DATA STRUCTURES AND ITS APPLICATIONS

Balanced Trees

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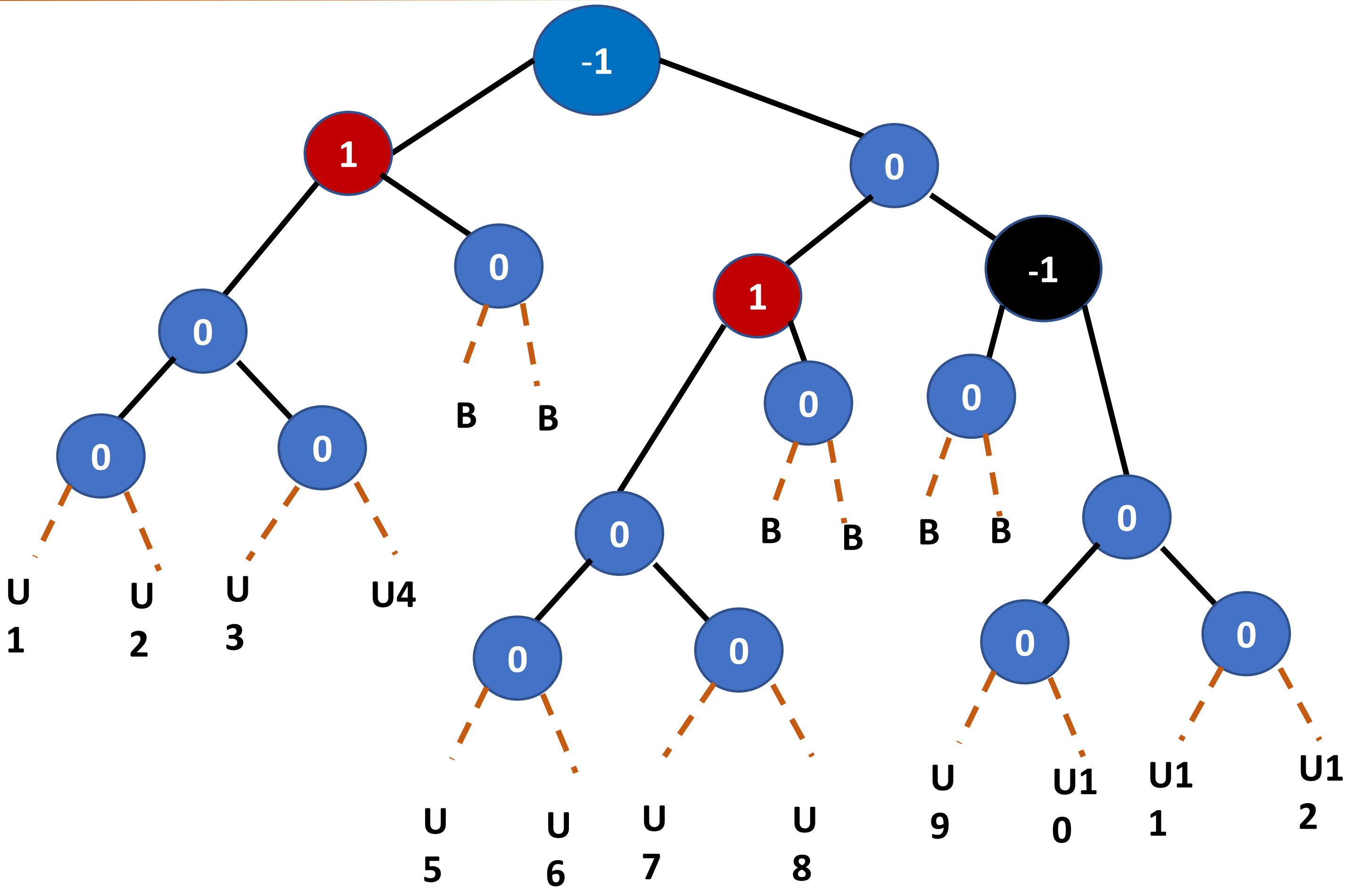
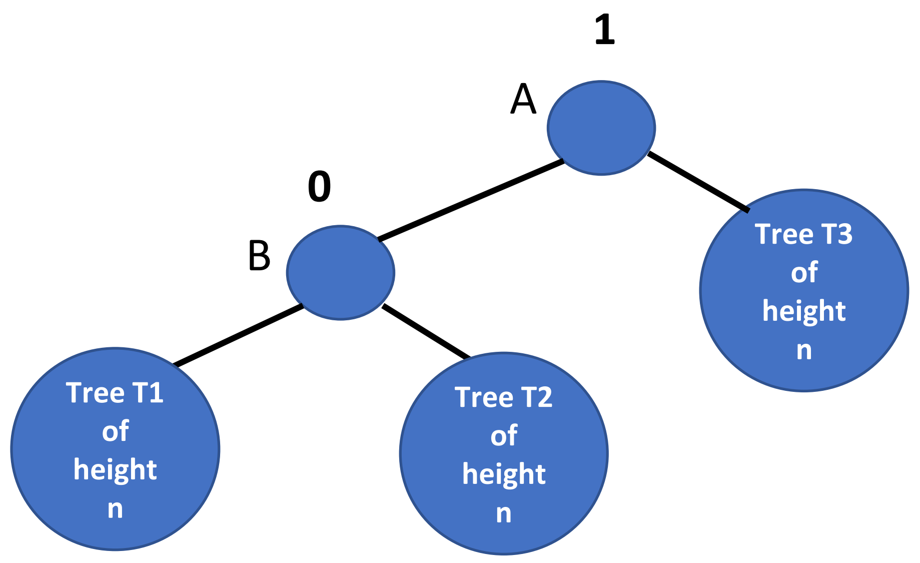
Possible insertion into AVL tree



- Unbalanced insertions are indicated by **U**
- Balanced insertions are indicated by **B**

Courtesy: "Data Structures using c and c++ " By Y Langsam, M. J. Augenstein and Aron M. Tenenbaum

AVL tree Insertions



Balance factor(A) = (n+1) – n =1

Balance factor(B) = n – n = 0

- Let us consider A is the youngest ancestor which becomes unbalanced
- Balance factor of A should be 1 before insertion
- A should have a left child B with the balance factor of 0

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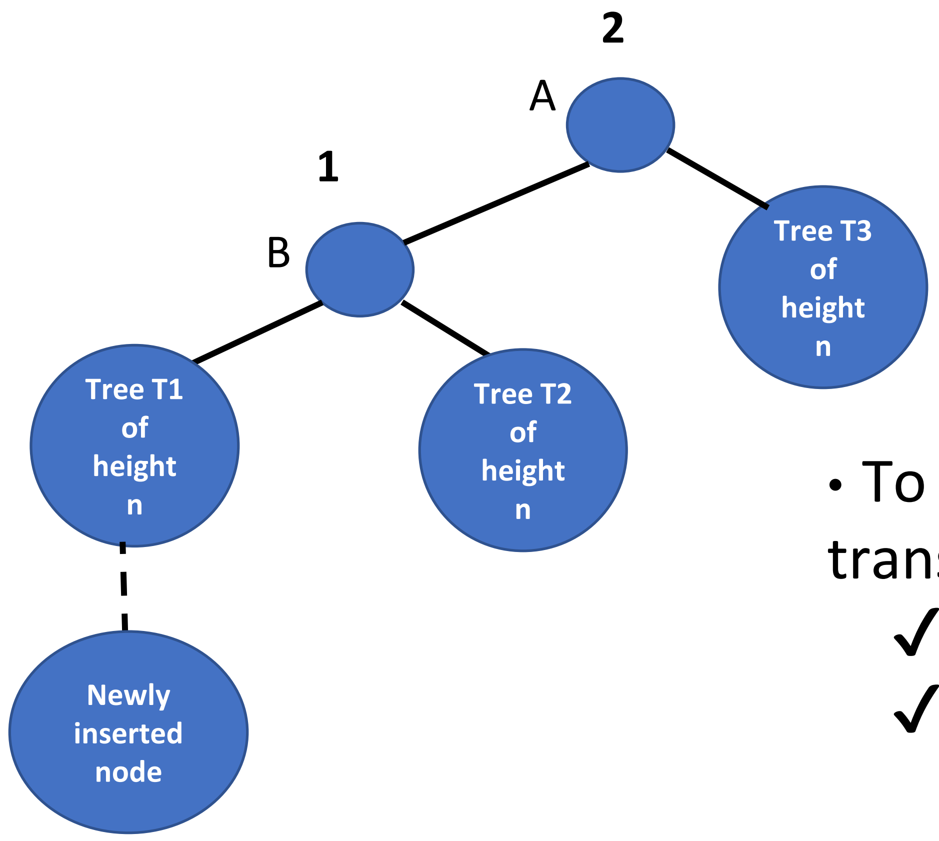
Unbalanced Tree after inserting a node to left subtree

