Modeling and Analysis of Complex Systems

Project Term Paper

by

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1 Uncovering the history of Easter Island

1.1 Answer to question 1,2,3

To capture the mysterious dynamics of Easter Island, with three state variables: Humans (H), Trees(T) and Rat(R), I explore a model which will have the following interactions between and within the three state variables:

- **Trees:** As clear from the reading the human population and the rat population are the primary factors causing decrease in the amount of the trees. But, Humans directly harvest trees while rats cause decrease in fertility. To put this difference into the model I build the model where rats eating the seeds of the tress effects the growth rate of trees represented by $g/(1 + fR)$ in model. The $1 + fR$ term in the denominator means in absence of rats (R=0) the trees would grow depending only on their natural growth rate g. Further, The growth rate of trees will also be logistic growth Dependent on the carrying capacity of the environment E. The humans decreasing trees by harvesting is modelled by parameter h showing direct effect of harvesting density dependent on the population of humans. I do not model the effect of trees dying due to a natural cause assuming this time would be long and a tree would be harvested before it can reach this age.

- **Humans:** I assume that the main resources available for the human population to survive comes from the trees. To keep the model simple, main resources available survival and growth for human come from trees and the human population growth will be a logistic equation, in which the carrying capacity is the population of trees. The humans also can interact with the rat population for example by trying to eradicate disease caused by plague or any other competition with the rat population. I model this interaction as with parameter e density dependent on the population of rats but do not effect the human survival directly. This allows me to set $e = 0$ to model no interaction between human and rats. The humans also have a natural growth rate b.

- **Rats:**Similar to the human population the rat population will have a natural growth rate r. As discussed earlier they will interact with humans which is modelled by e. Finally, the growth rate for their population is modelled as a logistic growth where the carrying capacity is dependent on the population of the trees.

  All interaction parameters discussed above g(growth rate of trees),f(Di-fertilization of trees caused by rats),h(human harvest of trees),E(carrying capacity of environment with respect to trees),e(eradication of rat population),b(growth rate of humans,r(growth rate of rats) are the model parameters in my model. Further all these parameters are positive and real valued.

The figure below represents the diagram schematics of this model.

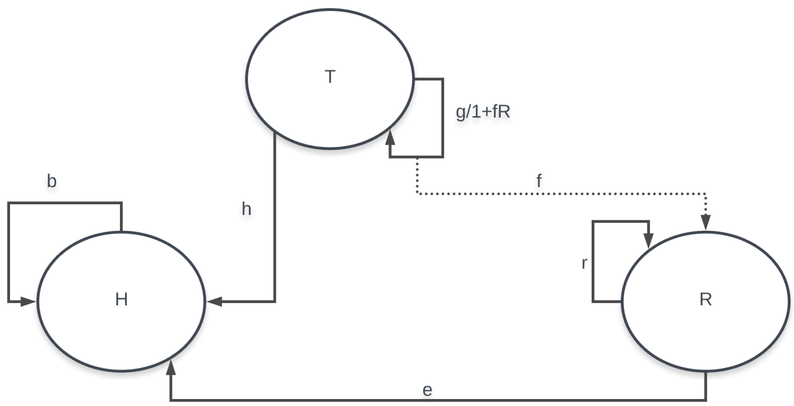


Figure 1: Qualitative representation of model

Using the pictorial representation above of the model, we can write the complete set of differential equations for the model as follows:

$$\frac{dP}{dt} = bP\left(1 - \frac{P}{T}\right) \tag{1}$$

$$dTdt = \frac{g}{1 + fR}T\left(1 - \frac{T}{E}\right) - hP \tag{2}$$

$$dRdt = rR\left(1 - \frac{R}{T}\right) - eR \tag{3}$$

1.2 Solution to question 4

The Python code is attached as ipynb file.

1.3 Solution to question 5

According to the given initial conditions and the differential equations that are defined above we ran three model using the odeint with changing the parameters .

