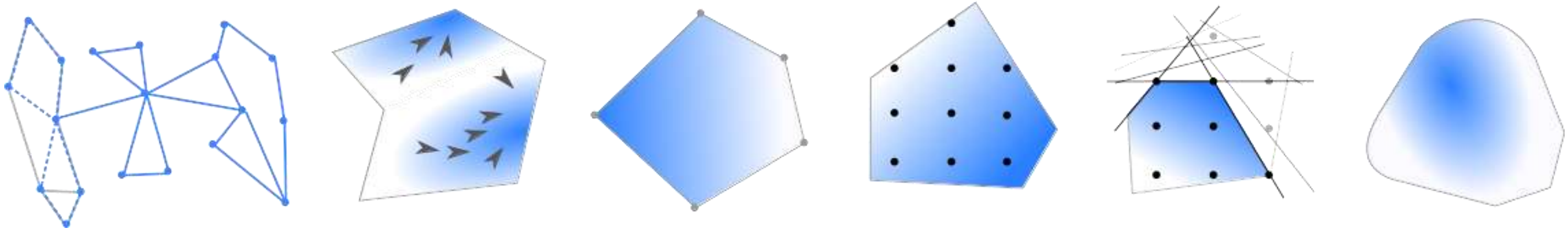
Optimisation

COMP4691 / 8691

Course Overview



Lecturer/Convener: **Felipe Trevizan**

Tutors

Jiawen Wang

Jerry Zhang

Course Outline

Main learning outcome of the course:

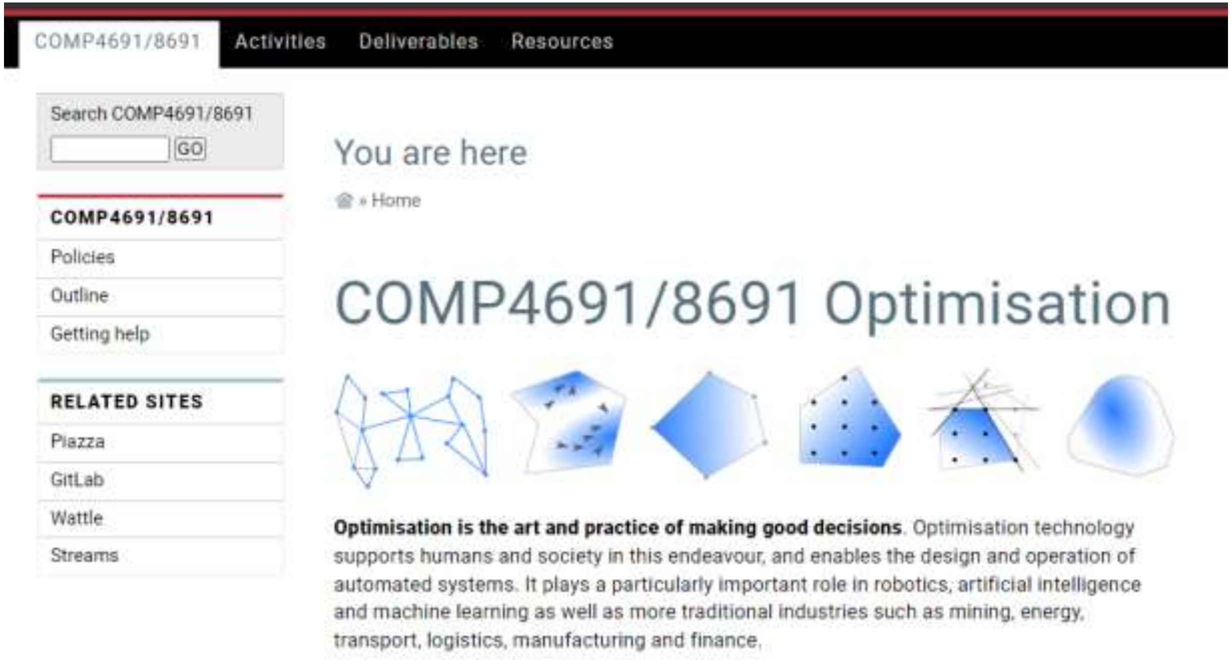
- **Modelling and implementation of optimisation problems and algorithms**

Main topics:

- Linear Programming
- Mixed-Integer Linear Programming NON-CONVEX
- Decomposition
- Convex Optimisation
- Local Search
- Metaheuristics
- Advanced Topics: Stochastic Opt., Multi-Objective Opt., Guest Lectures

COMP4691/8691 Website

All the information about the course can be found in this website: https://comp.anu.edu.au/courses/comp4691/



Activities

- Lectures
  - Tuesdays noon–2pm Bldg 48A Rm 1.23
  - Fridays 10–noon **Bldg 48 Lecture Theatre**
- Labs and Drop-ins
  - **Start next week**
  - Wednesdays 4-6 pm N111 CSIT
  - Thursdays 10-noon N109 CSIT
  - Thursdays 3:**30**-5:**30**pm N115/116 CSIT

Lectures

COMP4691/8691 Activities Deliverables Resources

Books and links

- We provide all the required course material
- If you require additional resources, there are a number of textbooks that we recommend in the resources section

