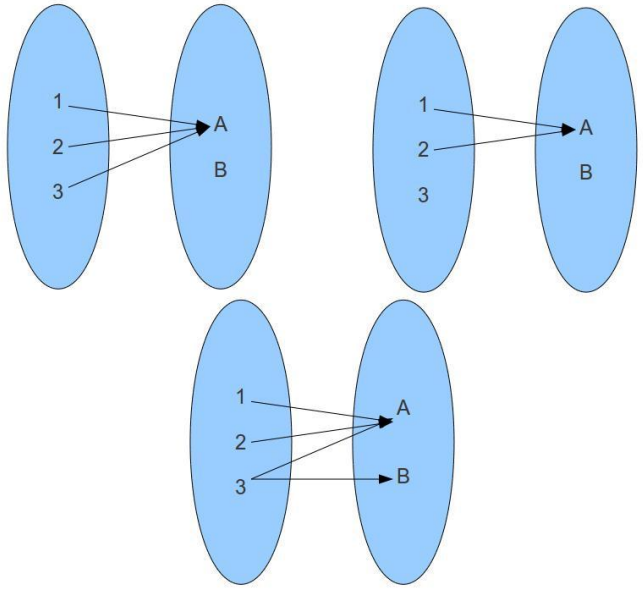


Figure 01

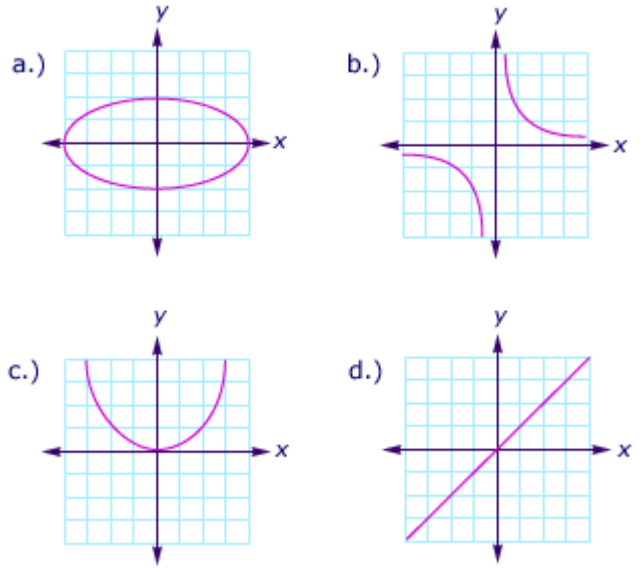


Figure 02

Inverse Function

- If f is one-to-one and onto then f^{-1} exists.

Definition :-

Suppose $f : X \to Y$ is a one to one correspondence; that is f is one to one and onto. Then, there is a function $f^{-1} : Y \to X$.

Given any element y in Y,

$f^{-1}(y)$ = that unique element x in X such that $f(x)$ equals y.

Example:

The function $f : R \to R$ is defined by the formula $f(x) = 4x - 3$ for all real numbers x.
Show that f is a one-to-one correspondence and find its inverse function.

Composition of Functions

- Function composition is a one way to combine existing functions.
- It is a function that depends on another function.
- A composite function is created when one function is substituted into another function.
- Example: Consider $f(x) = 3x + 2$ and $g(x) = x + 5$

$$(f \circ g)(x) / f(g(x)) = f(x+5)$$

$$= 3(x+5) + 2$$

$$= 3x + 17$$