

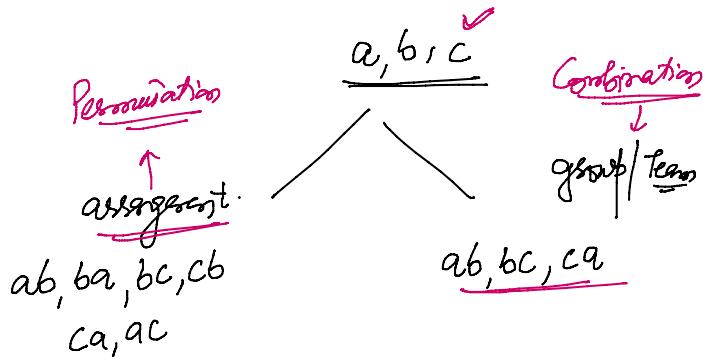
Permutation

arrangement of things in certain orders

Permutation of n objects (different) - taken

n objects at a time.
↓
choose

$$nP_r = P(n, r) = \frac{n!}{(n-r)!}$$



$$1! = 1$$

$$0! = 1$$

$$n! = n(n-1)!$$

How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

$$n=9$$

$$nP_3 = 9P_3 = \frac{9!}{(9-3)!} = 3 = 3$$

$$\frac{9 \times 8 \times 7 \times 6!}{6!} = 504$$

$$\begin{matrix} X & X & X \\ \downarrow & \downarrow & \downarrow \\ 9 \times 8 \times 7 = 504 \end{matrix}$$

Total
10 ✓

(1, 2, 3, 4, 5, 6, 7, 8, 9)

0123

How many 4-digit numbers are there with no digit repeated?

$$\begin{matrix} X & X & X & X \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 9 \times 9 \times 8 \times 7 \\ = 4536 \end{matrix}$$

$$\Rightarrow 10P_4 - 9P_3 = 4536$$

0XXX

$$\begin{array}{c}
 \begin{array}{l}
 \begin{array}{c}
 \begin{array}{c} \times \times \times \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ \checkmark \underline{9} \times \underline{9} \times 8 \times 7 \\ = 4536 \end{array}
 \end{array}
 \end{array}
 \end{array}
 \right| \Rightarrow \begin{array}{l}
 10P_4 - 9P_3 \\
 = \underline{\underline{4536}}
 \end{array}
 \quad (\text{XXXX})$$

- ① Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

$$\begin{array}{c}
 \begin{array}{l}
 \begin{array}{c}
 \begin{array}{c} \times \times \times \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 5 \times 4 \times 3 \times 2 \\ = 120 \end{array}
 \end{array}
 \end{array}
 \end{array}
 \left| \begin{array}{l}
 \begin{array}{c}
 \begin{array}{c} \times \times \times \cancel{\times} \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2 \times 3 \times 4 \times 2 \\ = 48 \end{array}
 \end{array}
 \end{array}
 \end{array}
 \end{array}$$

$\times \times \times 2$
 $24 \frac{4!}{1!} 4P_3$
 $\times \times \times 4$
 $+ 24 + 4P_3$

Permutation in such objects are not different. 48

Theorem 4 The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p_1! p_2! \dots p_k!}$.

$$\frac{n!}{p_1! p_2! p_3! \dots p_k!}$$

n objects
 \downarrow
 n (choose)

$$nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

$$\begin{array}{cccccc}
 & 1 & 1 & 2 & 2 & 3 & 4 \\
 \underline{+} & 1 & 2 & 1 & 2 & 3 & 4
 \end{array}$$

1 2 1 2 3 4

Find the number of permutations of the letters of the word ALLAHABAD.

$$= \frac{9!}{4! \times 2!} = 90$$

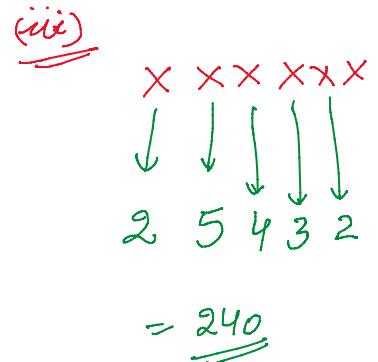
0, a

How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.

- (i) 4 letters are used at a time, (ii) all letters are used at a time,
 (iii) all letters are used but first letter is a vowel?

(i) $n=6$ $s=4$ ${}^n P_s = {}^6 P_4 = \frac{6!}{2!} = 360$

(ii) $n=6$ $s=6$ ${}^n P_s = {}^6 P_6 = 6! = 720$



In how many ways can the letters of the word PERMUTATIONS be arranged if the

- (i) words start with P and end with S, (ii) vowels are all together,
 (iii) there are always 4 letters between P and S?

(i) P S

10!
2!

(ii) Vowels e, i, o, u, a P m t n s

$$= \frac{8!}{2!} \times 5! \quad \checkmark$$

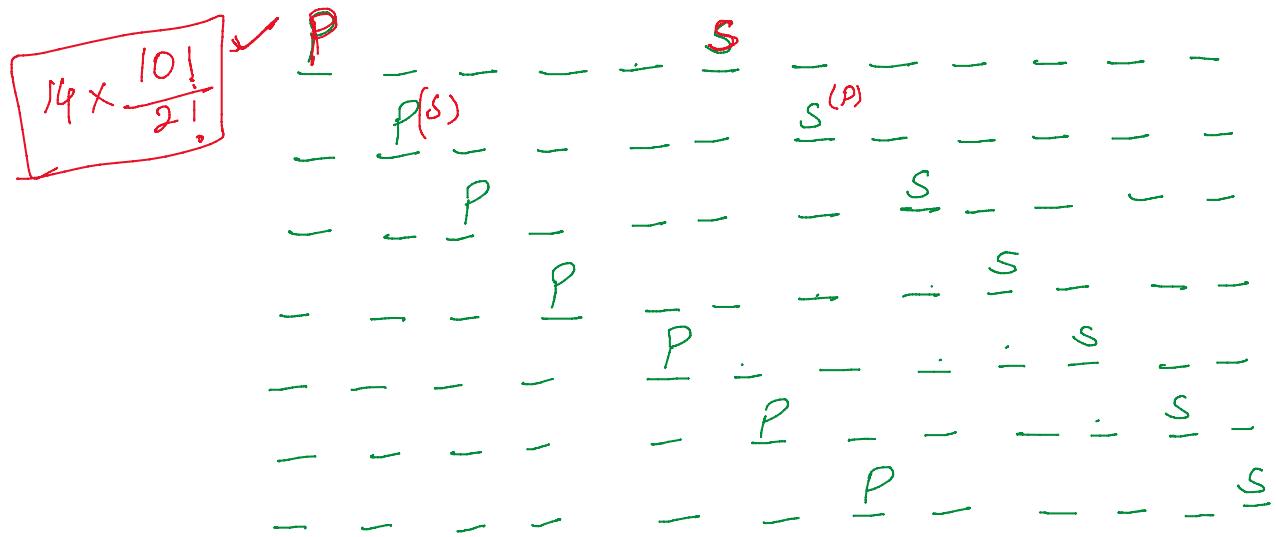
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(iii) Not together = Total - vowels are together

... together = total - vowels are together

$$\frac{12!}{2!} - \frac{8! \times 5!}{2!} = ?$$

PERMUTATIONS S



Example 11 How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of the digits is not allowed?