To Join or not to Join: Coalition Formation in Public Good Games

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Abstract

Commitment mechanisms such as coalitions can increase public good provisioning. This paper studies the role of social preference in a two stage public good game where heterogeneous agents have the option to join the coalition. In the first stage, agents choose whether to join a coalition and in the second stage, coalition votes on whether its members will contribute. I find that individuals with stronger social preferences are more likely to join the coalition and vote for the coalition to contribute to the public good. I further show that higher return from public good leads to more people joining the coalition and contributing to the public good. These results hold whether the coalition's decision is determined by a majority voting or a unanimous voting rule.

Keywords: Coalition, Social Preference, Public Good

JEL Classification: H4, C7

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Introduction

Conflict between social and private interests leads to under provisioning of public good. Coalitions where subgroups of individuals agree to act together to produce a public good presents a solution to this problem. Coordination is achieved because coalition members decide jointly on provisioning of public good based on the terms of coalition Marginal per capita return is an important determinant of contributions to public good. MPCR is the ratio of marginal benefit to marginal cost of privately providing a public good. For every dollar a person spends on privately providing the public good, the MPCR measures how much the individual gets back. MPCR is assumed to be less than 1. High marginal per capita return (MPCR) leads to increased cooperation and decreased free riding (Chaudhuri, 2011). However there is an inverse relationship between equilibrium number of coalition members and marginal per capita return (MPCR) (Kolstad, 2012). Using heterogeneity in social preferences, I study the conditions required to ensure a positive relationship between coalition size and MPCR.

My paper analyzes the decision problem of economic agents who have the option to join coalitions. International Environment Agreements (IEA) such as United Nations Framework Convention on Climate Change (UNFCCC), Kyoto Protocol are examples of existing coalitions. Although coalitions such as IEAs are observed in practice, the conditions under which they successfully form are not well understood. Additionally while social preferences, such as warm glow, altruism and inequality aversion, are well documented in other economic environments, their effect on the formation likelihood and size of coalitions to produce public goods is not well understood. In my paper I incorporate heterogeneity in social preferences which yields new conditions resulting in higher contributions to public goods.

I study the role of social preference in a two stage public good game. In the first stage, heterogeneous agents choose whether or not to join a coalition and in the second stage, the coalition votes whether its members will contribute or not. Social preferences are assumed to be Rawlsian where payoffs are strictly increasing in both own earnings and the payoff of the least-well off member of society. Following other well known social preference models I assume that utility is a weighted average of selfish and social preferences (Bolton and Ockenfels, 2000; Fehr and Schmidt, 1999). Individual's weight on monetary payoff and social preference is assumed to be private information.

My main findings are as follows. I find that individuals with pro-social preferences are likely to join the coalition and contribute to the public good. I also find that the likelihood of joining and contributing is increasing in the return from public good (MPCR). I extend the model to incorporate the additional benefit an individual receives from having the right to vote in a coalition. I find the coalition size increases as individuals join the coalition to avail this benefit. Interestingly , these gains are not translated towards contributing to the public good.

My research is motivated from the observation that while public goods theory predicts free riding and inefficient outcomes, experimental results find support for cooperation with contribution rates to public goods as high as 40-60 percent of the efficient level (Ledyard, 1995). These experimental results motivated research on the importance of mechanisms or institutional environment which can help in achieving the optimal outcome or reduce free riding. Communication between the participants regarding their strategies or intentions (Isaac and Walker (1988), Ostrom (2000)) can increase contributions to a public goods game. Chen (1996) and (Kurzban et al., 2001) use "pledge-to-contribute" as a "commitment" mechanism to increase cooperation. Punishment can also facilitate higher levels of cooperation by allowing people who contribute to punish "free-riders" (Ostrom et al., 1994).

My paper uses coalitions, a mechanism which uses commitment to increase cooperation (Barrett (1994),Hoel and Schneider (1997)). In a society where individuals have an incentive to free ride, existing models predict only few people will join coalition to maximize joint welfare. These standard models of coalition are unable to explain existence of large IEA. For instance, higher participation is possible if IEAs target lower abatement[1] such that global emissions reduce in equilibrium (Barrett (2002), Finus and Maus (2008)).

Other strand of literature studies the relationship between gains from coopera-

[1]as compared to optimal abatement, which would maximize joint welfare maximization

tion and coalition size. High marginal per capita return (MPCR) leads to increased cooperation and decreased free riding (Chaudhuri, 2011). Although gains from cooperation[2] increase in MPCR, size of the coalition decreases with MPCR (Barrett, 1994). Thus there exists an inverse relationship between coalition size and gains from cooperation (Komisar, 1969; Barrett, 1994; Kolstad, 2012). For instance, higher participation was seen in Montreal Protocol (IEA on ozone depletion) because gains to cooperation were very small (Barrett, 2003). Recently researchers have started studying the impact of MPCR on coalition size. Large coalitions can exist even with high MPCR ((Burger and Kolstad, 2009),Kosfeld, Okada and Riedl (2009)). However, Dannenberg, Lange and Sturm (2014) in their paper find a trade-off between participation and commitment, where coalitions with voluntary participation are less effective in facilitating cooperation compared to when all players are forced to participate. My paper derives the conditions required for existence of a large sized coalition even in the presence of high MPCR.

In my model I assume individuals have Rawlsian preferences, where they care about the least advantaged person in the society(Rawls, 2001). Voluntary contribution to public goods, points to the presence of social preferences. Individuals are willing to cooperate provided others in their group also cooperate (Fehr and Fischbacher, 2002; Charness and Rabin, 2002; Rabin, 1993). Individuals are also motivated to contribute towards public good because they care for equitable distribution of resources or equal outcomes (Fehr and Schmidt, 1999). To test models of various social preferences, Charness and Rabin (2002) designed a range of experiments and find that people are concerned about increasing social welfare, especially for low-payoff recipients. In their experiment, Engelmann and Strobel (2004) compare a number of social preferences and also find that a combination of efficiency concerns and maximin preferences can explain their findings. [3]

I use backward induction to derive contribution thresholds in Stage II both for

[2] gains from cooperation is defined as welfare difference between a full cooperative outcome and a noncooperative outcome.

[3] Pure Rawlsian social preferences are, interpretable as maximin preferences where utility depends only the welfare of the least well-off member of society.

the individuals who join the coalition and those who do not. Using this information, thresholds can be derived to determine who will join the coalition in stage one. I find that individuals with stronger social preferences are more likely to join the coalition and vote for the coalition to contribute to the public good. I further show that higher return from public good(MPCR) leads to more people joining the coalition and contributing to the public good. High MPCR also translates to a higher likelihood of the coalition contributing to the public good. These results hold whether the coalition's decision is determined by a majority voting or a unanimous voting rule or not.

I next extend the model by incorporating the benefit individuals receive from having the right to vote. I find the stage II threshold for contribution remains unchanged. However in stage I, the threshold for joining changes so that, if the benefit from voting is above a certain cutoff, the majority does not contribute to the public good and vice versa. In other words, higher benefits from voting increases the size of the coalition however the likelihood of a coalition contributing towards the public good decreases[4].

My paper contributes to three strands of literature. First, I show how incorporation of social preferences leads to an increase in coalition size. Closely related works include Kosfeld, Okada and Riedl (2009), Ringius, Torvanger and Underdal (2002) Kolstad (2014). In Ringius, Torvanger and Underdal (2002) identify 'fairness' as a motivation for countries in environmental negotiation. The study also analyzes various IEA's negotiations and finds considerations of fairness and equity to be building characteristics of these negotiations. Kosfeld, Okada and Riedl (2009) analyzes the impact of institution formation using the Fehr and Schmidt (1999) model of inequity aversion. The authors find that the presence of social preferences allows individuals to select the grand organization(full membership) as an equilibrium. Kolstad (2014) assumes homogeneous Charness and Rabin (2002) style preferences and shows that social preferences lower the threshold for cooperation but also reduces coalition size. This result stands in contrast to some of the experimental evidence and also does

[4]The results can also be interpreted in terms of the cost of isolation individuals face from not being in a group

not examine the effect of changing MPCR. In contrast, my study models agents with heterogeneous social preferences and examines outcomes when MPCR is allowed to vary. I find that coalition size, contributions and efficiency increases with MPCR and pro-social preferences.

Secondly my paper relates to research on group identity and voting. For individuals having both social and selfish preferences, social preferences dominate and makes it rational for individuals to vote even in large elections (Edlin, Gelman and Kaplan, 2007). Using this as motivation, in an extension, I assume individuals receive benefit from voting. My results suggest that additional benefit from voting leads to an increase in the size of the coalition but does not guarantee contribution to public goods. These results will be symmetric when we explore this in terms of an individual's cost for not being a part of a group instead of a benefit she enjoys from voting. This is motivated from various experiments (Chen and Li (2009), Ockenfels and Werner (2014)) finding evidence of in-group favoritism which arises because group identity impacts individuals choices and distribution of preferences. I find that individuals join the coalition to avoid the cost of isolation, however they might not end up contributing based on their preferences.

Third, my paper also uses heterogeneity in a society to derive the threshold for individuals to contribute. Heterogeneous preferences can explain variation in cooperation in a society (Gunnthorsdottir, Houser and McCabe, 2007). In their voluntary contribution mechanism (VCM) public goods experiment, authors classify the subjects into 'free riders' (contributes 30 % or less of his/her endowment) and 'cooperators' (contributes more than 30 %) based on their first round contribution. Heterogeneous preferences can also be used to explain the decline in cooperation in these experiments due to the presence of free riders Fischbacher and Gachter (2010). My results show that in a heterogeneous society, if individuals with pro-social preferences are in the majority, then public goods will be provided.

The paper is organized as follows: Section 1 theoretically analyzes the two stage public goods game. Section 2 extends the model to incorporate benefit from voting. Section 3 discusses the results. Section 4 concludes.

1 Model

Let $N = 1, 2, \cdots, n$ denote the set of players. Each player has a unit endowment and participates in a two-stage public goods game. In Stage I, each player must decide whether to join a coalition or not. Players that chose not to join the coalition are called *fringe members* and denoted by F. Before Stage II begins, size of the coalition is announced. Let the size of the coalition be denoted by M. Thus $N = M \cup F$ and $|M|$ and $|F|$ denote their respective cardinalities.

In Stage II of the game, using majority voting rule, members of the coalition decide whether or not they will contribute to the public good. Contributions are assumed to be binary, i.e., individuals in the coalition will either contribute their entire endowment or nothing depending on the outcome of the majority voting. Note that for $i \in M$, everyone follows the outcome of the majority voting procedure. For $i \in F$, however, each player independently decides whether they wish to contribute their entire endowment or not. Let F' denote the *fringe members* who contribute to the public good,i.e., $F' \subseteq F$. Let Q denote the total number of individuals contributing to the public good.[5]

Each individual's payoff is a convex combination of a pecuniary and non-pecuniary component. Let λ_i be the weight of the pecuniary component. I assume that λ_i is private information and is also uniformly distributed between 0 and 1 [6]. The non-pecuniary component is intended to capture social preferences. I will assume that social preferences are Rawlsian in the sense that players care about the player who gets the lowest pecuniary payoff.

Let's denote the action set of a player by $e_i \in \{0, 1\}$, where $e_i = 1$ implies that the players contribute to public good and $e_i = 0$, implies that they do not. The marginal per capita return (henceforth MPCR) for player i is denoted by $\gamma > 0$. Payoff from the public good when Q individuals contribute is given by γQ.

