Coalitions improve coordination and provision of public goods: Theory and experimental evidence

Sheryl Ball[1], Sudipta Sarangi[2], Sakshi Upadhyay[3]

[1]Department of Economics, Virginia Tech, email: sball@vt.edu
[2]Department of Economics, Virginia Tech, email: ssarangi@vt.edu
[3] Department of Economics, Washington and Lee University, email: supadhyay@wlu.edu

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Abstract

We study a public goods game with heterogeneous agents who care about their own payoff as well as of the player who receives the lowest payoff. The weight on own payoff varies across players, and is private information. We first develop a theoretical model and then test the predictions of our model in a laboratory setting under different parameter conditions. In both our model and experiments, introducing a coalition formation stage prior to making a contribution decision enables sorting of players according to their preferences, resulting in higher contributions to the public good. Additionally, we find that participants in our experiment take previous period outcomes into account while making current period decisions. These results help explain successful coalitions, like international environmental agreements, that are effective in creating real world public goods like reductions in carbon emission.

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1 Introduction

In this paper we study the provision of public goods by bringing together two stylized facts: (i) in practice, the provision of public goods is difficult because agents have heterogeneous preferences which are private information, and (ii) real world solutions often involve the creation of a coalition, an alliance formed when two or more parties agree to work together to achieve a common goal. The landmark 2015 Paris Agreement, for example, was adopted by 196 countries whose ongoing goal is to limit global warming by implementing country specific economic and social changes to reduce carbon emissions, with more developed countries providing financial support to developing countries. Taking these two styled facts into account we consider a two-stage public goods game with heterogeneous players who care about their own payoff as well as the payoff of the player with the lowest payoff. The weights on these two components of the payoff function are private information. In the first stage, players choose whether to join a coalition. In the second stage, coalition members vote on whether the coalition will contribute to the public good where the outcome of the vote is binding on all members of the coalition. Also in the second stage, non-members make independent decisions about whether to contribute. We show that contributions to the public good increase with both the number of participants who satisfy a derived threshold condition based on their social preferences, and on the marginal per capital return (MPCR). In the Bayesian Nash Equilibrium of the game, players who satisfy the threshold condition will join the coalition and anyone who joins the coalition in the first stage will vote to contribute to the public good in the second stage. Our experiments confirm our model's predictions.

In our model coalitions increase contributions by helping players to coordinate on an equilibrium. When coalitions are disallowed multiple equilibria are possible, and players may reduce contributions to avoid coordination failures in situations where they are the sole contributor. In the version of the game with coalitions, players learn the number of coalition members prior to deciding whether to vote to contribute with the outcome of the voting game determined by majority rule. Thus, it is impossible for a player to be the sole contributor unless they knowingly choose to be. At the same time, the coalition sorts players so that those who have stronger social preferences join the coalition while those with weaker social preferences do not. The conditions for joining the coalition are equivalent to the conditions for contributing, so players can reason ex ante that coalition members will vote to contribute. We also extend our theoretical model by incorporating benefits from voting (independent of public good) into an individual's utility function. This extension is motivated from literature on group identity, where individuals derive benefit from being part of a larger group or having the right to vote. We find more individuals are likely to join the coalition, but not all coalition members vote to contribute to public good. However, we do not test for theoretical predictions of this extension.

Our paper adds to a substantial literature on mechanisms for increasing voluntary contributions to public goods, only some of which work by coordinating individual's action. First, note that while experimental results on public good contribution stand in contrast to the standard free riding prediction and suggest higher contribution rates at 40-60% of the efficient level (Ledyard (1995)) many strategies have been tried to reach full efficiency. Making individual contributions observable, for example, increases contributions both in laboratory experiments (Savikhin Samek and Sheremeta (2014)) and field experiments (Rogers, Ternovski and Yoeli (2016)). Other strategies include adding a pre-play communication stage (Isaac and Walker (1988), Ostrom et al. (1994), Reischmann and Oechssler (2018)) and allowing punishment of free riders (Ostrom et al. (1994), Fehr and Gächter (2000), Ramalingam, Morales and Walker (2019), Kosfeld, Okada and Riedl (2009)) both of which facilitate higher contributions to the public good (Chaudhuri (2011)).

Specifically, we also add to the existing literature on the role of coalitions as a mechanism for coordination in public good games. Coalition formation helps in establishing conditions under which individuals will join a coalition and also contribute to the public good. Previous work established a minimum participation threshold for forming a coalition and shows that when the threshold is satisfied coalitions form and free riding behavior is mitigated (Burger and Kolstad (2009), Kosfeld, Okada and Riedl (2009), McEvoy (2010), Kolstad (2014), Weikard, Wangler and Freytag (2015)). Another approach is to require coalition members to announce their planned contribution levels and set a binding rule that all coalition members must contribute the minimum of these announced levels (Dannenberg, Lange and Sturm (2014), Schmidt and Ockenfels (2021)). These approaches not only help in coordinating individual's action but also results in higher contributions than expected under a VCM.

Since we assume that players have social preferences, this work also contributes to the literature in behavioral economics that has often focused on how social preferences affect the outcomes of games. In our model, players' social preferences are such that players care about both their own earnings and those of the lowest earning player. Models which incorporate social preferences provide a motivation for why individuals may contribute more to a public good than standard theory predicts (Chaudhuri (2011), Fehr and Fischbacher (2002)). For example, cooperation can increase when individuals are motivated by reciprocity (Rabin (1993), Fehr and Fischbacher (2002)), when players are altruistic (Andreoni, Harbaugh and Vesterlund (2010)), when they prefer equitable resource distributions (Fehr and Schmidt (1999)) or when they are inequality averse (Kosfeld, Okada and Riedl (2009)).

Finally, while in traditional models the MPCR does not affect expected outcomes of VCM games, experimental work finds that higher MPCR significantly increases the probability of joining the coalition and contributing to the public good (Ledyard (1995).Chaudhuri (2011)). While in our model higher MPCR leads to more people joining the coalition and contributing to the public good, this is not always the case. Some existing models (Komisar (1969), Barrett (1994)) suggest an inverse relationship between coalition size and MPCR. These results, however, stand in contrast to both our theory and experimental results, as well as to the existence of large sized coalitions, such as International Environmental Agreements such as the 2015 Paris Agreement.

In Section 2 we develop the theory model, in Section 3 we discuss our theoretical extension and in Section 4 we provide our experimental design and hypotheses. Section 5 describes our experimental results and Section 6 concludes.

2 Model

Let $N = \{1, 2, \cdots, n\}$ denote the set of players, each with a unit endowment to allocate in a two-stage public good game. Contributions are assumed to be binary, i.e., individuals must either contribute their entire endowment or nothing. We denote the action set of a player by $e_i \in \{0, 1\}$, where $e_i = 1$ implies that player i contributes to the public good and $e_i = 0$ implies that they do not. In Stage I of the game, each player decides whether or not to join a coalition, denoted by the set M. Players that choose not to join are called *fringe members* and belong to the set F. Thus $N = M \cup F$ where $|M|$ and $|F|$ denote the respective set cardinalities. At the end of stage I, all players learn the cardinality of each set. In Stage II, each fringe member independently decides whether to contribute, while members of the coalition vote on whether or not they will contribute to the public good, with the outcome determined by majority rule. Every $i \in N$ knows prior to the start of the game that the outcome of this vote is binding on all members of the coalition M.

Let F' denote the set of fringe members who contribute to the public good, and let Q denote the total number of individuals who contribute.[1] The MPCR of player $i \in N$ is denoted by $\gamma > 0$. Each individual's total payoff is a convex combination of their "own payoff" and a "social payoff" which captures their social preferences. Let λ_i be the weight on the own payoff component, where for all $i \in N$, λ_i is private information drawn from a uniform distribution on [0,1]. We assume that social preferences are such that players care about the player with the lowest own payoff. There are two possible interpretations of this assumption. First, the social component of total payoffs resembles Rawlsian preferences in the sense that individuals care about the player with the lowest own payoff. Since we will show the player with the lowest payoff is in that situation because they contributed their endowment to the public good, the second interpretation is that other players care about low payoff players because they are civic minded.

In our setup the lowest own payoff of any $i \in N$ is given by γQ, i.e., individuals for whom $e_i = 1$, while a higher payoff of $1 + \gamma Q$ is earned by those with $e_i = 0$. When nobody contributes and everyone keeps their endowment of 1, the public good is not provided. This also leads to the lowest total payoff. With slight abuse of notation let e_i denote the strategy of player i and e_{-i} the strategy of the remaining players. Assuming $Q > 1$, the total (weighted) payoff of a player i is given by:

$$\pi_i(e_i, e_{-i}) = \begin{cases} \lambda_i(\gamma Q) + (1 - \lambda_i)\gamma Q = \gamma Q & \text{for } e_i = 1, \\ \lambda_i(1 + \gamma Q) + (1 - \lambda_i)\gamma Q & \text{for } e_i = 0, \end{cases} \tag{1}$$

The first term in each expression is i's weighted own payoff while the second term is

[1] If the majority of coalition members vote to contribute then $Q = |M| + |F'|$ and $Q = |F'|$ otherwise.

their corresponding social payoff. Using a 3 player example we now illustrate the role of heterogeneous weights and coalitions in determining the equilibrium public good outcome.

Example Let $N = \{1, 2, 3\}$ and $\lambda_1 = 0.2$, $\lambda_2 = 0.4$ and $\lambda_3 = 0.6$ be the players' private information weights on their own payoffs. The payoff in the standard zero contribution equilibrium is 1 since everyone simply retains their endowment. In the example, for simplicity, we will assume that only the players who join the coalition contribute. If only one individual contributes to the public good, the payoff of this individual will be: $\pi_i = \lambda_i(\gamma) + (1 - \lambda_i)(\gamma) = \gamma$ which cannot be an equilibrium since $\gamma < 1$. If two people contribute their payoff will be 2γ which can now be an equilibrium when $\gamma > 1/2$. Similarly, if $\gamma > 1/3$, then all three players contributing is also an equilibrium. Thus, in the absence of a coordination mechanism there are multiple possible equilibria, and it is easy to see that players wish to avoid the coordination failure associated with being the sole contributor, and thus earning the lowest payoff. ■

This example illustrates how the introduction of a coalition formation stage can potentially help achieve a better equilibrium outcome by sorting individuals and facilitating coordination. As we will see in Proposition II, players will join a coalition if $\lambda_i \leq \gamma$. For instance, for MPCR (γ) = 0.5, individuals with $\lambda_i = 0.2$ and 0.4 will join the coalition and vote to contribute and for $\gamma > 0.6$, everyone will join the coalition and vote to contribute. Thus, even though λ_i is private information, the coalition stage automatically sorts the players and ensures a unique contribution equilibrium as long as all individuals with $\lambda_i \leq \gamma$ are in the coalition. Essentially, joining a coalition provides a coordination signal to the other players that those who join the coalition will contribute to the public good. Now we proceed to solving the two-stage game.

Stage II: Contributing to the public good We solve our two-stage game using backward induction. In Stage II the coalition contributes to public good only if the majority votes to do so. Let the voting decision of a player $i \in M$ be denoted by $v_i \in \{0, 1\}$ where $v_i = 1$ implies voting to contribute to public good, and $v_i = 0$ implies otherwise. With a slight abuse of notation, let v_i be the strategy of player i and v_{-i} be the strategy of the other players. Note that voting to contribute is not the same as contributing since the coalition decides based on the wishes of the majority. We now state our first proposition. Let $\lambda_M = \gamma M$ and $\lambda_F = \gamma$; note that below we use λ_F and γ interchangeably.

Proposition I: In equilibrium (i) For all $i \in M$, if $\lambda_i \leq \lambda_M$, then $v_i = 1$, otherwise $v_i = 0$. (ii) For all $i \in F$, if $\lambda_i \leq \lambda_F$, then $e_i = 1$, otherwise $e_i = 0$.

PROOF: See Appendix A.

Part (i) provides the threshold condition on λ_i which, when satisfied, implies that coalition members will contribute. These threshold values directly follow from the comparison of expected total payoffs of a coalition member from contributing versus not contributing.[2] The threshold value $\lambda_i \leq \gamma M$ can also be interpreted as the probability that $i \in M$ will vote to contribute to the public good. Observe that this probability is increasing in the size of the coalition (M) and the benefit of cooperation (γ). Likewise, part (ii) states the cutoff conditions on λ_i which, when satisfied, implies that fringe members will contribute. This follows from a similar expected total payoff comparison of fringe members and the condition can also be interpreted as the probability $i \in F$ will contribute to the public good. Observe that this threshold value is independent of the decisions of coalition members and is driven by the fact that the payoff of fringe members is an increasing function of the minimum payoff received by a player. Hence to increase the minimum payoff (which will be highest when everyone is contributing), fringe members contribute (independent of decisions by coalition members) when γ increases.

