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An analysis of the European sustainable investment regulation for new ways of entrepreneurship

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ABSTRACT

Entrepreneurs are often at the forefront of developing green technologies, circular economy models, and social enterprises. In addition, the European Union Regulation on sustainable investment aims to redirect capital flows toward environmentally sustainable activities. This paper aims to establish a conceptual and practical link between entrepreneurship and sustainable economic activities through a logical and mathematical analysis. More precisely, we investigate Regulation 2020/852 on sustainable investments, describing new ways to analyze this critical regulation to help entrepreneurs design new models adapted to regulation changes. The study follows a logical and quantitative approach, utilizing Boolean algebra and graph theory as the main tools of analysis. These tools were applied to a recent sustainable investments regulation to analyze the premises that led to some consequences and also to establish hierarchies. The results reveal the difficulty of meeting all the necessary conditions to classify an activity as environmentally sustainable and the identification of the most important articles by computing their centrality. These findings provide evidence of the existence of some mutually exclusive aspects that could be improved in the regulation, offering significant implications for entrepreneurs about critical aspects of regulation. These insights provide entrepreneurs with novel tools for assessing regulatory risk. Additionally, by identifying the most central articles, entrepreneurs can prioritize efforts in regulation compliance.

Introduction

Entrepreneurs play a key role in driving sustainable transformation by developing innovations in green technologies, circular economy models, and social enterprises. Their agility and vision enable them to identify and exploit opportunities that align economic growth with environmental stewardship and social impact. An important set of these opportunities arises from the regulation of sustainable investments. Within the European Union, the EU 2020/852 Taxonomy Regulation seeks to channel financial resources toward activities that contribute meaningfully to environmental objectives, such as climate change mitigation and biodiversity preservation. This regulatory framework not only incentivizes sustainable entrepreneurship, but also creates a structured environment to evaluate the ecological value of business initiatives. The purpose of this paper is to bridge the conceptual and practical dimensions of entrepreneurship and sustainable economic activities through a logical and mathematical analysis. By identifying regulation inconsistencies, we aim to support entrepreneurial decision-making through regulatory incentives and assessment criteria that can guide policy and investment strategies. Furthermore, by uncovering

hidden interactions and patterns, we offer a more rigorous foundation for understanding how entrepreneurial ventures can be optimized to meet sustainability goals. Through this lens, entrepreneurship emerges not only as a driver of innovation but also as a key element toward a more sustainable development.

However, codification is a foundational mechanism that supports the design, implementation, and enforcement of regulation. It brings structure and clarity to complex regulatory environments, which is vital for both compliance and governance. By establishing a formal structure, codification enables comprehensive governance over specific financial domains, reducing regulatory fragmentation, and improving predictability for entrepreneurs and other market participants, institutions, and regulators. As a result, it is generally accepted that norms can be judged from the point of view of their rationality (Alchourrón, 1969; Wright, 1981, 1991).

This process is grounded in fundamental principles and aims to ensure clarity, consistency, and practical utility in regulatory systems (Hart, 2012). More precisely, analyzing and comparing financial regulations through logical and quantitative systems offers a powerful

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framework for enhancing legal clarity, consistency, and transparency. For instance, an interesting line of research focuses on the automated synthesis of complex problems related to norms and regulations (Morales et al., 2013, 2015).

Methodologically, we believe that finance and economic policy need interdisciplinary analysis. However, the actual use of complexity models in regulatory analysis remains at an early stage despite recent insights (Battiston et al., 2016; Capelli, 2018; Gaffeo & Tamborini, 2011). A related interesting line of research includes the application of artificial intelligence in law and regulatory analysis (Surden, 2018). We diverge from existing research by emphasizing the use of logical and quantitative models in the analysis of environmental regulation. This approach is motivated by our commitment to maintaining a high level of explainability, which is widely recognized as essential for interpreting regulatory frameworks, particularly in the context of entrepreneurship.

Many regulatory concepts are inherently imprecise and context-dependent. By applying formal analysis, we aim to uncover inconsistencies, hidden structures, and dependencies within complex legal texts. This approach not only supports more objective evaluations of regulatory content, such as identifying the most central or influential provisions, but also offers a source of new opportunities for entrepreneurs. Ultimately, it empowers both legal scholars and policymakers to design, interpret, and refine regulations with greater precision and analytical rigor. However, while Boolean algebra can identify inconsistencies, it is incapable of modeling vagueness and imprecision. To this end, fuzzy logic offers a more nuanced alternative, allowing for degrees of truth and partial compliance (Zadeh, 1965).

