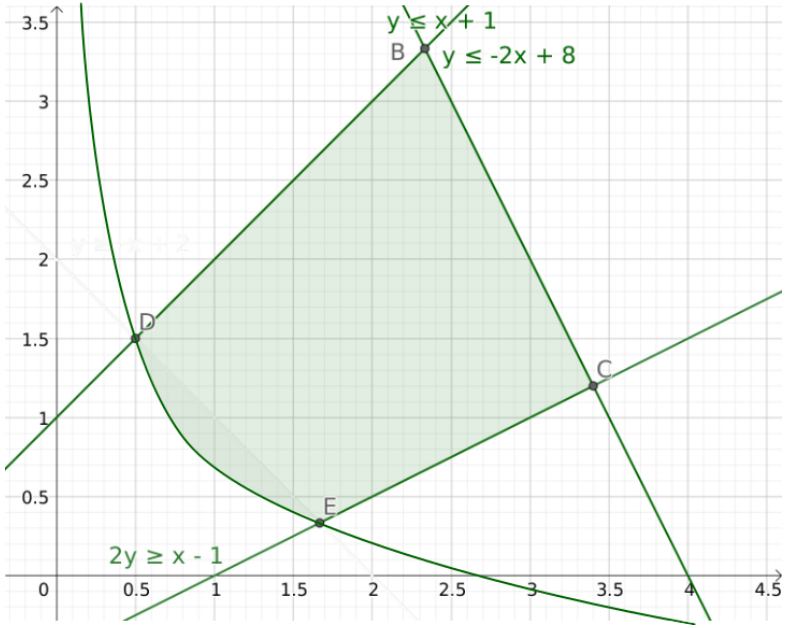
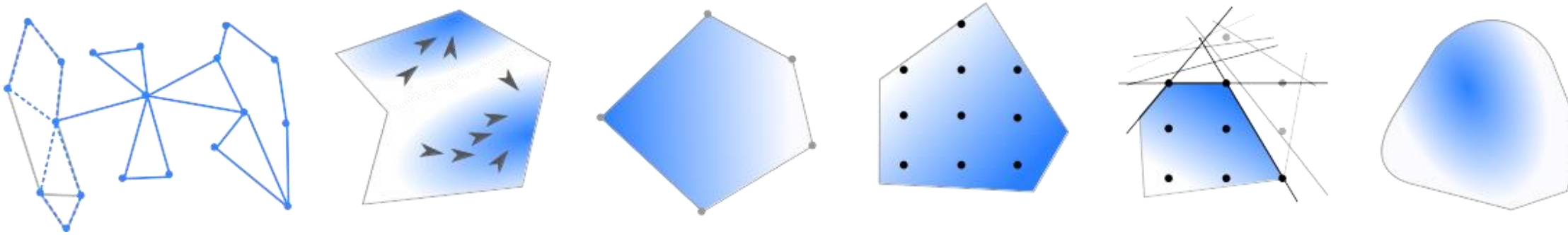
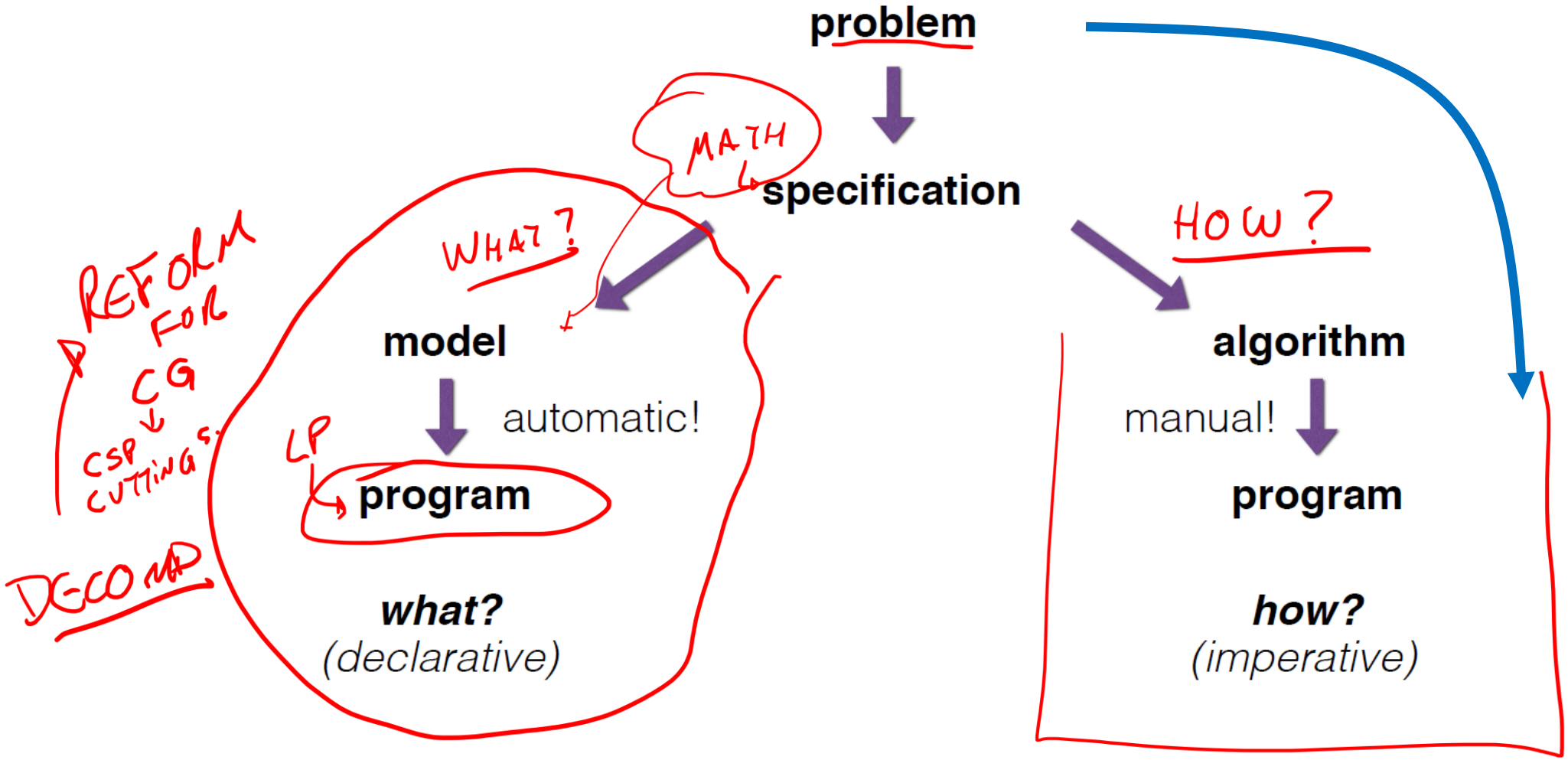
Construction Algorithms

COMP4691 / 8691





Previously on COMP4691(8691)

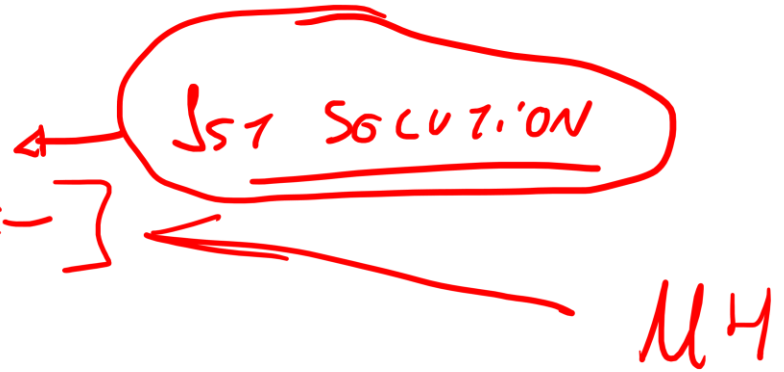


Previously on COMP4691(8691)

- Looked at a model-based approach to solving (MIPs)

Now
- Look at heuristic approaches

A classic
- Construct
- Improve



Today:
- Construct

SCHEDUCING
→ LNS
LARGE NEIGH SEARCH

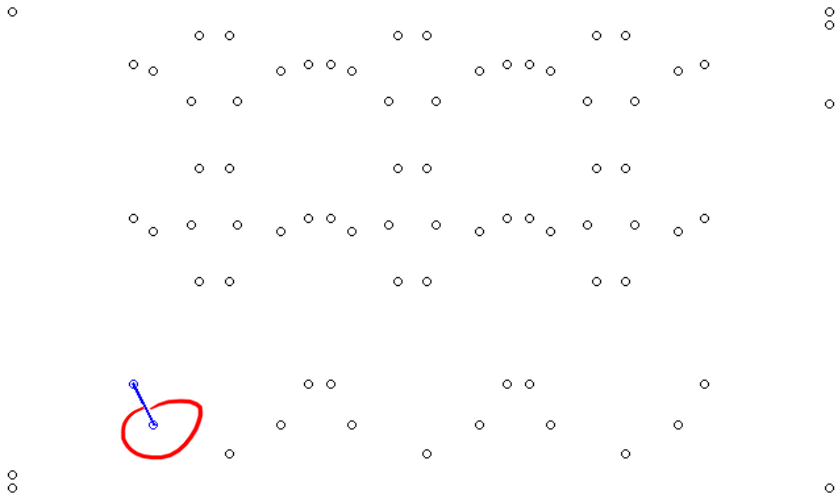
Construction algorithms

We need to start somewhere

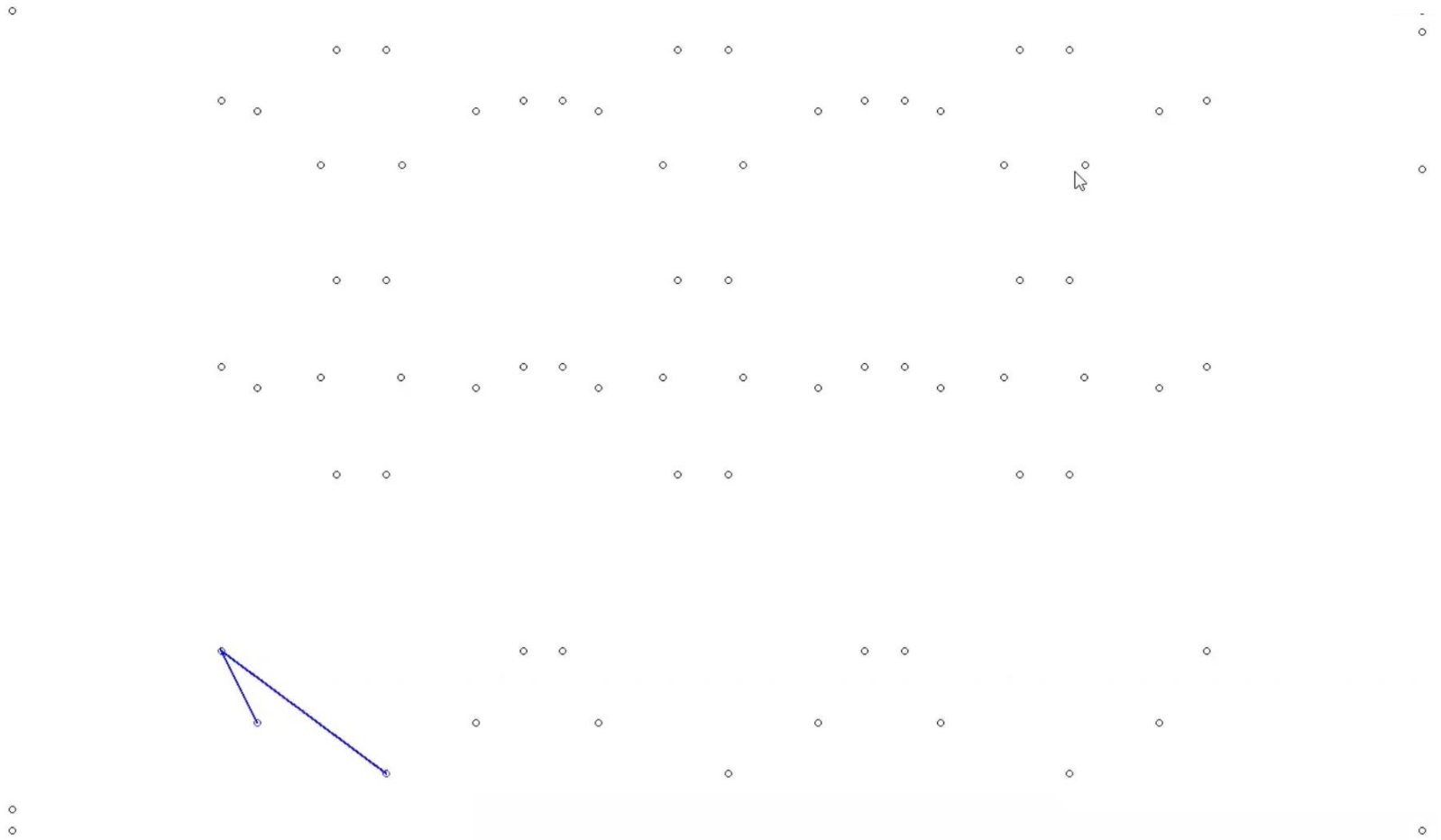
- When we first come to an instance we are looking at an empty page
  - In TSP, no cities are visited
  - In Knapsack, all the items are lying on the ground
- Initial solution often created by “constructing” one piece / element at a time

Construction algorithms – Nearest Neighbour

- For example: TSP
  - Starting with an empty tour, start in the “home” city
  - Repeat:
    - Add the city closest to the last-added city



Construction algorithms – Greedy



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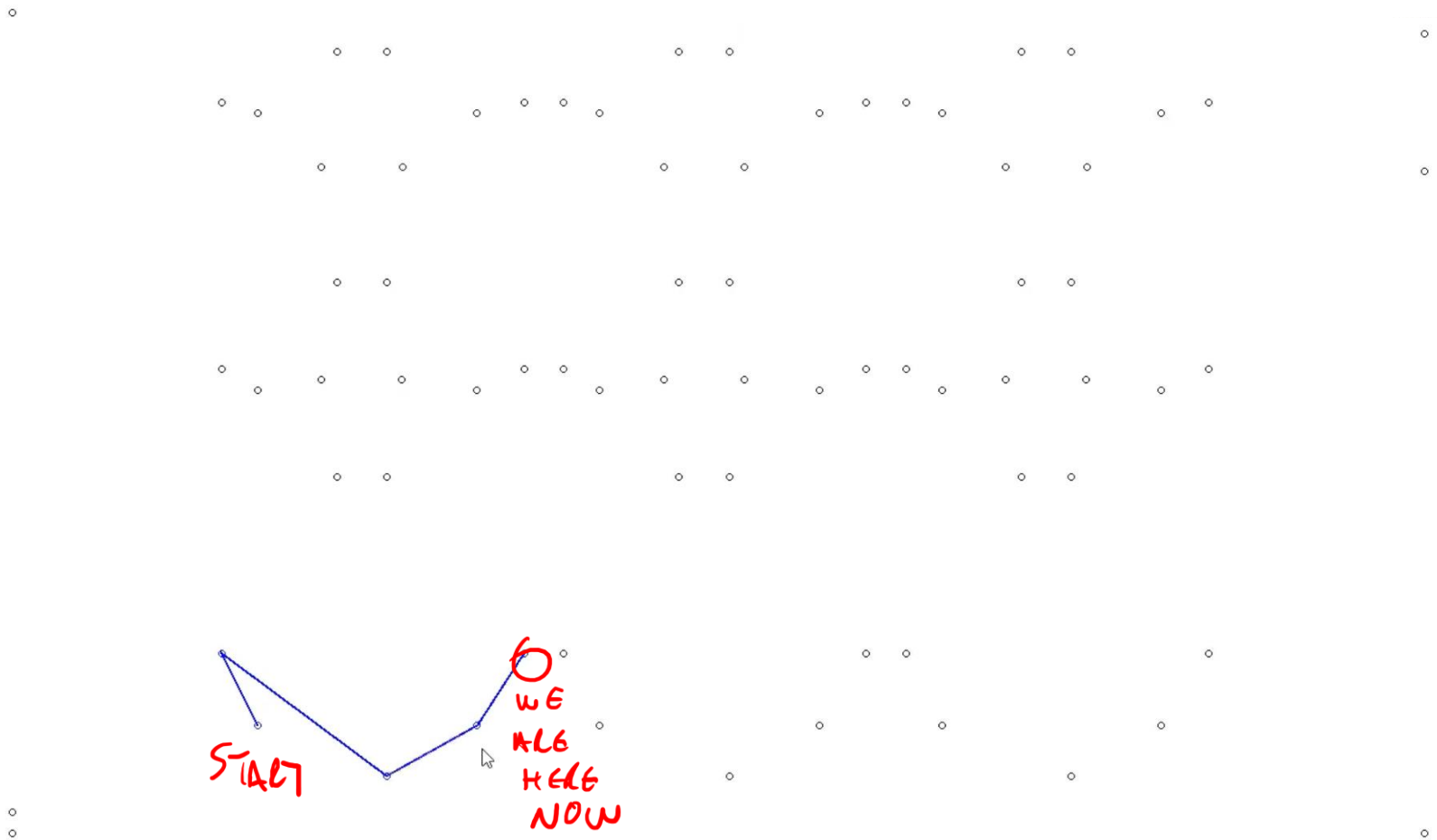

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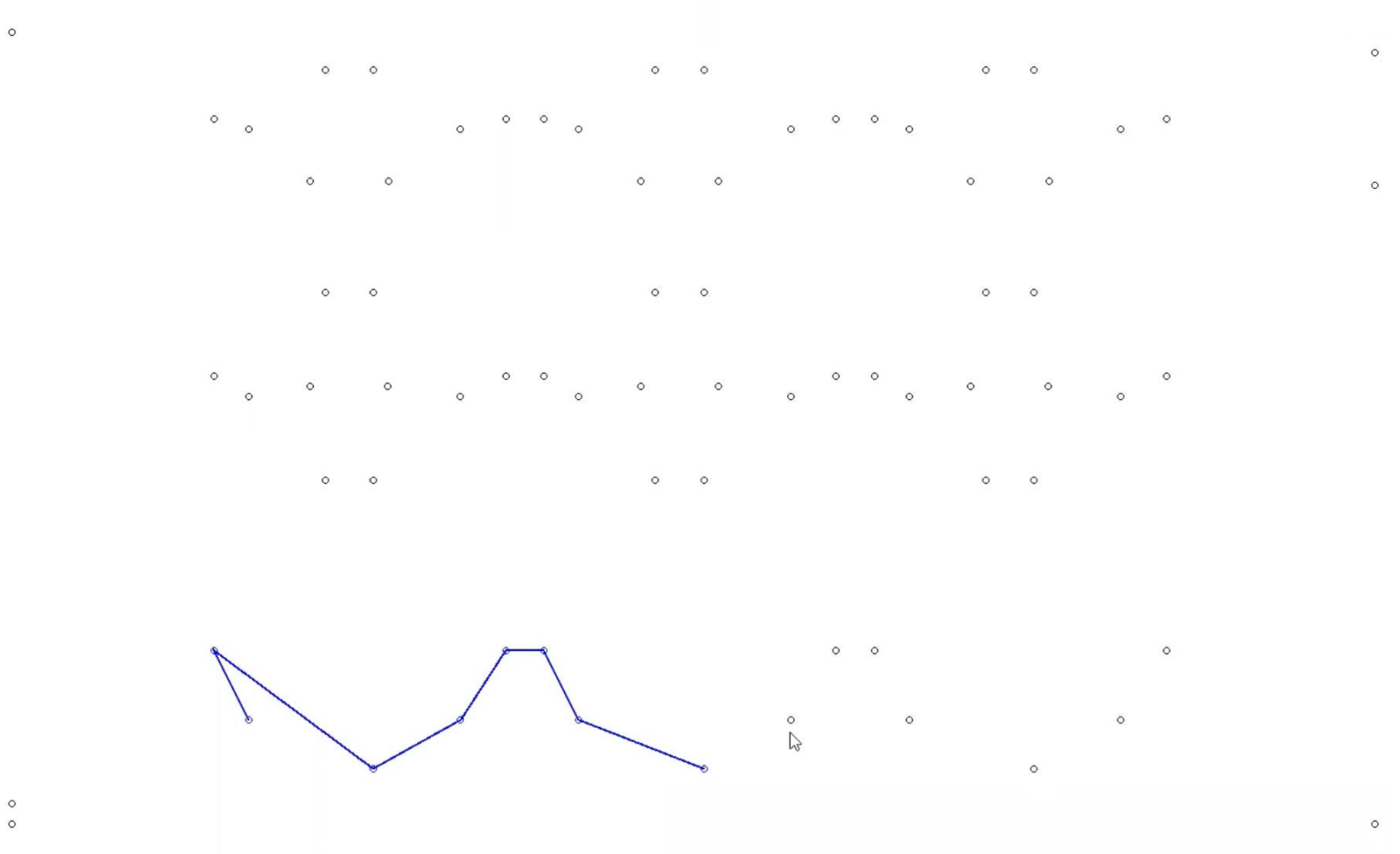

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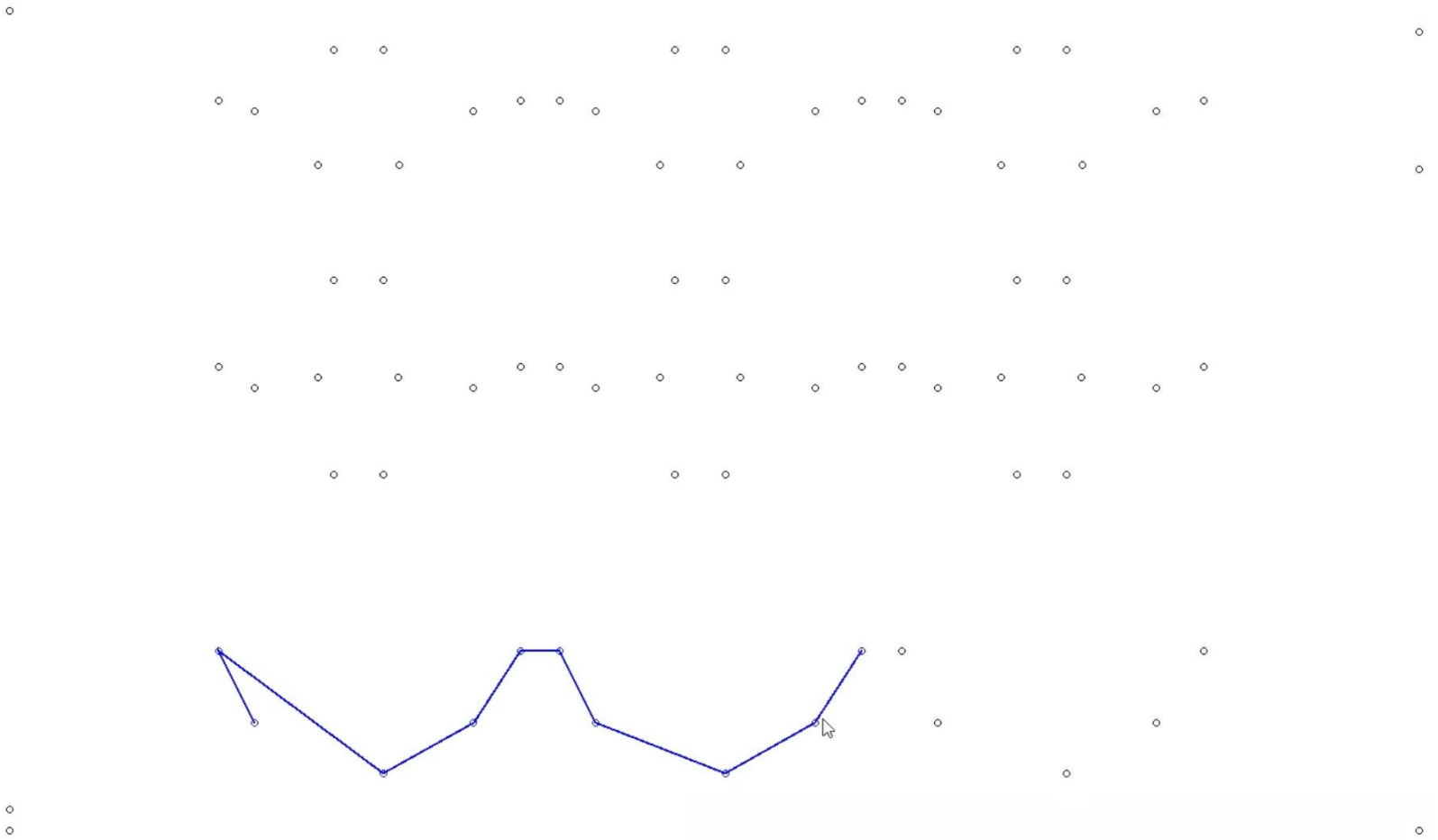

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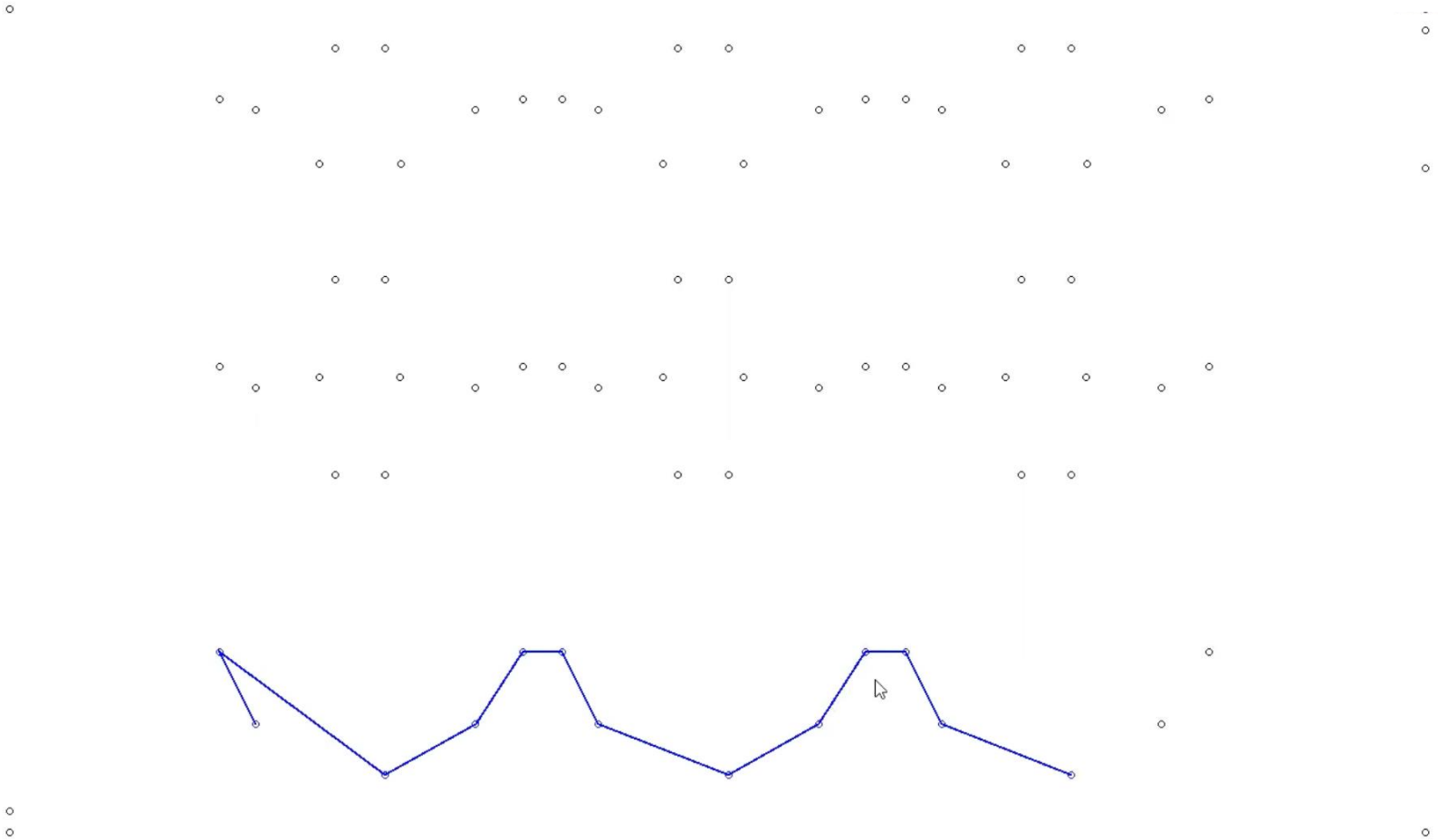

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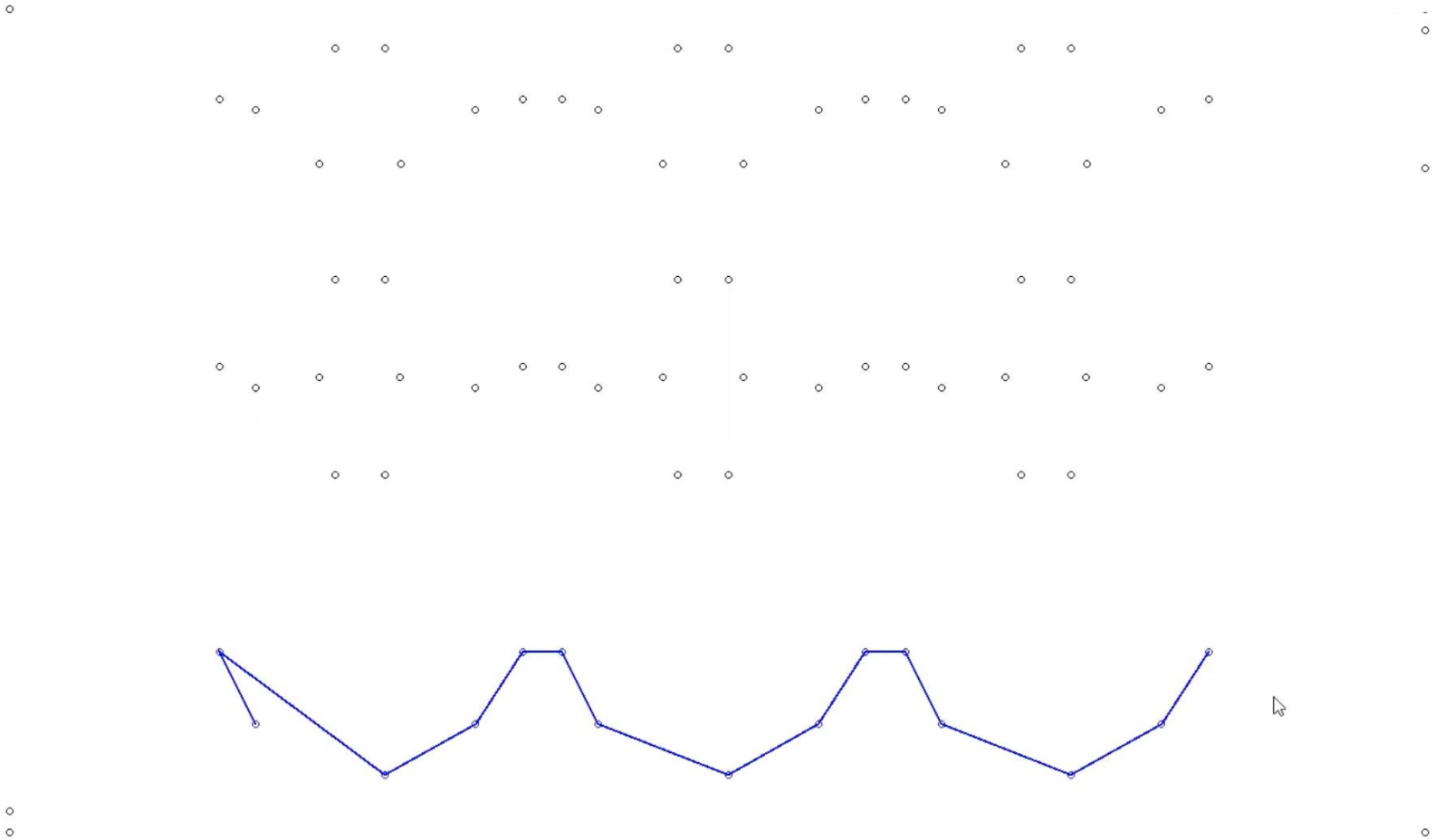

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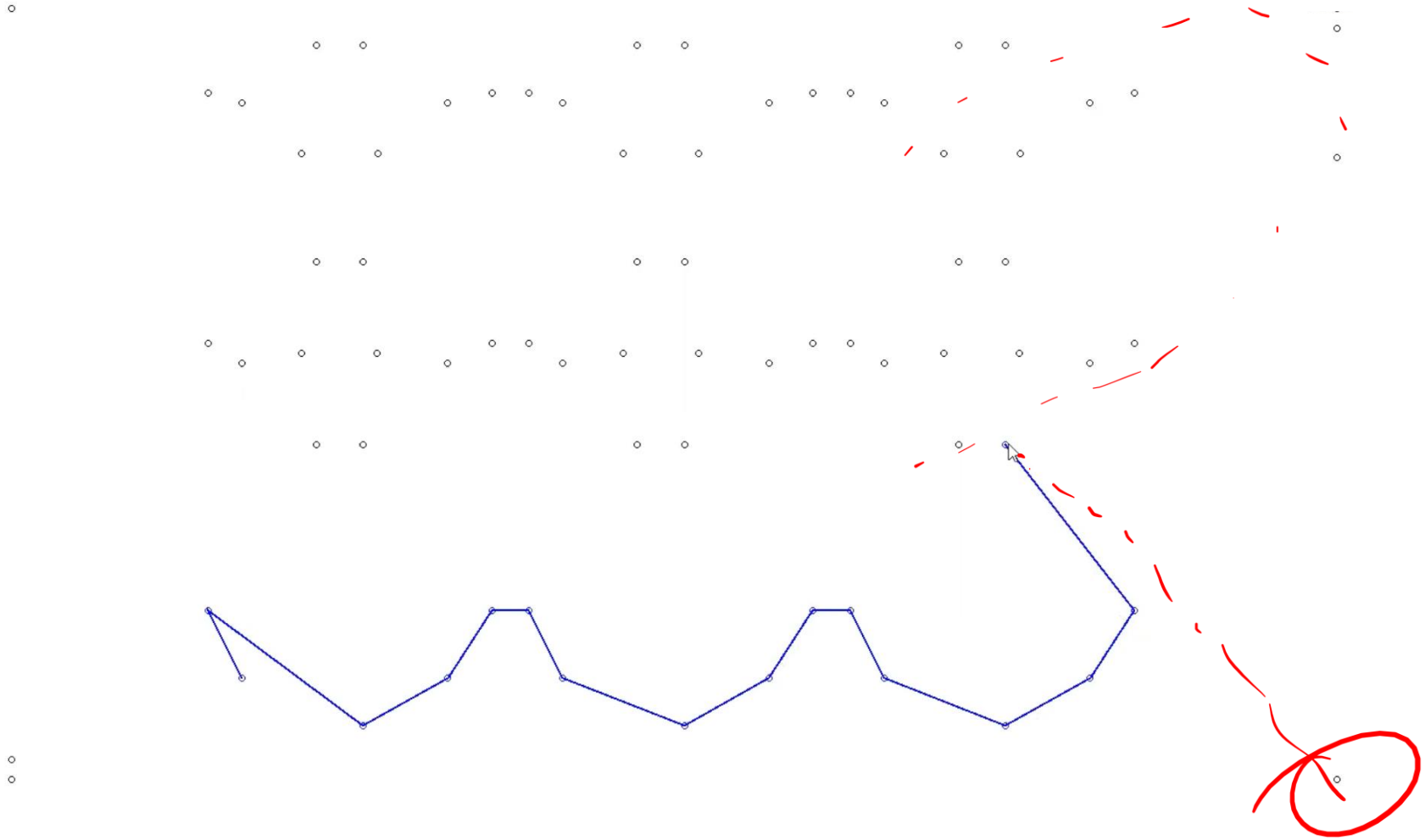

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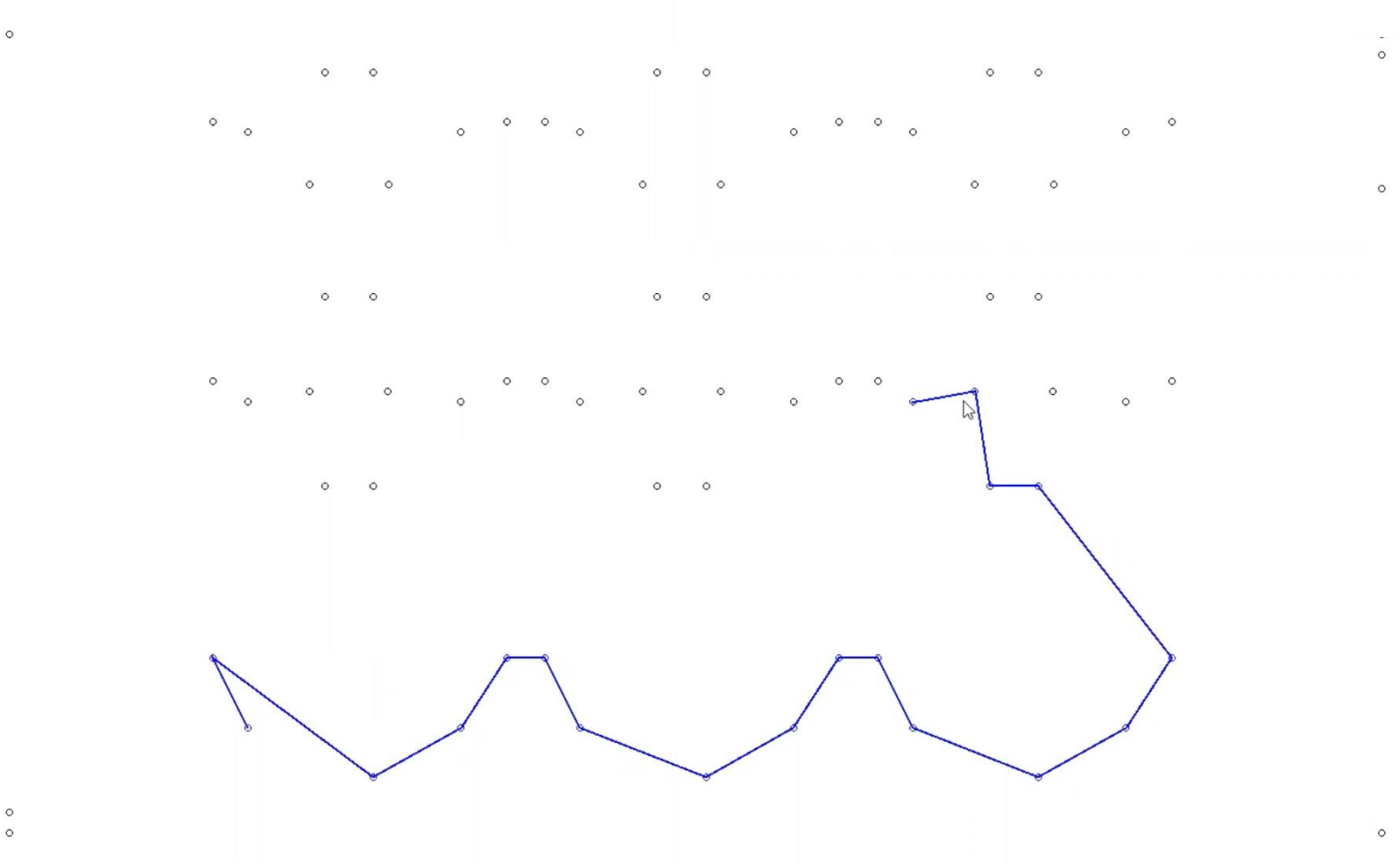

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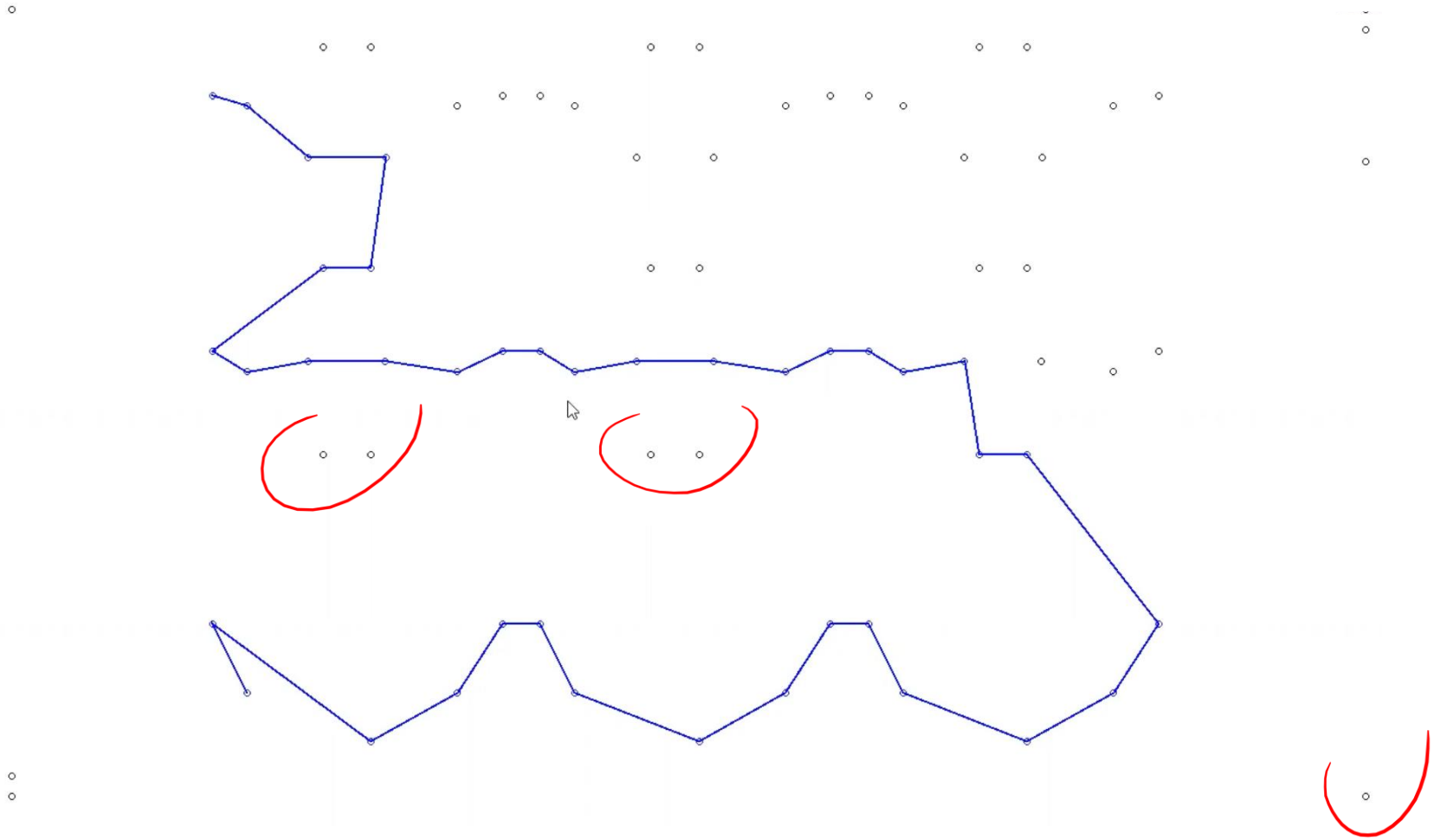

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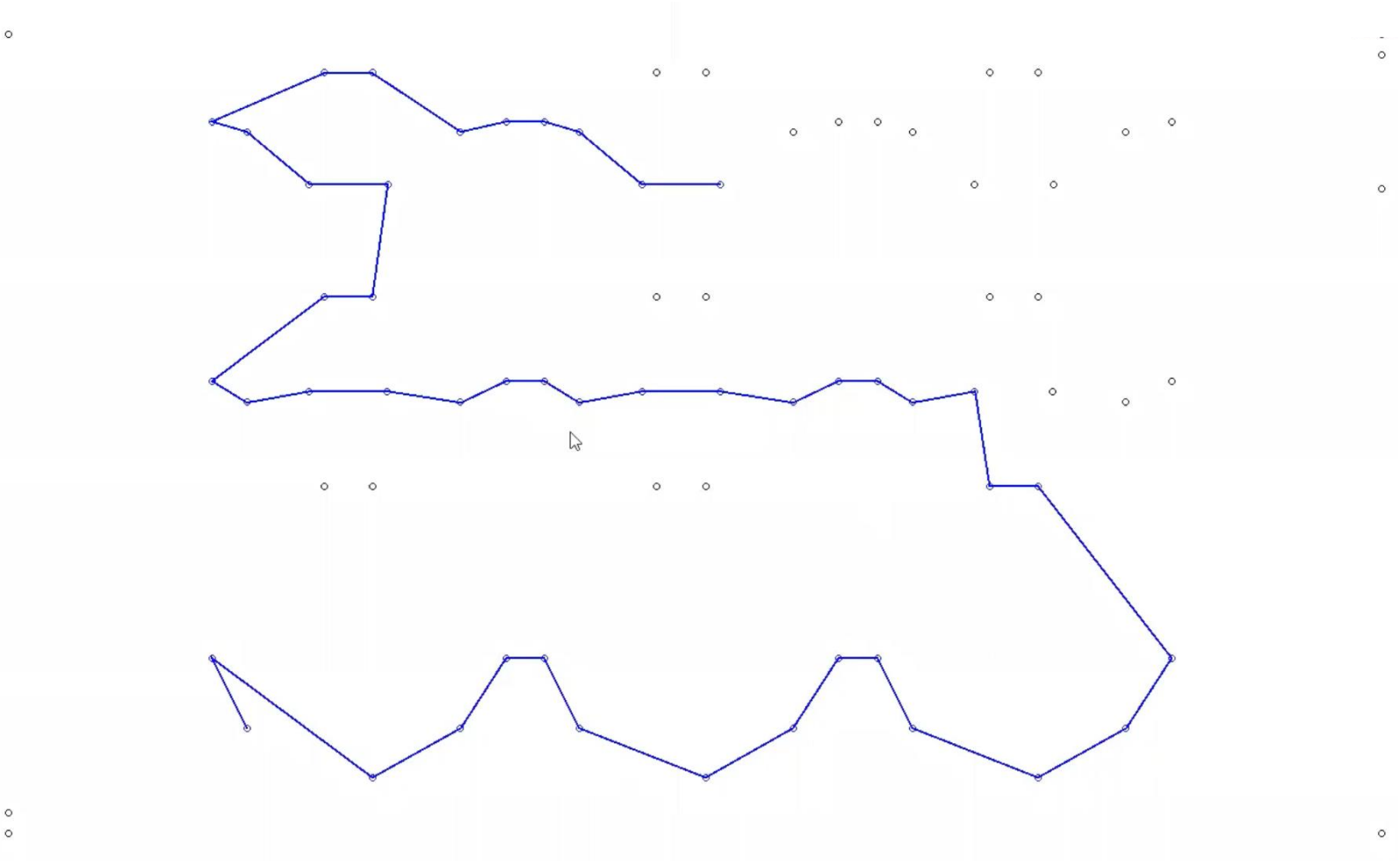

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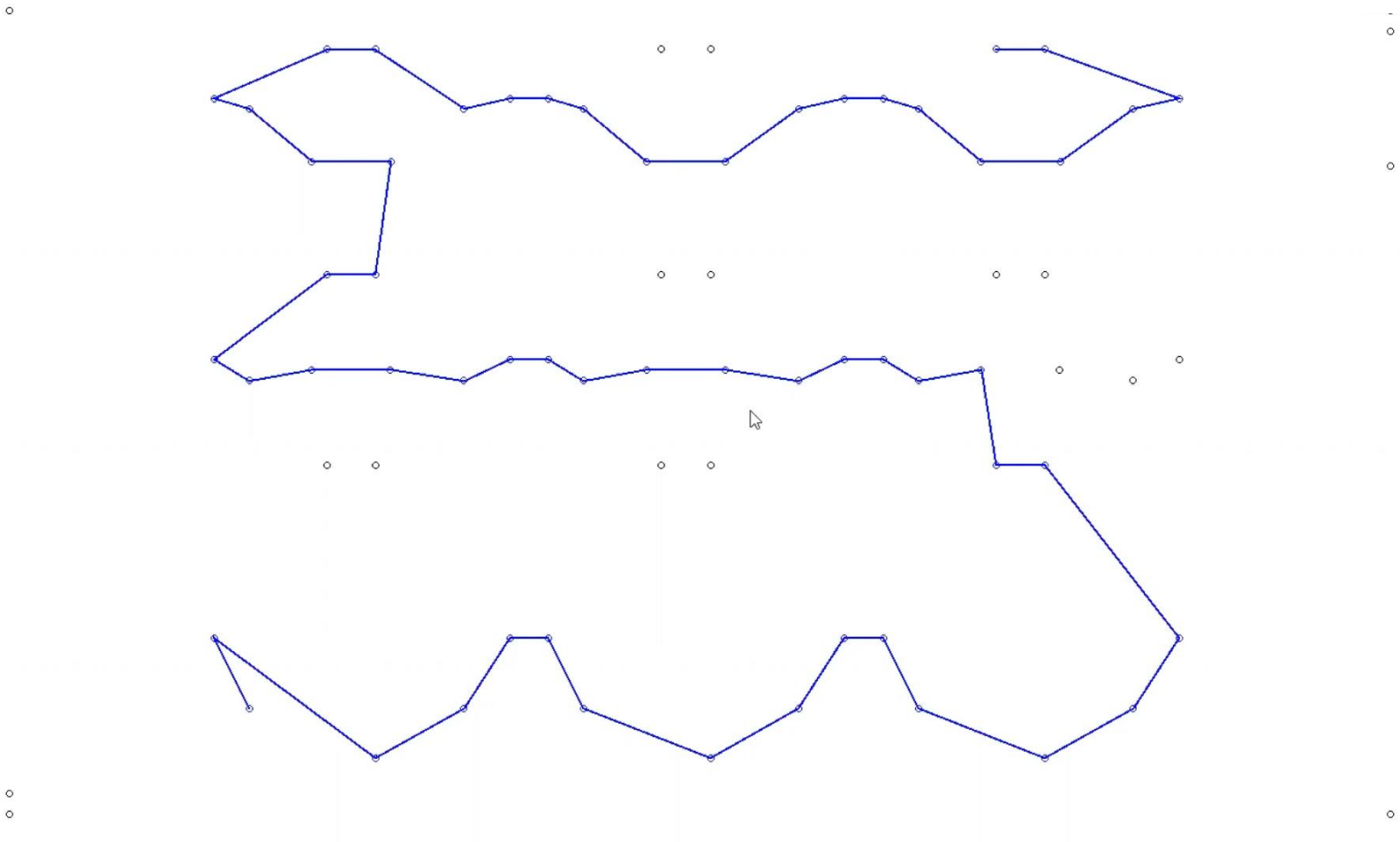