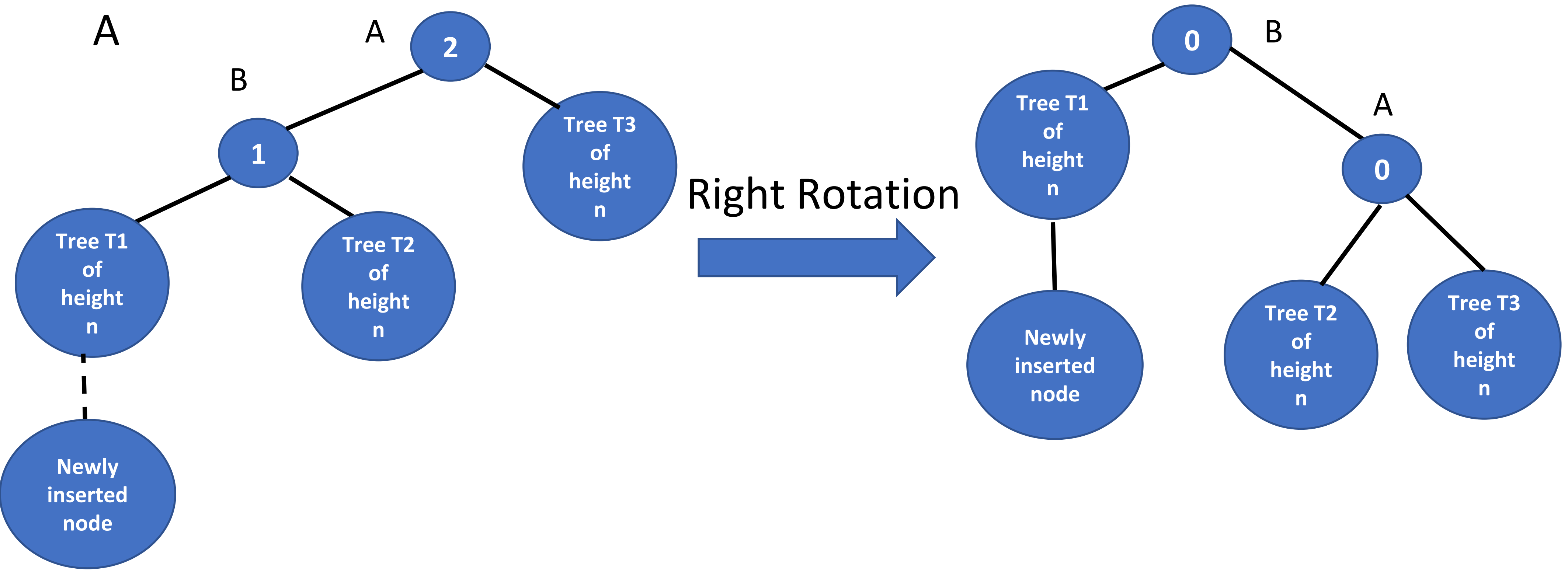
- Newly inserted node is left descendent of node A
- Changing the balance **B to 1 and A to 2**
- A is the youngest ancestor of the new node to become unbalanced



- To maintain the balance : Tree needs to be transformed
  - ✓ Transformed tree is balanced
  - ✓ Binary search tree property is maintained after transformation

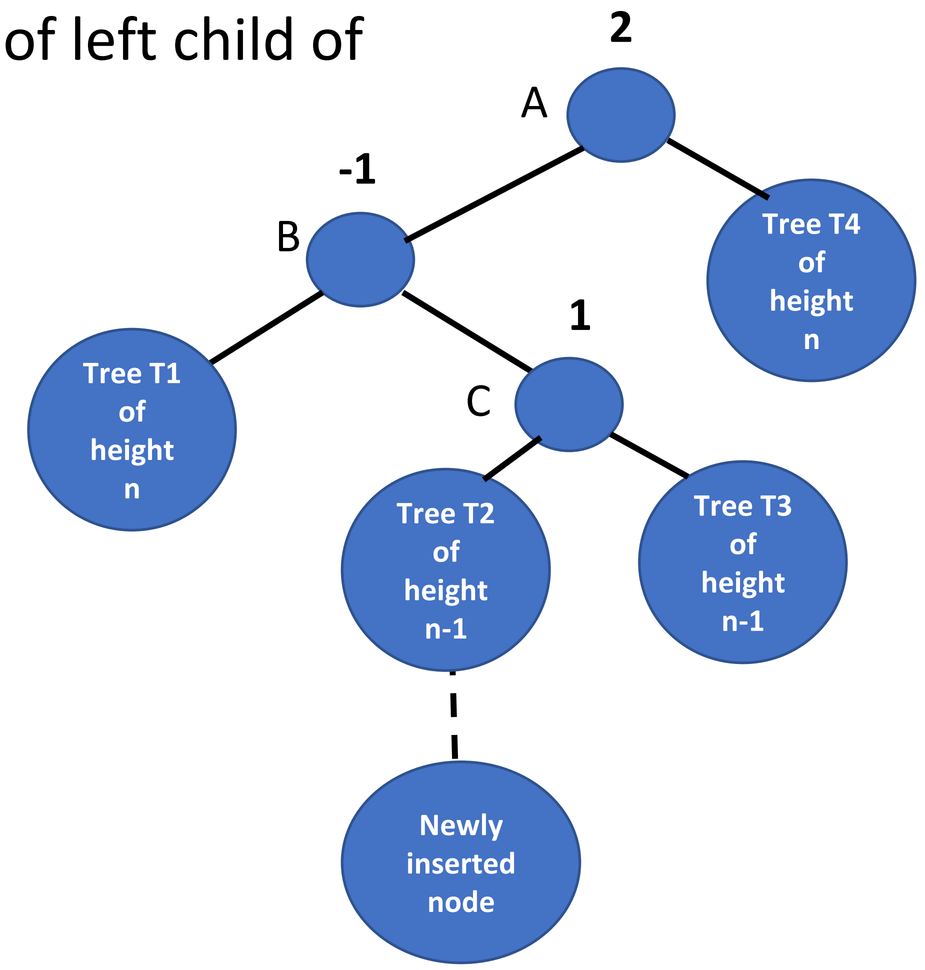
Transformed Balanced Tree after Rotations