A norm can be understood as an established, expected pattern of behavior (Wooldridge, 2009). As a result, the full financial impact of regulation depends on the entrepreneurs' analysis to support strategic planning by identifying opportunities to optimize the financial structure of companies. We argue that there is little logical and mathematical consideration of recent regulation. This paper attempts to address that shortfall. Along the lines of Fiedler (1968), regulatory frameworks are ideally designed to be complete, unambiguous, and internally consistent. Completeness implies that all necessary legal propositions can be derived from the codified rules without requiring ad hoc legal innovation. Unambiguity refers to the need for legal terms and provisions to have clear, precise, and singular meanings, minimizing interpretative uncertainty. Internal consistency ensures that the rules do not contradict each other or the broader legal system, thereby avoiding regulatory confusion and enforcement challenges.

To this end, we analyze a recent piece of financial regulation from a logical and mathematical perspective, describing new ways to analyze this critical regulation. More precisely, we focus on the Regulation (EU) 2020/852 (2020) of the European Parliament and of the Council of 18 June on the establishment of a framework to facilitate sustainable investment, and amending Regulation (EU) 2019/2088 (2019) of 27 November. Hereafter, we will refer to it as the Regulation or the Taxonomy Regulation. The establishment of the technical criteria that meet the requirements of the Regulation was postponed to Commission Delegated Regulation (EU) 2021/2139 (2021) of 4 June, supplementing Regulation (EU) 2020/852 (2020) of the European Parliament and of the Council by establishing the technical screening criteria for determining the conditions under which an economic activity qualifies as contributing substantially to climate change mitigation or climate change adaptation, and for determining whether that economic activity causes no significant harm to any of the other environmental objectives.

Our research seeks new insights into sustainable finance, contributing to a deeper understanding of finance regulation for entrepreneurship. We are conscious that in attempting to apply the insights of the mathematical theory to the practice of regulation, we take the risk of exploring the connection between two apparently very separate fields, such as mathematics and law. However, we hypothesize that the intersection of legal language and mathematical language is

non-empty. The intersection of legal language (L) and mathematical language (M) suggests that formal methods, while common in finance and mathematics, can also enhance the precision and reliability of legal codification, particularly in regulatory and compliance frameworks. Using mathematical language, it is possible to analyze issues of interest to legal scholars, such as completeness, univocity, or the absence of contradiction. It also allows for the study of relationships between different legal premises, the simplification of propositions, and the distinction between necessary and sufficient conditions for determining legal effects.

To analyze the regulation on sustainable investments, we employ two distinct strategies. First, using Boolean algebra (Boole, 1854), we establish the necessary conditions for classifying an economic activity as environmentally sustainable according to the general criteria, environmental objectives, and technical screening criteria. This formal analysis allows us to uncover inconsistencies in the Taxonomy regulation. Similarly, graph theory is a branch of discrete mathematics with multiple applications in economics, computer science, and other sciences (Diestel, 2024; West, 2001). It studies the properties of mathematical objects known as graphs. A graph is a model composed of vertices (or nodes) and edges (or arcs) that connect nodes. The use of graph theory not only reveals structures and hierarchies but also offers an objective quantitative assessment of centrality or article relevance. This assessment may lead to the detection of new business opportunities.

As a result of our logical-mathematical analysis of the Regulation, we aim to support entrepreneurial decision-making through the following insights:

1. Clarifying sustainability criteria. By applying logical models, entrepreneurs gain clearer guidance on how to align their ventures with regulatory expectations.
2. Addressing regulatory gaps. Through the identification of exclusions in the regulation, this insight enables entrepreneurs to proactively design business models that fill these gaps and advocate for more inclusive regulatory recognition.
3. Mapping article influence. By analyzing the relationships between articles, the study reveals the most central and influential components of the regulation. Entrepreneurs can use this information to prioritize compliance efforts.

For entrepreneurs, regulatory gaps and inconsistencies present both a challenge and an opportunity. In addition, the interaction between key elements of regulation can support entrepreneurs in designing innovative and compliant business models through the alignment with regulatory logic. The proposed analysis offers entrepreneurs a framework to deal with complex sustainability requirements. First, it addresses the limitations of traditional approaches by revealing hidden hierarchies and dependencies within regulatory texts. Second, it supports strategic decision-making by identifying the most central articles, enabling entrepreneurs to prioritize compliance efforts effectively. Third, the analysis uncovers mutually exclusive conditions that hinder sustainability classification, guiding the design of innovative business models aligned with regulatory expectations. Finally, it enhances investment planning by providing tools for assessing regulatory risk, helping entrepreneurs allocate resources more confidently in green ventures. Together, these contributions establish a direct and practical link between regulatory analysis and entrepreneurial strategy, empowering businesses to adapt, innovate, and thrive in the evolving landscape of sustainable economic activity.

