L31 19 April 2022

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\# Let G be a connected planar graph with v vertices and e edges and there is no circuit of length 3 in G. then

$$e \leq 2v-4.$$

Proof:- As G is connected planar graph

Euler formula $v - e + r = 2$.

$C_n,\ n \geq 3$

Min length of a cycle $= 3$

$\deg(r) \geq 4$ degree of every region ≥ 4.

Handshaking Lemma $\sum \deg(r) = 2e$

$2e = \sum \deg(r) \geq 4r$

$4r \leq 2e \Rightarrow \boxed{r \leq \frac{e}{2}}$

$v - e + r = 2$

$r = 2 - v + e \leq \frac{e}{2}$

$4 - 2v + 2e \leq e$

$2e - e \leq 2v - 4$

$\boxed{e \leq 2v-4}$

\# Is $K_{3,3}$ planar?

If $K_{3,3}$ is planar with $v = 6$ and $e = 9$

and there is no circuit of length 3.

then

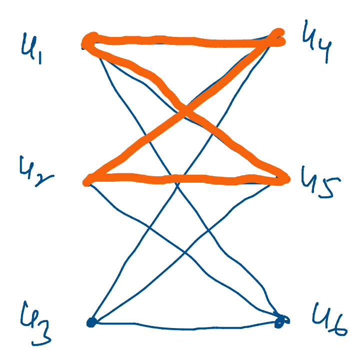
$e \leq 2v-4$.

$9 \leq 2(6)-4$

$9 \leq 12-4$

$9 \leq 8$ Not possible

$\Rightarrow$ $K_{3,3}$ is a non planar graph



Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?

