DIFFERENTIATION

Differentiation

- Basic Rules of Differentiation
- The Product and Quotient Rules
- The Chain Rule

BASIC DIFFERENTIATION RULES

1. $\boxed{\dfrac{d}{dx}(c) = 0 \quad (c \text{ is a constant})}$

Ex. $f(x) = 5$

$f'(x) = 0$

2. $\boxed{\dfrac{d}{dx}\left(x^n\right) = nx^{n-1} \quad (n \text{ is a real number})}$

Ex. $f(x) = x^7$

$f'(x) = 7x^6$

BASIC DIFFERENTIATION RULES

3. $$\boxed{\frac{d}{dx}\big(cf(x)\big) = c\frac{d}{dx}\big(f(x)\big) \quad (c \text{ is a constant})}$$

Ex. $f(x) = 3x^8$

$$f'(x) = 3\left(8x^7\right) = 24x^7$$

4. $$\boxed{\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]}$$

Ex. $f(x) = 7 + x^{12}$

$$f'(x) = 0 + 12x^{11} = 12x^{11}$$

MORE DIFFERENTIATION RULES

5. Product Rule

$$\boxed{\frac{d}{dx}\left[f(x)\cdot g(x)\right]=\frac{d}{dx}[f(x)]g(x)+\frac{d}{dx}[g(x)]f(x)}$$

Ex. $f(x)=\left(x^3+2x+5\right)\left(3x^7-8x^2+1\right)$

$$f'(x)=\left(3x^2+2\right)\left(3x^7-8x^2+1\right)+\left(x^3+2x+5\right)\left(21x^6-16x\right)$$

Derivative of the first function

Derivative of the second function

$$f'(x)=30x^9+48x^7+105x^6-40x^4-45x^2-80x+2$$

MORE DIFFERENTIATION RULES

6. Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Sometimes remembered as:

$$\frac{d}{dx}\left[\frac{\text{hi}}{\text{lo}}\right]=\frac{\text{lo }d[\text{hi}]-\text{hi }d[\text{lo}]}{\text{lo lo}}$$

MORE DIFFERENTIATION RULES

6. Quotient Rule (cont.)

Ex. $f(x) = \dfrac{3x+5}{x^2-2}$

Derivative of the numerator

Derivative of the denominator

$$f'(x) = \frac{3\left(x^2-2\right) - 2x(3x+5)}{\left(x^2-2\right)^2}$$

$$= \frac{-3x^2-10x-6}{\left(x^2-2\right)^2}$$

More Differentiation Rules

7. The Chain Rule

If $h(x) = g\left(f(x)\right)$ then

$$\boxed{h'(x) = g'\left(f(x)\right) \cdot f'(x)}$$

Note: $h(x)$ is a composite function.

Another Version:

If $y = h(x) = g\left(u\right)$, where $u = f(x)$, then

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

More Differentiation Rules

The Chain Rule leads to

The General Power Rule:

If $h(x) = \left[f(x)\right]^n \quad (n, \text{ real})$ then

$$\boxed{h'(x) = n\left[f(x)\right]^{n-1} \cdot f'(x)}$$

Ex. $f(x) = \sqrt{3x^2 + 4x} = \left(3x^2 + 4x\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left(3x^2 + 4x\right)^{-1/2}\left(6x + 4\right)$$

$$= \frac{3x + 2}{\sqrt{3x^2 + 4x}}$$

Chain Rule Example

Ex. $G(x)=\left(\frac{2x-1}{3x+5}\right)^7$

$$G'(x)=7\left(\frac{2x-1}{3x+5}\right)^6\left(\frac{(3x+5)2-(2x-1)3}{(3x+5)^2}\right)$$

$$G'(x)=7\left(\frac{2x-1}{3x+5}\right)^6\frac{13}{(3x+5)^2}=\frac{91(2x-1)^6}{(3x+5)^8}$$

Chain Rule Example

Ex. $y = u^{5/2},\ u = 7x^8 + 3x^2$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{5}{2}u^{3/2} \cdot \left(56x^7 + 6x\right)$$

Sub in for u

$$= \frac{5}{2}\left(7x^8 + 3x^2\right)^{3/2} \cdot \left(56x^7 + 6x\right)$$

$$= \left(140x^7 + 15x\right)\left(7x^8 + 3x^2\right)^{3/2}$$