In addition to this introduction, this work is structured into the following sections. Section "Materials and methods" provides a brief description of the Taxonomy Regulation of sustainable investments and basic background on Boolean algebra and graph theory. Section "Results" presents a logical and mathematical analysis of the Regulation. Finally, section "Conclusions" concludes this work by summarizing the most important findings and proposing future lines of research.

Materials and methods

This section begins with a brief description of the Taxonomy Regulation of sustainable investments. Additionally, it provides basic background on Boolean algebra and graph theory that will later be used in the analysis of the Regulation.

Brief description of the regulation (EU) 2020/852 on the taxonomy of sustainable investments

The Regulation (EU) 2020/852 of the European Parliament and of the Council of 18 June 2020 on the establishment of a framework to facilitate sustainable investments, and amending Regulation (EU) 2019/2088, emphasizes the need to redirect capital flows toward activities that promote economic development while respecting planetary boundaries. Its main objective is to define a unified system for classifying sustainable economic activities, namely, a taxonomy.

For this Regulation, an environmentally sustainable investment means an investment in one or several economic activities that qualify as environmentally sustainable under this Regulation. For the purposes of establishing the degree to which an investment is environmentally sustainable, Article 3 states that an economic activity shall qualify as environmentally sustainable when that economic activity: contributes substantially to one or more of the environmental objectives set out in Article 9 in accordance with Articles 10 to 16; does not significantly harm any of the environmental objectives set out in Article 9 in accordance with Article 17; is carried out in compliance with the minimum safeguards laid down in Article 18; and complies with technical screening criteria that have been established by the Commission in accordance with Article 10.

The environmental objectives outlined in Article 9 are as follows:

1. Climate change mitigation;
2. Climate change adaptation;
3. The sustainable use and protection of water and marine resources;
4. The transition to a circular economy;
5. Pollution prevention and control; and
6. The protection and restoration of biodiversity and ecosystems.

Climate change mitigation means the process of holding the increase in the global average temperature to well below 2 °C and pursuing efforts to limit it to 1.5 °C above pre-industrial levels, as laid down in the Paris Agreement. Similarly, climate change adaptation means the process of adjustment to actual and expected climate change and its impacts. Circular economy means an economic system whereby the value of products, materials, and other resources in the economy is maintained for as long as possible, enhancing their efficient use in production and consumption, reducing the environmental impact of their use, minimizing waste, and the release of hazardous substances at all stages of their life cycle.

Basic background on Boolean algebra

We stated in the introduction that the logical structure of legal norms has a strong affinity with mathematical logic, which is concerned with the study of formal structures and the interactions between different mathematical objects. Mathematical logic dates back to Aristotle's syllogisms (384–322 BC). Later on, Boole (1854) analyzed this type of logical reasoning and realized that the most important aspect was the class to which all individuals or elements belonged. Based on these elements, Boolean algebra can be formally defined as an algebraic structure of three elements (B, \vee, \wedge), consisting of a set B and two operations defined on B , where the following postulates or axioms are satisfied (Halmos & Givant, 2009; Whitesitt, 2010):

1. **Closure:** For all $X, Y \in B$, we have $X \vee Y \in B$ and $X \wedge Y \in B$.

2. **Identity elements:** There exist $0, 1 \in B$ such that:

$$X \vee 0 = X, \quad X \wedge 1 = X.$$

3. **Commutativity:** For all $X, Y \in B$:

$$X \vee Y = Y \vee X, \quad X \wedge Y = Y \wedge X.$$

4. **Distributivity:** For all $X, Y, Z \in B$:

$$X \wedge (Y \vee Z) = (X \wedge Y) \vee (X \wedge Z),$$

$$X \vee (Y \wedge Z) = (X \vee Y) \wedge (X \vee Z).$$

5. **Complement:** For every $X \in B$, there exists $X' \in B$ such that:

$$X \vee X' = 1, \quad X \wedge X' = 0.$$

Due to its historical development in the field of binary digital circuits and systems (Shannon, 1938), Boolean algebra is typically bivalent, meaning that the set B consists of only two values, for instance, 0 and 1, or *True* and *False*. Furthermore, Boolean algebra and set theory are related because, given an arbitrary set S , then $B = P(S)$, the power set of S , forms a Boolean algebra. The operations for any elements a and b of S are the usual set operations:

$$a \vee b = a \cup b, \quad a \wedge b = a \cap b, \quad a' = S \setminus a.$$

where \cup means union of classes, \cap means intersection of classes, and \setminus the complementary class.

