DATA STRUCTURES AND ITS APPLICATIONS

Heap: Definition and Implementation

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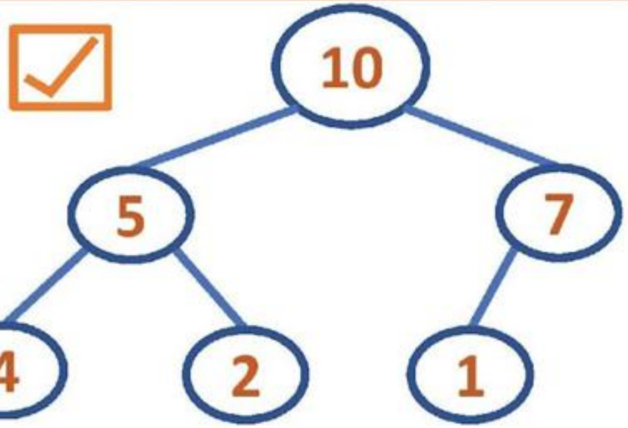
Heap Tree

Definition: A heap can be defined as a binary tree with keys assigned to its nodes (one key per node) provided the following two conditions are met:

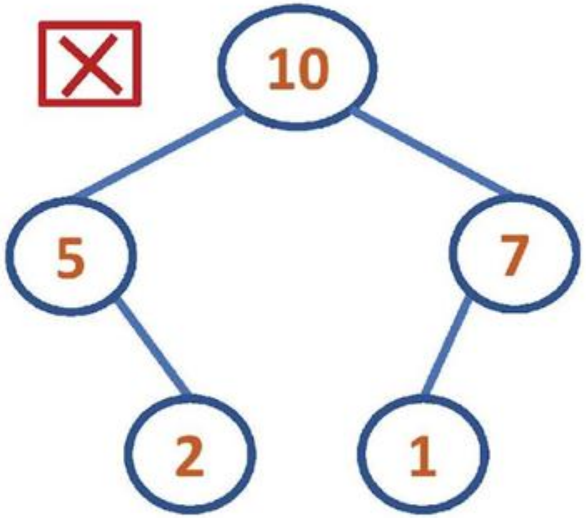
1. **The tree's shape requirement** - The binary tree is essentially complete, that is, all its levels are full except possibly the last level, where only some rightmost leaves may be missing
2. **The parental dominance requirement** - The key at each node is greater than or equal to the keys at its children. (This condition is considered automatically satisfied for all leaves.)

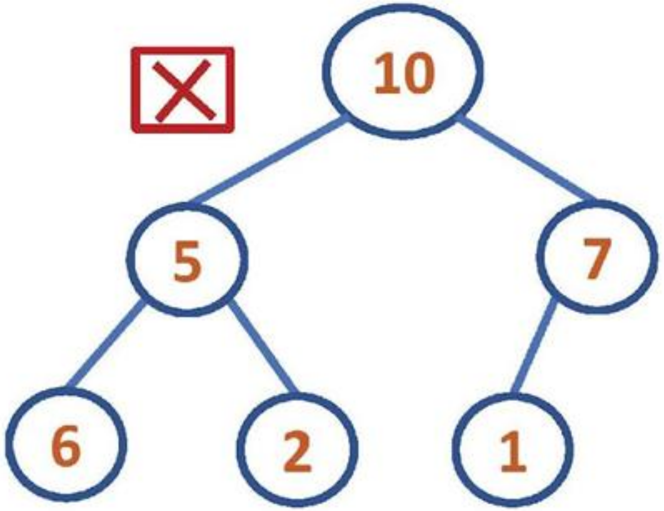
Heap Tree



Shape Requirement not satisfied

Parental dominance not satisfied





Only the topmost Binary Tree is a heap. Why?

Properties of Heap

1. There exists exactly one essentially complete binary tree with n nodes. Its height is equal to $\lfloor \log_2 n \rfloor$

2. The root of a heap always contains its largest element

3. A node of a heap considered with all its descendants is also a heap

4. A heap can be implemented as an array by recording its elements in the top-down, left-to-right fashion. It is convenient to store the heap's elements in positions 1 through n of such an array, leaving H[0] either unused or putting there a sentinel whose value is greater than every element in the heap.

