

### Right Triangle Trigonometry

$$1. \sin \theta = \frac{opp}{hyp}$$

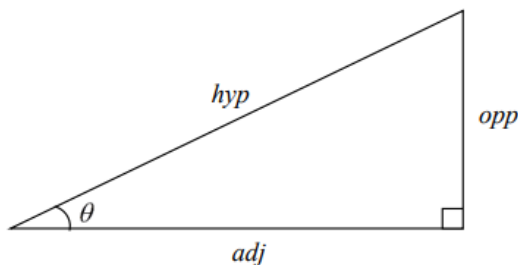
$$2. \cos \theta = \frac{adj}{hyp}$$

$$3. \tan \theta = \frac{opp}{adj}$$

$$4. \csc \theta = \frac{hyp}{opp}$$

$$5. \sec \theta = \frac{hyp}{adj}$$

$$6. \cot \theta = \frac{adj}{opp}$$



### Identities

#### Reciprocal

$$\sin \theta = \frac{1}{\csc \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

#### Ratio

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

#### Pythagorean

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

#### Even/Odd

$$\sin(-\theta) = -\sin \theta \quad \csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta \quad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \quad \cot(-\theta) = -\cot \theta$$

#### Cofunction

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta \quad \csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

### Identities

#### Double Angle

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

#### Half Angle

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

#### Sum and Difference

$$\sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi$$

$$\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$$

$$\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$$

$$\tan(\theta - \phi) = \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi}$$

$$\tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi}$$

Using the above trigonometric relationship verify the following identities

$$1. \cot \theta \sin \theta = \cos \theta$$

$$2. \frac{1}{\cos \theta} = \tan \theta \csc \theta$$

$$3. \sec \theta \sin \theta = \tan \theta$$

$$4. \sin \theta = \tan \theta \cos \theta$$

$$5. \cot \theta \sec \theta = \frac{1}{\sin \theta}$$

$$6. \frac{1}{\sin \theta} = \frac{\cot \theta}{\cos \theta}$$

$$7. \cos \theta \csc \theta = \cot \theta$$

$$8. \frac{1}{\cos \theta} = \frac{\tan \theta}{\sin \theta}$$

$$9. \csc^2 \theta (1 - \cos^2 \theta) = 1$$

$$10. 1 = \cos^2 \theta (1 + \tan^2 \theta)$$

$$11. \sin \theta (1 + \cot^2 \theta) = \csc \theta$$

$$12. \cos \theta = \sec \theta (1 - \sin^2 \theta)$$

$$13. \sin \theta (\csc \theta - \sin \theta) = \cos^2 \theta$$

$$14. \sin^2 \theta = \cos \theta (\sec \theta - \cos \theta)$$

$$15. \tan \theta (\cot \theta + \tan \theta) = \frac{1}{\cos^2 \theta}$$

$$16. \cot^2 \theta = \csc \theta (\csc \theta - \sin \theta)$$

$$17. \cot \theta \sec^2 \theta - \cot \theta = \tan \theta$$

$$18. \frac{1}{\sin \theta} = \sin \theta + \sin \theta \cot^2 \theta$$

$$19. \sin \theta \sec^2 \theta = \csc \theta \tan^2 \theta$$

$$20. \cos \theta = \cos \theta \csc^2 \theta - \cos \theta \cot^2 \theta$$

$$21. \tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta}$$

$$22. \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 - \cos \theta}$$

$$23. \csc \theta = \frac{\sec \theta + \csc \theta}{1 + \tan \theta}$$

$$24. \cos^3 \theta \csc^3 \theta \tan^3 \theta = \csc^2 \theta - \cot^2 \theta$$

$$25. 1 = \tan^2 \theta \cos^2 \theta + \cot^2 \theta \sin^2 \theta$$

$$26. \frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} = 2 \sec x$$

$$27. \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$28. 1 + \sin 2x = (\sin x + \cos x)^2$$

$$29. \sin 2x = 2 \cot x \sin^2 x$$

$$30. \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$31. \sec^2 x = \frac{2}{1 + \cos 2x}$$

$$32. \frac{1 + \cos 2x}{\sin 2x} = \cot x$$