Among all the properties of Boolean algebra, we will focus only on those relevant to the objective of this work. Let a and b be two legal assumptions or conditions. We will say that $a = 1$ if it is fulfilled or it is true. Similarly, we will say that $a = 0$ if it is not fulfilled or it is false.

Property 1. Boolean AND Function or Conjunctive Connection. Given the legal assumptions a and b , the conjunctive connection $a \wedge b$ holds (is equal to one) if and only if both a and b hold, that is, a and b are equal to one.

$$a \wedge b = 1 \iff a \wedge b = 1 \iff (a = 1) \wedge (b = 1).$$

Property 2. Boolean OR Function or Disjunctive Connection. Given the legal assumptions a and b , the disjunctive connection $a \vee b$ does not hold (is equal to zero) if and only if neither a nor b hold, that is, a and b are equal to zero.

$$a \vee b = 0 \iff a \vee b = 0 \iff (a = 0) \wedge (b = 0).$$

Property 3. Membership. If a legal assumption p leads to the legal consequence Q , denoted by $p \subset Q$, then the following holds:

$$p \subset Q \iff p \cap Q' = 0.$$

Property 3 can be equivalently rewritten as follows: if the legal assumption p holds, then the legal consequence Q also holds, and conversely, if the legal consequence Q holds, it is because the legal assumption p holds. This statement can be synthetically expressed as follows:

$$p = 1 \iff Q = 1.$$

Basic background on graph theory

Graph theory is a branch of discrete mathematics with multiple applications in economics, computer science, and other sciences. Graph theory studies the properties of mathematical objects known as *graphs*. A graph is a model composed of vertices (or nodes) and edges (or arcs) that connect nodes (Diestel, 2024; West, 2001).

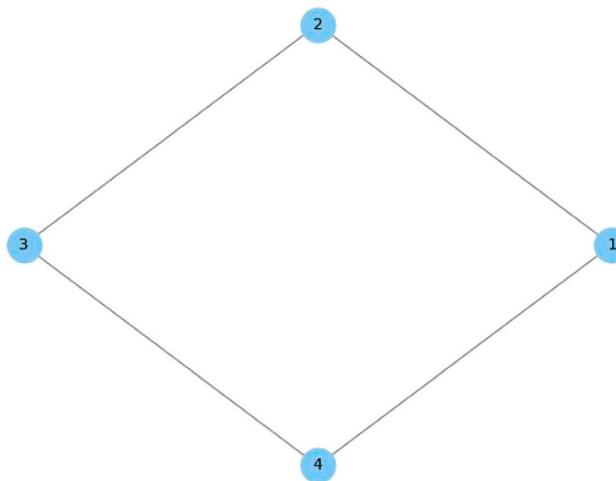


Fig. 1. Example of a graph with 4 nodes and 4 edges.

Formally, a graph $G = (V, E)$ is a pair of sets V and E , where V is a non-empty set of vertices or nodes, and E is a set of pairs of elements from V . When the pairs are ordered, they are usually called arcs and are represented by an arrow going from one node to another. This type of graph is called a directed graph. Alternatively, when the pairs are unordered, they are usually called edges and are represented by a line connecting the two nodes. In this work, we will focus only on undirected graphs.

The information needed to construct and represent a graph can be stored in an adjacency matrix. A matrix is a two-dimensional data structure, that is, a table with rows and columns. The element corresponding to row i and column j is denoted as a_{ij} . In the context of graph theory, both the rows and columns of an adjacency matrix represent the nodes of the graph. By convention, each element a_{ij} is equal to 1 if there is an edge connecting node i with node j , and 0 otherwise. For example, in a graph with four nodes and four edges, as shown in Fig. 1, the adjacency matrix A will be a 4×4 matrix like the following:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

The adjacency matrix A in an undirected graph is a symmetric matrix, meaning that $a_{ij} = a_{ji}$, because if node 1 is connected to node 2, then node 2 is also connected to node 1.

The importance or centrality of a node can be measured through its degree. In an undirected graph, the degree of a node is defined as the number of edges incident to that node. Formally, for a graph G with n nodes, the degree or centrality of node i can be calculated from the elements a_{ij} of the adjacency matrix using the following expression:

$$C_G(i) = \sum_{j=1}^n a_{ij} = \sum_{j=1}^n a_{ji}.$$

Thus, in the example shown in Fig. 2, all nodes in the graph have the same centrality because each of them has two incident edges. The centrality of each node is two.

