L17 07 Mar 2022

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Complement of an element :→

Let L be a bounded Lattice

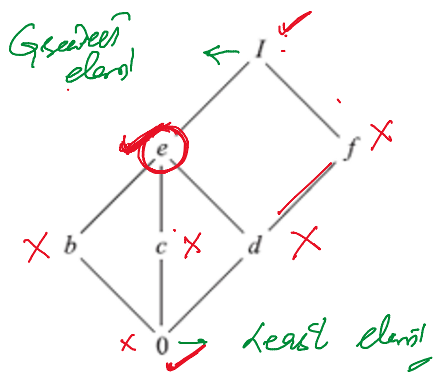
$a \in L$

The Complement of a is an element b such that

$a \vee b = 1$ and $a \wedge b = 0$, where 1 = Greatest element

0 = least element.

The complement of e and f



$l.u.b(f, x) = 1$ → Greatest element

$g.l.b(f, x) = 0$

$l.u.b(e, f) = I$

$g.l.b(e, f) = d$

a) None and b,c

b) f and e

c) None and d

d) d and b,c

$l.u.b(e, I) = I$

$l.u.b(0, e) = e$

$l.u.b(e, d) = e$

$l.u.b(e, c) = e$

$l.u.b(e, b) = e$

Complemented Lattice :→

A Lattice L is Complemented if every element of the Lattice has a Complement.

\# $A = \{a, b, c\}$

$\{P(A), \subseteq\}$ is a Complemented Lattice

Boolean Algebra :→

Let B be a set with two binary operations $+$ and $*$ elements

Let B be a set with two binary operations + and * elements 0 and 1, and a unary operation − such that these properties holds for all x, y, and z in B

① i) $x+y = y+x$
ii) $x*y = y*x$

Commutative Laws

② i) $(x+y)+z = x+(y+z)$
ii) $(x*y)*z = x*(y*z)$

Associative Laws.

③ i) $x+(y*z) = (x+y)*(x+z)$
ii) $x*(y+z) = (x*y)+(x*z)$

Distributive Laws

④ $x+\bar{x} = 0$
$x*\bar{x} = 1$

Complement Laws.

⑤ $x+0 = x$
$x*1 = x$

Identity Laws

Then B is called a Boolean Algebra

L is a set

$\vee$ join
$a \vee b = \text{l.u.b}(a,b)$

$\wedge$ meet
$a \wedge b = \text{g.l.b}(a,b)$

$I \rightarrow$ Greatest
$0 \rightarrow$ Least

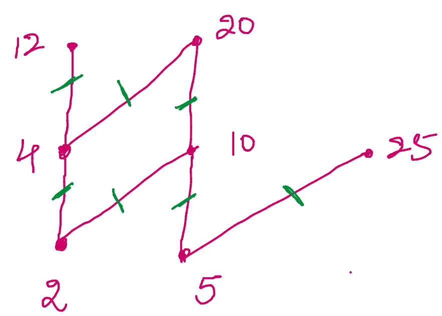
unary operation
$\rightarrow$ Complement operation
$\bar{a} = b$ means
$a \vee b = I$

$a \wedge b = 0$

L is a Boolean algebra under these operations

A bounded, distributive and the Complemented Lattice is a boolean algebra

({2, 4, 5, 10, 12, 20, 25}, |)



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