- To maintain a balance we need to rotate sub tree B rooted at node A



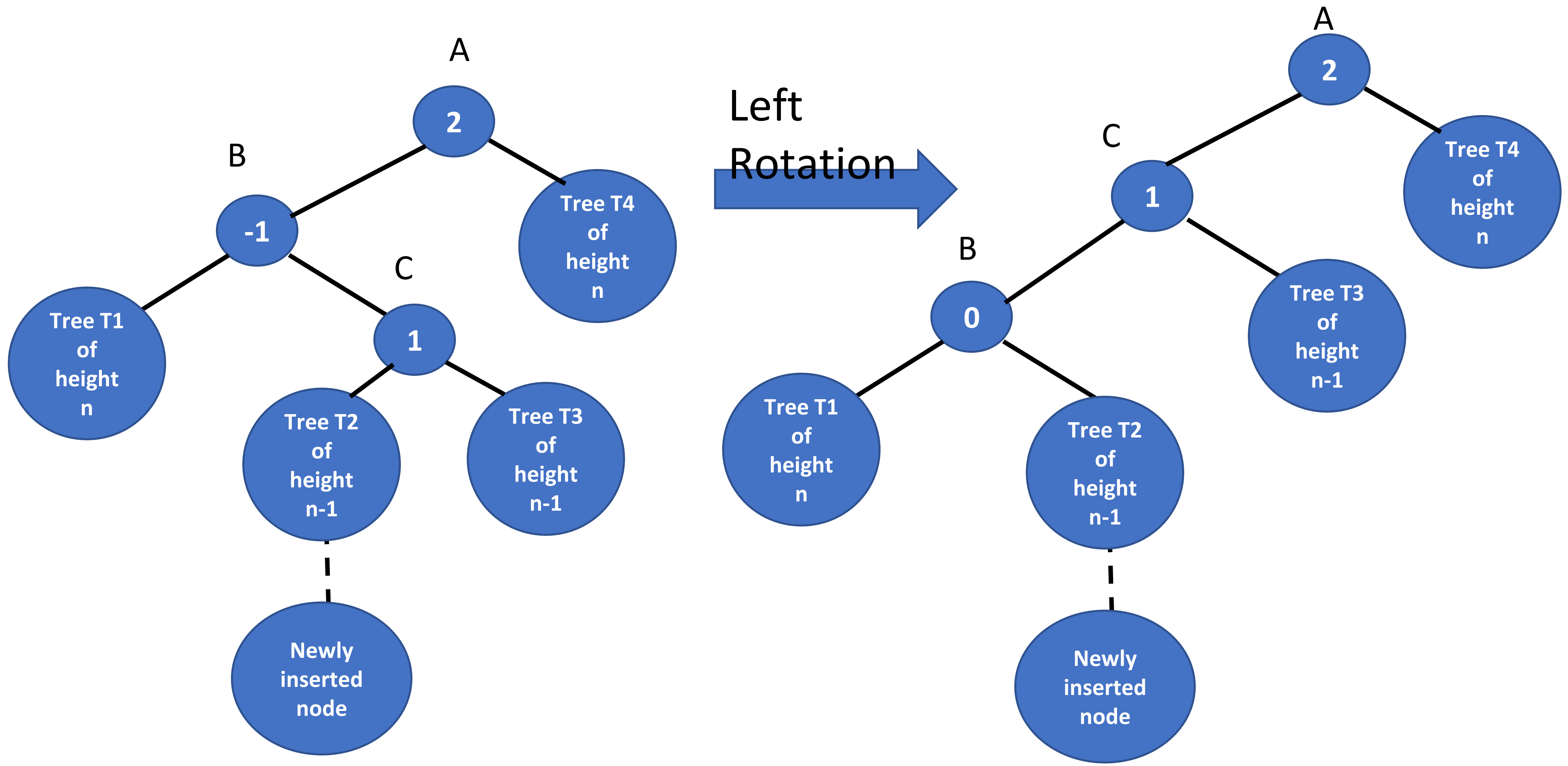
Unbalanced Tree after inserting a node to right subtree

- Newly inserted node is left descendent of the node A
- New node is inserted into right subtree of left child of A



Balance factor(C)= n-(n-1) = 1
Balance factor(B)= n-(n+1) = -1
Balance factor(A)= n+2-n= 2

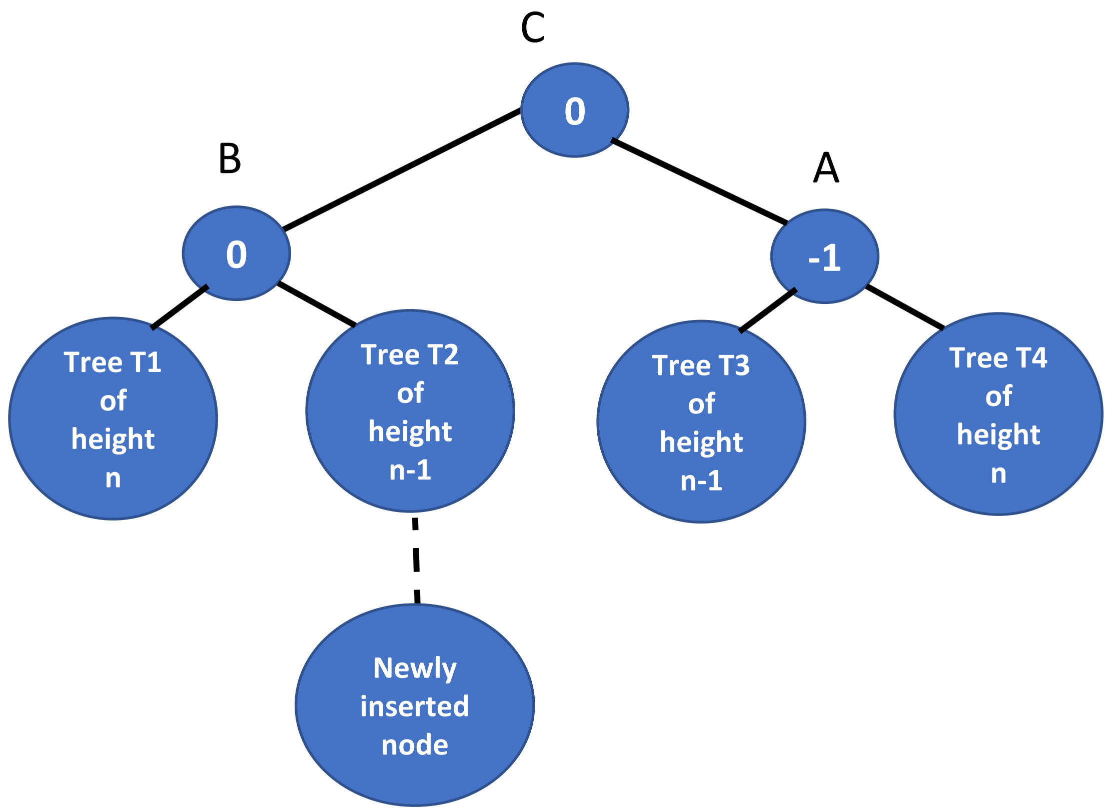
Transformed Balanced tree after Rotations



Courtesy: “Data Structures using c and c++ ” By Y Langsam, M. J. Augenstein and Aron M. Tenenbaum

Transformed Balanced tree after Rotations

Right Rotation



Courtesy: “Data Structures using c and c++ ” By Y Langsam, M. J. Augenstein and Aron M. Tenenbaum

AVL tree Insertion

- Insertion in AVL tree is performed using standard BST Insertion
- If tree becomes unbalanced, we rebalance the tree using left or right rotation
- If node X is inserted into balanced BST
- we need to find the youngest ancestor which becomes unbalanced

Four cases:

- IF(Balance factor of node) == 2 – unbalanced node(U)
  - ✓ case-1 : Left-Left case
    - IF((newly inserted key) < (key in the left subtree' root))
  - ✓ case-2: Left-Right case
    - IF((newly inserted key) > (key in the left subtree' root))