Properties of Heap

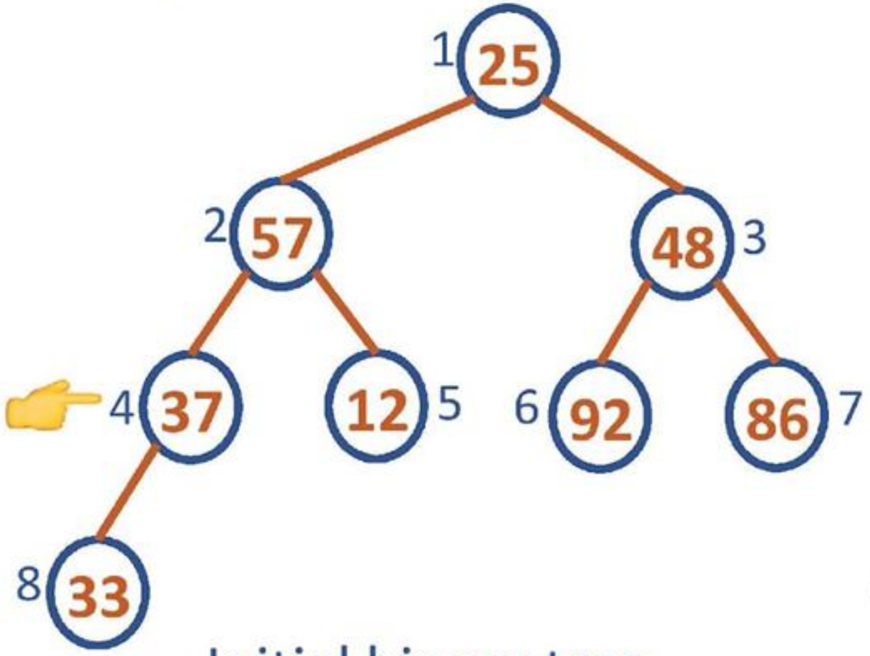
…

In such a representation,

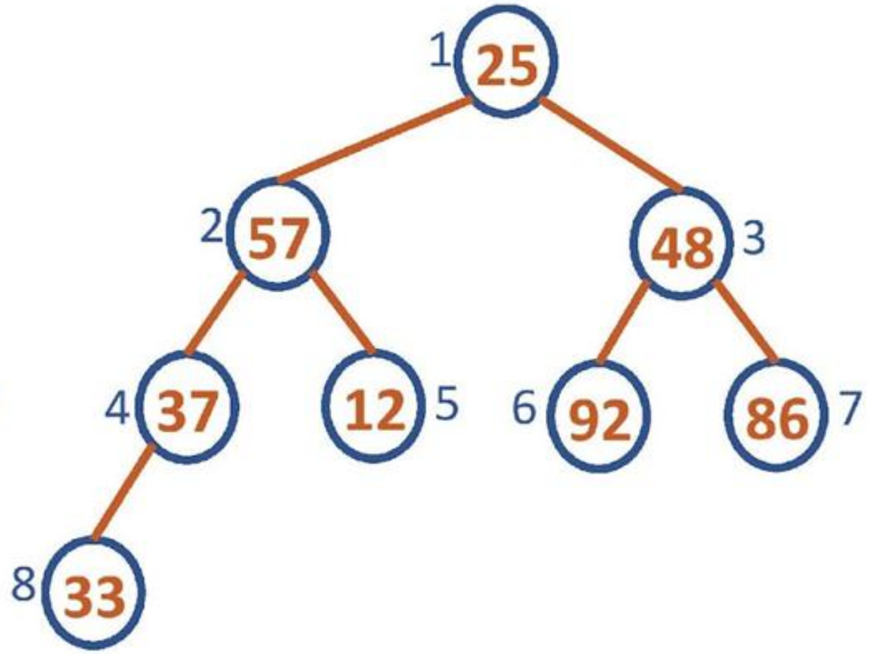
a) The parental node keys will be in the first $\lfloor n/2 \rfloor$ positions of the array, while the leaf keys will occupy the last $\lceil n/2 \rceil$ positions

b) The children of a key in the array's parental position i ($1 <= i <= \lfloor n/2 \rfloor$) will be in positions $2i$ and $2i + 1$, and, correspondingly, the parent of a key in position i ($2 <= i <= n$) will be in position $\lfloor i/2 \rfloor$

