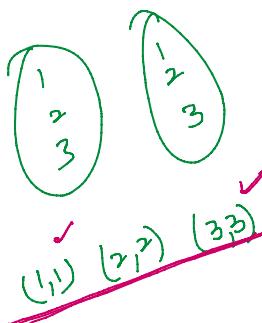


The no. of Reflexive relations on a set with 3 elements

$$= 2^{n^2-n} = 2^{n(n-1)}$$

$$\begin{aligned} \text{Total no. of relations} &= 2^{n(n-1)} \\ &= 2^9 = 512 \end{aligned}$$



$$= 2^{9-3} = 2^6$$

⑨

No. of Reflexive Relations on a set with n elements

$$= 2^{n^2-n} = 2^{n(n-1)}$$

|||

Partial ordering or Partial order

A Relation R on a set S is called a partial ordering or partial order if it is reflexive, antisymmetric and transitive.

Partial ordered set (Poset)

A set S together with a partial ordering R is called a poset.

It is denoted by (S, R)

"greater than or equal" is a partial ordering on the set of integers \mathbb{Z}

i) Reflexive: $a \geq a \quad \forall a \in \mathbb{Z}$

$$1 \geq 1, 2 \geq 2, -1 \geq -1, -100 \geq -100$$

ii) Antisymmetry aRb and $bRa \Rightarrow a = b$

$$a, b \in \mathbb{Z} \quad a \geq b, b \neq a$$

$$\begin{aligned} aRb \Rightarrow b \not\geq a \\ aRb \text{ and } bRa \\ \Rightarrow a = b \end{aligned}$$

$\forall a, b \in \mathbb{Z} \quad a \geq b, \quad b \neq a$

$3 \geq 1$

$$\begin{array}{c} aRb \text{ and } bRa \\ \Rightarrow a=b \end{array}$$

(iii) Transitive:- $aRb, bRc \Rightarrow aRc$

$a \geq b, \quad b \geq c \Rightarrow a \geq c$

$3 \geq 2, \quad 2 \geq 0 \Rightarrow 3 \geq 0$

yes

\geq is a partial order

$\Rightarrow (\mathbb{Z}, \geq)$ is a poset

* Which of these are posets?

✓ a) $(R, =)$

✗ b) $(R, <)$

✓ c) (R, \leq)

✗ d) (R, \neq)

i) Reflexive $a=a \forall a \in R$

ii) Antisymmetric $aRb \text{ and } bRa \Rightarrow a=b$

$a=b$

$2=4X$

iii) Transitive $aRb, bRc \Rightarrow aRc$

$2=3X$

i) Reflexive

$a \leq a$

$2 \leq 2$

Not Reflexive

✓ ii) Reflexive $a \leq a \forall a \in R$

iii) Antisymmetric $a \leq b \text{ and } b \leq a \Rightarrow a=b$

$\Rightarrow a=b$

$2 \leq 4, \quad 4 \leq 2X$

iv) Transitive

$a \leq b, \quad b \leq c \Rightarrow a \leq c$

v) Reflexive

$a \neq a$

NOT Reflexive

Antisymmetric

$aRb \text{ and } bRa \Rightarrow a=b$

$\Rightarrow a=b$?

$aRb \Rightarrow b \neq a$

(R, \leq)

$1, 3$

$1 \leq 3 \Rightarrow 3 \neq 1$

$R = \{ (1,1), (1,2), (1,3) \}$

Antisymmetric

$$R = \left\{ \begin{array}{c} \cancel{(1,1)} \\ (1,2) (1,3) \end{array} \right\}$$

Antisymmetric

$(a,b) \in R$ but $(b,a) \notin R$

$S = \{a, b, c\}$

$P(S)$ = the set contains all the subsets of the set.

$$P(S) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$$

\subseteq (inclusion relation) is a partial ordering on $P(S)$

$(P(S), \subseteq)$ is a poset

i) Reflexive

$$A \in P(S)$$

$$A \subseteq A$$

$$\{1, 2\} \subseteq \{1, 2, 3\}$$

(ii) antisymmetric

$$A \subseteq B \text{ and } B \subseteq A$$

$$\Rightarrow \boxed{A=B} \quad \checkmark$$

$$\{1, 2\} \subseteq \{1, 2, 3\} \quad \checkmark$$

(iii) Transitive

$$A \subseteq B, B \subseteq C$$

$$\Rightarrow A \subseteq C \quad \checkmark$$

$$\{1, 2, 3\} \subseteq \{1, 2, 3, 4\}$$