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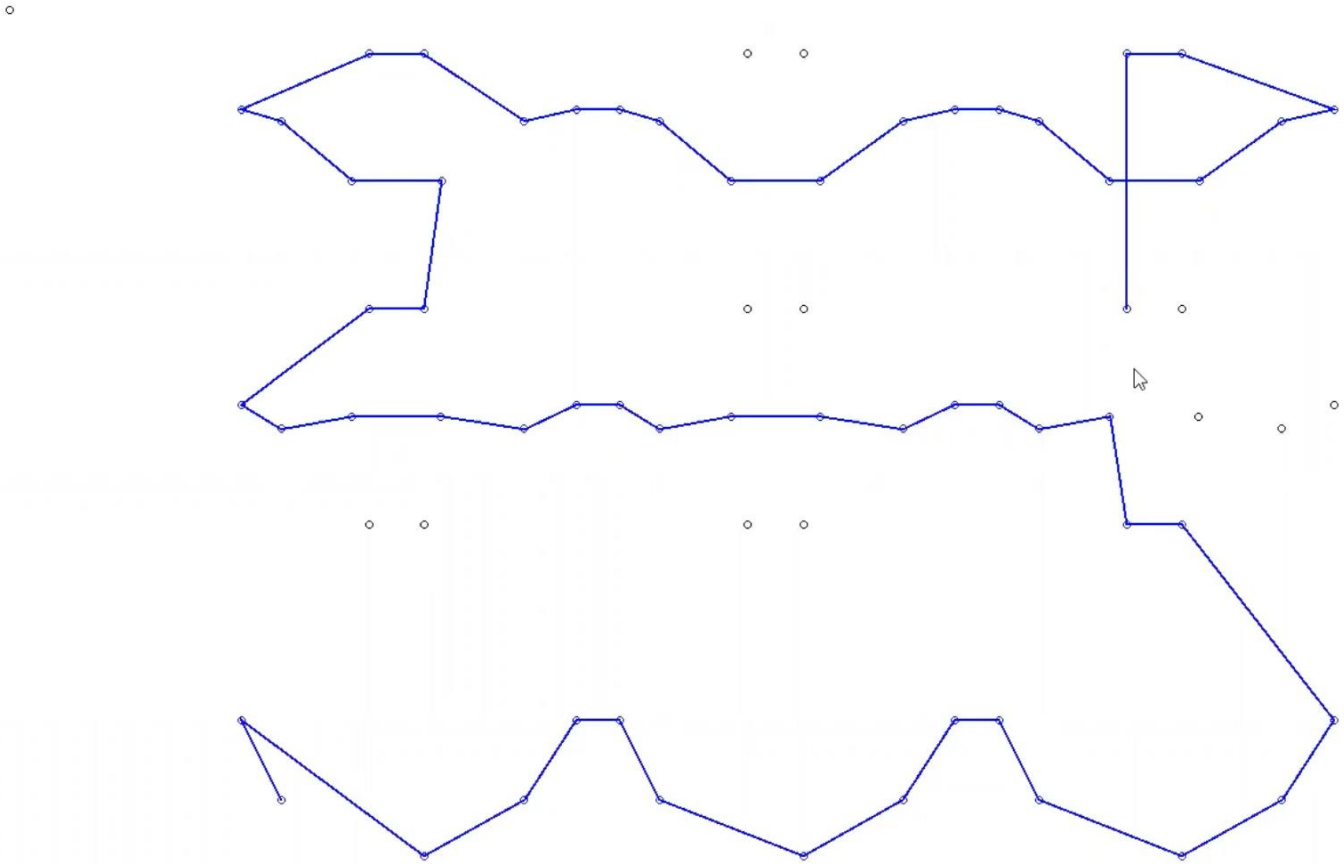

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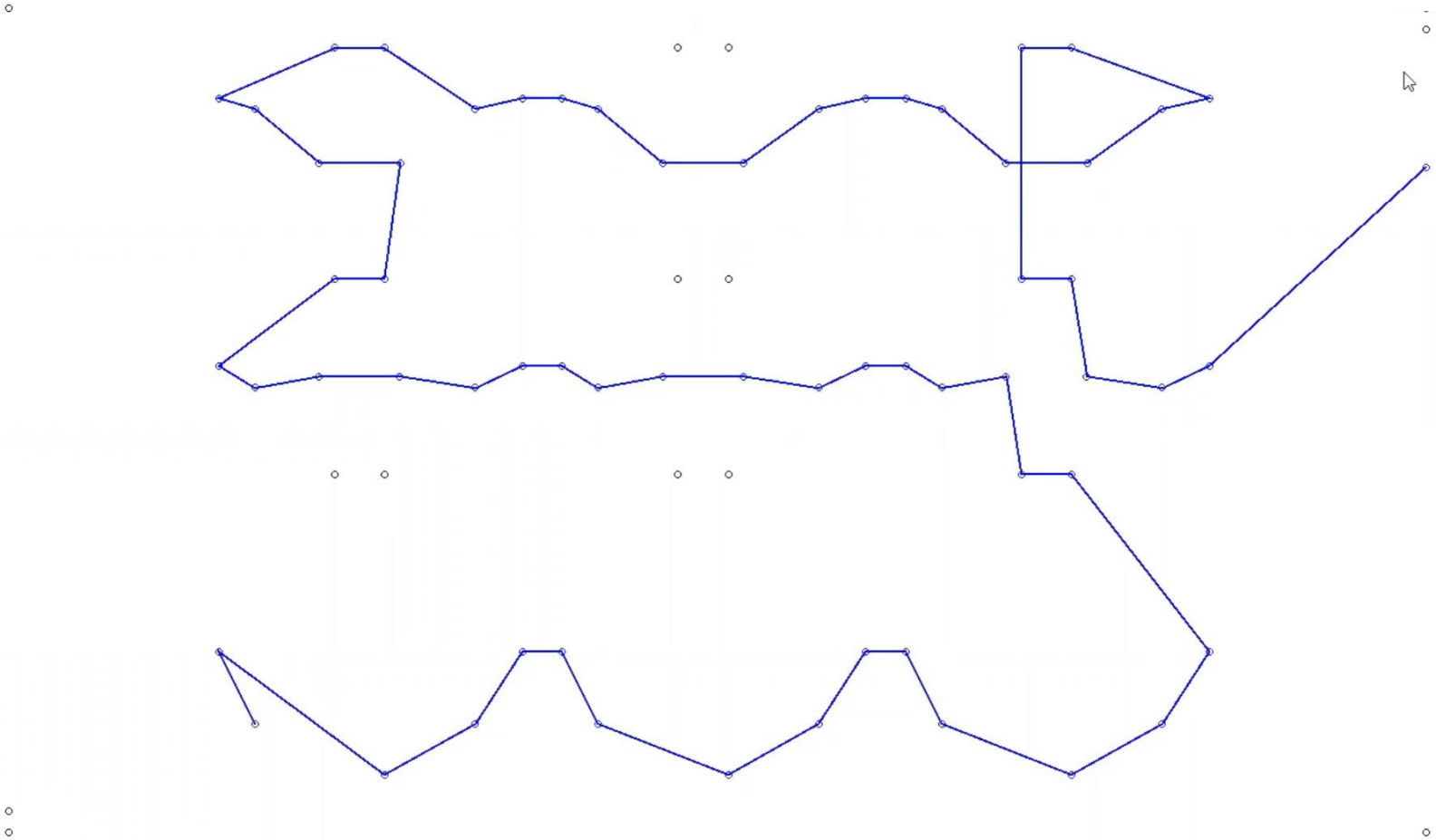

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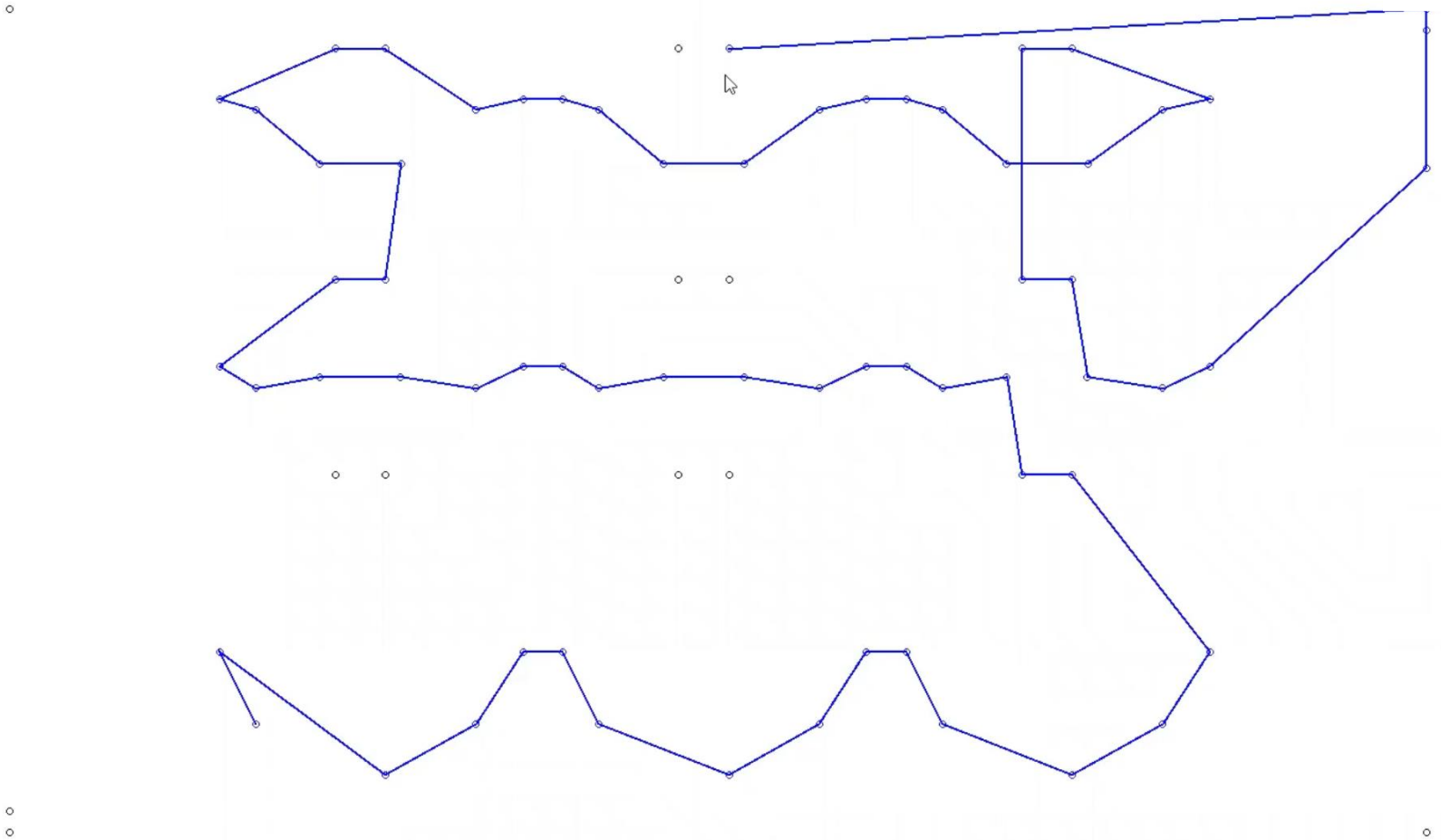

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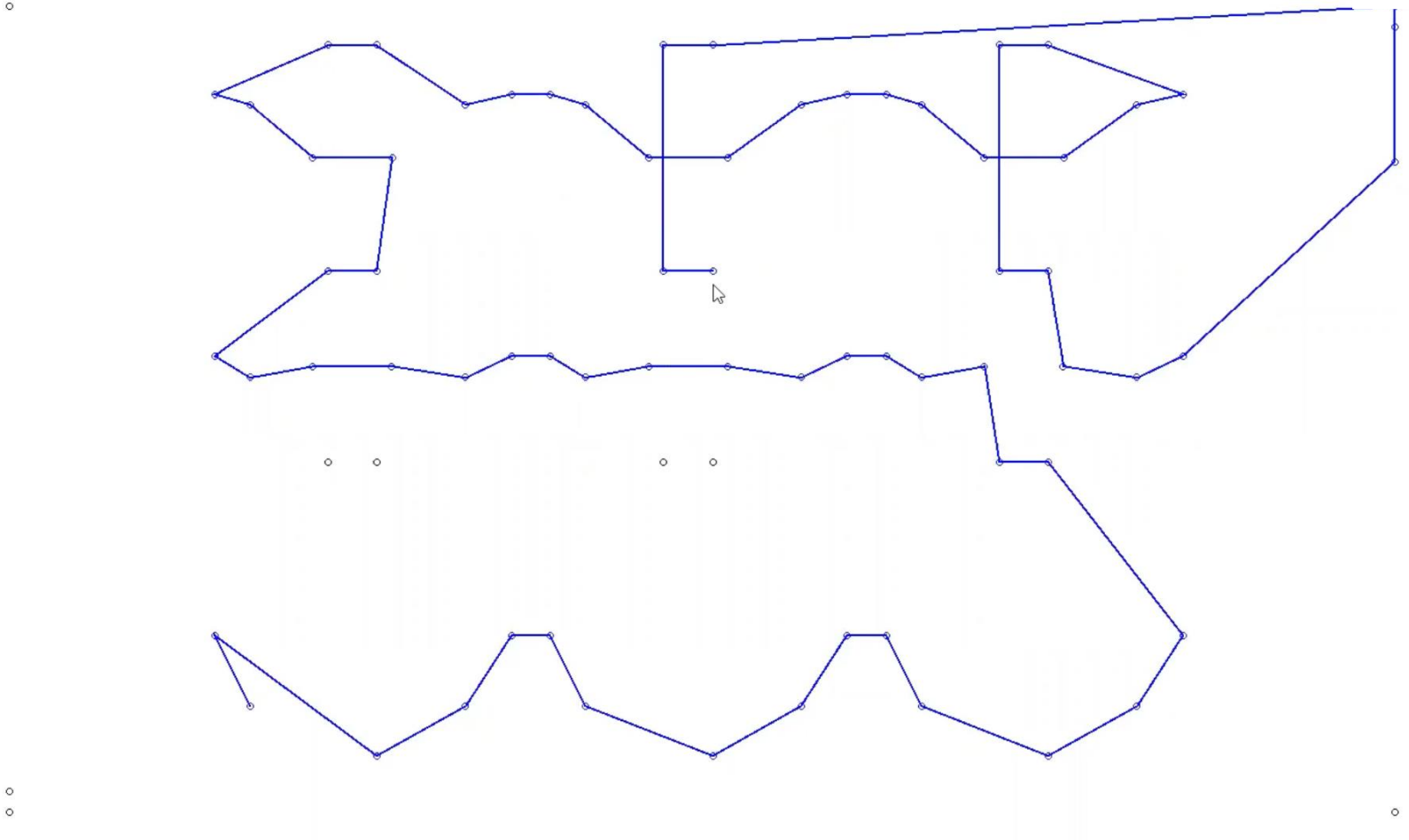

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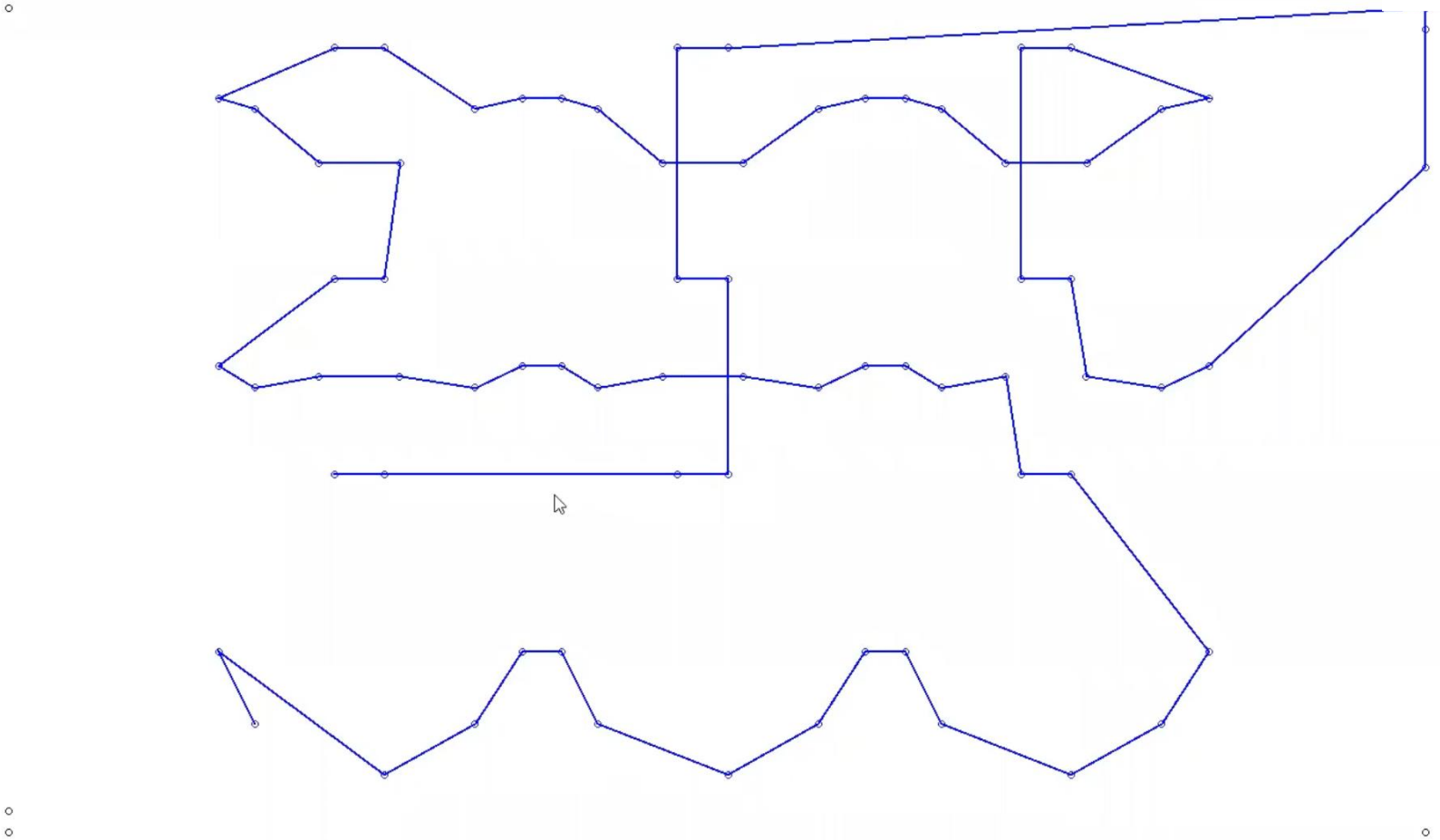

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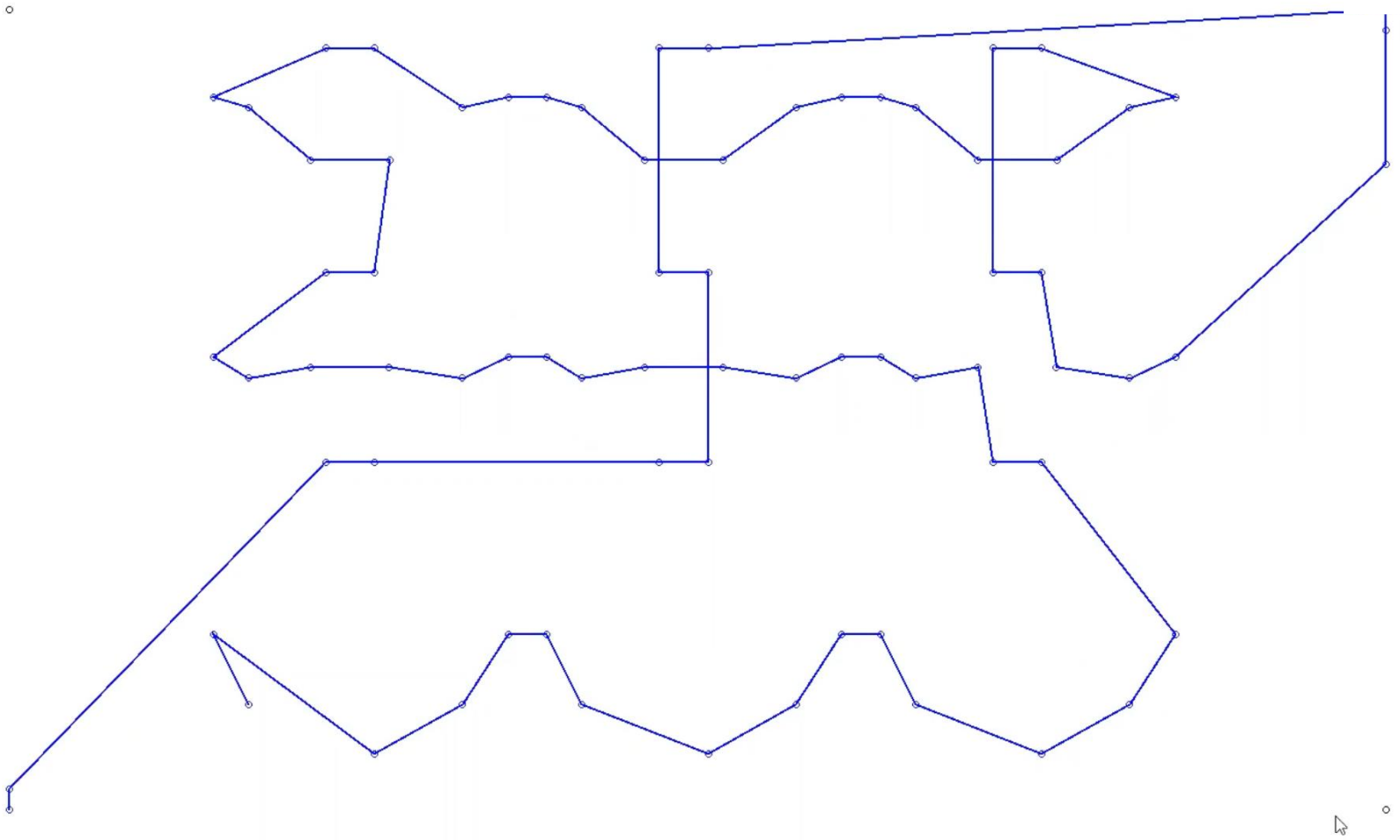

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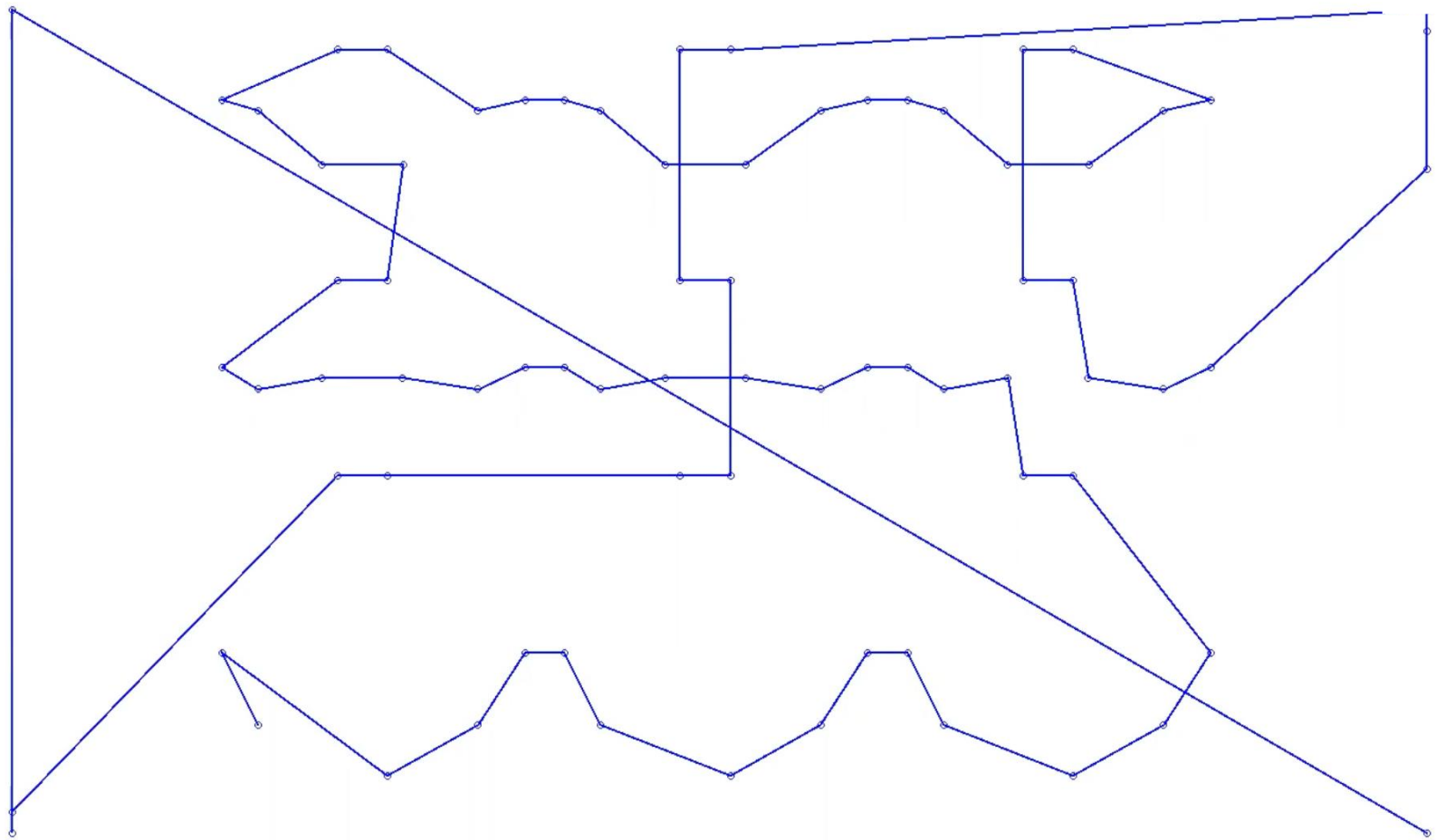


Construction algorithms – Greedy

- "Nearest Neighbour" is a classic greedy algorithm
- Greedy algorithms perform locally-optimal moves
- However, they often make "structural" errors.

- Greedy algorithms exist for many problems:
  - Knapsack: Insert the item with the best value/weight ratio
  - Scheduling: Schedule the item with the earliest deadline

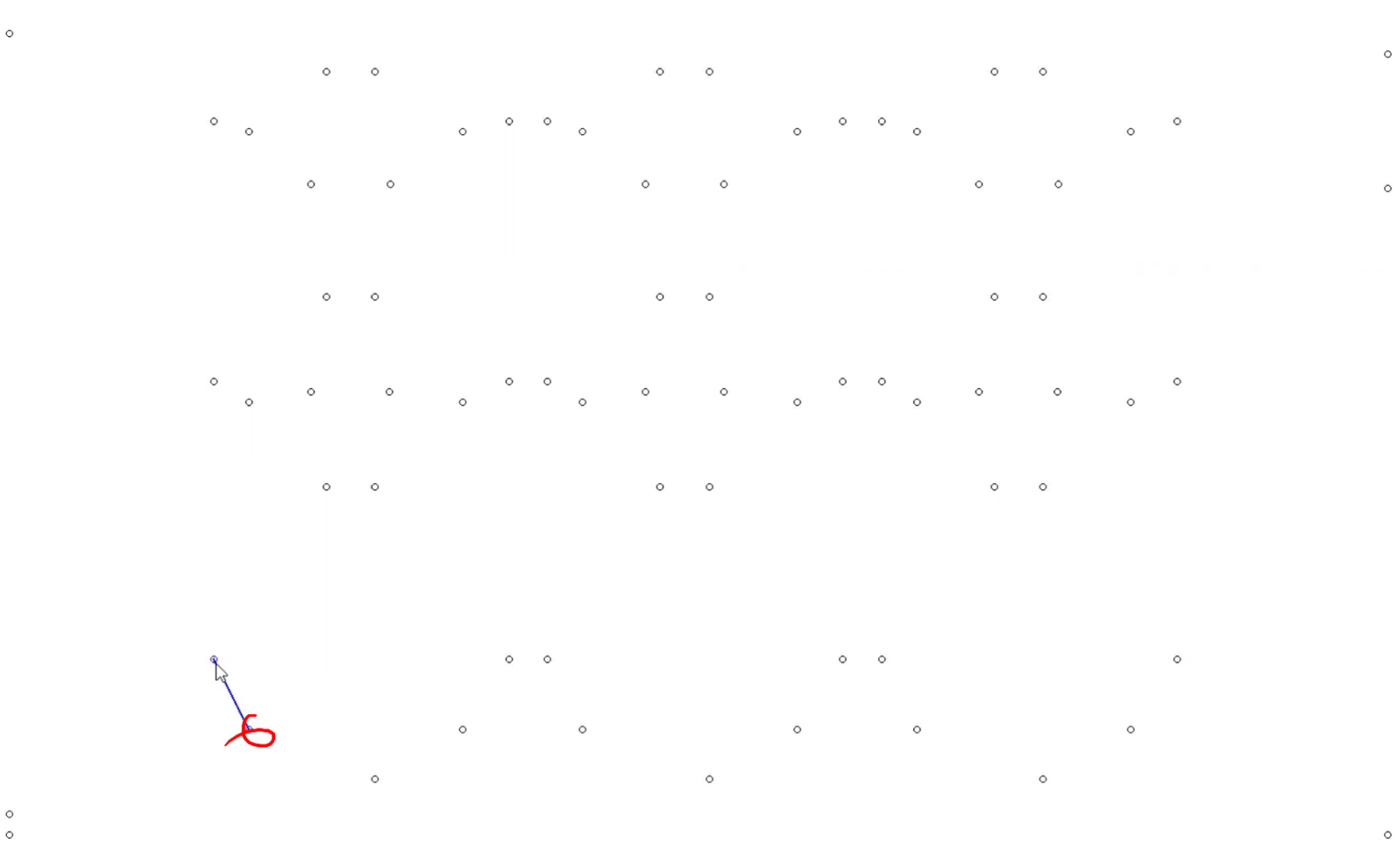
Construction algorithms – Minimum Insert Cost

A better greedy algorithm

- "Minimum Insert" is another greedy algorithm – but performs slightly better

- At each iteration
  - Find the customer that, when inserted, increases the cost by the least.
  - Insert it in the position that increases the cost by the least
  - Allows insert between *any* pair of customers – not just at the end

Construction algorithms – Minimum Insert Cost



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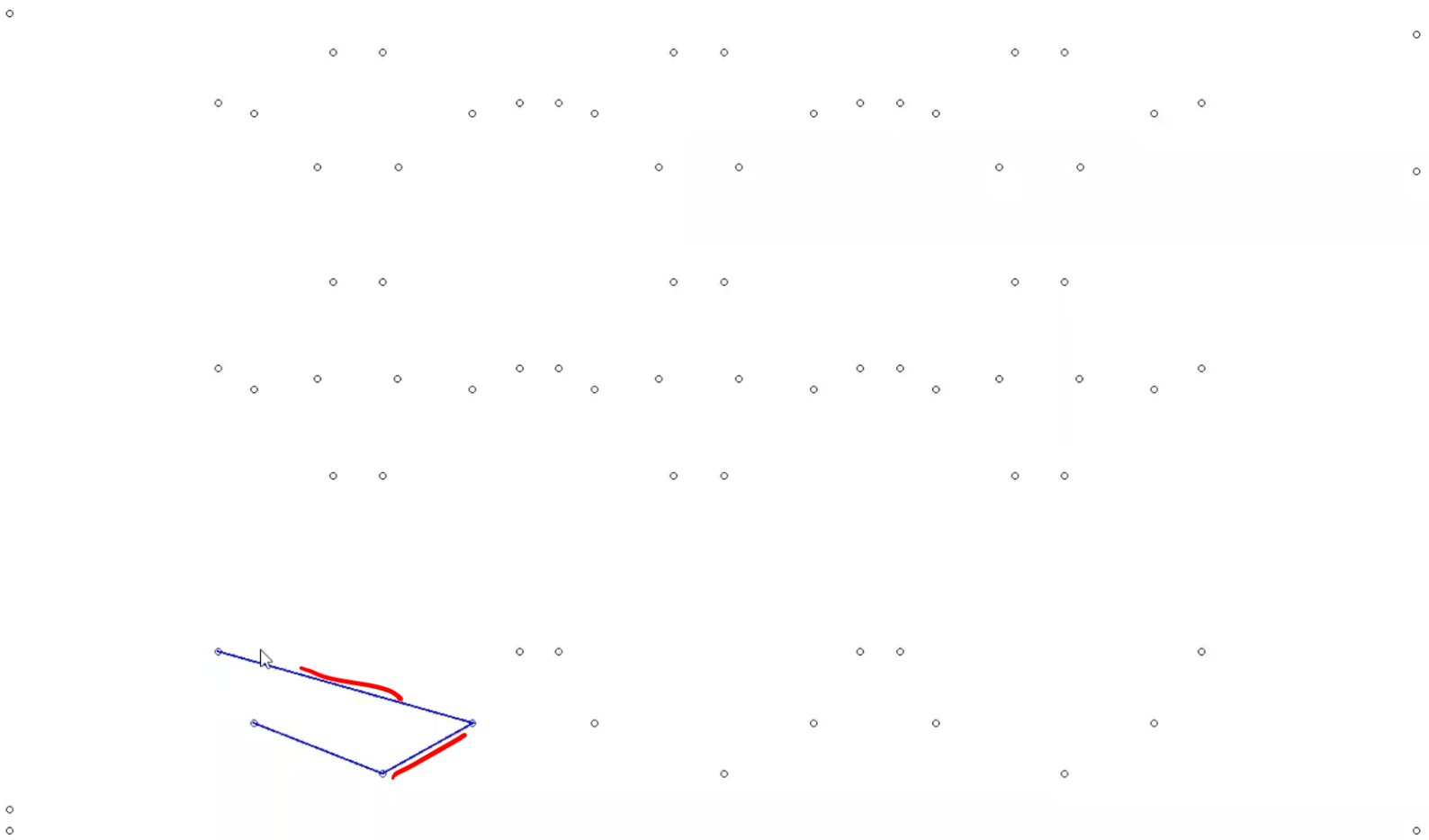

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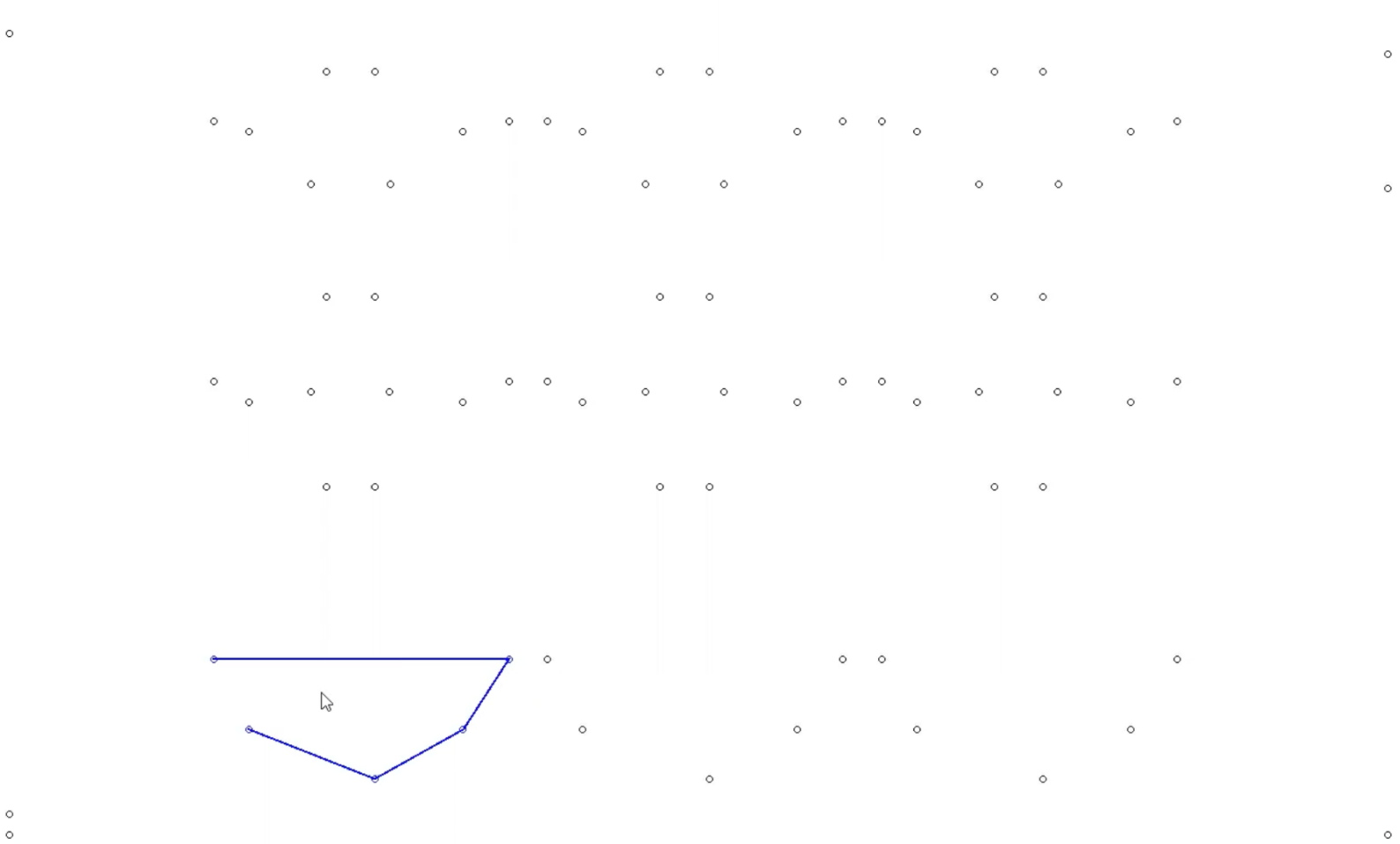

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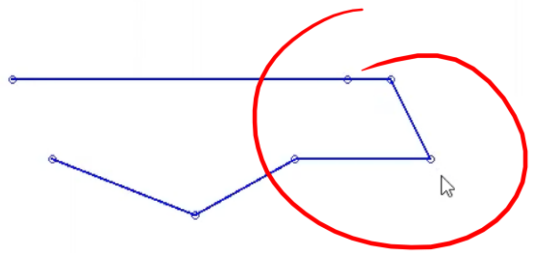

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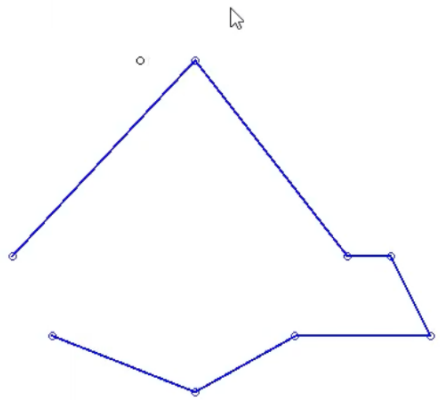

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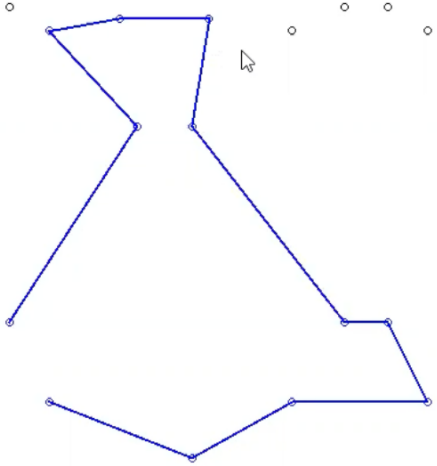

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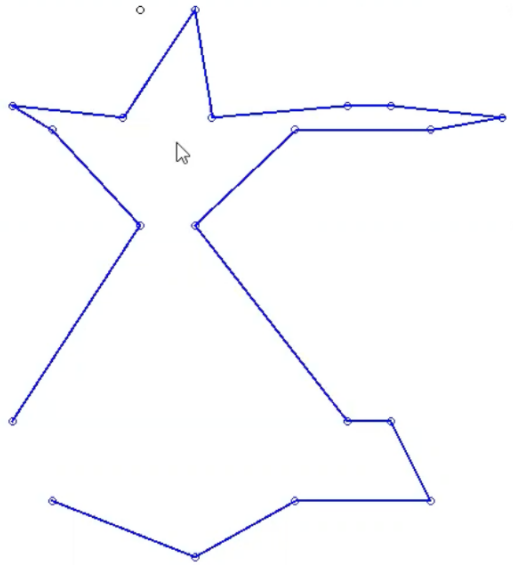

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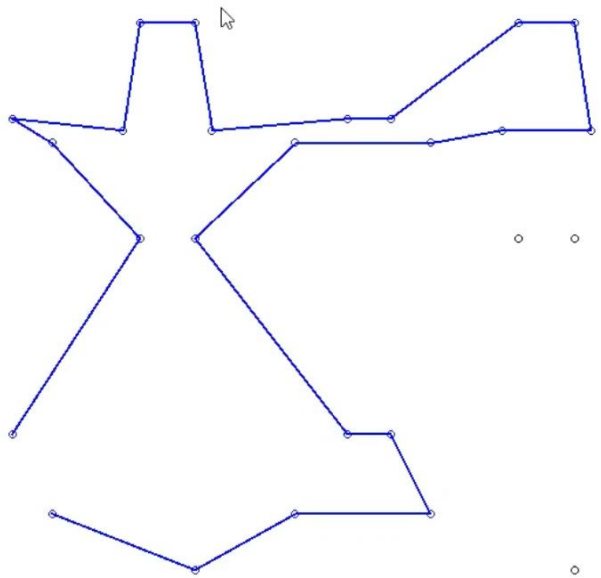

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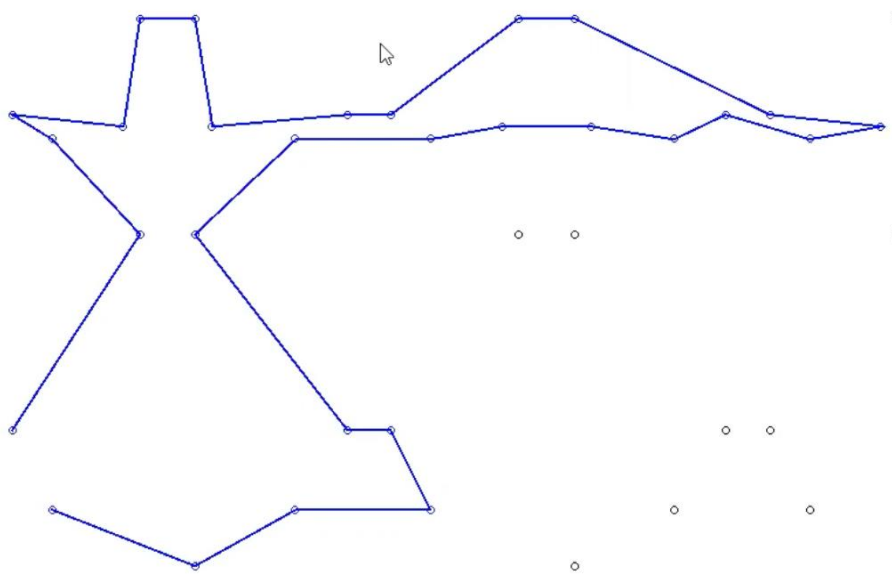

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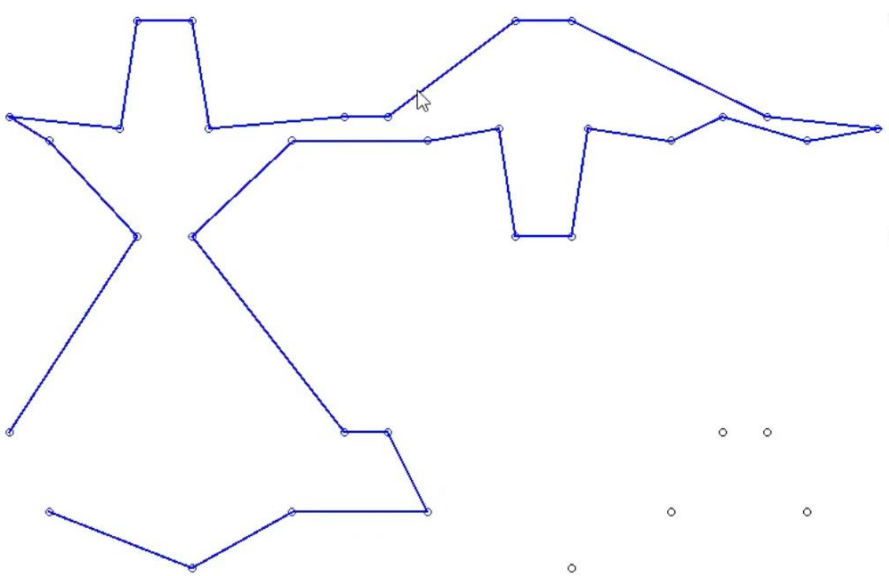

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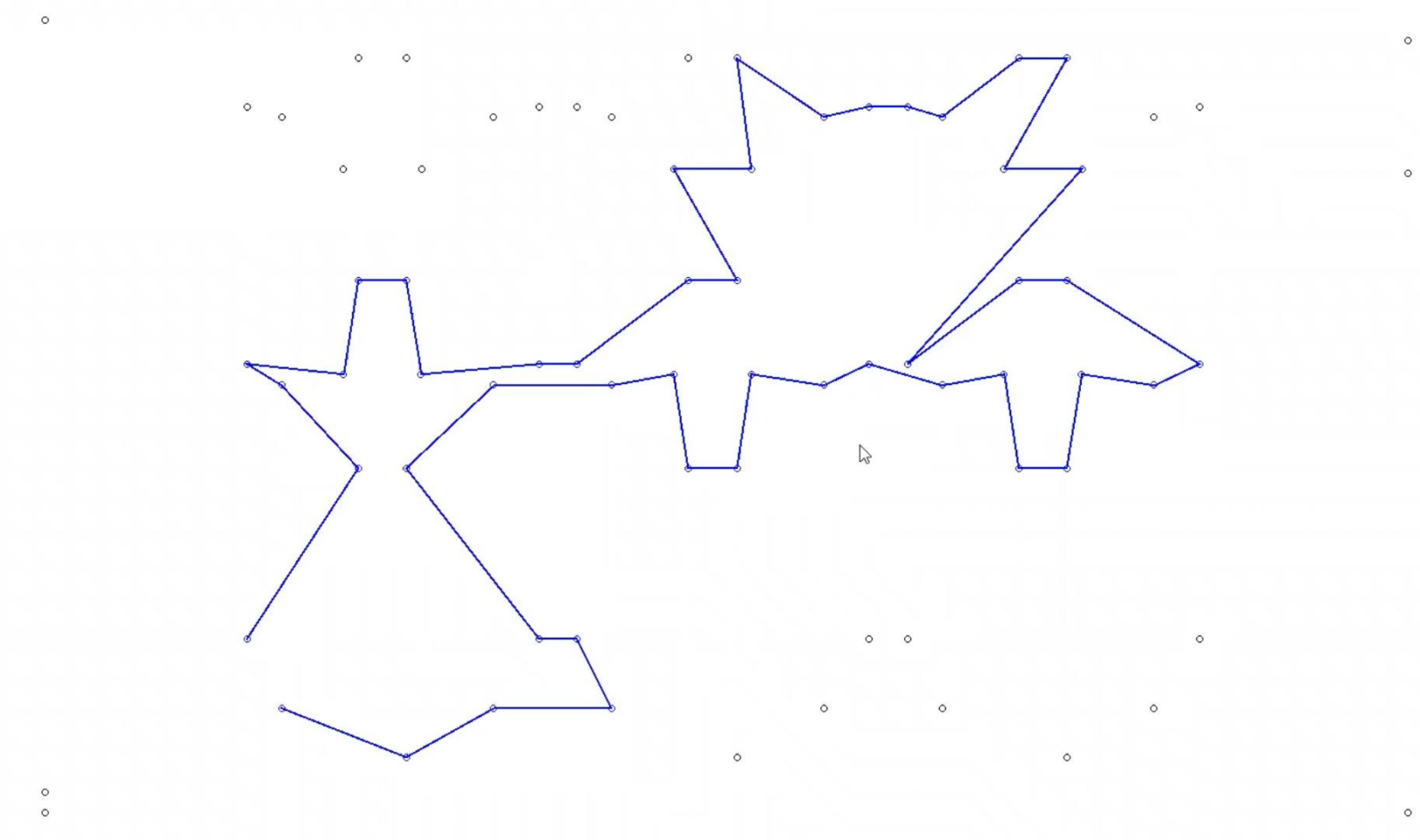