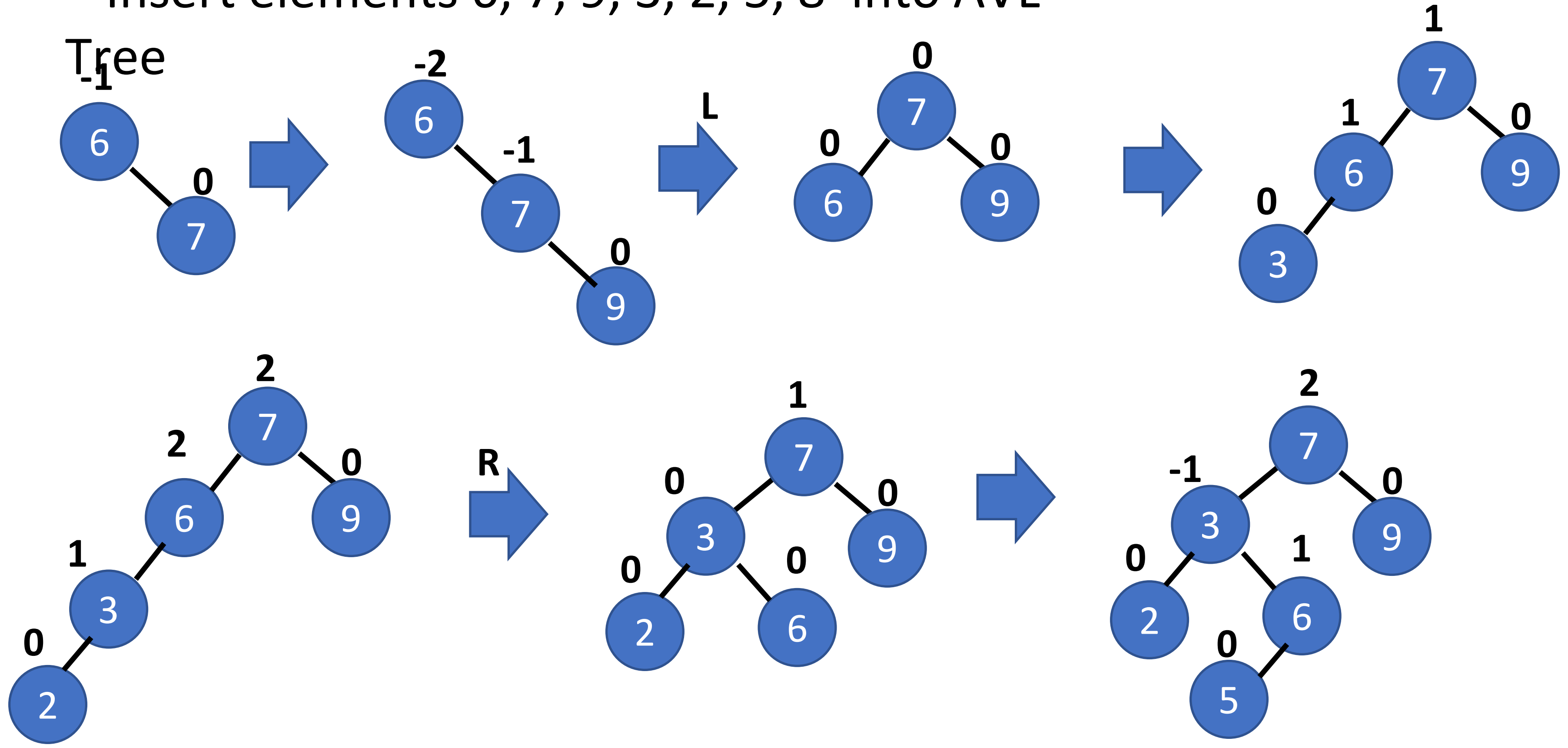
AVL tree Insertion

Four cases:

- IF((Balance factor of node)) == -2 – unbalanced node(U)
  - ✓ Case 3: Right-Right case
    - IF((newly inserted key) > (key in the right subtree' root))
  - ✓ Case 4:Right-Left case
    - IF((newly inserted key) < (key in the right subtree' root))

Examples – AVL Tree Insertions

- Insert elements 6, 7, 9, 3, 2, 5, 8 into AVL Tree



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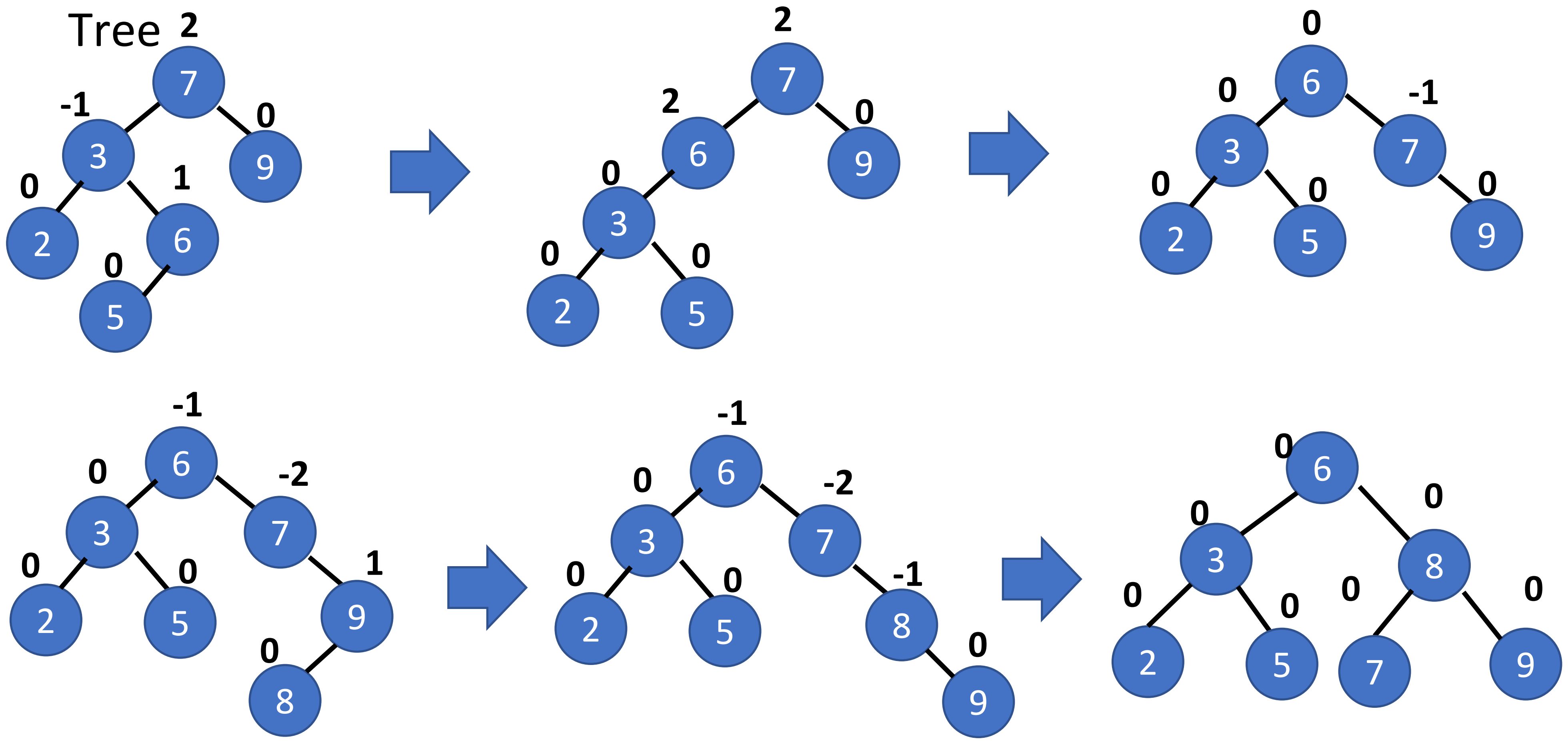
Example – AVL Tree Insertions

- Insert elements 6, 7, 9, 3, 2, 5, 8 into AVL Tree



Deletions in AVL tree

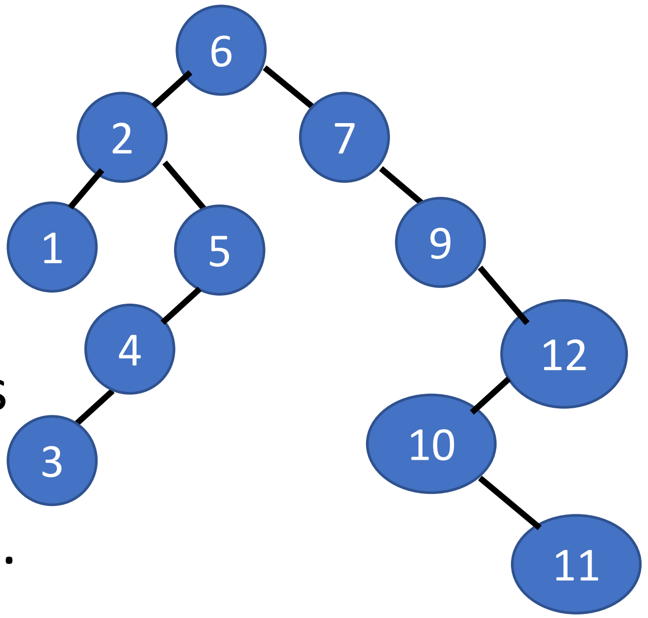
- Deletion in AVL tree is performed using standard BST Deletion
- If tree becomes unbalanced, we rebalance the tree using left or right rotation

BST Deletion: 3- case: Node to be delted

Case 1: Does not have any children

Case 2: has either left or right subtrees

Case 3: has both left and right subtrees.