To enable comparisons between graphs, this measure can be normalized by dividing the centrality $C_G(i)$ by the maximum number of edges incident to any node, thereby establishing a percentage-based comparison:

$$N_G(i) = \frac{C_G(i)}{\max_{u \in V} C_G(u)}.$$

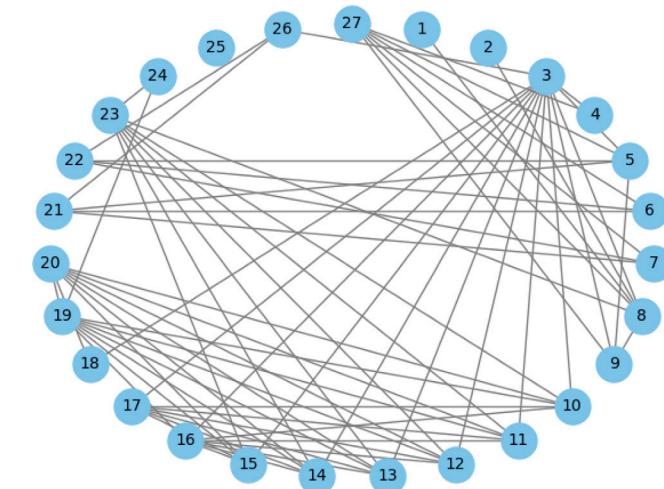


Fig. 2. The graph of the Taxonomy Regulation.

Results

This section presents the results derived from our logical analysis of the Taxonomy Regulation to identify its main characteristics and some inconsistencies from a logical point of view. In our mathematical analysis, we propose the use of the article as the basic unit of analysis in legal coding. We argue that this proposal facilitates the mathematical analysis of relationships between articles.

Results of the logical analysis of the taxonomy regulation

First, let us recall the literal wording of Article 3 of the Regulation, which defines the criteria applicable to environmentally sustainable economic activities. More specifically, an economic activity shall be considered environmentally sustainable when such activity:

- (a) Contributes substantially to one or more of the environmental objectives set out in Article 9, in accordance with Articles 10 to 16;
- (b) Does not cause significant harm to any of the environmental objectives set out in Article 9 in accordance with Article 17;
- (c) Is carried out in accordance with the minimum safeguards set out in Article 18;
- (d) Complies with the technical screening criteria established by the Commission in accordance with Article 10(3), Article 11(3), Article 12(2), Article 13(2), Article 14(2), or Article 15(2).

It is clear that Article 3 of the Regulation does not propose alternative paths for an activity to be considered environmentally sustainable, but rather establishes four legal conditions (described in sections (a), (b), (c), and (d)) that economic activities must simultaneously fulfill. This constitutes a necessary and sufficient condition. In other words, economic activities must substantially contribute to one or more objectives, while not causing significant harm to any other objectives, all in accordance with minimum safeguards and technical screening criteria.

According to the notation introduced in the previous section, the legal conditions required for the legal consequence (Q) of classifying an economic activity as sustainable under the Regulation are four, denoted by the same letters used in Article 3. Based on **Properties 1** and **3**, the entire chain of legal conditions a , b , c , and d must be simultaneously fulfilled for the legal consequence to occur:

$$a \wedge b \wedge c \wedge d \subset Q$$

Alternatively, this can be expressed as:

$$a \wedge b \wedge c \wedge d = 1 \iff Q = 1$$

This logical expression shows that Article 3 is highly stringent, in the sense that failure to meet any of the conditions (a), (b), (c), (d) implies that the economic activity in question would not be considered environmentally sustainable. Given that each condition may be fulfilled (or not fulfilled) independently of the others, the numerical exploration of all possible combinations of four conditions yields $2^4 = 16$ possibilities, and only one of them results in the desired legal consequence. In other words, only 1 out of 16 possible scenarios regarding the conditions would lead to the classification of the economic activity as environmentally sustainable.

However, we move one step forward to produce a more detailed logical analysis of conditions (a) and (b). Condition (a) states that the activity must substantially contribute to one or more objectives. On the other hand, condition (b) states that the activity must not cause significant harm to any of the objectives. Since both conditions must be fulfilled, substantial contribution and significant harm cannot occur simultaneously, or more precisely, such a possibility is not considered. This fact can be expressed as follows:

$$a \subset b \iff a \wedge b' = 0$$

where:

- a = substantial contribution
- b = no significant harm
- b' = significant harm.