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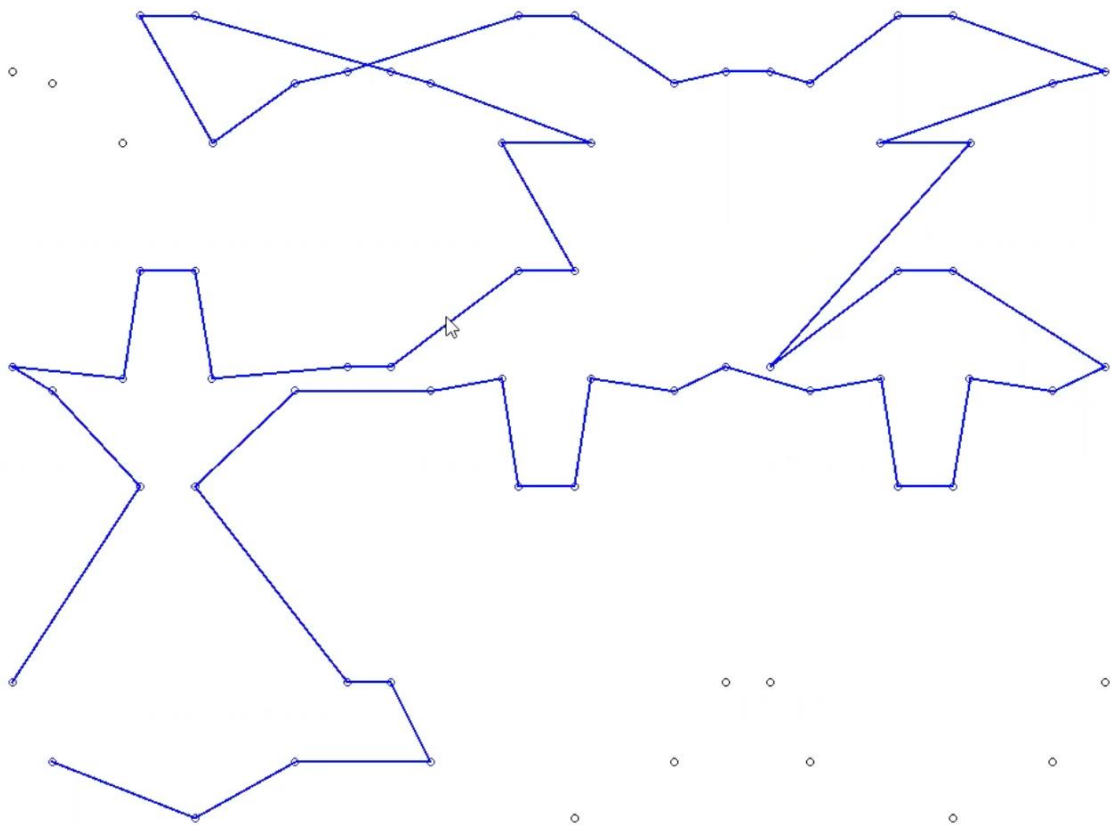

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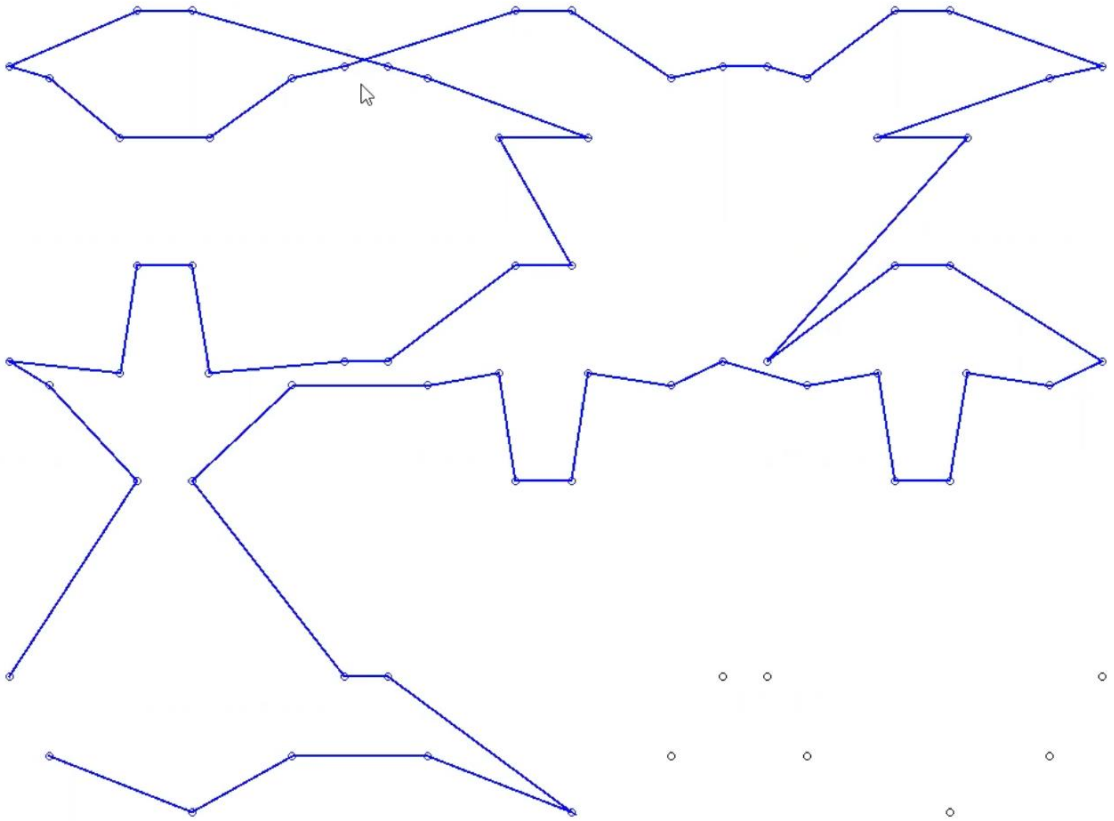

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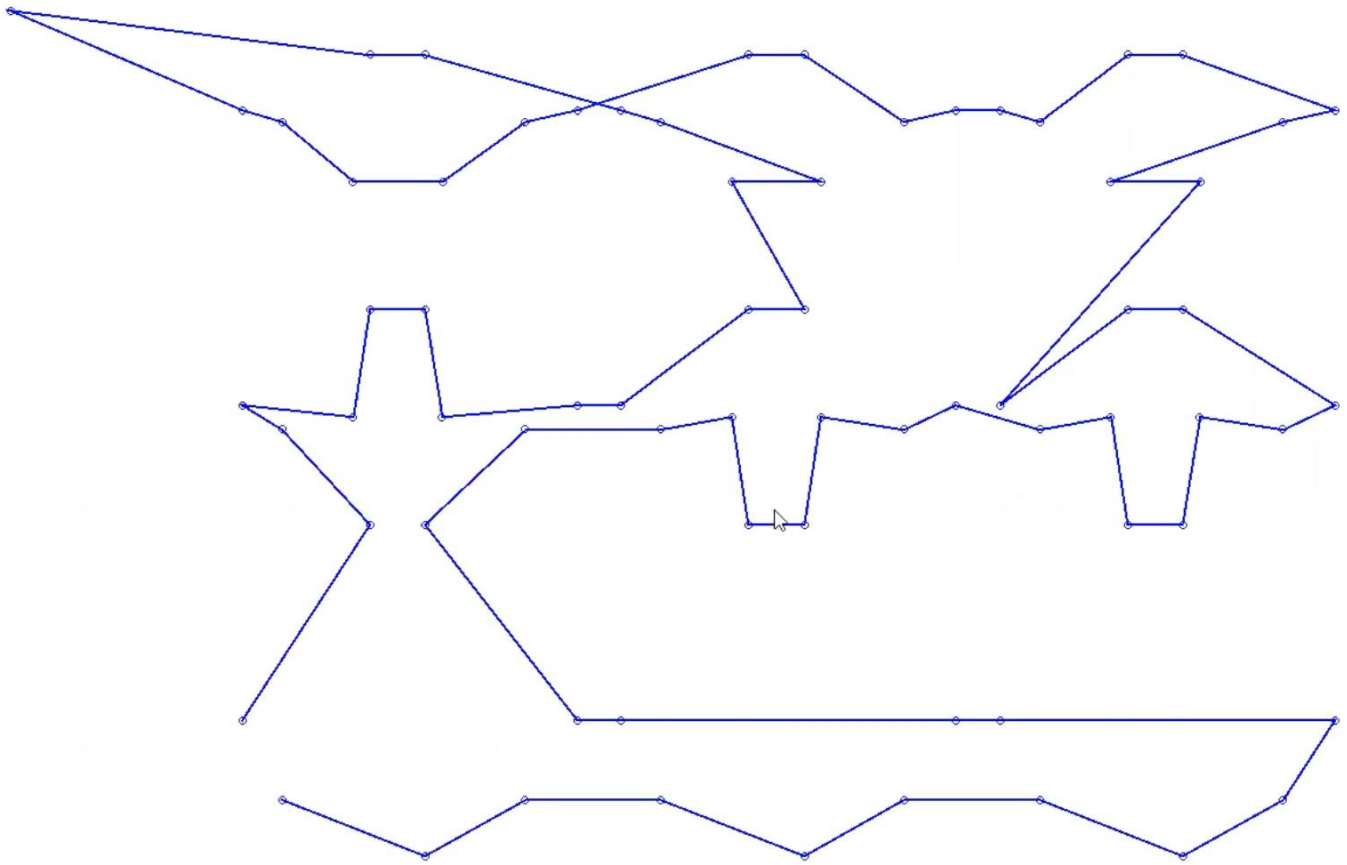

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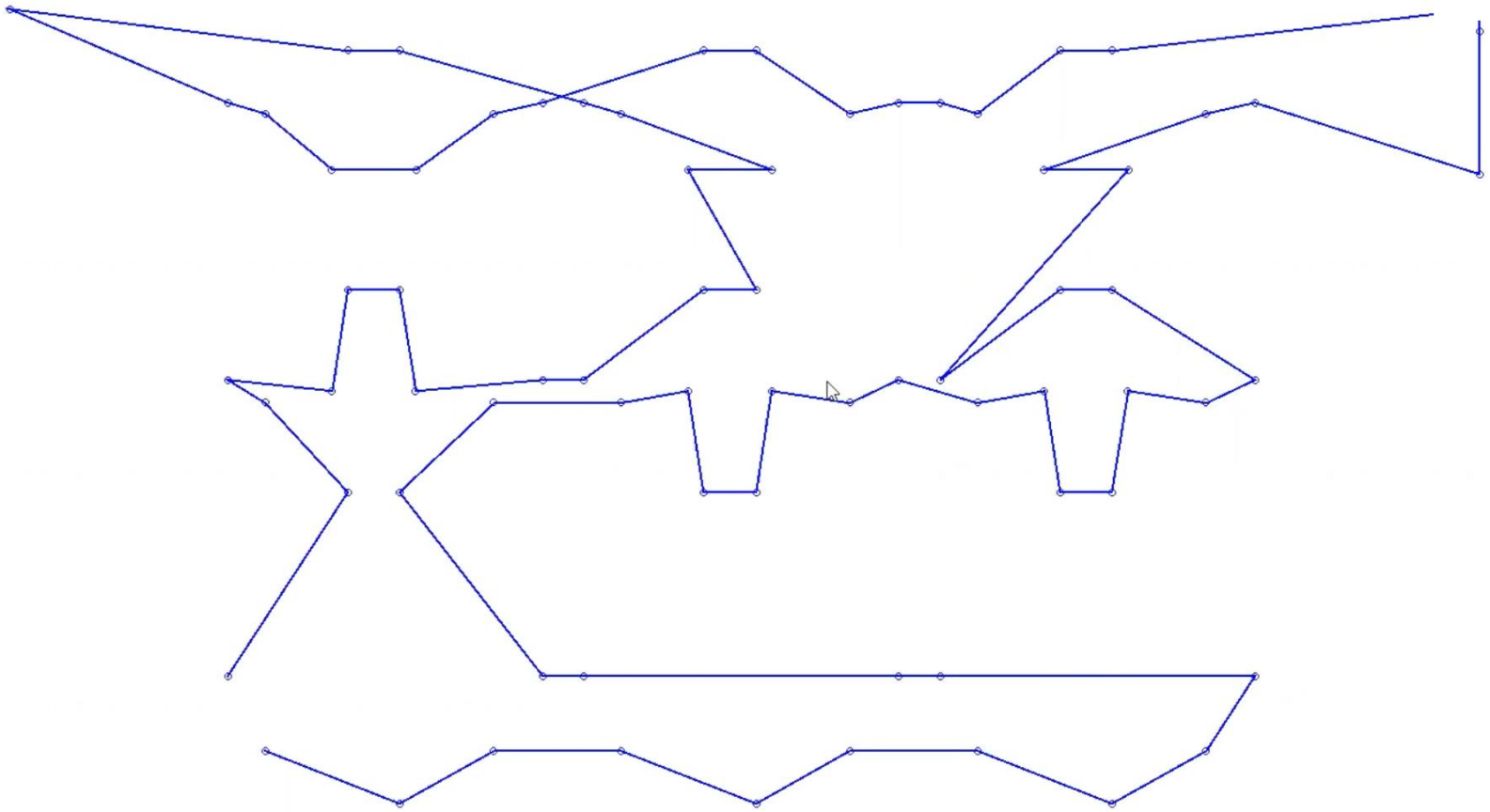

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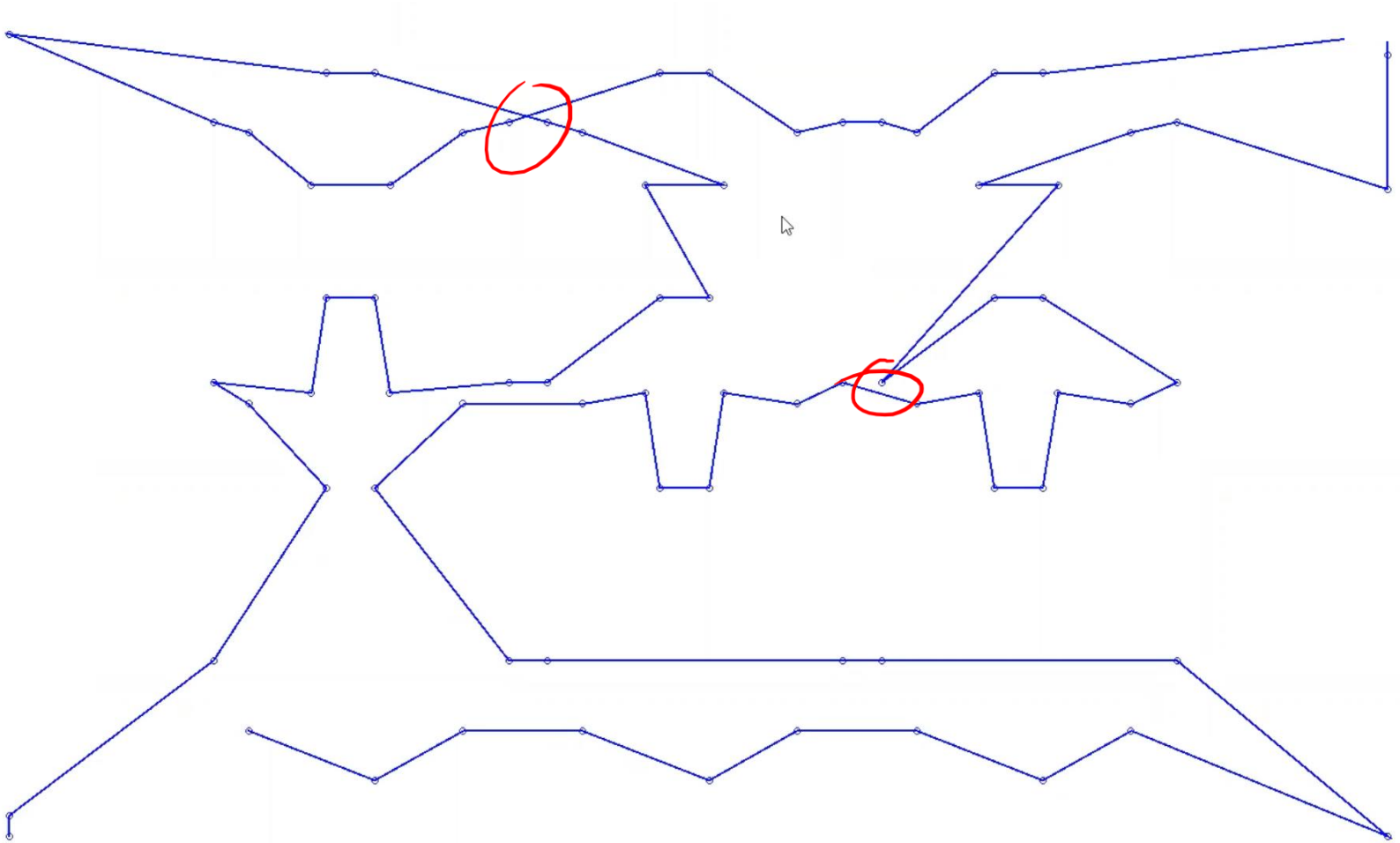
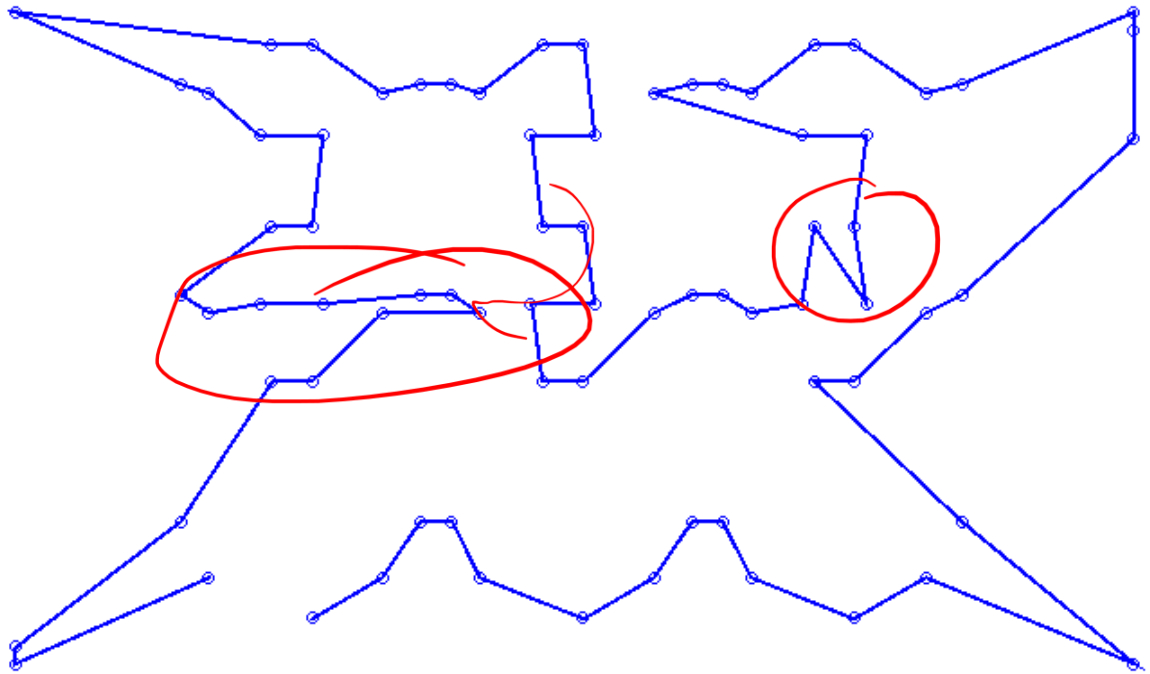


Construction algorithms – Randomisation

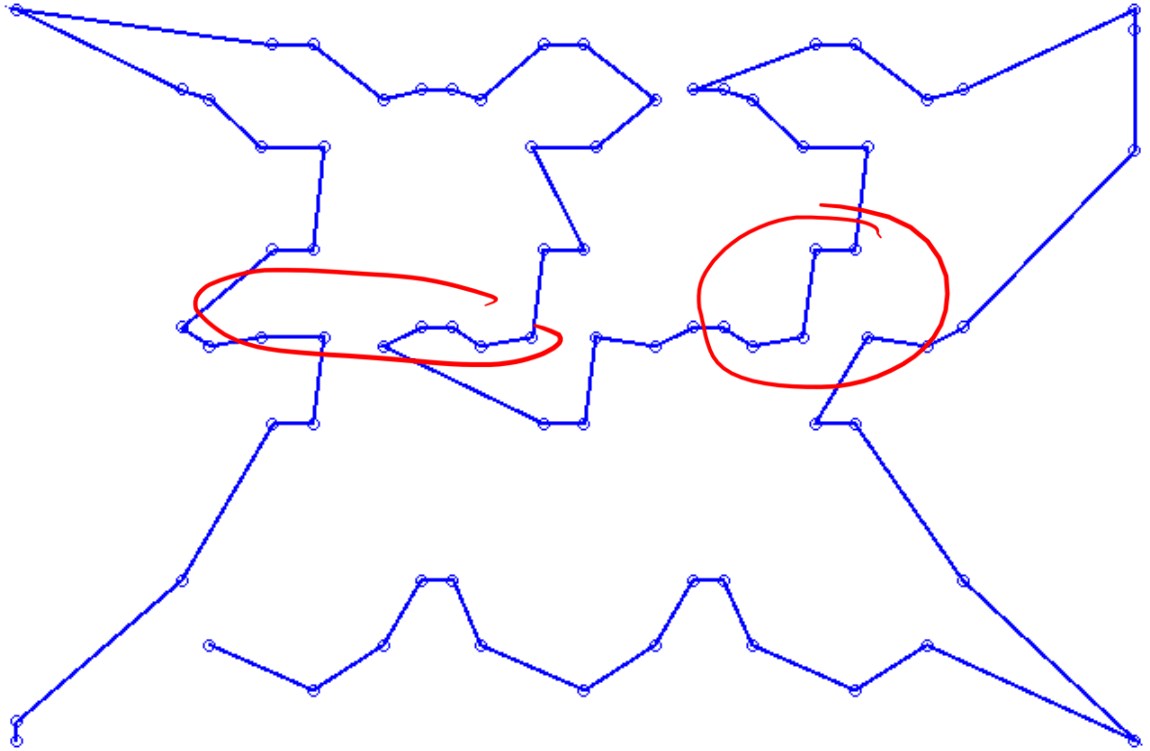
- At each iteration
  - Choose a **random customer**
  - **Insert it in the position that increases the cost by the least**
  - Allows insert between *any* pair of customers – not just at the end

Construction algorithms–Randomisation

ONE RANDOM TOUR



OPT. TOUR



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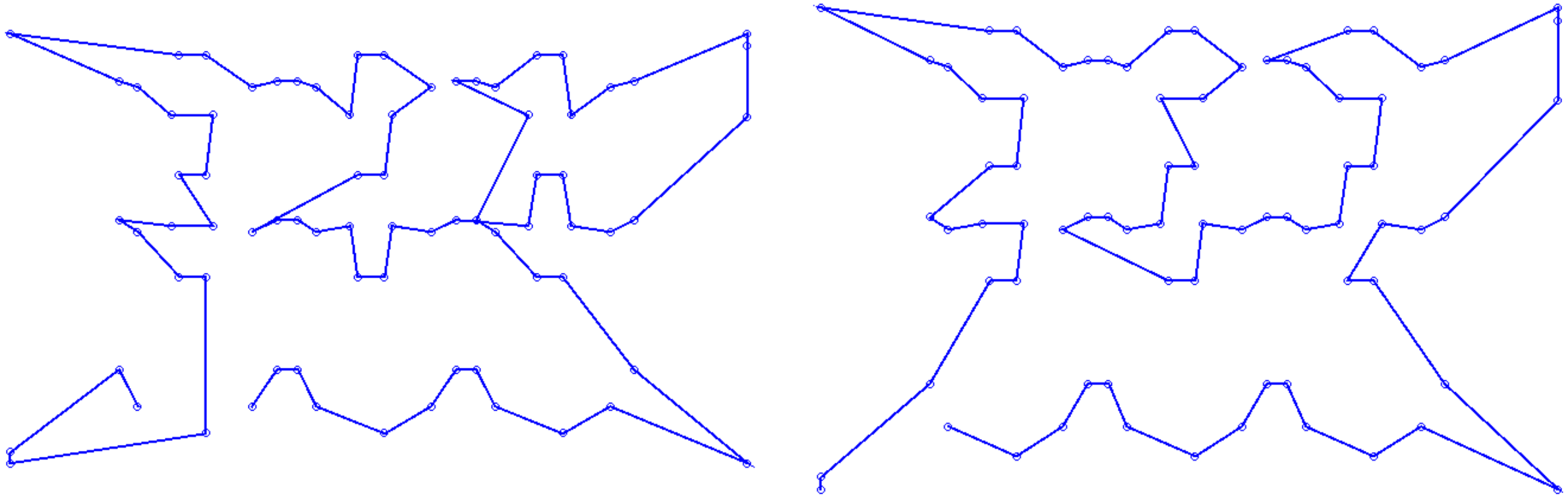

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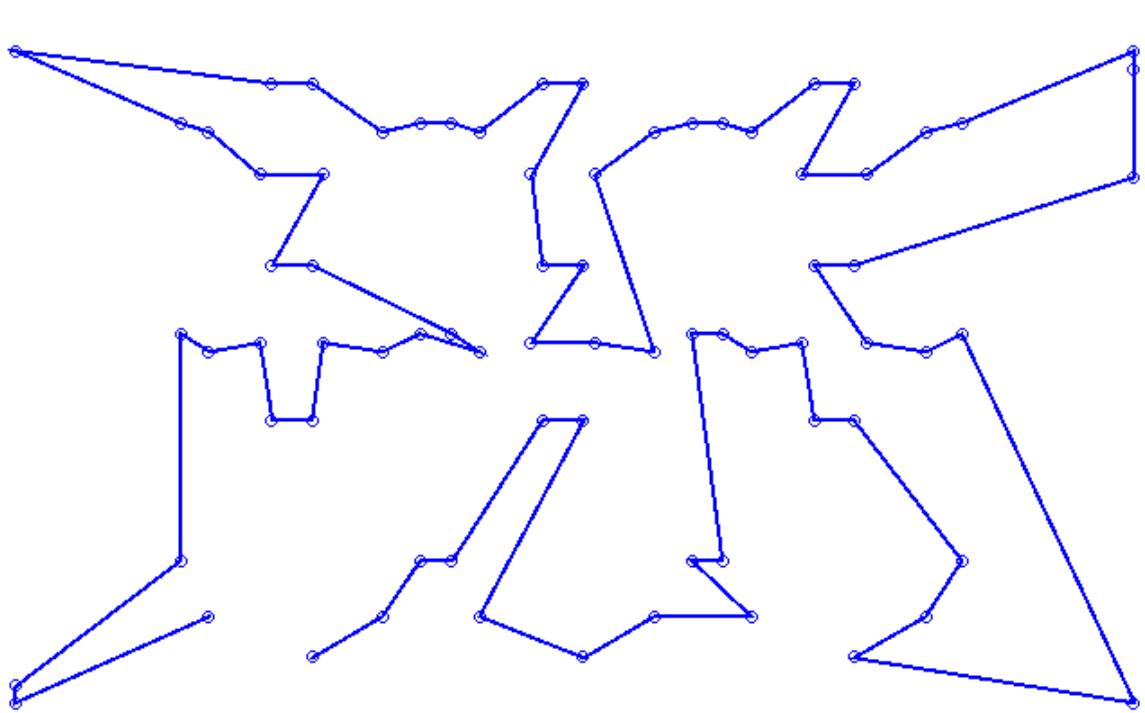

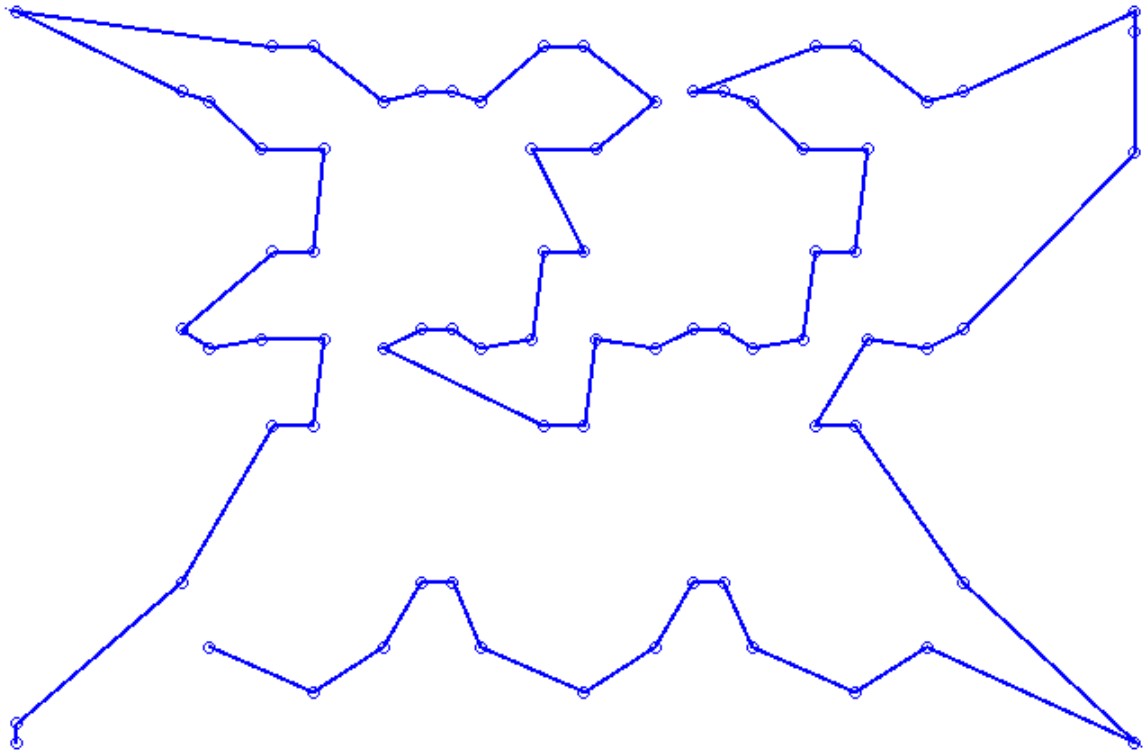

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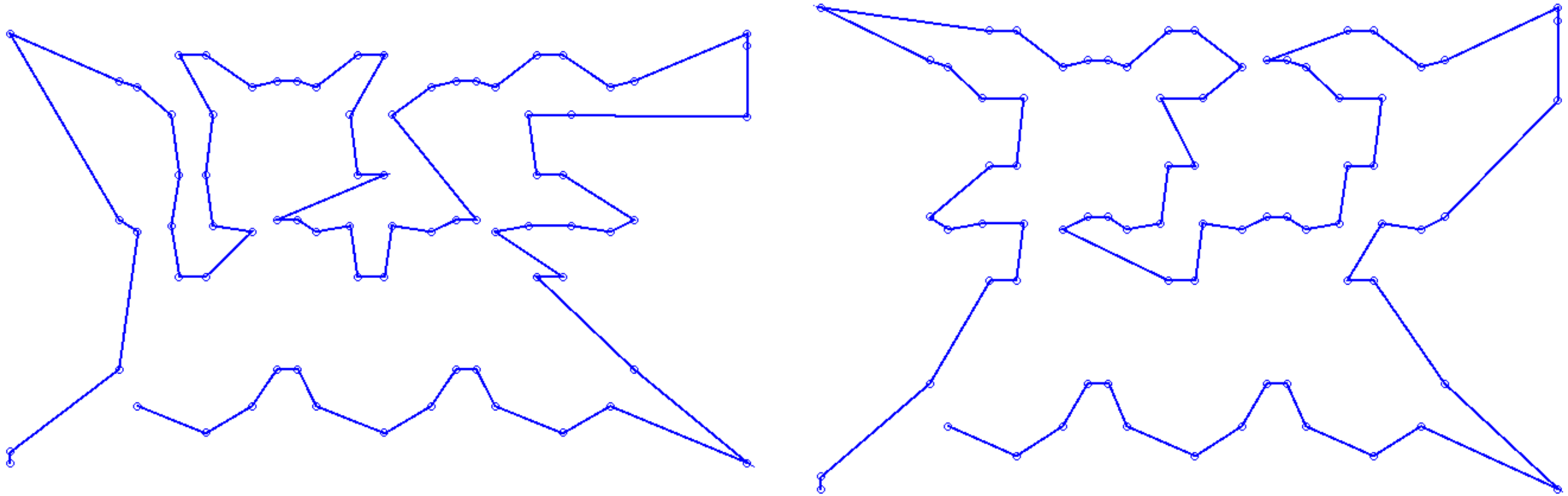

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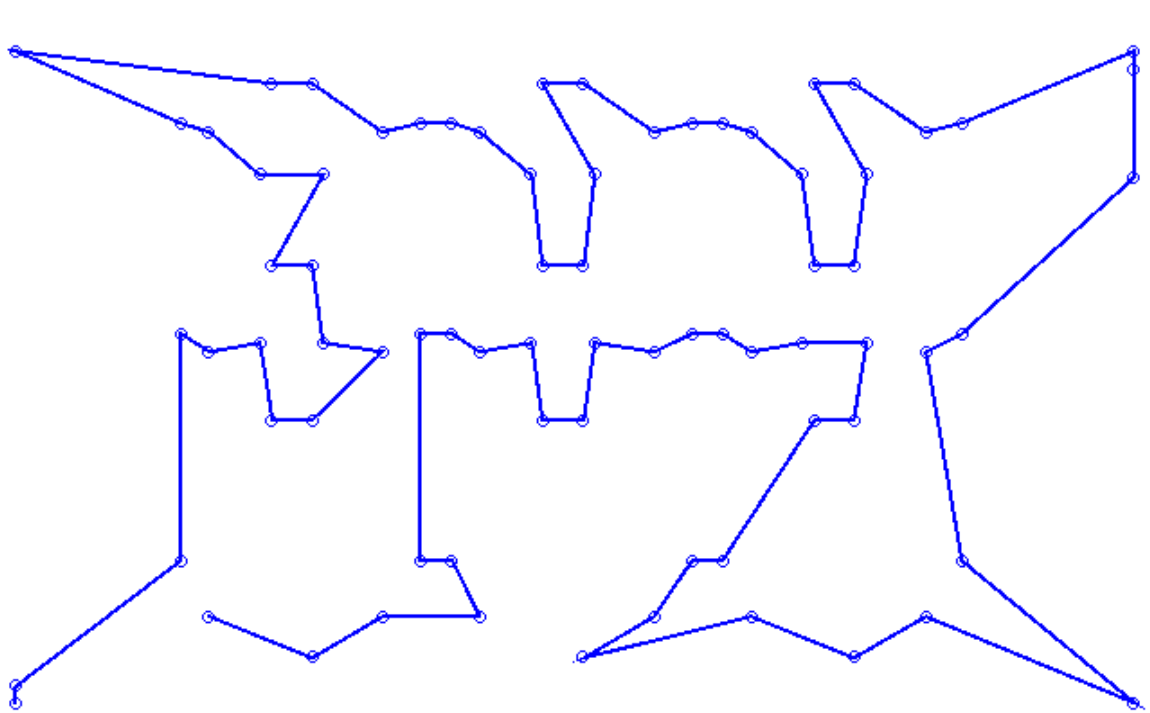

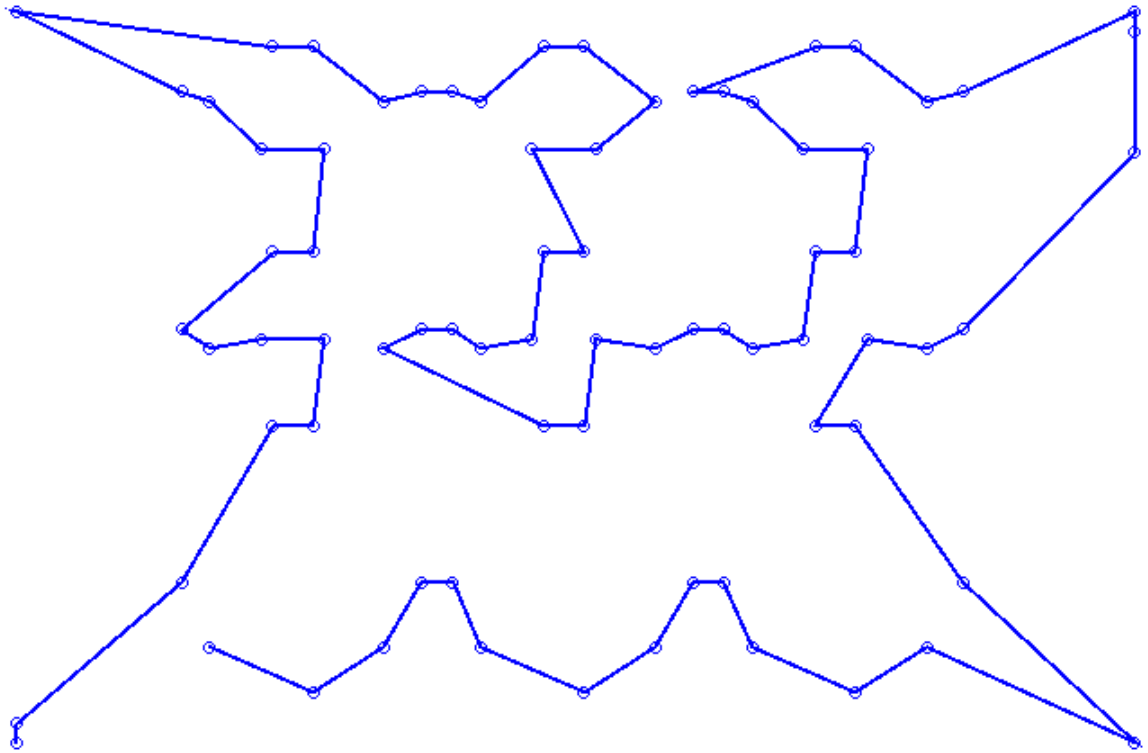

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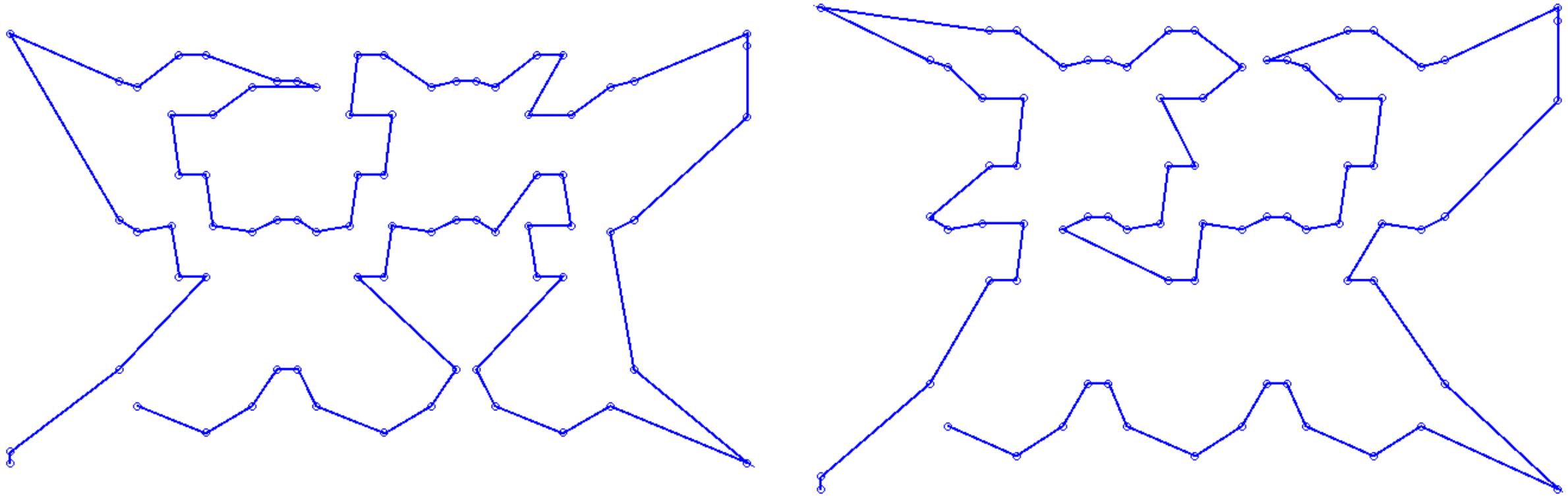
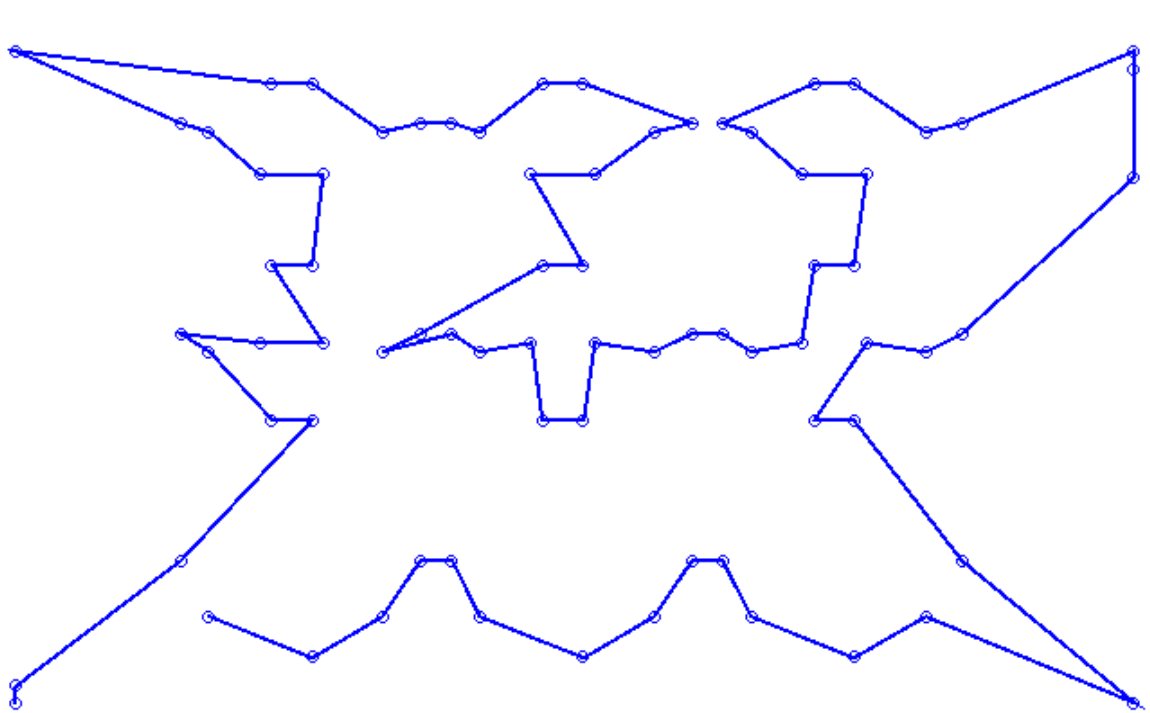
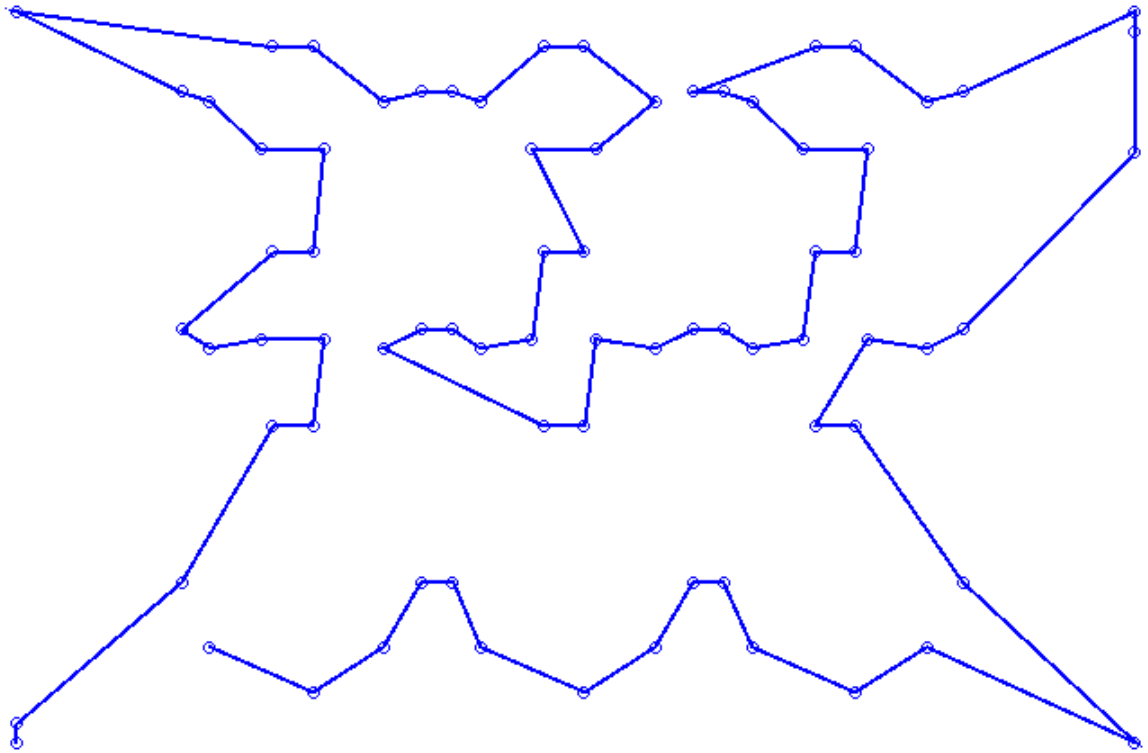