Bottom Up Heap Construction: 25, 57, 48, 37, 12, 92, 86, 33
Here, n=8

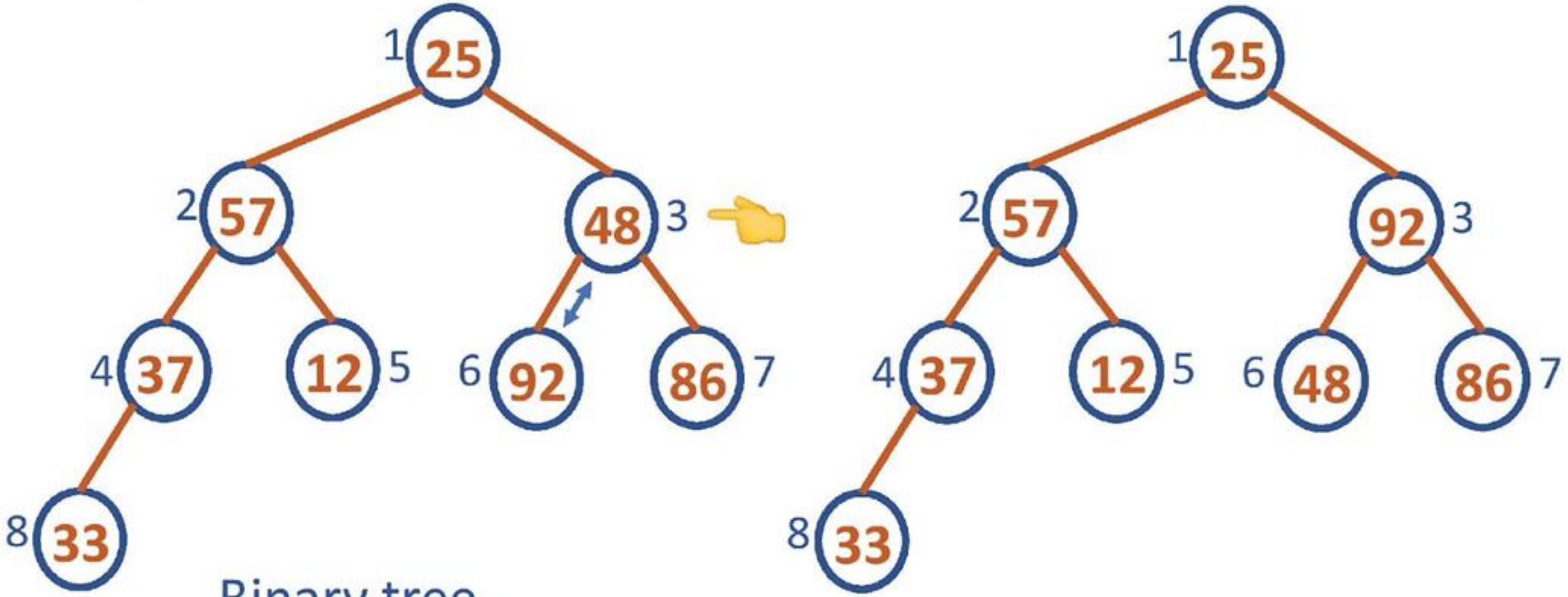


Initial binary tree



At k = 4, v = 37
Compare 37 with its only child 33
37 > 33, it's a heap at k=4

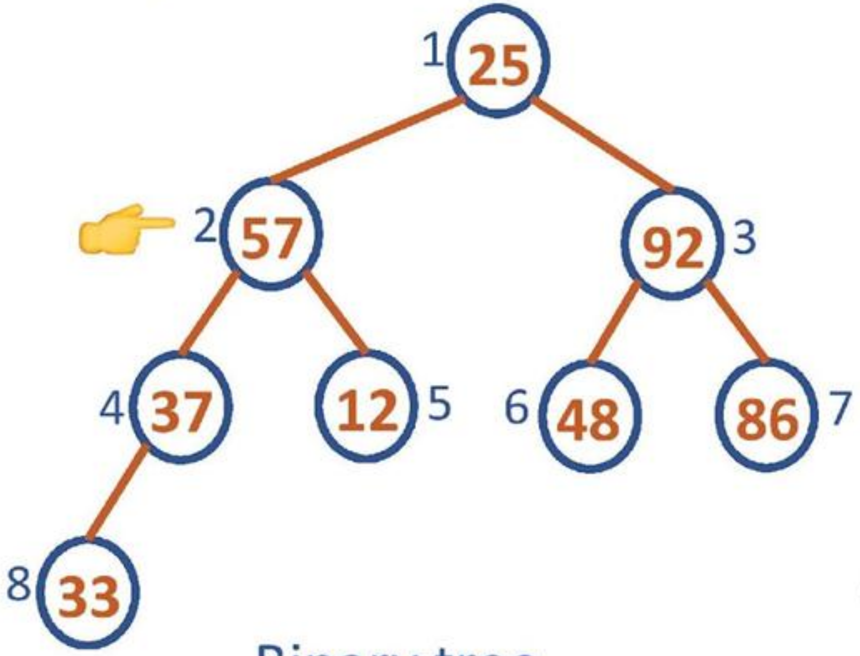
Bottom Up Heap Construction: 25, 57, 48, 37, 12, 92, 86, 33
Here, n=8

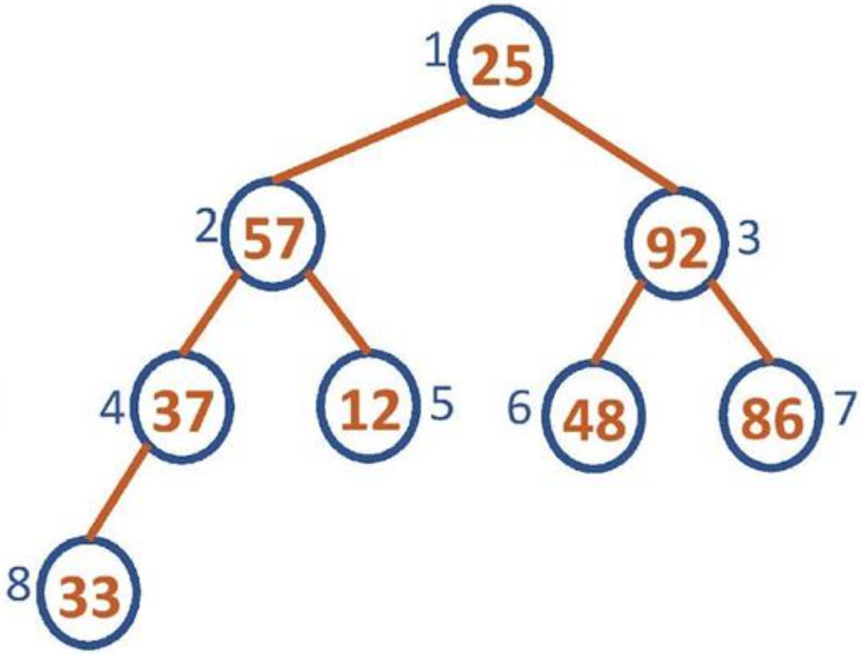


Binary tree
after one iteration at k=4

At k = 3, v = 48
Largest child: 92
Compare 48 with its largest child
48 < 92, Heapify

Bottom Up Heap Construction: 25, 57, 48, 37, 12, 92, 86, 33
Here, n=8





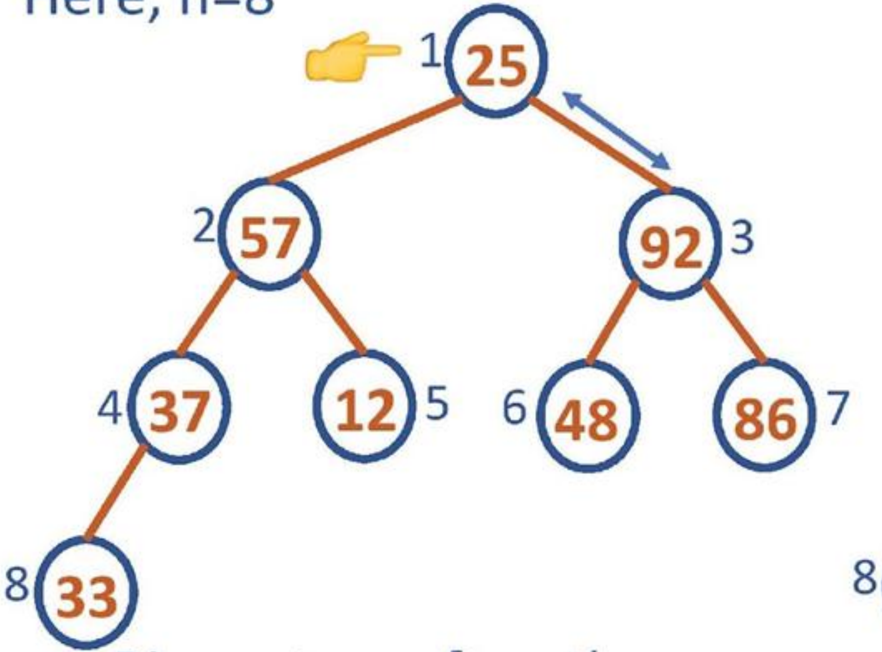
Binary tree
after two iterations at k=4, k=3