[5]It is easy to see that for coalitions members, if the majority votes to contribute, then Q consists of $|M|$ and the subset of fringe members $F' \subseteq F$ that chose to contribute to the public good. If the majority chooses not to contribute, then Q contains only those fringe members that contribute, i.e., $Q = |F'|$.

[6]The results are robust to specification for any distribution as I discuss in Discussion section

Individuals who contribute receive γQ as their payoff. Those who do not contribute earn the higher payoff of $1+\gamma Q$. Individuals who do not contribute are "free-riders" and receive benefit from contribution of other individuals in the society. Their total payoff is their endowment and the gains from public good (γQ)

Now let's see in detail how non-pecuniary payoff is calculated. Non-pecuniary payoff or social preference in the model is represented by pecuniary payoff of the least well off person. Note that when no one is contributing, everyone receives their endowment which is 1, in this case, payoff of the least well off person is also 1. In other scenarios, payoff of the least well off person is the total contributions from public good. Thus according to the setup the lowest pecuniary payoff of any $i \in N$ is given by γQ, i.e., individuals for whom $e_i = 1$. With slight abuse of notation let e_i denote the strategy of player i and e_{-i} the strategy of the remaining players. Then the payoff of a player i is given by:

$$\pi_i(e_i, e_{-i}) = \begin{cases} \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q = \gamma Q & \text{for } e_i = 1, Q > 1 \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q & \text{for } e_i = 0, Q > 1 \end{cases} \tag{1}$$

The first term in each expression is the weighted pecuniary payoff and the second term is the non-pecuniary payoff. Recall that the weights are private information. Effectively, this simplification means that players that contribute to the public good ($e_i = 1$) have payoff equivalent to standard public goods players while free-riders ($e_i = 0$) assign a weight to the payoffs of those who contribute ($e_i = 1$).[7]

Using a 3 player example I now illustrate the role of heterogeneous weights and coalitions in determining the equilibrium public good outcome.

Example 1 . Let $N = \{1, 2, 3\}$. and $\lambda_1 = 0.2$, $\lambda_2 = 0.4$ and $\lambda_3 = 0.6$. The payoff in a standard public good equilibrium where no one contributes is 1. If there is no coalition and one individual contributes to the public good, the payoff for the individual will be: $\pi_i = \lambda_i(\gamma) + (1-\lambda_i)(\gamma) = \gamma$. Thus, there is no equilibrium in

[7] An interesting possibility that can arise in this model is when the coalition decides not to contribute to the public good, i.e, $e_i = 0, \forall\ i \in M$, and some fringe members decide to contribute,i.e., $|F'| > 0$. Then the situation is reversed, since now the members of F' have the lowest pecuniary payoff and not members of M.

which only 1 player contributes, since $\gamma < 1$. If two people contribute their payoff will be 2γ[8]. It is beneficial for individuals to contribute if $\gamma > 1/2$. Similarly, if three individuals contribute the payoff will be 3. It is beneficial for everyone to contribute if $\gamma > 1/3$. Thus, when there is no coalition there can be multiple equilibria and everyone independently will not contribute as shown here.

This example illustrates how the coalition formation stage helps achieve a better equilibrium outcome by facilitating coordination. As we will see in Proposition II, players will join coalition if $\lambda_i \leq \gamma$. For instance, if MPCR $(\gamma) = 0.5$, individuals with $\lambda_i = 0.2$ and 0.4 will join the coalition and if $\gamma > 0.6$, everyone will join the coalition. Thus, although λ_i is private information, the coalition stage automatically sorts the players and ensures a unique equilibrium as individuals with $\lambda_i \leq \gamma$ will be in the coalition. Joining a coalition reduces uncertainty; it provides a signal to individuals about coordination.This makes sure that there will be no free riding among these players making it is worthwhile for individuals with $\lambda_i \leq \gamma$ to join the coalition.

1.1 Stage II: Contribute or not to the public good

I use backward induction to solve my two-stage public good game of incomplete information. Before Stage II begins, everyone knows the size of coalition (denoted by M). The coalition members use majority voting rule to decide if the coalition contributes to the public good. The fringe members decide independently if they would like to contribute. Let's move to the first proposition.

The action set of a player when they are voting is denoted by $v_i \in \{0, 1\}$ where $v_i = 1$ implies $i \in M$ votes to contribute to public good and $v_i = 0$ implies $i \in M$ does not vote. With slight abuse of notation, v_i is the strategy of player i and v_{-i} is the strategy of the other players. Vote to contribute is not the same as contributing since the coalition decides based on majority. The outcome from e_i will be the same as v_i when the individual votes to contribute and coalition contributes as well. Also when individual does not vote to contribute and the majority is not satisfied, the

[8]Here I assume that only players joining the coalition contribute.

outcomes will be the same.

Individual $i \in M$ will vote to contribute if the payoff from contributing to public good is higher than the payoff from not contributing to public good. Similarly a fringe member $i \in F$ will contribute to public good if payoff from contributing to public good is higher than not contributing to public good. Comparing these payoffs helps us derive he threshold for contribution and leads us to first proposition.

Proposition I: Assume that M is known.

a)If $\lambda_i \leq \hat{\lambda}$, where $\hat{\lambda} = \gamma M$, then $v_i = 1 \; \forall i \in M$.If $\lambda_i > \hat{\lambda}$ then $v_i = 0 \; \forall i \in M$.

b)If $\lambda_i \leq \hat{\lambda}$, where $\hat{\lambda} = \gamma$, then $e_i = 1 \; \forall i \in F$. If $\lambda_i > \hat{\lambda}$, then $e_i = 0 \; \forall i \in F$.

PROOF:Appendix

In part a of the proposition, if the given cutoff is satisfied, the individual will vote yes to contribute to the public good. The cutoff $\lambda_i \leq \gamma M$ can be interpreted as the probability $i \in M$ will vote yes to contribute to the public good. An increase in the return from public good (γ), leads to change in the cutoff such that the probability of voting yes for contribution increases. The probability is also increasing in the size of coalition (M). As I explain in the appendix, the cutoff $\lambda_i \leq \gamma M$ can also be interpreted as the comparison between gain to the pivotal voter from voting to contribute ($\gamma M'$) and gain from not voting (λ_i). The pivotal voter decides whether the coalition will contribute or not hence if the pivotal voter contributes then the coalition contributes, thereby leading to a payoff of γM as the marginal gain from contribution. If the pivotal voter does not contribute then the coalition members will marginally earn from their private account $\lambda_i(1) = \lambda_i$. This interpretation is assuming fringe members contributing is fixed at F'.

I also check for the robustness of this equilibrium. Keeping actions of other players fixed, I check for deviation by an individual. Deviation refers to change in player i's actions. I allow for deviation by letting player i revise his/her actions with probability r. For instance if player i decides to join the coalition and also votes to contribute to public good, however does not contribute his/her endowment for public good. I find that $\lambda_i \leq \hat{\lambda}$ where $\hat{\lambda} = \gamma M$ guarantees no deviation by individual i[9].

From Equation 1, we can see the marginal gain from free riding is λ_i whereas marginal from contributing(independently) is γ. The cutoff in Proposition II can be interpreted as the comparison between fringe member's marginal gain from free riding(λ_i) and fringe member's marginal gain from contributing (γ). An individual i contributes if gain from contribution is higher than the gain from free riding i.e. $\lambda_i \leq \gamma$. This is also a general result from a public goods game without coalition.

The cutoff $\lambda_i \leq \gamma$ in part b can be interpreted as the probability $i \in F$ will contribute to the public good. As γ increases, the cutoff will increase. This will lead to an increase in the likelihood that fringe member will contribute to the public good. The cutoff is independent of the coalition members. This is because payoff of fringe members is an increasing function of the minimum payoff received by anyone. In order to increase the minimum payoff, which will be highest when everyone is contributing, the fringe members contribute irrespective of the coalition members.

1.2 Stage I: Decision to join the coalition

I now use the cutoffs from Stage II to derive cutoffs for joining the coalition in Stage I. Based on the cutoff $\lambda_i \leq \gamma M$ (the coalition members vote yes to contribute) and $\lambda_i \leq \gamma$(the fringe members will contribute), we have the following three possible range for values of λ_i:

Case(i) $\lambda_i \leq \gamma$ which also implies $\lambda_i \leq \gamma M$

Case(ii) $\gamma < \lambda_i \leq \gamma M$

Case(iii) $\lambda_i > \gamma M$.

Here M refers to any size of the coalition which will be formed. Again M is assumed to be odd here and for the majority at least $\frac{M+1}{2} = m'$ people should vote to contribute to the public good. A coalition decides to contribute or not based on majority rule, thus if there are people who have the cutoff $\lambda_i \leq \gamma M$ in majority, then the coalition will contribute.

I again compare the payoff when $i \in N$ decides to join the coalition and $i \in N$

[9]Proof in Appendix

decides not to join the coalition. The payoffs will incorporate the outcomes from stage II. From stage I we derived the probability that $i \in M$ will contribute to public good is given by $\lambda_i \leq \gamma M$. I will use these values in Stage I to derive the cutoff for joining the coalition since the final payoff depends on if coalition contributes to public good or not.

If payoff from joining coalition is higher than the payoff from not joining the coalition then $i \in N$ joins the coalition. Comparing payoffs in all the three cases described above leads us to the second Proposition of this paper.

Let's denote the action set of a player when they are deciding to join the coalition by $j_i \in \{0, 1\}$ where $j_i = 1$ implies $i \in M$ decides to join the coalition and $j_i = 0$ implies $i \in M$ does not join the coalition. With slight abuse of notation, j_i is the strategy of player i and j_{-i} is the strategy of the other players.

Proposition II:In the subgame perfect Nash equilibrium, if $\lambda_i \leq \gamma$ then $j_i = 1$. If $\lambda_i > \gamma$ then $j_i = 0$.

PROOF:APPENDIX

From Proposition II, we find that individuals with relatively lower weight on pecuniary payoff ($\lambda_i \leq \gamma$) will join the coalition and individuals with higher weight on pecuniary payoff will not join the coalition. Further note that size of equilibrium coalition size M is given by individuals who satisfy $\lambda_i \leq \gamma$. Therefore λ_i can be interpreted as the proportion of individuals in the population that will contribute to the public good.

As γ increases, more individuals satisfy the cutoff ($\lambda_i \leq \gamma$), thus leading to more people joining the coalition. Higher γ also leads to an increase in the cutoff of the people who will vote yes to contribute to public good in Stage II ($\lambda_i \leq \gamma M$). Thus an increase in γ or higher benefits of cooperation can increase the size of a coalition and also increase the likelihood by which an existing coalition will contribute to the public good. Hence the results show that an increase in return from public good i.e. MPCR leads to a bigger coalition size which also has a higher likelihood to contribute to the public good.

2 Model Extension

2.1 Benefit from voting

I motivate this extension from the literature "Why do people vote". In their paper Edlin, Gelman and Kaplan (2007) show that it is rational for people to vote if they care for others and even a self interested individual who votes will vote for the common good and not for individual interests. The authors argue that model of vote choice will work on social utility function and not selfish utility function. The authors also provide evidence for their model from survey on potential voters: "preferences on national candidates and issues are strongly correlated with views on what would be desirable for the country, and more weakly correlated with opinions about personal gain". In political science literature, voters consider themselves as part of large groups (Uhlaner, 1989). Such sense of belonging pushes them to vote in the hope of benefitting others in his/her network (Fowler, 2005). This social benefit model is better applied to groups rather than individuals.