Comparing λ_F and λ_M, we arrive at three possible ranges of λ_i: $0 < \lambda_i \leq \lambda_F(= \gamma)$, $\lambda_F(= \gamma) < \lambda_i \leq \lambda_M$, $\lambda_M < \lambda_i < 1$. We use these three threshold values to derive Proposition II.[3]

Stage I: Deciding whether to join the coalition Based on the two cutoffs from Stage II, we have three possible cases: (i) $\lambda_i \leq \gamma$ which also implies $\lambda_i \leq \gamma M$, (ii) $\gamma < \lambda_i \leq \gamma M$ and (iii) $\lambda_i > \gamma M$. For simplicity, M is assumed to be odd and we need at least $\frac{M+1}{2} = m'$ people to vote in favor of the public good for contributions to occur. Hence, if $\lambda_i \leq \gamma M$ for the majority, then the coalition will contribute. Given that λ_i is uniformly distributed between 0 and 1, the probability that $\lambda_i \leq \gamma M$ is given by γM and the probability that $\lambda_i > \gamma M$ can be expressed as $1 - \gamma M$. We also know from stage II that each $i \in F'$ satisfies $\lambda_i \leq \gamma$. Comparing expected total payoffs in the three cases described above leads us to the second result of our paper. Let the action set of a player be denoted by $j_i \in \{0, 1\}$ where $j_i = 1$ implies $i \in N$ joins the coalition, and $j_i = 0$ implies $i \in N$ does not join the coalition.

Proposition II: In the Bayesian Nash equilibrium, when $\lambda_i \leq \gamma$ then $j_i = 1$, otherwise $j_i = 0$.

PROOF: See Appendix A.

From Proposition II, we see that only individuals with relatively lower weight on their own payoff ($\lambda_i \leq \gamma$) will join the coalition. Thus, for the sorting mechanism to work, we need individuals to be civic minded or to care about others. Further, note that size of equilibrium coalition M is determined by individuals for whom $\lambda_i \leq \gamma$. Finally, from Propositions I and II, we can see that an increase in γ increases both the probability of joining the coalition and the probability of contributing to public good. From these results it also follows that an increase in MPCR leads to a larger coalition. It is also

[2]Expected total payoff takes into account the total payoff conditional on the probability of the coalition contributing, for more details, please see the Appendix.

[3]Note that the $\gamma \leq \lambda_i \leq \gamma M$ case does not arise in Proposition I, because we endogenously derive these cutoffs through relevant payoff comparisons. This follows from backward induction: since coalition members know the size of the coalition before making their contribution voting decision, their cutoff is based on M.

always true that everyone who joins the coalition will satisfy $\lambda_i \leq \gamma$ as well as $\lambda_i \leq \gamma M$, which means they will also vote to contribute to the public good. Thus it is a dominant strategy for anyone who joins the coalition to also vote to contribute to the public good.

Recall from Example 1 that multiple equilibria are possible in the game without coalitions. It is now easy to see how the coalition formation stage helps sort individuals and facilitates coordination. From Proposition II, players will join a coalition when $\lambda_i \leq \gamma$. Hence for MPCR $(\gamma) = 0.5$, individuals with $\lambda_i = 0.2$ and 0.4 will join the coalition, and if $\gamma > 0.6$, all three players will join the coalition. Thus, although λ_i is private information, Stage I automatically sorts the players and ensures a unique equilibrium where individuals with $\lambda_i \leq \gamma$ will join the coalition and contribute.

3 Theoretical extension

In this section we extend our baseline model to incorporate benefits from voting. This extension is motivated from existing literature on group-identity. Akerlof and Kranton (2000) propose a general utility function which incorporates group-identity as a motivation for human behavior. Group-identity accounts not only for a person's self-image, but also the prevailing social differences in a society. Such sense of attachment in a person will likely overcome his/her selfish motivations and usher in behaviour consistent with the entire group. Authors also apply the results from their paper to study gender discrimination in the workplace, the economics of poverty and social exclusion, and the household division of labor. Chen and Li (2009) use laboratory experiments to measure impact of group identity on participant's social preference in a wide variety of games and find that group identity has significant impact on the distribution of preferences. For instance, participants put a higher weight on in-group match's payoff in their utility function and are more likely to reward an in-group match for good behavior as compared to out-group match. Ockenfels and Werner (2014) also find evidence of in-group favouritism in their dictator game experiment as dictators transfer substantially more to participants who are in their group, compared to out group members. Another reason to join the coalition could be the benefit individuals receive from voting. Edlin, Gelman and Kaplan (2007) show that for individuals with both selfish and social preference, social preference dominates and makes it rational for a person to vote even in large elections. One of the plausible explanation in the political science literature talks about how voters consider themselves as part of large groups (Uhlaner, 1989). Such sense of belonging pushes them to vote in the hope of benefiting others in his/her network (Fowler, 2005).

Thus there can be various incentives for an individual to join a coalition: group identity or the benefit they receive from voting. In our model, although individuals who are part of the coalition have the right to vote to contribute towards the public good, there is no benefit from this voting hitherto. However, individuals could join a coalition to be associated with a group or due to the benefit they derive from voting and thereby the utility function can incorporate this benefit from being in a coalition. Using group identity or the benefit from voting as a motivation, our model now assimilates the right to vote as an added benefit for the individuals who are part of coalitions. The equations can be re-written as:

$$\pi_i(e_i, e_{-i}) = \begin{cases} \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q = \gamma Q & \text{for } e_i = 1, Q > 1, i \in F \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q & \text{for } e_i = 0, Q > 1, i \in F \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q + \varepsilon & \text{for} e_i = 0, Q > 1, i \in M \\ \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q + \varepsilon & \text{for} e_i = 1, Q > 1, i \in M \end{cases} \tag{2}$$

We find that Stage II results remain unchanged. The coalition members vote to contribute if $\lambda_i \leq \gamma M$ or $\lambda_i \leq \lambda M$. [4] The fringe members contribute if $\lambda_i \leq \gamma$ or $\lambda_i \leq \gamma F$. [5] This is because only the coalition members enjoy the benefit, irrespective of their vote.

However, Stage I cutoff changes:

$i \in N$ will join the coalition if $\lambda_i \leq \gamma + \varepsilon$, $i \in N$ will not join if $\lambda_i > \gamma + \varepsilon$.

Note that if the benefit from right-to-vote changes from ε to ε_i, the stage I cutoff for joining changes to $\lambda_i \leq \gamma + \varepsilon_i$. Benefit in terms of ε_i describes the case when every individual derives a unique benefit from voting.

Suppose the benefit from voting increases with the size of the coalition, then the benefit of each individual is εM. This implies that every individual receives higher benefit from voting as the size of the coalition increases. The equations can be re-written as:

$$\pi_i(e_i, e_{-i}) = \begin{cases} \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q = \gamma Q & \text{for } e_i = 1, Q > 1, i \in F \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q & \text{for } e_i = 0, Q > 1, i \in F \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q + \varepsilon M & \text{for} e_i = 0, Q > 1, i \in M \\ \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q + \varepsilon M & \text{for} e_i = 1, Q > 1, i \in M \end{cases} \tag{3}$$

Stage II cutoff remains unchanged, however Stage I cutoff changes:

$i \in N$ will join the coalition if $\lambda_i \leq \gamma + \varepsilon M$, $i \in N$ will not join if $\lambda_i > \gamma + \varepsilon M$.

Note that if the benefit from right to vote changes from εM to $\varepsilon_i M$, the Stage I cutoff for joining changes to $\lambda_i \leq \gamma + \varepsilon_i M$. Benefit in terms of $\varepsilon_i M$ would mean that each individual receives a different marginal benefit from voting and the overall benefit increases with the size of the coalition.

From the above analysis we have the following proposition for the extended model:

Proposition III: In the Bayesian Nash equilibrium, when $\lambda_i \leq \gamma + \varepsilon$ then $j_i = 1$, otherwise $j_i = 0$.

PROOF: See Appendix A

[4] It is easy to see that after inclusion of benefit from voting equation 11 appears as: $\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][\gamma(M+F') + \varepsilon - (\lambda_i + \gamma F' + \varepsilon)] \geq 0$. This leads to no change in the cutoff for joining the coalition in equation 12.

[5] There is no change in the cutoff for joining the coalition in equation 15 as they do not enjoy any benefit.

Here the benefit from voting is given in terms of ε. However, it can also be interpreted in terms of ε_i, εM or $\varepsilon_i M$ as discussed above.

Recall in the benchmark model, where we had no tenable benefit from voting, individuals who join the coalition (cutoff $\lambda_i \leq \gamma$) always contribute to the public good ($\lambda_i \leq \gamma M$). However with the addition of benefit from voting, Stage II cutoff stays the same, however Stage I cutoff changes. According to Stage I cutoff, individuals who satisfy $\lambda_i \leq \gamma + \varepsilon$ will join the coalition. However individuals with $\lambda_i \leq \gamma M$ vote to contribute to the public good. The relative difference in these two cutoffs will determine whether a coalition formed contributes to the public good or not. From this comparison we have the following proposition:

Proposition IV: If $\varepsilon > \gamma(2M-1)$, then a coalition of size less than or equal to M will not contribute to the public good.

PROOF: See Appendix A

Higher benefit from voting attracts more people to join the coalition(as compared to the benchmark model) but it is not necessary that the coalition contributes to the public good. However, as the size of the coalition increases or the benefit from cooperation increases, even with a higher benefit from voting we can expect majority to contribute. This is because as γ or M increases, the benefit from contributing: γM also increases.

It is easy to see that when $M = 2$ and $\varepsilon \leq 3\gamma$ then the coalition of size 2 contributes to the public good. Since the least value M can take is 2, we have the following corollary:

Corollary I: If $0 \leq \varepsilon \leq 3\gamma$, then majority in coalition of any size can be sustained.

We can also interpret this extension to model the behavior when individuals feel isolated if they are not part of a group/coalition. We refer to this as home alone cost. This cost of isolation is represented by κ. The payoff function can be rewritten as:

$$\pi_i(e_i, e_{-i}) = \begin{cases} \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q = \gamma Q & \text{for } e_i = 1, Q > 1, i \in M \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q & \text{for } e_i = 0, Q > 1, i \in M \\ \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q - \kappa & \text{for } e_i = 0, Q > 1, i \in F \\ \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q - \kappa & \text{for } e_i = 1, Q > 1, i \in F \end{cases} \tag{4}$$

In the above equation, the equations in blue denote the cost κ an individual $i \in F$ faces when he/she is not a part of the group/ coalition.

Stage II results remain unchanged. The coalition members vote to contribute if $\lambda_i \leq \gamma M$[6].The fringe members contribute if $\lambda_i \leq \gamma$[7].

However, Stage I cutoff changes:

$i \in N$ will join the coalition if $\lambda_i \leq \gamma + \kappa$, $i \in N$ will not join if $\lambda_i > \gamma + \kappa$.

[6]There is no change in the cutoff for joining the coalition in equation 12 as they do not face any cost of isolation.

[7]It is easy to see that after inclusion of cost from isolation appears in both equation 13 and equation 14 . This leads to no change in the cutoff for being a fringe member in equation 15

Note that if the cost changes from κ to κ_i, the Stage I cutoff for joining changes to $\lambda_i \leq \gamma + \kappa_i$. Cost in terms of κ_i describes the case when every individual derives a unique cost from isolation. From the above analysis we arrive at the following proposition:
Proposition V: In the Bayesian Nash equilibrium, when $\lambda_i \leq \gamma + \kappa$ then $j_i = 1$, otherwise $j_i = 0$.

PROOF: See Appendix A

Here the cost from voting is given in terms of κ but it can also be interpreted in terms of κ_i as discussed above.

Stage I cutoff says that individuals who satisfy $\lambda_i \leq \gamma + \kappa$ will join the coalition. However individuals with $\lambda_i \leq \gamma M$ vote to contribute to the public good. The relative difference in these two cutoffs will determine whether a coalition formed contributes to the public good or not. From this comparison we have the following proposition:
Proposition VI: If $\kappa > \gamma(2M - 1)$, then a coalition of size less than or equal to M will not contribute to the public good.

PROOF: See appendix A

Free riders are likely to join the coalition to get rid of the cost of isolation. Presence of these free riders in the coalition leads to the coalition not contributing to the public good as the majority is not satisfied. However, as γ or M increases, the benefit from contributing to the public good γM also increases, thus we need even a higher cost for the coalition to not contribute. In other words, if the cost from isolation is small, then the benefit from contributing dominates and leads to the coalition contributing to the public good.

When $M = 2$ and $\kappa \leq 3\gamma$ then the coalition of size 2 contributes to the public good. Since the least value M can take is 2, we have the corollary also holding true in this case.
Corollary II: If $0 \leq \kappa \leq 3\gamma$, then majority in coalition of any size can be sustained.

Unlike our benchmark model, in this model voting plays a strategic role. The threshold for a coalition contributing will not be the same in case of unanimous voting. The position of λ_i will determine if the coalition contributes to the public good. A future extension is to see what happens when we include both ε and κ in our model. The relative difference in ε and κ will also be a determining factor for a coalition to contribute.

4 Experimental Methods and Hypotheses

The experiment was divided into two parts. In Part I, participants played a standard, 8-round, Voluntary Contribution Mechanism (VCM) public good game to familiarize them with the public goods setting. Payoffs for participants in Part I were calculated using equation 1, with $\lambda_i = 0$ and $e_i = 10$ if individuals contribute to the public good and $e_i = 0$ otherwise. After completion of Part I, participants were given separate instructions for Part II and then played 12 rounds of the two stage "coalition" version of the public goods game described in our model.[8] In both parts of the experiment, participants were randomly re-assigned to groups consisting of 6 members at the start of each round. In every round, each participant received an endowment of 10 points which could be entirely allocated to either their individual (private) account or the group project (public good). As is common in public goods experiments, we chose two different values of MPCR (γ) for both Part I and Part II to explore results in a low MPCR ($\gamma = 0.3$) and high MPCR ($\gamma = 0.7$) environment. At the end of each round, participants learned their earnings from both the group and individual accounts. At the conclusion of the experiment participants were paid for one randomly chosen round each from Part I and Part II.