At k = 2, v = 57
Largest child: 37
Compare 57 with its largest child
57 > 37, it's a heap at k=2

Heap Construction – Bottom Up

Bottom Up Heap Construction: 25, 57, 48, 37, 12, 92, 86, 33

Here, n=8



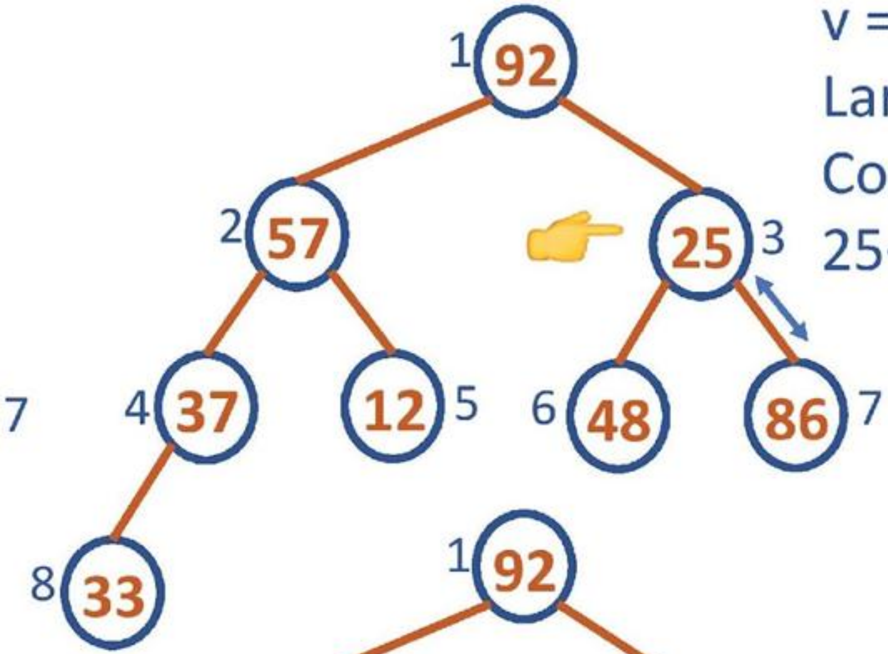
Binary tree after three iterations at k=4, k=3, k=2

