# Matrices

# Introduction

***What is a matrix..?***

A *matrix* is simply a rectangular table of numbers written in either ( ) or [ ] brackets. Matrices have many applications in science, engineering and Computing.

# Matrix Arithmetic

i) Scalar Multiplication
ii) Addition
iii) Subtraction
iv) Matrix multiplication

# Matrix Multiplication

**Multiplication process**

**Ex:**

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times3} \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix} = \begin{bmatrix} (au+bw+cy) & (av+bx+cz) \\ (du+ew+fy) & (dv+ex+fz) \end{bmatrix}_{2\times2}$$

# Types of Matrices

i) Square matrix
ii) Zero matrix
iii) Diagonal matrix
iv) Identity matrix (unit matrix)
v) Transpose of a matrix

# Equality of matrices

**For two matrices to be equal, they must be of the same size and the corresponding elements are equal.**

# Determinant

A **determinant of a matrix** represents a single number. We obtain this value by multiplying and adding its elements in a special way

# Determinants

## Determinants that are Zero

The determinant of a matrix will be zero if

1. An entire row is zero.
2. Two rows or columns are equal.
3. A row or column is a constant multiple of another row or column

# Inverse matrix

## Inverse matrix for 2 x 2 matrix.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

1. Solve the equation $A = B$ where

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \\ -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & x \\ y-x & 1 \\ -1 & 2 \end{bmatrix} \text{ for } x \text{ and } y.$$

2. Given that

$$A = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & 2 \end{bmatrix}$$

Find the matrices $A + B$, $A - B$, and $2A - 3B$.

3. Taking 3 appropriate matrices verify the distributive law $A(B+C) = AB + AC$ and the associative law $A(BC) = (AB)C$

4. Show that $A + A^T$ is a symmetric matrix, and that $A - A^T$ is skew symmetric.

If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ 1 & -1 & 1 \end{bmatrix}$

# Questions

5. Let

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$$

Write down $A^T$, and find the products $AA^T$ and $A^TA$

6. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & -1 & 0 \\ b & c & 1 \end{bmatrix}$$ Find $A^2$

For what relation between $a$, $b$ and $c$ is $A^2 = I_3$?