T6 G1 04 Feb 2022

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Negating Quantified Expressions

"Every student in your class has taken a course in Discrete Maths"

$\forall x\, P(x)$

where $P(x)$:- x has taken a course in Discrete Maths and Domain consists of all the students in the class

Negation :- It is false that every student in your class has taken a course in Discrete Maths

or

There is a student in your class that has not taken a course in Discrete Maths.

$\exists x\, \neg P(x)$

Conclude that

$$\boxed{\neg \forall x\, P(x) \equiv \exists x\, \neg P(x)}$$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Hly

$$\boxed{\neg \exists x\, P(x) \equiv \forall x\, \neg P(x)}$$

De Morgan Laws for Quantifiers

\# "There is a student in your class who has taken a course in Discrete Maths"

$\exists x\, P(x)$

$P(x)$:- x has taken a course in Discrete Maths

Domain consists of all the students of the class.

$\neg \exists x\, P(x)$

$\neg \exists x P(x)$
↓
Negation :→ Every student in your class has not taken a Course in Discrete Maths

Domain Consists of all the students of the class.

$$\forall x \neg P(x)$$

$$\boxed{\neg \exists x P(x) \equiv \forall x \neg P(x)}$$

What is the negation of the statement $\forall x\ (x^2 > x)$?

(a) ~~$\forall x\ (x^2 < x)$~~
(b) $\exists x\ (x^2 = x)$
(c) $\exists x\ (x^2 \leq x)$
(d) ~~$\forall x\ (x^2 \leq x)$~~

$$\neg \forall x (x^2 > x) \equiv \exists x \neg (x^2 > x)$$
$$\exists x (x^2 \leq x)$$

Negation of $\exists x (x^2 = 2)$

(a) $\forall x (x^2 = 2)$ (b) $\exists x (x^2 \neq 2)$ (c) $\forall x (x^2 \neq 2)$ (d) $\exists x (x^2 > 2)$

$$\neg \exists x (x^2 = 2) \equiv \forall x \neg (x^2 = 2)$$
$$\forall x (x^2 \neq 2)$$

"No Rabbit Knows Maths"

↓

Domain :- all Rabbits

P(x) :- "x Knows Maths"

$$\forall x \neg P(x) \equiv \neg \exists x P(x)$$

6. The statement,"Every comedian is funny" where C(x) is "x is a comedian" and F (x) is "x is funny" and the domain consists of all people.

a) $\exists x(C(x) \wedge F(x))$ ✗

b) $\forall x(C(x) \wedge F(x))$ ✗

c) $\exists x(C(x) \rightarrow F(x))$

d) $\forall x(C(x) \rightarrow F(x))$ ✓

If x is comedian then x is funny

Nested Quantifiers :→

Two quantifiers are nested if one is with in the scope of the other. such as

$$\boxed{\forall x \exists y (x+y=0)}$$

$\forall x\, Q(x)$ where $Q(x) = \exists y\, (x+y=0)$

$= \exists y\, P(x,y)$

$$\forall x \forall y \big((x>0) \wedge (y<0) \rightarrow (xy<0) \big)$$

Domain be set of real nos