In my model, although individuals who are part of the coalition have the right to vote to contribute towards the public good, there is no benefit from this voting hitherto. My model now assimilates the right to vote as an added benefit for the individuals who are part of coalitions. Based on the literature discussed, the benefit captures the sense of belonging to a larger group, utility from contributing to the common goal, etc. After incorporating this benefit Equation 1 can be re-written as:

$$\pi_i(e_i, e_{-i}) = \begin{cases} \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q = \gamma Q & \text{for } e_i = 1, Q > 1, i \in F \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q & \text{for } e_i = 0, Q > 1, i \in F \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q + \varepsilon & \text{for} e_i = 0, Q > 1, i \in M \\ \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q + \varepsilon & \text{for} e_i = 1, Q > 1, i \in M \end{cases} \tag{2}$$

The benefit from voting is added for the coalition members. I repeat the same analysis for the extended model. Using the backward induction I first calculate the cutoffs for Stage II and then for Stage I.

For Stage II results, I compare the payoff $i \in M$ receives when h/she votes to contribute v/s the payoff h/she receives when they do not contribute. Stage II results remain unchanged[10]. The coalition members vote to contribute if $\lambda_i \leq \gamma M$. The fringe members contribute if $\lambda_i \leq \gamma$. This is intuitive because only the coalition members enjoy the benefit, [11].

For Stage I, I compare the payoff when $i \in N$ decides to joint the coalition v/s the payoff when $i \in N$ decides not to join the coalition. Comparing these payoffs I derive the cutoff for Stage I which leads us to the third proposition.

Proposition III: $i \in N$ will join the coalition if $\lambda_i \leq \gamma + \varepsilon$, $i \in N$ will not join if $\lambda_i > \gamma + \varepsilon$.
PROOF: APPENDIX

Thus higher the benefit from voting, more people are likely to satisfy this cutoff and join the coalition. Note that if the benefit from right-to-vote changes from ε to ε_i, the stage I cutoff for joining the coalition changes to $\lambda_i \leq \gamma + \varepsilon_i$. Benefit in terms of ε_i describes the case when every individual derives a unique benefit from voting. Suppose the benefit from voting increases with the size of the coalition, then the benefit of each individual is εM. This implies that every individual receives higher benefit from voting as the size of the coalition increases. In this case the cutoff to joining the coalition is given by $\lambda_i \leq \gamma + \varepsilon M$. Note that if the benefit from right to vote changes from εM to $\varepsilon_i M$, the stage I cutoff for joining changes to $\lambda_i \leq \gamma + \varepsilon_i M$. Benefit in terms of $\varepsilon_i M$ would mean that each individual receives a different marginal benefit from voting and the overall benefit increases with the size of the coalition.

Recall in the benchmark model, where I had no tenable benefit from voting, individuals who join the coalition (cutoff $\lambda_i \leq \gamma$) always contribute to the public good ($\lambda_i \leq \gamma M$). However with the addition of benefit from voting, Stage II cutoff stays the same, however Stage I cutoff changes. According to Stage I cutoff, individuals who satisfy $\lambda_i \leq \gamma + \varepsilon$ will join the coalition. However individuals with $\lambda_i \leq \gamma M$

[10]Proof: Appendix

[11]There is no change in the cutoff for fringe members as they do not enjoy any benefit

vote to contribute to the public good. The relative difference in these two cutoffs will determine whether a coalition formed contributes to the public good or not. From this comparison I have the following proposition:

Proposition IV:Let M be the size of the coalition. If $\varepsilon > \gamma(2M-1)$, then a coalition of size M will not contribute to the public good.

PROOF: APPENDIX

Higher benefit from voting attracts more people to join the coalition(as compared to the benchmark model) but it is not necessary that the coalition contributes to the public good. However, as the size of the coalition increases or the benefit from cooperation increases, even with a higher benefit from voting we can expect majority to contribute. This is because as γ or M increases, the benefit from contributing: γM also increases.

It is easy to see that when $M = 2$ and $\varepsilon \leq 3\gamma$ then the coalition of size 2 contributes to the public good. Since the least value M can take is 2, we have the following corollary:

Corollary I: If $0 \leq \varepsilon \leq 3\gamma$, then majority in coalition of any size can be sustained.

Given the additional benefit from voting lies in this range, any coalition will contribute based on the majority voting rule.

2.2 Home-Alone effect

Another reason to join the coalition can be motivated using "group -identity". In their paper, Akerlof and Kranton (2000) propose a general utility function which incorporates group-identity as a motivation for behavior. Group-identity accounts not only for a person's self-image, but also the prevailing social differences in a society. Such sense of attachment in a person will likely overcome his/her selfish motivations and usher in behaviour consistent with the entire group. Through a game theoretic model, the authors establish connection between economic interactions and identity and also provide a comparative static analysis on identity related parameters. The paper also adapts these models to study gender discrimination in the workplace, the economics of poverty and social exclusion, and the household division of labor

Chen and Li (2009) use laboratory experiments to measure impact of group identity on participant social preference in a wide variety of games by evaluating effects of various components in creating group identity. The authors find that group identity has significant impact on the distribution of preferences. For instance, participants put a higher weight on in-group match's payoff in their utility function and are more likely to reward an in-group match for good behavior as compared to out-group match. Ockenfels and Werner (2014) also find evidence of in-group favouritism in their dictator game experiment as dictators transfer substantially more to participants who are in their group, compared to out group members.

My model now assumes the individuals who are not part of coalition face a cost of isolation. Based on the literature discussed the cost captures the loss from feeling sense of attachment, no group identity. This cost of isolation is represented by κ. After incorporating this cost Equation 1 can be re-written as:

$$\pi_i(e_i, e_{-i}) = \begin{cases} \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q = \gamma Q & \text{for } e_i = 1, Q > 1, i \in M \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q & \text{for } e_i = 0, Q > 1, i \in M \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q - \kappa & \text{for } e_i = 0, Q > 1, i \in F \\ \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q - \kappa & \text{for } e_i = 1, Q > 1, i \in F \end{cases} \tag{3}$$

The cost of isolation is faced by the fringe members and hence shows up in their payoff. I repeat the same analysis for the extended model. Using the backward induction I first calculate the cutoffs for Stage II and then for Stage I.

For Stage II results, I compare the payoff $i \in M$ receives when h/she votes to contribute v/s the payoff h/she receives when they do not contribute. I also compare the payoff $i \in F$ receives when h/she contribute v/s the payoff h/she receives when they do not contribute. The coalition members vote to contribute if $\lambda_i \leq \gamma M$. This is intuitive as they do not face any cost of isolation. The fringe members contribute if $\lambda_i \leq \gamma$. Stage II results remain unchanged [12].

For Stage I, I compare the payoff when $i \in N$ joins the coalition v/s the payoff when $i \in N$ decides not to join the coalition. Comparing these payoffs I derive the cutoff for Stage I which leads us to the fifth proposition.

Proposition V: $i \in N$ will join the coalition if $\lambda_i \leq \gamma + \kappa$, $i \in N$ will not join if $\lambda_i > \gamma + \kappa$.

PROOF: APPENDIX

Thus higher the cost from isolation, more people are likely to satisfy this cutoff and join the coalition. Note that if the cost of isolation changes from κ to κ_i, the stage I cutoff for joining the coalition changes to $\lambda_i \leq \gamma + \kappa_i$. Cost in terms of κ_i describes the case when every individual faces a unique cost from not being int he group. Suppose the cost from isolation increases as the size of coalition/group rises, then the cost each individual faces is κM. This implies that every individual faces a higher cost as the size of the coalition increases. In this case the cutoff to joining the coalition is given by $\lambda_i \leq \gamma + \kappa M$. Note that if this cost changes from κM to $\kappa_i M$, the stage I cutoff for joining changes to $\lambda_i \leq \gamma + \kappa_i M$. Cost in terms of $\kappa_i M$ would mean that each individual faces a unique cost from isolation and the cost increases with the size of the coalition/group.

Recall in the benchmark model, where individuals faced no cost from isolation, we saw that individuals who join the coalition (cutoff $\lambda_i \leq \gamma$) always contribute to the public good ($\lambda_i \leq \gamma M$). However in the new model, Stage II cutoff stays the same, however Stage I cutoff changes. Stage II cutoff says that individuals who satisfy $\lambda_i \leq \gamma + \kappa$ will join the coalition. However individuals with $\lambda_i \leq \gamma M$ vote to contribute to the public good. The relative difference in these two cutoffs will determine whether a coalition formed contributes to the public good or not. From this comparison we have the following proposition:

[12] Proof: appendix

Proposition VI:Let the size of the coalition be M. If $\kappa > \gamma(2M - 1)$, then a coalition of size less than or equal to M will not contribute to the public good.

PROOF:APPENDIX

Free riders are likely to join the coalition to get rid of the cost of isolation. Presence of these free riders in the coalition leads to the coalition not contributing to the public good as the majority is not satisfied. However, as γ or M increases, the benefit from contributing to the public good γM also increases, thus we need even a higher cost for the coalition to not contribute. In other words, if the cost from isolation is small, then the benefit from contributing dominates and leads to the coalition contributing to the public good.

When $M = 2$ and $\kappa \leq 3\gamma$ then the coalition of size 2 contributes to the public good. Since the least value M can take is 2, we have the corollary also holding true in this case.

Corollary II: If $0 \leq \kappa \leq 3\gamma$, then majority in coalition of any size can be sustained.

Given the cost of isolation lies int his range, individuals who join the coalition will also contribute as the majority will be satisfied.

Unlike in my benchmark model, in this model voting plays a strategic role. The threshold for a coalition contributing will not be the same in case of unanimous voting. The position of λ_i will determine if the coalition contributes to the public good. A future extension is to see what happens when I include both ε and κ in our model. The relative difference in ε and κ will also be a determining factor for a coalition to contribute.

3 Discussion

The results of my paper establish a relation between coalition formation in public goods game along with voting rule. From the results we know that individuals with

$\lambda_i \leq \gamma$ will join the coalition. Individuals who satisfy $\lambda_i \leq \gamma$ also satisfy $\lambda_i \leq \gamma M$. In other words, individuals who join the coalition will also satisfy the cutoff for contributing in Stage II. Individuals can predict from stage II that if there are people with cutoff : $\lambda_i \leq \gamma M$ in majority , coalition will contribute to the public good.

It is easy to see that an individual who joins the coalition under majority voting will also contribute to the public good under the unanimous voting rule. This is because the result is independent of how many people vote to join a contribution. It is always true that $i \in N$ with preference $\lambda_i \leq \gamma$ will join the coalition. Since everyone who will be in the coalition will have the preference $\lambda_i \leq \gamma$ and also satisfy $\lambda_i \leq \gamma M$, they will also vote to contribute to the public good. As MPCR increases, more people join the coalition and everyone will vote to contribute. Thus, the public good will be provided under unanimous voting as well.

The cutoff for joining the coalition makes sure that everyone in the coalition will be contributing to public good. This also incentivizes the individuals to contribute to the public good as they know from Stage II that all the individuals in the coalition will contribute to the public good. The chances of there being a free rider is reduced and this leads to increased cooperation. This is also consistent with Gunnthorsdottir, Houser and McCabe (2007) who find that cooperators contribution decreases when the frequency of free riders increases.