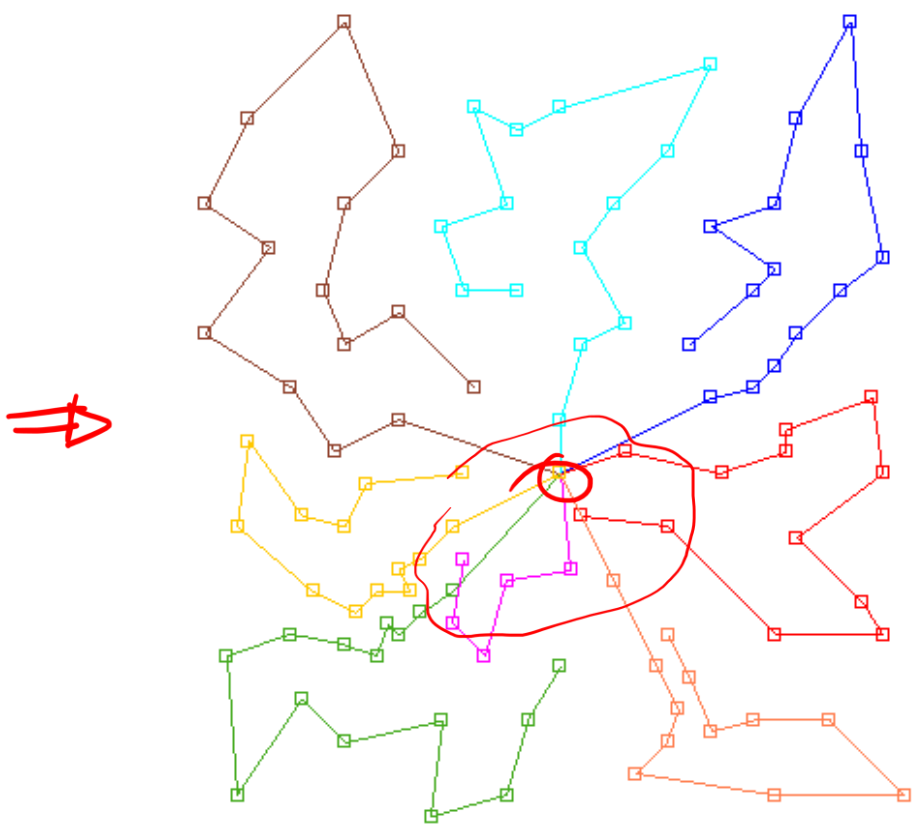
Construction algorithms–Randomisation

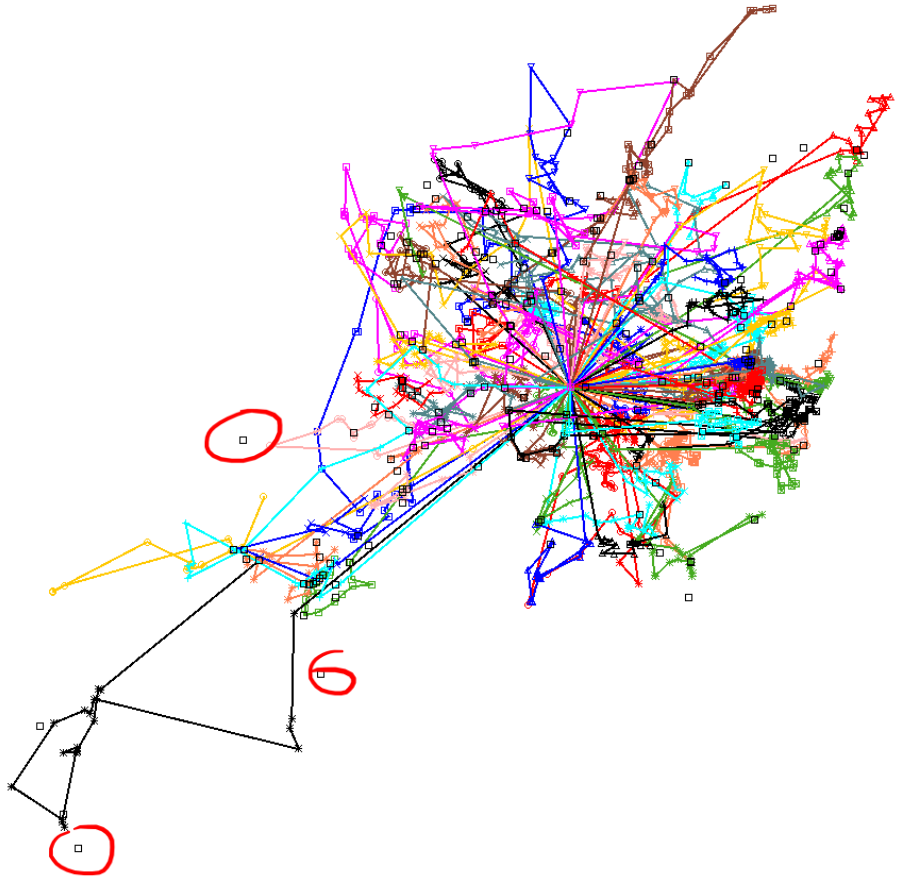




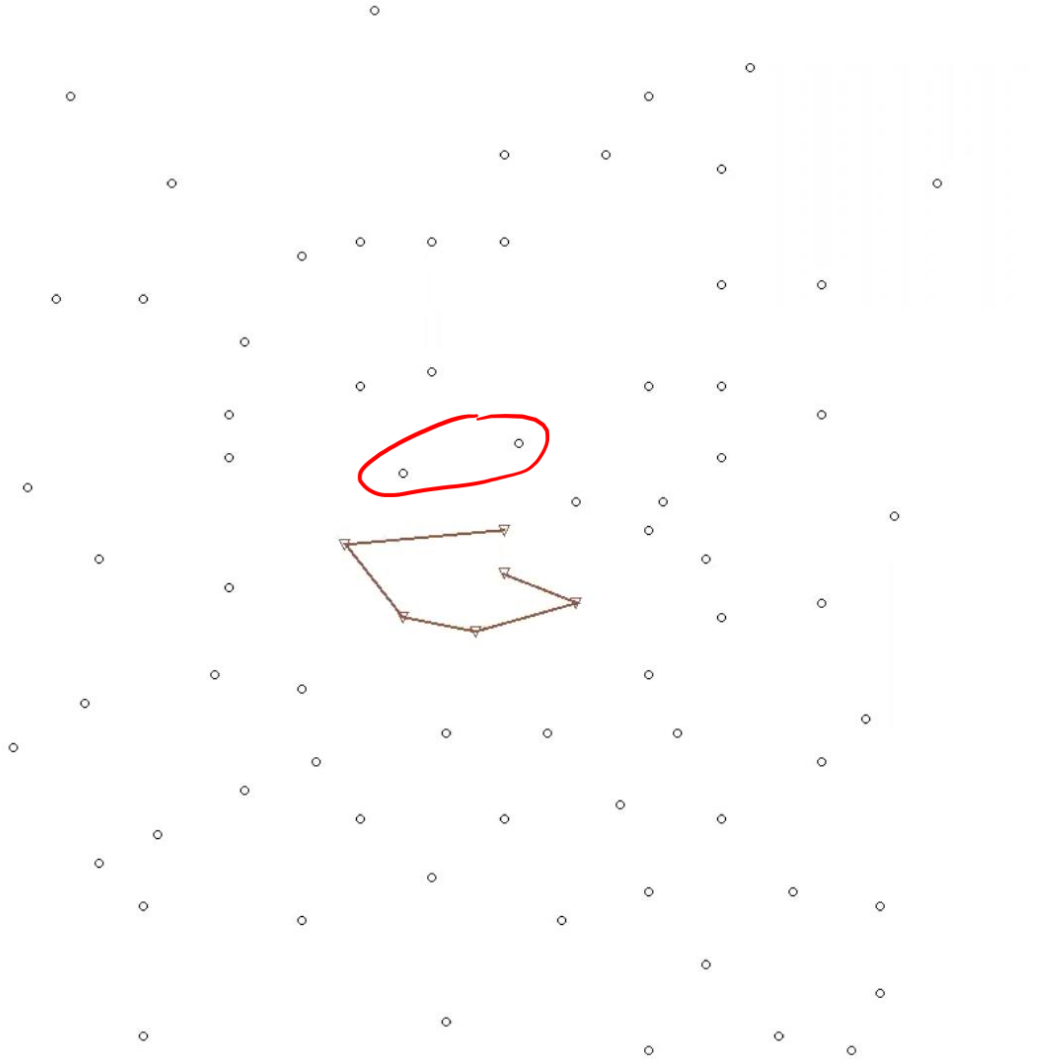
Vehicle Routing Problem (VRP)

Given a set of customers, and a fleet of vehicles to make deliveries, find a set of routes that services all customers at minimum cost

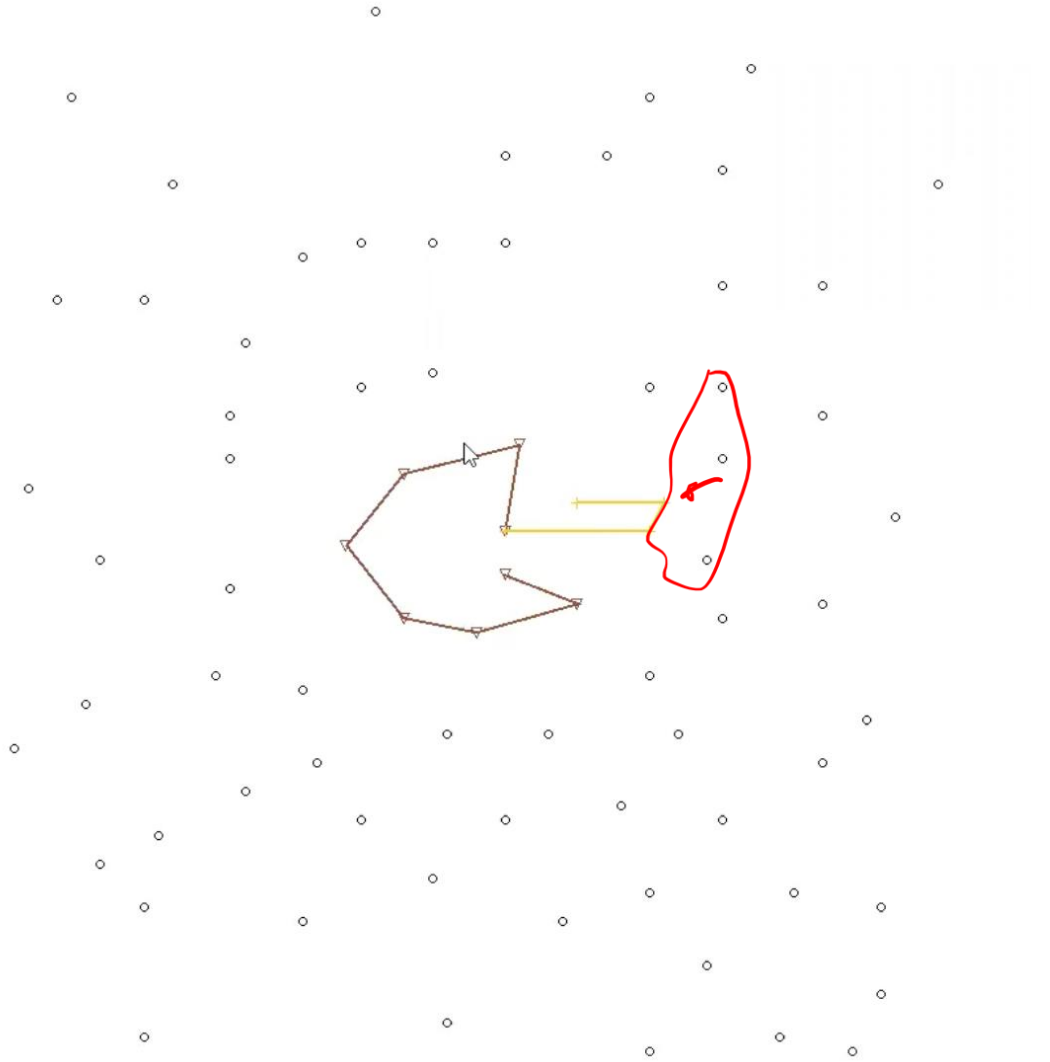




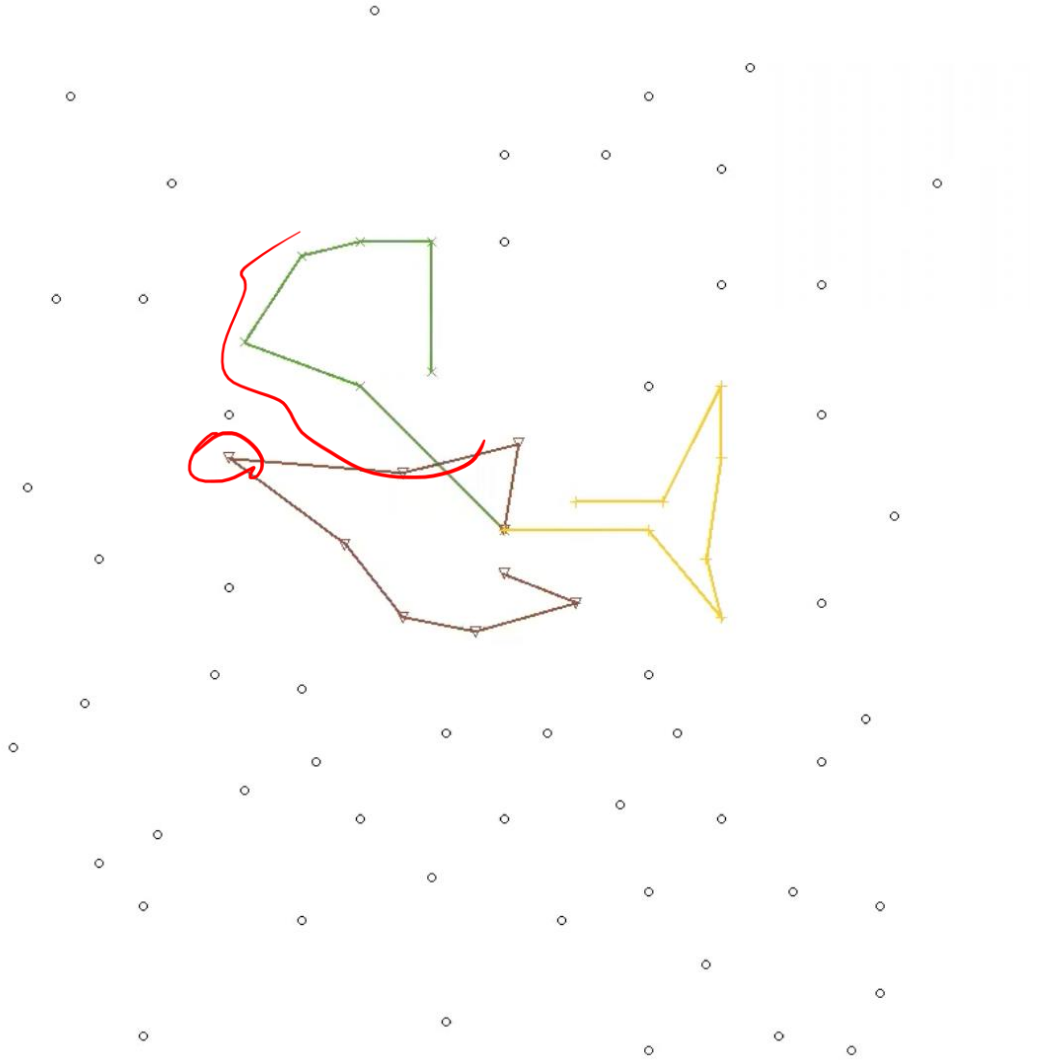
Minimum Insert Cost – VRP



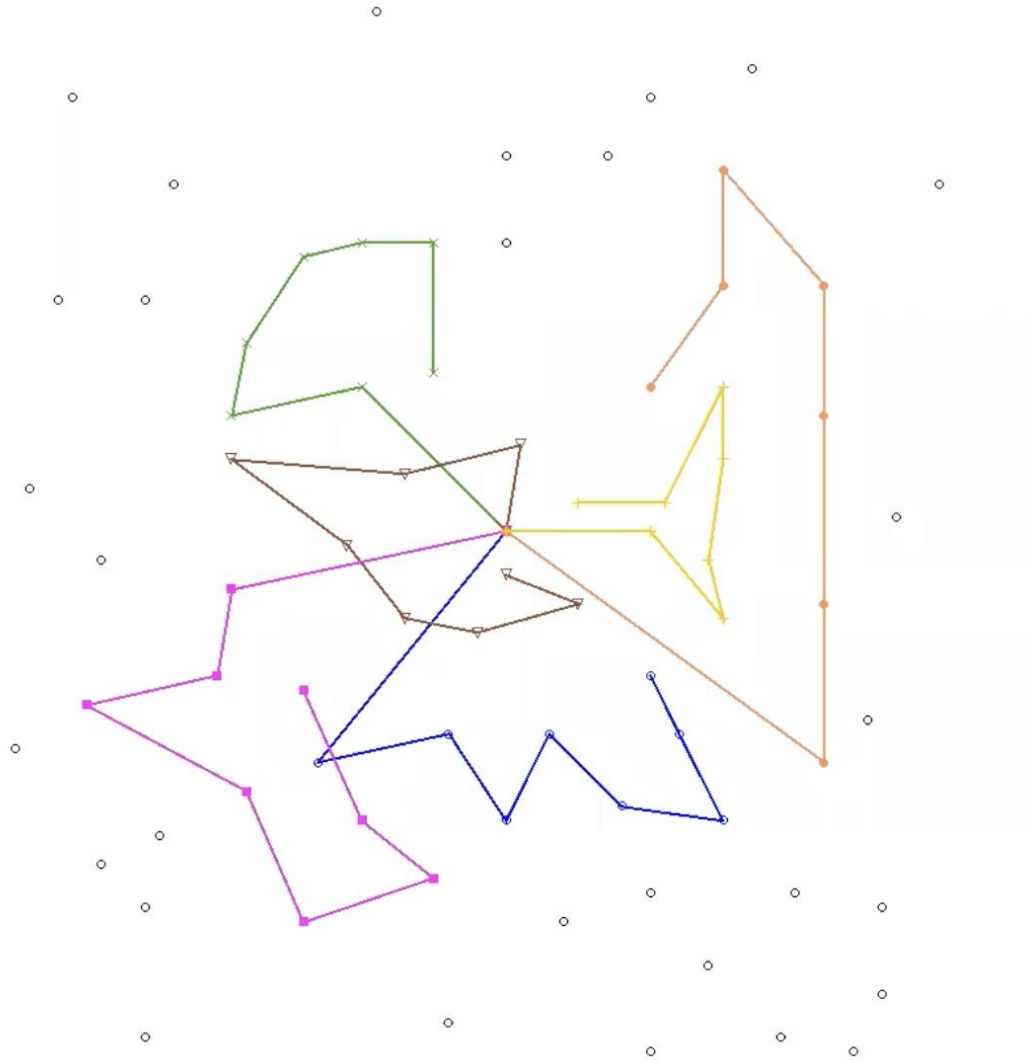
Minimum Insert Cost – VRP



Minimum Insert Cost – VRP

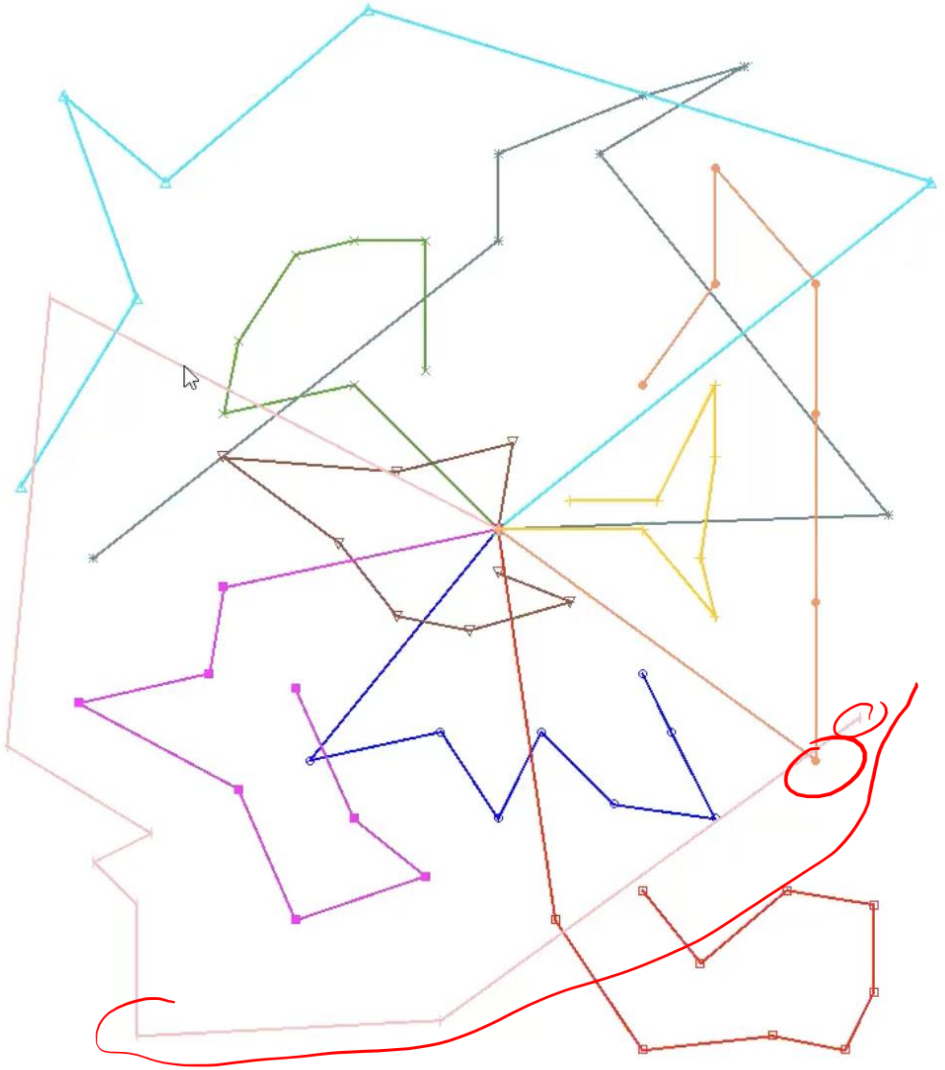


Minimum Insert Cost – VRP



Minimum Insert Cost – VRP

COMBINING
H/MH + MATH OPT
AI: LNS ← NGT WGGK
LARGE \ N. \ SEARCH

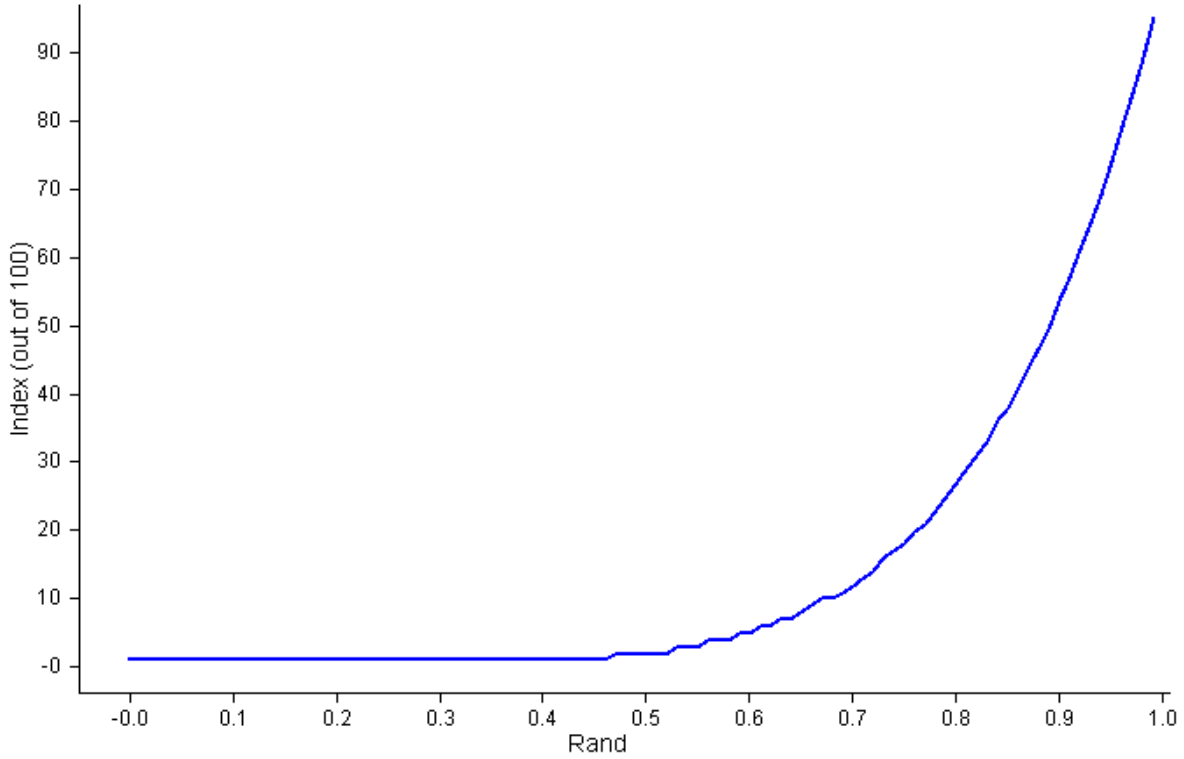


randomising the order does not help ☹

Construction algorithms

- Combine “greedy” and “random”
- Don’t always make the very “greediest” choice

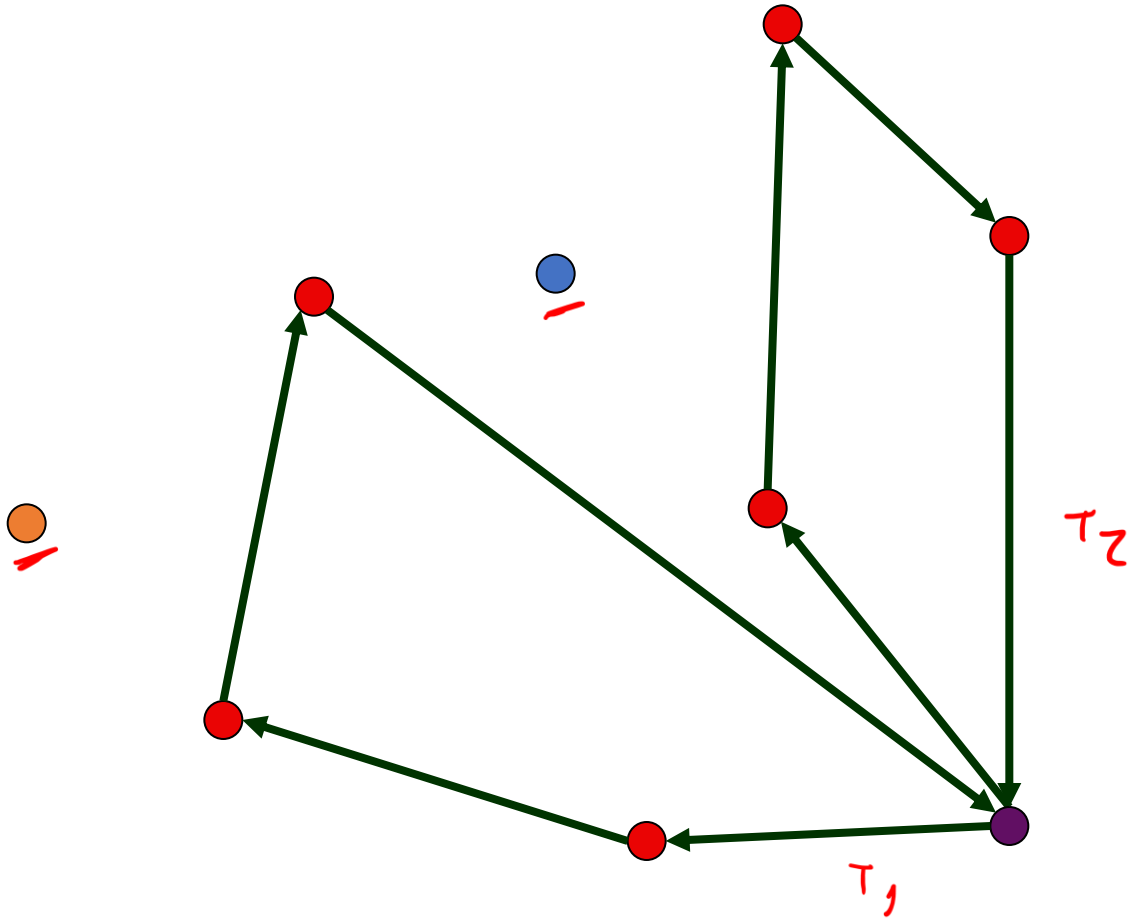
- E.g. choose the r^{th} best of n choices
- $r = n * (\text{uniform}(0,1))^{y}$



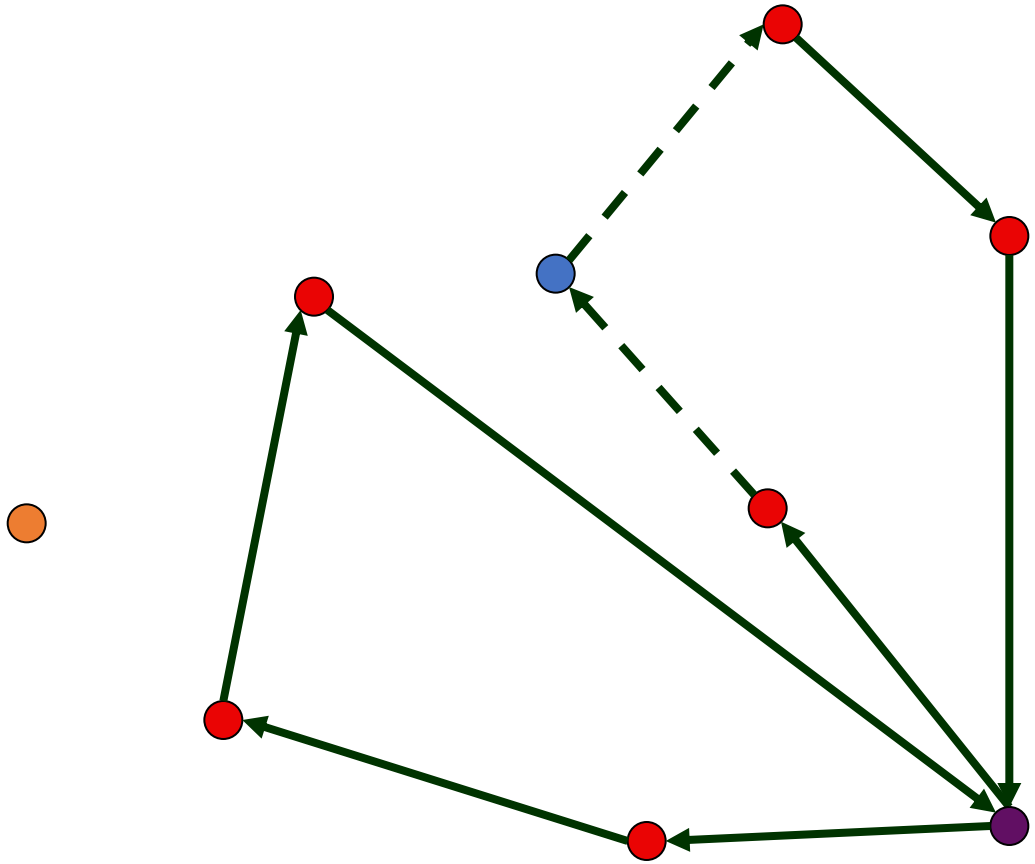
Construction algorithms

- We have seen that greedy algorithms lack “foresight”
- Next step: Regret

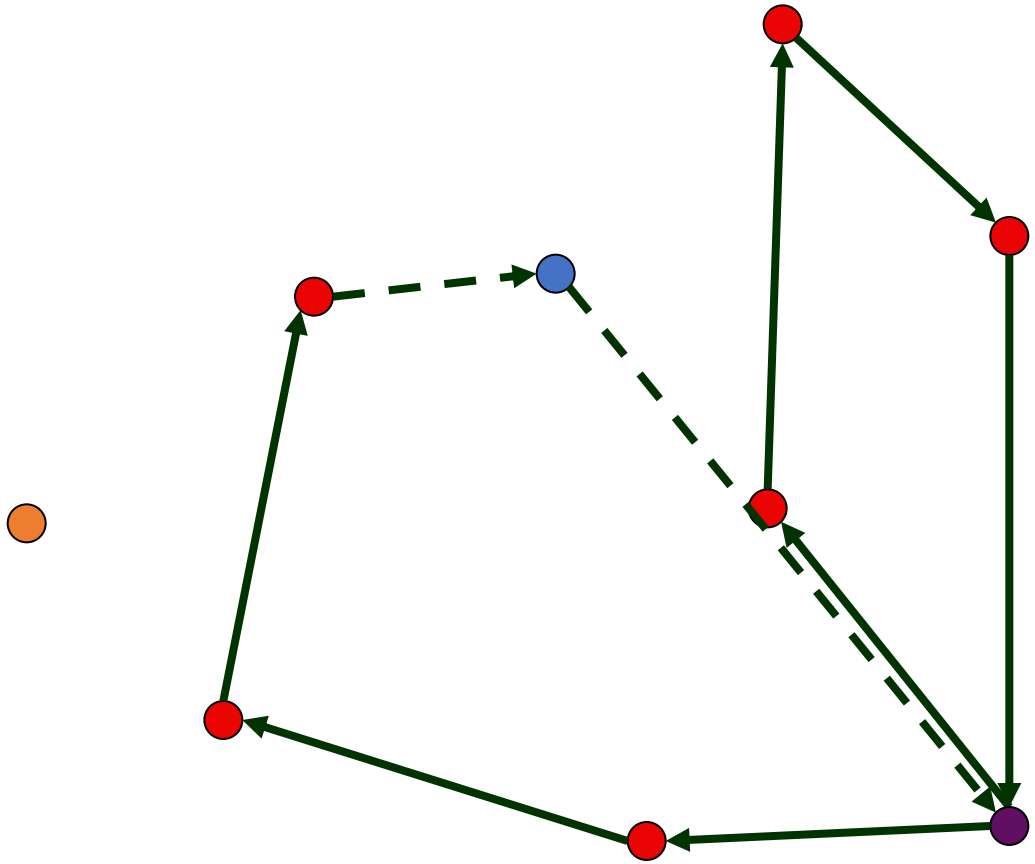
Regret – VRP



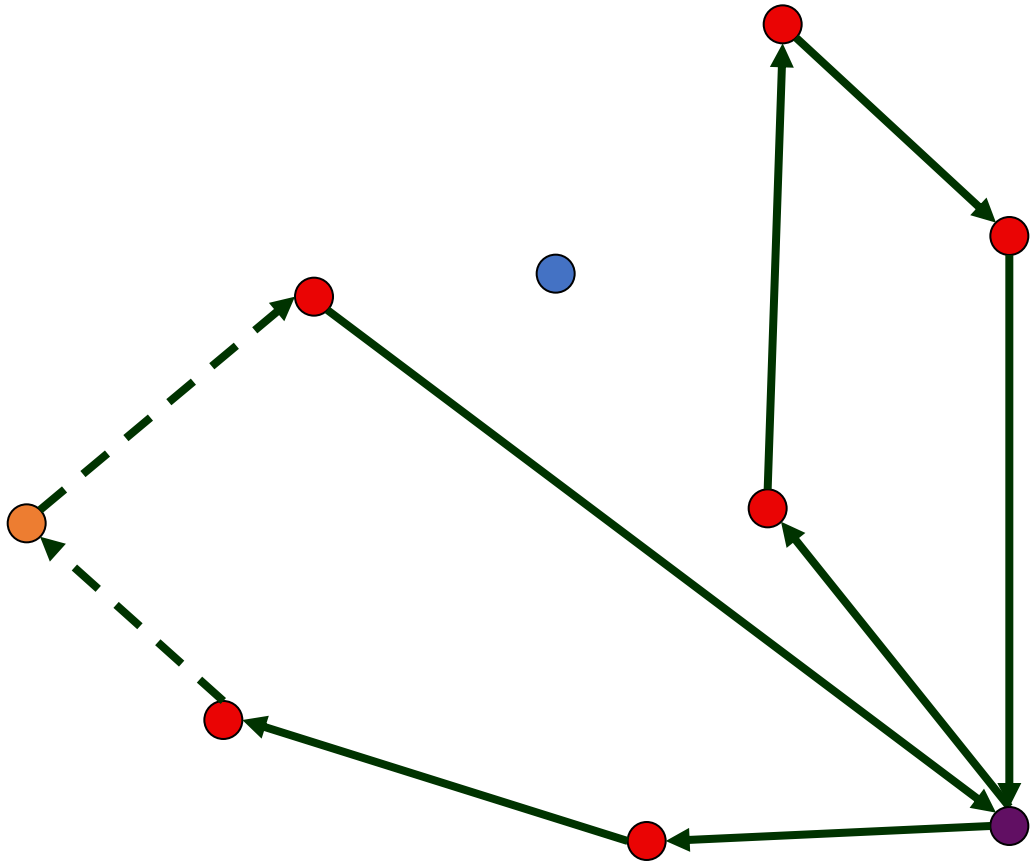
Regret – VRP



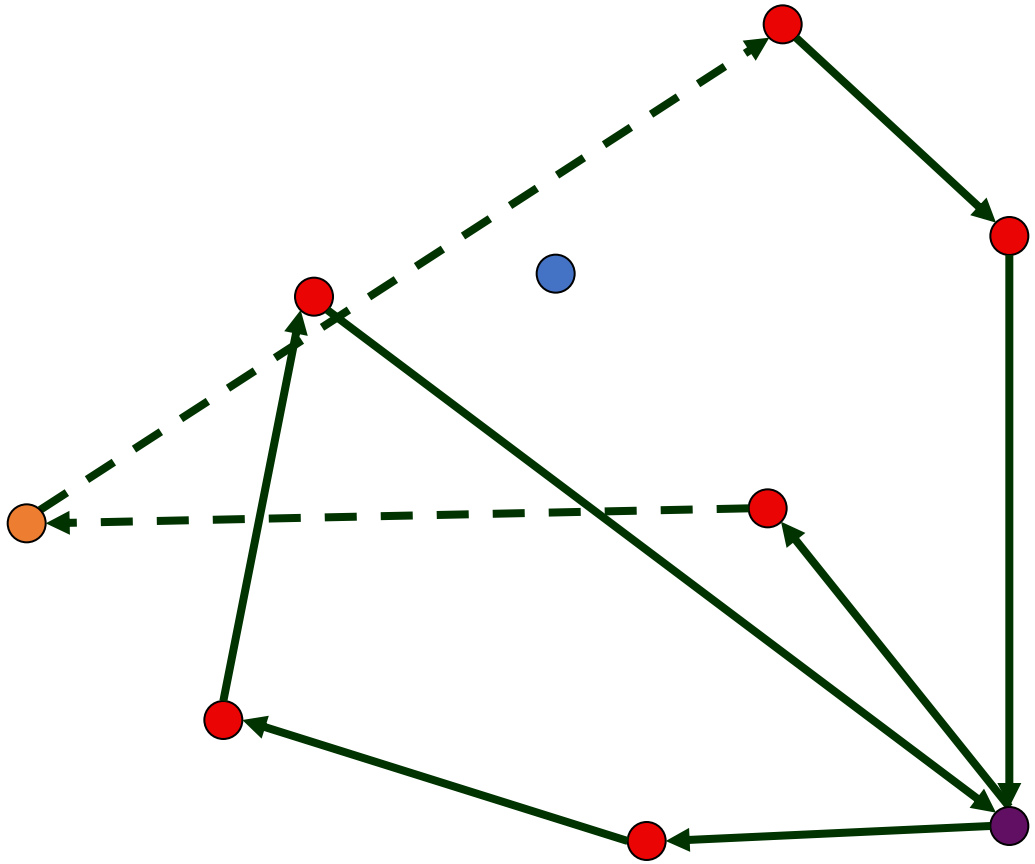
Regret – VRP



Regret – VRP



Regret – VRP



Regret

regret(i) = C(insert i in 2nd-best route) – C(insert i in best route)

= $f(2,i) - f(1,i)$

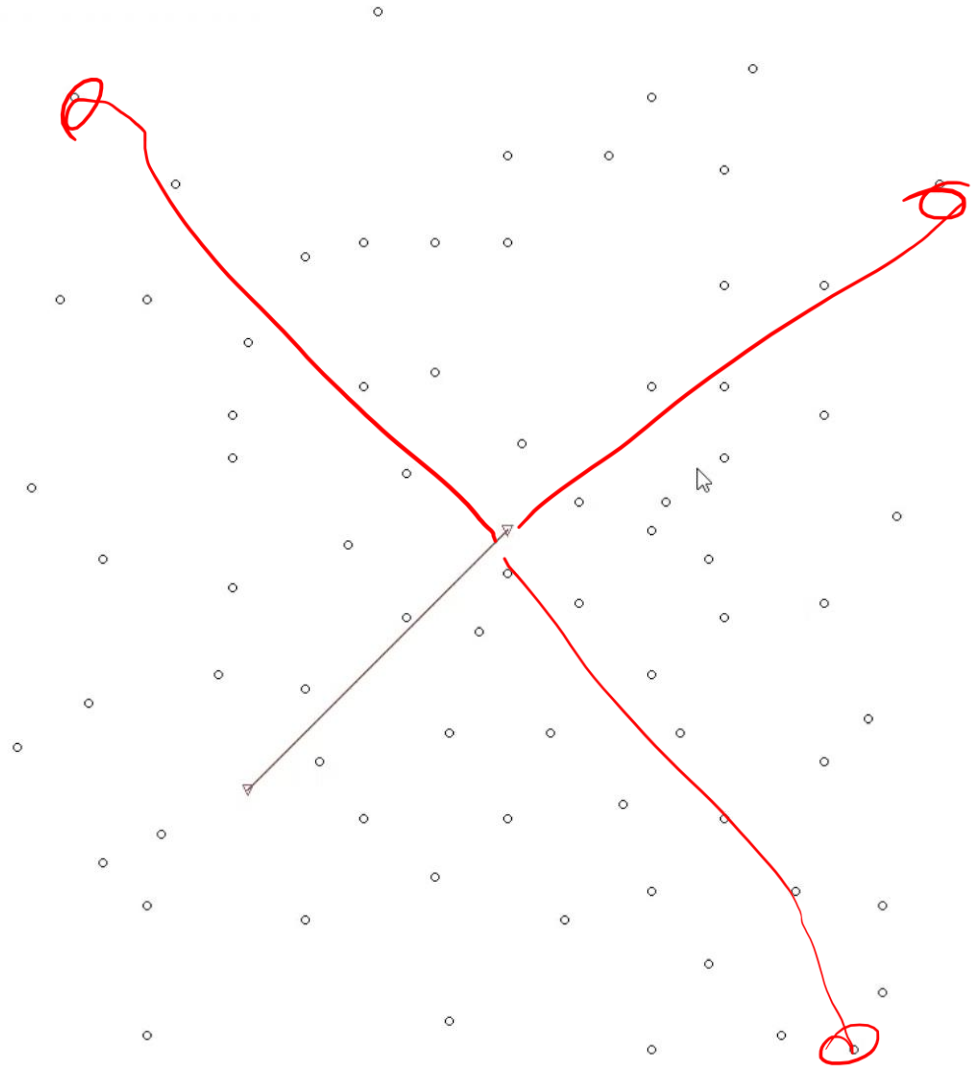
Select the choice i with largest regret

- VRP: next customer to be served is the one with largest regret

Can be extended to the k-th best routes:

k-regret (i) = $\sum_{j=2}^{k} (f(j,i) - f(1,i))$

Regret



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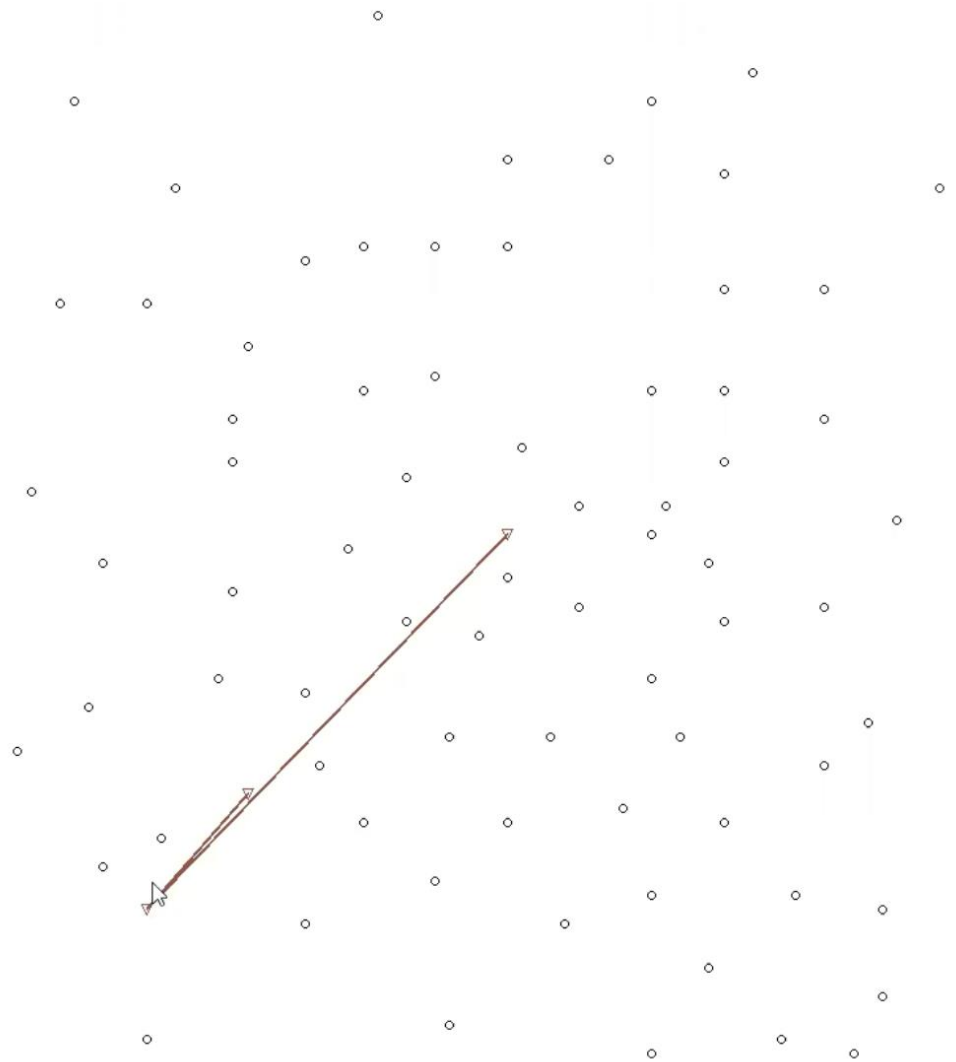

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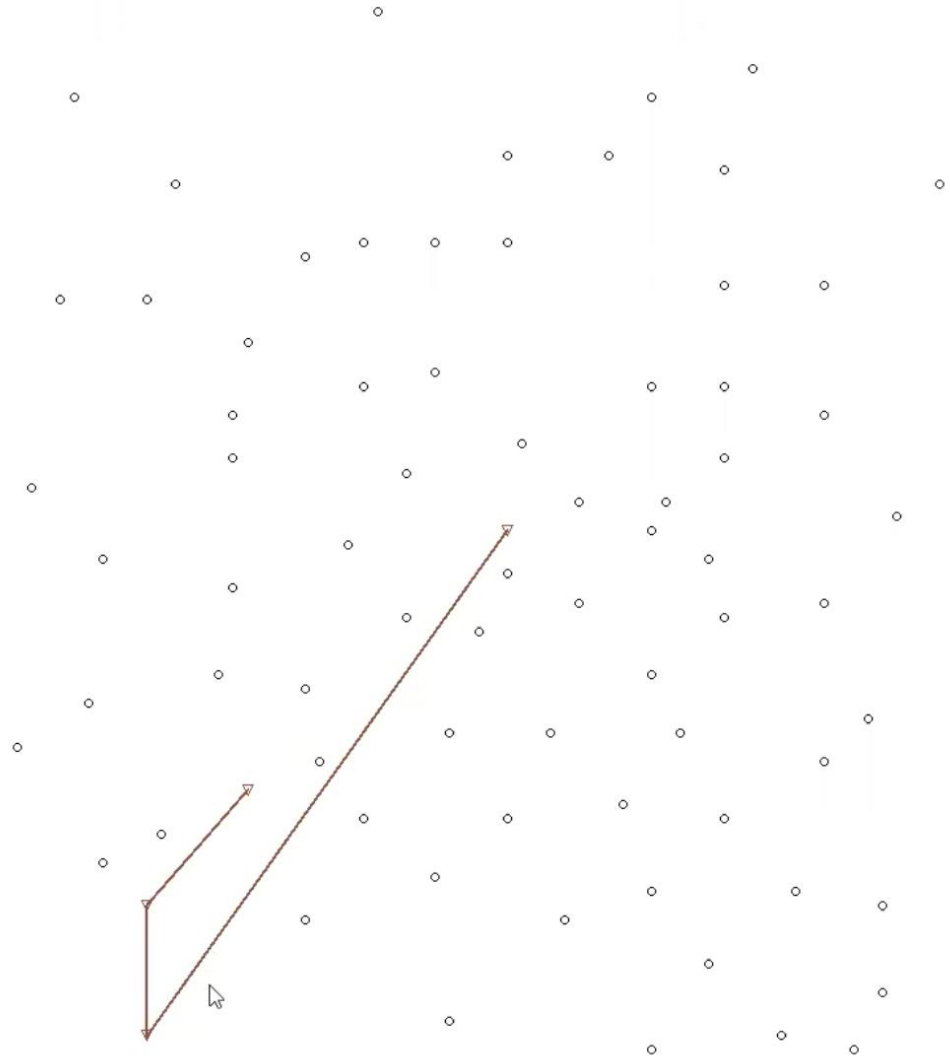

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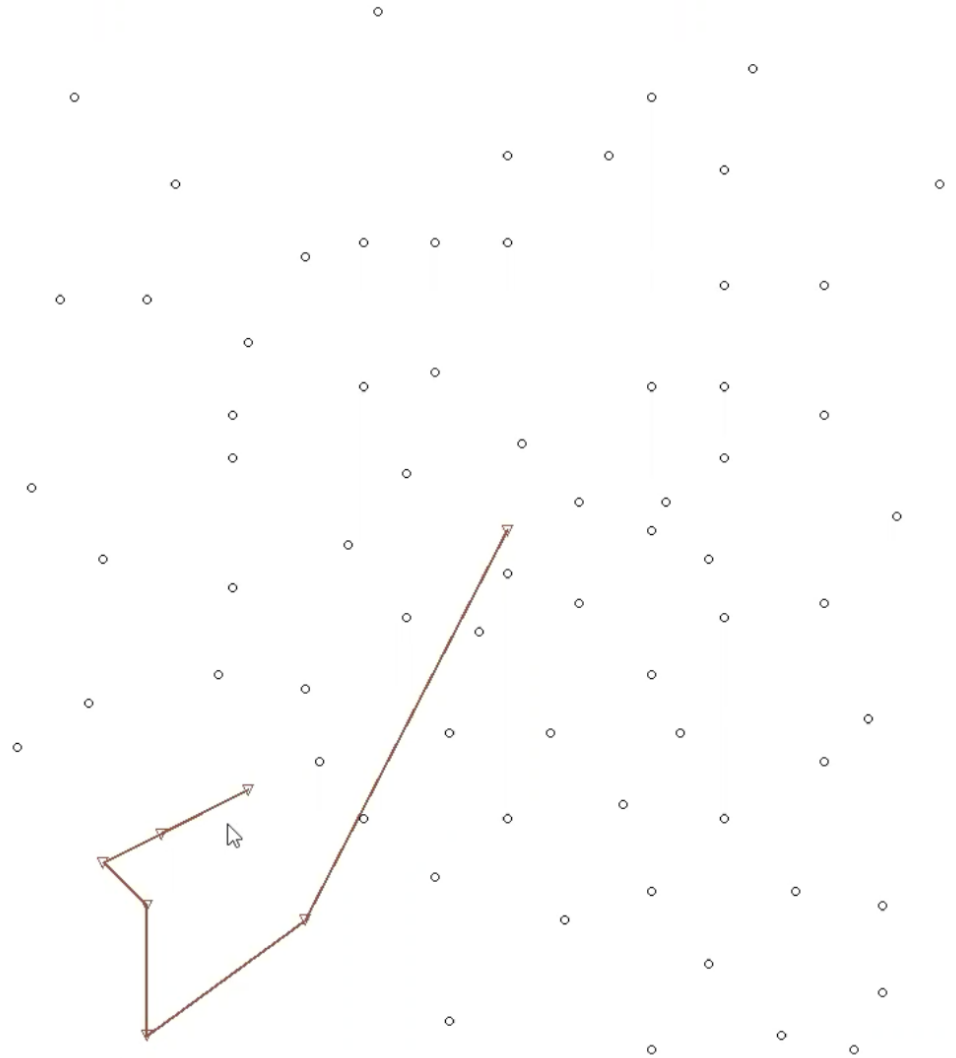

