L7 02 Feb 2022

02 February 2022 10:08

All the elements in the domain are $x_1, x_2, x_3, \ldots x_n$

universal quantification $\forall x\, P(x)$

$$P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge P(x_4) \wedge \ldots \wedge P(x_n)$$

$T \wedge T \wedge T \wedge T \wedge \ldots \wedge T \rightarrow$ True

$T \wedge T \wedge F \wedge T \wedge \ldots \wedge T \rightarrow$ False

$\forall x$

Existential quantification:-

The existential quantification of $P(x)$ is the proposition

"There exists an element x in the domain such that $P(x)$"

Notation:- $\exists x\, P(x)$

$\exists x$ → existential quantifier

Let $P(x)$ denotes the statement "$x > 3$"

What is the truth value of the quantification $\exists x\, P(x)$ → True

when the domain consists of all real nos?

$P(4)$: $4 > 3$ True

Domain elements $x_1, x_2, x_3, \ldots x_n$

Existential quantification $\exists x\, P(x)$

$$P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots \vee P(x_n)$$

If any one is True then it is True

If all are false then it is false

If all are false then it is false

Let $Q(x)$ denote the statement "$x = x + 1$." What is the truth value of the quantification $\exists x\, Q(x)$, where the domain consists of all real numbers?

False

$x + 1 = x$?

What is the truth value of $\exists x\, P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4?

Domain $= \{1, 2, 3, 4\}$

$P(1) = 1^2 = 1 > 10$ ✗

$P(2) = 2^2 = 4 > 10$ ✗

$P(3) = 3^2 = 9 > 10$ ✗

$P(4) = 4^2 = 16 > 10$ ✓ True

$P(1) \vee P(2) \vee P(3) \vee P(4)$

$F \vee F \vee F \vee T$

$=$ True

Statement	True (Domain)	False
Universal $\forall x\, P(x)$	$P(x)$ is true for all values of x	There is an x for which $P(x)$ is false.
Existential $\exists x\, P(x)$	There is an x for which $P(x)$ is true	$P(x)$ is false for all values of x.

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\exists x(x^2 = 2)$

b) $\exists x(x^2 = -1)$

c) $\forall x(x^2 + 2 \geq 1)$

d) $\forall x(x^2 \neq x)$

(a) $\exists x (x^2 = 2)$ → True

$x = \sqrt{2}$ $(\sqrt{2})^2 = 2$

(b) $\exists x (x^2 = -1)$

↓

False

$i = \sqrt{-1}$

↳ imaginary

(c) $\forall x (x^2 + 2 \geq 1)$

$\forall x (x^2 \geq 1 - 2)$

$\forall x (x^2 \geq -1)$

True

(d) $\forall x (x^2 \neq x)$

false