This situation excludes the possibility that an economic activity simultaneously contributes to and harms the same objective. However, regardless of how an economic activity is evaluated under conditions (c) and (d), substantial contribution and significant harm may occur simultaneously. For example, the vast majority of economic activities harm the objective of climate change mitigation due to greenhouse gas (GHG) emissions, i.e., their carbon footprint. This circumstance excludes from the classification of environmentally sustainable any activity that harms climate change mitigation through GHG emissions. According to this interpretation of the Regulation, a reduction in GHG emissions from such activities may substantially contribute to climate change mitigation, but if it significantly harms the same objective, it would be excluded from sustainability classification under our logical analysis. Strictly speaking, the conclusion is that any activity that harms one of the objectives must be excluded, leaving such activities with the only option of ceasing operations to avoid harm.

It is clear that this is not the regulator's intention. Apparently, there is no contradiction between condition (a), substantial contribution, and condition (b), significant harm. However, there is a clear lack of completeness in the Regulation, since substantial contribution and significant harm are incompatible in the Regulation, whereas they are not incompatible in the technical and economic reality that the Regulation aims to govern.

Another criticism of the Regulation is a lack of univocity, i.e., that terms have precise meanings. The adjectives *substantial* and *significant* may convey a sense of relevance, but they cannot be considered precise. Articles 10 to 15 attempt to establish what should be considered a substantial contribution for each of the six objectives listed in Article 9. However, the degree of precision does not improve significantly. Specifically, Article 10 states:

"An economic activity shall be considered to contribute substantially to climate change mitigation where that activity contributes substantially to stabilizing greenhouse gas concentrations in the atmosphere at a level that prevents dangerous anthropogenic interference with the climate system, in line with the long-term temperature goal of the Paris Agreement, through the avoidance or reduction of greenhouse gas emissions or the enhancement of greenhouse gas removals".

Despite the detrimental self-reference in the definition, it can be understood that substantial contribution may be achieved through a reduction in greenhouse gas emissions, regardless of the quantity. Therefore, the use of the adjective *substantial* is highly questionable.

Continuing our analysis of the climate change mitigation objective, section (a) of Article 17 of the Regulation states:

"Taking into account the life cycle of the products supplied and services provided by an economic activity, in particular evidence from existing life cycle assessments, an economic activity shall be considered to cause significant harm where that activity leads to considerable greenhouse gas emissions".

Replacing the term *significant* with *considerable* does not improve precision either.

Article 19 of the Regulation sets out a series of requirements applicable to the technical screening criteria. Specifically, paragraph (a) states that the technical screening criteria must "identify the most relevant potential contributions to a given environmental objective." Paragraph (b) states that the technical screening criteria must "specify the minimum requirements that must be met to avoid significant harm to any of the relevant environmental objectives".

The establishment of technical screening criteria that meet the requirements of Article 19 was deferred to Delegated Regulation (EU) 2021/2139. However, the level of precision regarding what constitutes a substantial contribution or significant harm does not clearly improve. Furthermore, the Delegated Regulation postpones the inclusion of major economic activities such as agriculture or the textile industry to future regulations, resulting in additional challenges to entrepreneurs in these sectors.

From our logical analysis of the Regulation, a reasonable need arises to propose a simplification of the criteria applicable to environmentally sustainable economic activities as set out in Article 3. In more detail, paragraph (b) could be removed and paragraph (a) modified as follows:

(a) generates an overall positive contribution to the environmental objectives set out in Article 9, in accordance with Articles 10 to 16;

This approach aims to address the potential exclusion of activities that, while causing some harm to one of the objectives, have a net positive overall contribution to the set of environmental objectives. Naturally, the proposed amendment to Article 3 would require additional modifications to all articles that refer to significant harm.

In our view, the benefit of such a modification would be twofold. On the one hand, the Regulation would be significantly improved from a logical standpoint by avoiding the exclusion of activities that, in one way or another, impact one of the objectives but still make a net positive contribution to environmental improvement. On the other hand, the range of economic activities that could benefit from access to qualified financing would be broader.

Results of the mathematical analysis using graph theory

The origin of graph theory dates back to Leonhard Euler (1707–1783) and the city of Königsberg (now Kaliningrad, part of Russia since 1945). Königsberg was located at the mouth of the Pregel River, and its inhabitants wondered whether it was possible to start at a certain point and cross all seven bridges over the river exactly once, returning to the starting point. In 1736, Euler mathematically proved that this was not possible, marking the birth of graph theory. Nearly two centuries later, Shannon (1948), following a suggestion by statistician John W. Tukey, introduced the concept of the bit, a contraction of binary digit, as the minimal unit for encoding information. For communication to occur, a code must be employed; that is, a finite yet sufficiently rich set of symbols combined according to rules known by both the sender and the receiver.