Week	Dates	Lecture A	Lecture B	Labs A/B
1	22/7 - 26/7	01-course-overview -and-introduction	02-LP-modelling	-
2	29/7 - 2/8	03-LP-feasibility -and-optimality	04-LP-simplex	Lab LP: Q1,11,12
3	5/8 - 9/9	05-LP-approximations -and-duality	06-MIP-relaxation -and-modelling	Drop-in
4	12/8 - 16/8	07-MIP-branch	08-MIP-cutting	MIP: Q2,4,7
5	19/8 - 23/8	09-decomp-column-gen	10-decomp-benders	Drop-in
6	26/8 - 30/8	11-cvx-convexity	**NO-LECTURE**	
7	16/9 - 20/9	12-cvx-optimisation -and-lagrangian	13-cvx-interior	Lab Decomposition
8	23/9 - 27/9	14-construction	15-local-search	CVX: Q2,5,8
9	30/9 - 4/10	16-metaheuristics-1	17-metaheuristics-2	
10	7/10 - 11/10	18-stochastic opt	19-multi-objective opt	Lab Metaheuristics
11	14/10 - 18/10	20-network-flow	21-path-planning	Drop-in
12	21/10 - 25/10	Guest Lecture A TBD SAT	Guest Lecture B TBD	

Computer labs/drop-ins

5 Labs (weeks 2, 4, 7, 8, 10):

- 30 minutes drop-in at the start
- 90 minutes working in a **set of problems** to practice your skills, test your knowledge, and prepare you for the assignments and exam.
- The GitLab link to **problem sets** is available on the website.

BGF . AJ & AZ

3 Drop-ins only (weeks 3, 5, 11):

- 2 hours of drop-in
- Use it to ask questions about the lab’s problem sets, deliverables, and lectures

Deliverables

Assessment marks

Assessment	COMP4691	COMP8691
Final Exam	50% (40% hurdle)	50% (40% hurdle)
Assignment 1	10%	10%
Assignment 2	10%	10%
5 Quizzes	30%	20%
Seminar	–	10%

Assignments

General instructions

- Assignment 1: LPs and MILPs due: **Friday 30-08-2024 18:00**
- Assignment 2: Meta-heuristics, due: **Week 12**

Quizzes:

- In-person at the end of the lectures
- 30-45 minutes

Timeline

Week	Dates	Due	Released	Quiz
1	22/7 - 26/7		Seminar (COMP8691-only)	
2	29/7 - 2/8		Simplex - LP	LP
3	5/8 - 9/9			Quiz-1
4	12/8 - 16/8		Assignment 1	MID
5	19/8 - 23/8			Quiz-2
6	26/8 - 30/8	Assignment 1		
7	16/9 - 20/9	Seminar: Group and Topic Selection		DEC CUX
8	23/9 - 27/9			Quiz-3
9	30/9 - 4/10		Assignment 2	LS MH
10	7/10 - 11/10			Quiz-4
11	14/10 - 18/10			ADUT
12	21/10 - 25/10	Assignment 2 Seminar: Video delivery		Quiz-5

Deliverables – 8691 Only

Seminar Overview (COMP8691 ONLY)

Students will work individually or in groups of up to 3 to research an optimisation topic that is not covered in the course material. Topics may include:

- A particularly interesting application of optimisation
- An optimisation algorithm or solving technique

The groups first need to propose a topic that will then be approved (or not) by the course lecturers. Be sure to do this as soon as you can so there is enough time to find another topic if your first choice is taken or inappropriate.

Once the topic is approved, the group will then further research the topic and put together a 15 minute video presentation on it.

- Week 1: Seminar project released
- Week 7: Group and topic selection deadline: **20-09-2024 18:00**
- Week 12: Video delivery deadline: **25-10-2024 18:00**

Resources

RESOURCES

- FAQ
- Books and Links
- Software

RELATED SITES

- Ed Discussions
- GitLab
- Wattle Gradebook

» Resources

FAQ

Some questions which come up, well... frequently

» read more

Books and Links

Books, links and other resources

» read more

Software

Software setup and usage

» read more

Communication

Admin questions: ← the whole COMP4691 team

- Use Ed Discussions
- make it a private message if needed

Admin questions regarding private matters: ← just me

- email the convener directly

Class Representative

The role of Student Representatives is to provide ongoing constructive feedback on behalf of the student cohort to Course Conveners and to Associate Directors (Education) for continuous improvements to the course.

- Act as the official **liaison between your peers and convener.**
- Be available and proactive in **gathering feedback from your classmates**.
- Attend regular meetings, and **provide reports on course feedback** to your course convener
- Close the feedback loop by **reporting back to the class the outcomes of your meetings**.

Note: Class representatives will need to be comfortable with their contact details being made available via Wattle to all students in the class.