In my model,I assumed that λ_i is uniformly distributed between 0 and 1. To check the robustness of the results, lets assume that λ_i is normally distributed between 0 and 1. Recall that from Stage I we find that $i \in N$ with preference $\lambda_i \leq \gamma$ will join the coalition. The results were not based on the probability and hence on the distribution. Thus irrespective of the distribution of λ_i, $i \in N$ with preference $\lambda_i \leq \gamma$ will join the coalition.

I also test my model for a different social preference. Let us assume individuals are concerned about the social surplus in the economy and the minimum payoff received by someone(Charness and Rabin, 2002). The utility function can be defined as:

$$\pi_i(e_i, e_{-i}) = \lambda_i(\gamma Q) + (1-\lambda_i)(\sum_{i=1}^{Q} \gamma Q + \sum_{i=N-Q}^{N} (1+\gamma Q)), Q > 1 \qquad (4)$$

In the above payoff function, individual is concerned about the least payoff anyone receives and also the social surplus available in the economy. Social Surplus is described by $(\sum_{i=1}^{Q} \gamma Q + \sum_{i=N-Q}^{N}(1+\gamma Q))$ where the first term refers to the payoff received by the individuals who contribute to the public good (Q) and the second term is for the individuals who free ride ($N-Q$).

I compare this situation to Example 1. Suppose $i \in N$ contributes to the public good, his/her payoff will be: $\lambda_i\gamma + (1-\lambda_i)(3\gamma+2)$. $i \in N$ will contribute if: $\lambda_i\gamma + (1-\lambda_i)(3\gamma+2) > 1$. This inequality gives us that $i \in N$ will contribute if $\lambda_i < 0.5$ Thus individuals with $\lambda_1 = 0.2$ and $\lambda_2 = 0.4$ will contribute to the public good. This is different from the result we had earlier where no one contributed alone.

Now let us see what happens when a coalition is formed. In a coalition we need to have at-least 2 people. We know that $i \in N$ will contribute to the public good, thus their payoff from contributing in a coalition of 2 will be the same. $i \in N$ with $\lambda_3 = 0.6$ will join the coalition, then his/her payoff is: $\lambda_i(3\gamma) + (1-\lambda_i)(9\gamma)$. The payoff when $i \in N$ with $\lambda_3 = 0.6$ will not join the coalition: $\lambda_i(2\gamma)+(1-\lambda_i)(6\gamma+1)$. This will be true if $\gamma > 1/3$, thus $i \in N$ with $\lambda_3 = 0.6$, he/she will join the coalition. Recall in example 1, $i \in N$ with $\lambda_3 = 0.6$ will not join the coalition.

Thus the cutoff for an individual contributing to the public good, is reduced as compared to my benchmark model. Here also coalition helps in increasing cooperation.Future work would involve testing the propositions for other social preferences which might be very different from the Rawlsian inequality.

The theoretical predictions of baseline model are also tested through a human subjects experiment in another paper. In the experiment preferences are exogenous and are pre-assigned to participants by incorporating preferences into their payoff. Sessions were conducted for both homogeneous and heterogeneous preferences. Predictions of the model hold true and individuals with stronger social preferences are more likely to join the coalition and contribute to the public good. The results show that higher MPCR leads to increase in the coalition size. With high return from public good, probability of joining the coalition is significantly higher. As the theory model predicts, individuals who join the coalition also contribute to public good. Higher payoff in the last period reduces the incentive to join the coalition, however higher payoff of the least well off person in last period increases the probability of joining the coalition. This result suggest subjects in the sample care about the least well off person[13].

4 Conclusion

In this paper, I explore the disparity between experimental results and the theoretical findings regarding the relation between the size of the coalition and return from public good(MPCR). The proposed model in my paper takes a novel approach, allowing for heterogeneity in privately known social preferences between members of society as well as variation in individual returns from the public good. The thresholds in my model determine the individuals who will join the coalition and contribute to public good when weight on social preferences is private information. The results from paper also shed light on the importance of heterogeneous individuals in a society to ensure provisioning of public good.

Incorporating social preferences in a public good game with coalition explains the existence of large sized IEA. Heterogeneity of social preferences predicts who will contribute to the public good in a diverse population. In my two stage public good game, in the first stage, heterogeneous agents choose whether or not to join a coalition. In the next stage, the coalition votes whether its members will contribute or not. Using backward induction, I first derive Stage II cutoffs for both the fringe members and coalition members which if satisfied will lead to contribution. After incorporating these critical values, the cutoffs are derived which must be satisfied by individuals who will join the coalition in stage I. I find that individuals with relatively lower weight on pecuniary payoff satisfy the cutoff for both joining the

[13] Draft coming soon

coalition and contributing as a coalition member.

As MPCR increases, more individuals satisfy the cutoff. This results in coalition of larger size and higher likelihood that a coalition will contribute for the public good. Existence of individuals who satisfy the cutoff (those who have lower weight on pecuniary payoff) in majority can lead to the large sized coalitions contributing for the public good and henceforth increase cooperation. The results also apply to unanimous voting rules since joining the coalition guarantees contribution to the public good. Increase in the return from public good leads to more individuals joining the coalition and all of them contributing towards the public good.

I extend the model by incorporating benefits from voting for coalition members. The results show that individuals with stronger social preferences are more likely to join the coalition and vote to contribute to the public good. Higher benefits from voting also increases the size of the coalition but decreases likelihood of a coalition to contribute to the public good. Upon including cost of isolation in the benchmark model, it has been shown that higher costs will lead to a larger coalition and higher likelihood of majority not being satisfied. These results suggest that adding a benefit from voting to coalition members (which also acts as a cost of isolation to those who are not part of the group) can lead to larger participation by individuals in order to enjoy the benefit(abstain the cost). However these gains are not translated into contribution by the individuals. In order to guarantee the coalition contributes to the public good, the MPCR or the size of the coalition need to be high enough to surpass the additional gain received from voting.

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Appendix

Appendix for Theoretical Model

Proof for Proposition I

Assuming M to be odd and hence the majority is given by: $m' = \frac{M+1}{2}$. I solve for Stage II by comparing the expected payoff from contributing v/s expected payoff from not contributing, for which we will be using the probabilities of contribution. Heterogeneity in preferences leads to a different probability for contributing to the public good.

Probability of contributing to the public good for each $i \in M$ can be ordered as: $p_1, p_2, \cdots, p_M$. Here p_1 is the probability of the person who is most likely to contribute to the public good. p_M is the probability of the person who is least likely to contribute to the public good and let it be represented by p. For technical reasons and without loss of generality, we substitute all the probabilities with p_M i.e. p. Using the least probability,i.e, p will give us the least expected payoff from contributing.

p is the least probability for each $i \in M$ saying yes to contribution to the public good. It follows a binomial distribution and $0 \leq p \leq 1$ since contribution is a binary decision. A coalition member will contribute to the public good if the payoff from contribution is at-least equal to the payoff from not contributing. Similarly a fringe member will contribute to the public good if the payoff from contribution is at-least equal to the payoff from not contributing. Comparing these set of equations I derive the first proposition.

Proof of part a:

I define the terms which I will be using in the analysis when $i \in M$ votes yes to contribute to public good:

$$\theta^v = \Bigg[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'} + \binom{M-1}{m'}(p)^{m'}(1-p)^{M-m'-1} + \cdots + \binom{M-1}{M-1}(p)^{M-1}\Bigg]$$
$$= \left[\sum_{i=m'-1}^{M-1}\binom{M-1}{i}(p)^i(1-p)^{M-(i+1)}\right] \tag{5}$$

$$\eta^v = \Bigg[\binom{M-1}{0}(1-p)^{M-1} + \binom{M-1}{1}(p)(1-p)^{M-2} + \cdots + \binom{M-1}{m'-2}(p)^{m'-2}(1-p)^{M-m'+1}\Bigg]$$
$$= \left[\sum_{i=0}^{m'-2}\binom{M-1}{i}(p)^i(1-p)^{M-(i+1)}\right] \tag{6}$$

θ^v sums all the cases when at least $m'-1$ other players vote yes to contribute to the public good. In this case, a yes by individual i will lead to at least the majority voting yes to contribute and hence the coalition will contribute to the public good. η^v sums all the cases when the majority is not satisfied, and as a result the coalition does not contribute to public good. Note even though i votes yes to contribute, majority is not satisfied and as a result he/she does not contribute.

Now let's define the terms which I will be using in the analysis when individual $i \in M$ does not vote yes to contribute to public good:

$$\theta^{nv} = \Bigg[\binom{M-1}{m'}(p)^{m'}(1-p)^{M-m'-1} + \binom{M-1}{m'+1}(p)^{m'+1}(1-p)^{M-m'-2} + \cdots + \binom{M-1}{M-1}(p)^{M-1}\Bigg]$$
$$= \left[\sum_{i=m'}^{M-1}\binom{M-1}{i}(p)^i(1-p)^{M-i-1}\right] \tag{7}$$

$$\eta^{nv} = \left[\binom{M-1}{0} (1-p)^{M-1} + \binom{M-1}{1} (p)(1-p)^{M-2} + \right.$$
$$\left. \cdots + \binom{M-1}{m'-1} (p)^{m'-1} (1-p)^{M-m'} \right]$$
$$= \left[\sum_{i=0}^{m'-1} \binom{M-1}{i} (p)^i (1-p)^{M-(i+1)} \right] \quad (8)$$

θ^{nv} sums all the cases when at least the majority votes to contribute to the public good and as a result the coalition contributes to public good. Note that the majority does not include $i \in M$ here but i has to contribute being part of the coalition. η^{nv} sums all the cases when the majority is not satisfied and as a result the coalition does not contribute to public good.

The expected payoff when $i \in M$ chooses $v_i = 1$ (votes to contribute):

$$\begin{aligned} \pi^v(v_i, v_{-i}) &= [\theta^v] \left[\lambda_i(\gamma(M+F')) + (1-\lambda_i)(\gamma(M+F')) \right] \\ &+ [\eta^v] \left[\lambda_i(1+\gamma F') + (1-\lambda_i)\gamma(F') \right] \\ &= [\theta^v] \left[\gamma(M+F') \right] + [\eta^v] \left[\lambda_i + \gamma F' \right] \end{aligned} \quad (9)$$

The term adjacent to θ^v is the payoff $i \in M$ receives when $Q = M \cup F'$. The term adjacent to η^v is the payoff when $Q = F'$(majority was not satisfied in this case). Recall that Q denotes the total number of individuals contributing to the public good and F' denotes the fringe members contributing to public good. I make no assumption on the number of fringe members contributing to public good, only size of M is announced before Stage II begins.

The expected payoff when $i \in M$ chooses $v_i = 0$ (does not vote to contribute):

$$\begin{aligned} \pi^{nv}(v_i, v_{-i}) &= [\theta^{nv}] \left[\lambda_i(\gamma(M+F')) + (1-\lambda_i)(\gamma(M+F')) \right] \\ &+ [\eta^{nv}] \left[\lambda_i(1+\gamma F') + (1-\lambda_i)\gamma(F') \right] \\ &= [\theta^{nv}] \left[\gamma(M+F') \right] + [\eta^{nv}] \left[\lambda_i + \gamma F' \right] \end{aligned} \quad (10)$$

The term adjacent to θ^{nv} is the payoff $i \in M$ receives when $Q = M \cup F'$. Note that majority does not include $i \in M$ here but i has to contribute being part of the coalition. The term adjacent to η^{nv} is the payoff $i \in M$ receives when $Q = F'$.