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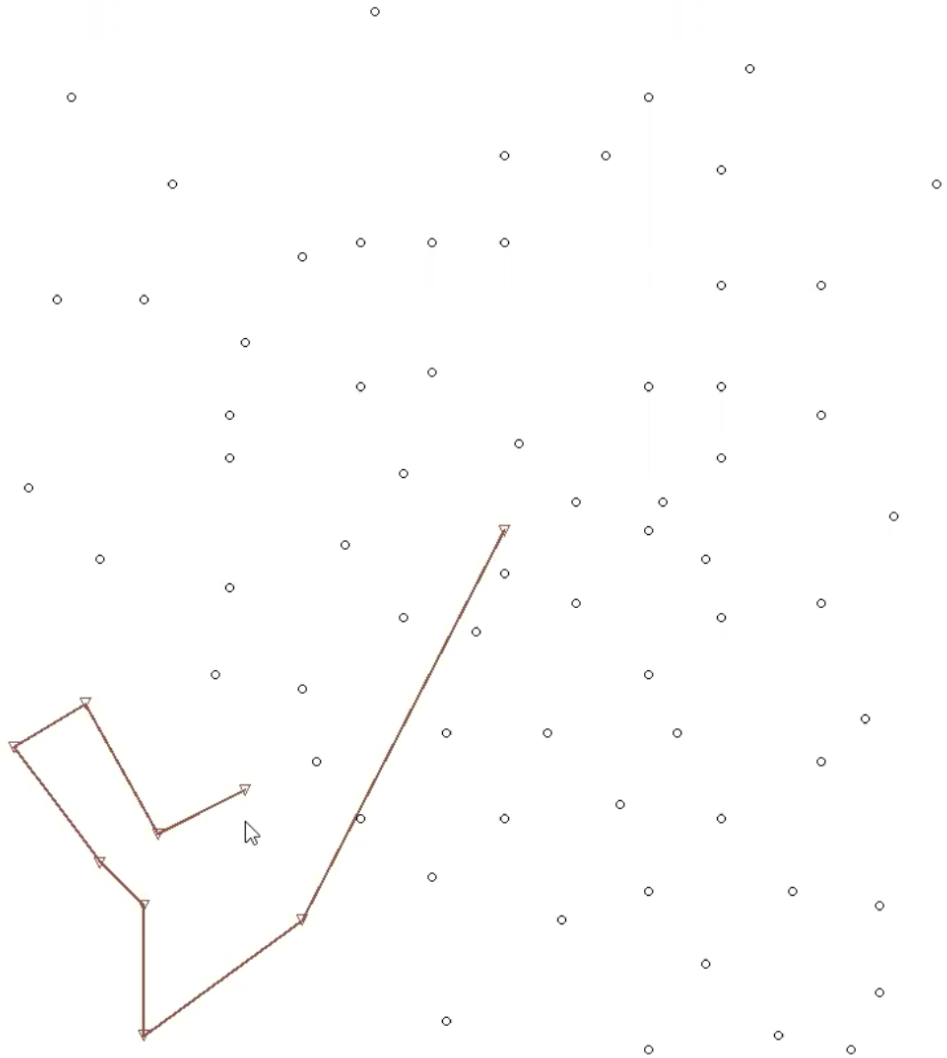

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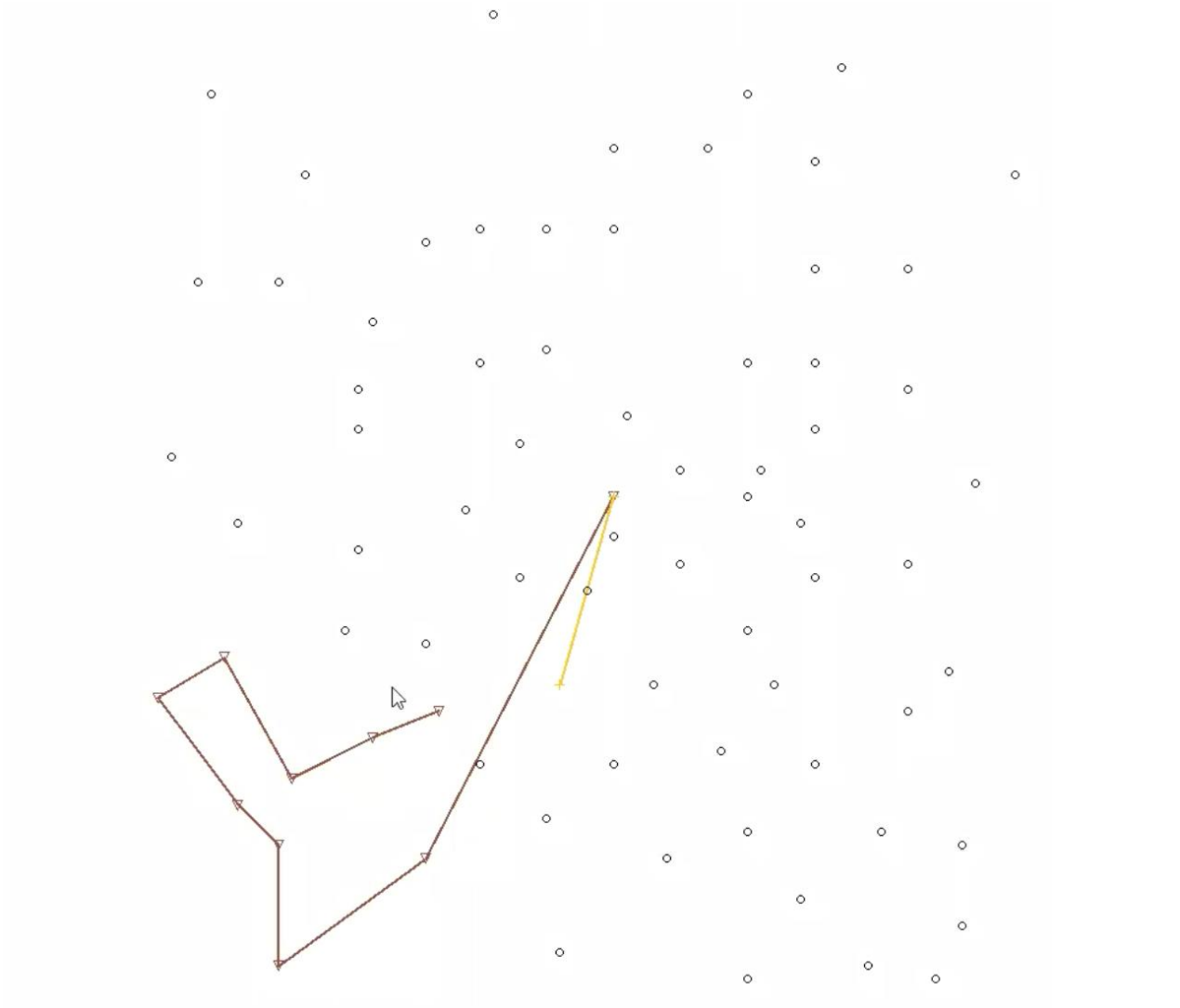

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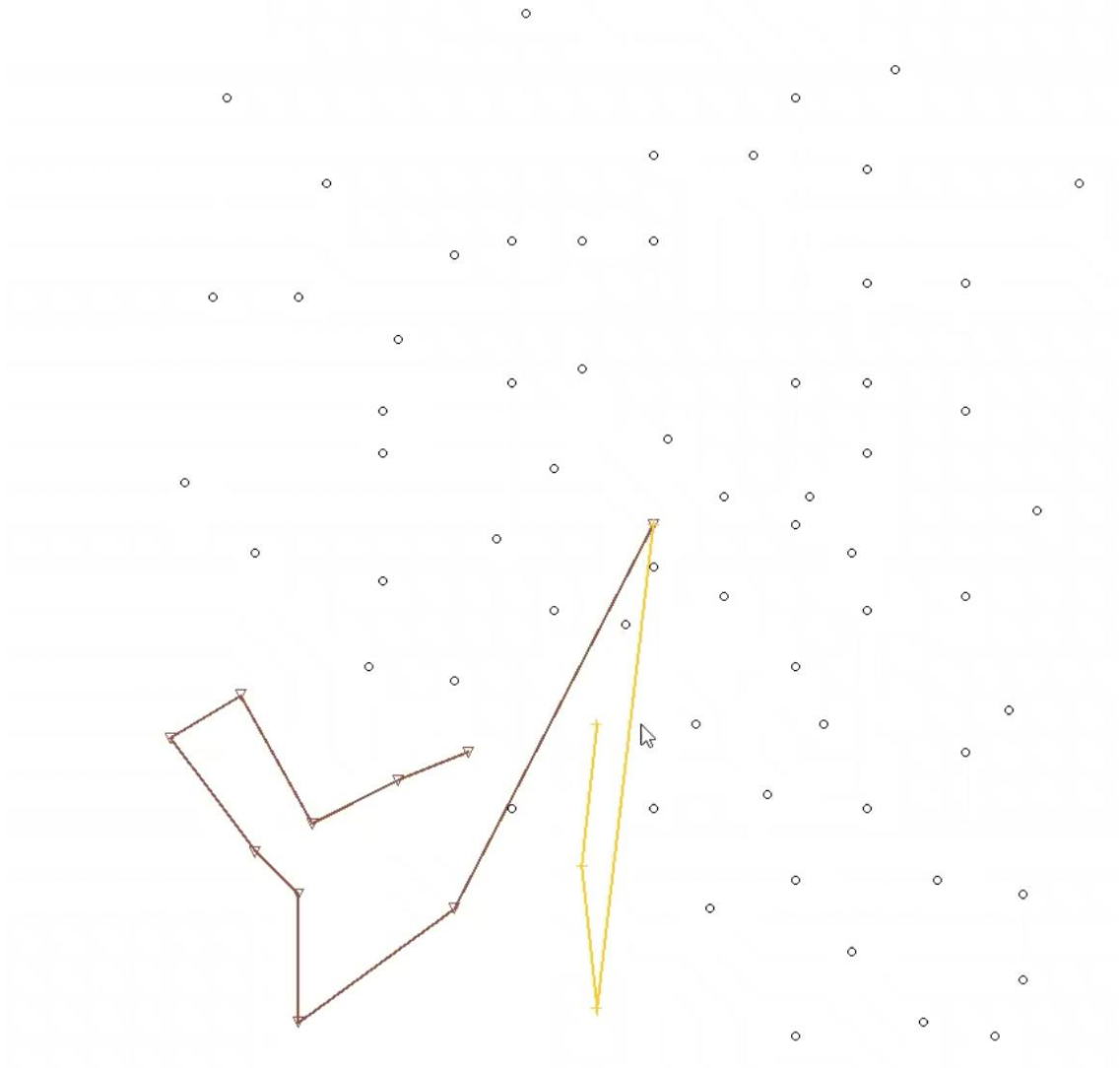

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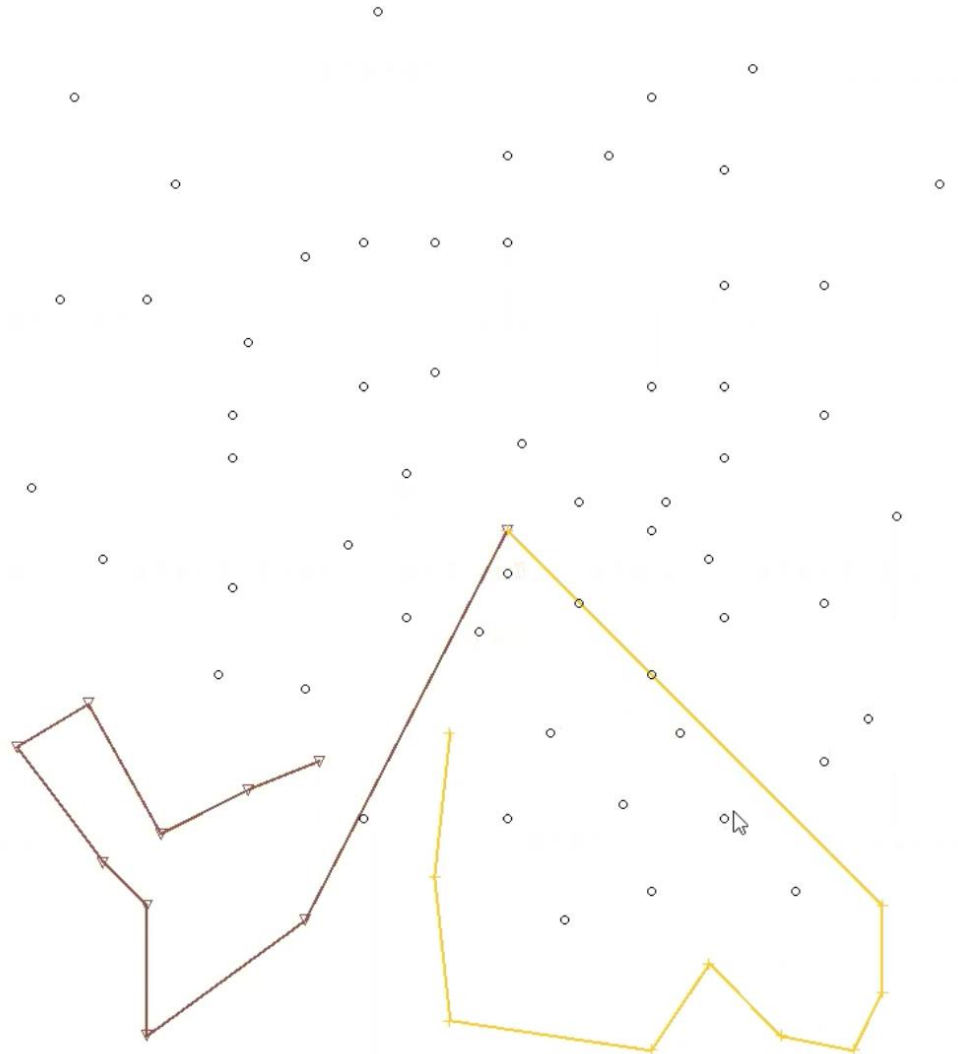

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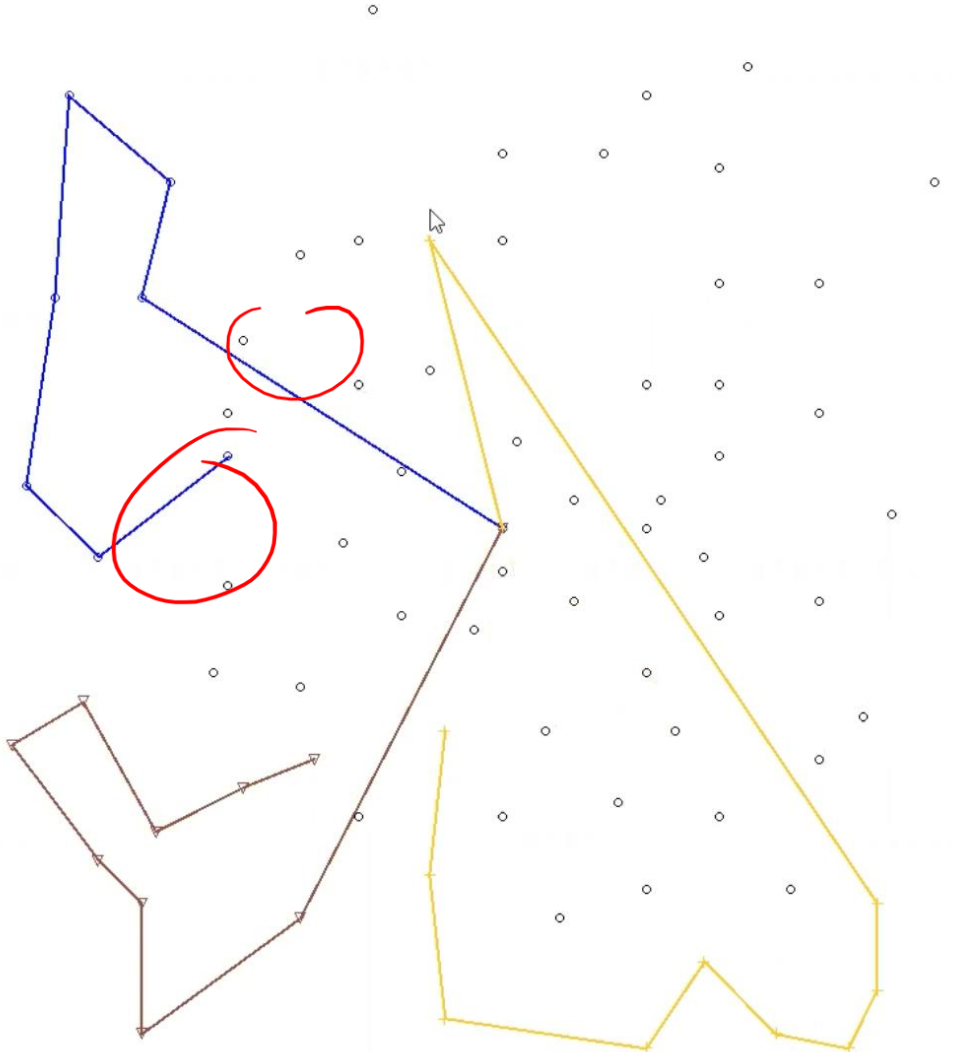

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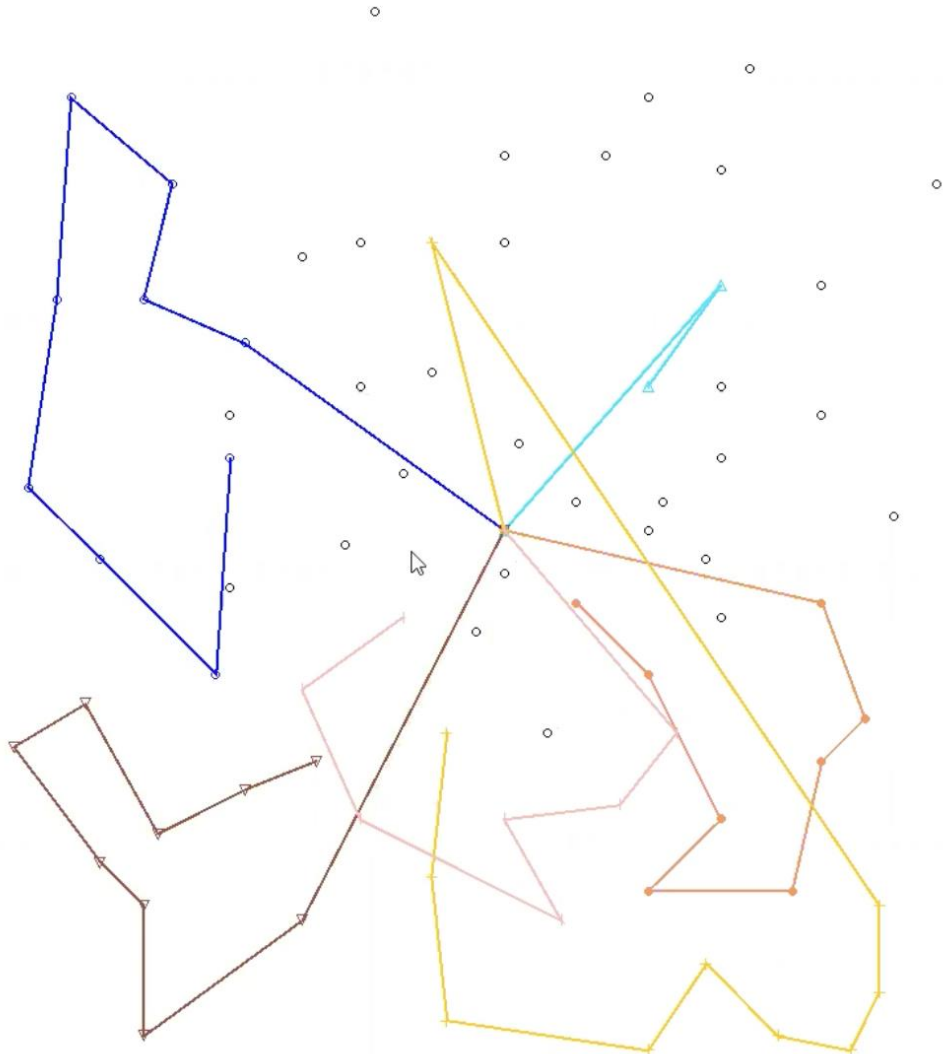

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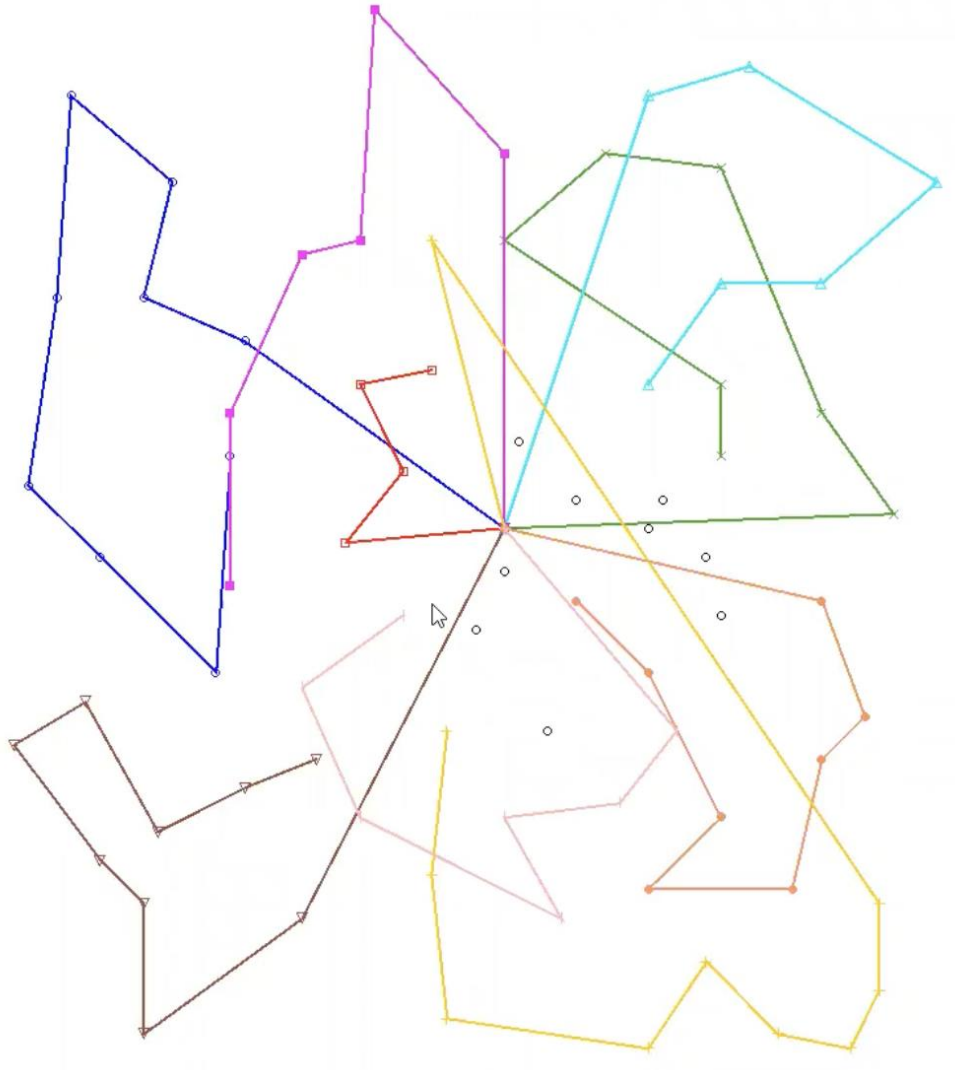

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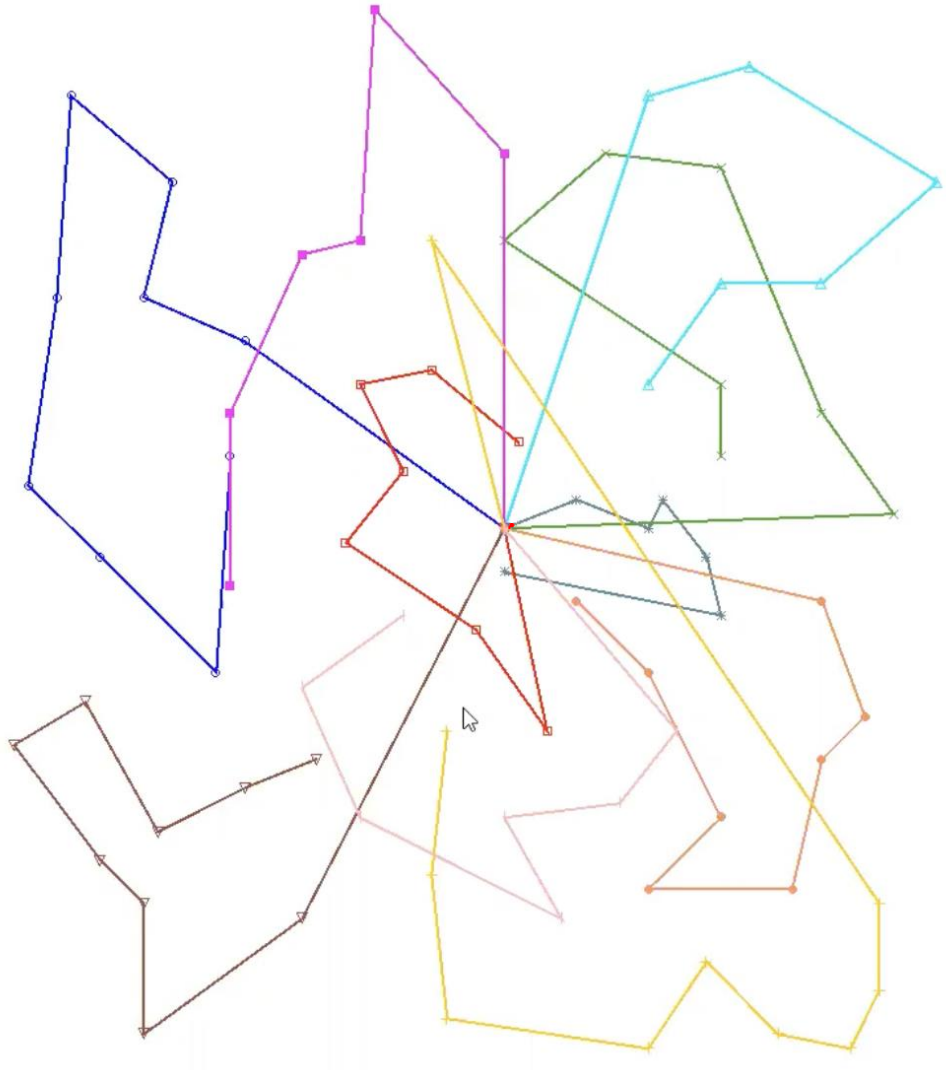
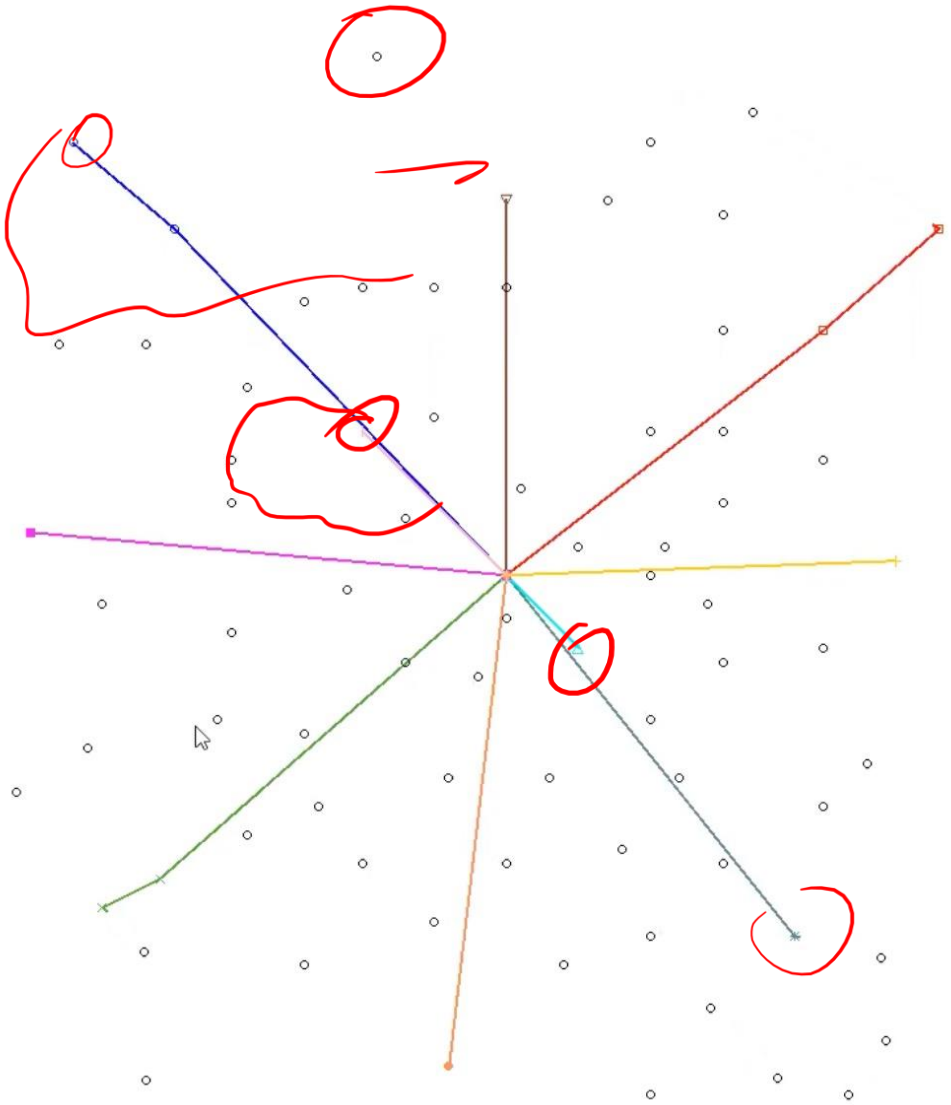


Regret

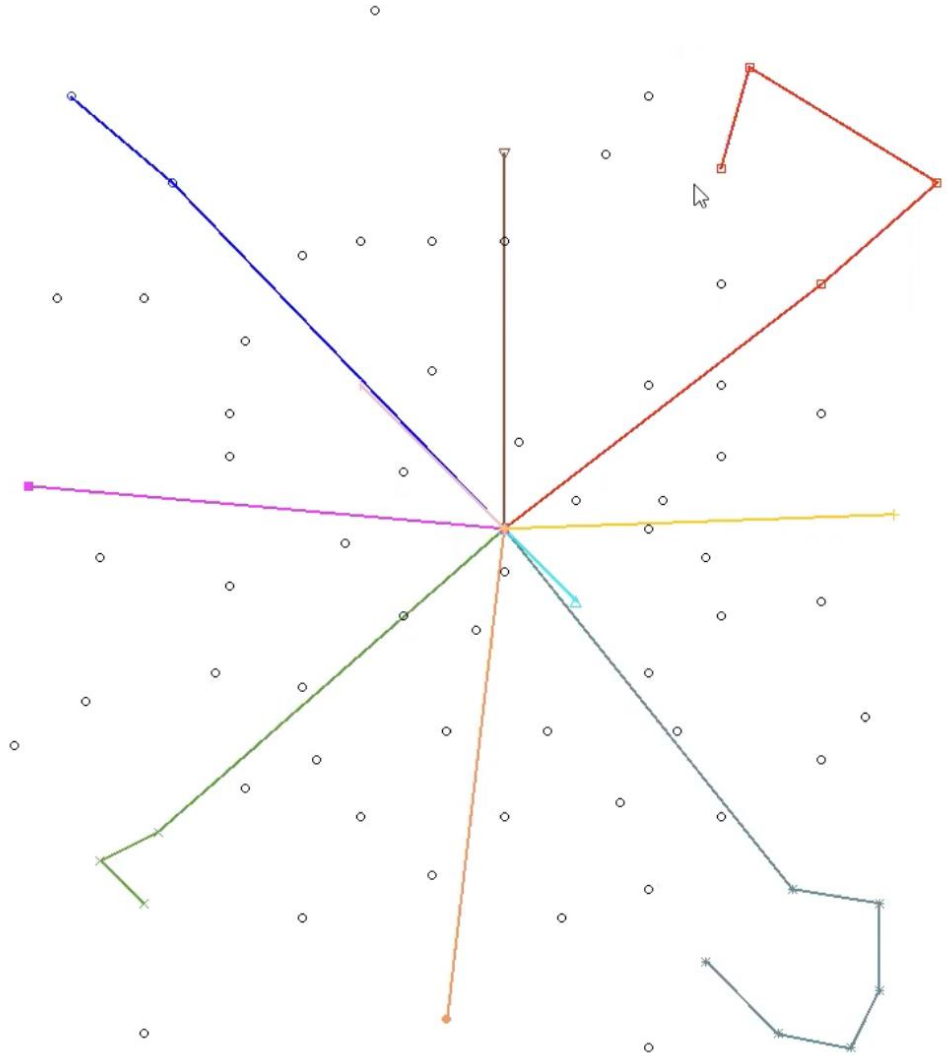
- And a little more structure…
- Regret works best when some structure is already present
- Use Seeds

- Seed 1: The customer with max (dist to depot)
- Seed 2: The customer with max (dist to Seed 1)
- Seed 3: The customer with max (dist to Seed 1 + dist to Seed 2)
- Seed 4: The customer with max (dist to Seed 1 + dist to Seed 2 + dist to Seed 3)
- Etc

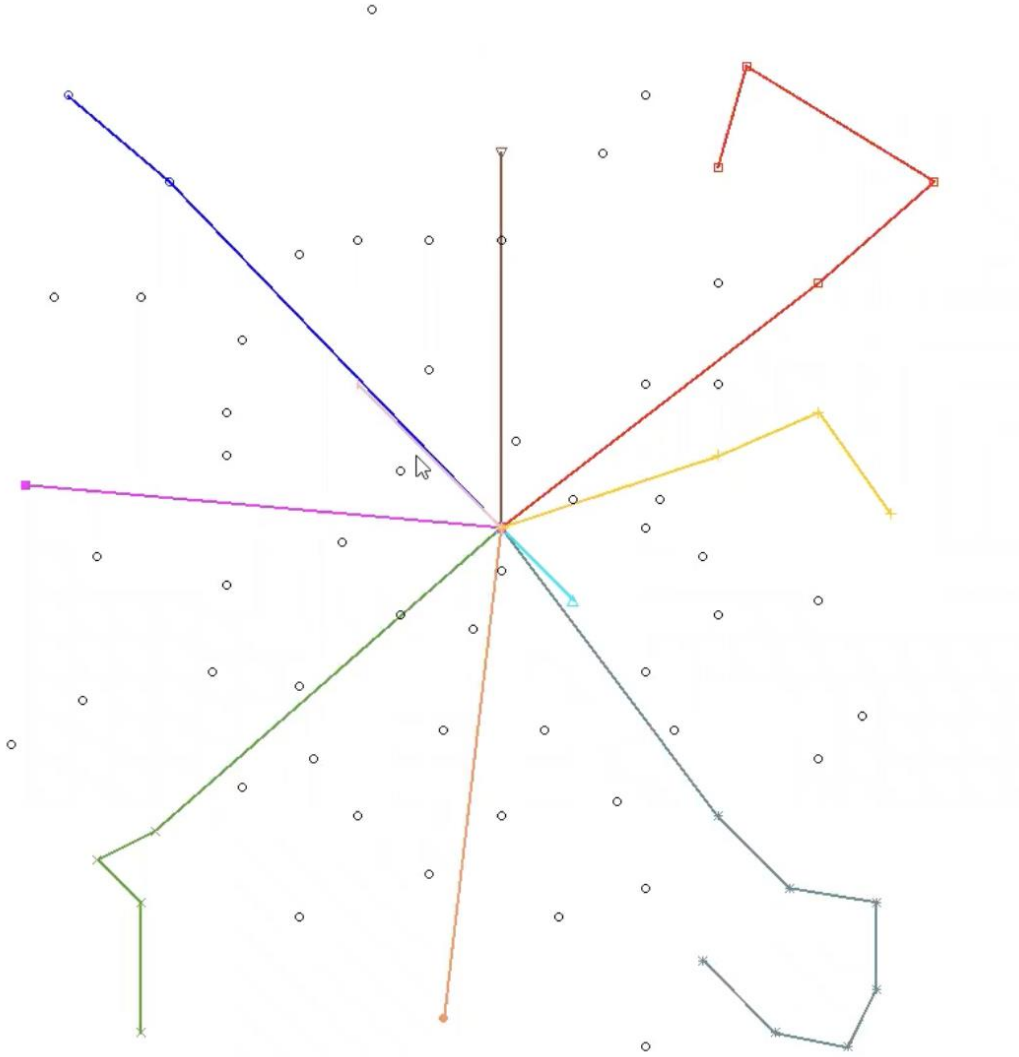
Regret with seeds



Regret with seeds



Regret with seeds



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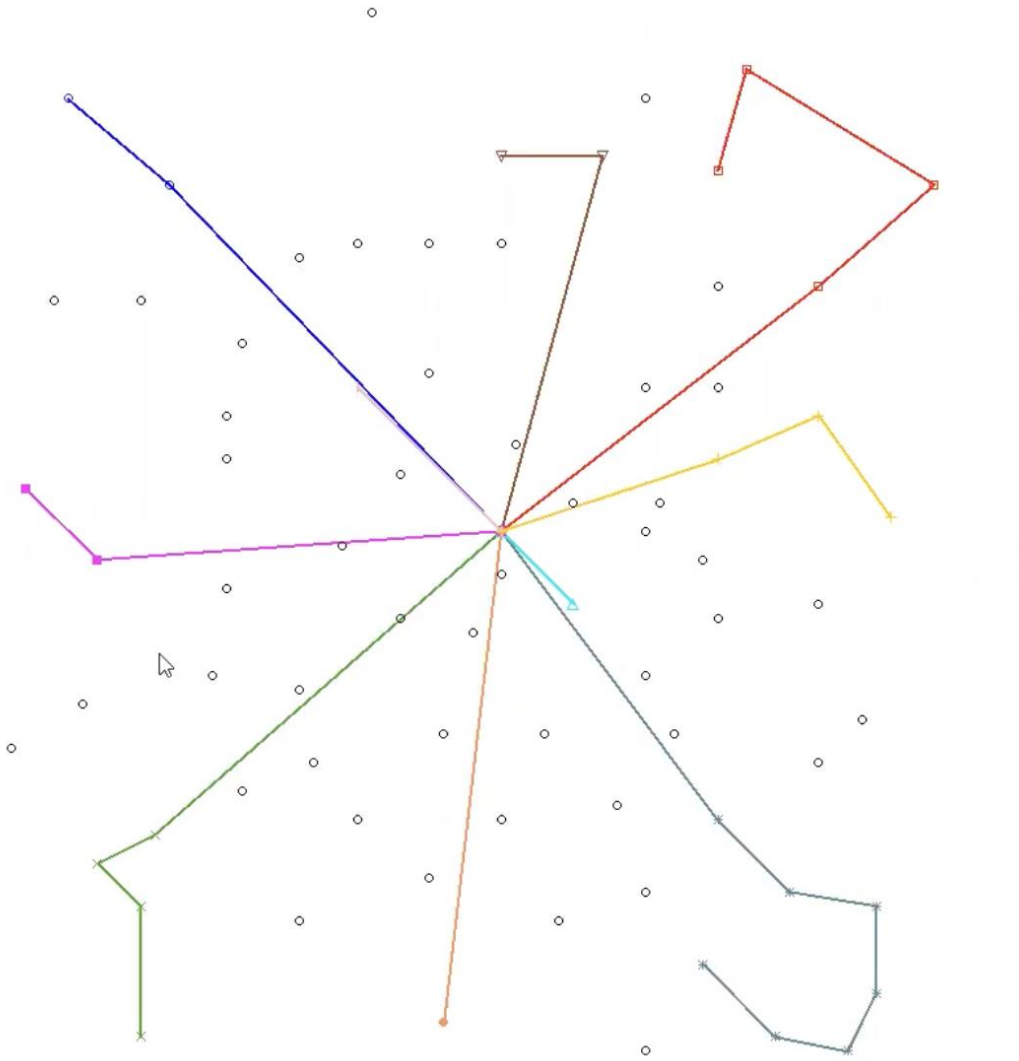

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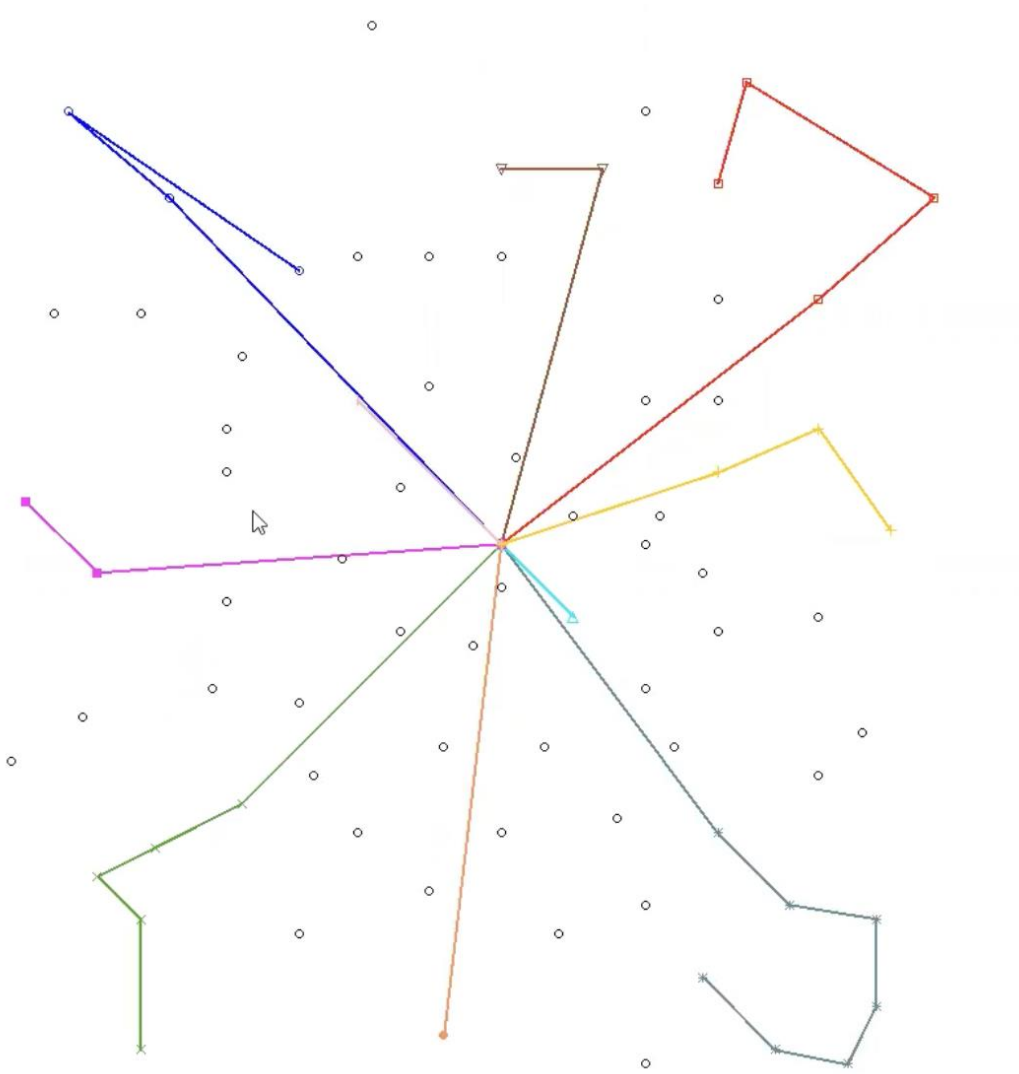

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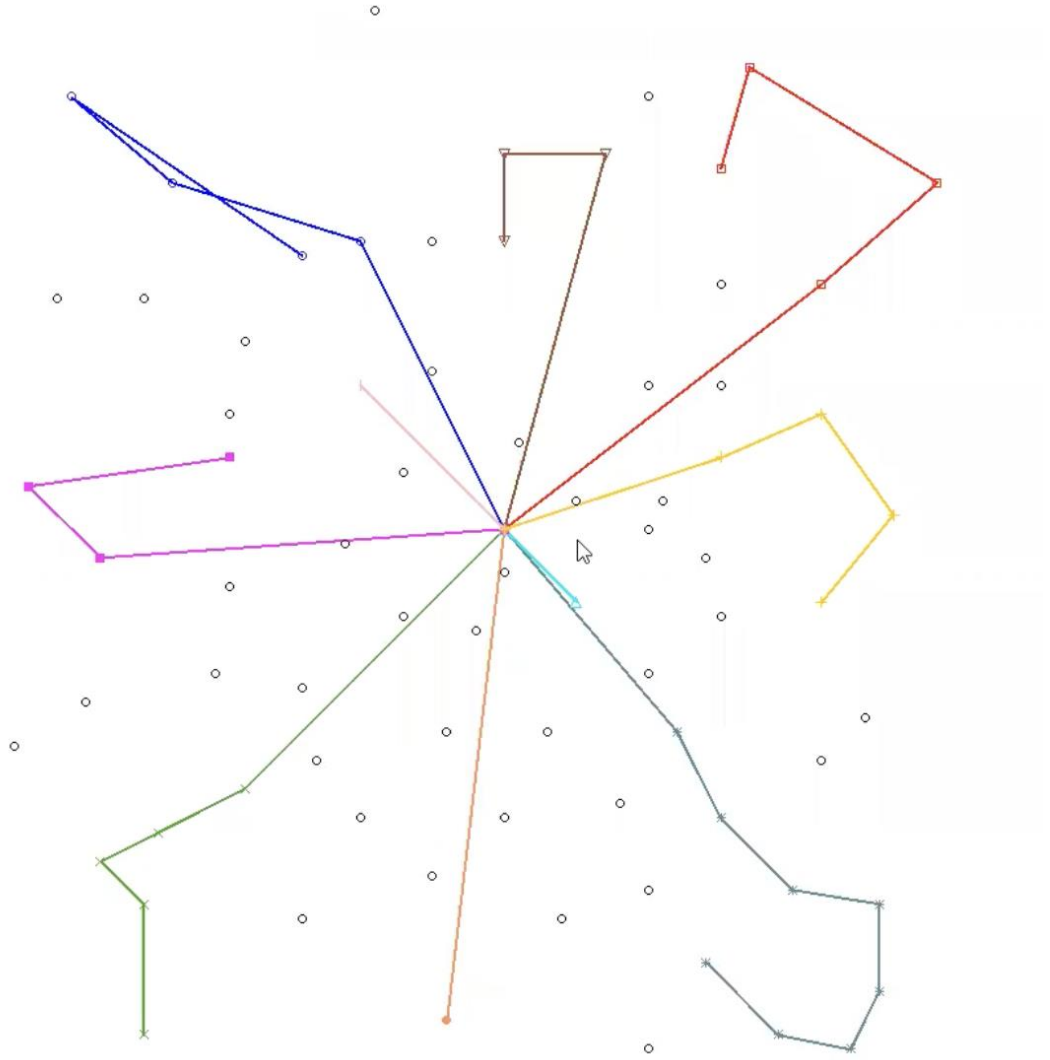