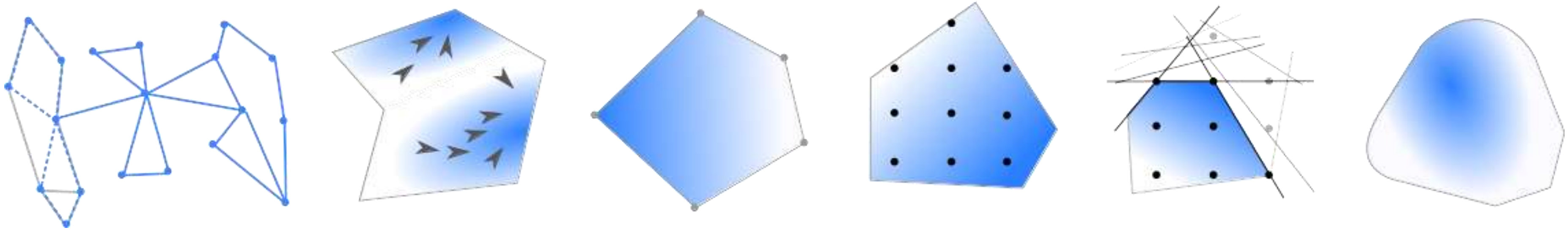
For more info regarding roles and responsibilities, contact: sa.cecs@anu.edu.au

Please nominate yourself by sending a private message in ED

Optimisation

COMP4691 / 8691

Introduction to optimisation



Agenda

- What is optimisation?
- Examples of real-world optimisation problems
- How do we solve optimisation problems?
- Problem Formulation
- Methods to solve optimisation problems

Optimisation

… is the selection of a best element, with regard to some criterion, from some set of available alternatives …

Wikipedia

… finding the best solution out of a very large set of possible solutions.

Google

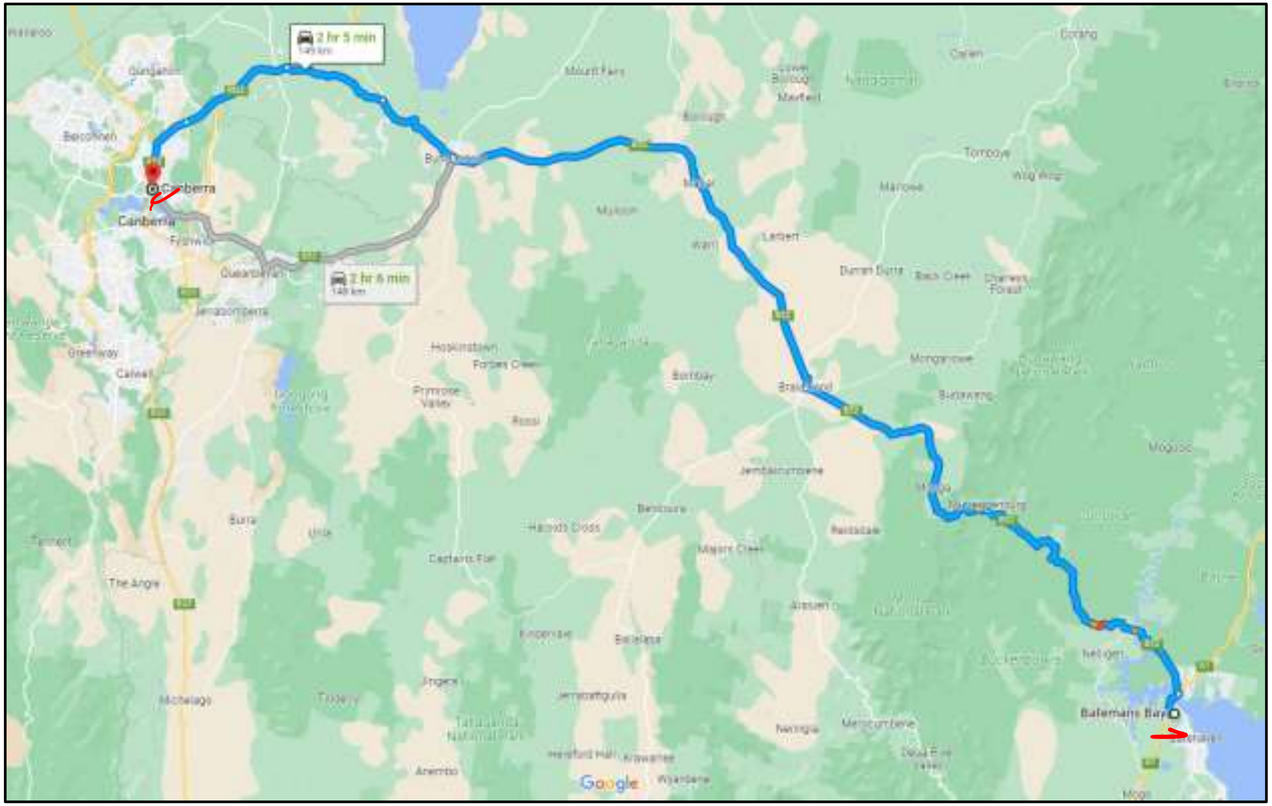
The art of making choices, when good choices lead to good solutions.

