Tutorial - 4

1. $T(n) = 3T(n/2) + n^2$

$\rightarrow$ $a = 3, b = 2, f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

$\therefore n^{\log_b a} < f(n)$

$\therefore T(n) = \theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$\rightarrow$ $a = 4, b = 2, f(n) = n^2$

$n^{\log_b a} = n^{\log_2 4} = n^{\log_2 2^2}$

$= n^2$

$\therefore n^{\log_b a} = f(n)$

$\therefore T(n) = \theta(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

$\rightarrow$ $a = 1, b = 2, f(n) = 2^n$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$n^{\log_b a} < 2^n$

$\therefore T(n) = \theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$\rightarrow$ $\because$ a is a function

$\Rightarrow$ Master's theorem is not possible

5. $T(n) = 16T(n/4) + n$

$\rightarrow$ $a = 16, b = 4, f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^2$

$\therefore n^{\log_b a} > f(n)$

$\therefore T(n) = \theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n$

$\rightarrow$ $a=2, b=2, f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$f(n) > n^{\log_b a}$

$\therefore T(n) = \theta(n\log n)$

7. $T(n) = 2T(n/2) + n/\log n$

$\rightarrow$ $a=2, b=2, f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$\therefore n^{\log_b a} < f(n)$

$\therefore T(n) = \theta(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$\rightarrow$ $a=2, b=4, f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.51}$

$\therefore n^{\log_b a} < f(n)$

$\therefore T(n) = \theta(n^{0.51})$

9. $T(n) = 0.5T(n/2) + 1/n$

$\rightarrow$ $\because a < 1$

$\therefore$ Master's theorem not applicable

10. $T(n) = 16T(n/4) + n!$

$\rightarrow$ $a=16, b=4, f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^2$

$\because n^2 < n!$ i.e. $n^{\log_b a} < f(n)$

$\therefore T(n) = \theta(n!)$

11. $T(n) = 4T(n/2) + \log n$

$\rightarrow$ $a=4, b=2, f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \theta(n^2)$

12. $T(n) = sqrt(n)\, T(n/2) + \log n$

$\rightarrow$ $\because$ a is not constant

$\therefore$ Master's theorem not applicable

13. $T(n) = 3T(n/2) + n$

$\rightarrow$ $a = 3, b = 2, f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{\log_b a} > f(n)$

$\therefore T(n) = \theta(n^{1.58})$

14. $T(n) = 3T(n/3) + \sqrt{n}$

$\rightarrow$ $a = 3, b = 3, f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$\therefore n^{\log_b a} > f(n)$

$\Rightarrow T(n) = \theta(n)$

15. $T(n) = 4T(n/2) + cn$

$\rightarrow$ $a = 4, b = 2, f(n) = c * n$

$n^{\log_b a} = n^{\log_2 4} = n^{\log_2 2^2} = n^2$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \theta(n^2)$

16. $T(n) = 3T(n/4) + n\log n$

$\rightarrow$ $a = 3, b = 4, f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$\because n^{\log_b a} < f(n)$

$\therefore T(n) = \theta(n\log n)$

17. $T(n) = 3T(n/3) + n/2$

$\rightarrow$ $a = 3, b = 3, f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \theta(n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$\rightarrow$ $a = 6$, $b = 3$, $f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{\log_b a} < f(n)$

$\Rightarrow T(n) = \Theta(n^2 \log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$\rightarrow$ $a = 4$, $b = 2$, $f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$\because n^{\log_b a} > f(n)$

$\Rightarrow T(n) = \Theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

$\rightarrow$ Master's theorem is not applied as $f(n)$ is not increasing function.

21. $T(n) = 7T(n/3) + n^2$

$\rightarrow$ $a = 7$, $b = 3$, $f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$\because n^{\log_b a} < f(n)$

$\Rightarrow T(n) = \Theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

$\rightarrow$ Master's Theorem is not applied $\because$ regularity condition is isolated in case 3.