Individual $i \in M$ will compare π^v and π^{nv} to choose between the strategies $v_i = 1$ (vote to contribute) or $v_i = 0$ (vote not to contribute). $i \in M$ will choose to contribute to public good if payoff from contributing is at least greater than the payoff from not contributing. After comparison we observe that $\pi^v \geq \pi^{nv}$ if:

$$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][\gamma(M+F') - (\lambda_i + \gamma F')] \geq 0 \tag{11}$$

$$\lambda_i \leq \gamma M \tag{12}$$

Equation 12 gives us the cutoff for part a of Proposition I. ■

For interpretation purpose, equation 11 can be re-written as:

$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][\gamma(M+F')]$ - $\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][(\lambda_i + \gamma F')]$

$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right]$ denotes that $m'-1$ people are contributing to the public good. Thus i has the deciding vote, in other words i is the *pivotal voter* and equation 11 shows the least expected marginal benefit a pivotal voter receives from contributing.

The result is also intuitive since the pivotal member's vote decides whether a coalition will contribute or not. The first term in the above equation depicts the expected payoff when individual $i \in M$ or the pivotal voter, votes to contribute. Note that, a vote by i leads to majority being satisfied and hence the coalition contributes. Everyone in the coalition receives the payoff $\gamma(M+F')$, since $Q = M \cup F'$. The second term depicts the expected payoff when the coalition does not contribute. This is because individual $i \in M$ does not vote and hence the coalition only receives $m'-1$ votes. The resulting payoff is $\lambda_i + \gamma F'$ and $Q = F'$. The gain to pivotal voter from voting to contribute is $\gamma M'$ and the gain from not voting is λ_i. $i \in M$ will vote to contribute if the gain from voting is more than the gain from not voting to contribute which leads us to the threshold $\lambda_i \leq \gamma M$ in equation 12.

Checking for deviation by player i

I now check if the derived threshold is robust to deviation by player $i \in M$. Let r be the probability with which individual i revises his/her action.

Let's assume player i **does not deviate**.

Let's assume the coalition size to be M. If individuals deviate, the coalition will not contribute if the majority is not satisfied. Using equation 5 we can define this as following:

$$\begin{aligned}
\theta^{novote} &= \Bigg[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{m'-2}\binom{m'-1}{1}(r)(1-r)^{m'-2} \\
&\quad + \binom{M-1}{m'}(p)^{m'}(1-p)^{M-m'-1}\binom{m'}{2}(r^2)(1-r)^{m'-2} + \\
&\quad \cdots + \binom{M-1}{M-1}(p)^{M-1}\binom{M-1}{M-(m'-1)}(r^{M-(m'-1)})(1-r)^{m'-2})\Bigg] \\
&= \left[\sum_{i=m'-1}^{M-1}\binom{M-1}{i}(p)^i(1-p)^{M-(i+1)}\right]\Bigg(\binom{m'-1}{1}(r)(1-r)^{m'-2} \\
&\quad + \binom{m'}{2}(r^2)(1-r)^{m'-2} + \cdots + \binom{M-1}{M-(m'-1)}(r^{M-(m'-1)}(1-r)^{m'-2}\Bigg)
\end{aligned} \tag{13}$$

θ^{novote} describes all the scenarios where the coalition does not contribute. since majority is not satisfied due to deviation by coalition members. We can rewrite equation 13 in terms of equation 5:

$$\begin{aligned}
\theta^{novote} &= \left[\sum_{i=m'-1}^{M-1}\binom{M-1}{i}(p)^i(1-p)^{M-(i+1)}\right]\Bigg(\binom{m'-1}{1}(r)(1-r)^{m'-2} \\
&\quad + \binom{m'}{2}(r^2)(1-r)^{m'-2} + \cdots + \binom{M-1}{M-(m'-1)}(r^{M-(m'-1)})(1-r)^{m'-2}\Bigg) \\
&= \theta^v\Bigg(\binom{m'-1}{1}(r)(1-r)^{m'-2} \\
&\quad + \binom{m'}{2}(r^2)(1-r)^{m'-2} + \cdots + \binom{M-1}{M-(m'-1)}(r^{M-(m'-1)}(1-r)^{m'-2}\Bigg) \\
&= \theta^v(x)
\end{aligned} \tag{14}$$

Similarly, probability with which the coalition will vote even if individuals are allowed to deviate, is given by:

$$1 - \theta^v(x) \tag{15}$$

The above equation 15 covers all the possibilities when even allowing for deviation of any individuals can still lead to coalition vote for public good.

The expected payoff if i does not deviate:

$$\theta^v(x)(\lambda_i + \gamma F') + (1 - \theta^v(x))(\gamma(M + F')) \tag{16}$$

Now lets consider the case when i also **deviates** and changes vote from yes to no.

Let's assume the coalition size to be M. If individuals deviate, the coalition will not contribute if the majority is not satisfied. Using equation 5 we can define this as following:

$$\begin{aligned}
\theta^{novote} &= \Bigg[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'} \\
&\quad + \binom{M-1}{m'}(p)^{m'}(1-p)^{M-m'-1}\binom{m'}{1}(r)(1-r)^{m'-1} + \\
&\quad \cdots + \binom{M-1}{M-1}(p)^{M-1}\binom{M-1}{M-m'}(r^{M-m'}(1-r)^{m'-1})\Bigg] \\
&= \left[\sum_{i=m'-1}^{M-1}\binom{M-1}{i}(p)^i(1-p)^{M-(i+1)}\right]\Bigg(1 + \binom{m'}{1}(r)(1-r)^{m'-1} + \cdots \\
&\quad + \binom{M-1}{M-m'}(r^{M-m'}(1-r)^{m'-1}\Bigg)
\end{aligned} \tag{17}$$

Here θ^{novote} describes all the cases when the coalition does not contribute because majority is not satisfied. Equation 17 can be re written in terms of equation 5.

$$= \left[\sum_{i=m'-1}^{M-1} \binom{M-1}{i} (p)^i (1-p)^{M-(i+1)}\right] \left(1 + \binom{m'}{1}(r)(1-r)^{m'-1} + \cdots \right.$$
$$\left. + \binom{M-1}{M-m'} (r^{M-m'}(1-r)^{m'-1}\right)$$
$$= \theta^v \left(1 + \binom{m'}{1}(r)(1-r)^{m'-1} + \cdots \right.$$
$$\left. + \binom{M-1}{M-m'} (r^{M-m'}(1-r)^{m'-1}\right) = \theta^v(z) \quad (18)$$

Similarly, probability with which the coalition will vote even if individuals are allowed to deviate, is given by:

$$1 - \theta^v(z) \tag{19}$$

The above equation 19 covers all the possibilities when even allowing for deviation of any individuals can still lead to coalition vote for public good.

The expected payoff if i deviates

$$\theta^v(z)(\lambda_i + \gamma F') + (1 - \theta^v(z))(\gamma(M + F')) \tag{20}$$

Individual i will not deviate if Equation 16 $\geq$ equation 20

$$\theta^v(x)(\lambda_i + \gamma F') + (1 - \theta^v(x))(\gamma(M + F')) \geq \theta^v(z)(\lambda_i + \gamma F') + (1 - \theta^v(z))(\gamma(M + F'))$$
$$\implies (\lambda_i + \gamma F')(\theta^v(x) - \theta^v(z)) \geq (\gamma(M + F'))(\theta^v(x) - \theta^v(z))$$
$$\implies \lambda_i \leq \gamma M \quad (21)$$

This condition is exactly the same condition which needs to be specified for individuals to vote yes to contribute when they are in the coalition. In other words the condition is sufficient to guarantee no deviation .

Proof of part b:

The payoff a *fringe member* $i \in F'$ receive when h/she decides to contribute.[14] In

[14]We can also calculate the payoff for $i \in F$, keeping F' fixed. In that case F' increases by 1, when $i \in F$ contributes.

other words payoff when $i \in F'$ chooses $e_i = 1$:

$$\begin{aligned}\pi^c(e_i, e_{-i}) &= [\alpha]\,[\lambda_i(\gamma(M+F')) + (1-\lambda_i)\gamma(M+F')] \\ &\quad + [\beta]\,[\lambda_i(\gamma F')_{+}(1-\lambda_i)\gamma(F')] \\ &= [\alpha]\,[\gamma(M+F')] + [\beta]\,[\gamma F']\end{aligned} \tag{22}$$

In the above equations α depicts the case when coalition of size M is contributing to the public good as the majority rule is satisfied. The corresponding term is the payoff $i \in F'$ receives when a coalition of size M and F' fringe members are contributing ($Q = M \cup F'$). β depicts the case when M is not contributing and thus $Q = F'$. The adjacent term is the payoff $i \in F'$ receives when h/she is contributing with the other fringe members.

The payoff $i \in F'$ receives when he/she decides not to contribute to the public good ($e_i = 0$):

$$\begin{aligned}\pi^{nc}(e_i, e_{-i}) &= [\alpha]\,[\lambda_i(1+\gamma(M+F'-1)) + (1-\lambda_i)\gamma(M+F'-1)] \\ &\quad + [\beta]\,[\lambda_i(1+\gamma(F'-1)_{+}(1-\lambda_i)\gamma(F'-1)] \\ &= [\alpha]\,[\gamma(M+F'-1)+\lambda_i] + [\beta]\,[\lambda_i + \gamma(F'-1)]\end{aligned} \tag{23}$$

In this case i is a free rider and size of the *fringe members* is reduced to $F' - 1$. α and β are defined as above. The term adjacent to α is the payoff i receives when h/she is free riding and coalition of size M and $F' - 1$ *fringe members* are contributing ($Q = M \cup F' - 1$). The term corresponding to β is the payoff i receives when $Q = F' - 1$.

$i \in F'$ will compare π^c and π^{nc} to decide whether to contribute ($e_i = 1$)or not ($e_i = 0$) as a fringe member. After comparison we see that $\pi^c \geq \pi^{nc}$ if:

$$\lambda_i \leq \gamma \tag{24}$$

Equation 24 gives us the cutoff for part b in Proposition I. ■

We can interpret equation 24 as the comparison between gain from contribution and the gain from being a free rider for a fringe member. From equation 1,Fringe member's marginal gain from free riding will be λ_i. The marginal gain from contribution will be γ. An individual contributes if gain from contribution is higher than the benefit from free riding,i.e., $\lambda_i \leq \gamma$ which is the threshold in the above equation. This is also a general result from a public goods game without coalition.

Let $\hat{\lambda} = \gamma$, if $\lambda_i \leq \hat{\lambda}$ is satisfied then the fringe member will contribute for the public good. This leads us to part b of Proposition I.