The legal system of a society is structured through legal norms that regulate the relationships among its various agents. Understanding the legal system implies understanding the norms, and understanding the norms requires analyzing both their structure and the relationships established externally, between different norms, and internally. Among the different elements of a single norm.

Following Shannon (1948), where the bit is considered the basic unit of information encoding, it seems reasonable to assume that the article is the basic unit of analysis in normative coding. In this analysis, we will focus on the internal relationships among the different elements of a norm. To this end, we will use the article as the basic unit of analysis. The motivation for this choice is based on three pillars:

1. **Historical or customary reasons.** Since the earliest legal codes, such as the Spanish Commercial Code of 1829, the article has been used as the fundamental unit of normative structuring. This tradition has persisted in contemporary legislation, becoming a standard.
2. **Organizational or structural reasons.** Using the article as the basic unit allows for an orderly and comprehensible structuring of norms. Each article focuses on a specific provision, facilitating the understanding of the norm. A typical hierarchy is established, beginning with the law, regulation, or code, followed by the title, chapter, section, article, and paragraph. However, it is the article that serves to address specific provisions, while the higher levels in the hierarchy group articles together. Below the article, paragraphs are only relevant when accompanied by the article to which they belong. This structuring into articles facilitates the amendment of norms and, ultimately, of the legal system, without the need to modify an entire norm.
3. **Referencing reasons.** Since norms are divided into articles, they can be referenced or cited precisely within the norm itself and later in legal analysis and argumentation.

The referencing reason leads us to conduct a relational analysis using graph theory. We will illustrate this approach through an analysis of the relationships established within the Taxonomy Regulation. This quantitative analysis will allow us to objectively identify the most relevant articles within a norm.

Likely, that an expert accustomed to working daily with codes, laws, and regulations can identify the most important articles through a quick reading, without the need for mathematical tools. This work does not question that possibility; rather, it offers tools to complement the expert's judgment with quantitative evaluations of an article's centrality or the relative relevance of certain articles compared to others.

Returning to the Taxonomy Regulation, we observe that it contains 27 articles. To establish relationships between articles, we follow a simple criterion: if one article references another, we assume a direct relationship between them, regardless of how many times it is cited. Based on these references or citations, we construct a 27×27 adjacency matrix, where each element a_{ij} is set to one if there is a relationship between article i and article j , and zero if there is no relationship. From the adjacency matrix, we can construct the graph that shows the relationships among the articles of the regulation. The result is shown in Fig. 2.

At first glance, two aspects stand out. First, it is worth mentioning the nodes that exhibit a higher degree of relationships with the rest. Without yet entering into quantitative evaluations, node 3, related to the criteria applicable to economic activities, and articles 10 to 20, concerning contributions, significant harm, guarantees, selection criteria requirements, and the sustainable finance platform, are particularly notable.

Also noteworthy is Article 25, which amends Regulation 2019/2088 on the disclosure of sustainability related information in the financial services sector. This article appears isolated in the graph shown in Figure 3, as it neither cites nor is cited by any other article in the regulation under analysis.

Table 1
Centrality and normalized centrality of articles.

Node	Centrality	Normalized centrality
3	14	100%
19	9	64%
16	8	57%
20	8	57%
23	8	57%
17	7	50%
8	6	43%
10	6	43%
11	6	43%
12	6	43%
13	6	43%
14	6	43%
15	6	43%
27	6	43%
5	5	36%
9	4	29%
21	4	29%
22	4	29%
6	3	21%
7	3	21%
26	3	21%
4	2	14%
18	2	14%
24	2	14%
1	1	7%
2	1	7%
25	0	0%

We now proceed to a quantitative evaluation of the relationships between the articles of the regulation in order to establish a ranking based on centrality $C_G(i)$ and normalized centrality $N_G(i)$. The results of this ranking are shown in Table 1. The visual assessment made from the graph in Fig. 2 is now complemented by a quantitative evaluation of the node centrality.

Quantitatively, Article 3, related to the criteria applicable to economic activities, is the node with the highest centrality, followed by Article 19, related to the requirements applicable to selection criteria. It is striking that Article 9, which outlines environmental objectives, ranks low in the centrality ordering. This seemingly counterintuitive result, based on a reading of the regulation, becomes evident when analyzing the relationships between articles using graph theory. This tool may help entrepreneurs identify key elements of the regulation as a source for business opportunities.