Part II of the experiment added a first stage to the public goods game as well as adding social preferences to participant utility functions. Rather than participants making decisions based on their own innate social preferences we assigned each participant one of three weights for their payoffs. In particular, we use $\lambda_i = 0.2$ or $\lambda_i = 0.5$ or $\lambda_i = 0.8$, to represent high, medium and low social preferences, respectively. Instructions included a number of examples of how preferences were calculated, and participants maintained the same social preferences throughout Part II (instructions in Appendix C). This allowed us to more cleanly explore the effect of experience in previous rounds and to help avoid participant confusion. We conducted sessions with both heterogeneous and homogeneous weights on players' own utility, to examine if participants would be sensitive to these variations. In the homogeneous sessions there was only one player type, although player types varied across these sessions. In the heterogeneous sessions, a participant's social preference weights were randomly assigned by the computer while ensuring that all types were equally represented in each session. Payoffs for participants were calculated using equation 1. Incorporating the two values of MPCR ($\gamma = 0.3$ and $\gamma = 0.7$) into our model generates predictions about when an individual will join the coalition and contribute.

We conducted a total of 9 sessions, 6 homogeneous session and 3 heterogeneous, with 12 participants in each session for a total 108 participants. Experimental protocols were approved by the Virginia Tech Institutional Review Board, and all participants provided informed consent at the start of their session. Experiments were computerized using z-Tree (Fischbacher (2007)). Participants were primarily undergraduate students, recruited through the Virginia Tech Economic Research Lab's SONA system. Participants earned an average of \$20, which included both a \$10 show up fee and their earnings from Part I and Part II of the experiment, and which was paid to them in cash at the end of the experiment.

Based upon our model in section 2 we offer two hypotheses that organize our data analysis.

Hypothesis 1: *Individuals with a threshold value that satisfies $\lambda_i \leq \gamma$ both join the coalition and contribute to public good.*

[8] Since the objective of Part I was to familiarize subjects with the public goods setting we did not switch the two parts in any session. Moreover, Ashley, Ball and Eckel (2010) find no differences in individual behavior between the two halves of an experiment consisting of two different 10-round VCM games. Hence we did not randomize the order of the parts of the experiment.

Proposition 2 requires that individuals who join the coalition ($\lambda \leq \gamma$) also satisfy the cutoff for contributing to public good ($\lambda_i \leq \gamma M$) from Proposition 1.

Hypothesis 2: *When MPCR (γ) is high, we observe larger coalitions and more contributions to the public good.*

Hypothesis 2 follows from Proposition 2, which says that as γ increases, more individuals satisfy the cutoff for joining the coalition and Proposition 1, which says that high MPCR ($\gamma = 0.7$) leads to more individuals contributing to public good.

5 Results

We begin our analysis by examining data and summary statistics from the experiment. Recalling that the first 8 periods of play made up Part I of the experiment and consisted of a VCM, while coalitions were only possible in Part II of the experiment which began with period 9, Figure 1 illustrates the observed pattern of contributions to the public good. First note that, as is typical in VCM experiments (Ledyard, 1995), average contributions in Part I are high in the first period and then decline. In Part II of the experiment when coalitions are possible, however, neither contributions by coalition members nor fringe members appear to decline. The percentage of players who contribute after joining a coalition is significantly higher in Part II than Part I (94% compared to 55%), whereas the percentage of fringe players who join is significantly lower (24% compared to 55%, Kruskal-Wallace, p=0.0001).

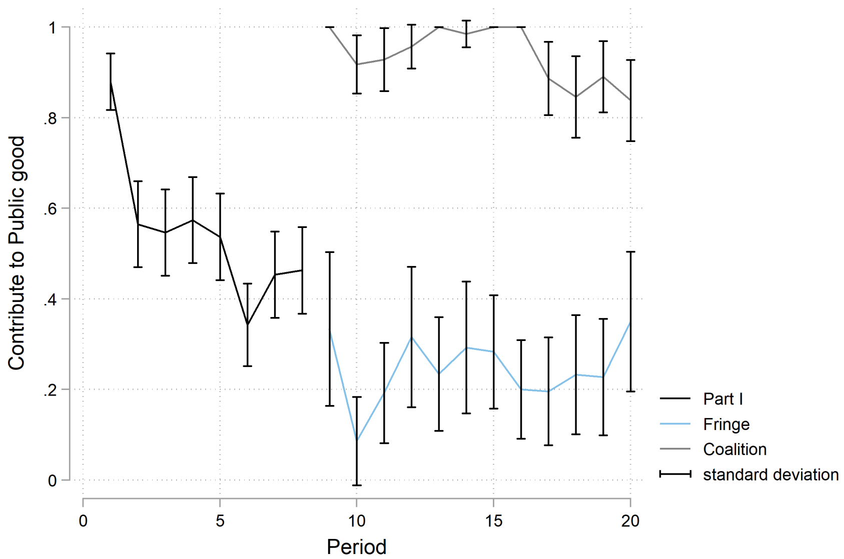

Figure 1: Comparison of contributions between Part 1 and Part 2

Note that since participants do not know how many others are likely to contribute in Part I because social preferences are private information, participants who prefer to contribute only when others also contribute choose to opt for the zero contribution equi-

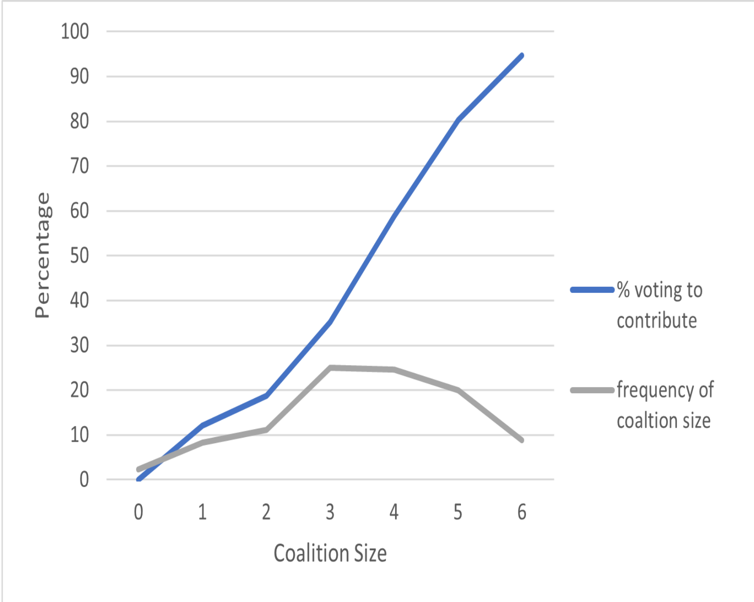

Figure 2: Percentage of votes conditional on coalition size

librium. When coalitions are possible, however, participants have information on how many other participants likely plan to contribute before they vote on the coalition's contribution decision. Every participant who joins a coalition recognizes that participants who join the coalition likely satisfy the contribution threshold, which increases their confidence that they will not be the only contributor in their group. Figure 2 illustrates the relationship between coalition size and the frequency with which participants vote to contribute. In our model the optimal coalition size varies based on the number of participants who satisfy the threshold. If everyone in a group meets the threshold then the optimal coalition size is six, whereas in a heterogeneous session where two players meet the threshold the optimal coalition size is two. Moreover, every such participant that joins the coalition should vote in favor of the public good and contribute. Interestingly, while 12 percent of participants vote to contribute when they are the only member of the coalition, this rate rises to 94.7 percent for coalitions with six members. These results are consistent with previous observations that many people care about equity in outcomes, thus conditionally cooperate in public goods games (Reischmann and Oechssler (2018); Oechssler, Reischmann and Sofianos (2022)).

Another standard result in the public goods literature is that the MPCR affects contribution levels (Ledyard, 1995). In Table 1 we report summary statistics on play in both the high and low MPCR conditions, and show that in Part I, both contributions ($p < 0.001$) and payoffs ($p < 0.001$) increase in the high MPCR condition. In Part II, the likelihood of joining the coalition is higher when the MPCR is high ($p < 0.001$, see Figure 3) as are both contributions by coalitions ($p < 0.001$, see Figure 4) and fringe members ($p < 0.001$). It follows, therefore, that payoffs to coalition members, fringe members and the of least well off person in each group are higher when MPCR is high ($p < 0.001$ for all three tests). These results are consistent with Hypothesis 2 as well as previous

Table 1: Descriptive statistics: low and high MPCR

	High MPCR	Low MPCR	Difference
Part I			
Player contributes to public good	0.782	0.308	0.475***
			(0.0298)
Payoff of participants	35.04	12.46	22.57***
			(0.398)
Part II			
Player joins the coalition	0.718	0.469	0.248***
			(0.0264)
Contribute to public good by coalition members	0.974	0.878	0.0959***
			(0.0177)
Contribute to public good by fringe members	0.344	0.189	0.155***
			(0.0387)
Payoff to coalition members	35.44	12.23	23.21***
			(0.439)
Payoff to fringe members	32.55	12.21	20.34***
			(0.548)
Payoff of least well off person	33.54	10.24	23.30***
			(0.372)
Coalition size	4.306	2.815	1.491***
			(0.0707)

Notes: Standard errors in parentheses. $***p < 0.01, **p < 0.05, *p < 0.10$
t-test for the difference of means.

experimental results on the relationship between MPCR and coalition size. (Burger and Kolstad (2009); Kosfeld, Okada and Riedl (2009)).

We next conduct more formal regression analysis to tease apart some of the factors that influence contributions. When decisions are binary, as in the decision to contribute, we use Probit, otherwise we use Ordinary Least Squares (OLS), and control for random or fixed effects as appropriate. To test Hypothesis 1, which provides the threshold conditions under which players join the coalition and contribute, we first divide our sample between individuals who satisfy the threshold for joining ($\lambda_i \leq \gamma$) and those who do not (Table 2). Note that players with $\lambda_i = 0.2$ always satisfy the threshold, while players with $\lambda_i = 0.5$ satisfy the threshold when $\gamma = 0.7$, and players for whom $\lambda_i = 0.8$ never satisfy the threshold so should never join. We find that players who satisfy the threshold condition are both more likely to join the coalition ($p < 0.001$) and contribute to the public good. Other factors also influence players' decisions. We find that, while our experiment consists of a series of one-shot games, players do respond to their previous game experience. The payoff of the least well off player in the previous game is negatively associated with both the decision to both join the coalition ($p < 0.01$) and to contribute ($p < 0.001$). This is consistent with our Rawlsian style social preferences in the sense that players increase their contributions in response to seeing the earnings of the least well off member of the group decline. In addition, coalition size in the previous period has a positive effect on both decision to join the coalition ($p < 0.001$) and decision to contribute ($p < 0.001$). We

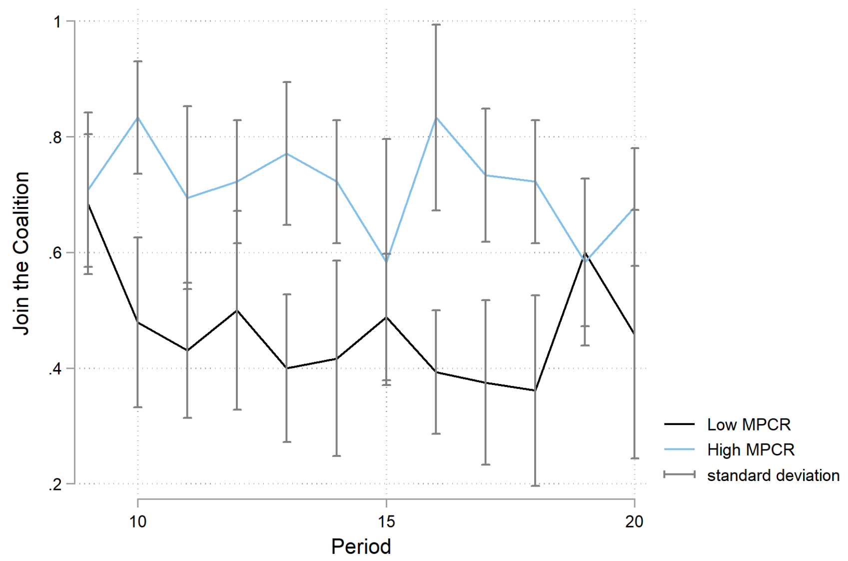

Figure 3: Effect of MPCR on decision to join the coalition in Part II

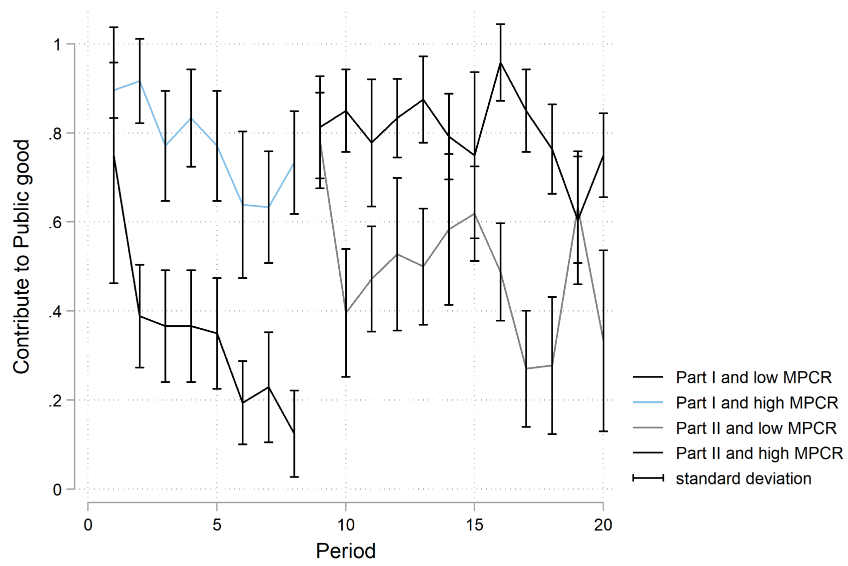

Figure 4: Effect of MPCR on decision to contribute

also test our results by including the lag of the number of contributors and find similar results (Appendix B Table B.2).[9]