Proof for Proposition II:

Case (i): $\lambda_i \leq \gamma$:

The payoff whether you decide to join the coalition or not is dependent on whether the coalition contributes or not. I use the thresholds derived in Stage II to calculate the probability coalition contributes to public good. Let's define the terms which we will be using when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to join the coalition.

$$\begin{aligned}\phi = \Bigg[\binom{M-1}{m'-1}(\gamma M)^{m'-1}(1-\gamma M)^{M-m'} + \binom{M-1}{m'}(\gamma M)^{m'}(1-\gamma M)^{M-m'-1} + \\ \cdots + \binom{M-1}{M-1}(\gamma M)^{M-1}\Bigg] \\ = \left[\sum_{i=m'-1}^{M-1}\binom{M-1}{i}(\gamma M)^{i}(1-\gamma M)^{M-i-1}\right] \quad (25)\end{aligned}$$

$$\begin{aligned}1-\phi = \Bigg[\binom{M-1}{0}(1-\gamma M)^{M-1} + \binom{M-1}{1}(\gamma M)(1-\gamma M)^{M-2} + \\ \cdots + \binom{M-1}{m'-2}(\gamma M)^{m'-2}(1-\gamma M)^{M-m'+1}\Bigg] \\ = \left[\sum_{i=0}^{m'-2}\binom{M-1}{i}(\gamma M)^{i}(1-\gamma M)^{M-(i+1)}\right] \quad (26)\end{aligned}$$

Here ϕ sums all the cases when at-least $m'-1$ will vote to contribute to the public good. In other words there are at-least $m'-1$ individuals who have the preference $\lambda_i \leq \gamma M$. From stage I, we know $i \in M$ will vote to contribute if $\lambda_i \leq \gamma M$, since λ_i follows a uniform distribution, this is given by γM.Similarly, $i \in M$ will not vote to contribute if $\lambda_i > \gamma M$ and this is given by $1-\gamma M$. A vote by $i \in N$ will lead to at least the majority contributing to the public good. Since $i \in N$ satisfies $\lambda_i \leq \gamma$ which also implies $\lambda_i \leq \gamma M$, h/she will vote to contribute once they join coalition.

$1-\phi$ sums all the cases when individuals in coalition have preferences such that majority does not vote to contribute to public good. In other words majority of the individuals who join the coalition have preference which satisfies $\lambda_i > \gamma M$.

We define the terms which we will be using when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to not join the coalition.

$$\phi' = \Bigg[\binom{M-1}{m'-1}(\gamma(M-1))^{m'-1}(1-\gamma(M-1))^{M-m'}+$$
$$\binom{M-1}{m'}(\gamma(M-1))^{m'}(1-\gamma(M-1))^{M-m'-1}+\cdots+\binom{M-1}{M-1}(\gamma(M-1))^{M-1}\Bigg]$$
$$=\left[\sum_{i=m'-1}^{M-1}\binom{M-1}{i}(\gamma(M-1))^{i}(1-\gamma(M-1))^{M-i-1}\right] \quad (27)$$

$$1-\phi' = \Bigg[\binom{M-1}{0}(1-\gamma(M-1))^{M-1}+\binom{M-1}{1}(\gamma(M-1))(1-\gamma(M-1))^{M-2}+$$
$$\cdots+\binom{M-1}{m'-2}(\gamma(M-1))^{m'-2}(1-\gamma(M-1))^{M-m'+1}\Bigg]$$
$$=\left[\sum_{i=0}^{m'-2}\binom{M-1}{i}(\gamma(M-1))^{i}(1-\gamma(M-1))^{M-(i+1)}\right] \quad (28)$$

Now the size of coalition is reduced to $M-1\ \forall M$. The probability of a coalition member contributing, is now given by the cutoff $\lambda_i \leq \gamma(M-1)$ which is $\gamma(M-1)$. Similarly probability of a coalition member not contributing is given by the cutoff: $\lambda_i > 1-\gamma(M-1)$ i.e. $1-\gamma(M-1)$. ϕ' sums all the cases when at-least majority of the $M-1$ members of the coalition vote to contribute i.e. majority of the people in the coalition have preference: $\lambda_i \leq \gamma(M-1)$. 1- ϕ' sums all the cases when majority of the people who join the coalition do not satisfy $\lambda_i \leq \gamma(M-1)$ and as a result the coalition does not contribute.

I denote the action set of a player joining the coalition by $j_i \in \{0,1\}$ where $j_i = 1$ implies $i \in N$ joins the coalition and $j_i = 0$ implies $i \in N$ does not join the coalition. With slight abuse of notation, j_i is the strategy of player i and j_{-i} is the strategy of the other players. Payoff when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to join the coalition ($j_i = 1$):

$$\pi_i(j_i, j_{-i}) = [\phi]\,[(\gamma(M+F')] + [1-\phi]\,[\lambda_i + \gamma F'] \quad (29)$$

The term adjacent to ϕ shows the payoff as a result of the coalition of any size M and F' fringe members contributing i.e. $Q = M \cup F'$. The term adjacent to $1-\phi$ is the payoff when the coalition is not contributing and only the fringe members who meet the cutoff ($\lambda_i \leq \gamma$) contribute i.e $Q = F'$. Note even though $i \in N$ wants to contribute to public good, he/she has to follow the decision of coalition.

Payoff when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to not join the coalition ($j_i = 0$):

$$\pi_i(j_i, j_{-i}) = [\phi'] \left[(\gamma(M + F')\right] + [1 - \phi'] \left[\gamma(F' + 1)\right] \tag{30}$$

Note that $i \in N$ satisfies: $\lambda_i \leq \gamma$, thus he/she will contribute as a fringe member, and as a result the size of fringe member increases to $F' + 1$, also now the coalition is of size $M - 1$. In equation 30 term adjacent to ϕ' is the payoff received when a coalition of size $M - 1$ and $F' + 1$ fringe members are contributing. The term adjacent to $1-\phi'$ shows the payoff when only $F'+1$ fringe members are contributing.

$i \in N$ will compare equation 29 and equation 30 to decide whether to contribute to the public good or not. The payoff functions changes values based on three sub-cases. I will be using the three sub cases to derive solution for case (ii) and case(iii) as well. The three sub cases are given by:

a) $\gamma(M - 1) \geq 1$, $\gamma M > 1$.

b) $\gamma(M - 1) < 1$, $\gamma M = 1$.

c) $\gamma(M - 1) < 1$, $\gamma M < 1$.

Case (i) (a) $\gamma(M - 1) \geq 1$**,** $\gamma M > 1$.

Given M is assumed to be odd, M needs to be at-least 3. The probability individual satisfies $\lambda_i \leq \gamma M$ i.e. γM will be 1 and probability any individual has cutoff $\lambda_i > \gamma M$ which is $1 - \gamma M = 0$, since $\lambda_i < 1$. Similarly probability individual satisfies $\lambda_i \leq \gamma(M - 1)$ i.e. $\gamma(M - 1)$ will be 1 and probability any individual has cutoff $\lambda_i > \gamma(M - 1)$ which is $1 - \gamma(M - 1) = 0$. Substituting these values in equation 29 and equation 30, one finds that $\phi = \phi' = 1$ and $1 - \phi = 1 - \phi' = 0$. Equation 29 can be re-written as $\gamma(M + F')$, equation 30 can be re written as $\gamma(M + F')$. Comparing these equations shows that the individual in the case($\lambda_i \leq \gamma$) will be indifferent between joining the coalition and not joining the coalition. Based on our definition of joining the coalition, the individual joins the coalition if h/she is indifferent.

Case (i) (b) $\gamma(M-1) < 1$**,** $\gamma M = 1$

Note $\phi = 1$ and $1 - \phi = 0$. Thus equation 29 can be expressed as :$\gamma(M + F')$. $i \in N$ will join the coalition if equation 29 $\geq$ equation 30. This implies:

$\gamma(M + F') \geq [\phi'] [(\gamma(M + F')] + [1 - \phi'] [\gamma(F' + 1)]$

$\rightarrow (1 - \phi')\gamma(M + F') \geq [1 - \phi'] [\gamma(F' + 1)]$

$\rightarrow M \geq 1$

$M = 1$ is a trivial condition and recall that we showed in the example, that if there is only 1 individual contributing, then $i \in N$ is better off by not contributing. The above condition implies $i \in N$ will join if there is at least one person contributing.

Case (i) (c) $\gamma(M-1) < 1$**,** $\gamma M < 1$.

$i \in N$ will join if equation 29 $\geq$ equation 30. This implies:

$\phi[\gamma(M + F')] + [1 - \phi][\lambda_i + \gamma F'] \geq [\phi'] [(\gamma(M + F')] + [1 - \phi'] [\gamma(F' + 1)]$

$\rightarrow [\phi - \phi'][\gamma(M + F')] + [1 - \phi][\lambda_i] \geq +[\phi - \phi'] [\gamma F'] + [1 - \phi']\gamma$

$\rightarrow [\phi - \phi'][\gamma M] \geq [1 - \phi']\gamma - [1 - \phi][\lambda_i]$

Replacing λ_i *with* γ *on the R.H.S. since* $\lambda_i \leq \gamma$

$\rightarrow [\phi - \phi'][\gamma M] \geq [1 - \phi']\gamma - [1 - \phi][\gamma]$

$\rightarrow [\phi - \phi'][\gamma M] \geq [1 - \phi']\gamma - [1 - \phi][\gamma]$

$\rightarrow [\phi - \phi'][\gamma M] \geq [\phi - \phi']\gamma$

$\rightarrow M \geq 1$

Again $M = 1$ is a trivial case and $i \in N$ will join the coalition if there is at least one other member in the coalition.

Thus $i \in N$ with preference $\lambda_i \leq \gamma$ will join the coalition.

Case (ii):$\gamma < \lambda_i \leq \gamma M$

The payoff when $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ decides to join the coalition ($j = 1$):

$$\pi_i(j_i, j_{-i}) = [\phi]\,[(\gamma(M + F')] + [1 - \phi]\,[\lambda_i + \gamma F'] \tag{31}$$

ϕ and $1-\phi$ are expressed by equation 25 and equation 26. $i \in N$ has preference $\gamma < \lambda_i \leq \gamma M$ and hence would vote to contribute to the public good. The term adjacent to ϕ shows the payoff as a result of the coalition when $Q = M \cup F'$. The term adjacent to $1 - \phi$ is the payoff when the coalition does not contribute i.e. $Q = F'$. Note even though $i \in N$ has preference $\gamma < \lambda_i \leq \gamma M$, he/she does not contribute as majority is not satisfied.

The payoff when $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ decides to not join the coalition ($j_i = 0$):

$$\pi_i(j_i, j_{-i}) = [\phi']\,[\lambda_i + \gamma(M - 1 + F')] + [1 - \phi']\,[\lambda_i + \gamma(F')] \tag{32}$$

ϕ' and $1 - \phi'$ are expressed by equation 27 and equation 28. Note that $i \in N$ has preference $\gamma < \lambda_i \leq \gamma M$ and hence will not contribute as a fringe member. In equation 32, the term adjacent to ϕ' is the payoff received by $i \in N$ when coalition of size $M - 1$ and F' fringe members are contributing. The term adjacent to $1 - \phi'$ depicts the payoff $i \in N$ receives when only F' fringe members contribute to the public good.

An individual will compare equation 31 and equation 32 for each of the sub cases to decide whether to join the coalition or not.

Case(ii) (a) $\gamma(M - 1) \geq 1$**,** $\gamma M > 1$

In this case we again have $\phi = \phi' = 1$ and $1 - \phi = 1 - \phi' = 0$. Plugging these values in equation 31 and equation 32 shows that individual will not join the coalition as equation 31 < equation 32: $\gamma(M + F') < \lambda_i + \gamma(M - 1 + F')$

$\rightarrow \gamma(M + F') < \lambda_i + \gamma(M + F') - \gamma$

$\rightarrow \gamma < \lambda_i$

$\gamma < \lambda_i$ hold true in case(ii) thus, $i \in N$ will not join the coalition.

