L11 15 Feb 2022

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The no. of Reflexive relations on a set with 3 elements

$= 2^{n^2-n} = 2^{n(n-1)}$

Total no. of relations $= 2^{n(A \times A)}$

$= 2^9 = 512$?

1, 2, 3   1, 2, 3

$= 2^{9-3} = 2^6$ 9

(1,1) (2,2) (3,3)

No. of Reflexive Relations on a set with n elements

$= 2^{n^2-n} = 2^{n(n-1)}$

Partial ordering or Partial order

A Relation R on a set S is called a partial ordering or partial order if it is reflexive, antisymmetric and transitive.

Partial ordered set (Poset)

A set S together with a partial ordering R is called a poset.

It is denoted by (S, R)

"greater than or equal" is a partial ordering on the set of integer."

$\geq$ $\mathbb{Z}$

(i) Reflexive:→ $a \geq a \quad \forall a \in \mathbb{Z}$

$1 \geq 1, \; 2 \geq 2 \quad -1 \geq -1 \quad -100 \geq -100$

(ii) Antisymmetric: aRb and $bRa \Rightarrow a=b$

$a, b \in \mathbb{Z} \quad a \geq b, \quad b \not\geq a$

$aRb \Rightarrow b \not R a$

aRb and bRa

$\Rightarrow a=b$

$a, b \in \mathbb{Z}$ $a \geq b$, $b \not\geq a$

$3 \geq 1$

aRb and $bRa \Rightarrow \boxed{a=b}$

(iii) Transitive :- $aRb, bRc \Rightarrow aRc$

$a \geq b, b \geq c \Rightarrow a \geq c$ yes

$3 \geq 2$ $2 \geq 0 \to 3 \geq 0$

$\geq$ is a partial order

$\Rightarrow$ $(\mathbb{Z}, \geq)$ is a poset

\# Which of these are posets?

(a) $(\mathbb{R}, =)$ (b) $(\mathbb{R}, <)$ (c) $(\mathbb{R}, \leq)$ ~~(d)~~ $(\mathbb{R}, \neq)$

(a)

i) Reflexive $a=a$ $\forall a \in \mathbb{R}$

ii) antisymmetric aRb and $bRa \Rightarrow a=b$

$a=b$

$2=4$ X

(iii) Transitive $a,b,c \in \mathbb{R}$

$aRb, bRc \Rightarrow aRc$

$2=b$ X

(b)

i) Reflexive

$a<a$ X

$2<2$ X

Not Reflexive

(c)

Reflexive $a \leq a$ $\forall a \in \mathbb{R}$

Antisymmetric

$a \leq b$ and $b \leq a$

$\Rightarrow \boxed{a=b}$

$2 \leq 4$, $4 \leq 2$ X

Transitive $a \leq b, b \leq c$

$\Rightarrow a \leq c$

$2 \leq 4, 4 \leq 6$

$2 \leq 6$

(d)

Reflexive

$a \neq a$ X

Not Reflexive

Antisymmetric aRb and bRa

$\Rightarrow \boxed{a=b}$?

$aRb \Rightarrow b \not R a$

$(\mathbb{R}, \leq)$ $1, 3$

$1 \leq 3 \Rightarrow 3 \not\leq 1$

$R = \{ \cancel{(1,1)} (1,2) (1,3) \}$ Antisymmetric

$R = \{ \boxed{\cancel{(1,1)}} \ (1,2) \ (1,3) \}$ Antisymmetric

$(a,b) \in R$ but $(b,a) \notin R$

$S = \{a, b, c\}$

$P(S) =$ the set contains all the subsets of the set.

$P(S) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$

$\subseteq$ (inclusion relation) is a partial ordering on $P(S)$

$(P(S), \subseteq)$ is a poset

i) Reflexive $A \in P(S)$

$A \subseteq A$

$\{1,2\} \subseteq \{1,2,3\}$

(ii) antisymmetric $A \subseteq B$ and $B \subseteq A$

$\Rightarrow \boxed{A = B}$

$\{1,2\} \subseteq \{1,2,3\}$

$\{1,2,3\} \subseteq \{1,2,3,4\}$

(iii) Transitive $A \subseteq B$, $B \subseteq C$

$\Rightarrow A \subseteq C$