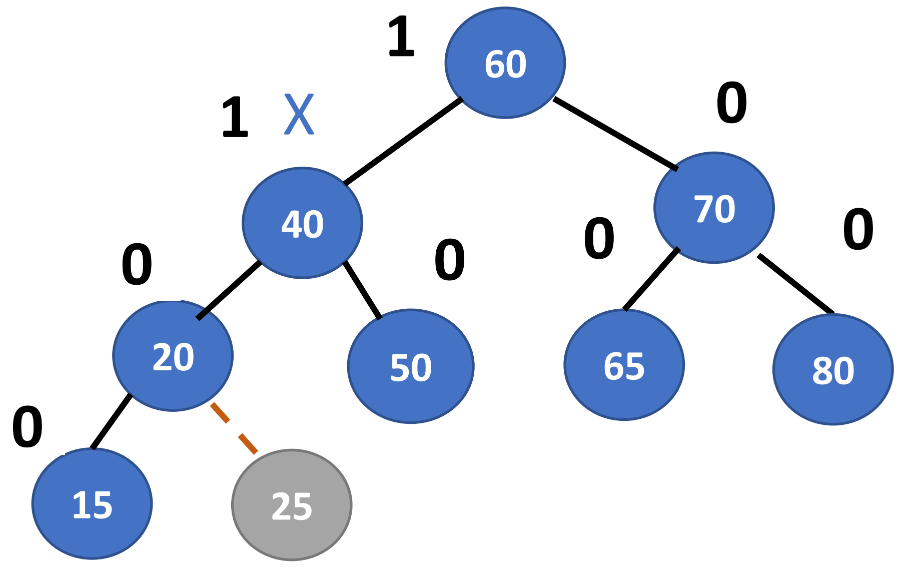


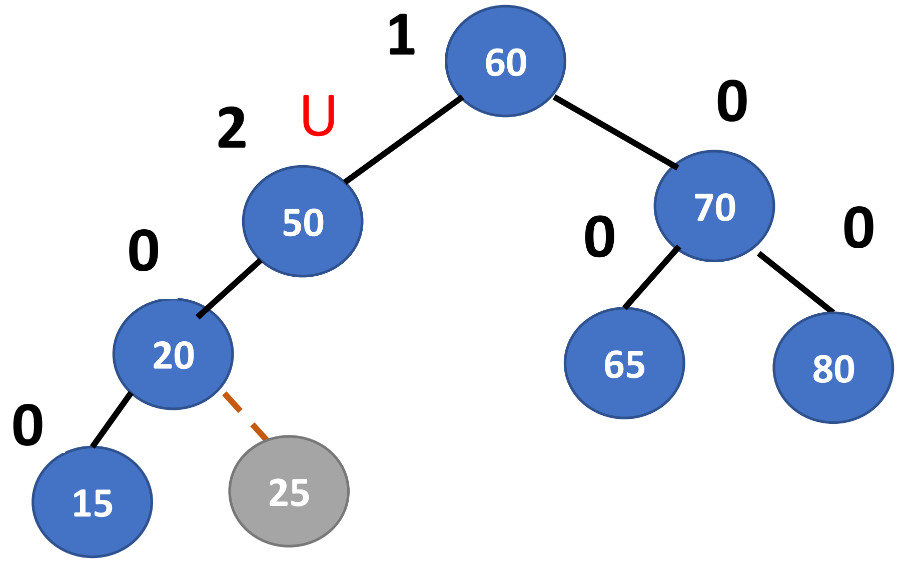
Deletions in AVL tree

- If node X is deleted from the BST
- we need to find the youngest ancestor which becomes unbalanced

Four cases: Case-1

- IF((Balance factor of a node) == 2) - unbalanced node(U)
  - ✓ Left-Left case:
    - IF(Balance factor of left subtree's root) >= 0

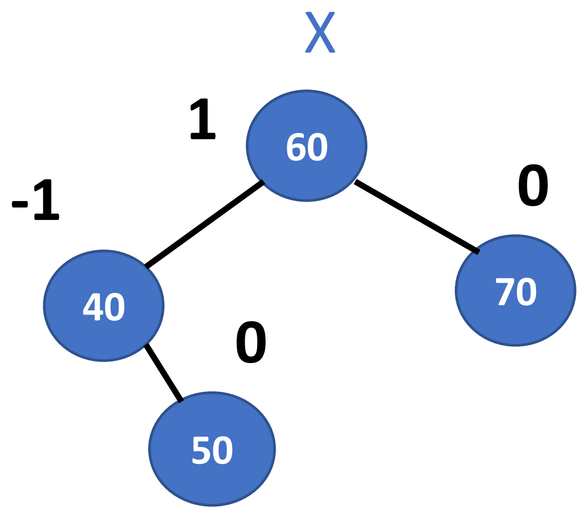


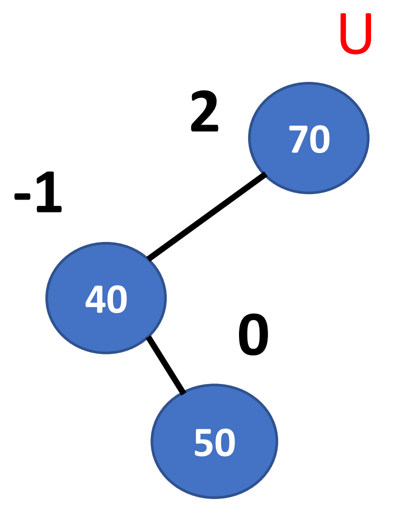


Deletions in AVL tree

Case-2:

- IF(Balance factor of a node == 2) – unbalanced node(U)
  - ✓Left-Right case
    - IF(Balance factor of left subtree's root) < 0

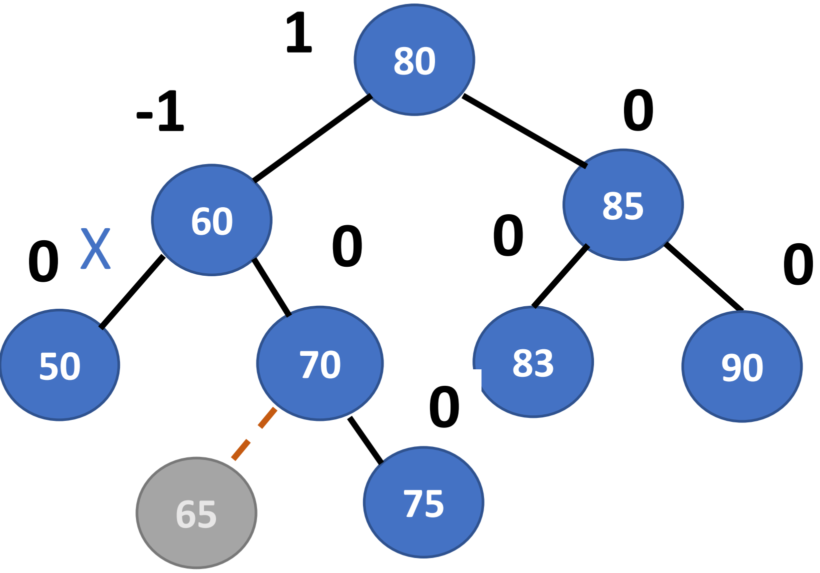


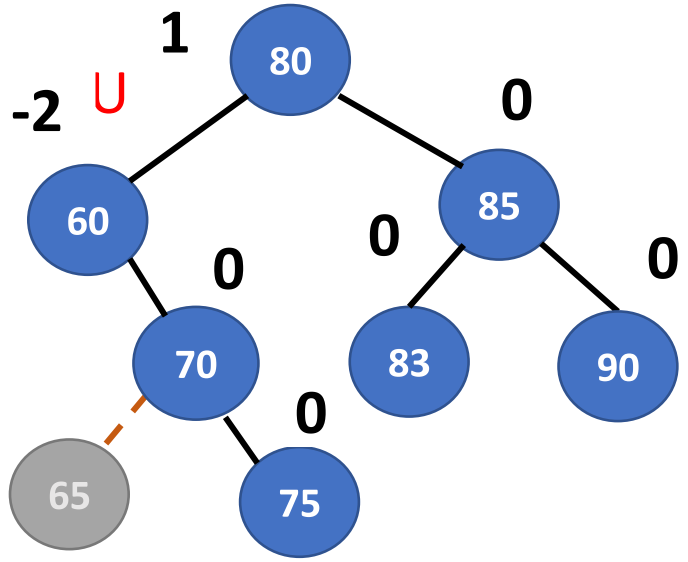


Deletions in AVL tree

Case-3:

- If ((Balance factor of a node) == -2) unbalanced node(U)
  - ✓Right-Right case
    - IF(Balance factor of Right subtree's root) <= 0





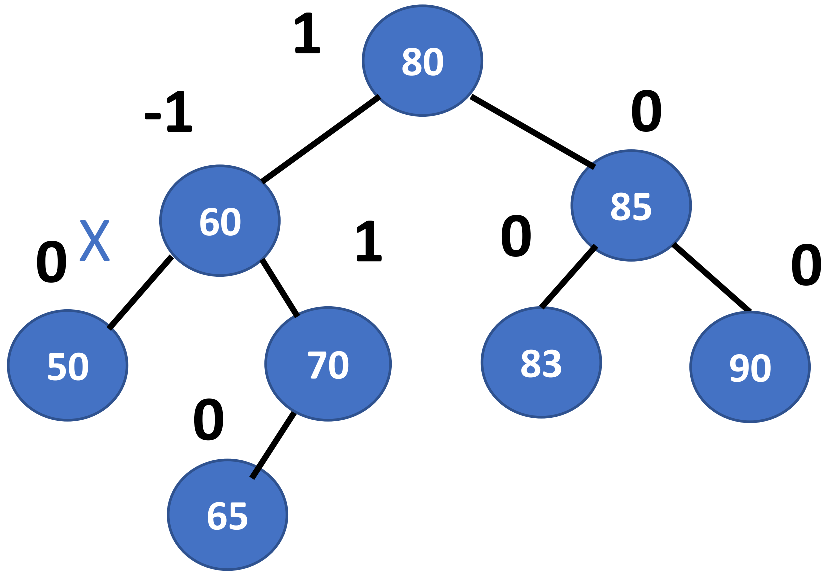
Deletions in AVL tree

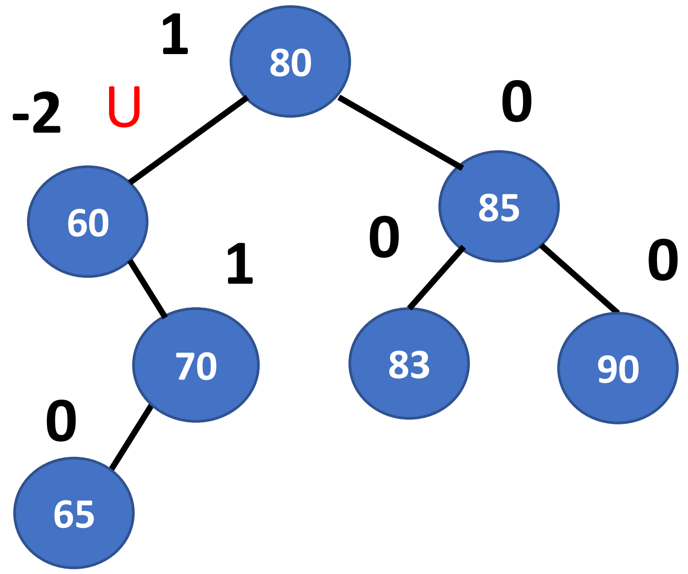
Case-4:

IF((Balance factor of unbalanced node) == -2) unbalanced node(U)

✓Right-Left case

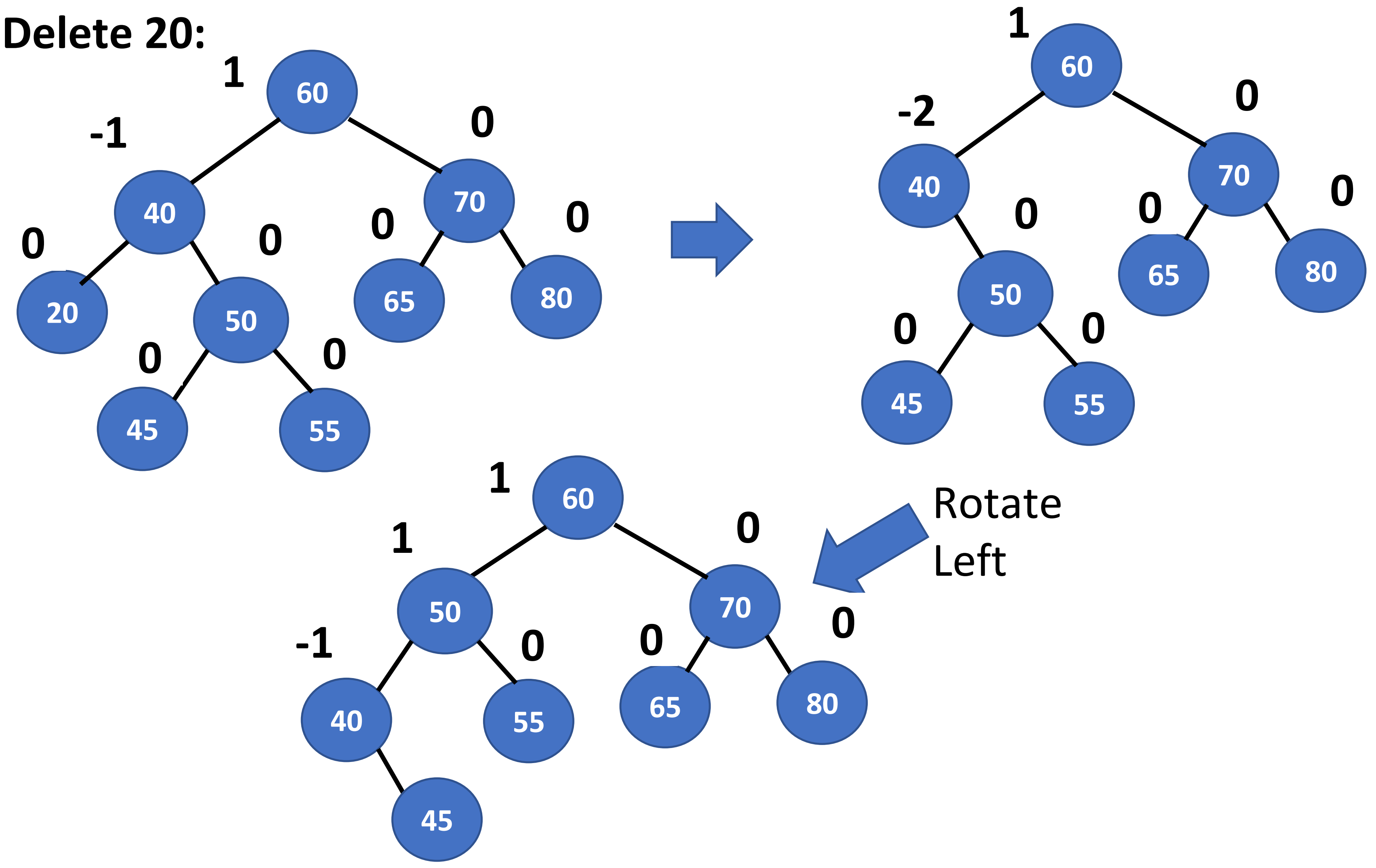
- IF(Balance factor of Right sub tree's root) > 0