Table 2: Probit estimates of average marginal effect on the decision to join and contribute to the coalition

	(1) Decision to join	(2) Decision to contribute
Threshold satisfied	0.202***	0.214***
	(0.0422)	(0.0362)
Lagged payoff of least well off person	-0.00445**	-0.00458**
	(0.00150)	(0.00144)
Lagged coalition size	0.0398**	0.0429**
	(0.0150)	(0.0138)
Controls	Yes	Yes
Observations	1188	1188

Notes: Dependent variable is decision to join/contribute (1:yes,0:no), margins reported here. Results on threshold satisfied are in comparison to threshold not satisfied. Controls include gender dummy, political orientation and number of Economics classes taken. Full table available in Appendix B (Table B.1). Standard errors in parentheses and clustered at participant level. $***p < 0.01, **p < 0.05, *p < 0.10$

Hypothesis 2 posits a positive relationship between MPCR and the decision to join the coalition and contribute. Since coalitions were only allowed in Part II of the experiment, Table 3 reports results on the decision to join the coalition and contribute only for data from Part II. In support of Hypothesis 2, we find that High MPCR has a positive and significant marginal effect on joining the coalition ($p < 0.001$) and contributing to public good ($p < 0.001$). This is to be expected both based on prior experimental results and, specific to our formulation of the problem, the fact that as MPCR increases more players satisfy the threshold. Since some players with ($\lambda_i = 0.5$) satisfy the threshold while others do not, we only include ($\lambda_i = 0.8$) as a regressor in this equation. We find that selfish preferences ($\lambda_i = 0.8$) are associated with reduced likelihood of both joining the coalition ($p < 0.01$) and contributing ($p < 0.001$). Inclusion of participants with heterogeneous preferences had no significant effect on participants' behavior, a finding that is not surprising since our model says preference heterogeneity should not matter. We again find a negative effect of the payoff of the least well off player in the previous period on both both the decision to join the coalition ($p < 0.01$) and to contribute ($p < 0.01$). Finally, coalition size in the previous period has a marginally significant positive effect on decision to join the coalition ($p < 0.10$) and a significant effect on the

[9] The lag of the number of contributors and coalition size are highly correlated, hence we use different regression models for Appendix B Table B.2 and Table B.4.

Table 3: Average Marginal Effect of key explanatory variables on the decision to join and contribute to coalition: Estimates from Probit Regression(with coalition size)

	(1) Decision to join	(2) Decision to contribute
High MPCR	0.269***	0.308***
	(0.0327)	(0.0281)
$\lambda_i = 0.8$	-0.158**	-0.170***
	(0.0566)	(0.0499)
Homogeneous session	0.0569	0.0601
	(0.0527)	(0.0426)
Lagged Payoff of least well off person	-0.00467**	-0.00499***
	(0.00150)	(0.00134)
Lagged Coalition Size	0.0325*	0.0340**
	(0.0153)	(0.0130)
Controls	Yes	Yes
Observations	1188	1188

Notes: Dependent variable is decision to join/contribute(1:yes,0:no), margins reported here. Results on $\lambda_i = 0.8$ are in comparison to low social preferences where $\lambda_i = 0.2$ and $\lambda = 0.5$ are clubbed together. Results on High MPCR are in comparison to low MPCR. Results in homogeneous session are in comparison to heterogeneous session. Controls include gender dummy, political orientation and number of Economics classes taken. Full table available in Appendix B (Table B.3). Standard errors in parentheses and clustered at participant level. $***p < 0.01, **p < 0.05, *p < 0.10$

decision to contribute ($p < 0.01$). We also test our results by including the lag of the number of contributors and find similar results (Appendix B Table B.4).

6 Discussion

In this paper, we explore a novel role of coalitions for public good provisioning, namely that in a society where social preferences are heterogeneous and private information, coalitions can act as a self selection mechanism to sort agents according to the strength of their social preferences. In the absence of coalitions multiple equilibria exist, meaning that coordination failure in the form of low contributions is the most likely result. We show that that individuals with relatively lower weight on their own payoff opt to join the coalition and contribute to the public good. Even though the weight on an individual's own payoff is private information, the coalition stage sorts players and insures a unique equilibrium where players who satisfy a threshold condition join the coalition and contribute. Moreover, high MPCR increases the likelihood of joining the coalition and contributing to the public good. Unlike some previous theoretical findings (Komisar (1969), Barrett (1994), Kolstad (2012), Kolstad (2014)) our framework ensures the existence of large sized coalitions even for high MPCR.

We also conduct lab experiments to test the theoretical predictions of our model and find that the data support for our model. First, we find that individuals who satisfy the threshold condition shown in our model are more likely to both join the coalition and contribute to the public good. As in our model, both coalition size and contributions to the public good increase with MPCR, a result consistent with previous experimental work (Burger and Kolstad (2009), Kosfeld, Okada and Riedl (2009)). The role of coalitions as a signaling device is also supported since we find that if more people join the coalition a higher the percentage of coalition members vote to contribute to the public good.

Our experiment reveals that the probability of joining the coalition and contributing to public good can be affected by two additional factors not captured by our one-shot public goods game — the previous period's payoff of the least well off person, and the previous period's coalition size. These results are consistent with existing work showing that decision makers who care about justice, inequality, and those who are conditional cooperators (cooperate when others cooperate) and support the success of coalitions (Kosfeld, Okada and Riedl (2009); Lin (2018)). These results are also consistent with research which shows that participants who observe that they are earning more or less than their peers may adjust their contributions in subsequent periods (see for instance, Lin (2018)).

Our model can be generalized in a number of ways. For example, we can show that a unanimous voting rule can serve as an alternative to majority rule. It is easy to see that an individual who contributes under majority rule will also contribute under a unanimous rule, because everyone who joins the coalition satisfies $\lambda_i \leq \gamma$, thereby also satisfying $\lambda_i \leq \gamma M$, meaning they will vote to contribute. Thus, any public good provided under majority rule will also be provided under unanimous voting. It is also easy to check that our results will hold when λ_i is normally distributed. Future research can explore testing the theoretical predictions of our extensions: whether having some benefit from joining a coalition (independent of the public good) lowers the threshold for joining the coalition and voting to contribute.

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Appendix A: Model details and proofs

Proof of Proposition I

Assume M is odd, so the number of coalition members needed for the coalition to contribute is given by: $m' = \frac{M+1}{2}$.[10] We solve for Stage II by comparing the expected total payoff for contributing to the expected total payoff for not contributing, for which we will be using the probabilities of contribution. Heterogeneity in preferences leads to a different probability for contributing to the public good.

The probability of contributing to the public good for each $i \in M$ can be ordered as: $p_1, p_2, \cdots, p_M$. Here p_1 is the probability of the person who is most likely to contribute to the public good. p_M is the probability of the person who is least likely to contribute to the public good and let it be represented by p. For technical reasons and without loss of generality, we substitute all the probabilities with p_M i.e. p. Using the least probability, i.e, p will give us the least expected total payoff from contributing. p is the least probability for each $i \in M$ voting yes to contribute to the public good. It follows a binomial distribution and $0 \leq p \leq 1$ since contribution is a binary decision. A coalition member will vote to contribute to the public good if the expected total payoff from contribution is at-least equal to the expected total payoff from not contributing. Similarly a fringe member will contribute to the public good if the expected total payoff from contributing is at-least equal to the expected total payoff from not contributing. Comparing these set of equations we derive the first proposition.

Proof of part a:

We define the terms which we will be using in the analysis when $i \in M$ votes yes to contribute to public good:

$$\theta^v = \left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'} + \binom{M-1}{m'}(p)^{m'}(1-p)^{M-m'-1} + \cdots + \binom{M-1}{M-1}(p)^{M-1}\right] = \left[\sum_{i=m'-1}^{M-1}\binom{M-1}{i}(p)^i(1-p)^{M-(i+1)}\right] \quad (5)$$

$$\eta^v = \left[\binom{M-1}{0}(1-p)^{M-1} + \binom{M-1}{1}(p)(1-p)^{M-2} + \cdots + \binom{M-1}{m'-2}(p)^{m'-2}(1-p)^{M-m'+1}\right] = \left[\sum_{i=0}^{m'-2}\binom{M-1}{i}(p)^i(1-p)^{M-(i+1)}\right] \quad (6)$$

[10] The majority when M is even is given by $m' = \frac{M}{2}$. The results will also hold true if M is assumed to be even.

θ^v sums all the cases when at least $m' - 1$ other players vote yes to contribute to the public good. In this case, a yes by individual i will lead to at least the majority voting yes to contribute and hence the coalition will contribute to the public good. η^v sums all the cases when the majority is not satisfied, and as a result the coalition does not contribute to public good. Note even though i votes yes to contribute, when majority is not satisfied the coalition does not contribute and as a result he/she does not contribute.

Now let's define the terms which we will be using in the analysis when individual $i \in M$ does not vote yes to contribute to public good:

$$\begin{aligned}\theta^{nv} = \Bigg[\binom{M-1}{m'}(p)^{m'}(1-p)^{M-m'-1} + \binom{M-1}{m'+1}(p)^{m'+1}(1-p)^{M-m'-2} + \\ \cdots + \binom{M-1}{M-1}(p)^{M-1}\Bigg] \\ = \left[\sum_{i=m'}^{M-1}\binom{M-1}{i}(p)^{i}(1-p)^{M-(i+1)}\right] \quad (7)\end{aligned}$$

$$\begin{aligned}\eta^{nv} = \Bigg[\binom{M-1}{0}(1-p)^{M-1} + \binom{M-1}{1}(p)(1-p)^{M-2} + \\ \cdots + \binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\Bigg] \\ = \left[\sum_{i=0}^{m'-1}\binom{M-1}{i}(p)^{i}(1-p)^{M-(i+1)}\right] \quad (8)\end{aligned}$$

θ^{nv} sums all the cases when at least the majority votes to contribute to the public good and as a result the coalition contributes to public good. Note that the majority does not include $i \in M$ here but i has to contribute being part of the coalition. η^{nv} sums all the cases when the majority is not satisfied and as a result the coalition does not contribute to public good.

The expected total payoff when $i \in M$ chooses $v_i = 1$ (votes to contribute):

$$\begin{aligned}\pi^v(v_i, v_{-i}) &= [\theta^v]\left[\lambda_i(\gamma(M+F')) + (1-\lambda_i)(\gamma(M+F'))\right] \\ &+ [\eta^v]\left[\lambda_i(1+\gamma F') + (1-\lambda_i)\gamma(F')\right] \\ &= [\theta^v]\left[\gamma(M+F')\right] + [\eta^v]\left[\lambda_i + \gamma F'\right]\end{aligned} \quad (9)$$

The term adjacent to θ^v is total payoff $i \in M$ receives when $Q = M \cup F'$. The term adjacent to η^v is total payoff $i \in M$ receives when $Q = F'$(majority was not satisfied in this case).[11] Recall that Q denotes the total number of individuals contributing to the public good and F' denotes the fringe members contributing to public good. We make no assumption on the number of fringe members contributing to public good, only size of M is announced before Stage II begins.

[11]Recall total payoff comprises of own payoff(corresponding to λ_i) and social payoff(corresponding to $1 - \lambda_i$).

The expected total payoff when $i \in M$ chooses $v_i = 0$ (does not vote to contribute):

$$\begin{aligned}\pi^{nv}(v_i, v_{-i}) &= [\theta^{nv}]\,[\lambda_i(\gamma(M+F')) + (1-\lambda_i)(\gamma(M+F'))]\\ &\quad + [\eta^{nv}]\,[\lambda_i(1+\gamma F') + (1-\lambda_i)\gamma(F')]\\ &= [\theta^{nv}]\,[\gamma(M+F')] + [\eta^{nv}]\,[\lambda_i + \gamma F']\end{aligned} \tag{10}$$

The term adjacent to θ^{nv} is total payoff $i \in M$ receives when $Q = M \cup F'$. Note that majority does not include $i \in M$ here but i has to contribute being part of the coalition. The term adjacent to η^{nv} is the total payoff $i \in M$ receives when $Q = F'$.