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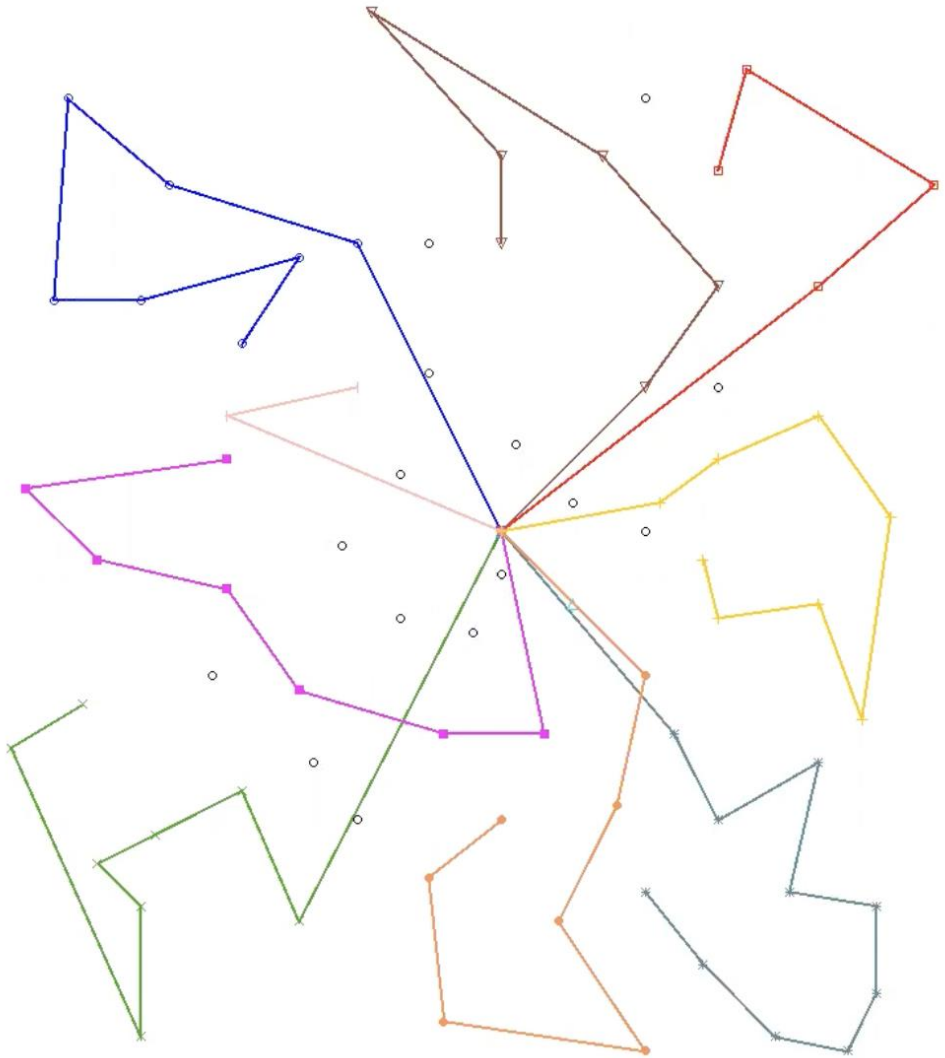
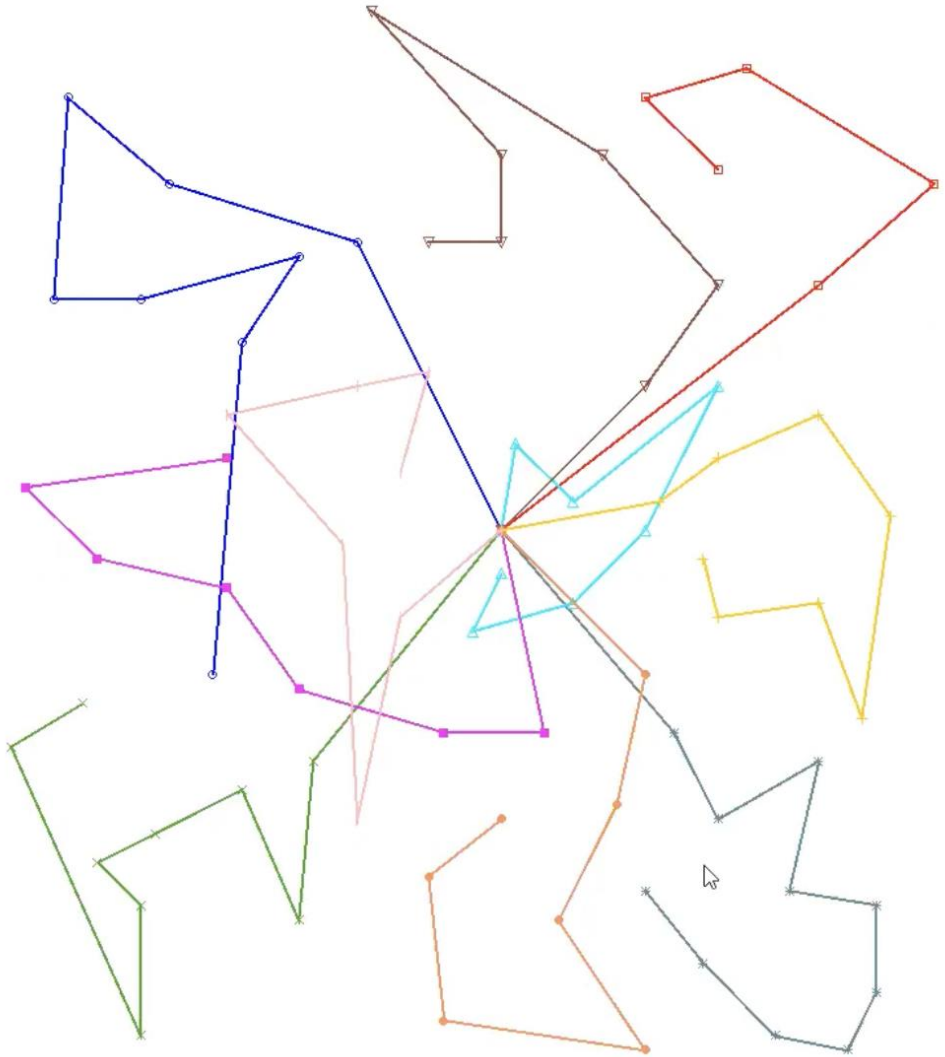
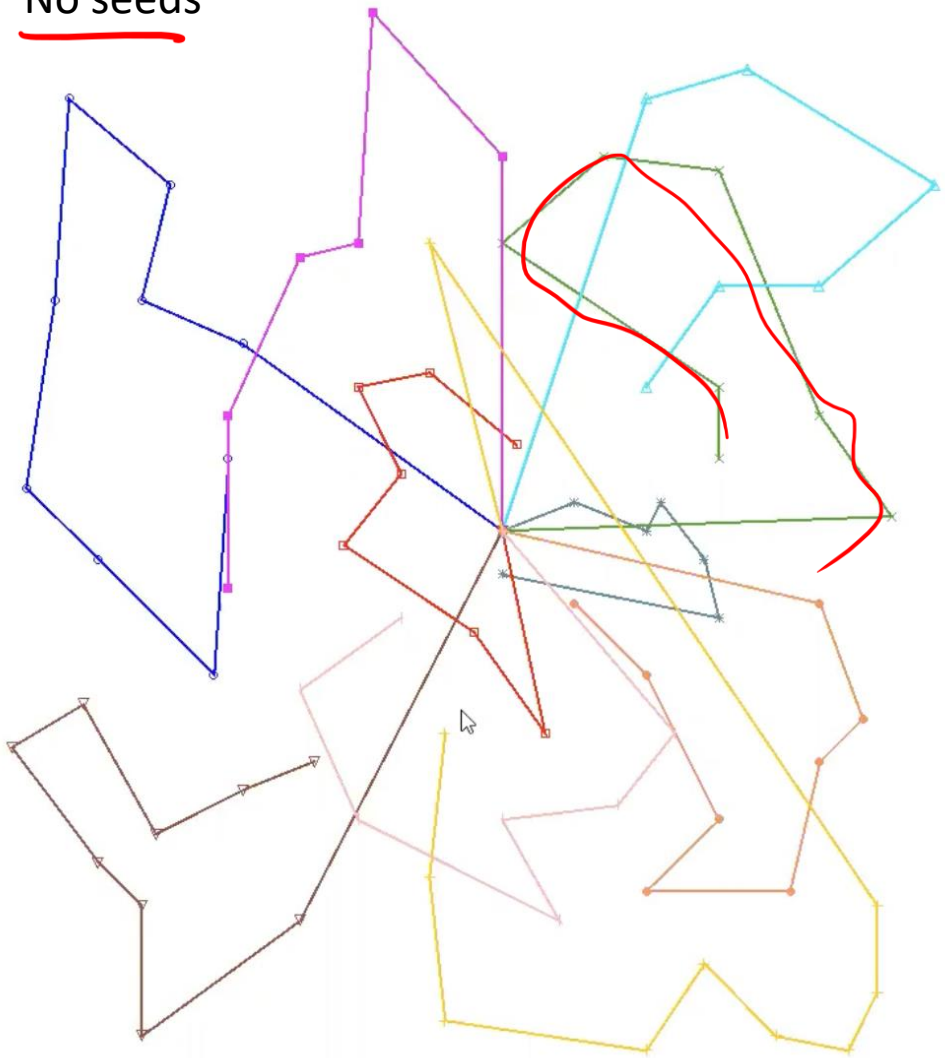


Regret with seeds

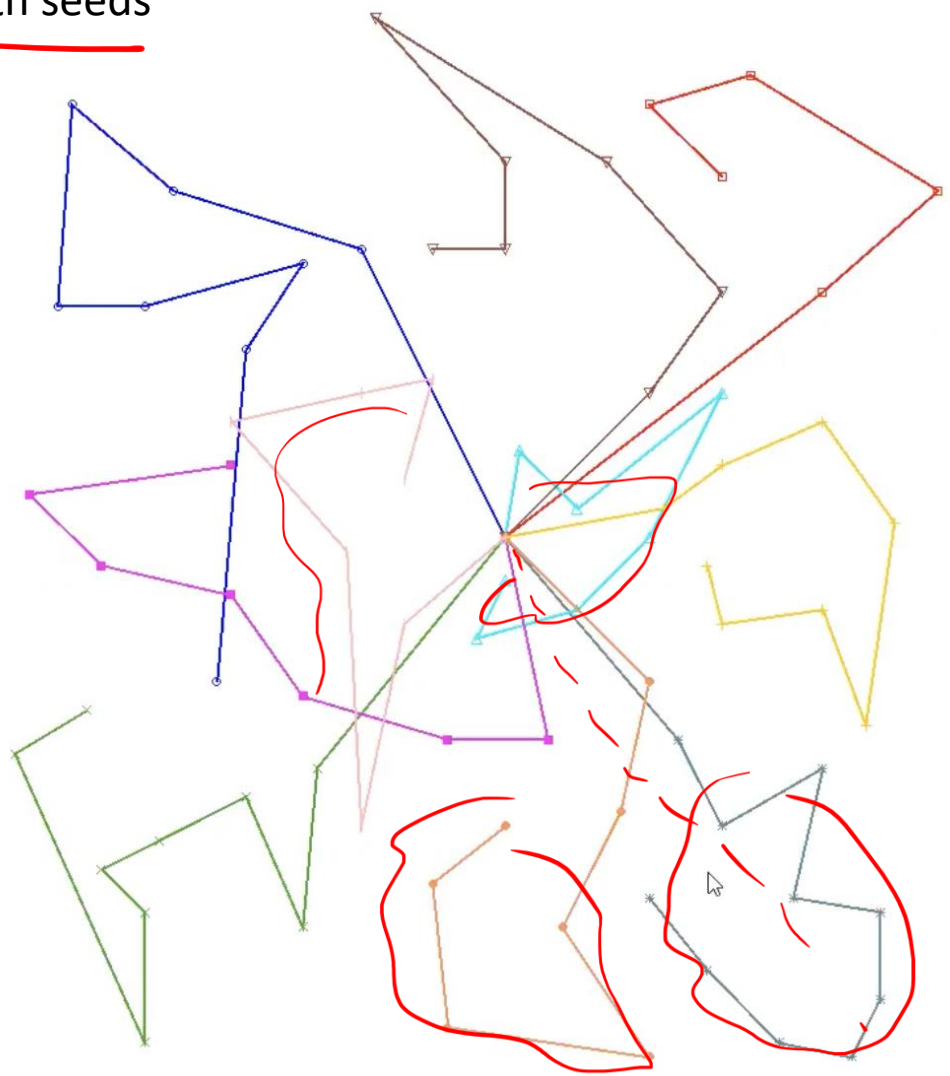


Regret with seeds

No seeds



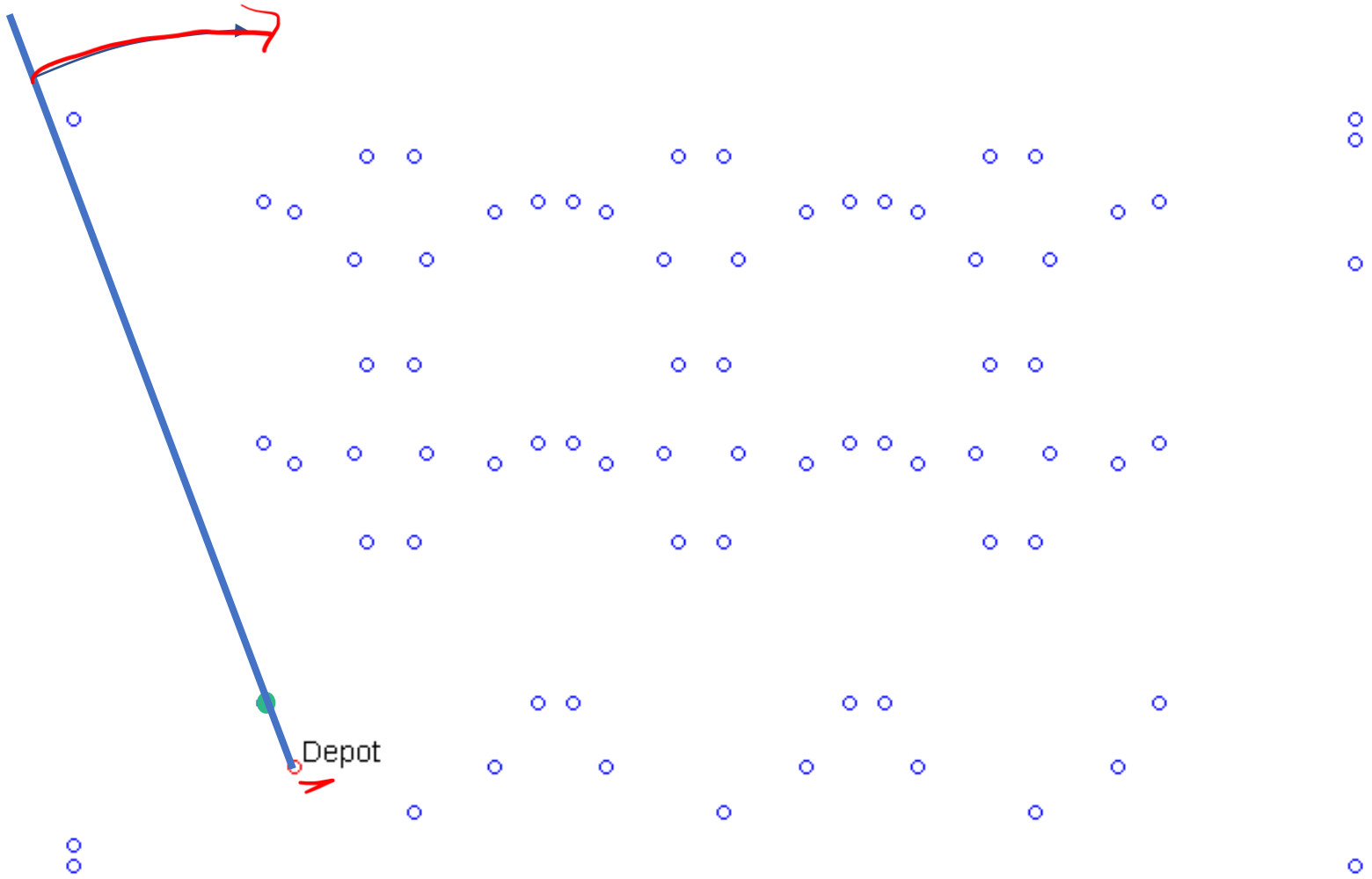
With seeds



Other construction methods (for the VRP)

Bespoke methods

- Eg “Sweep”



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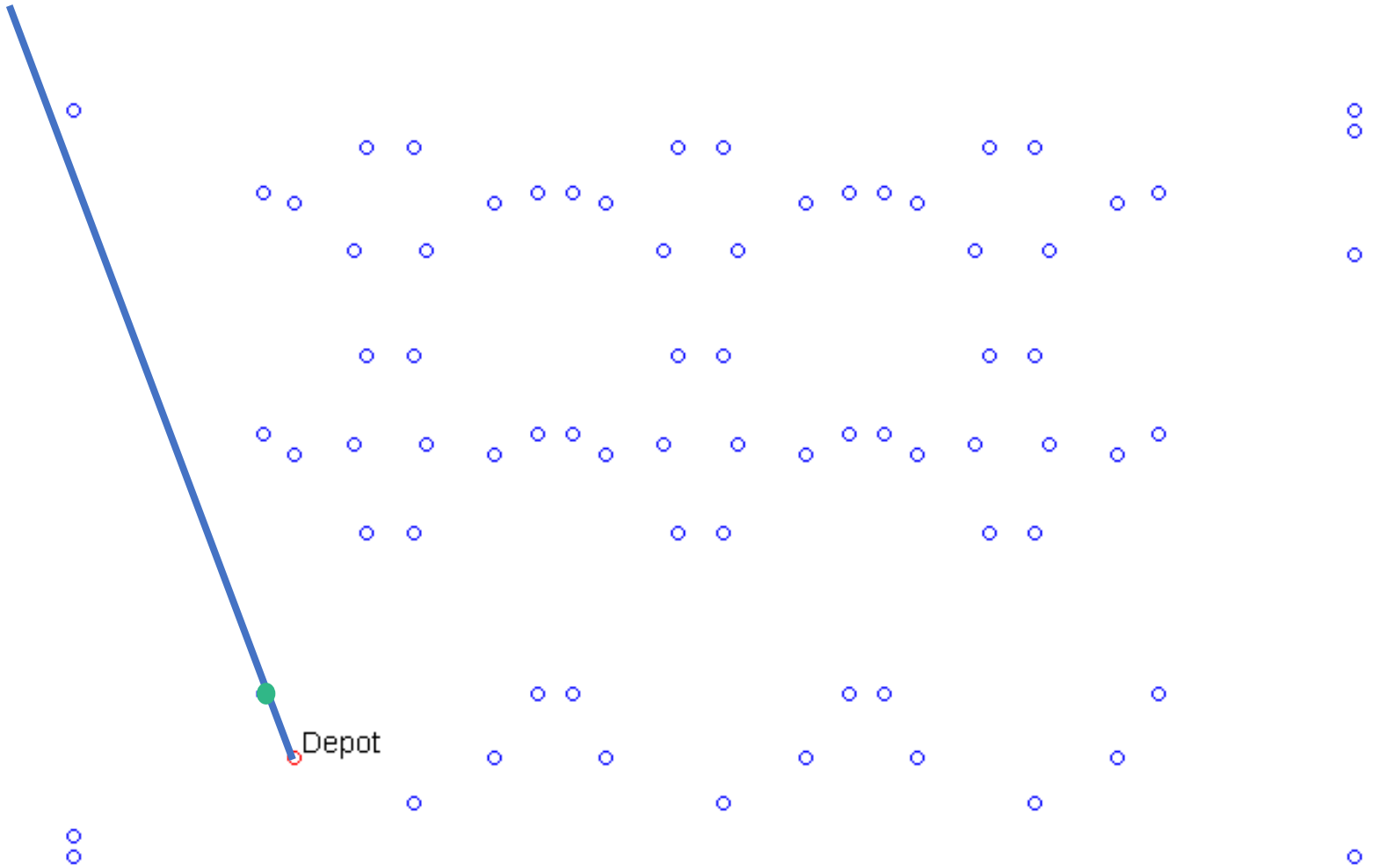

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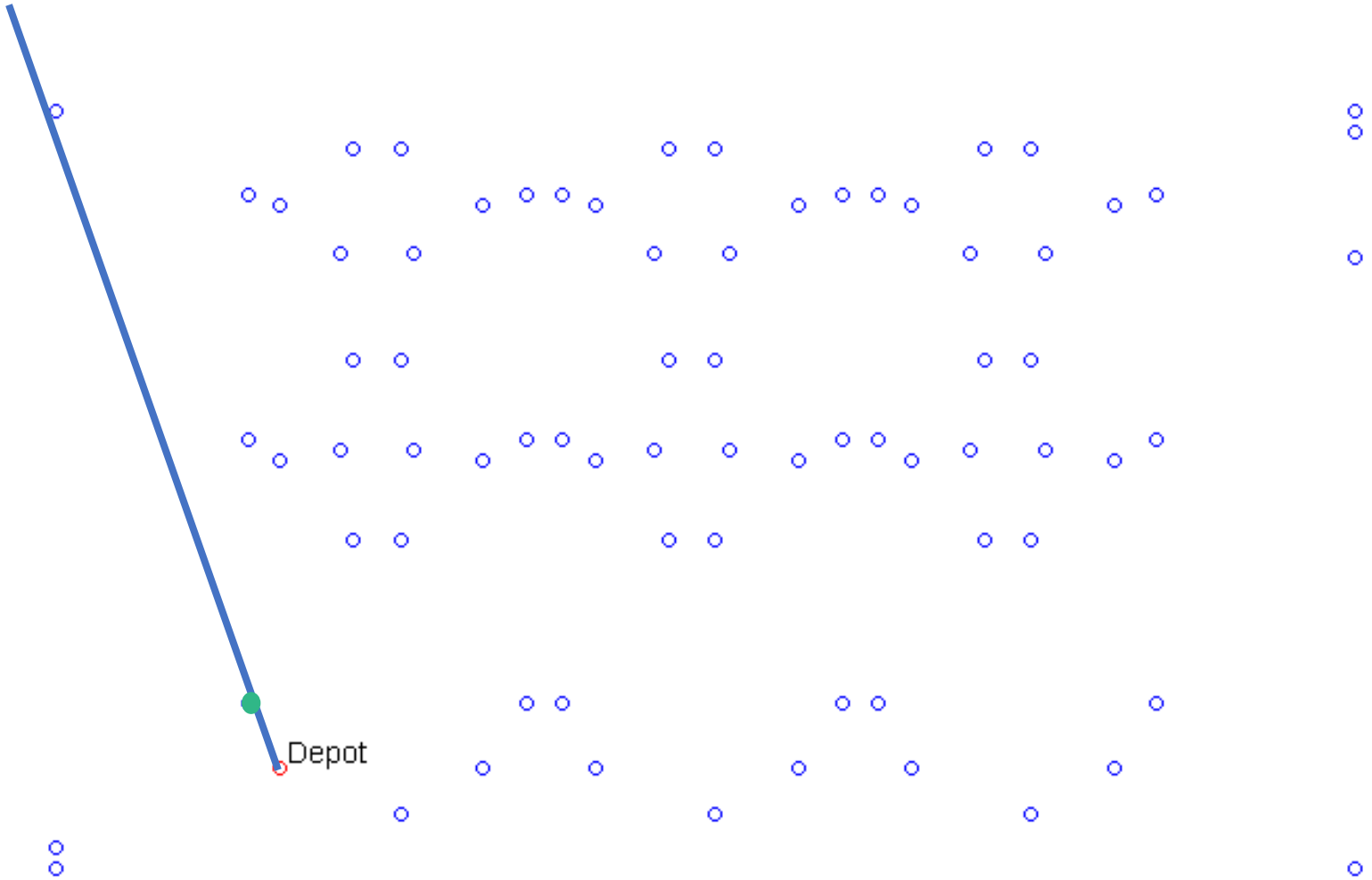

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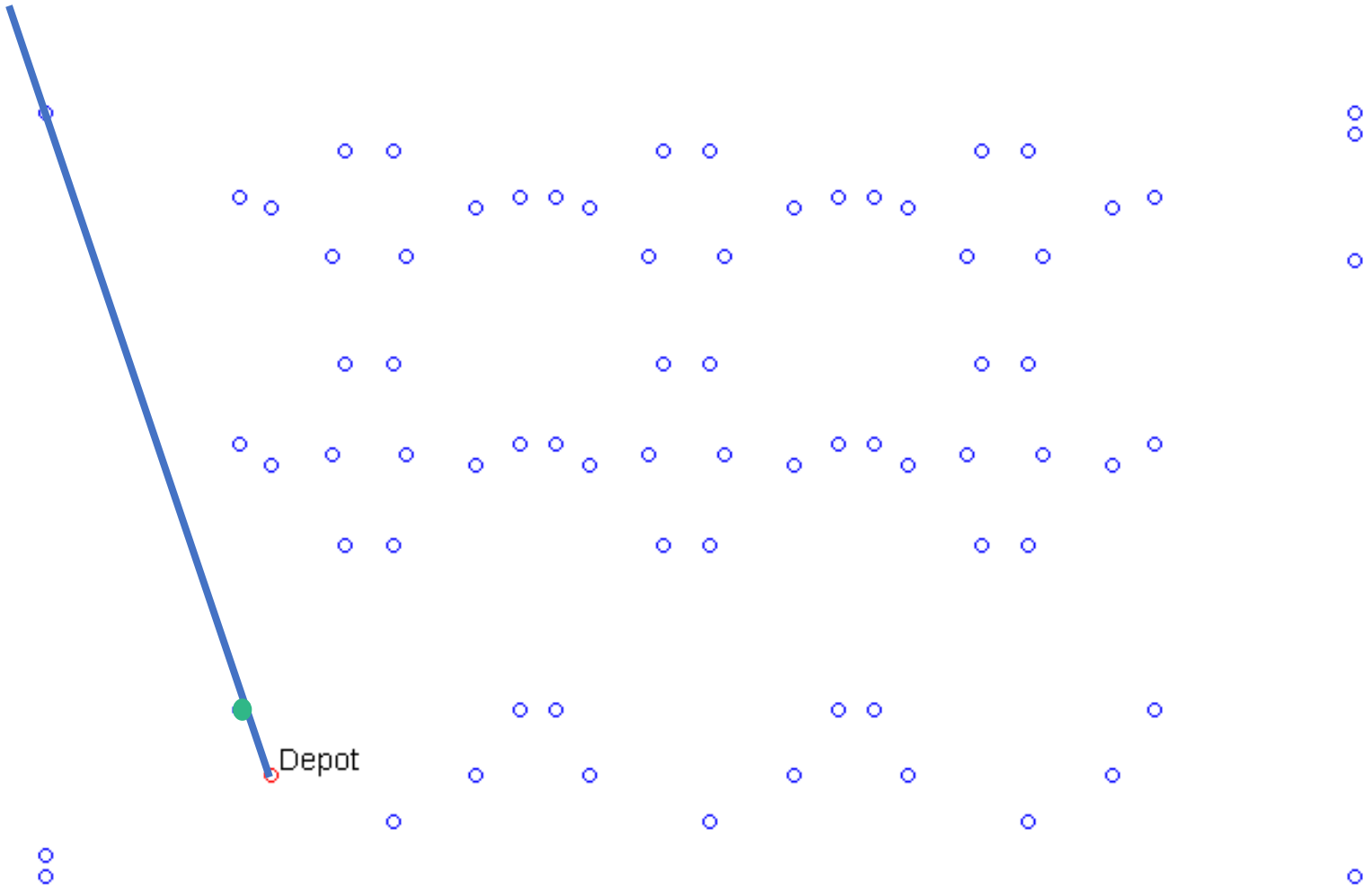
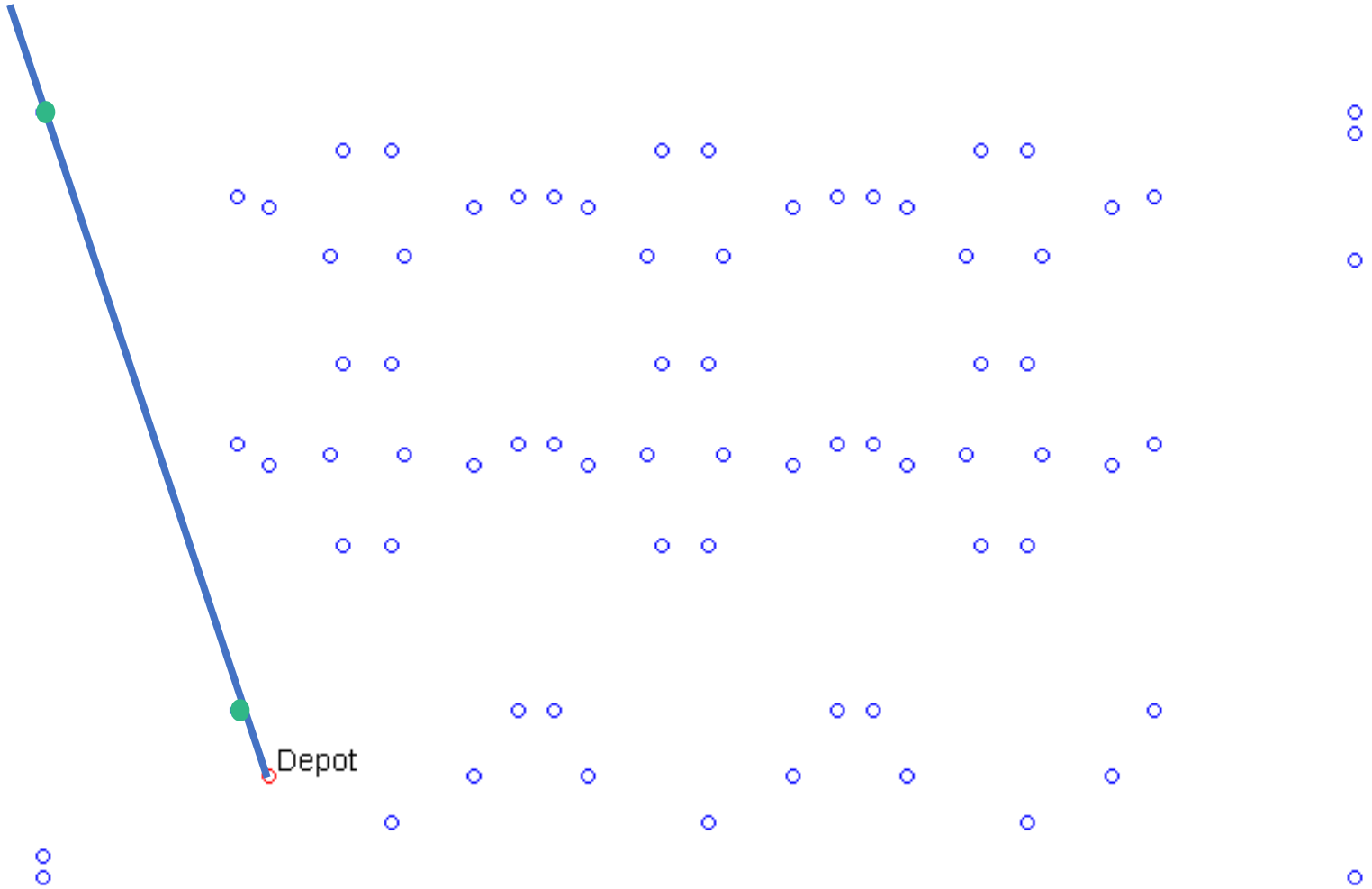
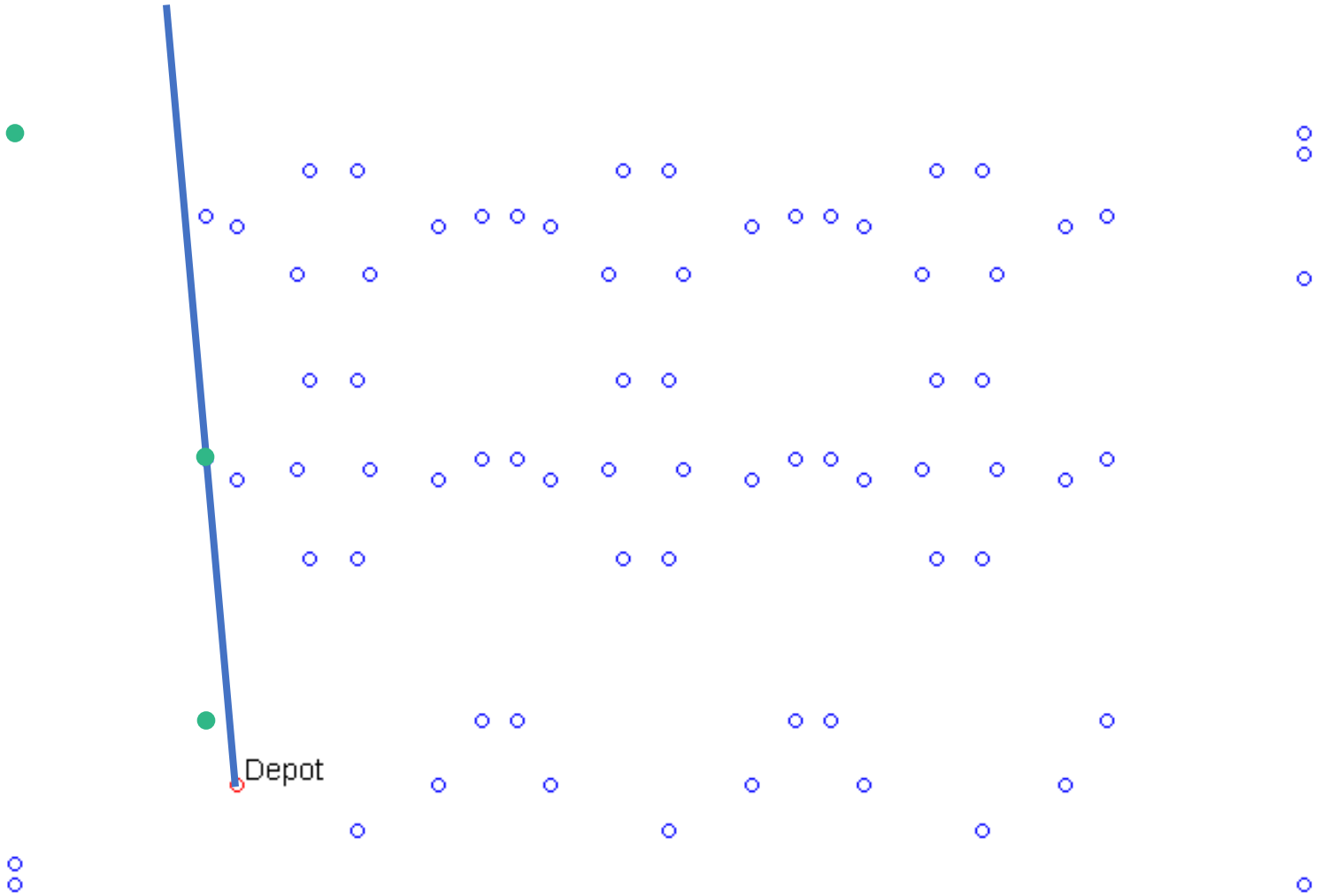


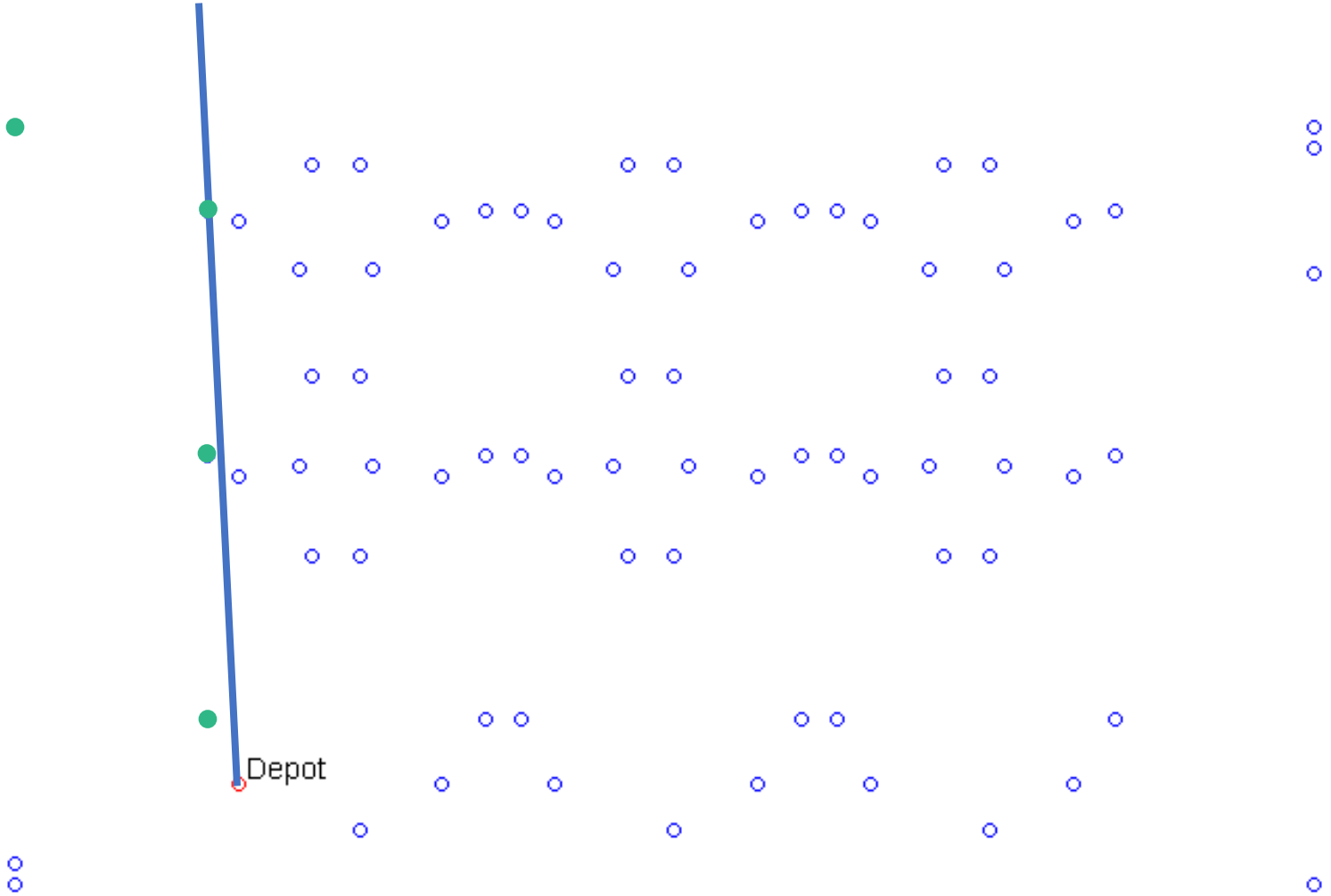
Other construction methods (for the VRP)



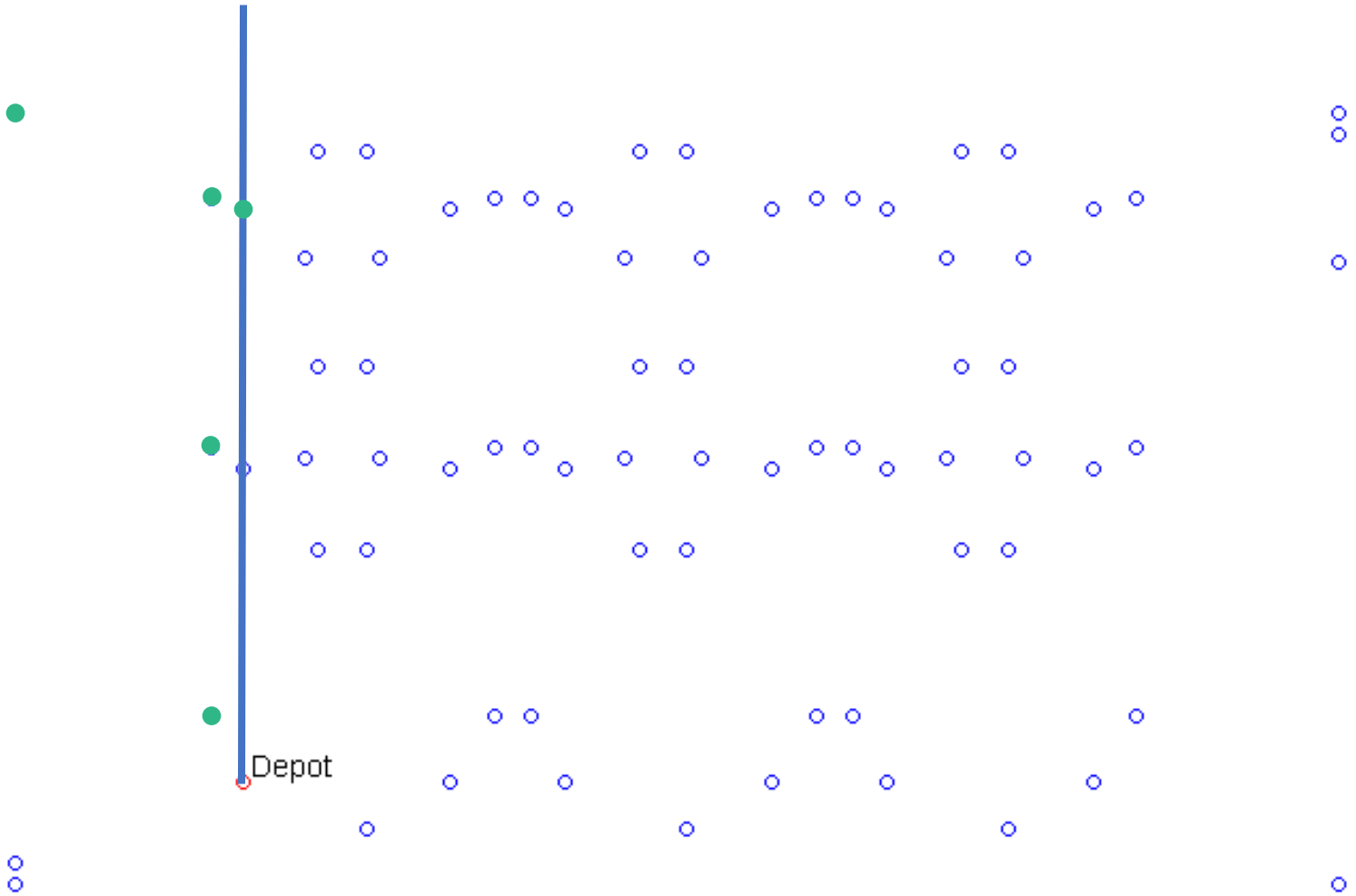
Other construction methods (for the VRP)



Other construction methods (for the VRP)



Other construction methods (for the VRP)



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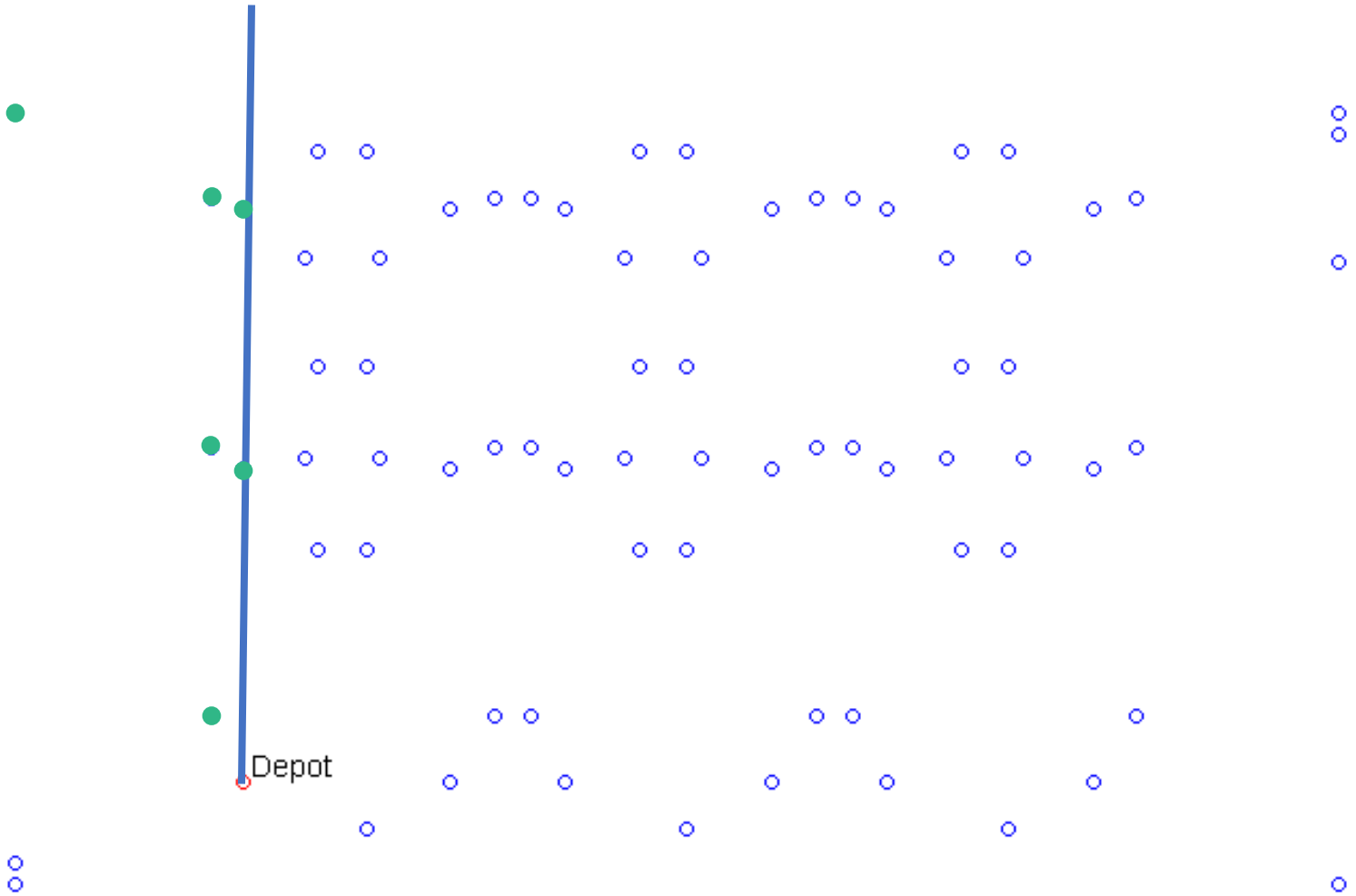
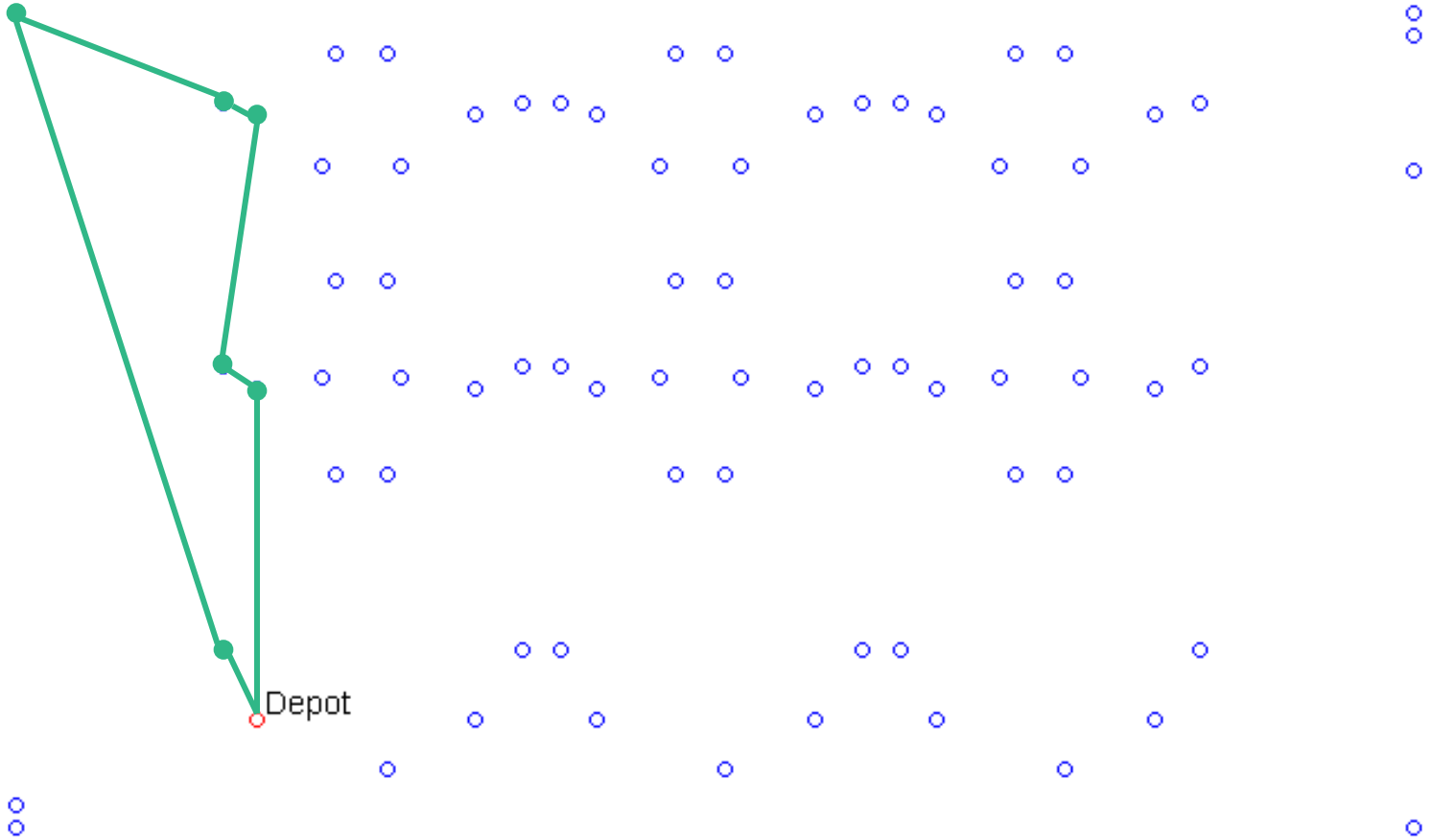


Other construction methods (for the VRP)