Conclusions

In this final section, we provide a synthesis of the findings and elaborate on the implications for entrepreneurship and future research.

Synthesis

This study provides entrepreneurs with a novel analytical framework to better understand and respond to sustainability regulations. By applying Boolean logic and graph theory to Regulation (EU) 2020/852 (2020), we offer tools that clarify how legal conditions determine whether an activity qualifies as environmentally sustainable. Entrepreneurs can use this logic-based approach to assess whether their business models meet regulatory thresholds and to identify risky elements.

Furthermore, the identification of inconsistencies, such as the exclusion of activities that both contribute to and harm environmental objectives, enables entrepreneurs to anticipate regulatory gaps and design models that emphasize net positive environmental impact. The proposed analysis offers entrepreneurs a framework to deal with complex sustainability requirements. For instance, the analysis of mutually exclusive conditions that hinder sustainability classification may guide

entrepreneurs in the design of innovative business models aligned with regulatory expectations. Finally, it enhances investment planning by providing tools for assessing regulatory risk.

The graph-based analysis reveals which regulatory articles are most central, helping entrepreneurs prioritize compliance efforts and monitor key legal developments. It addresses the limitations of traditional approaches by revealing hidden hierarchies and dependencies within regulatory texts. By analyzing the relationships between articles, the study reveals the most central and influential components of the regulation. Entrepreneurs can use this information to prioritize compliance efforts and the alignment with regulatory requirements.

Boolean algebra and graph theory offer complementary insights into the structure and logic of sustainability regulations. On the one hand, Boolean analysis reveals the strict logical conditions imposed by Article 3, where failure to meet any requirement results in disqualification of an activity as environmentally sustainable. Graph theory, meanwhile, identifies Article 3 as the most central node in the regulatory network, underscoring its influence. The logical stringency of Article 3, derived from the logical analysis using Boolean algebra, is reinforced by the graph analysis, which identifies Article 3 as the node with the highest normalized centrality (100%). As a result, the convergence of high centrality and stringent logic highlights Article 3 as both pivotal and challenging for entrepreneurs to understand and address. This result confirms that the application criterion is not only the most structurally important element of the regulation, but also the most restrictive, making it a critical focus for strategic compliance and business model design. On the other hand, Article 9's low centrality ranking suggests that, for entrepreneurs, operability and access to sustainable financing depend less on the declaration of environmental objectives and more on compliance with detailed technical criteria outlined in Articles 3 and 19.

Implications

By integrating logical and mathematical analysis into regulatory interpretation, this approach offers entrepreneurs a clearer, more structured understanding of complex legal requirements. Unlike conventional legal analysis, it translates regulatory conditions into precise logical expressions and visualizes article relationships through graph theory. This enables entrepreneurs to identify key compliance priorities, detect regulatory gaps, and design business models that align with both legal expectations and sustainability goals.

As an additional result, our analysis has demonstrated the hypothesis that the intersection between legal language and mathematical language is a non-empty set. To reach this conclusion, we started from elements of legal language and showed that, after applying certain transformations, these elements are also common components of mathematical language. This conclusion suggests the existence of additional elements and transformations that may be useful to develop new tools for regulation analysis in an attempt to identify opportunities for new business models. Although the analysis described in this work is limited to the Taxonomy Regulation on sustainable investments, the approach and strategies based on mathematical logic are applicable to the study of the logical structure of other legal norms, especially in the fields of commercial, tax, and financial law.

The method supports scalable applications across various legal domains opening pathways for innovation in legal technology. Mathematical approaches offer entrepreneurs and policymakers powerful tools to navigate complex regulations with greater precision and strategic insight. Future research could explore the integration of mathematical logic with environmental, social and governance metrics to create more robust, transparent frameworks for evaluating sustainability performance. Additionally, combining this approach with artificial intelligence augmented legal reasoning could enable automated interpretation and analysis of complex legal texts.

Boolean algebra provides a powerful framework for identifying logical inconsistencies. However, its binary nature limits its ability to model the vagueness and imprecision often present in legal language. Many regulatory concepts, such as "significant harm" or "substantial contribution", are inherently imprecise. To address this limitation, integrating fuzzy logic into regulatory analysis would enable the consideration of degrees of truth and partial compliance.

CRediT authorship contribution statement

Francisco Salas-Molina: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Methodology, Investigation, Formal analysis, Conceptualization.
Eduardo Miranda-Ribera: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

We have no conflicts of interest to disclose.

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