At k = 1, v = 25

Largest child: 92

Compare 25 with its largest child

25 < 92, Heapify

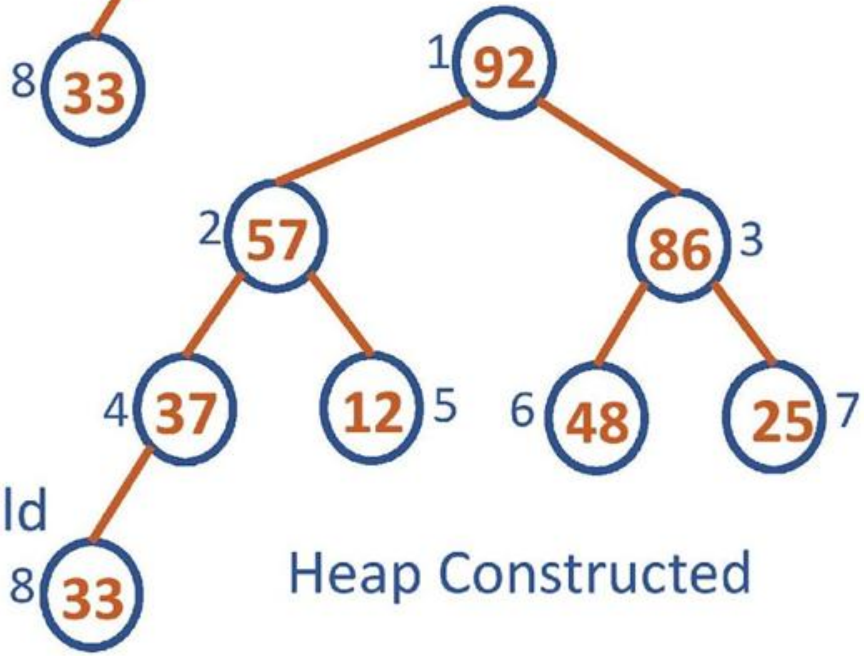


v = 25, Now at k = 3,

Largest child: 86

Compare 25 with its largest child

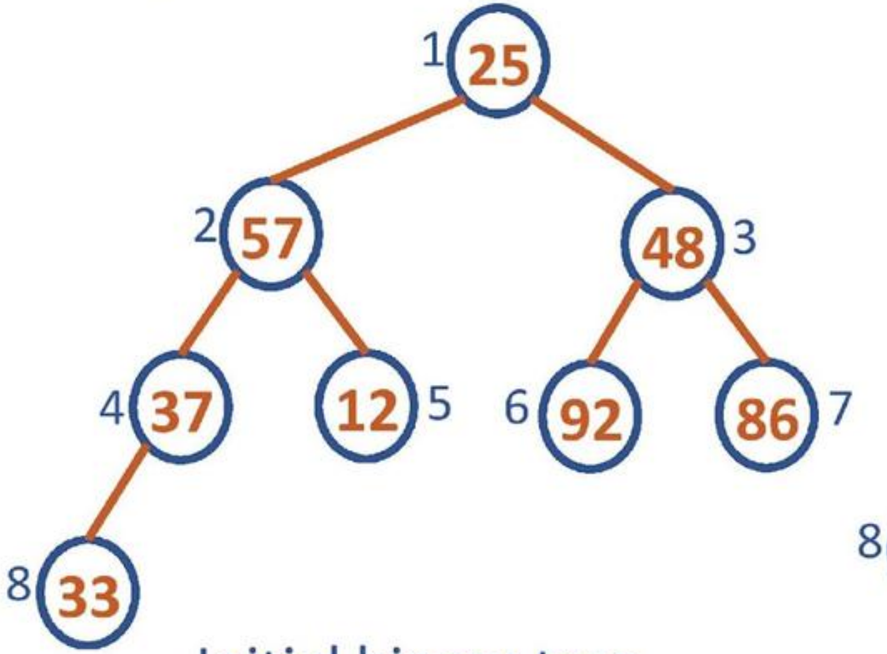
25<86, Heapify



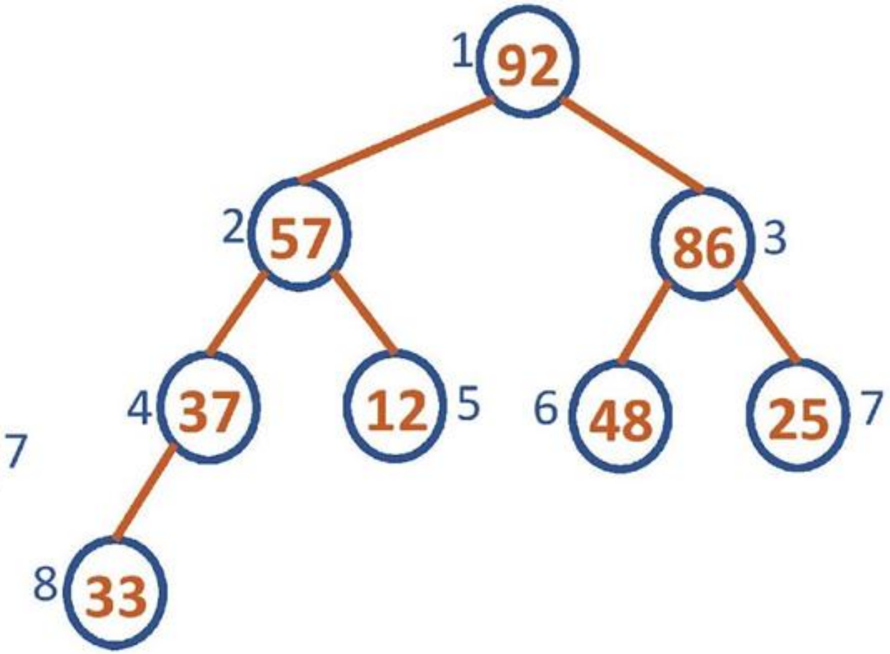
Heap Constructed

Heap Construction – Bottom Up

Bottom Up Heap Construction: 25, 57, 48, 37, 12, 92, 86, 33
Here, n=8



Initial binary tree



Bottom Up Heap Constructed

```
ALGORITHM HeapBottomUp(H[1…n])
//Constructs a heap from the elements of a given array by bottom-up algorithm
//Input: An array H[1…n] of orderable items
//Output: A heap H[1…n]
for i ← ⌊n/2⌋ downto 1 {
    k ← i
    v ← H[k]
    heap ← false
    while not heap and 2*k ≤ n {
        j ← 2*k
        if j < n                         //if there are two children
           if H[j] < H[j+1]
                j ← j+1                  //find position of largest child
        if v ≥ H[j]          //if key of parent node ≥ key of largest child
           heap ← true                   //it's a heap
        else {                           //heapify
                H[k] ← H[j]
                k ← j
        }      //end of else
    }   //end of while
    H[k] ← v
}   //end of for
```

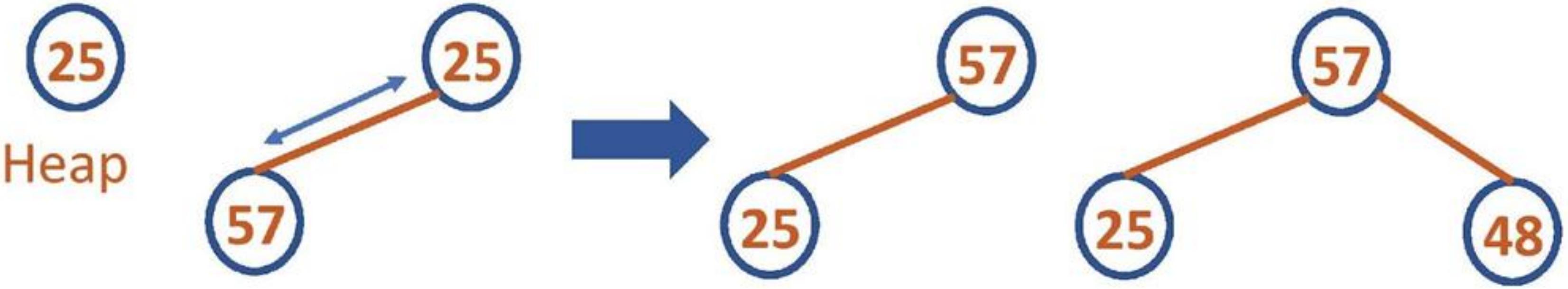
Heap Construction – Bottom Up

```
for(i=n/2-1;i>=0;i--)
 {
  k=i;
  v=h[k];
  heap=0;
  while(!heap && 2*k+1<=n-1)
   {
     j=2*k+1;
     if(j+1<=n-1)
      if(h[j+1]>h[j])  j=j+1;
    if(v>h[j])
      heap=1;
    else
     {
      h[k]=h[j];
      k=j;
      }
    }
   h[k]=v;
  }
```