Phil Kilby

Examples of optimisation problems

Shortest path problem

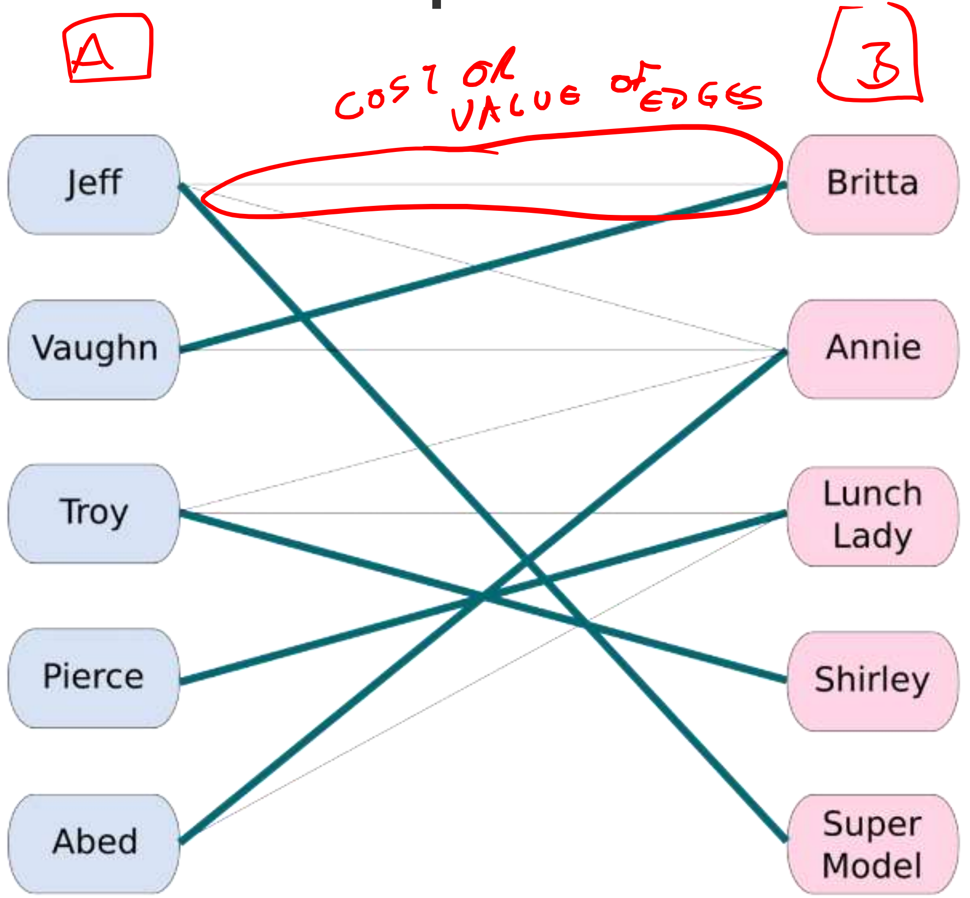
- Find shortest path between two locations



Examples of optimisation problems

Matching problems

- Find the pairing with minimum cost (or maximal satisfaction)



Examples of optimisation problems

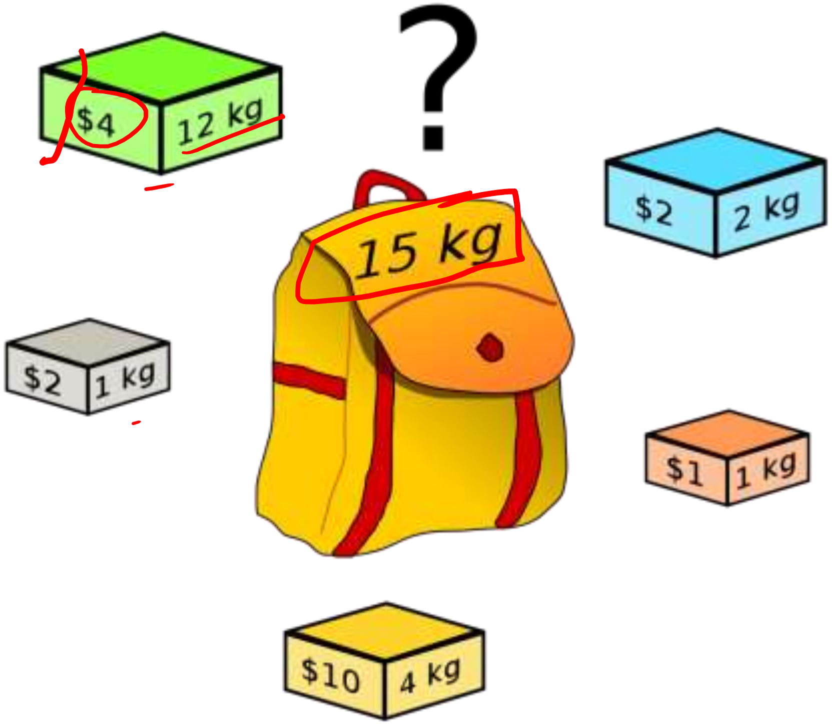
Knapsack / Bin Packing problem

- Choose items to give the maximum value within a given capacity

WEIGHT CAP

VOLUME CAP

2 { - REG
  - REFRIG CO.

