Set Theory

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Definitions and Notations

- A *Set* is any well defined collection of "objects."
- The *elements* of a set are the objects in a set.
- Usually we denote sets with upper-case letters, elements with lower-case letters. The following notation is used to show set membership

- $x \in A$ means that *x* is a member of the set A
- $x \notin A$ means that *x* is not a member of the set A.

Describing Sets

- List the elements

$$A=\{1,2,3,4,5,6\}$$

- verbal description:

  "A is the set of all integers from 1 to 6, inclusive"

- Give a mathematical inclusion rule

$$A=\{\text{Integers } x \mid 1 \le x \le 6\}$$

Special Sets

- The Null Set or Empty Set. This is a set with no elements, often symbolized by

$$\varnothing$$

- The Universal Set. This is the set of all elements currently under consideration, and is often symbolized by

$$\Omega \; or \; U \; or \; \varepsilon$$

Membership Relationships

- Subset of a set is notated as follows:

$$A \subseteq B \quad \text{“A is a subset of B”}$$

- We say “A is a subset of B” if $x \in A \Rightarrow x \in B$, i.e., all the members of A are also members of B.
- Example:

A={1, 2, 3, 4} and B={2, 4}

$$\Longrightarrow B \subseteq A$$

Proper Subset

- "A is a proper subset of B" is notated as,

$$A \subset B$$

- We say "A is a proper subset of B" if all the members of A are also members of B, but in addition there exists at least one element *c* such that $c \in B$ but $c \notin A$.
- Example:

A={1, 2, 3, 4} and B={2, 4}

$$\Longrightarrow B \subset A$$

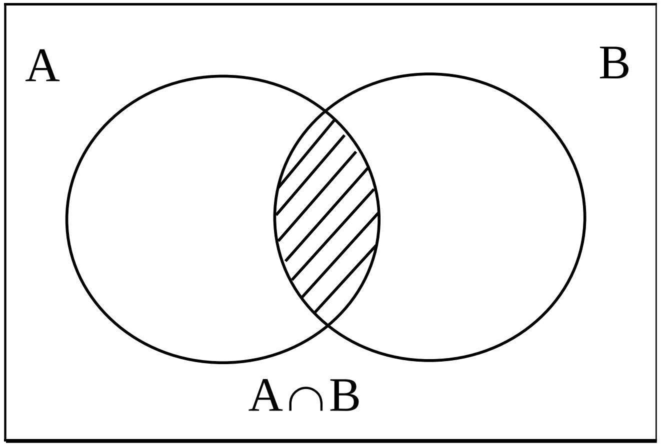
Note: A is not a proper subset of A

Set Intersection

- "*A* intersect *B*" is the set of all elements that are in *both* *A* and *B*.
- Notated as :

$$A \cap B$$

- This is similar to the logical "and"
- Venn diagram:

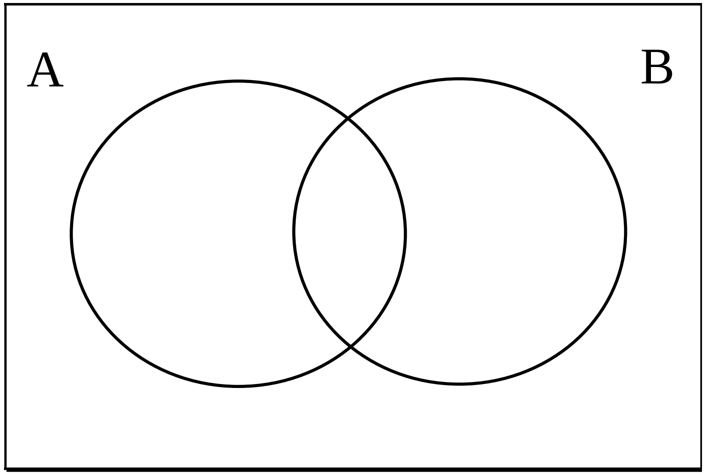


Set Union

- "*A* union *B*" is the set of all elements that are in *A*, or *B*, or both.
- Notated as :

$$A \cup B$$

- This is similar to the logical "or" operator.
- Draw the Venn diagram:



Set Complement

- "*A* complement," or "not *A*" is the set of all elements not in *A*.
- Notated as:

$$A^c \, or \, \bar{A}$$

- The complement of A^c is A.

- Question: Draw the Venn diagram for A^c

Set Difference

- The set difference "A minus B" is the set of elements that are in A, with those that are in B subtracted out. Another way of putting it is, it is the set of elements that are in A, *and* not in B.
- Notated as :

$$A - B$$

- Questions:
    1. Draw the Venn diagram of $A - B$
    2. Using Venn diagrams show that,

$$A - B = A \cap \overline{B}$$

Examples

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2, 3\} \qquad B = \{3, 4, 5, 6\}$$

$$A \cap B = \{3\} \qquad A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$B - A = \{4, 5, 6\} \qquad \overline{B} = \{1, 2\}$$

Disjoint Sets

- We say that two sets A and B are *disjoint* iff,

$$A \cap B = \emptyset$$

- In other words, the sets have no elements in common.

- Ex: Set of odd numbers and set of even numbers are disjoint sets.

Cartesian product of sets.

- Given two sets A and B, the Cartesian product of A and B, denoted A x B (read "A cross B"), is the set of all ordered pairs (a, b), where a is in A and b is in B
- Notation:

  $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

- Example:

  $A = \{a, b\}$

  $B = \{1,2,3\}$

  $A \times B = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$

Questions

- Find the equal sets for the following sets.

1. $A \cup \varnothing$
2. $A \cup \bar{A}$
3. $A \cap \varnothing$
4. $A - \bar{A}$
5. $A \cap \bar{A}$
6. $A \cup \Omega$
7. $A \cap \Omega$

Power Set

- Given a set A, the power set of A, denoted P (A), is the set of all subsets of A.

- Example:

  The power set of {x, y},

  $P(\{x, y\}) = \{\varnothing,\{x\},\{y\},\{x ,y\}\}$

Note: If the set has ***n*** elements then the power set of that set has $\boldsymbol{2^n}$ elements

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Set Identities

Let all sets referred to below be subsets of a universal set U.

1. Commutative Laws: For all sets A and B,

   (a) $A \cap B = B \cap A$
   (b) $A \cup B = B \cup A$.

2. Associative Laws: For all sets A, B, and C,

   (a) $(A \cap B) \cap C = A \cap (B \cap C)$
   (b) $(A \cup B) \cup C = A \cup (B \cup C)$

3. Distributive Laws: For all sets, A, B, and C,
   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. Intersection with U (U Acts as an Identity for $\cap$): For all sets A,

   $$A \cap U = A$$

5. Double Complement Law: For all sets A
   $(A^c)^c = A$.
6. Idempotent Laws: For all sets A,

   (a) $A \cap A = A$
   (b) $A \cup A = A$

Set Identities Cont….

7. De Morgan's Laws: For all sets A and B,

   (a) $(A \cup B)^c = A^c \cap B^c$

   (b) $(A \cap B)^c = A^c \cup B^c$

8. Union with U (U Acts as a Universal Bound for U):

   $A \cup U = U$

9. Absorption Laws: For all sets A and B,

   (a) $A \cup (A \cap B) = A$

   (b) $A \cap (A \cup B) = A$

10. Alternate Representation for Set Difference: For all sets A and B,

    $A - B = A \cap B^c$

Thank You!