Heap Construction – Top Down

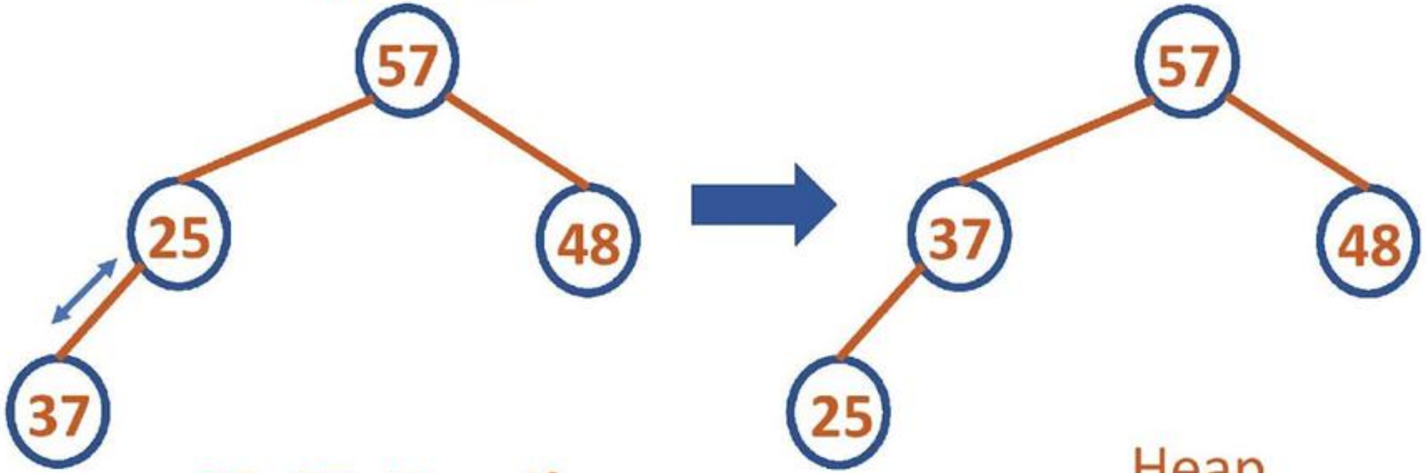
Top Down Heap Construction: 25, 57, 48, 37, 12, 92, 86, 33
Here, n=8

Insert 25 | Insert 57 | Insert 48



Heap | 25<57, Heapify | Heap | Heap

Insert 37



25<37, Heapify | Heap

Heap Construction – Top Down

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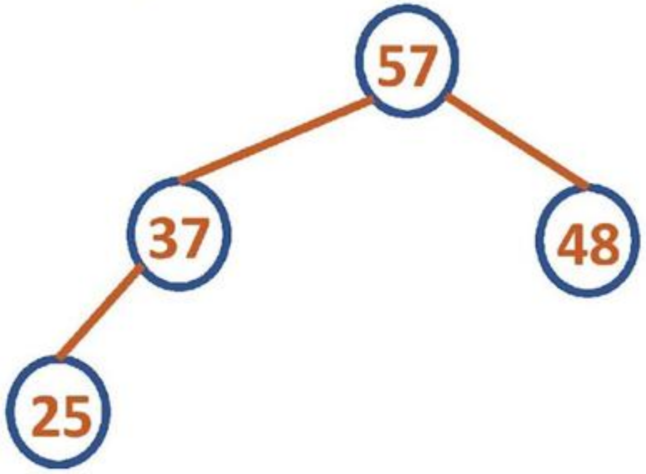
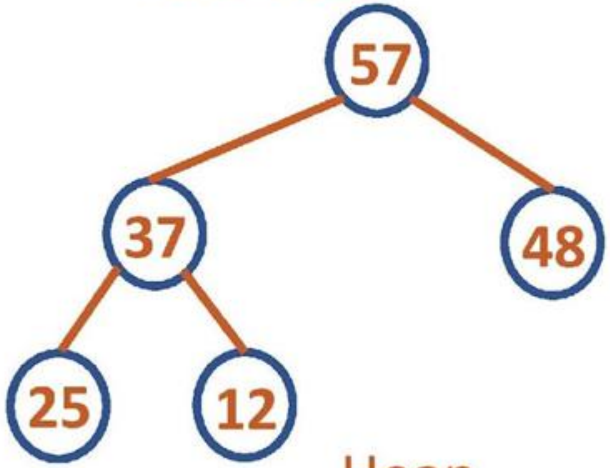
Here, n=8