Case (ii) (b) $\gamma(M - 1) < 1$**,** $\gamma M = 1$

$i \in N$ will not join the coalition if equation 31 < equation 32:

$\gamma(M+F') < [\phi'][\lambda_i + \gamma(M-1+F')] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [1-\phi'][\gamma(M+F')] < \phi'[\lambda_i - \gamma] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [1-\phi'][\gamma M] < \lambda_i - \phi'\gamma$

Replacing λ_i *with* γM *since* $\lambda_i \leq \gamma M$

$\rightarrow [1-\phi'][\gamma M] < \gamma M - \phi'\gamma$

$\rightarrow \phi'\gamma M > \phi'\gamma$

$\rightarrow M > 1$

Case(ii) (c) $\gamma(M-1) < 1$**,** $\gamma M < 1$

$i \in N$ will not join the coalition if equation 31 < equation 32:

$\phi[\gamma(M+F')] + [1-\phi][\lambda_i + \gamma F'] < [\phi'][\lambda_i + \gamma(M-1+F')] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi-\phi'][\gamma(M+F')] < \phi'[\lambda_i - \gamma] + [\phi-\phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi-\phi'][\gamma M] < \phi'\lambda_i - \phi'\gamma + \phi\lambda_i - \phi'\lambda_i$

$\rightarrow [\phi-\phi'][\gamma M] < \phi\lambda_i - \phi'\gamma$

Replace λ_i *with* γM *since* $\lambda_i \leq \gamma M$

$\rightarrow [\phi-\phi'][\gamma M] < \phi\gamma M - \phi'\gamma$

$\rightarrow \phi\gamma M - \phi'\gamma M < \phi\gamma M - \phi'\gamma$

$\rightarrow \gamma M > \gamma$

$\rightarrow M > 1$

In both the sub-cases we find that $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ will not join the coalition if $M > 1$. M needs to be more than 1 for it to be a coalition and hence $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ will not join the coalition.

Case (iii): $\lambda_i > \gamma M$

I now define the terms to describe the payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to join the coalition.

$$\phi'' = \Bigg[\binom{M-1}{m'}(\gamma M)^{m'}(1-\gamma M)^{M-m'-1} + \binom{M-1}{m'+1}(\gamma M)^{m'+1}(1-\gamma M)^{M-m'-2} + \cdots + \binom{M-1}{M-1}(\gamma M)^{M-1}\Bigg]$$
$$= \left[\sum_{i=m'}^{M-1}\binom{M-1}{i}(\gamma M)^{i}(1-\gamma M)^{M-i-1}\right] \quad (33)$$

$$
\begin{aligned}
1-\phi'' = \Bigg[\binom{M-1}{0}(1-\gamma M)^{M-1} + \binom{M-1}{1}(\gamma M)(1-\gamma M)^{M-2} + \\
\dots + \binom{M-1}{m'-1}(\gamma M)^{m'-1}(1-\gamma M)^{M-m'}\Bigg] \\
= \left[\sum_{i=0}^{m'-1}\binom{M-1}{i}(\gamma M)^{i}(1-\gamma M)^{M-(i+1)}\right] \quad (34)
\end{aligned}
$$

Here $i \in N$ with preference $\lambda_i > \gamma M$ will not be willing to contribute to public good. ϕ'' sums all the cases where at least the majority of the individuals who join the coalition contribute to the public good. Also, the majority does not include $i \in N$. In other words at least m' individuals with preference $\lambda_i \leq \gamma M$ join the coalition. Note although $i \in N$ has preference $\lambda_i > \gamma M$, he/she will contribute if the majority is satisfied. $1 - \phi''$ sums all the cases where individuals in coalition have preferences such that majority does not vote to contribute to public good. There are less than m' individuals with preference $\lambda_i \leq \gamma M$.

Payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to join the coalition ($j_i = 1$).

$$
\pi_i(j_i, j_{-i}) = [\phi''][(\gamma(M + F')] + [1 - \phi''][\lambda_i + \gamma F'] \quad (35)
$$

The term adjacent to ϕ'' shows the payoff when the coalition of size M and F' fringe members contribute to the public good i.e $Q = M \cup F'$. The term adjacent to $1 - \phi''$ is the payoff received when $Q = F'$.

The payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to not join the coalition ($j_i = 0$):

$$
\pi_i(j_i, j_{-i}) = [\phi'][\lambda_i + (\gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma(F')] \quad (36)
$$

Note $i \in N$ has preference $\lambda_i > \gamma M$ and hence will not contribute as a fringe member and size of the coalition reduces by 1. ϕ' and $1 - \phi'$ are expressed by equation 27 and equation 28. The term adjacent to ϕ' is the payoff $i \in N$ receives when $Q = M - 1 \cup F'$ and the term adjacent to $1 - \phi'$ is the payoff $i \in N$ receives when $Q = F'$.

An individual will compare equation 35 and equation 36 for the three sub-cases to decide whether they want to join the coalition.

Case (iii) (a) $\gamma(M-1) \geq 1, \gamma M > 1$

For case III, we can see that $\phi'' = \phi' = 1$ and $1-\phi'' = 1-\phi' = 0$.Plugging these values and comparing equation 35 and equation 36, shows that individual will not join the coalition as equation 35 $<$ equation 36:

$\gamma(M+F') < \lambda_i + \gamma(M-1+F')$

$\rightarrow \gamma(M+F') < \lambda_i + \gamma(M+F') - \gamma$

$\rightarrow \gamma < \lambda_i$

$\gamma < \lambda_i$ hold true in case(iii) thus, $i \in N$ will not join the coalition.

Case (iii) (b) $\gamma(M-1) < 1$**,** $\gamma M = 1$

Note now $\phi'' = 1$, $1-\phi'' = 0$ since $\gamma M = 1$, thus equation 35 can be re-written as: $\gamma(M+F')$.

$i \in N$ will not join the coalition if equation 35 $<$ equation 36:

$\gamma(M+F') < [\phi'][\lambda_i + \gamma(M-1+F')] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [1-\phi'][\gamma(M+F')] < \phi'[\lambda_i - \gamma] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [1-\phi'][\gamma M] < \lambda_i - \phi'\gamma$

$\rightarrow [1-\phi'][\gamma M] + \phi'\gamma < \lambda_i$

Replace γM *with* γ *on the right side as* $\gamma M > \gamma$

$\rightarrow [1-\phi'][\gamma] + \phi'\gamma < \lambda_i$

$\rightarrow \gamma < \lambda_i$

Case (iii) (c) $\gamma(M-1) < 1$**.** $\gamma M < 1$

$i \in N$ will not join the coalition if equation 35 $<$ equation 36.

$\phi''[\gamma(M+F')] + [1-\phi''][\lambda_i + \gamma F'] < [\phi'][\lambda_i + \gamma(M-1+F')] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi''-\phi'][\gamma(M+F')] < \phi'[\lambda_i - \gamma] + [\phi''-\phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi''-\phi'][\gamma M] < \phi'[\lambda_i - \gamma] + [\phi''-\phi']\lambda_i$

$\rightarrow [\phi''-\phi'][\gamma M] < \phi'\lambda_i - \phi'\gamma + \phi''\lambda_i - \phi'\lambda_i$

$\rightarrow [\phi''-\phi'][\gamma M] < \phi''\lambda_i - \phi'\gamma$

Replace γM *with* γ *on the right side as* $\gamma M > \gamma$

$\rightarrow [\phi''-\phi']\gamma + \phi'\gamma < \phi''\lambda_i$

$\rightarrow \phi''\gamma < \phi''\lambda_i$

$\rightarrow \gamma < \lambda_i$

Since $\lambda_i > \gamma$ in Case(iii) $i \in N$ with preference $\lambda_i > \gamma M$ will not join the coalition. One can see from Case (iii) (b) and (c) that the solution applies to Case(ii) (b) and (c) since these cases also assume $\lambda_i > \gamma$ and $\gamma M > \gamma$ also holds true. There is another interpretation for Case (ii) and Case(iii), part (b) and (c) which I have explained below.

$\blacksquare$

Case(ii) (b) $\gamma(M-1) < 1$, $\gamma M = 1$

$i \in N$ will not join the coalition if equation 31 < equation 32:

$\gamma(M + F') < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma(M + F')] < \phi'[\lambda_i - \gamma] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma M] < \lambda_i - \phi'\gamma$

Replacing γM with 1

$\rightarrow 1 - \phi' < \lambda_i - \phi'\gamma$

$\rightarrow \phi' > \frac{1-\lambda_i}{1-\gamma}$

Thus $i \in N$ will not join the coalition if probability of a coalition contributing without individual i is above the threshold $\frac{1-\lambda_i}{1-\gamma}$.

Case(ii) (c) $\gamma(M-1) < 1$, $\gamma M < 1$

$i \in N$ will not join the coalition if equation 31 < equation 32:

$\phi[\gamma(M + F')] + [1 - \phi][\lambda_i + \gamma F'] < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

Replacing λ_i with γ

$\rightarrow [\phi - \phi'][\gamma(M + F')] + [1 - \phi][\gamma + \gamma F'] < \phi'[\lambda_i - \gamma] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi - \phi']\gamma M < \phi'[\lambda_i - \gamma] + [1 - \phi']\lambda_i - [1 - \phi]\gamma$

$\rightarrow [\phi - \phi']\gamma M < [\phi - \phi']\gamma + \lambda_i - \gamma$

$\rightarrow [\phi - \phi'][\gamma M - \gamma] < \lambda_i - \gamma$

$\rightarrow [\phi - \phi'] < \frac{\lambda_i - \gamma}{\gamma M - \gamma}$

Thus $i \in N$ will not join the coalition if the difference in probability of a coalition contributing when i is in the coalition(ϕ) and the probability of a coalition contributing when $i \in N$ is not in the coalition (ϕ') is less than the threshold $\frac{\lambda_i - \gamma}{\gamma M - \gamma}$
.

Since the structure of equation is the same for Case (iii) (b) and (c), we can get similar cutoffs in these cases as well.

Proof for model extension

Benefit from voting

It is easy to see that after inclusion of benefit from voting equation 11 appears as: $\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right]\left[\gamma(M+F')+\varepsilon-(\lambda_i+\gamma F'+\varepsilon)\right]\geq 0$. This leads to no change in the cutoff for joining the coalition in equation 12. Hence no change in Stage I results.

Proof for Proposition III

Case (i): $\lambda_i \leq \gamma$: Payoff when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to join the coalition ($j_i = 1$) changes.Equation 29 after incorporation of benefit will be as follows:

$$\pi_i(j_i, j_{-i}) = [\phi]\left[(\gamma(M+F')+\varepsilon\right] + [1-\phi]\left[\lambda_i + \gamma F' + \varepsilon\right] \tag{37}$$

Payoff when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to not join the coalition ($j_i = 0$) remains the same as equation 30:

$$\pi_i(j_i, j_{-i}) = [\phi']\left[(\gamma(M+F')\right] + [1-\phi']\left[\gamma(F'+1)\right] \tag{38}$$

$i \in N$ will compare equation 37 and equation 38 to decide whether to contribute to the public good or not. The payoff functions changes values based on three sub-cases. I will be using the three sub cases to derive solution for case (ii) and case(iii) as well. The three sub cases are given by:

a) $\gamma(M-1) \geq 1$, $\gamma M > 1$.

b) $\gamma(M-1) < 1$, $\gamma M = 1$.

c) $\gamma(M-1) < 1$, $\gamma M < 1$.

Case (i) (a) $\gamma(M-1) \geq 1$**,** $\gamma M > 1$

$\phi = \phi' = 1$ and $1-\phi = 1-\phi' = 0$. Equation 37 can be re-written as $\gamma(M+F')+\varepsilon$, equation 38 can be re written as $\gamma(M+F')$. Comparing these equations shows that the individual in the case($\lambda_i \leq \gamma$) will be joining the coalition since $\gamma(M+F')+\varepsilon > \gamma(M+F')$.