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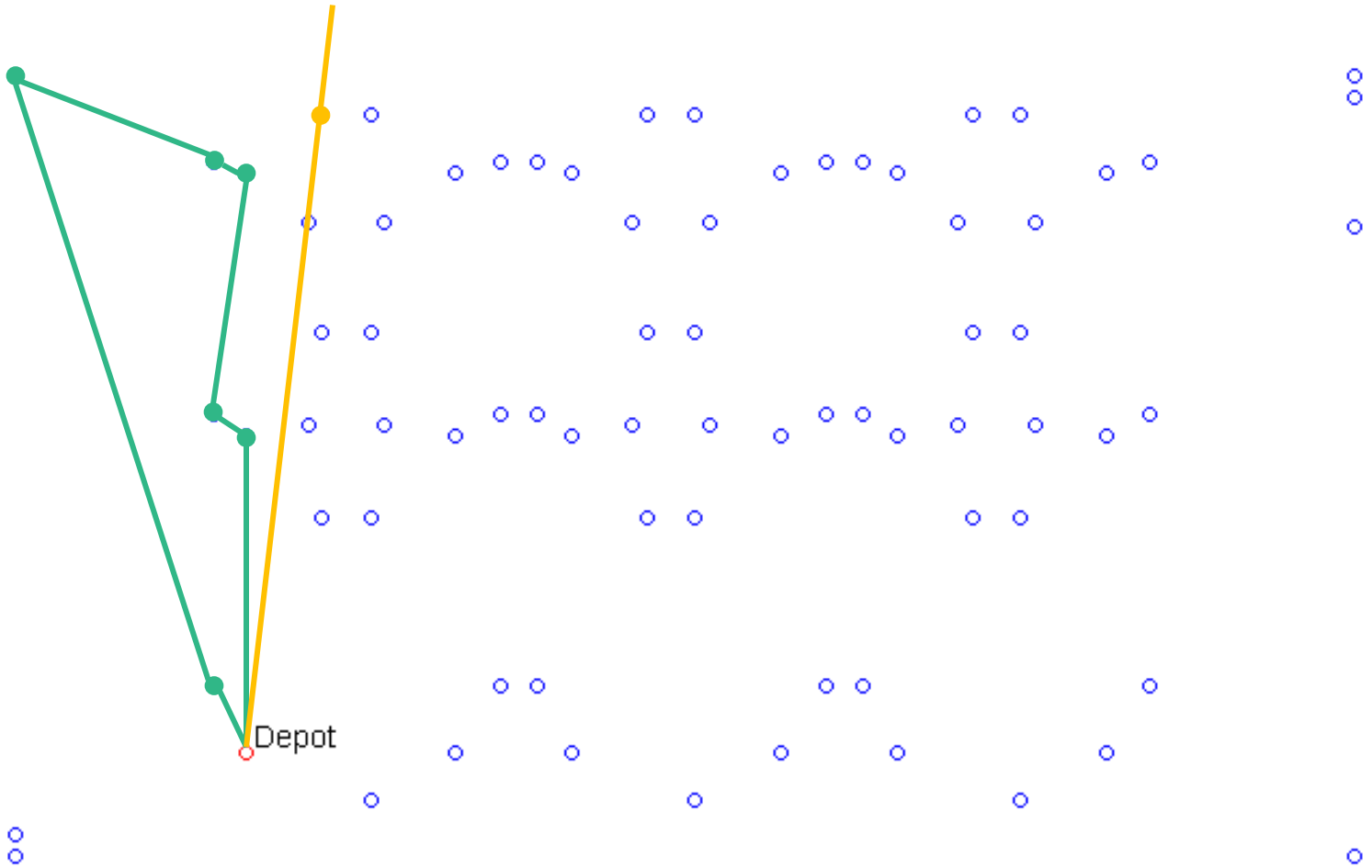
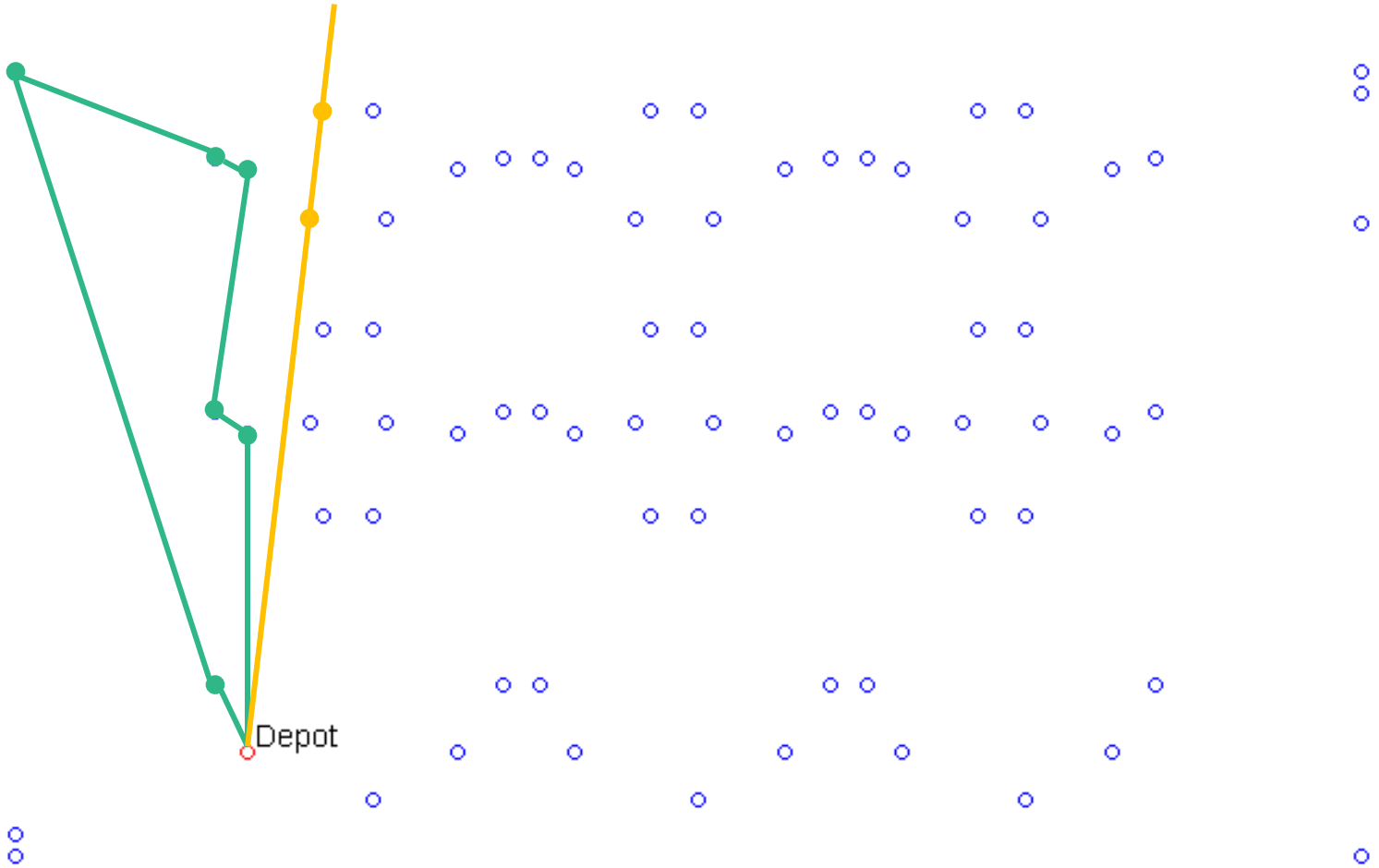
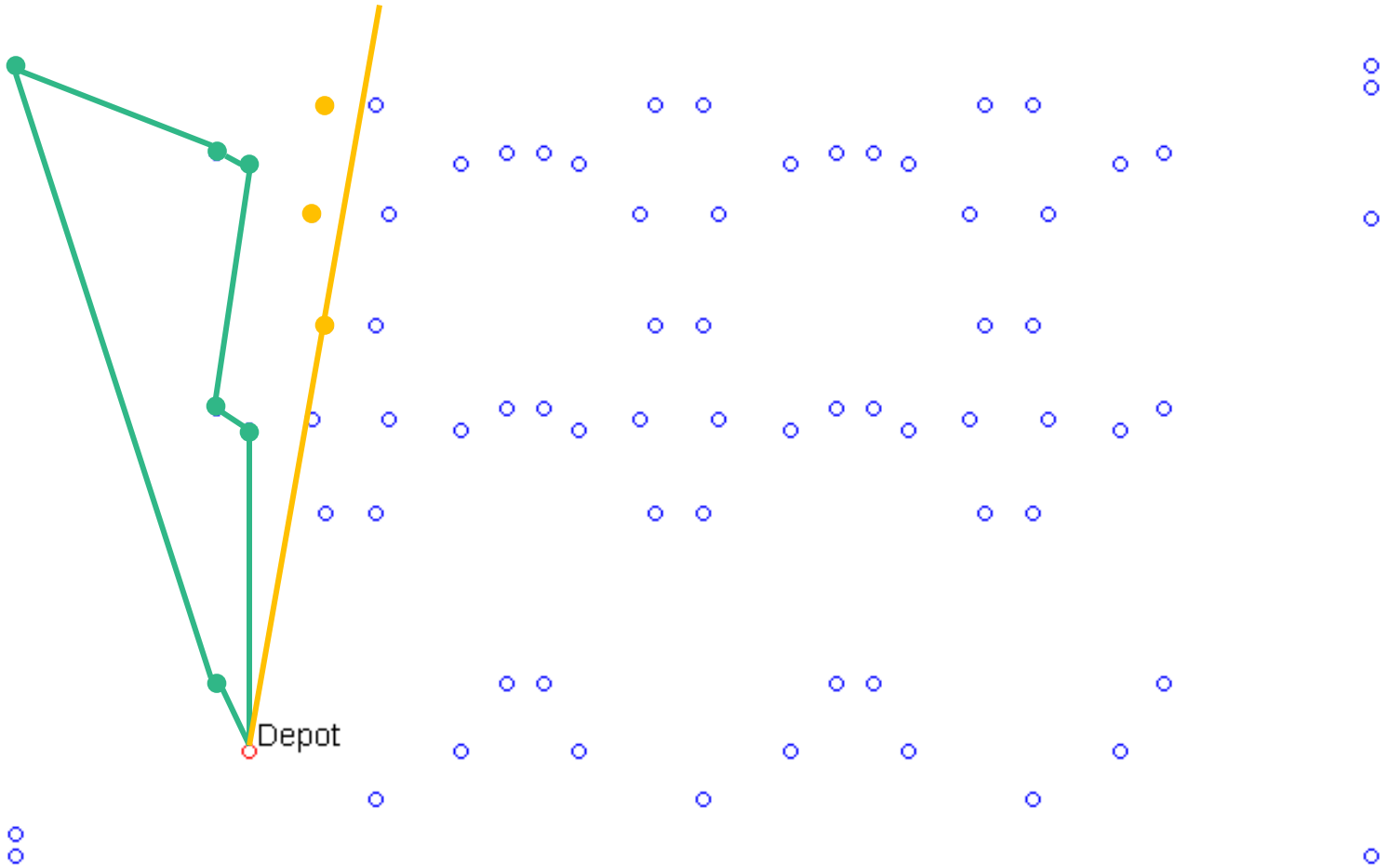


Other construction methods (for the VRP)



Other construction methods (for the VRP)



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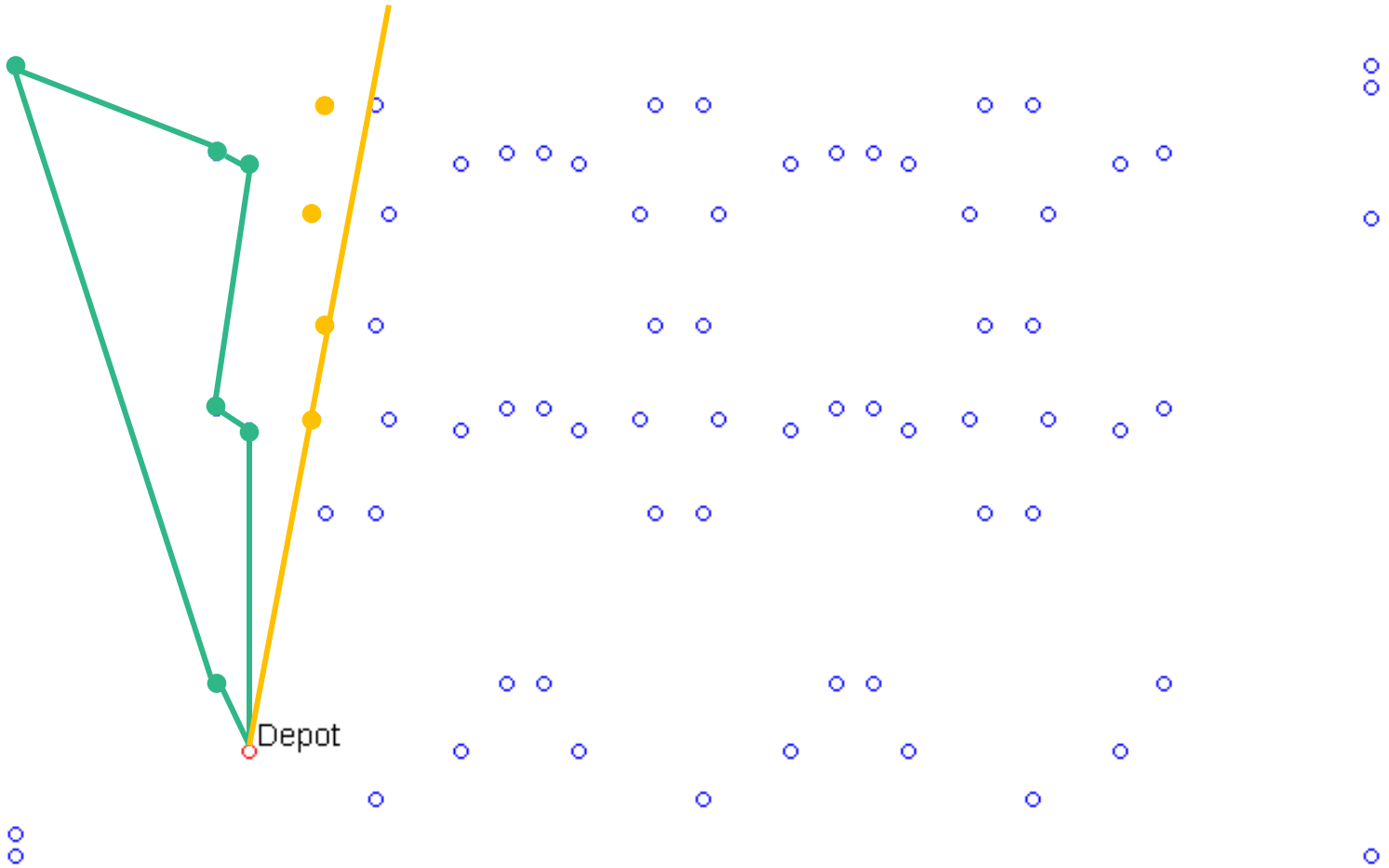

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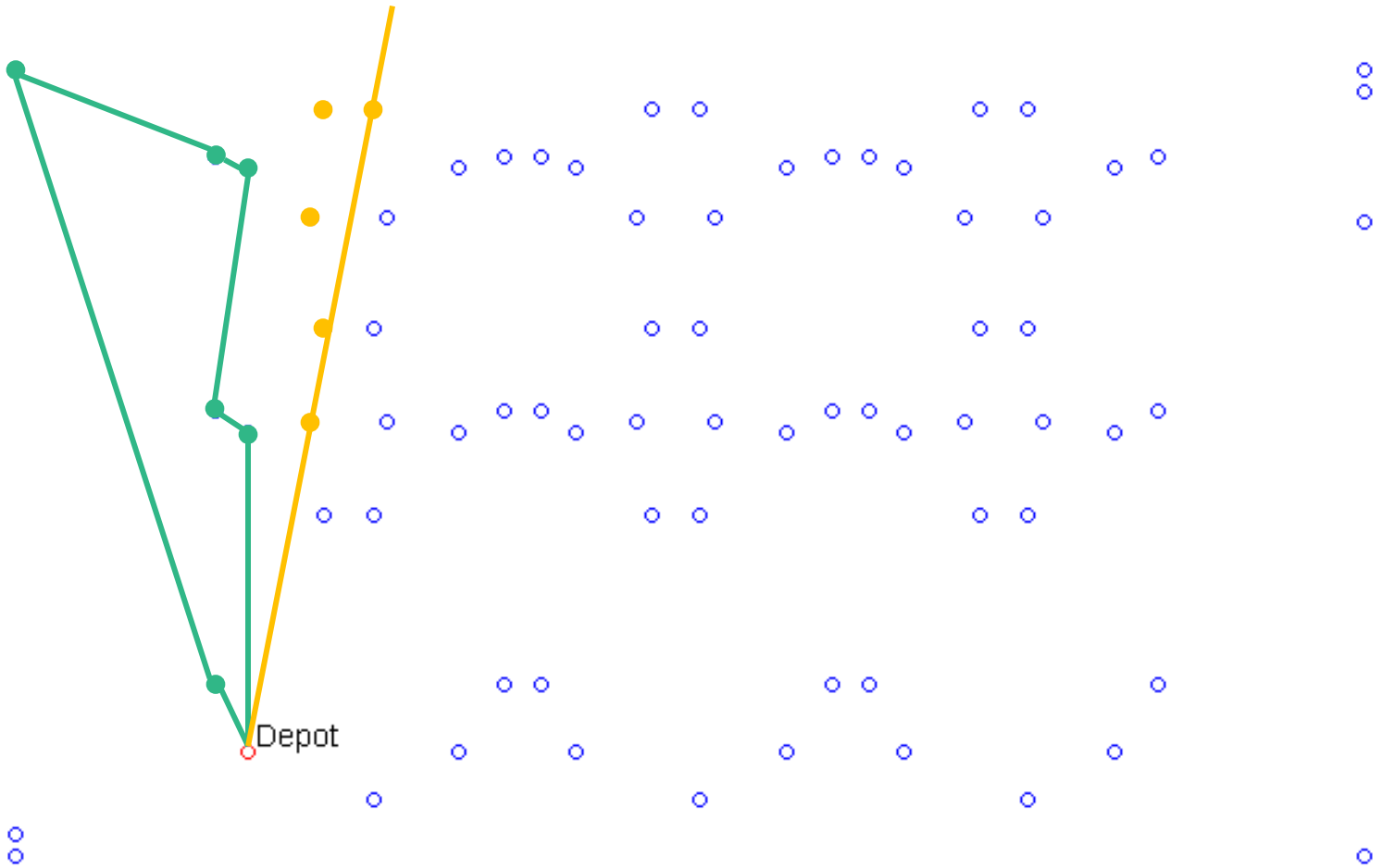

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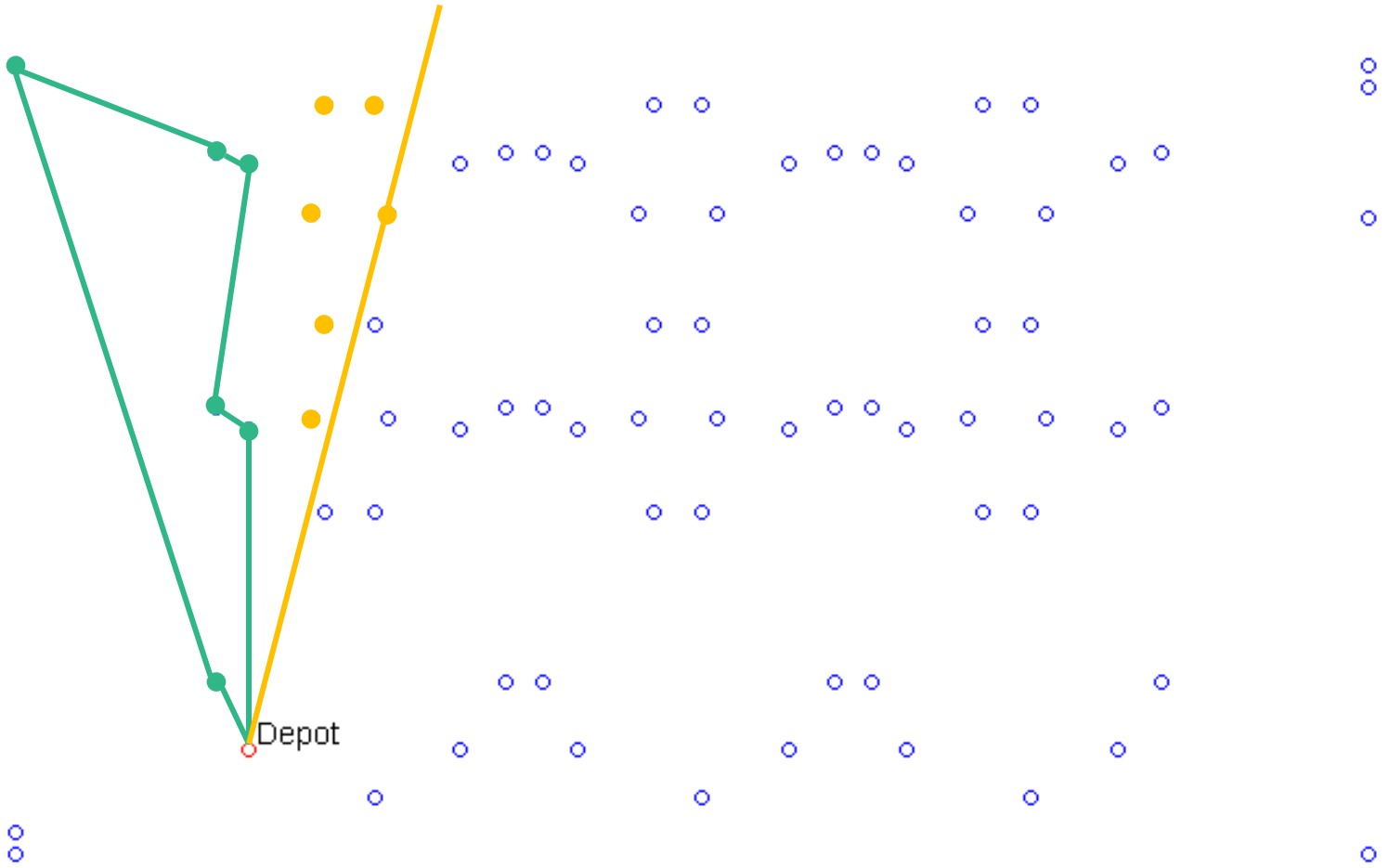

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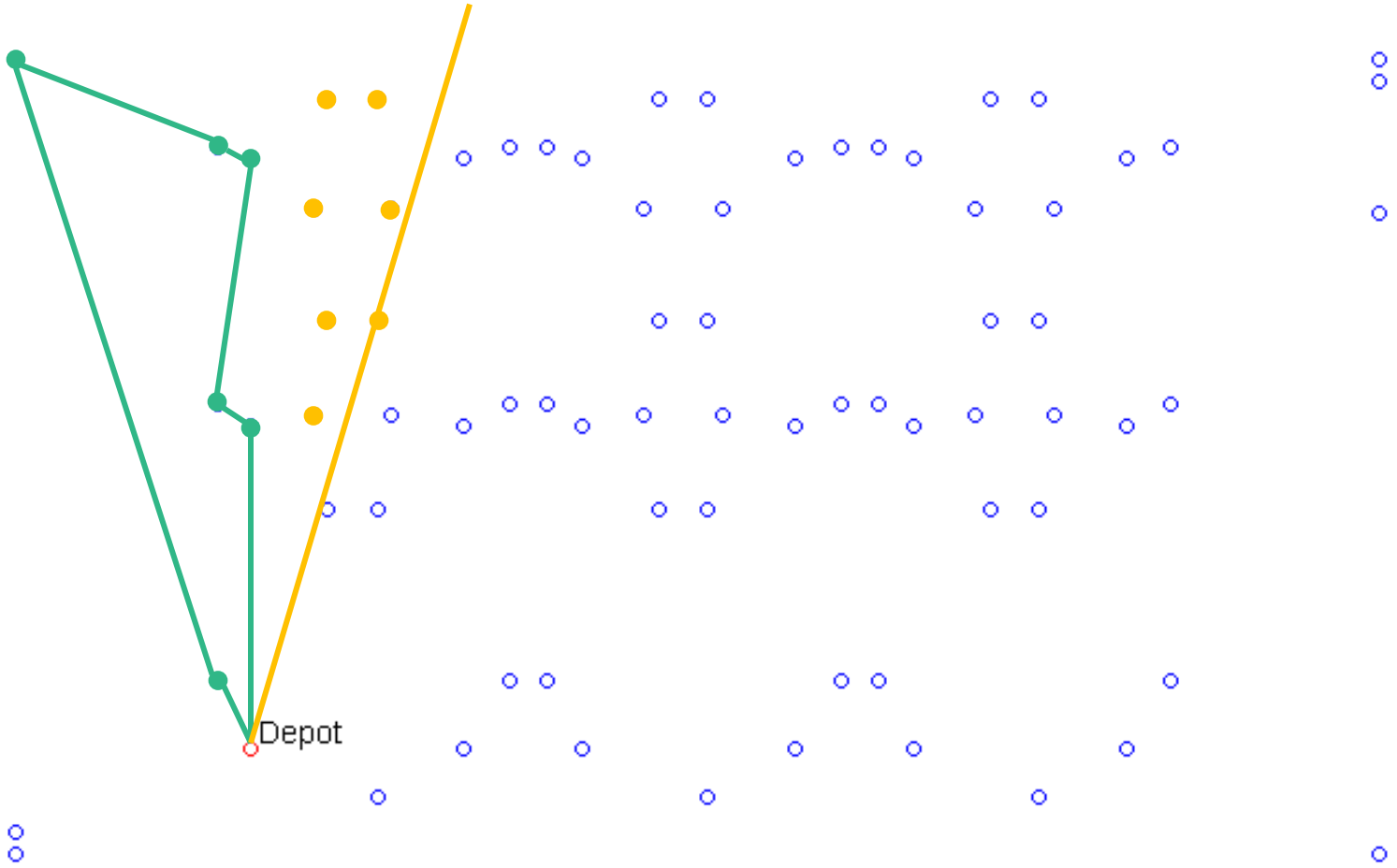

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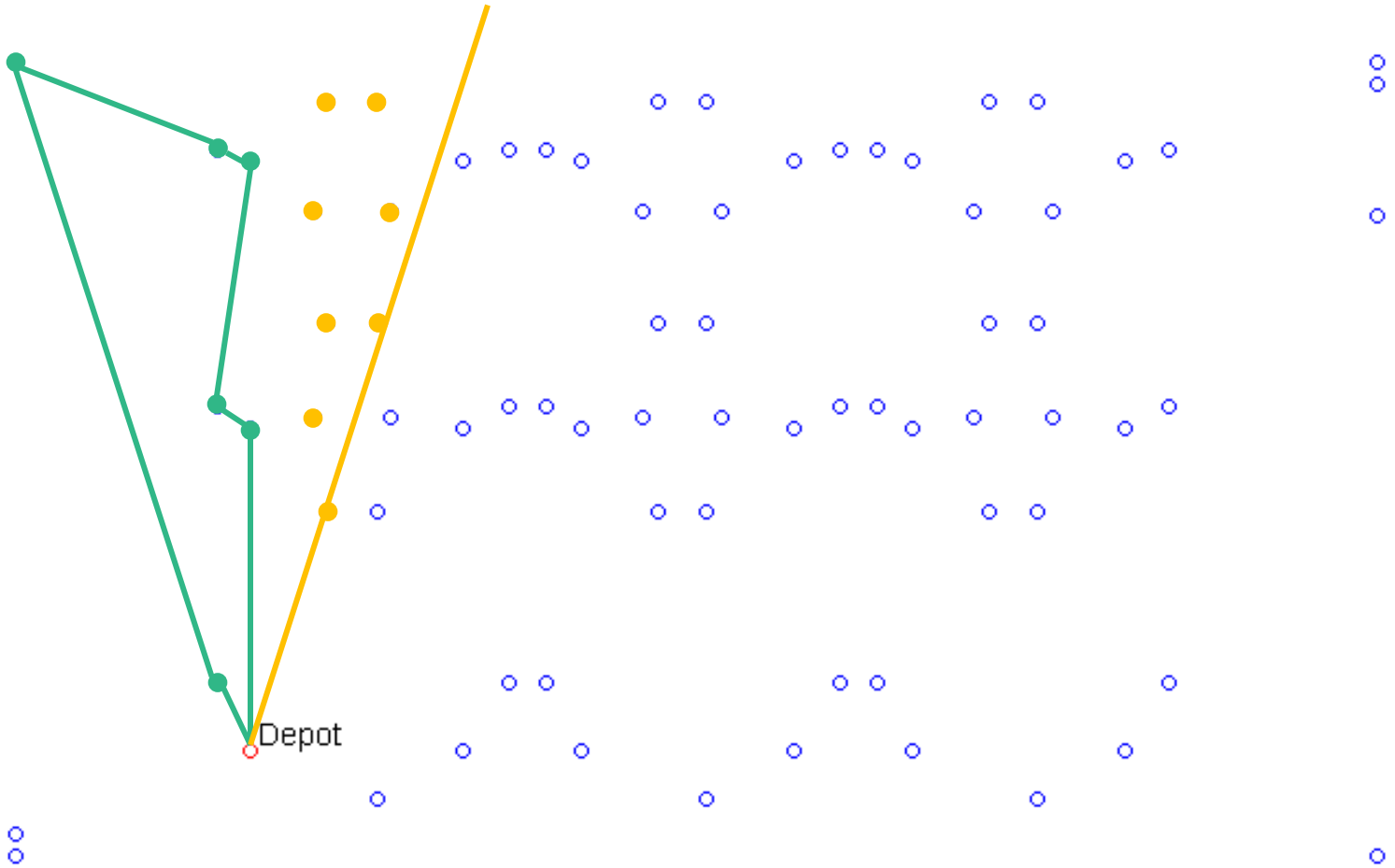

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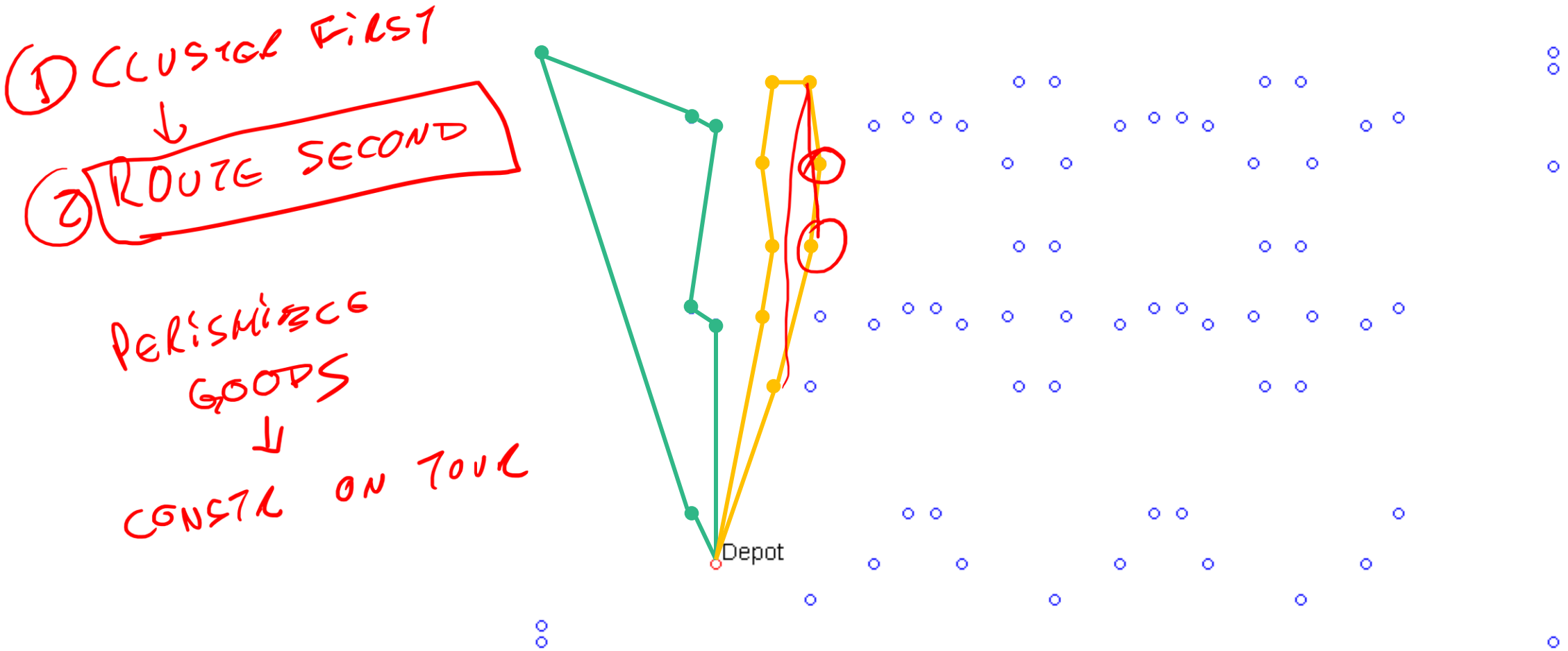

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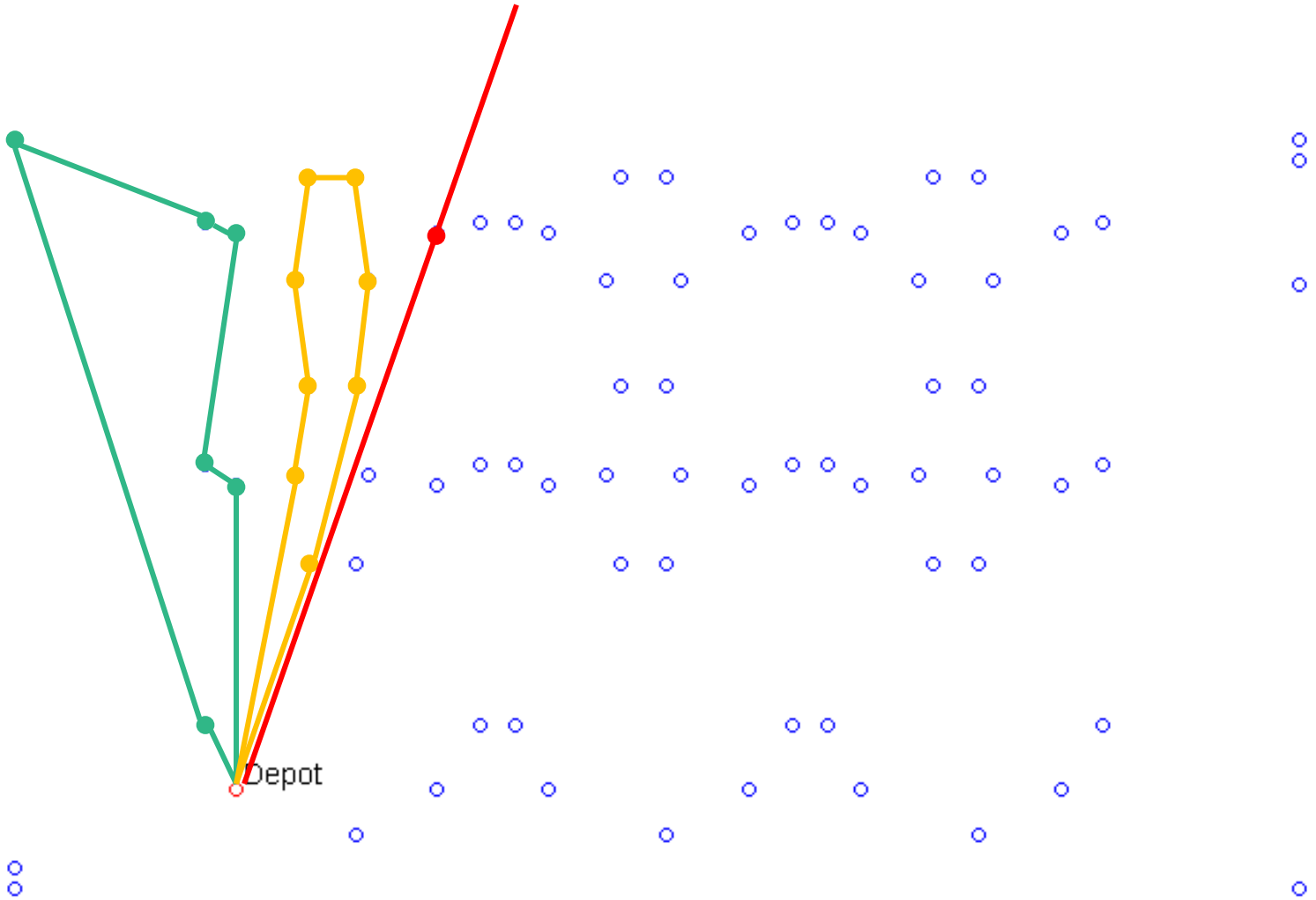


Other construction methods

3D Packing – Knapsack in 3D

- Working with a large mining co.
- They truck stuff from distributions centres near Perth to their mines
- Lots of stuff

3D Packing – Knapsack in 3D

- Working with a large mining co.
- They truck stuff from distributions centres near Perth to their mines
- Lots of stuff
- Lots and lots of stuff

The problem

- The company use 3PLs (Third-Party Logistics Providers) to move their stuff
- 3PLs are paid by the truck
- 3PLs *say* they pack their trucks well, **but** there is no incentive to do so

The problem

- The company *know* they are being ripped off
- They just don't know how much

- This is a "size of opportunity" project
- Aim: Answer the question

If 3PLs packed their trucks really, really well, how much could we save?

- Answer: $5M – Meh
- Answer: $20M – Let's have a closer look

Problem statement

Given a collection of items and a fleet of trucks,
fit as many items as possible into the trucks.

Data:

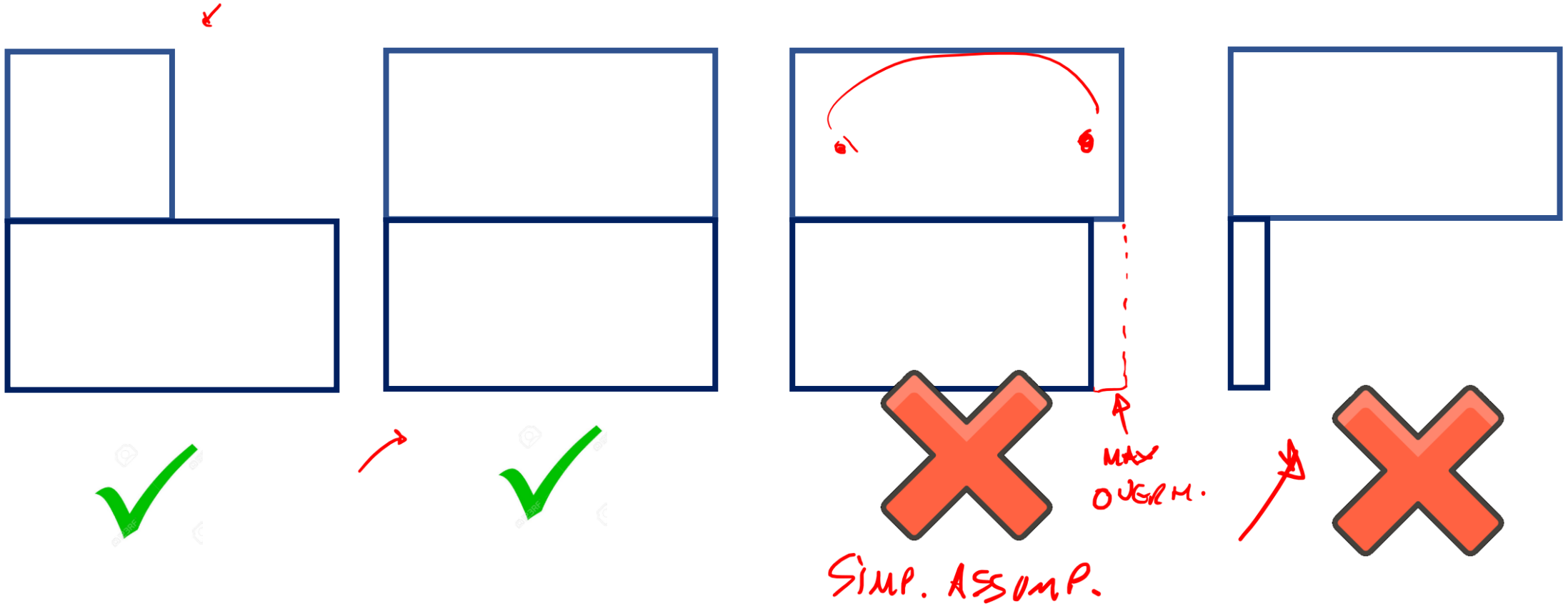
- Each item has
  - dimensions WxHxD
  - a weight
  - a due date
  - a value
- Each vehicle has
  - multiple compartments with known dimensions WxHxD
  - a maximum load (weight)
  - a departure date

Problem statement

Constraints

- The vehicle's weight capacity constraint must be observed
- Items cannot be placed on trucks departing after their due date
- The items must be arranged so that they fit into the truck
- They may be rotated the around the vertical axis, but can't be placed on their side
- Items may be placed on top of each other
  - Some may be marked "*top-load only*"
    - They must be supported from below with *no overhang*

No overhang



(Side view)

Problem statement

What we don't look at:

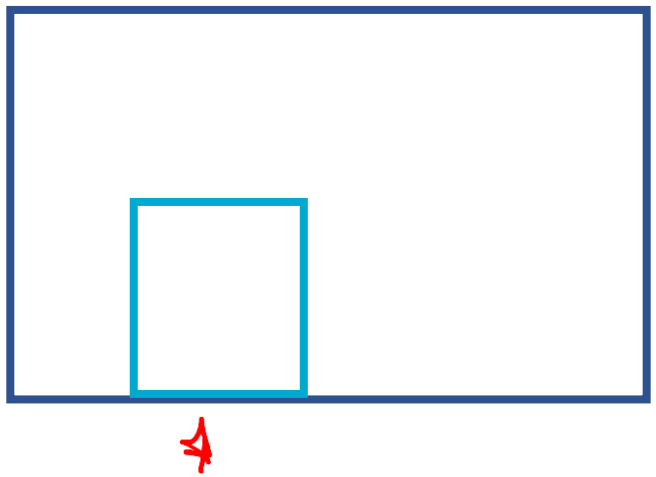
- Weight distribution / axle loading constraints
- Stacking constraints (e.g. item above cannot weigh more than item below)
- Tipping constraints (don't track centre of gravity)

Solution

- We will use a *metaheuristic* to solve the problem
- It will need two related subroutines:
  - Given a stack of items and a number of trucks, how can I distribute the items amongst the trucks
  - Given a truck and a stack of items, can I fit the items into the truck?
- We know that the metaheuristic will call these subroutines many thousands of times.
  - So, packing must be **FAST**

Solution

- The solution is based on a few observations.
- Observation 1: Might as well pack in corners



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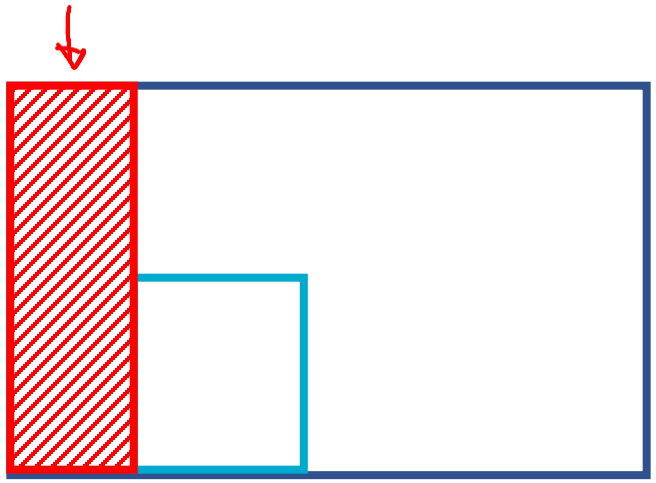

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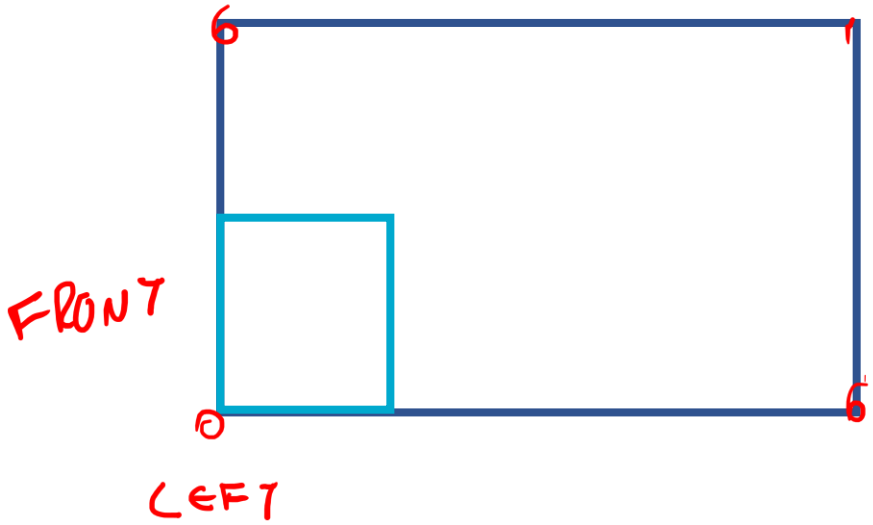
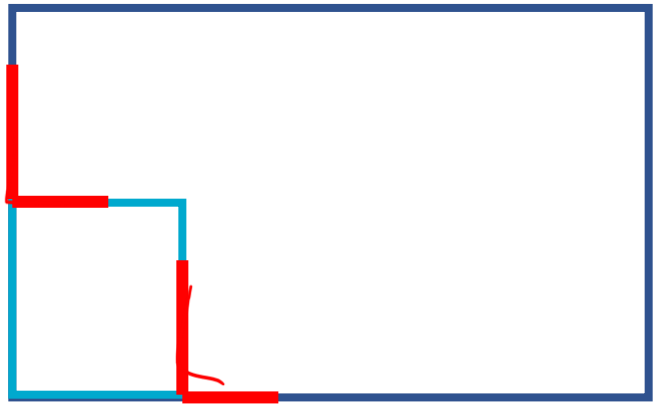


Solution

- The solution is based on a few observations.
- Observation 2: Corners are easy to track



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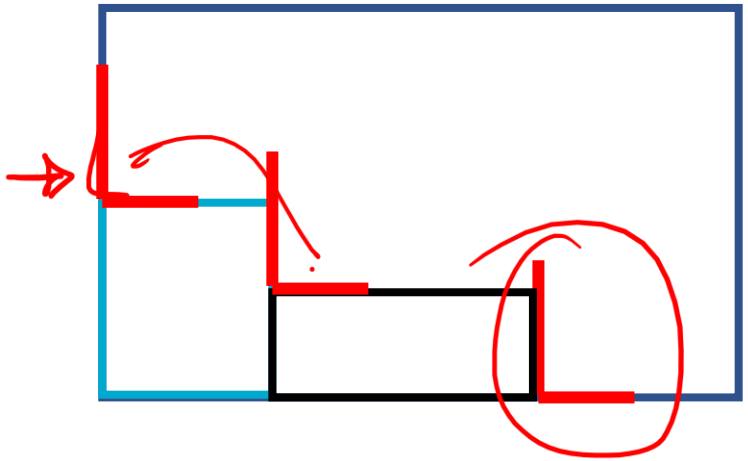
Side view



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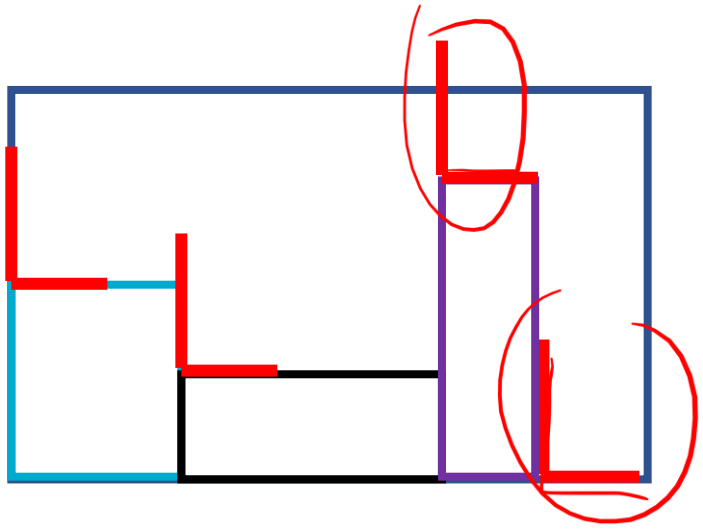

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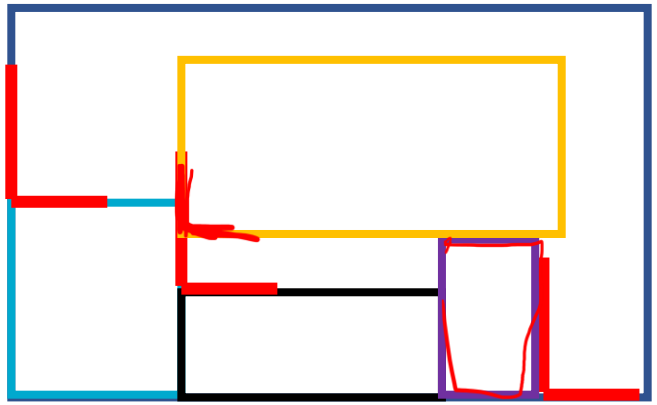
- Observation 2’: Corners are mostly easy to track



Recall that this is the view from the top:

- **Yellow is by the side of purple** (not on top)

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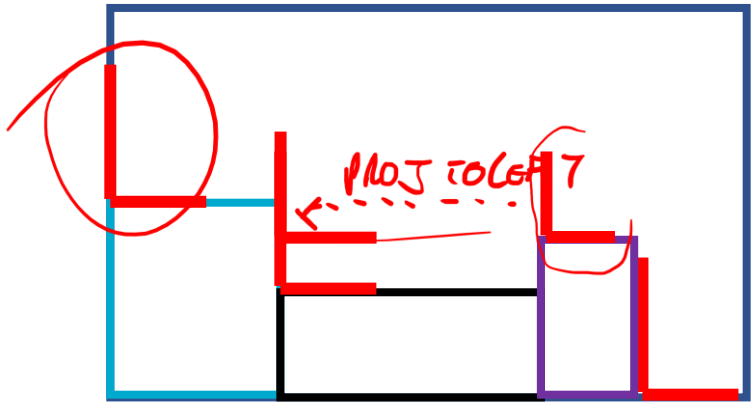