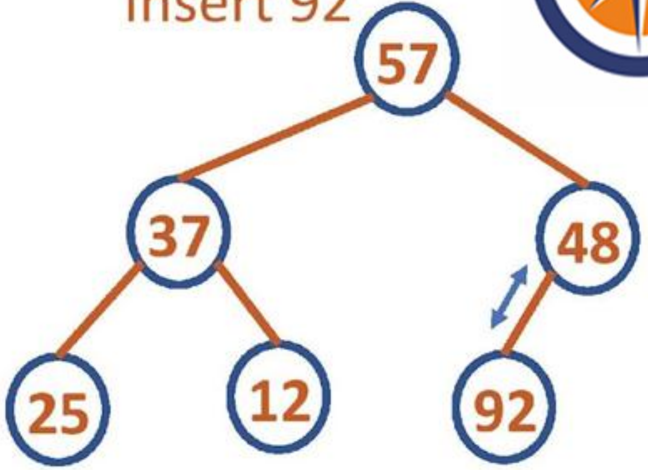


Insert 12

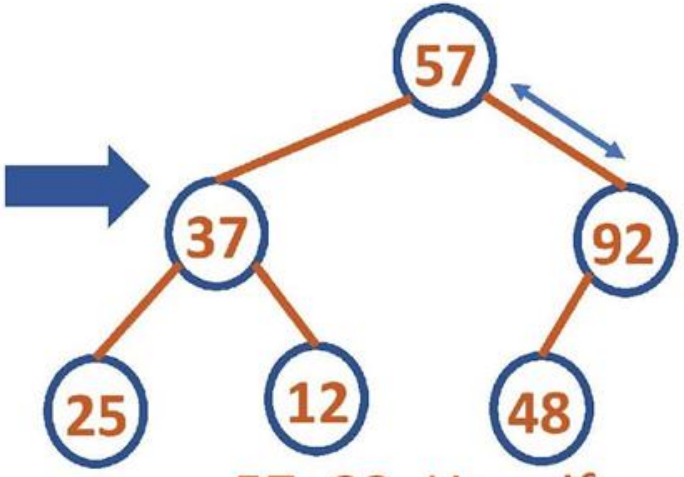


Heap

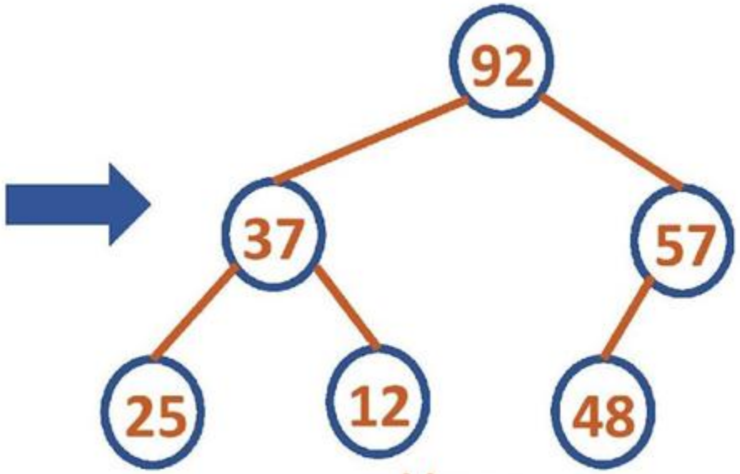
Insert 92



48<92, Heapify



57<92, Heapify



Heap

Heap Construction – Top Down

Top Down Heap Construction: 25, 57, 48, 37, 12, 92, 86, 33
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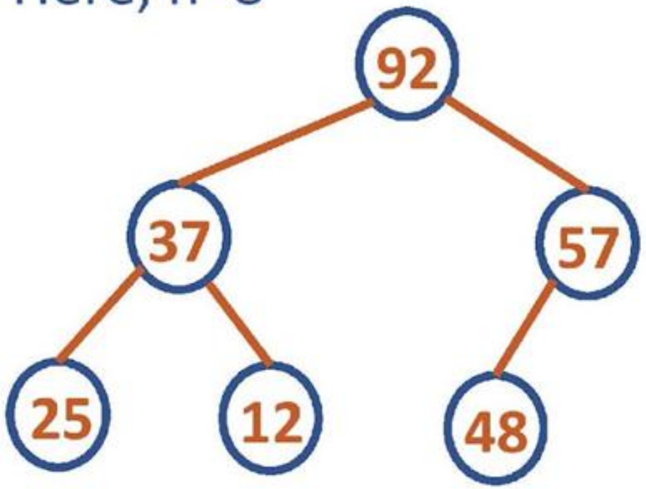
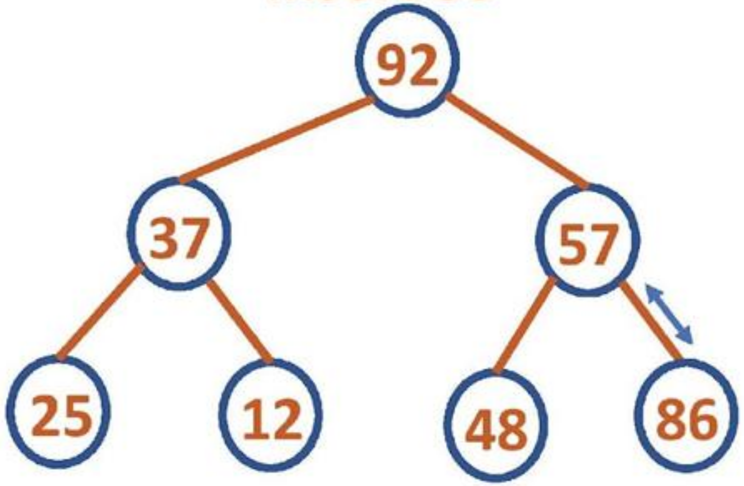
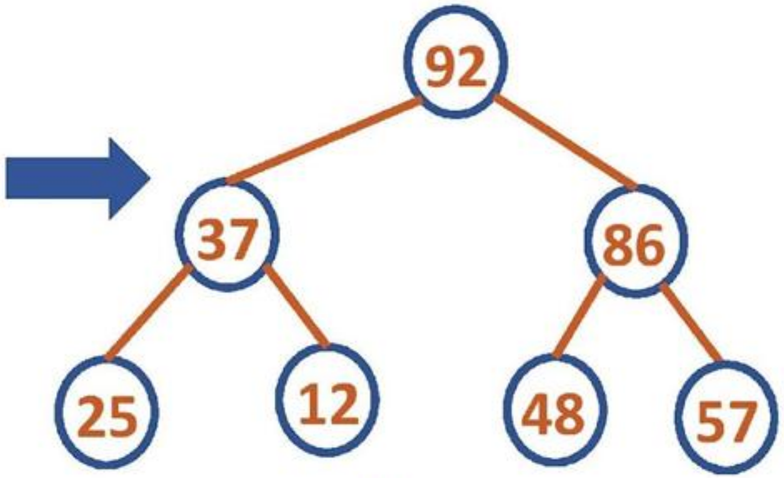