Individual $i \in M$ will compare π^v and π^{nv} to choose between the strategies $v_i = 1$ (vote to contribute) or $v_i = 0$ (vote not to contribute). $i \in M$ will choose to contribute to public good if expected total payoff from contributing is at least greater than the expected total payoff from not contributing. After comparison we observe that $\pi^v \geq \pi^{nv}$ if:

$$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][\gamma(M+F') - (\lambda_i + \gamma F')] \geq 0 \tag{11}$$

$$\lambda_i \leq \gamma M \tag{12}$$

■

Equation 12 gives us the cutoff for part a of Proposition I, which can also be re-written as $\lambda_i \leq \lambda_M$, where $\lambda_M = \gamma M$
For interpretation purpose, equation 11 can be re-written as:
$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][\gamma(M+F')]$ - $\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][(\lambda_i + \gamma F')] \geq 0$

$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right]$ denotes that $m'-1$ people are contributing to the public good. Thus i has the deciding vote, in other words i is the *pivotal voter* and equation 11 shows the least expected marginal benefit a pivotal voter receives from contributing. The result is also intuitive since the pivotal member's vote decides whether a coalition will contribute or not. The first term in the above equation depicts the total payoff when individual $i \in M$ or the pivotal voter, votes to contribute. Note that, a vote by i leads to majority being satisfied and hence the coalition contributes. Everyone in the coalition receives the total payoff $\gamma(M+F')$, since $Q = M \cup F'$. The second term depicts the total payoff when the coalition does not contribute. This is because individual $i \in M$ does not vote and hence the coalition only receives $m'-1$ votes. The resulting total payoff is $\lambda_i + \gamma F'$ and $Q = F'$. The marginal gain to pivotal voter from voting to contribute is $\gamma M'$ and the marginal gain from not voting is λ_i. $i \in M$ will vote to contribute if the marginal gain from voting is more than the marginal gain from not voting to contribute which leads us to the threshold $\lambda_i \leq \lambda_M$ where $\lambda_M = \gamma M$ in equation 12.

Proof of part b:
The expected total payoff a *fringe member* $i \in F'$ receive when h/she decides to con-

tribute.[12] In other words expected total payoff when $i \in F'$ chooses $e_i = 1$:

$$\begin{aligned}\pi^c(e_i, e_{-i}) &= [\alpha]\,[\lambda_i(\gamma(M+F')) + (1-\lambda_i)\gamma(M+F')] \\ &+ [\beta]\,[\lambda_i(\gamma F')_{+}(1-\lambda_i)\gamma(F')] \\ &= [\alpha]\,[\gamma(M+F')] + [\beta]\,[\gamma F']\end{aligned} \tag{13}$$

In the above equations α depicts the case when coalition of size M is contributing to the public good as the majority rule is satisfied. The corresponding term is the total payoff $i \in F'$ receives when a coalition of size M and F' fringe members are contributing ($Q = M \cup F'$). β depicts the case when M is not contributing as majority is not satisfied and thus $Q = F'$. The adjacent term is the total payoff $i \in F'$ receives when h/she is contributing with the other fringe members.

The expected total payoff $i \in F'$ receives when he/she decides not to contribute to the public good ($e_i = 0$):

$$\begin{aligned}\pi^{nc}(e_i, e_{-i}) &= [\alpha]\,[\lambda_i(1+\gamma(M+F'-1)) + (1-\lambda_i)\gamma(M+F'-1)] \\ &+ [\beta]\,[\lambda_i(1+\gamma(F'-1)_{+}(1-\lambda_i)\gamma(F'-1)] \\ &= [\alpha]\,[\gamma(M+F'-1)+\lambda_i] + [\beta]\,[\lambda_i + \gamma(F'-1)]\end{aligned} \tag{14}$$

In this case i is a free rider and size of the *fringe members* is reduced to $F' - 1$. α and β are defined as above. The term adjacent to α is the total payoff i receives when h/she is free riding and coalition of size M and $F' - 1$ *fringe members* are contributing ($Q = M \cup F' - 1$). The term corresponding to β is the total payoff i receives when $Q = F' - 1$ or only fringe members contribute.

$i \in F'$ will compare π^c and π^{nc} to decide whether to contribute ($e_i = 1$)or not ($e_i = 0$) as a fringe member. After comparison we see that $\pi^c \geq \pi^{nc}$ if:

$$\lambda_i \leq \gamma \tag{15}$$

■

Equation 15 gives us the cutoff for part b in Proposition I and can be rewritten as $\lambda_i \leq \lambda_F$ where $\lambda_F = \gamma$.

We can interpret equation 15 as the comparison between marginal gain from contribution and the marginal gain from being a free rider for a fringe member. From equation 1, Fringe member's marginal gain from free riding will be λ_i. The marginal gain from contribution will be γ. An individual contributes if marginal gain from contribution is higher than the marginal gain from free riding, i.e., $\lambda_i \leq \gamma$ which is the threshold in the above equation. This is also a general result from a public good game without coalition.

Let $\lambda_F = \gamma$, if $\lambda_i \leq \lambda_F$ is satisfied then the fringe member will contribute for the public good. This leads us to part b of Proposition I.

Proof for Proposition II:

Case (i): $\lambda_i \leq \gamma$:

[12]We can also calculate the expected total payoff for $i \in F$, keeping F' fixed. In that case F' increases by 1, when $i \in F$ contributes.

Individuals will compare the expected total payoff they receive when they join the coalition and the expected total payoff they receive when they do not join the coalition. The expected total payoff in both the scenarios is dependent on whether the coalition contributes or not. We use the thresholds derived in Stage II to calculate the probability coalition contributes to public good. Let's define the terms which we will be using when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to join the coalition.

$$\begin{aligned}\phi = \Bigg[\binom{M-1}{m'-1} (\gamma M)^{m'-1} (1-\gamma M)^{M-m'} + \binom{M-1}{m'} (\gamma M)^{m'} (1-\gamma M)^{M-m'-1} + \\ \cdots + \binom{M-1}{M-1} (\gamma M)^{M-1} \Bigg] \\ = \left[\sum_{i=m'-1}^{M-1} \binom{M-1}{i} (\gamma M)^i (1-\gamma M)^{M-(i+1)} \right]\end{aligned} \tag{16}$$

$$\begin{aligned}1-\phi = \Bigg[\binom{M-1}{0} (1-\gamma M)^{M-1} + \binom{M-1}{1} (\gamma M)(1-\gamma M)^{M-2} + \\ \cdots + \binom{M-1}{m'-2} (\gamma M)^{m'-2} (1-\gamma M)^{M-m'+1} \Bigg] \\ = \left[\sum_{i=0}^{m'-2} \binom{M-1}{i} (\gamma M)^i (1-\gamma M)^{M-(i+1)} \right]\end{aligned} \tag{17}$$

Here ϕ sums all the cases when at-least $m'-1$ will vote to contribute to the public good. In other words there are at-least $m'-1$ individuals who have the preference $\lambda_i \leq \gamma M$. From stage I, we know $i \in M$ will vote to contribute if $\lambda_i \leq \gamma M$, since λ_i follows a uniform distribution, this threshold is given by γM. Similarly, $i \in M$ will not vote to contribute if $\lambda_i > \gamma M$ and this is given by $1-\gamma M$. A vote by $i \in N$ will lead to at least the majority contributing to the public good. Since $i \in N$ satisfies $\lambda_i \leq \gamma$ which also implies $\lambda_i \leq \gamma M$, h/she will vote to contribute once they join coalition.

$1-\phi$ sums all the cases when individuals in coalition have preferences such that majority does not vote to contribute to public good. In other words majority of the individuals who join the coalition have preference which satisfies $\lambda_i > \gamma M$.

We define the terms which we will be using when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to not join the coalition.

$$\begin{aligned}\phi' = \Bigg[\binom{M-1}{m'-1} (\gamma(M-1))^{m'-1} (1-\gamma(M-1))^{M-m'} + \\ \binom{M-1}{m'} (\gamma(M-1))^{m'} (1-\gamma(M-1))^{M-m'-1} + \cdots + \binom{M-1}{M-1} (\gamma(M-1))^{M-1} \Bigg] \\ = \left[\sum_{i=m'-1}^{M-1} \binom{M-1}{i} (\gamma(M-1))^i (1-\gamma(M-1))^{M-(i+1)} \right]\end{aligned} \tag{18}$$

$$
\begin{aligned}
1-\phi' = \Bigg[\binom{M-1}{0}(1-\gamma(M-1))^{M-1} + \binom{M-1}{1}(\gamma(M-1))(1-\gamma(M-1))^{M-2} + \\
\cdots + \binom{M-1}{m'-2}(\gamma(M-1))^{m'-2}(1-\gamma(M-1))^{M-m'+1}\Bigg] \\
= \left[\sum_{i=0}^{m'-2}\binom{M-1}{i}(\gamma(M-1))^{i}(1-\gamma(M-1))^{M-(i+1)}\right] \quad (19)
\end{aligned}
$$

Now the size of coalition is reduced to $M-1$ $\forall M$. The probability of a coalition member contributing, is now given by the cutoff $\lambda_i \leq \gamma(M-1)$ which is $\gamma(M-1)$. Similarly probability of a coalition member not contributing is given by the cutoff: $\lambda_i > 1-\gamma(M-1)$ i.e. $1-\gamma(M-1)$. ϕ' sums all the cases when at-least majority of the $M-1$ members of the coalition vote to contribute i.e. majority of the people in the coalition have preference: $\lambda_i \leq \gamma(M-1)$. 1- ϕ' sums all the cases when majority of the people who join the coalition do not satisfy $\lambda_i \leq \gamma(M-1)$ and as a result the coalition does not contribute.

We denote the action set of a player joining the coalition by $j_i \in \{0,1\}$ where $j_i = 1$ implies $i \in N$ joins the coalition and $j_i = 0$ implies $i \in N$ does not join the coalition. With slight abuse of notation, j_i is the strategy of player i and j_{-i} is the strategy of the other players. Expected total payoff when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to join the coalition ($j_i = 1$):

$$\pi_i(j_i, j_{-i}) = [\phi]\,[(\gamma(M+F')] + [1-\phi]\,[\lambda_i + \gamma F'] \quad (20)$$

The term adjacent to ϕ shows the total payoff as a result of the coalition of any size M and F' fringe members contributing i.e. $Q = M \cup F'$. The term adjacent to $1-\phi$ is the total payoff when the coalition is not contributing and only the fringe members who meet the cutoff ($\lambda_i \leq \gamma$) contribute i.e $Q = F'$. Note when $i \in N$ joins the coalition, he/she has to follow the decision of coalition regarding contributing to public good.

Expected total payoff when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to not join the coalition ($j_i = 0$):

$$\pi_i(j_i, j_{-i}) = [\phi']\,[(\gamma(M+F')] + [1-\phi']\,[\gamma(F'+1)] \quad (21)$$

Note that $i \in N$ satisfies: $\lambda_i \leq \gamma$, thus he/she will contribute as a fringe member, and as a result the size of fringe member increases to $F'+1$, also now the coalition is of size $M-1$. In equation 21 term adjacent to ϕ' is the total payoff received when a coalition of size $M-1$ and $F'+1$ fringe members are contributing. The term adjacent to $1-\phi'$ shows the total payoff when only $F'+1$ fringe members are contributing.

$i \in N$ will compare equation 20 and equation 21 to decide whether to contribute to the public good or not. The expected total payoff functions changes values based on three sub-cases. We will also be using the three sub cases to derive solution for case (ii) and case(iii) as well. The three sub cases are given by:

a) $\gamma(M-1) \geq 1$, $\gamma M > 1$.
b) $\gamma(M-1) < 1$, $\gamma M = 1$.
c) $\gamma(M-1) < 1$, $\gamma M < 1$.

Case (i) (a) $\gamma(M-1) \geq 1$**,** $\gamma M > 1$.

Given M is assumed to be odd, M needs to be at-least 3. The probability individual satisfies $\lambda_i \leq \gamma M$ i.e. γM will be 1 and probability any individual has cutoff $\lambda_i > \gamma M$ which is $1-\gamma M = 0$, since $\lambda_i < 1$. Similarly probability individual satisfies $\lambda_i \leq \gamma(M-1)$ i.e. $\gamma(M-1)$ will be 1 and probability any individual has cutoff $\lambda_i > \gamma(M-1)$ which is $1-\gamma(M-1) = 0$. Substituting these values in equation 20 and equation 21, one finds that $\phi = \phi' = 1$ and $1-\phi = 1-\phi' = 0$. Equation 20 can be re-written as $\gamma(M+F')$, equation 21 can be re written as $\gamma(M+F')$. Comparing these equations shows that the individual in the case($\lambda_i \leq \gamma$) will be indifferent between joining the coalition and not joining the coalition. Based on our definition of joining the coalition, the individual joins the coalition if h/she is indifferent.

Case (i) (b) $\gamma(M-1) < 1$**,** $\gamma M = 1$

Note $\phi = 1$ and $1-\phi = 0$. Thus equation 20 can be expressed as :$\gamma(M+F')$. $i \in N$ will join the coalition if equation 20 $\geq$ equation 21. This implies:
$\gamma(M+F') \geq [\phi'] [(\gamma(M+F')] + [1-\phi'] [\gamma(F'+1)]$
$\rightarrow (1-\phi')\gamma(M+F') \geq [1-\phi'] [\gamma(F'+1)]$
$\rightarrow M \geq 1$
$M \geq 1$ is a trivial condition and implies $i \in N$ will join if there is at least one person joining.

Case (i) (c) $\gamma(M-1) < 1$**,** $\gamma M < 1$.

$i \in N$ will join if equation 20 $\geq$ equation 21. This implies:
$\phi[\gamma(M+F')] + [1-\phi][\lambda_i + \gamma F'] \geq [\phi'] [(\gamma(M+F')] + [1-\phi'] [\gamma(F'+1)]$
$\rightarrow [\phi-\phi'][\gamma(M+F')] + [1-\phi][\lambda_i] \geq [\phi-\phi'] [\gamma F'] + [1-\phi']\gamma$
$\rightarrow [\phi-\phi'][\gamma M] \geq [1-\phi']\gamma - [1-\phi][\lambda_i]$
Replacing λ_i *with* γ *on the R.H.S. since* $\lambda_i \leq \gamma$
$\rightarrow [\phi-\phi'][\gamma M] \geq [1-\phi']\gamma - [1-\phi][\gamma]$
$\rightarrow [\phi-\phi'][\gamma M] \geq [\phi-\phi']\gamma$
$\rightarrow M \geq 1$
Again $M \geq 1$ is a trivial case and $i \in N$ will join the coalition if there is at least one other member in the coalition.

