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Tutorial - 4

Masters theorem

$T(n) = aT(n/b) + f(n)$

$a \geq 1$ & $b > 1$ & $f(n)$ is asymptotically positive

1. If $f(n) < \Theta(n^{\log_b a})$ n^c

   $T(n) = \Theta(n^c)$

2. $f(n) = n^c$

   $T(n) = \Theta(n^c \log n)$

3. $f(n) > n^c$

   $T(n) = \Theta(f(n))$

$c = \log_b a$

Compare n^c & $f(n)$

1) $T(n) = 3T(n/2) + n^2$

$a = 3, b = 2$

$c = \log_2 3 = 1.5$

$f(n) = n^c$

$n^2 > n^{1.5}$

$\therefore T(n) = \Theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2$

$c = \log_2 4$

$c = 2$

$f(n) = n^c$

$n^2 = n^2$

$T(n) = \Theta(n^2 \log n)$

3) $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$c = \log_2 1$

$c = 0 \quad (n^0 = 1)$

$2^n > 1$

$T(n) = \Theta(2^n)$

4) $T(n) = 2^n T(n/2) + 2^n$

$a = 2^n, b = 2$

$c = \log_2 2^n$

$c = n$ $\quad n^c = n^n$

$f(n) < n^c$ $\quad \Theta(n^n)$

5) $T(n) = 16\,T(n/4) + n$

$a = 16,\ b = 4$

$c = \log_4^{16}$

$c = 2$

$f(n) < n^2$

$T(n) = \Theta(n^2)$

6) $T(n) = 2T(n/2) + n\log n$

$c = \log_2^2 = 1$

$f(n) > n^c$

$n\log n > n$

$T(n) = \Theta(n\log n)$

7) $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2$

$c = \log_2^2 \quad c = 1$

$\frac{n}{\log n} < n^1$

$f(n) < n^c$

$\Theta(n)$

8) $T(n) = 2T(n/4) + n^{0.5}$

$a = 2,\ b = 4$

$c = \log_4^2 \quad c = 1/2$

$f(n) = n^c$

$n^{0.5} = n^{0.5}$

$T(n) = \Theta(n^{0.5}\log n)$

9) $T(n) = 0.5T(n/2) + 1/n$

$c = \log_2^{1/2} = -1$

$n^{-1} = n^{-1}$

$T(n) = \Theta(n^{-1}\log n)$

10) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4$

$c = \log_4^{16} \quad c = 2$

$n! \quad n^2$

if $(n \geq 3)\ n! > n^2$

$\Theta(n!)$

if $(n < 3)\ n^2 > n!$

$\Theta(n^2)$

11) $T(n) = 4T(n/2) + \log n$

$a = 4,\ b = 2$

$c = \log_2^4 \quad c = 2$

$\log n < n^2$

$T(n) = \Theta(n^2)$

12) $T(n) = \sqrt{n}\,T(n/2) + \log n$

$a = \sqrt{n},\ b = 2$

$c = \log_2^{n^{1/2}}$

(13) $T(n) = 3T(n/2) + n$

$a=3, b=2$

$c = \log_2^3 = 1.5$

$f(n) < n^c$

$n < n^{1.5}$

$T(n) = \theta(n^{1.5})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a=3, b=3$

$c = \log_b^a = 1$

$f(n) = n^{1/2}$

$n^c = n^1$

$f(n) < n^c$

$T(n) = \theta(n)$

(15) $T(n) = 4T(n/2) + cn$

$a=4, b=2$

$c = \log_2^2 \Rightarrow 2$

$f(n) < n^c$

$cn < n^2$

$T(n) = \theta(n^2)$

(16) $T(n) = 3T(n/3) + n/2$

$a=3, b=3$

$c = \log_b^a = 1$

$f(n) = n/2$

$n^{c'} = n$

$f(n) < n^c$

$T(n) = n$

(17) $T(n) = 3T(n/4) + n\log n$

$a=3, b=4$

$c = \log_4^3$

$f(n) = n\log n$

$n^c = n^{0.7}$

$f(n) > n^c$

$T(n) = n\log n$

(18) $T(n) = 6T(n/3) + n^2\log n$

$a=6, b=3$

$c = \log_3^6 = 1.6$

$f(n) > n^c$

$T(n) = \theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n\log n$

$a=4, b=2$

$c = \log_2^4 = 2$

$f(n) = n\log n$

$n^c = n^2$

$n\log n < n^2$ $T(c) = \theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2\log n$

$a=64, b=8$

$c = \log_8^{64} \rightarrow 2$

$f(n) \rightarrow n^2$

$n^2\log n > n^2$

$T(c) = \theta(n^2 \log n)$

(21) $T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3$

$c = \log_3^7 = 1.77$

$f(n) > n^c$

$n^2 > n^{1.77}$

$T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2$

$\log_2^1 = 0$

$f(n) > n^c$

$n(2 - \cos n) > n^0$

$T(n) = \Theta(n(2 - \cos n))$