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Parameter Estimation Assignment

Q1 normal distribution

given mean $= \theta_1$

variance $= \theta_2$

$$PDF = f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mathcal{L} = \prod_{i=1} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\mathcal{L} = \left(\frac{1}{2\pi\sigma^2}\right)^n \cdot e^{-1/2\left(\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)}$$

Take ln

$$\ln(\mathcal{L}) = n.\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) + \ln\left(e^{-\frac{1}{2\sigma^2}\cdot(x_i-\mu)^2}\right)$$

$$= n\left(\ln(1\,0) - \ln(\sqrt{2\pi\sigma^2}) - \frac{1}{2\sigma^2}\sum(x_i-\mu)^2\right)$$

$$= -n\ln(\sqrt{2\pi\sigma^2}) - \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$$

For μ

$$\frac{\partial \ln(L)}{\partial\mu} = 0 \Rightarrow \frac{-1}{2\sigma^2}\sum 2(x_i-\mu)(-1)$$

$$= \frac{1}{\sigma^2}\sum(x_i-\mu) = 0$$

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$\mu = \frac{\sum (x_i)}{n} = \bar{x} = $ Sample mean.

For σ^2

$$\frac{\partial \ln(l)}{\partial \sigma^2} = 0$$

$$\Rightarrow -\frac{n}{2} \times \frac{1}{2\pi\sigma^2} \times 2\pi + \frac{1}{2\sigma^4} \sum (x_i - \mu)^2 = 0$$

$$\sigma^2 = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n}$$

Q2 $\quad \text{pdf} = {}^m C_{x_i} \theta^{x_i} (1-\theta)^{m - x_i}$

$$L = \prod_{i=1}^{n} {}^m C_{x_i} \theta^{x_i} (1-\theta)^{m - x_i}$$

$$\ln(L) = \sum_{i=1}^{n} \left(\ln ({}^m C_{x_i}) + x_i \ln \theta + ((m - x_i) \ln(1-\theta) \right)$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{(m - x_i)}{1-\theta} \right) = 0$$

$$\frac{\sum x_i}{\theta} - \frac{n.m - \sum x_i}{1-\theta} = 0$$

$$\frac{\sum x_i}{\theta} = \frac{n.m - \sum x_i}{1-\theta}$$

$$\theta = \frac{\sum x_i}{n.m}.$$