Thus $i \in N$ with preference $\lambda_i \leq \gamma$ will join the coalition.

Case (ii):$\gamma < \lambda_i \leq \gamma M$

The expected total payoff when $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ decides to join the coalition ($j = 1$):

$$\pi_i(j_i, j_{-i}) = [\phi]\,[(\gamma(M + F')] + [1 - \phi]\,[\lambda_i + \gamma F'] \tag{22}$$

ϕ and $1 - \phi$ are expressed by equation 16 and equation 17. $i \in N$ has preference $\gamma < \lambda_i \leq \gamma M$ and hence would vote to contribute to the public good. The term adjacent to ϕ shows the total payoff as a result of the coalition when $Q = M \cup F'$. The term adjacent to $1 - \phi$ is the total payoff when the coalition does not contribute i.e. $Q = F'$. Note even though $i \in N$ has preference $\gamma < \lambda_i \leq \gamma M$, he/she does not contribute as majority is not satisfied.

The expected total payoff when $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ decides to not join the coalition ($j_i = 0$):

$$\pi_i(j_i, j_{-i}) = [\phi']\,[\lambda_i + \gamma(M - 1 + F')] + [1 - \phi']\,[\lambda_i + \gamma(F')] \tag{23}$$

ϕ' and $1 - \phi'$ are expressed by equation 18 and equation 19. Note that $i \in N$ has preference $\gamma < \lambda_i \leq \gamma M$ and hence will not contribute as a fringe member. In equation 23, the term adjacent to ϕ' is the total payoff received by $i \in N$ when coalition of size $M - 1$ and F' fringe members are contributing. The term adjacent to $1 - \phi'$ depicts the total payoff $i \in N$ receives when only F' fringe members contribute to the public good.

An individual will compare equation 22 and equation 23 for each of the sub cases to decide whether to join the coalition or not.

Case(ii) (a) $\gamma(M - 1) \geq 1$**,** $\gamma M > 1$

In this case we again have $\phi = \phi' = 1$ and $1 - \phi = 1 - \phi' = 0$. Plugging these values in equation 22 and equation 23 shows that individual will not join the coalition as equation 22 < equation 23: $\gamma(M + F') < \lambda_i + \gamma(M - 1 + F')$

$\rightarrow \gamma(M + F') < \lambda_i + \gamma(M + F') - \gamma$

$\rightarrow \gamma < \lambda_i$

$\gamma < \lambda_i$ hold true in case(ii) thus, $i \in N$ will not join the coalition.

Case (ii) (b) $\gamma(M - 1) < 1$**,** $\gamma M = 1$

$i \in N$ will not join the coalition if equation 22 < equation 23:

$\gamma(M + F') < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma(M + F')] < \phi'[\lambda_i - \gamma] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma M] < \lambda_i - \phi'\gamma$

Replacing λ_i *with* γM *since* $\lambda_i \leq \gamma M$

$\rightarrow [1 - \phi'][\gamma M] < \gamma M - \phi'\gamma$

$\rightarrow \phi'\gamma M > \phi'\gamma$

$\rightarrow M > 1$

Case(ii) (c) $\gamma(M - 1) < 1$**,** $\gamma M < 1$

$i \in N$ will not join the coalition if equation 22 < equation 23:

$\phi[\gamma(M + F')] + [1 - \phi][\lambda_i + \gamma F'] < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi - \phi'][\gamma(M + F')] < \phi'[\lambda_i - \gamma] + [\phi - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi - \phi'][\gamma M] < \phi'\lambda_i - \phi'\gamma + \phi\lambda_i - \phi'\lambda_i$
$\rightarrow [\phi - \phi'][\gamma M] < \phi\lambda_i - \phi'\gamma$
Replace λ_i *with* γM *since* $\lambda_i \leq \gamma M$
$\rightarrow [\phi - \phi'][\gamma M] < \phi\gamma M - \phi'\gamma$
$\rightarrow \phi\gamma M - \phi'\gamma M < \phi\gamma M - \phi'\gamma$
$\rightarrow \gamma M > \gamma$
$\rightarrow M > 1$

In both the sub-cases we find that $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ will not join the coalition if $M > 1$. M needs to be more than 1 for it to be a coalition and hence $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ will not join the coalition.

Case (iii): $\lambda_i > \gamma M$

We now define the terms to describe the payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to join the coalition.

$$\begin{aligned}\phi'' = \Bigg[\binom{M-1}{m'}(\gamma M)^{m'}(1-\gamma M)^{M-m'-1} + \binom{M-1}{m'+1}(\gamma M)^{m'+1}(1-\gamma M)^{M-m'-2} + \\ \cdots + \binom{M-1}{M-1}(\gamma M)^{M-1}\Bigg] \\ = \left[\sum_{i=m'}^{M-1}\binom{M-1}{i}(\gamma M)^i(1-\gamma M)^{M-i-1}\right] \qquad (24)\end{aligned}$$

$$\begin{aligned}1-\phi'' = \Bigg[\binom{M-1}{0}(1-\gamma M)^{M-1} + \binom{M-1}{1}(\gamma M)(1-\gamma M)^{M-2} + \\ \cdots + \binom{M-1}{m'-1}(\gamma M)^{m'-1}(1-\gamma M)^{M-m'}\Bigg] \\ = \left[\sum_{i=0}^{m'-1}\binom{M-1}{i}(\gamma M)^i(1-\gamma M)^{M-(i+1)}\right] \qquad (25)\end{aligned}$$

Here $i \in N$ with preference $\lambda_i > \gamma M$ will not be willing to contribute to public good. ϕ'' sums all the cases where at least the majority of the individuals who join the coalition contribute to the public good. Also, the majority does not include $i \in N$. In other words we need at least m' individuals with preference $\lambda_i \leq \gamma M$ to join the coalition for the coalition to contribute. Note although $i \in N$ has preference $\lambda_i > \gamma M$, he/she will contribute if the majority is satisfied. $1 - \phi''$ sums all the cases where individuals in coalition have preferences such that majority does not vote to contribute to public good. There are less than m' individuals with preference $\lambda_i \leq \gamma M$.

Expected total payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to join the coalition ($j_i = 1$).

$$\pi_i(j_i, j_{-i}) = [\phi''][(\gamma(M + F')] + [1 - \phi''][\lambda_i + \gamma F'] \qquad (26)$$

The term adjacent to ϕ'' shows the total payoff when the coalition of size M and F' fringe members contribute to the public good i.e $Q = M \cup F'$. The term adjacent to $1 - \phi''$ is the total payoff received when only fringe members contribute or $Q = F'$.

The expected total payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to not join the coalition ($j_i = 0$):

$$\pi_i(j_i, j_{-i}) = [\phi'] \left[\lambda_i + (\gamma(M - 1 + F')\right] + [1 - \phi'] \left[\lambda_i + \gamma(F')\right] \tag{27}$$

Note $i \in N$ has preference $\lambda_i > \gamma M$ and hence will not contribute as a fringe member and size of the coalition reduces by 1. ϕ' and $1 - \phi'$ are expressed by equation 18 and equation 19. The term adjacent to ϕ' is the total payoff $i \in N$ receives when $Q = M-1 \cup F'$ and the term adjacent to $1 - \phi'$ is the total payoff $i \in N$ receives when $Q = F'$.

An individual will compare equation 26 and equation 27 for the three sub-cases to decide whether they want to join the coalition.

Case (iii) (a) $\gamma(M-1) \geq 1, \gamma M > 1$

For case III, we can see that $\phi'' = \phi' = 1$ and $1 - \phi'' = 1 - \phi' = 0$. Plugging these values and comparing equation 26 and equation 27, shows that individual will not join the coalition as equation 26 < equation 27:

$\gamma(M + F') < \lambda_i + \gamma(M - 1 + F')$

$\rightarrow \gamma(M + F') < \lambda_i + \gamma(M + F') - \gamma$

$\rightarrow \gamma < \lambda_i$

$\gamma < \lambda_i$ hold true in case(iii) thus, $i \in N$ will not join the coalition.

Case (iii) (b) $\gamma(M-1) < 1$**,** $\gamma M = 1$

Note now $\phi'' = 1$, $1 - \phi'' = 0$ since $\gamma M = 1$, thus equation 26 can be re-written as: $\gamma(M + F')$.

$i \in N$ will not join the coalition if equation 26 < equation 27:

$\gamma(M + F') < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma(M + F')] < \phi'[\lambda_i - \gamma] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma M] < \lambda_i - \phi'\gamma$

$\rightarrow [1 - \phi'][\gamma M] + \phi'\gamma < \lambda_i$

Replace γM *with* γ *on the right side as* $\gamma M > \gamma$

$\rightarrow [1 - \phi'][\gamma] + \phi'\gamma < \lambda_i$

$\rightarrow \gamma < \lambda_i$

Case (iii) (c) $\gamma(M-1) < 1$**,** $\gamma M < 1$

$i \in N$ will not join the coalition if equation 26 < equation 27.

$\phi''[\gamma(M + F')] + [1 - \phi''][\lambda_i + \gamma F'] < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi'' - \phi'][\gamma(M + F')] < \phi'[\lambda_i - \gamma] + [\phi'' - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi'' - \phi'][\gamma M] < \phi'[\lambda_i - \gamma] + [\phi'' - \phi']\lambda_i$

$\rightarrow [\phi'' - \phi'][\gamma M] < \phi'\lambda_i - \phi'\gamma + \phi''\lambda_i - \phi'\lambda_i$

$\rightarrow [\phi'' - \phi'][\gamma M] < \phi''\lambda_i - \phi'\gamma$

Replace γM *with* γ *on the right side as* $\gamma M > \gamma$

$\rightarrow [\phi'' - \phi']\gamma + \phi'\gamma < \phi''\lambda_i$

$\rightarrow \phi''\gamma < \phi''\lambda_i$

$\rightarrow \gamma < \lambda_i$

Since $\lambda_i > \gamma$ in Case(iii) $i \in N$ with preference $\lambda_i > \gamma M$ will not join the coalition. One can see from Case (iii) (b) and (c) that the solution applies to Case(ii) (b) and (c) since these cases also assume $\lambda_i > \gamma$ and $\gamma M > \gamma$ also holds true. ■

Proof for Proposition III

Case (i): $\lambda_i \leq \gamma$: Payoff when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to join the coalition ($j_i = 1$) changes.Equation 20 can be re-written as

$$\pi_i(j_i, j_{-i}) = [\phi] \left[(\gamma(M + F') + \varepsilon\right] + [1 - \phi] \left[\lambda_i + \gamma F' + \varepsilon\right] \tag{28}$$

Payoff when $i \in N$ with preference $\lambda_i \leq \gamma$ decides to not join the coalition ($j_i = 0$) remains the same as equation 21:

$$\pi_i(j_i, j_{-i}) = [\phi'] \left[(\gamma(M + F')\right] + [1 - \phi'] \left[\gamma(F' + 1)\right] \tag{29}$$

$i \in N$ will compare equation 28 and equation 29 to decide whether to contribute to the public good or not. The payoff functions changes values based on three sub-cases. We will be using the three sub cases to derive solution for case (ii) and case(iii) as well. The three sub cases are given by:

a) $\gamma(M - 1) \geq 1$, $\gamma M > 1$.

b) $\gamma(M - 1) < 1$, $\gamma M = 1$.

c) $\gamma(M - 1) < 1$, $\gamma M < 1$.

Case (i) (a) $\gamma(M-1) \geq 1$, $\gamma M > 1$. $\phi = \phi' = 1$ and $1-\phi = 1-\phi' = 0$. Equation 28 can be re-written as $\gamma(M + F')+$, equation 29 can be re written as $\gamma(M + F')n$. Comparing these equations shows that the individual in the case($\lambda_i \leq \gamma$) will be joining the coalition.

Case (i) (b) $\gamma(M - 1) < 1$, $\gamma M = 1$.

Note $\phi = 1$ and $1 - \phi = 0$. Thus equation 28 can be expressed as :$\gamma(M + F') + \varepsilon$. $i \in N$ will join the coalition if equation 28 $\geq$ equation 29:. This implies:

$\gamma(M + F') + \varepsilon \geq [\phi'] \left[(\gamma(M + F')\right] + [1 - \phi'] \left[\gamma(F' + 1)\right]$

$\rightarrow (1 - \phi')\gamma(M + F') + \varepsilon \geq [1 - \phi'] \left[\gamma(F' + 1)\right]$

$\rightarrow (1 - \phi')\gamma M + \varepsilon \geq [1 - \phi']\gamma$

This is true since $\gamma M > \gamma$

The above condition implies $i \in N$ will join the coalition.

Case (i) (c) $\gamma(M - 1) < 1$, $\gamma M < 1$.