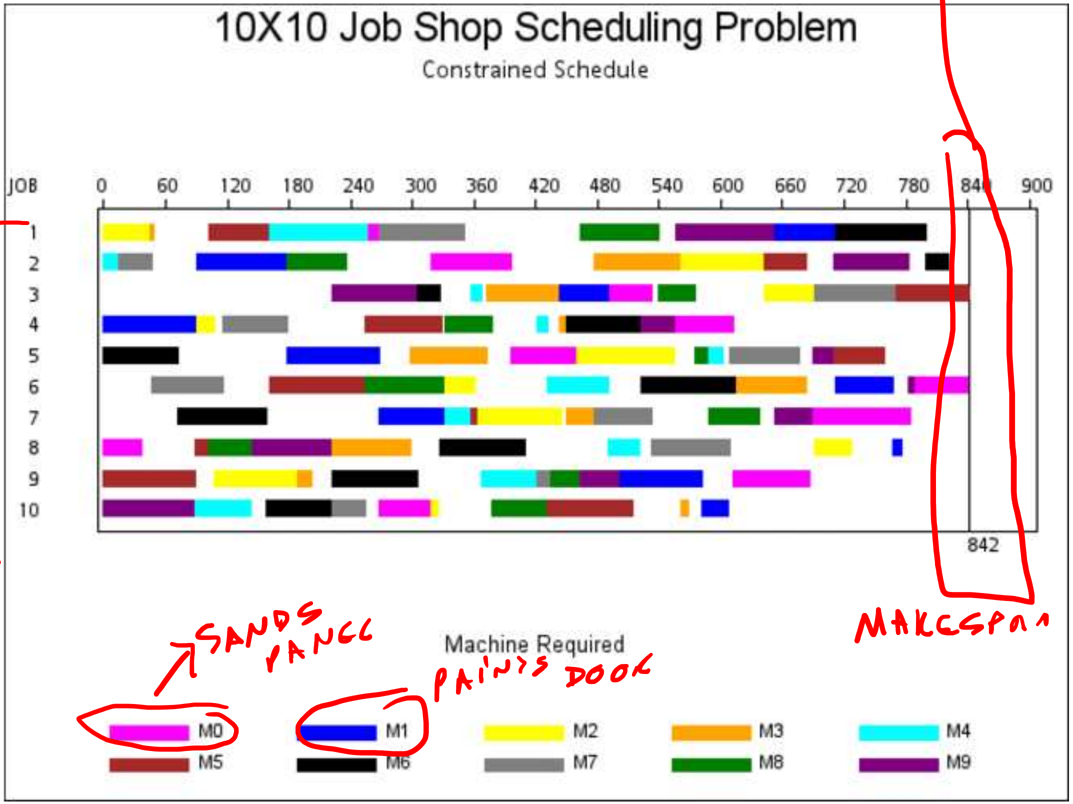


Examples of optimisation problems

Job-shop scheduling problem

- Schedule jobs on machines to minimise total time

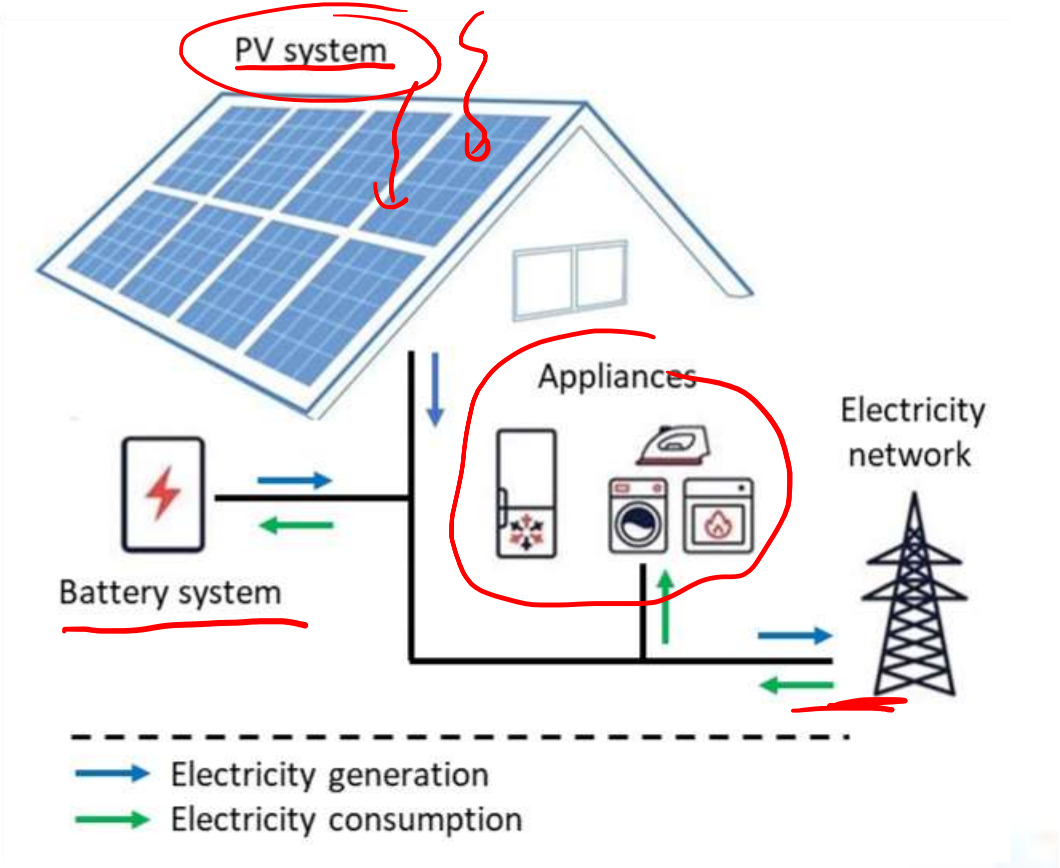
TIME TO COMPLETIONS



Examples of optimisation problems

Smart home scheduling problem

- Scheduling the operation of a smart home with a PV-battery system



Examples of optimisation problems

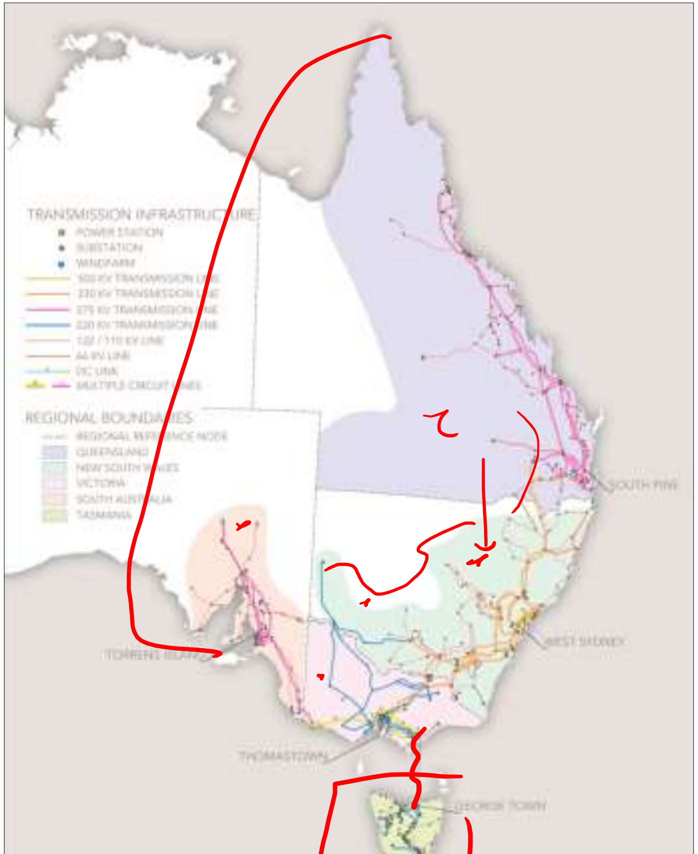