Insert 86



57<86, Heapify



Heap

Heap Construction – Top Down

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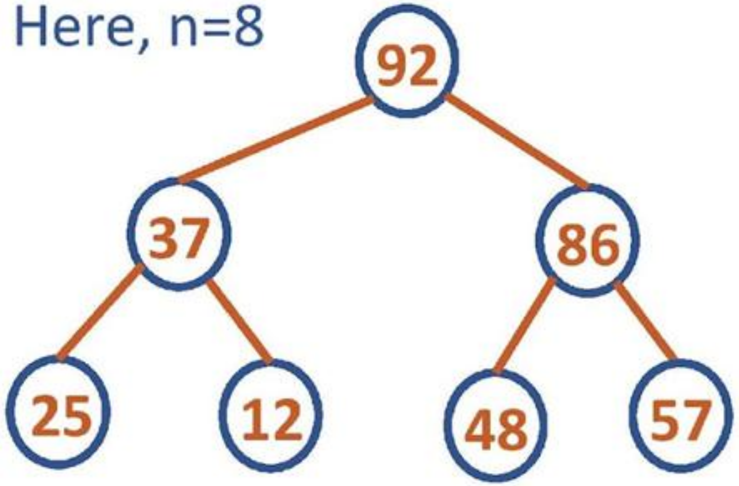

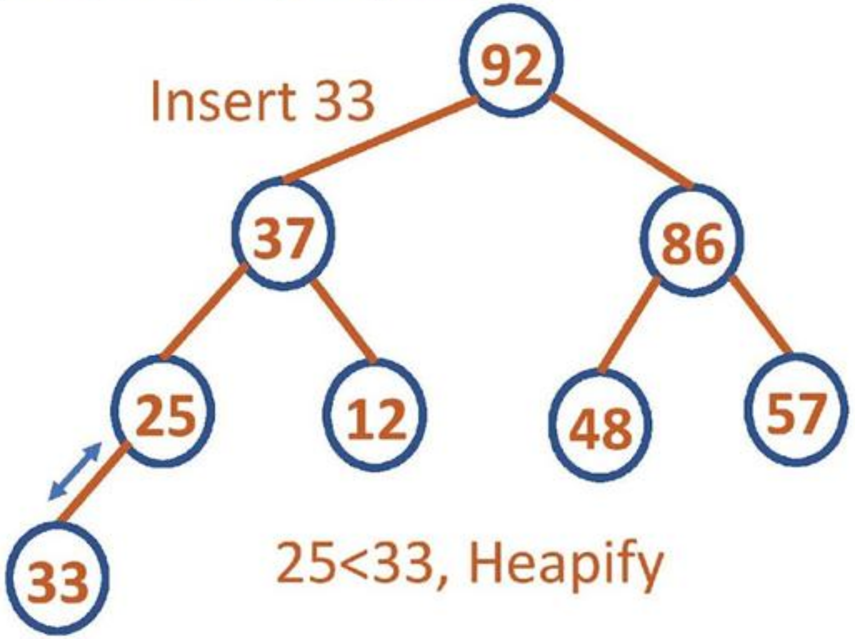

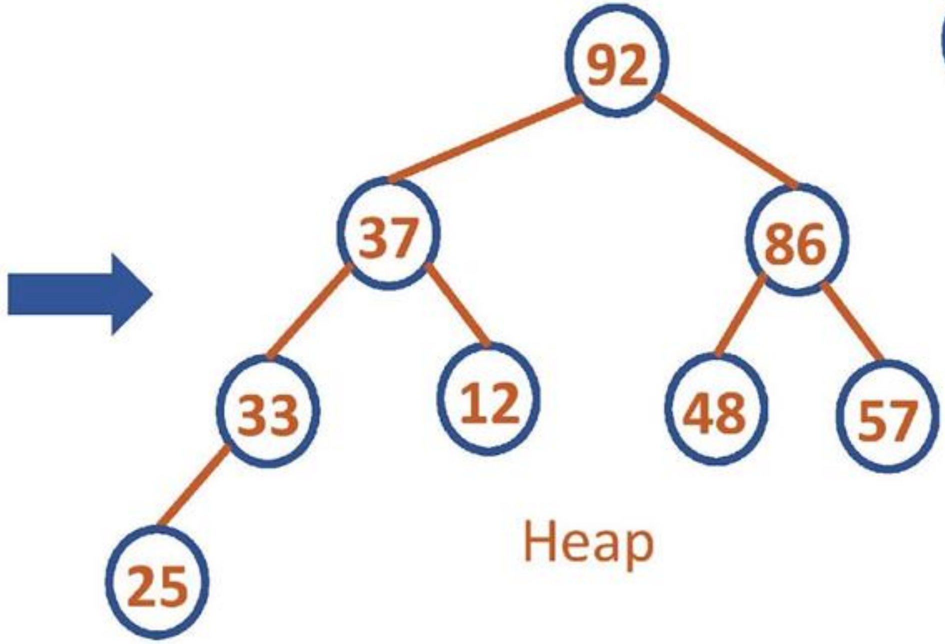

Heap Construction – Top Down

1. First, attach a new node with key *K* in it after the last leaf of the existing heap
2. Then sift *K* up to its appropriate place in the new heap as follows
3. Compare *K* with its parent's key: if the latter is greater than or equal to *K,* stop (the structure is a heap);
4. otherwise, swap these two keys and compare *K* with its new parent
5. This swapping continues until *K* is not greater than its last parent or it reaches the root

1. In a max-heap, the value of the parent node is always:

A) Less than its children

B) Equal to its children

C) Greater than or equal to its children

D) None of the above

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2. In an array representation of a heap, the parent of node at index i is at: (Assume Heap elements are stored at index positions 1…..n)

A) i/2

B) (i-1)/2

C) 2i

D) 2i + 1

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A) i/2
B) (i-1)/2
C) 2i
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3. Which method of heap construction is more efficient for a large array?

A) Incremental insertion

B) Heapify (bottom-up)

C) Preorder traversal

D) BFS insertion

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4. Heapify-down (bubble-down) is used when:

A) Inserting a new element
B) Deleting the root element
C) Searching for an element
D) Traversing the heap

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5. After inserting an element in a max-heap, which operation ensures the heap property is maintained?

A) Heapify-down
B) Heapify-up (bubble-up)
C) BFS traversal
D) Sorting the array

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THANK YOU

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