Case (i) (b) $\gamma(M-1) < 1$**,** $\gamma M = 1$

Note $\phi = 1$ and $1-\phi = 0$. Thus equation 37 can be expressed as :$\gamma(M+F')+\varepsilon$. $i \in N$ will join the coalition if equation 37 $\geq$ equation 38:. This implies:

$\gamma(M+F')+\varepsilon \geq [\phi'] [(\gamma(M+F')] + [1-\phi'] [\gamma(F'+1)]$

$\rightarrow (1-\phi')\gamma(M+F')+\varepsilon \geq [1-\phi'] [\gamma(F'+1)]$

$\rightarrow (1-\phi')\gamma M+\varepsilon \geq [1-\phi']\gamma$

This is true since $\gamma M > \gamma$

The above condition implies $i \in N$ will join the coalition.

Case (i) (c) $\gamma(M-1) < 1$**,** $\gamma M < 1$

$i \in N$ will join if equation 37 $\geq$ equation 38. This implies:

$\phi[\gamma(M+F')+\varepsilon]+[1-\phi][\lambda_i+\gamma F'+\varepsilon] \geq [\phi'] [(\gamma(M+F')] + [1-\phi'] [\gamma(F'+1)]$

$\rightarrow [\phi-\phi'][\gamma(M+F')] + [1-\phi][\lambda_i] + \varepsilon \geq +[\phi-\phi'] [\gamma F'] + [1-\phi']\gamma$

$\rightarrow [\phi-\phi'][\gamma M] + \geq [1-\phi']\gamma - [1-\phi][\lambda_i]$

Replacing λ_i *with* γ *on the R.H.S. since* $\lambda_i \leq \gamma$

$\rightarrow [\phi-\phi'][\gamma M] + \varepsilon \geq [1-\phi']\gamma - [1-\phi][\gamma]$

$\rightarrow [\phi-\phi'][\gamma M] + \varepsilon \geq [1-\phi']\gamma - [1-\phi][\gamma]$

$\rightarrow [\phi-\phi'][\gamma M] + \varepsilon \geq [\phi-\phi']\gamma$

Since$\gamma M > \gamma$ this holds true

Thus $i \in N$ with preference $\lambda_i \leq \gamma$ will join the coalition.

Case (ii):$\gamma < \lambda_i \leq \gamma M$

The payoff when $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ decides to join the coalition ($j = 1$) changes. Equation 31 changes as:

$$\pi_i(j_i, j_{-i}) = [\phi] [(\gamma(M+F')+\varepsilon] + [1-\phi] [\lambda_i+\gamma F'+\varepsilon] \tag{39}$$

The payoff when $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ decides to not join the coalition ($j_i = 0$) remains the same as equation 32

$$\pi_i(j_i, j_{-i}) = [\phi'] [\lambda_i+\gamma(M-1+F')] + [1-\phi'] [\lambda_i+\gamma(F')] \tag{40}$$

An individual will compare equation 39 and equation 40 for each of the sub cases to decide whether to join the coalition or not.

Case(ii) (a) $\gamma(M-1) \geq 1$**,** $\gamma M > 1$

$\phi = \phi' = 1$ and $1-\phi = 1-\phi' = 0$. Plugging these values in equation 39 and equation 40 shows that individual will not join the coalition as equation 39 $<$ equation 40: $\gamma(M+F')+\varepsilon < \lambda_i+\gamma(M-1+F')$

$\rightarrow \gamma(M + F') + \varepsilon < \lambda_i + \gamma(M + F') - \gamma$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case (ii) (b) $\gamma(M - 1) < 1$**,** $\gamma M = 1$

Note $\phi = 1$ and $1 - \phi = 0$. $i \in N$ will not join the coalition if equation 39 $<$ equation 40:

$\gamma(M + F') + \varepsilon < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma(M + F')] + \varepsilon < \phi'[\lambda_i - \gamma] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma M] + \varepsilon < \lambda_i - \phi'\gamma$

Replace γM *with* γ *on the right side as* $\gamma M > \gamma$

$\rightarrow [1 - \phi'][\gamma] + \phi'\gamma + \varepsilon < \lambda_i$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case(ii)(c) $\gamma(M - 1) < 1$**,** $\gamma M < 1$

$i \in N$ will not join the coalition if equation 39 $<$ equation 40.

$\phi[\gamma(M+F')] + [1-\phi][\lambda_i + \gamma F'] + \varepsilon < [\phi'][\lambda_i + \gamma(M-1+F')] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi - \phi'][\gamma(M + F')] + \varepsilon < \phi'[\lambda_i - \gamma] + [\phi - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon < \phi'[\lambda_i - \gamma] + [\phi - \phi']\lambda_i$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon < \phi'\lambda_i - \phi'\gamma + \phi\lambda_i - \phi'\lambda_i$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon < \phi\lambda_i - \phi'\gamma$

Replace γM *with* γ *on the right side as* $\gamma M > \gamma$

$\rightarrow [\phi - \phi']\gamma + \phi'\gamma + \varepsilon < \phi\lambda_i$

Since $\phi'\varepsilon < \varepsilon$

$\rightarrow \phi' + \ < \phi'\lambda_i$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case (iii): $\lambda_i > \gamma M$

Payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to join the coalition ($j_i = 1$) changes. Equation 35 changes as:

$$\pi_i(j_i, j_{-i}) = [\phi''] \left[(\gamma(M + F') + \varepsilon\right] + [1 - \phi''] \left[\lambda_i + \gamma F' + \varepsilon\right] \tag{41}$$

The payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to not join the coalition ($j_i = 0$) remains the same as equation 36:

$$\pi_i(j_i, j_{-i}) = [\phi'] \left[\lambda_i + (\gamma(M - 1 + F')\right] + [1 - \phi'] \left[\lambda_i + \gamma(F')\right] \tag{42}$$

An individual will compare equation 41 and equation 42 for each of the sub cases to decide whether to join the coalition or not.

Case(iii) (a) $\gamma(M-1) \geq 1$**,** $\gamma M > 1$

$\phi'' = \phi' = 1$ and $1 - \phi'' = 1 - \phi' = 0$. Plugging these values in equation 41 and equation 42 shows that individual will not join the coalition as equation 41 < equation 42: $\gamma(M+F') + \varepsilon < \lambda_i + \gamma(M-1+F')$

$\rightarrow \gamma(M+F') + \varepsilon < \lambda_i + \gamma(M+F') - \gamma$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case (iii) (b) $\gamma(M-1) < 1$**,** $\gamma M = 1$

$\phi'' = 1$ and $1 - \phi'' = 0$. $i \in N$ will not join the coalition if equation 41 < equation 42:

$\gamma(M+F') + \varepsilon < [\phi'][\lambda_i + \gamma(M-1+F')] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [1-\phi'][\gamma(M+F')] + \varepsilon < \phi'[\lambda_i - \gamma] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [1-\phi'][\gamma M] + \varepsilon < \lambda_i - \phi'\gamma$

Replace γM with γ on the right side as $\gamma M > \gamma$

$\rightarrow [1-\phi'][\gamma] + \phi'\gamma + \varepsilon < \lambda_i$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case(iii)(c) $\gamma(M-1) < 1$**,** $\gamma M < 1$

$i \in N$ will not join the coalition if equation 41 < equation 42.

$\phi''[\gamma(M+F')] + [1-\phi''][\lambda_i + \gamma F'] + \varepsilon < [\phi'][\lambda_i + \gamma(M-1+F')] + [1-\phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi'' - \phi'][\gamma(M+F')] + \varepsilon < \phi'[\lambda_i - \gamma] + [\phi'' - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi'' - \phi'][\gamma M] + \varepsilon < \phi'[\lambda_i - \gamma] + [\phi'' - \phi']\lambda_i$

$\rightarrow [\phi'' - \phi'][\gamma M] + \varepsilon < \phi'\lambda_i - \phi'\gamma + \phi''\lambda_i - \phi'\lambda_i$

$\rightarrow [\phi'' - \phi'][\gamma M] + \varepsilon < \phi''\lambda_i - \phi'\gamma$

Replace γM with γ on the right side as $\gamma M > \gamma$

$\rightarrow [\phi'' - \phi']\gamma + \phi'\gamma + \varepsilon < \phi''\lambda_i$

Since $\phi''\varepsilon < \varepsilon$

$\rightarrow \phi''\gamma + \phi''\varepsilon < \phi''\lambda_i$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Thus $i \in N$ will not join the coalition with $\lambda_i > \gamma + \varepsilon$. $i \in N$ joins the coalition when $\lambda_i \leq \gamma$, which also holds true for $\lambda_i \leq \gamma + \varepsilon$ since $\varepsilon > 0$. Thus there are some individuals in case(ii) and case (iii), with $\lambda_i > \gamma$ however $\lambda_i \leq \gamma + \varepsilon$ who join the coalition.

Proof for Proposition IV

Comparing the cutoff for joining the coalition($\lambda_i \leq \gamma + \varepsilon$) and the cutoff for contributing ($\lambda_i \leq \gamma M$), we can have the following two situations:

1. $\gamma + \varepsilon \leq \gamma M$: In this case the cutoff for joining the coalition is less than the cutoff for contributing to the public good. Thus everyone who joins the coalition will be contributing to the public good.

2. $\gamma + \varepsilon > \gamma M$: In this case everyone who joins the coalition might not contribute. It depends whether majority of individuals have the weight $\lambda_i \leq \gamma M$.

If majority of the individuals have the weight $\lambda_i \leq \gamma M$ then the coalition contributes to the public good. If majority of the individuals have the weight $\lambda_i > \gamma M$ then the coalition does not contribute to the public good.

When majority will contribute

Since λ_i is uniformly distributed, if there is more mass in the interval $0 < \lambda_i \leq \gamma M$ than in the remaining interval ($\lambda_i \leq \gamma + \varepsilon - \gamma M$), then the majority will contribute to the public good. In other words, if $\gamma M \geq \gamma + \varepsilon - \gamma M$ then the majority of the coalition vote yes to contribute.

$\gamma M \geq \gamma + \varepsilon - \gamma M$

$\rightarrow 2\gamma M \geq \gamma + \varepsilon$

$\rightarrow \varepsilon \leq \gamma(2M - 1)$

$\therefore$ If $\varepsilon \leq \gamma(2M - 1)$, then coalition contributes to the public good.

Majority will not contribute

Since λ_i is uniformly distributed, if there is more mass in the interval $\lambda_i > \gamma M$ then the majority will not contribute to the public good. In other words if $\gamma M < \gamma + \varepsilon - \gamma M$ then the majority of the contributors will not vote yes to contribute.

$\gamma M < \gamma + \varepsilon - \gamma M$

$\rightarrow 2\gamma M < \gamma + \varepsilon$

$\rightarrow \varepsilon > \gamma(2M - 1)$

$\therefore$ If $\varepsilon > \gamma(2M - 1)$, then the coalition does not contribute to the public good.

Cost from isolation

It is easy to see that cost from isolation appears in both equation 22 and equation 23 . This leads to no change in the cutoff for being a fringe member in equation 24. Hence no change in Stage I results.

Proof for Proposition V

The proof is similar to the proof of Proposition III. Instead of ε appearing on the left hand side of the equations, we have κ being subtracted from the right hand side of the equations.

Proof for Proposition VI

The proof is very similar to the proof of Proposition IV. Again I compare if there is more mass in the interval $\lambda_i \leq \gamma M$ or in the interval $\lambda_i \leq \gamma + \kappa - \gamma M$.