Electricity market dispatch problem

- Dispatch the operation of generators to meet demand by maximising social-welfare (or minimising costs)

CONSTRAINTS: DEMAND MUST BE MET

LP

EU: MILP



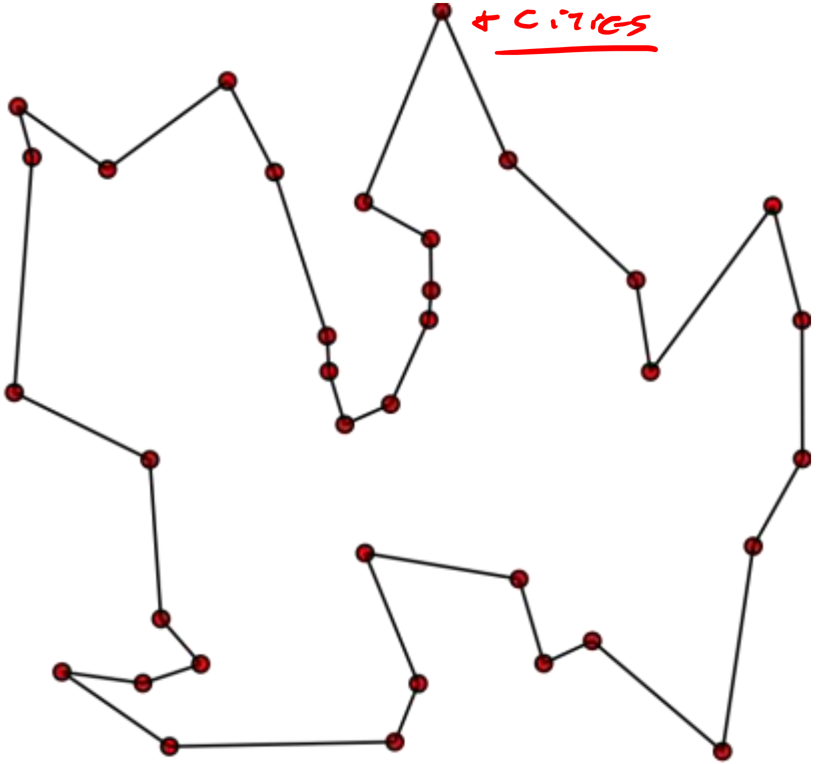
Source: AEMO

Examples of optimisation problems

Travelling salesman problem

- The travelling salesman problem asks the following question:

  "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?"