$i \in N$ will join if equation 28 $\geq$ equation 29. This implies:

$\phi[\gamma(M + F') + \varepsilon] + [1 - \phi][\lambda_i + \gamma F' + \varepsilon] \geq [\phi'] \left[(\gamma(M + F')\right] + [1 - \phi'] \left[\gamma(F' + 1)\right]$

$\rightarrow [\phi - \phi'][\gamma(M + F')] + [1 - \phi][\lambda_i] + \varepsilon \geq +[\phi - \phi'] \left[\gamma F'\right] + [1 - \phi']\gamma$

$\rightarrow [\phi - \phi'][\gamma M]+ \geq [1 - \phi']\gamma - [1 - \phi][\lambda_i]$

Replacing λ_i with γ on the R.H.S. since $\lambda_i \leq \gamma$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon \geq [1 - \phi']\gamma - [1 - \phi][\gamma]$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon \geq [1 - \phi']\gamma - [1 - \phi][\gamma]$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon \geq [\phi - \phi']\gamma$

Since$\gamma M > \gamma$ this holds true

Thus $i \in N$ with preference $\lambda_i \leq \gamma$ will join the coalition.

Case (ii):$\gamma < \lambda_i \leq \gamma M$

The payoff when $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ decides to join the coalition ($j = 1$) changes. Equation 22 changes as:

$$\pi_i(j_i, j_{-i}) = [\phi]\,[(\gamma(M + F') + \varepsilon] + [1 - \phi]\,[\lambda_i + \gamma F' + \varepsilon] \tag{30}$$

The payoff when $i \in N$ with preference $\gamma < \lambda_i \leq \gamma M$ decides to not join the coalition ($j_i = 0$) remains the same as equation 23

$$\pi_i(j_i, j_{-i}) = [\phi']\,[\lambda_i + \gamma(M - 1 + F')] + [1 - \phi']\,[\lambda_i + \gamma(F')] \tag{31}$$

An individual will compare equation 22 and equation 31 for each of the sub cases to decide whether to join the coalition or not.

Case(ii) (a) $\gamma(M-1) \geq 1$, $\gamma M > 1$. $\phi = \phi' = 1$ and $1-\phi = 1-\phi' = 0$. Plugging these values in equation 30 and equation 31 shows that individual will not join the coalition as equation 30 < equation 31: $\gamma(M + F') + \varepsilon < \lambda_i + \gamma(M - 1 + F')$

$\rightarrow \gamma(M + F') + \varepsilon < \lambda_i + \gamma(M + F') - \gamma$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case (ii) (b) $\gamma(M - 1) < 1$, $\gamma M = 1$

$i \in N$ will not join the coalition if equation 30 < equation 31:

$\gamma(M + F') + \varepsilon < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma(M + F')] + \varepsilon < \phi'[\lambda_i - \gamma] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma M] + \varepsilon < \lambda_i - \phi'\gamma$

Replace γM with γ on the right side as $\gamma M > \gamma$

$\rightarrow [1 - \phi'][\gamma] + \phi'\gamma + \varepsilon < \lambda_i$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case(ii)(c) $\gamma(M - 1) < 1$, $\gamma M < 1$

$i \in N$ will not join the coalition if equation 30 < equation 31.

$\phi[\gamma(M + F')] + [1 - \phi][\lambda_i + \gamma F'] + \varepsilon < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi - \phi'][\gamma(M + F')] + \varepsilon < \phi'[\lambda_i - \gamma] + [\phi - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon < \phi'[\lambda_i - \gamma] + [\phi - \phi']\lambda_i$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon < \phi'\lambda_i - \phi'\gamma + \phi\lambda_i - \phi'\lambda_i$

$\rightarrow [\phi - \phi'][\gamma M] + \varepsilon < \phi\lambda_i - \phi'\gamma$

Replace γM with γ on the right side as $\gamma M > \gamma$

$\rightarrow [\phi - \phi']\gamma + \phi'\gamma + \varepsilon < \phi\lambda_i$

Since $\phi'\varepsilon < \varepsilon$

$\rightarrow \phi' + \ < \phi'\lambda_i$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case (iii): $\lambda_i > \gamma M$

Payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to join the coalition ($j_i = 1$) changes. Equation 26 changes as:

$$\pi_i(j_i, j_{-i}) = [\phi'']\,[(\gamma(M + F') + \varepsilon] + [1 - \phi'']\,[\lambda_i + \gamma F' + \varepsilon] \tag{32}$$

The payoff when $i \in N$ with preference $\lambda_i > \gamma M$ decides to not join the coalition ($j_i = 0$) remains the same as equation 27:

$$\pi_i(j_i, j_{-i}) = [\phi'] [\lambda_i + (\gamma(M - 1 + F')] + [1 - \phi'] [\lambda_i + \gamma(F')] \tag{33}$$

An individual will compare equation 32 and equation 33 for each of the sub cases to decide whether to join the coalition or not.

Case(iii) (a) $\gamma(M-1) \geq 1$, $\gamma M > 1$.
$\phi = \phi' = 1$ and $1 - \phi = 1 - \phi' = 0$. Plugging these values in equation 32 and equation 33 shows that individual will not join the coalition as equation 32 < equation 33: $\gamma(M + F') + \varepsilon < \lambda_i + \gamma(M - 1 + F')$

$\rightarrow \gamma(M + F') + \varepsilon < \lambda_i + \gamma(M + F') - \gamma$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case (iii) (b) $\gamma(M-1) < 1$, $\gamma M = 1$

$i \in N$ will not join the coalition if equation 32 < equation 33:

$\gamma(M + F') + \varepsilon < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma(M + F')] + \varepsilon < \phi'[\lambda_i - \gamma] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [1 - \phi'][\gamma M] + \varepsilon < \lambda_i - \phi'\gamma$

Replace γM with γ on the right side as $\gamma M > \gamma$

$\rightarrow [1 - \phi'][\gamma] + \phi'\gamma + \varepsilon < \lambda_i$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Case(iii)(c) $\gamma(M-1) < 1$, $\gamma M < 1$

$i \in N$ will not join the coalition if equation 32 < equation 33.

$\phi''[\gamma(M + F')] + [1 - \phi''][\lambda_i + \gamma F'] + \varepsilon < [\phi'][\lambda_i + \gamma(M - 1 + F')] + [1 - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi'' - \phi'][\gamma(M + F')] + \varepsilon < \phi'[\lambda_i - \gamma] + [\phi'' - \phi'][\lambda_i + \gamma F']$

$\rightarrow [\phi'' - \phi'][\gamma M] + \varepsilon < \phi'[\lambda_i - \gamma] + [\phi'' - \phi']\lambda_i$

$\rightarrow [\phi'' - \phi'][\gamma M] + \varepsilon < \phi'\lambda_i - \phi'\gamma + \phi''\lambda_i - \phi'\lambda_i$

$\rightarrow [\phi'' - \phi'][\gamma M] + \varepsilon < \phi''\lambda_i - \phi'\gamma$

Replace γM with γ on the right side as $\gamma M > \gamma$

$\rightarrow [\phi'' - \phi']\gamma + \phi'\gamma + \varepsilon < \phi''\lambda_i$

Since $\phi''\varepsilon < \varepsilon$

$\rightarrow \phi''\gamma + \phi''\varepsilon < \phi''\lambda_i$

$\rightarrow \gamma + \varepsilon < \lambda_i$

Proof for Proposition IV Comparing the cutoff for joining the coalition($\lambda_i \leq \gamma + \varepsilon$) and the cutoff for contributing ($\lambda_i \leq \gamma M$), we can have the following two situations:

1. $\gamma + \varepsilon \leq \gamma M$: In this case the cutoff for joining the coalition is less than the cutoff for contributing to the public good. Thus everyone who joins the coalition will be contributing to the public good.

2. $\gamma + \varepsilon > \gamma M$: In this case everyone who joins the coalition might not contribute. It depends whether majority of individuals have the weight $\lambda_i \leq \gamma M$.

If majority of the individuals have the weight $\lambda_i \leq \gamma M$ then the coalition contributes to the public good. If majority of the individuals have the weight $\lambda_i > \gamma M$ then the coalition does not contribute to the public good.

When majority will contribute

Since λ_i is uniformly distributed, if there is more mass in the interval $0 < \lambda_i \leq \gamma M$ than in the remaining interval ($\lambda_i \leq \gamma + \varepsilon - \gamma M$), then the majority will contribute to the public good. In other words, if $\gamma M \geq \gamma + \varepsilon - \gamma M$ then the majority of the coalition vote yes to contribute.

$\gamma M \geq \gamma + \varepsilon - \gamma M$

$\rightarrow 2\gamma M \geq \gamma + \varepsilon$

$\rightarrow \varepsilon \leq \gamma(2M - 1)$

$\therefore$ If $\varepsilon \leq \gamma(2M - 1)$, then coalition contributes to the public good.

Majority will not contribute

Since λ_i is uniformly distributed, if there is more mass in the interval $\lambda_i > \gamma M$ then the majority will not contribute to the public good. In other words if $\gamma M < \gamma + \varepsilon - \gamma M$ then the majority of the contributors will not vote yes to contribute.

$\gamma M < \gamma + \varepsilon - \gamma M$

$\rightarrow 2\gamma M < \gamma + \varepsilon$

$\rightarrow \varepsilon > \gamma(2M - 1)$

$\therefore$ If $\varepsilon > \gamma(2M - 1)$, then the coalition does not contribute to the public good.

Proof for Proposition V

The proof is similar to the proof of Proposition III. Instead of ε appearing on the left hand side of the equations, we have κ being subtracted from the right hand side of the equations.

Proof for Proposition VI

The proof is very similiar to the proof of Proposition IV. We compare if there is more mass in the interval $\lambda_i \leq \gamma M$ or in the interval $\lambda_i \leq \gamma + \kappa - \gamma M$.

Appendix B: Experimental results

Table B.1: Average Marginal Effect of key explanatory variables on the decision to join and contribute to coalition: Estimates from Probit Regression(with coalition size)

	(1) Decision to join	(2) Decision to contribute
Threshold satisfied	0.202***	0.214***
	(0.0422)	(0.0362)
Gender (Male)	0.0662	0.0792
	(0.0504)	(0.0427)
Political orientation	0.00753	0.0215
	(0.0226)	(0.0200)
Number of Econ classes	-0.0189	-0.0200
	(0.0206)	(0.0187)
Lagged payoff of least well off person	-0.00445**	-0.00458**
	(0.00150)	(0.00144)
Lagged coalition size	0.0398**	0.0429**
	(0.0150)	(0.0138)
Observations	1188	1188

Notes: Standard errors in parentheses and clustered at participant level. Dependent variable is decision to join/contribute(1:yes,0:no), margins reported here. Results on threshold satisfied are in comparison to threshold not satisfied. Political orientation take values from 0 to 4 where 0 is Complete Conservative and 4 is Complete Liberal. Standard errors in parentheses and clustered at participant level. $***p < 0.01, **p < 0.05, *p < 0.10$

Table B.2: Average Marginal Effect of key explanatory variables on the decision to join and contribute to coalition: Estimates from Probit Regression(with number of contributors)

	(1)	(2)
	Decision to join	Decision to contribute
Threshold satisfied	0.198^{***}	0.213^{***}
	(0.0418)	(0.0358)
Gender (Male)	0.0651	0.0780
	(0.0509)	(0.0432)
Political orientation	0.00530	0.0192
	(0.0227)	(0.0200)
Number of Econ classes	-0.0178	-0.0190
	(0.0208)	(0.0188)
Lagged payoff of least well off person	-0.00457^{**}	-0.00417^{**}
	(0.00160)	(0.00141)
Lagged number of contributors (coalition)	0.0342^{**}	0.0310^{**}
	(0.0129)	(0.0116)
Observations	1188	1188

Notes: Standard errors in parentheses and clustered at participant level. Dependent variable is decision to join/contribute(1:yes,0:no), margins reported here. Results on threshold satisfied are in comparison to threshold not satisfied. Political orientation take values from 0 to 4 where 0 is Complete Conservative and 4 is Complete Liberal. Standard errors in parentheses and clustered at participant level. $***p < 0.01, **p < 0.05, *p < 0.10$

Table B.3: Probit estimates of average marginal effect of key explanatory variables on the decision to join and contribute to coalition (with coalition size)

	(1) Decision to join	(2) Decision to contribute
$\lambda_i = 0.8$	-0.158**	-0.170***
	(0.0566)	(0.0499)
High MPCR	0.269***	0.308***
	(0.0327)	(0.0281)
Homogeneous session	0.0569	0.0601
	(0.0527)	(0.0426)
Gender (male)	0.0508	0.0619
	(0.0476)	(0.0392)
Political orientation	0.0107	0.0248
	(0.0217)	(0.0193)
Number of Econ classes	-0.0177	-0.0191
	(0.0202)	(0.0175)
Lagged payoff of least well off person	-0.00467**	-0.00499***
	(0.00150)	(0.00134)
Lagged coalition Size	0.0325*	0.0340**
	(0.0153)	(0.0130)
Observations	1188	1188

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes:Dependent variable is decision to join/contribute(1:yes,0:no), margins reported here. Results on $\lambda_i = 0.8$ are in comparison to low social preferences where $\lambda_i = 0.2$ and $\lambda = 0.5$ are clubbed together. Results on High MPCR are in comparison to low MPCR. Results in homogeneous session are in comparison to heterogeneous session. Political orientation take values from 0 to 4 where 0 is Complete Conservative and 4 is Complete Liberal. Standard errors in parentheses and clustered at participant level. $***p < 0.01, **p < 0.05, *p < 0.10$

Table B.4: Average Marginal Effect of key explanatory variables on the decision to join and contribute to coalition: Estimates from Probit Regression(with number of contributers)

	(1)	(2)
	Decision to join	Decision to contribute
$\lambda_i = 0.8$	-0.160**	-0.177***
	(0.0568)	(0.0501)
High MPCR	0.266***	0.306***
	(0.0327)	(0.0281)
Homogeneous session	0.0577	0.0621
	(0.0532)	(0.0428)
Gender (male)	0.0504	0.0616
	(0.0482)	(0.0399)
Political orientation	0.00883	0.0229
	(0.0219)	(0.0195)
Number of Econ classes	-0.0170	-0.0188
	(0.0204)	(0.0177)
Lagged payoff of least well off person	-0.00443**	-0.00401**
	(0.00158)	(0.00137)
Lagged number of contributors (coalition)	0.0244	0.0177
	(0.0130)	(0.0107)
Observations	1188	1188

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes:Dependent variable is decision to join/contribute(1:yes,0:no), margins reported here. Results on $\lambda_i = 0.8$ are in comparison to low social preferences where $\lambda_i = 0.2$ and $\lambda = 0.5$ are combined. Results on High MPCR are in comparison to low MPCR. Results in homogeneous session are in comparison to heterogeneous session. Political orientation take values from 0 to 4 where 0 is Complete Conservative and 4 is Complete Liberal. Standard errors in parentheses and clustered at participant level.$***p < 0.01, **p < 0.05, *p < 0.10$

Appendix C: Instructions

Page 1: Instructions

Welcome to the experimental session. You will be paid $10 for participating, you may earn additional money based on the decisions you make in the experiment. Your earnings will be paid in cash at the end of the session. You will complete a number of rounds in the experiment and your earnings from two randomly determined rounds will be paid to you. **You are not allowed to communicate with others during the experiment.**

Violation of this rule will lead to the exclusion from the experiment and all payments. If you have questions, please raise your hand. A member of the experimenter team will come to you and answer them in private. Cell phones are not allowed during the experiment.