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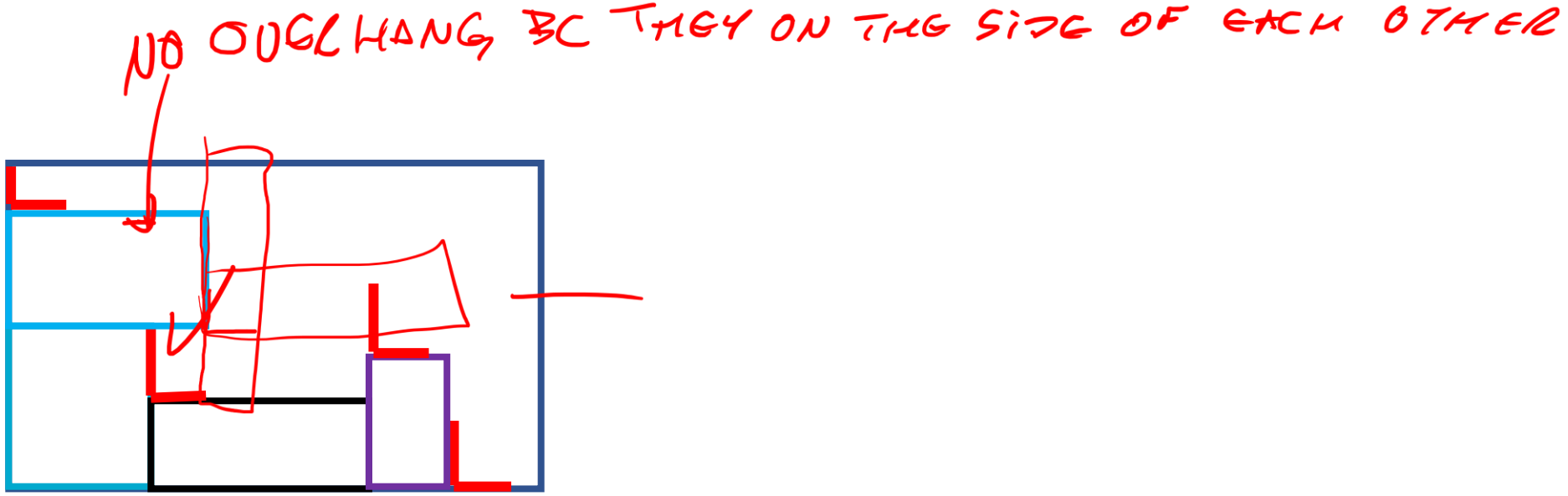

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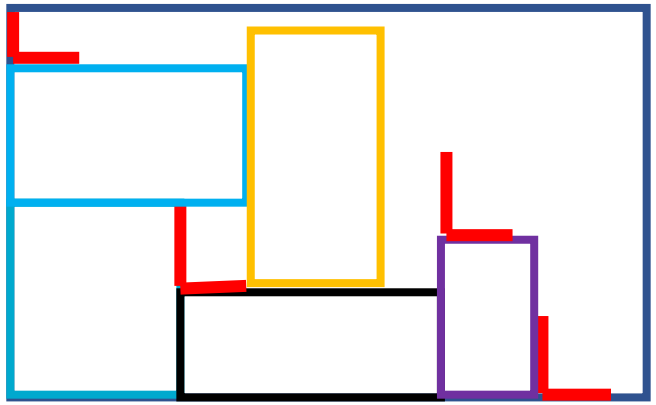

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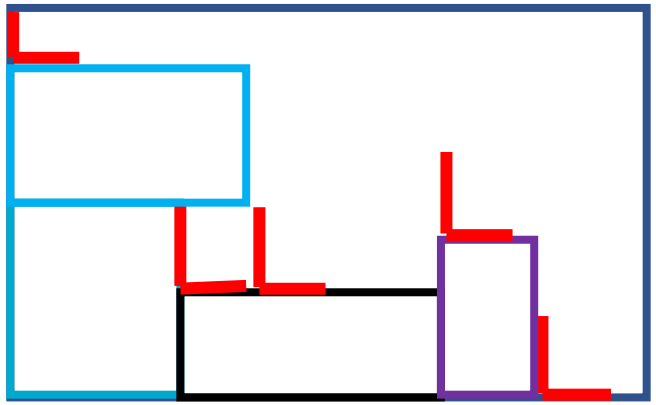

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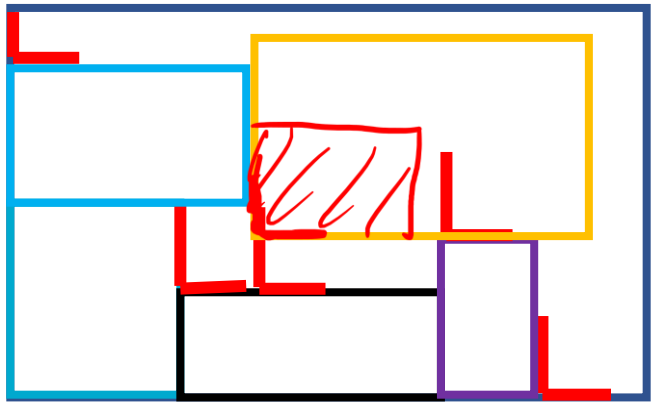

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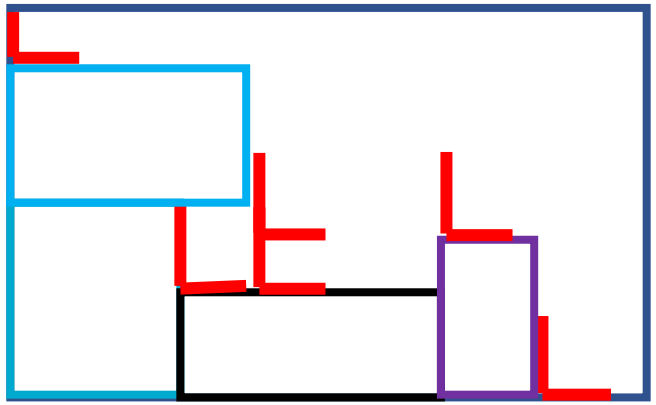

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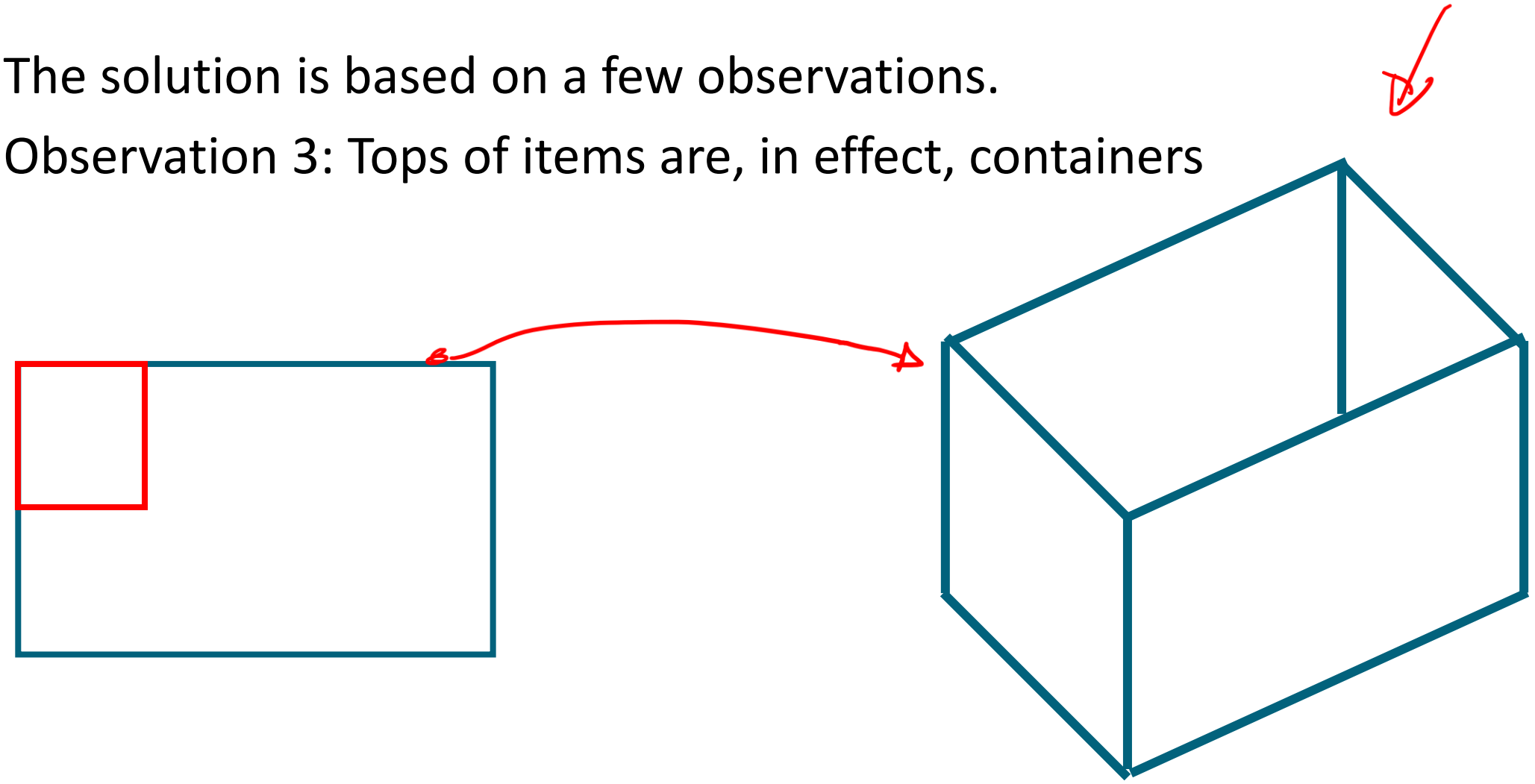
- The solution is based on a few observations.
- Observation 3: Tops of items are, in effect, containers



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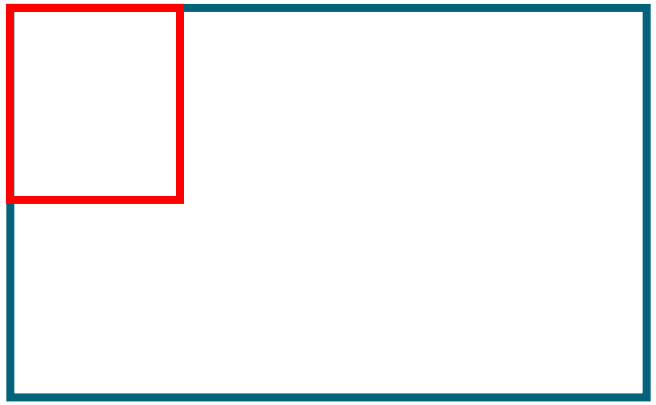

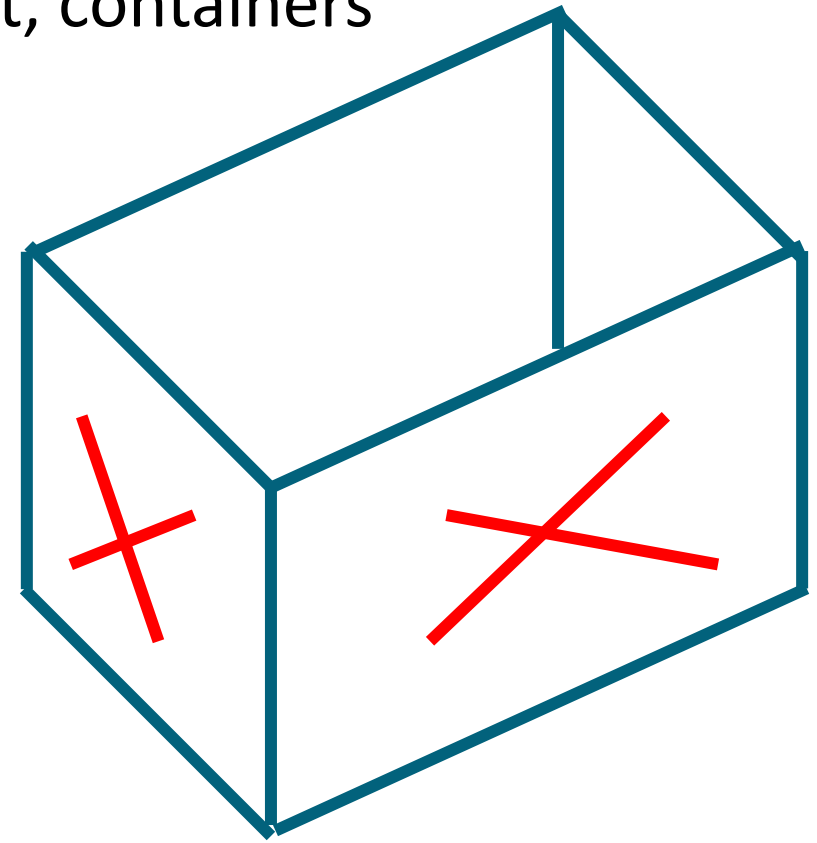

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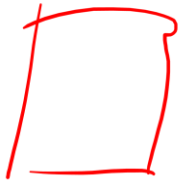

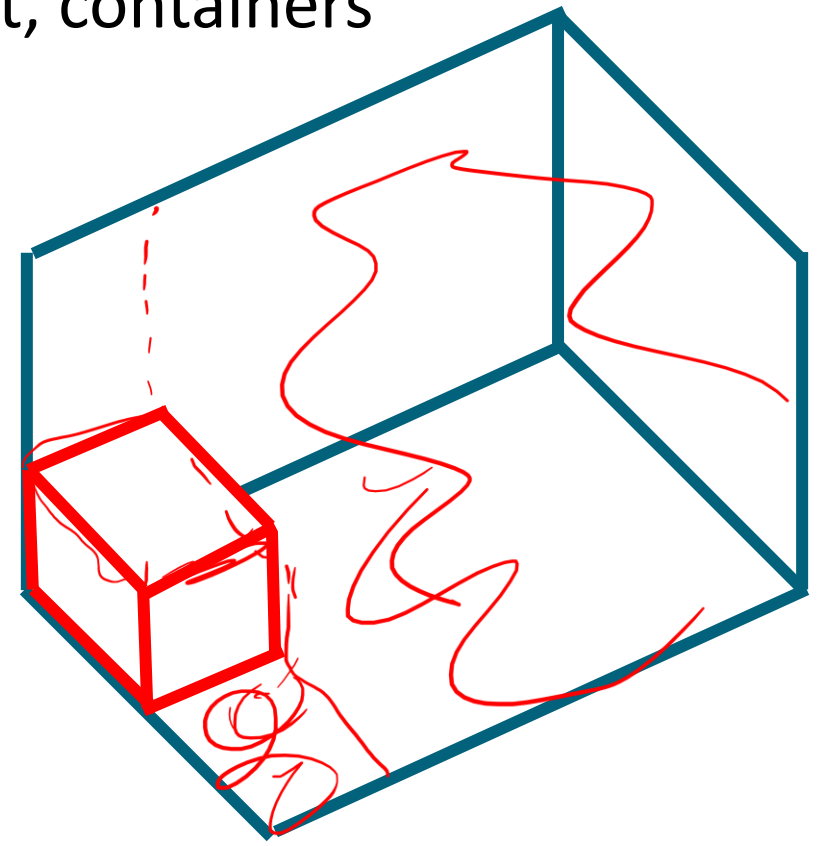

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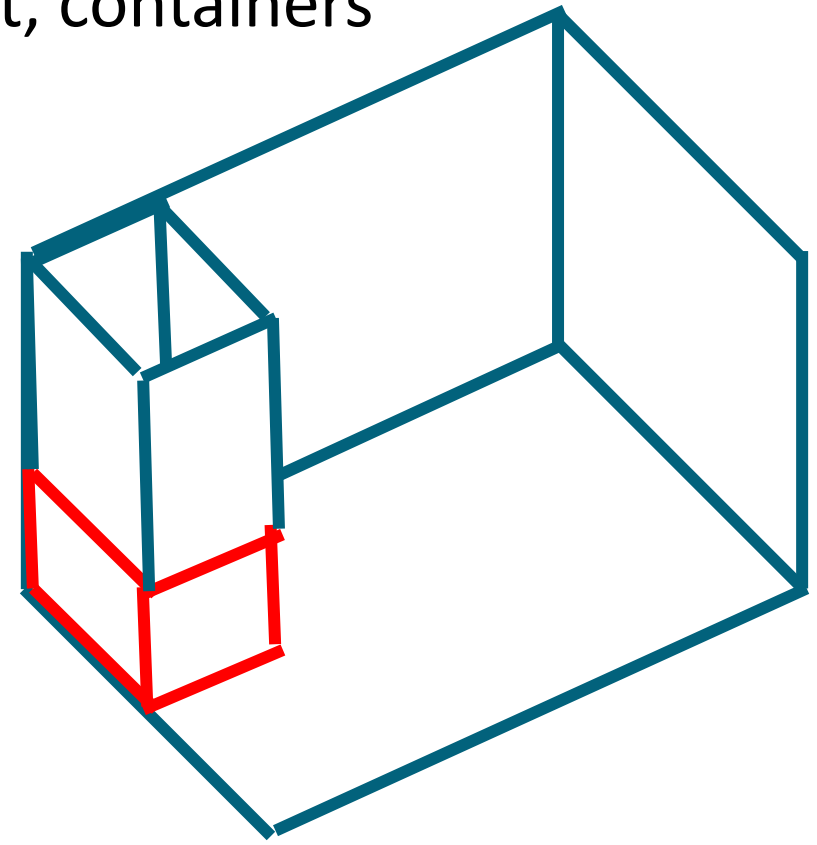

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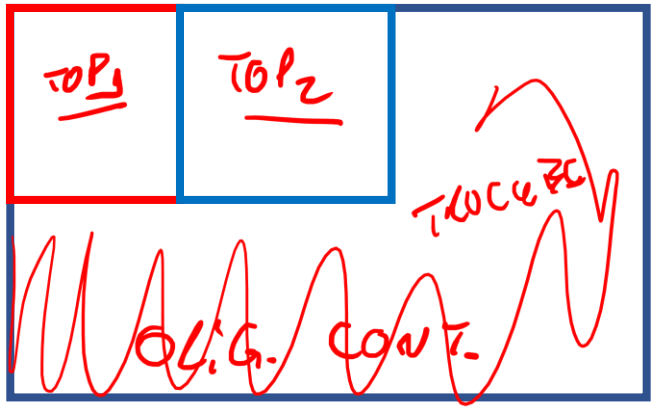
- The solution is based on a few observations.
- Observation 4: Adjoining tops (at the same height) can be joined



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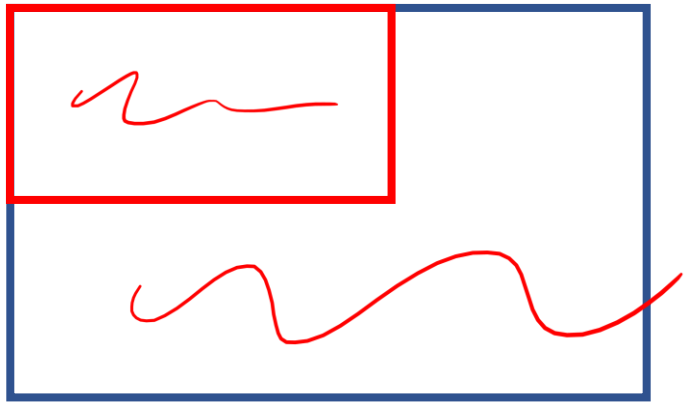

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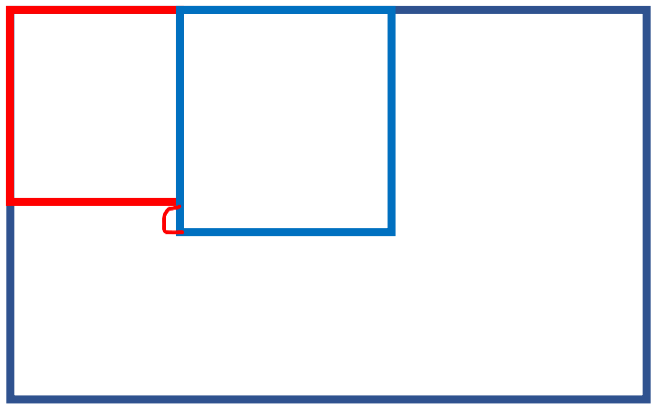

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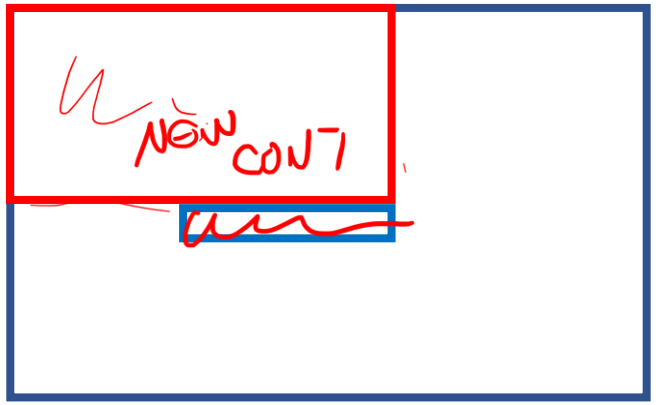

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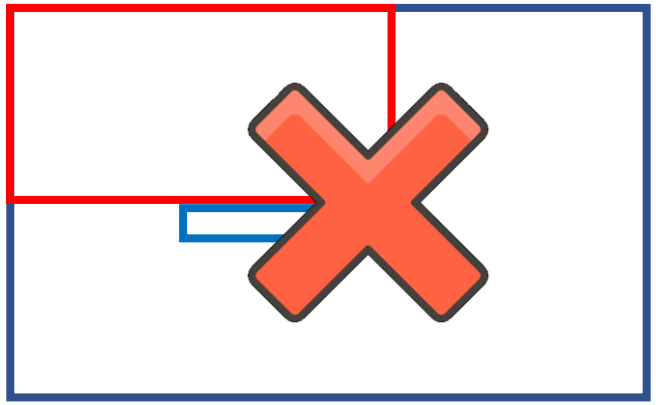
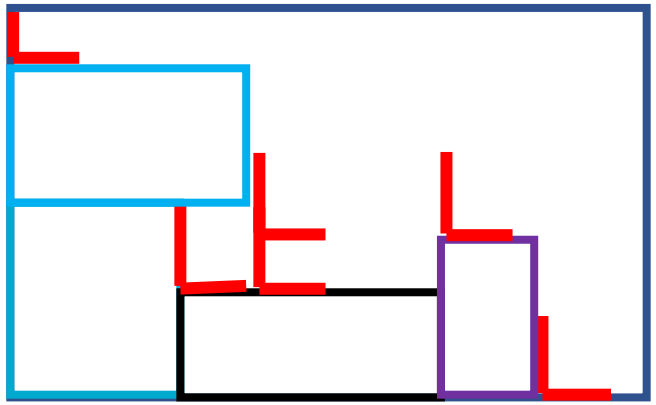


Solution

- The solution is based on a few observations.
- Observation 5: Taking things out of the middle is hard

EMPTY THE CAN?



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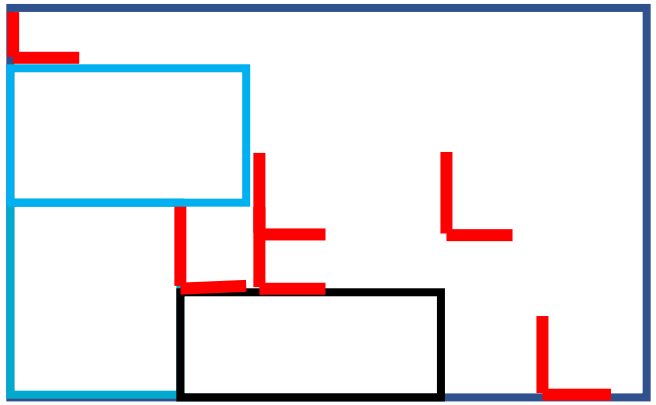
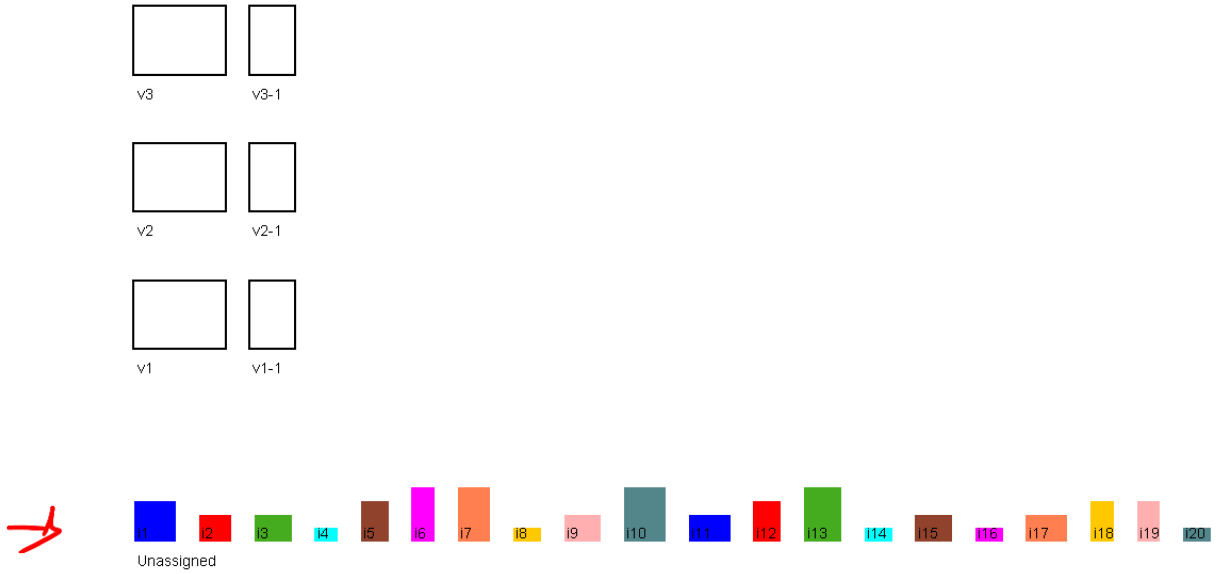


Solution

- The solution is based on a few observations.
- Observation 5: Taking things out of the middle is hard, so we just empty the container

Solution

- Let’s see it run …



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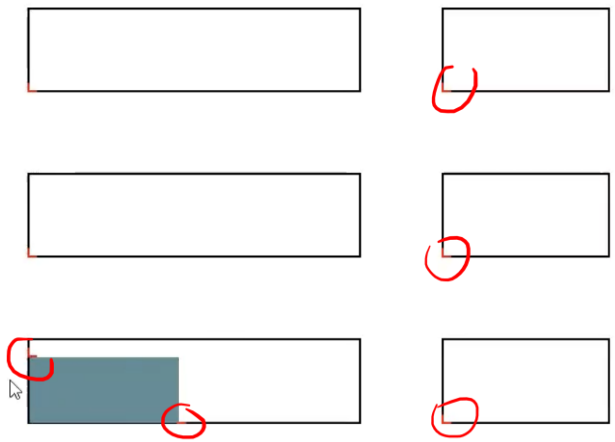

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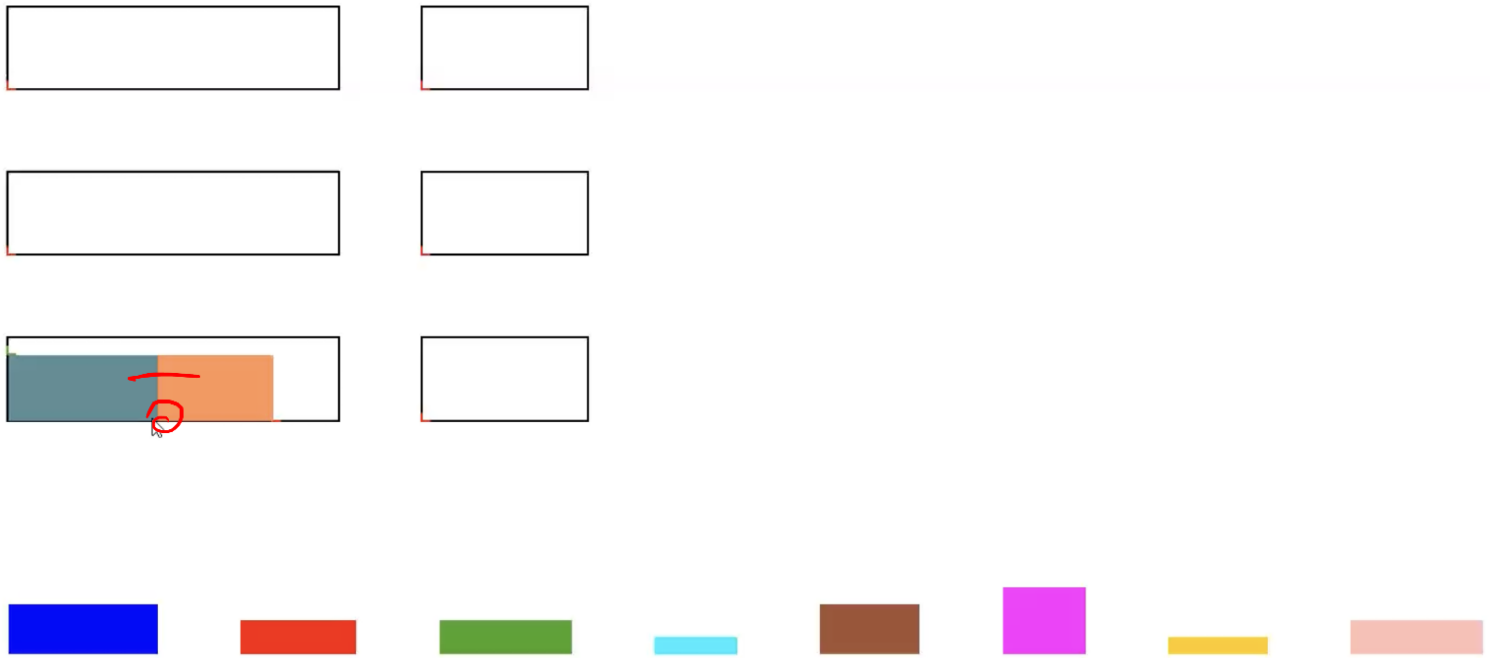

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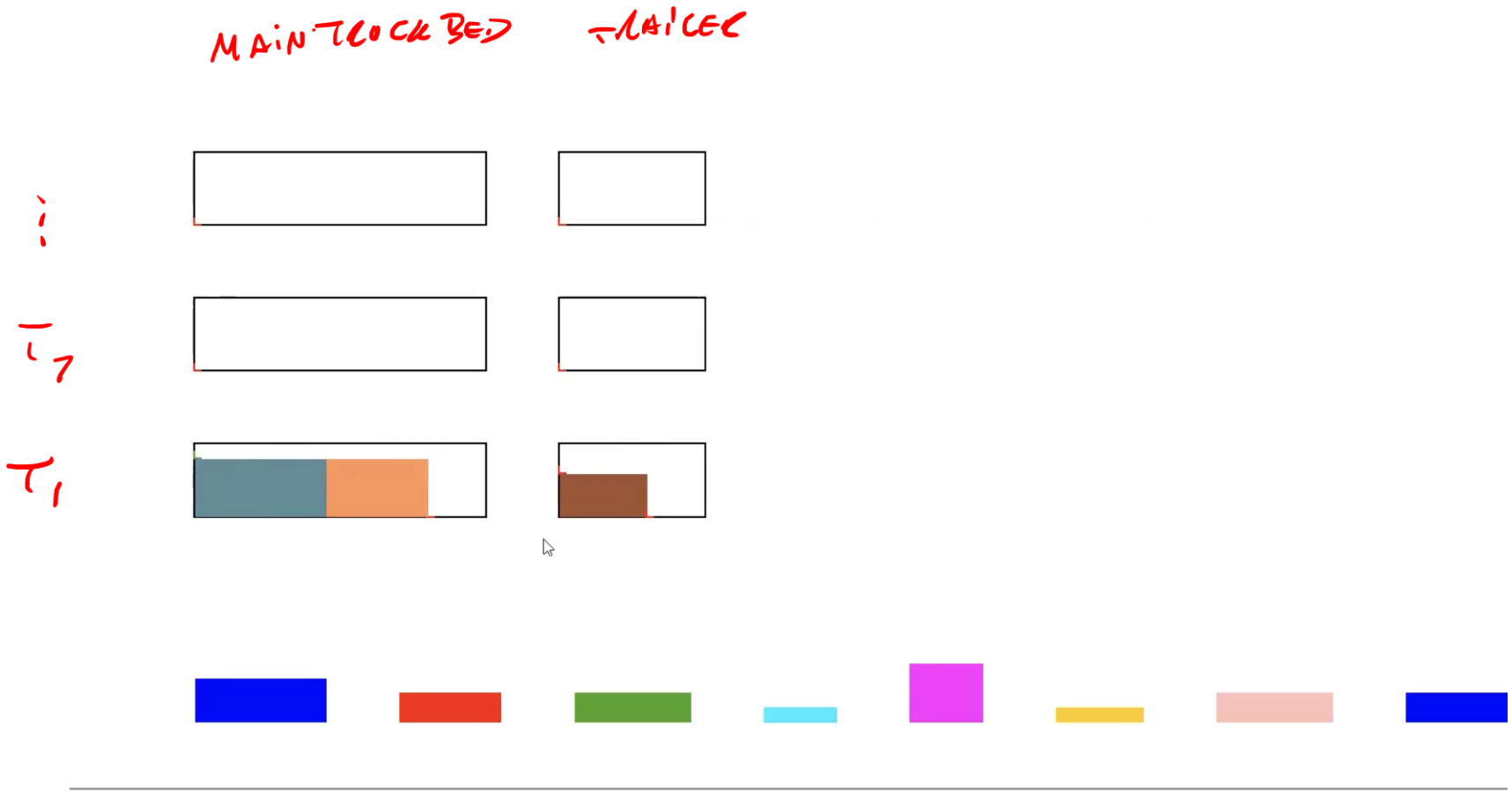

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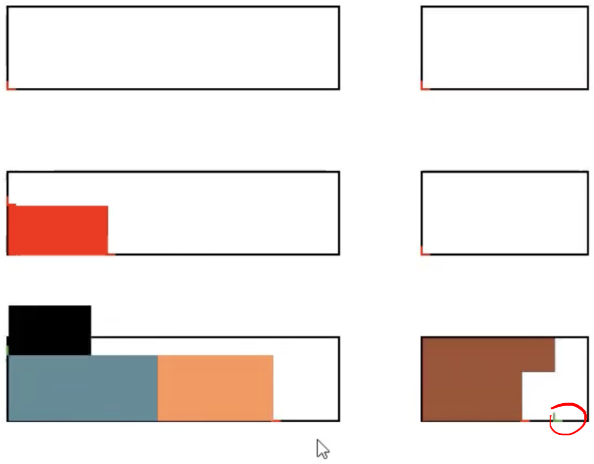

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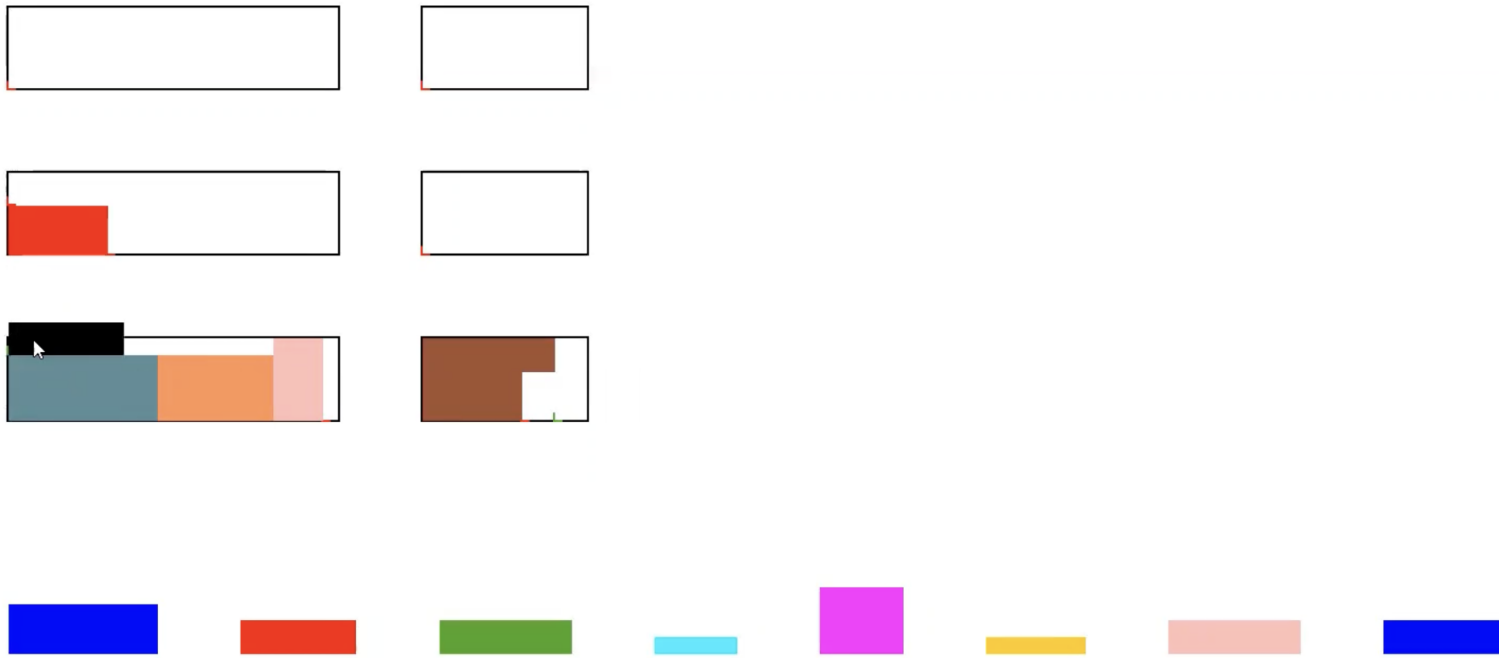

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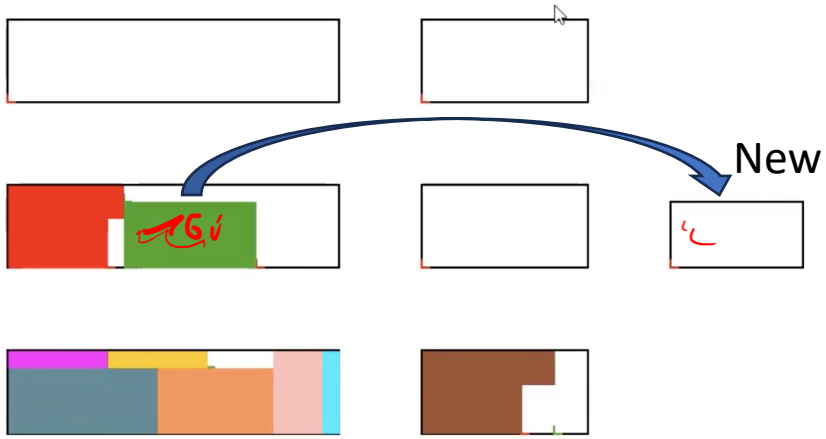
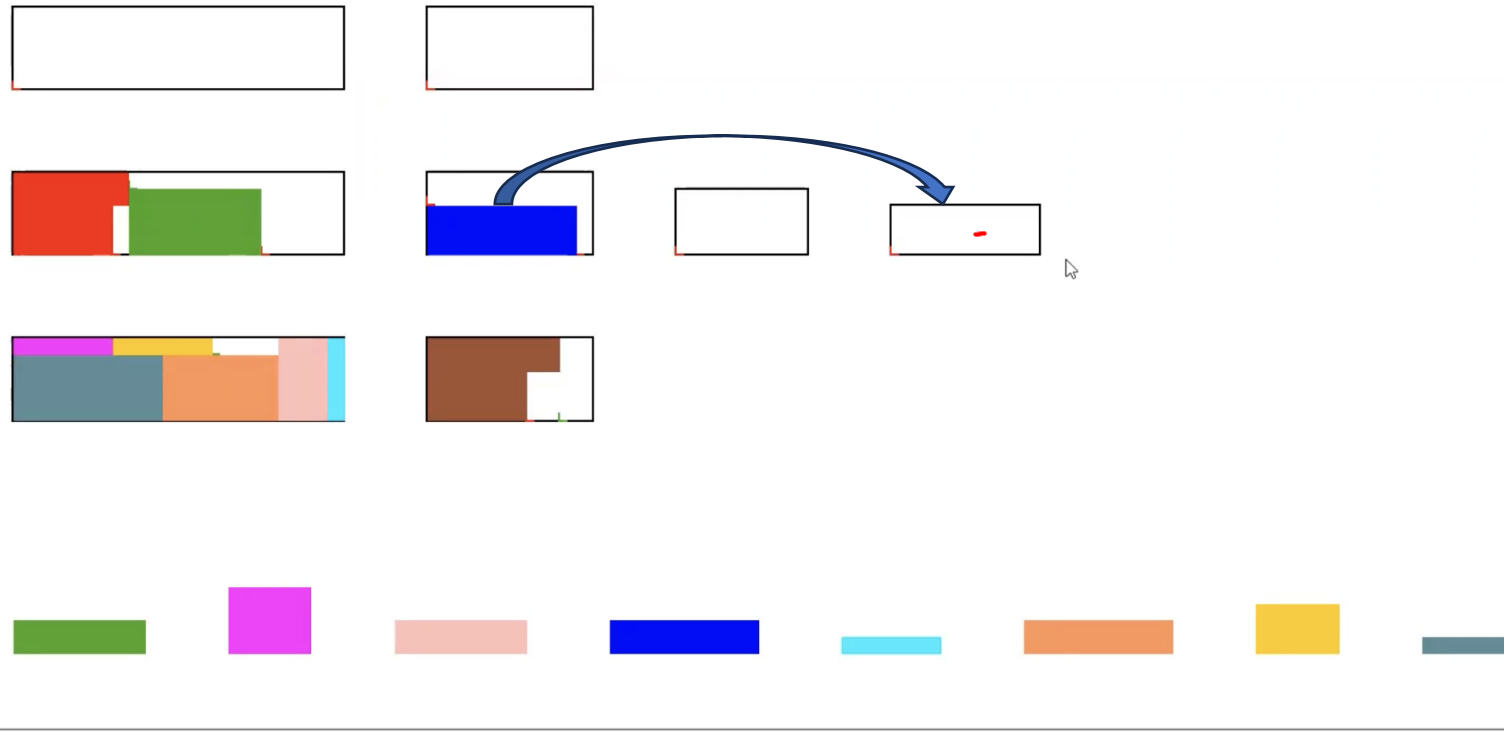
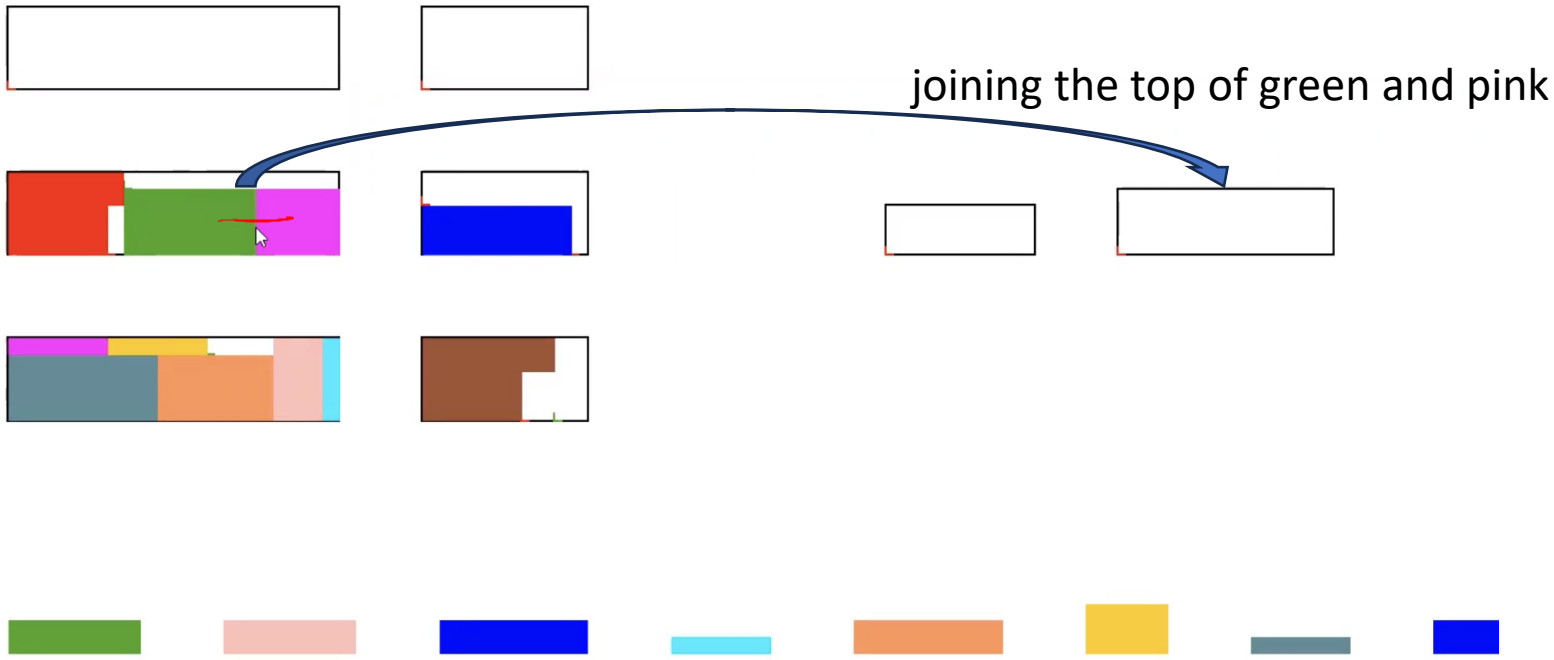


New container representing the top of green

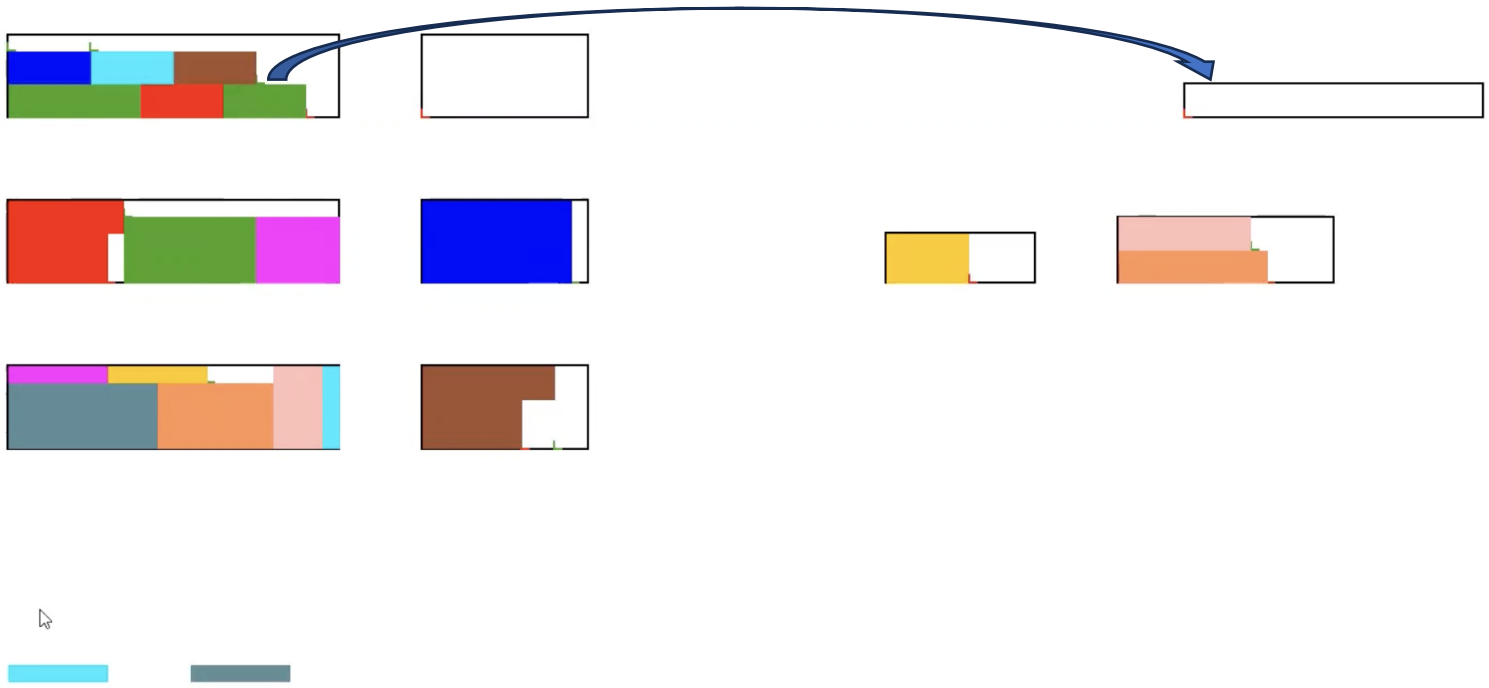
Solution



Solution



Solution



Conclusion

- Constructing a solution is the first step in solving a combinatorial optimisation problem
- Building up a solution one element at a time is often a good place to start
- Greedy heuristics can give half-good solutions, but are often structurally unsound
- Some techniques can give a heuristic a bit of foresight, such as Regret

LOOK AHEAD

Next time

- Now we have our first solution … can we improve it?

FRIDAY QUIZ