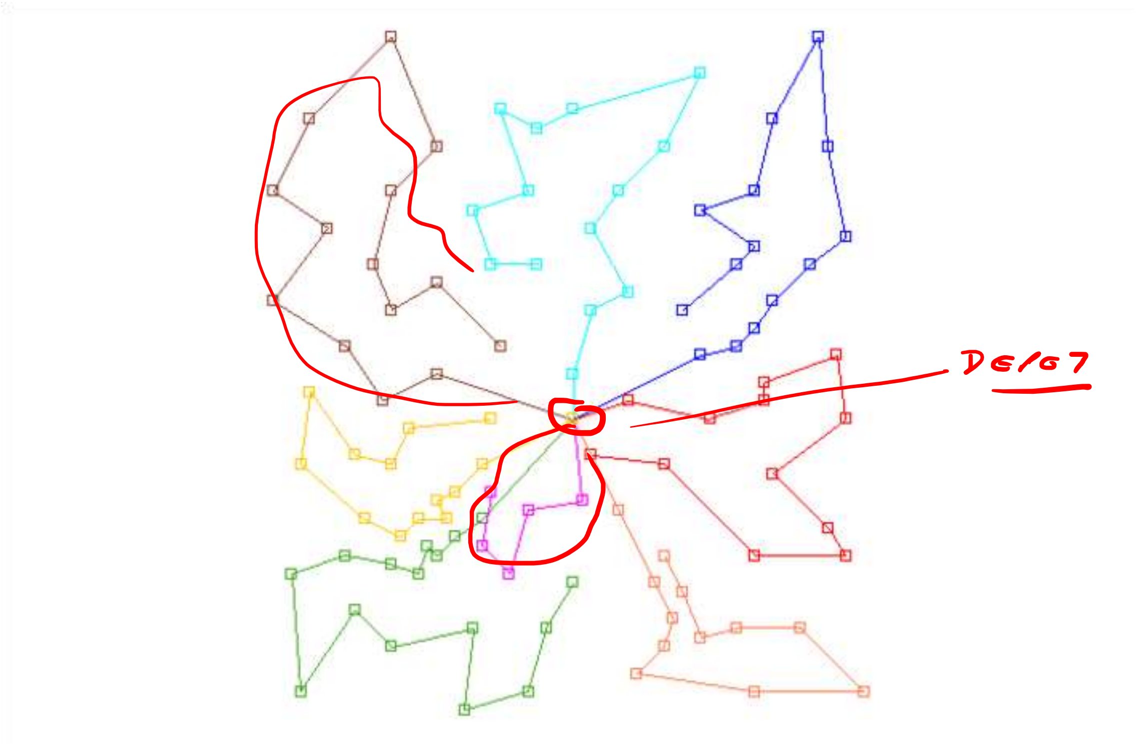


cities

Examples of optimisation problems

Vehicle routing problem

- Given a set of customers, and a fleet of vehicles to make deliveries, find a set of routes that services all customers at minimum cost



What optimisation problems did you learn on AI (COMP3620)?

MODEL BASED

PLANNING

$\boxed{\text{STRIPS (REP)} \rightarrow \text{PDDL (LANG)}}$

ALG BASED

SCHEDULING CLASSES

→ LNS

How do we solve these optimisation problems?

How do we solve these optimisation problems?

Typically, we need to follow these steps:

1. Formulate the optimisation problem ENGLISH → MATH
2. Select the method to solve the problem

MODEL BASED

PLANNING

ALG BASED → A^* + STATE REPR

Problem Formulation

Definition

- Translating the statement of a problem into a mathematical formulation

Key elements

- Decision variables
- Parameters
- Constraints
- Objective(s)

Decision variables

- Variables that we have control over (that affect our solution)

- Example of decision variables in the smart home scheduling problem:
  - Power generation of the PV system
  - Discharging and charging power of the battery system

- Example of decision variables for the job-shop scheduling problem:
  - Variables to select which machines operate
  - Starting time of the machines

Parameters

- Elements we have no control over (but do effect the outcome)
- **They are constant**

- Example of parameters for the job-shop scheduling problem:
  - Number of machines k
  - Execution time of each job

x_i FOR $i \in \{1, \dots, k\}$ 10

$x_1, x_2 \dots x_{10}$

DECISION VARS

- Example of parameters in the smart home scheduling problem:
  - Efficiency of the battery
  - Capacity of the battery

$\boxed{D.V. \leq \boxed{CAP_i}}$ $\forall i \in$ BATTERIES

Constraints

CONSTANT

$\boxed{\text{PARAM}}$

- Define:
  - the limits of the decision variables – $\underbrace{\text{ALL YOU BUY}}_{}$ ≤ MAX ENER FROM GRID

    D.V

  - relationships between variables
  - relationships between variables and parameters

- Example of constraints in the smart home scheduling problem:
  - Maximum charging and discharging power of the battery
  - Exchanged power with the network

- Example of constraints for the job-shop scheduling problem:
  - Assign jobs to machines → JOB MUST BE DONE BY ONE-OF {M0, M2, M7, ...}
  - Coordinate the operation of machines: machine M0 starts before than M3

Objective(s)