Page 2: Instructions

We will not speak of Dollars during the experiment, but rather of points. Your whole income will first be calculated in points. At the end of the experiment, the total amount of points you earned will be converted to Dollars. Each 4 points is worth $1. So, if you earn 40 points you will receive $10 in addition to the $10 you get for participating.

The experiment consists of 2 parts and in each part there will be a number of rounds. You will be paid for one random rounds in each part. We will start by explaining the first part. You will receive separate instructions for part 2 after you have finished part 1.

Page 3: Instructions for **Part I**

This part will have 8 rounds, and, in each round, you will be required to make a decision.

The decision situation.

You will be a member of a group consisting of **6 people**. In each round everyone in your group will be given 10 points. Each group member has to decide on how to invest their 10 points in each round. You can invest all 10 points into your **private account**, or all 10 points into a **group project**. The points cannot be split between private account and the group project.

Page 4: Instructions for Part I

Your earnings from your **private account**. **You will earn one point for each point you put into your private account**. You can either put 0 points or 10 points into your private account. For example, if you put 10 points into your private account (and therefore do not invest in the group project), your earnings from private account will amount to exactly 10 points. If you put 0 points into your private account, your earnings from private account will be 0 points. No one except you earns something from your private account.

Page 5: Instructions for Part I

Here is information about your earnings from the group project. Both group members who do put their points in the group project and those who do not put their points in the group project will receive an equal number of points from the group project.

The earnings for each group member will be determined through a conversion rate.

There will be two conversion rates in the experiment as described below.

Case 1) Earnings from the project = 30% multiplied by the sum of all contributions. Example 1: If everyone in your group of 6 participants contributes 10 points then, the sum of all contributions to the project is 60 points. Here the conversion rate is 30%. You and the other members of your group will each earn 30% (0.3) multiplied by 60, which is18 points (30% of 60 points = $0.3x60 = 18$).

Example 2: If four members of the group contribute 10 points each, then sum of contributions is 40 points. You and everyone in your group each earn 30% of the total contributions of 40, which is 12 points.

Case 2) Earnings from the project = 70% (0.7) multiplied by the sum of all contributions.

Example 1: If everyone in your group contributes 10 points then, the sum of all contributions to the project is 60 points. Here the conversion rate is 70%. You and the other members of your group will each earn 70% of the total contribution (60 points), which is 42 points (0.7x60).

Example 2: If four members of the group contribute 10 points each, then the sum of contributions is 40 points. You and everyone in your group each earn 70% of the total contributions (40 points), which is 28 points.

Remember that you also get earnings from your private account, so in any round Total Earnings = earnings from your private account + earnings from the group project

Page 6: Instructions for Part I

Points to remember

Case 1 (30% earnings from the group project) and Case 2 (70% earnings from the group project) will occur in random order for the 8 rounds. Please pay attention to the amount of earnings from the group project in each round.

Your total earning: Your total earnings is the sum of your earnings from your private account and that from the project.

If you contribute to the project: In this case, you would have invested nothing in your private account and your earnings will solely depend on the earnings from the group project. Example 1: Total earnings= Earnings from your private account (0 points) + Earnings from the project (30%of sum of all contributions). Example 2: Total earnings= Earnings from your private account (0 points) +Earnings from the project (70% of sum of all contributions).

If you do not contribute to the project: In this case, you would have invested the 10 points in your private account and your earnings will include that 10 points in addition to the earnings from the groups project.

Example 1: Total earnings= Earnings from your private account (10 points) +Earnings from the project (30%of sum of all contributions). Example 2: Total earnings = Earnings from your private account (10 points) +Earnings from the project (70%of sum of all contributions).

To reiterate, income from the project goes up if more people contribute to the project. On the other hand, Income from your private account is only dependent on your contribution. At the end of each round, you will be informed about your earnings and how many people contributed to the project.

Page 7: Instructions for Part II

We now move to Part II of our experiment. In the previous part of the experiment, you were making choices independently. There are two differences in Part II of the experiment compared to Part I:

1. group members can join a coalition, and

2. your earnings will be calculated differently

Let's discuss coalitions first.

In this part, you all will have an option to join a coalition which decides together whether every coalition member will contribute to the group project or not. In each round of Part 1 you had only one decision to make in each round, but in Part II you will make two decisions in each round.

As in Part 1, you will be a member of a group consisting of 6 members. Part II will have 12 rounds with two stages in each round.

In **stage 1**, you will decide whether or not to join a coalition. In **stage 2**, your decision will depend on whether you decided to join the coalition or not. **If you do not join the coalition**, your decision is exactly the same as in Part 1: you decide independently whether to invest 10 points in your private account or the group project. **If you do join the coalition**, then members of the coalition collectively decide whether all members will contribute or not contribute to the group project.

Page 8: Instructions for Part II

Remember that in stage 1 every group member decides whether or not to join the coalition. At the end of stage 1, you will know how many members in the group of 6 participants have decided to join the coalition.

How do coalitions make decisions in stage 2?

Every group member who joins the coalition, will vote on whether members of the coalition will invest all 10 points in the group project or not. **If half or more than half the people in the coalition vote to put "points in the group project" then everyone's points in the coalition go into the group project**.

Example 1: If the coalition consists of 5 members and 3 of them vote to contribute their 10 points, then each of the 5 members will put 10 points in the group project.

Example 2: If the coalition consists of 4 members and 2 of them vote to contribute their 10 points, then each of the 4 members will put 10 points in the group project.

Once you join the coalition, you remain in it until the round ends. You are free to make a different decision in each round about whether to join the coalition.

Stage 2 instructions

Page 9: Instructions for Part II

As in Part I, your earnings depend on how much you put in your private account and how much you contribute to the group project. As a reminder: You will earn one point for each point you put into your private account . No one except you earns something from your private account. Each group member will profit equally from the amount you put into the project. The earnings from the group project for each group member will be determined through a conversion rate as before (30%or 70%). Recall the earnings from the project can be determined in the following ways:

Case 1) Earnings from the group project = 30% multiplied by the sum of all contributions=0.3x sum of all contributions. Case 2) Earnings from the group project =30% multiplied by the sum of all contributions=0.7x sum of all contributions.

Case 1 (30% earnings from the group project) and Case 2 (70% earnings from the group project) will occur in random order for the 8 rounds. Please pay attention to the amount of earnings from the group project in each round.

Page 10: Instructions for Part II

The second difference in Part II compared to Part I, is the earning calculation. In Part II, your total earnings in a round are based on

1) A percentage of your earnings from your private account,

2) A percentage of your earnings from the group project and

3) A percentage of the earnings of the person in your group **who earns the least** in that round of the experiment.

Here is how to analyze who earns the least in your group a. If everyone in the group puts all of their points in their private account, then everyone is tied for lowest earner in the group b. If everyone in the group puts all of their points in the group account, then everyone is tied for lowest earner in the group c. If some people put all of their points in their private account, and some put all of their points in the group account, then the people who put their points in their private account earn more. That means that all of the people who put their points in the group account are tied for lowest earner in the group

The percentages you use to calculate your total earnings for the round will remain the same throughout Part II of the experiment. Other members in your group may or may not have the same percentages as you. **You will not be informed about the percentages other participants use to calculate their total earnings**.

Page 11: Instructions for Part II

To illustrate this earnings calculation, let's assume that the percentage of the earnings you receive from both your private account and the group project is 20% (0.2) and the percentage of the earnings you receive based on the earnings of the least well-off person in your group is 80% (0.8).

The formula for calculating your earnings for a round is: .2x(earnings from private account) + .2x(earnings from group project) + .8x(earnings of the least well off person in your group)

Now suppose the group project's conversion rate is 30%.

If you contribute to the project then your earnings for the round will be .2x(Earnings from your private account or 0 points) + .2x(Earnings from the group project or 30% of the sum of everyone's contributions) + .8x(the earnings of the least well-off person)

Remember that when you contribute to the project; your total earnings will be the same as the earnings of the least well-off person who is also contributing to the project. **Thus, when you contribute to the group project, your earnings for the round turn out to be the same as in Part 1 of the experiment**.

If you do not contribute to the project then your earnings for the round will be .2x(Earnings from your private account or 10 points) + .2x(Earnings from the group project or 30% of the sum of everyone's contributions) + .8x(the earnings of the least well-off person)

Remember, when you do not contribute to the project, your total earnings will not be the same as earnings of the least well-off person who is contributing to the project. **Thus, when you do not contribute to the group project, your earnings for the round will not be the same as in Part 1 of the experiment**.

Page 12: Instructions for Part II

Let's give you some scenarios as examples and your earnings in each one of them.

Suppose that the group project's conversion rate is 30%.

Suppose the percentage of the earnings you receive from both your private account and the group project is 20% (0.2) and the percentage of the earnings you receive based on the income of the least well-off person in your group is 80% (0.8).

Also, suppose three people in the group join the coalition and that two out of these three people vote to contribute to the group project. In this case, **everyone in the coalition contributes their 10 points to the group project**.

Last, suppose that 2 out of the remaining 3 people the group who are not in the coalition also contribute to the group project. Thus, there are now 5 people contributing to the group project. The sum of everyone's contributions is 50 (5 multiplied by 10=5x10).

Scenario 1: you are a member of the coalition of 3 people that collectively decided to contribute to the group project.

Since you contributed to the project, your earnings for the round will be .2x(Earnings from your private account = 0 points) + .2x(Earnings from the group project or 30% of the sum of everyone's contributions=0.3x50) + .8x(the earnings of the least well-off person=0.3x50) **Earnings for the round**=0.2x(0) +0.2x(15) +0.8x(15) =15 points

Suppose you **do not join the coalition** and there are four members in your group (other than you) who are contributing to the project. If you **decide to contribute** to the project, **there are now 5 people contributing to the group project**. The sum of everyone's contributions is 50 (5 multiplied by 10=5x10). In this scenario your calculation for **Earnings for the round will exactly be the same as above**.

Recall when you contribute to the group project, your earnings for the round turn out to be the same as in Part 1 of the experiment.

Page 13: Instructions for Part II

- **Suppose three people in a group join the coalition including you** . Suppose only 1 out of the 3 people in your coalition votes to contribute to the group project, **in this case the coalition of 3 people will not contribute to the group project**.

- Also, suppose 2 out of the remaining 3 people in your group who are not in the coalition also contribute to the group project. Thus, **there are now 2 people contributing to the group project**. Sum of everyone's contributions is 20 (2 multiplied by 10=2x10).

Since you do not contribute to the project, your earnings for the round will be .2x(Earnings from your private account =10 points) + .2x(Earnings from the group project or 30% of the sum of everyone's contributions=0.3x20) + .8x(the earnings of the least well-off person=0.3x20)

Earnings for the round=0.2x(10) +0.2x(6) +0.8x(6) =3.2+4.8=8 points

Suppose you **do not join the coalition** and there are two members in your group (other than you) who are contributing to the project. Now you decide not to contribute to the project, **there are now 2 people contributing to the group project**. Sum of everyone's contributions is 20 (2 multiplied by 10=2x10). In this scenario your calculation for **Earnings for the round will exactly be the same as above**.

Recall when you do not contribute to the group project, your earnings for the round will not be the same as in Part 1 of the experiment.

Page 14: Points to remember

Summary of the changes in from Part 1 to Part 2:

1. In each round you have the option to join a coalition. The coalition decides together whether to contribute to the group project or not. **If half or more than half the people in the coalition vote to put "points in the group project" then everyone's points in the coalition go into the group project**.

2. In Part II, your total earnings in a round are based on 1) A percentage of your earnings from your private account, 2) A percentage of your earnings from the group project and 3) A percentage of the earnings of the person in your group **who earns the least**in that round of the experiment. This percentage will be fixed for you until the end of the experiment. You will not be informed about the percentage other participants have attached to their total earnings and earnings of the least well-off person.

3. **At the end of each round, you will be informed about your earning and how many people contributed to the project**.

Participants play a quiz before they began to play the game in Part II