$r = ?$

$v = 20$

$e = 30$

$v - e + r = 2$

$20 - 30 + r = 2$

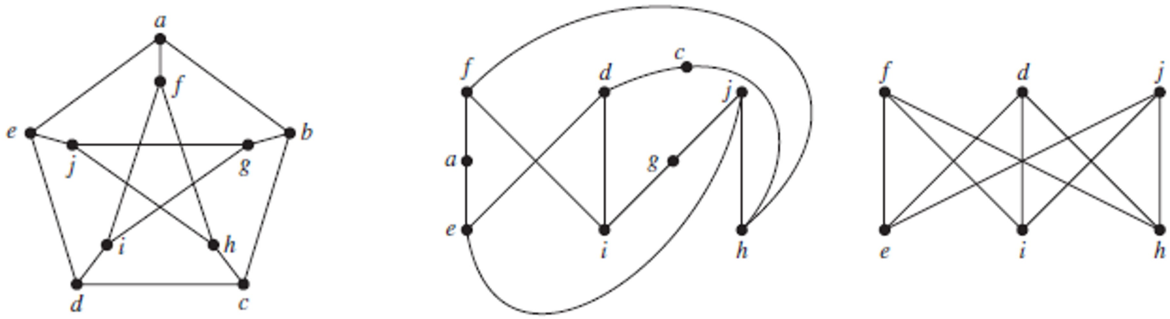
$-10 + r = 2$

$\boxed{r = 2 + 10 = 12}$

$\sum \deg(v) = 2e$

$3(\cancel{20}) = \cancel{2}e$

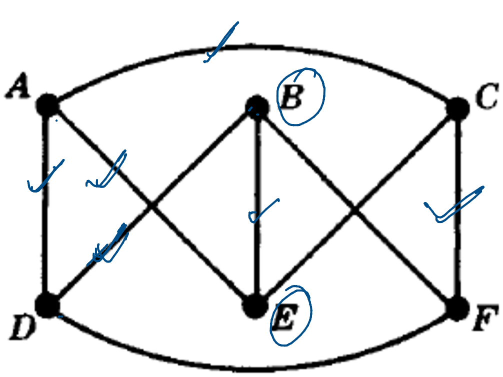
$\boxed{e = 30}$



(b) H (c) $K_{3,3}$

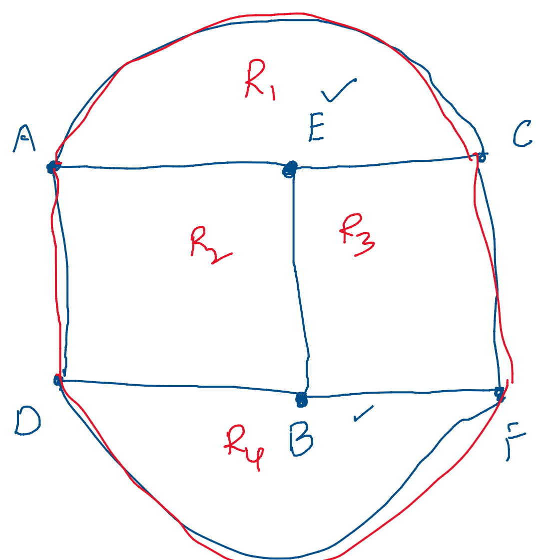
Kuratowski's Theorem

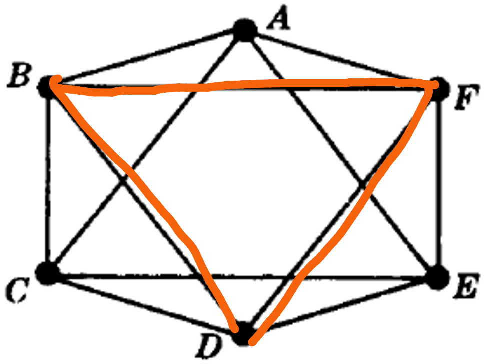
A graph is nonplanar if and only if it contains a subgraph homeomorphic to $K_{3,3}$ or K_5.



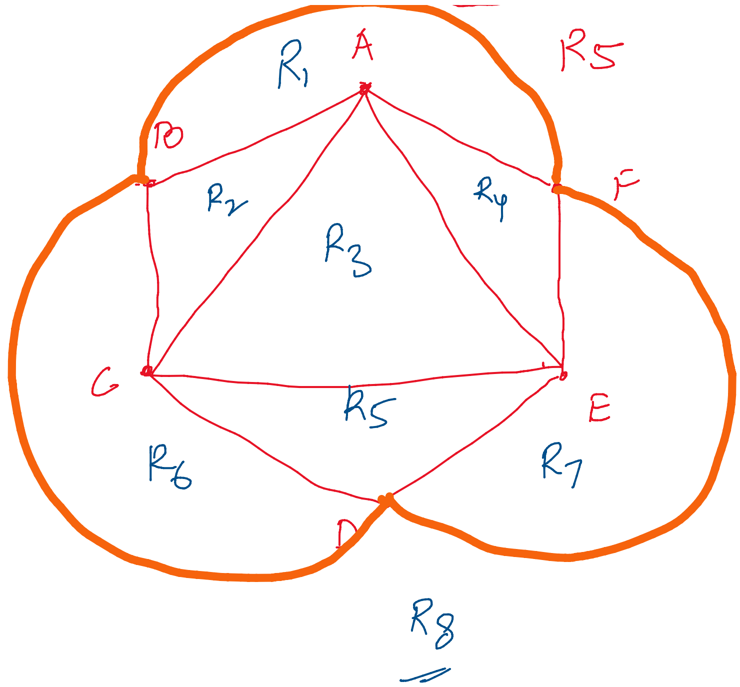
(a)

$\rightarrow$



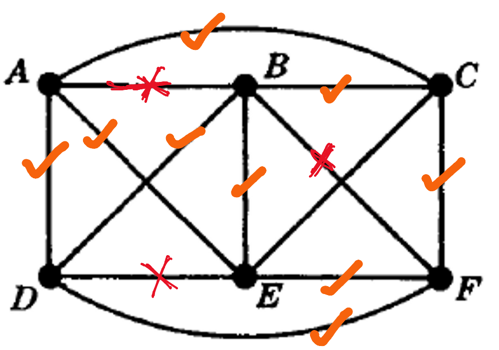


(b)

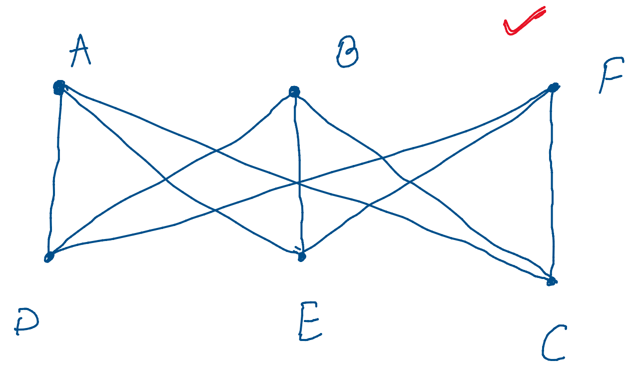


planar graph

Remove the edge (1) A-B
(2) B-F (3) D-E



(c)



Non planar graph

Subgraph of graph (c)
↓
$K_{3,3}$

THE FOUR COLOR THEOREM The chromatic number of a planar graph is no greater than four.