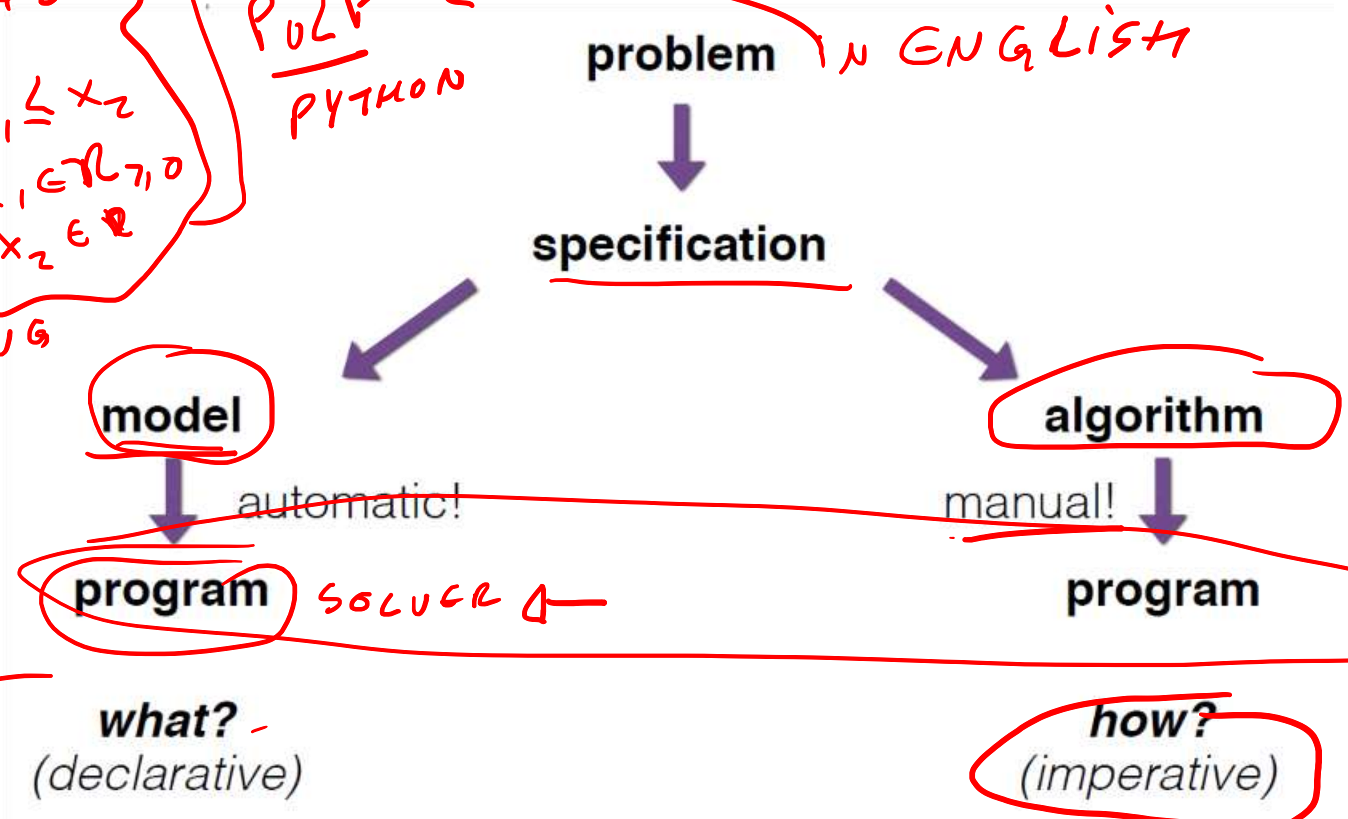
- What makes a solution “good”?
- What characteristics are we seeking in a solution?
  - Smart home problem: minimise the electricity cost of the house
  - Scheduling problem: minimise the maximum completion time over all jobs (makespan)
- Sense is minimisation (e.g., of cost) or maximisation (e.g., of profit)
- Multiple objectives:
  - Smart home problem: minimise electricity cost and environmental impacts
  - Shortest path problem: minimise time and fuel

Methods to solve optimisation problems

$$\min_{x_1, x_2} \; x_1 + 2x_2$$
$$x_1 \le x_2$$
$$x_1 \in \mathbb{R}_{\ge 0}$$
$$x_2 \in \mathbb{R}$$

PULP
PYTHON

MODELING
IN ENGLISH



SOLVER

A* + STATE REPR

PLANNING AND PDDL

RUN CODE

SOLVING OPT PROB.

Model-based methods

Examples of model-based methods:

- LP (Linear programming) – we will start this next class
- MILP (Mixed-integer linear programming) – we will study this later
- SOCP (Second-order cone programming)
- QP (Quadratic programming)
- SDP (Semidefinite programming)

GENERAL CASE → CONVEX OPT

These methods can handle a class of problems

The models are inputs to the solvers

Model-based methods

Advantages

- Formulate the problem once
- Separates problem formulation from input data
- Automatic solution techniques (e.g., solvers)
- We can add / remove constraints easily
- Guarantees of optimal solution in convex problems

Disadvantages

- It can be slower than algorithm-based methods
- Less control over the solution method

A^*

WHAT ACTION TO DO

↳ CANNOT REPR. CONSTR. ON THE PLANS

Algorithm-based methods

A* : HEURISTIC SEARCH
GBFS | INFORMED SEARCH
GUIDED "

Algorithm-based methods

- Method is specifically designed for a particular problem (or class of problems)
- Often rely on "rules of thumb" to guide the method to a good solution
- Examples: ≠ HEURISTICS IN AI
  - OPT → Heuristics (often problem-dependent) – we will study this later
  - Metaheuristics (problem-independent techniques that can be applied to a class of problems) – we will study this later

LNS

Algorithm-based methods

Advantages

- It can usually solve the problem much faster
- Tailor solution method

Disadvantages

- Tailored to a specific problem statement
- New constraint → rewrite algorithm?
- Time-consuming to develop
- Trial-and-error – intuition does not always work

Summary

We've looked at

- Some optimisation problems
- Formulating a problem
- Model-based and algorithm-based methods

→ OPEN SOURCE LIB. → L.S. MH

We will look at most of these aspects in more detail over the coming weeks.

We will start with linear problems.