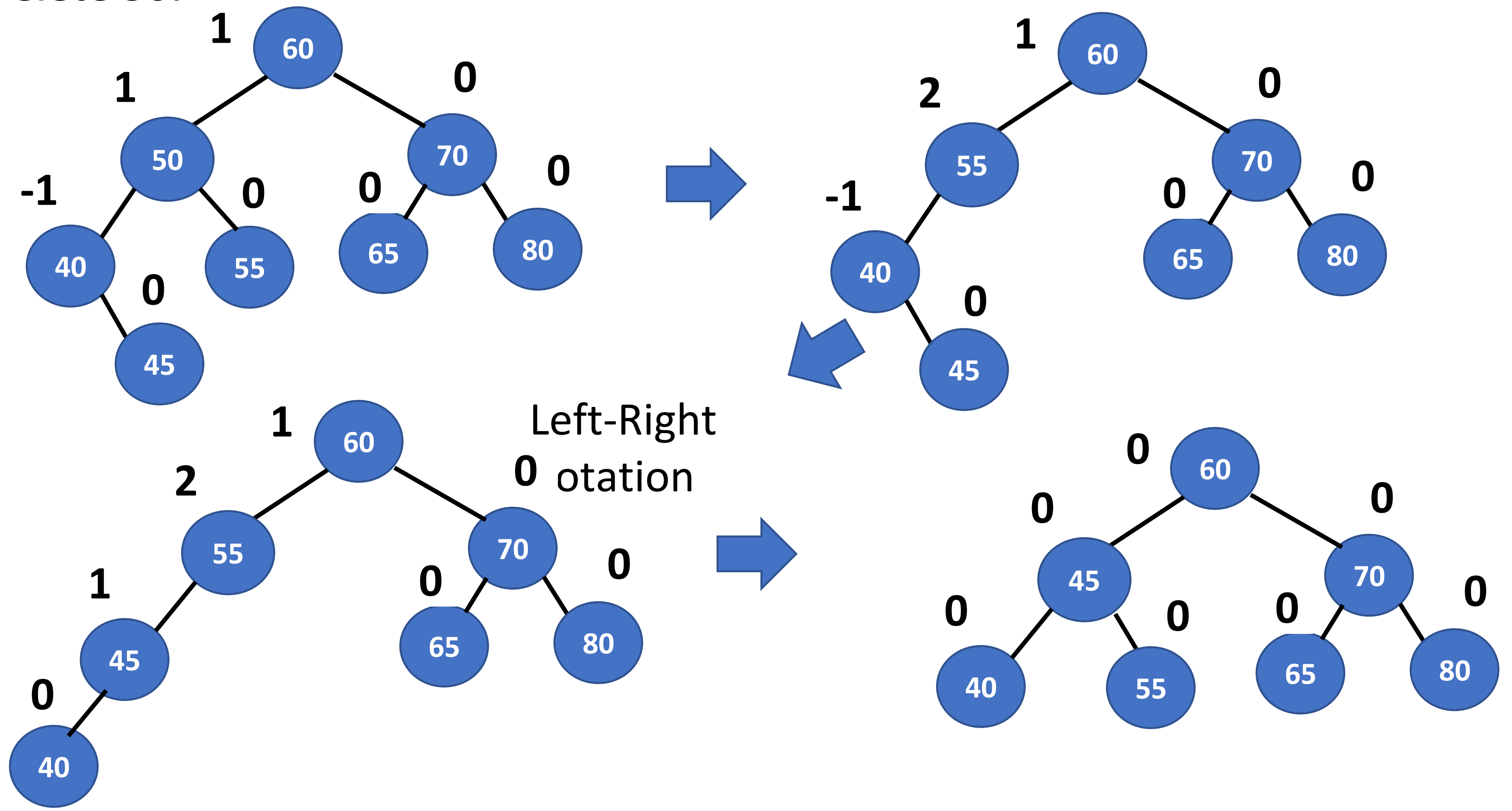
Example – Deletions in AVL tree

Delete 20:



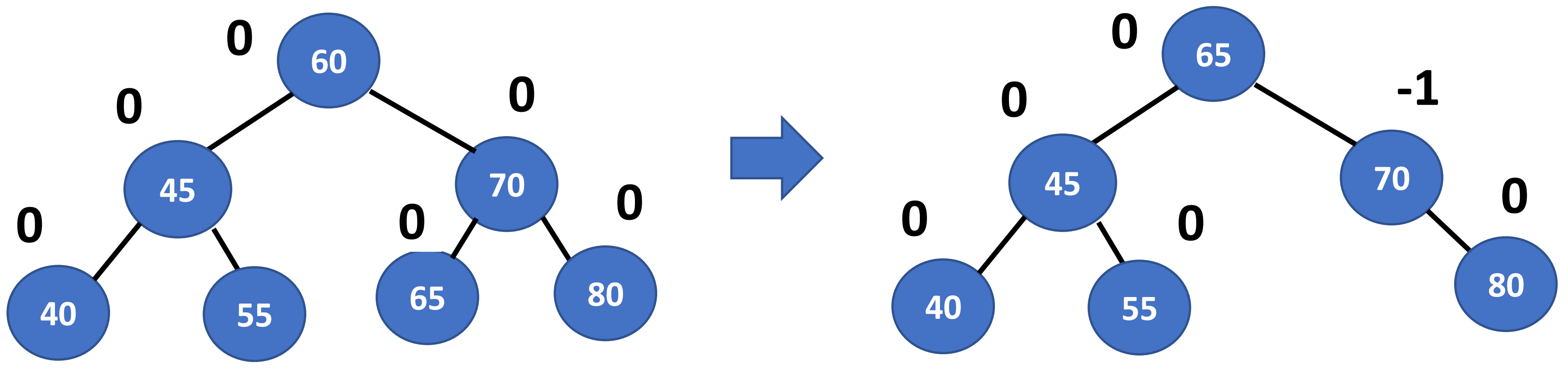
Example – Deletions in AVL tree

Delete 50:

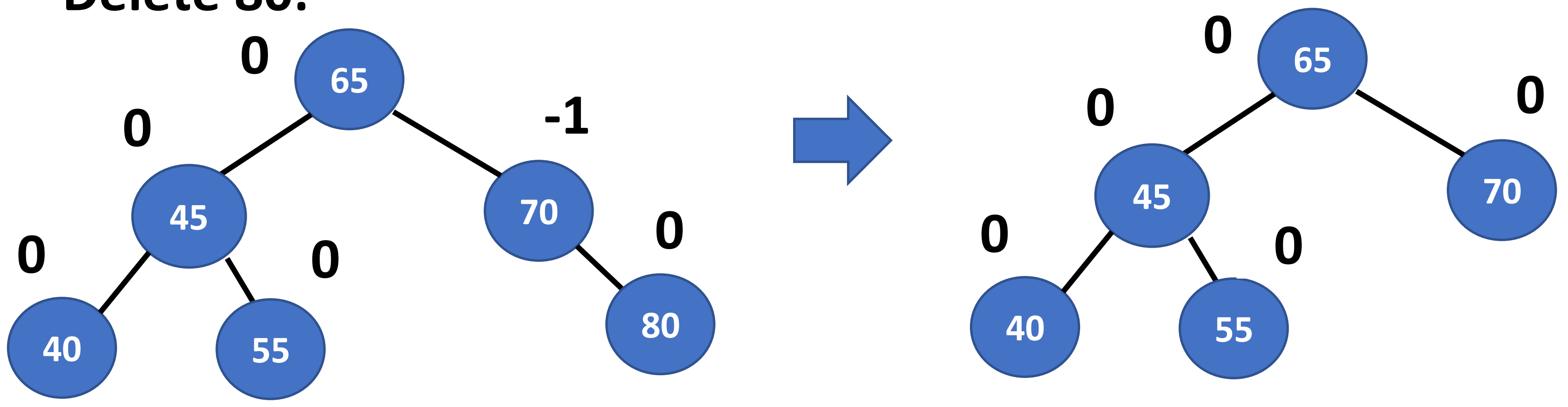


Example – Deletions in AVL tree

Delete 60:

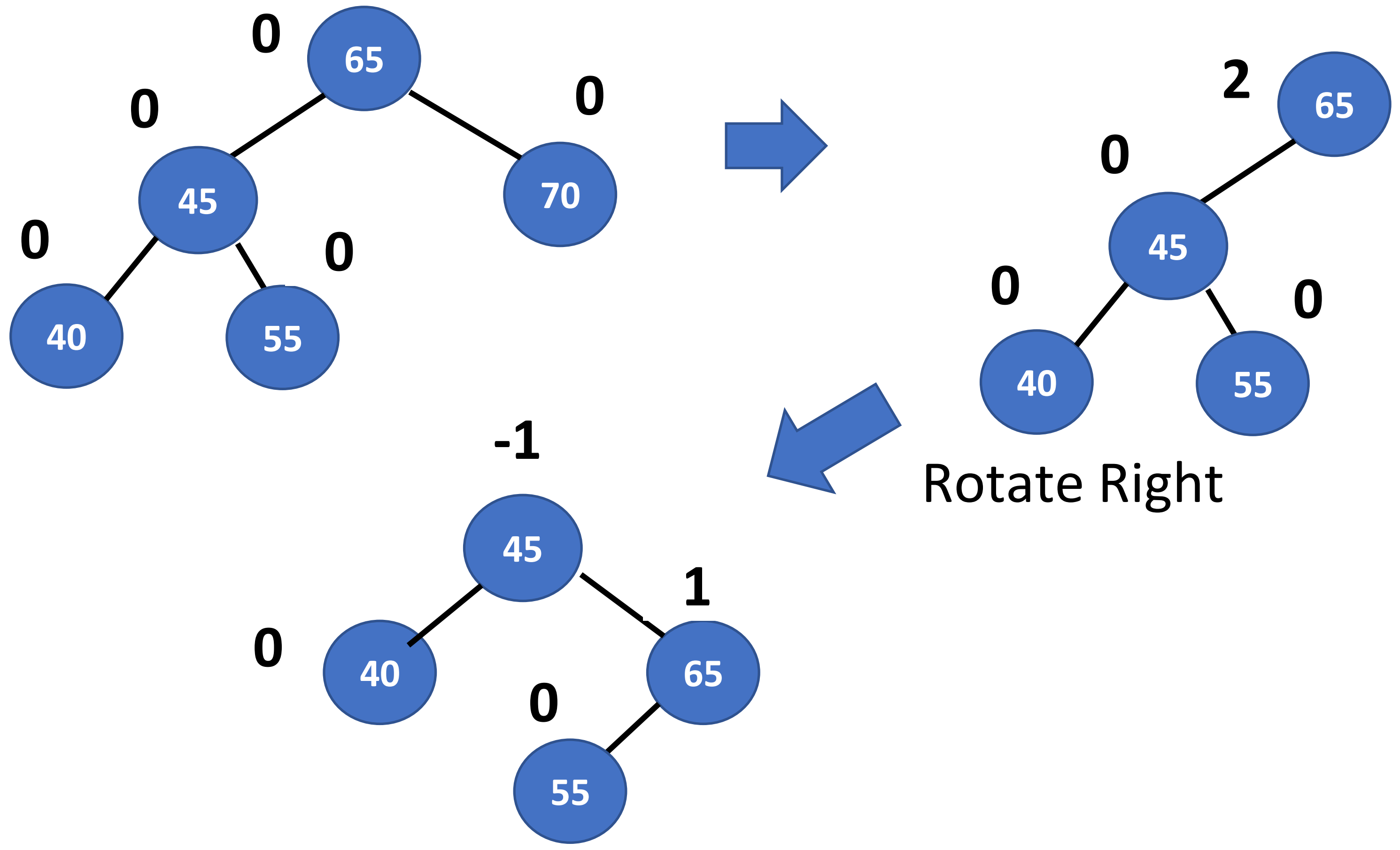


Delete 80:



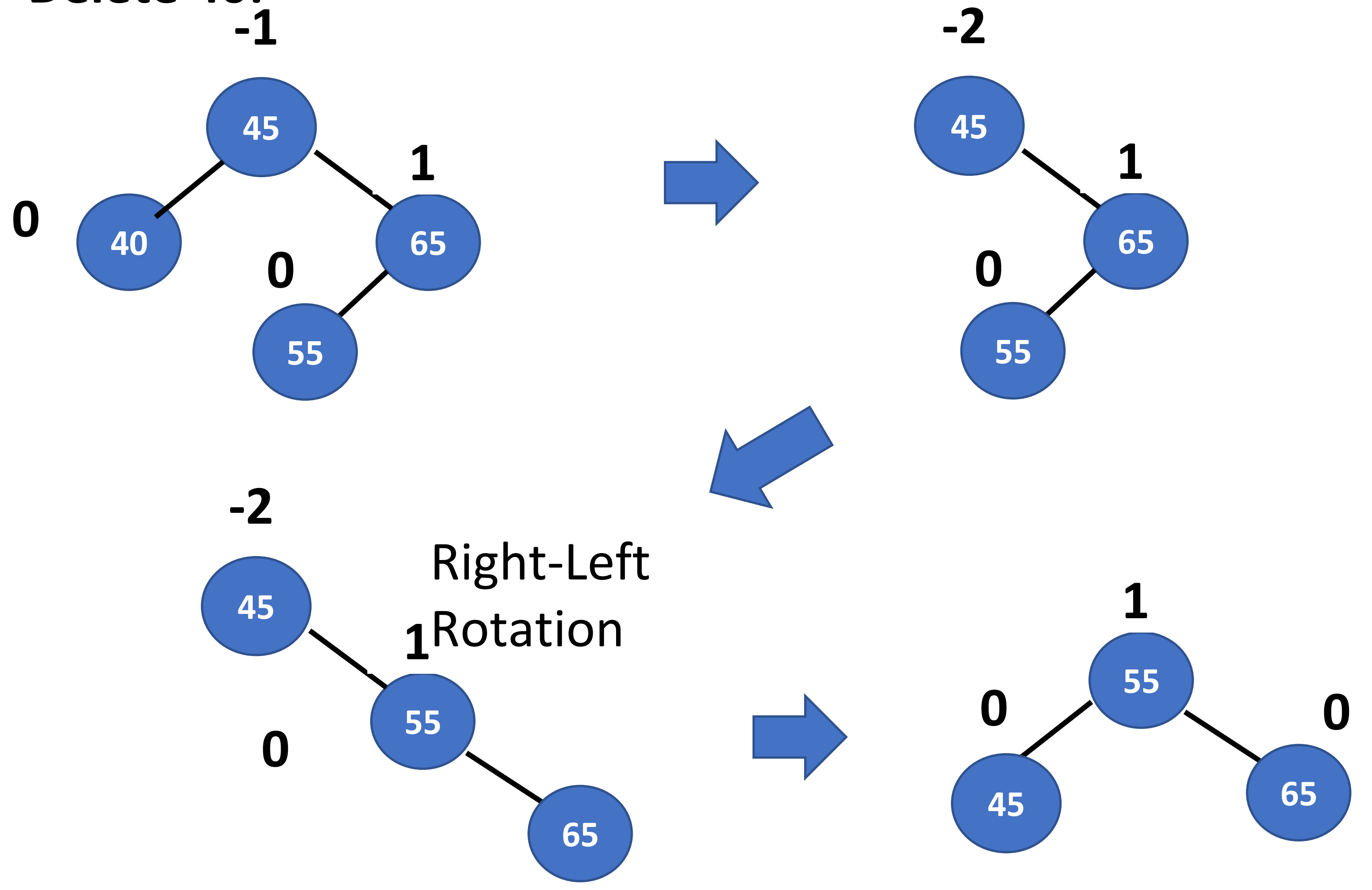
Example – Deletions in AVL tree

Delete 70:



Example – Deletions in AVL tree

Delete 40:



1. During insertion in an AVL tree, imbalance occurs when the balance factor becomes:

A) −1 or +1
B) −2 or +2
C) 0
D) None of the above

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1. During insertion in an AVL tree, imbalance occurs when the balance factor becomes:

A) −1 or +1
B) −2 or +2
C) 0
D) None of the above

(Soln: In an AVL tree, imbalance occurs when the balance factor of any node becomes less than −1 or greater than +1, i.e., when it reaches −2 or +2.)

2. Insertion into the left subtree of the left child of a node causes which imbalance?
A) LL case
B) RR case
C) LR case
D) RL case

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A) LL case
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3. After inserting nodes 10, 20, 30 in sequence into an empty AVL tree, the root becomes:

A) 10
B) 20
C) 30
D) 15

3. After inserting nodes 10, 20, 30 in sequence into an empty AVL tree, the root becomes:

A) 10
B) 20
C) 30
D) 15

4. After deleting a node from an AVL tree, which of the following is TRUE?

A) Balance factor may become −2 or +2 before rebalancing

B) Rebalancing is never required

C) Deletion requires more rotations than insertion in worst case

D) Both A and C

4. After deleting a node from an AVL tree, which of the following is TRUE?

A) Balance factor may become −2 or +2 before rebalancing

B) Rebalancing is never required

C) Deletion requires more rotations than insertion in worst case

D) Both A and C

THANK YOU

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