These are the parameters used to define our first model:

- **E = 8200000** Here we assume the environment was almost on its carrying capacity initially and to define this term I google density of trees per km2 and multiplied it with area of Easter Island (163.6 km$\hat{2}$) resulting amount was almost similar to $8,200,000$ initial palm trees present on island,
- **b = 0.05** Growth rate of humans assuming net increase growth of 5% each year

- **r= 10** growth rate of rats rodents grow fast!!!
- **g= 1** Growth rate for trees assuming naturally trees can double each year
- **f = 0.1** Rate with which rats effect fertilization of trees assuming one percent of total palm seeds consumed by rats
- **h = 0.25** Assuming 25 percent of trees are harvested by humans
- **e = 0** Humans do not interact with rats at all.

We also need to define the growth function of the trees which is dependent on the Rat population at each time step.

$$fg(R) : \frac{g}{1 + f * R} \tag{4}$$

The results for the models are as follows:

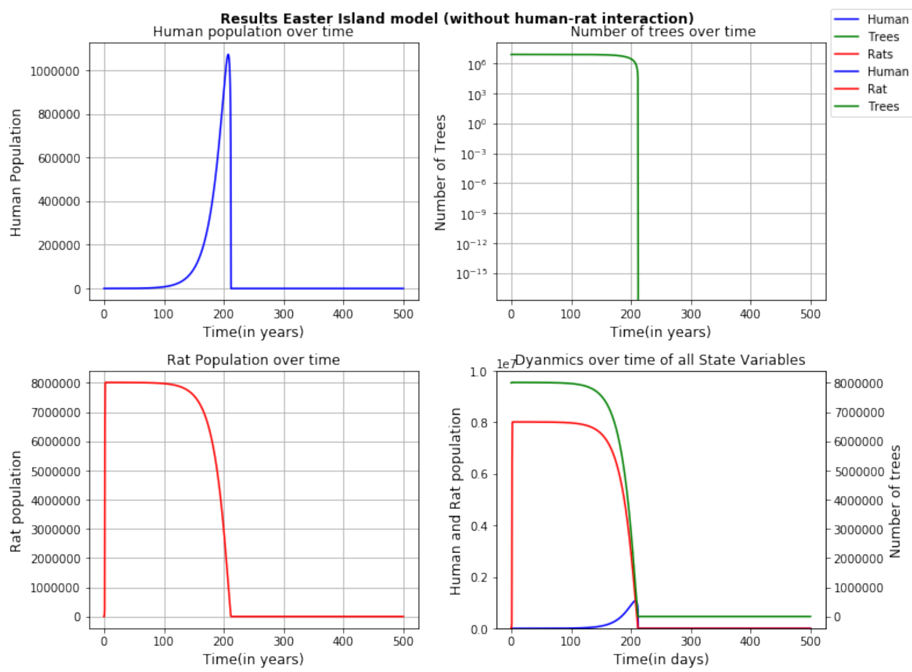


Figure 2: Results of Easter Island with no human rat interaction

For the second model we increased the human interaction with rats taking (e=4) and keeping rest of the parameters and initial condition same. The results for the models are shown below:

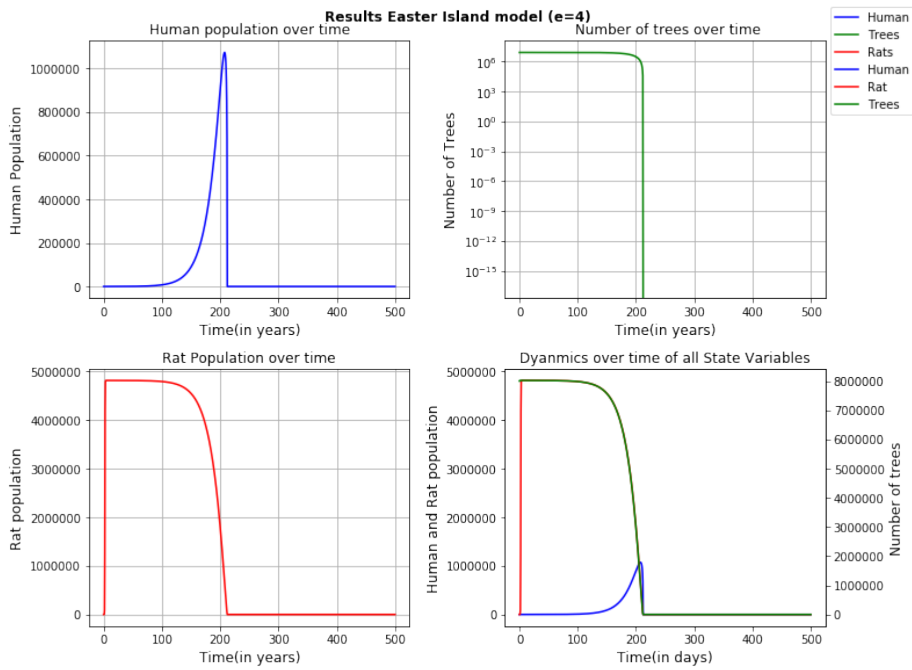


Figure 3: Results of Easter Island with e=4

For the last model we tried to force our model by increasing the human-rat interaction and causing the extinction for the rats in the early stage. Below are the following result produced using value of e=11:

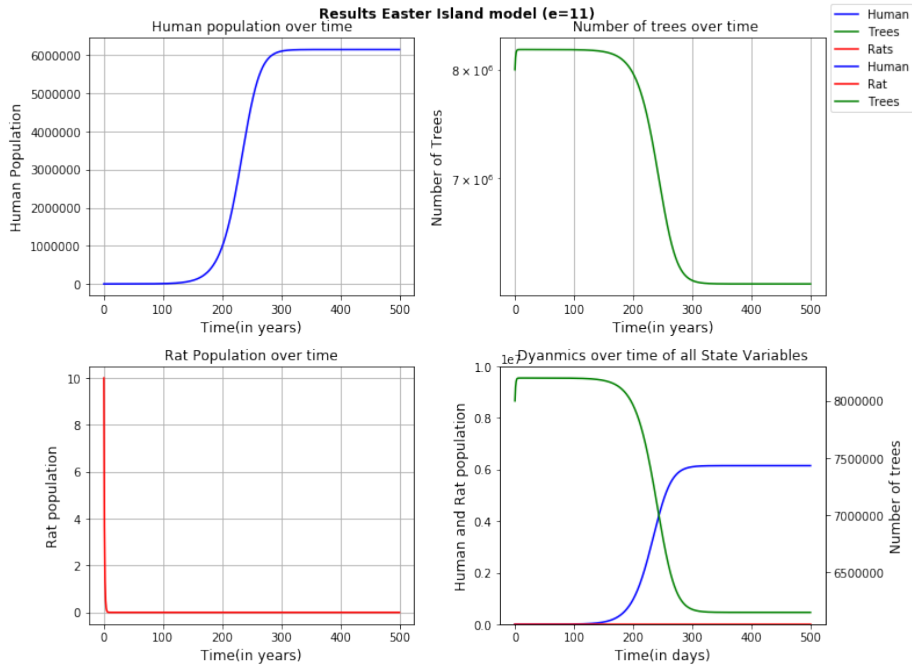
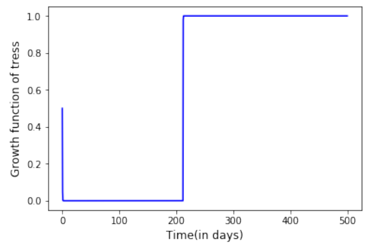


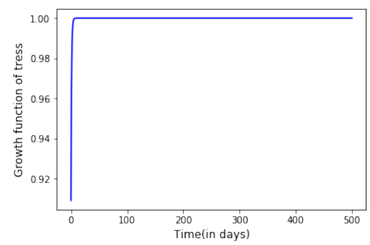
Figure 4: Results of Easter Island with e=11

From the results shown above, the initial model where humans and rats do not interact generates a ecocide narrative. In Fig1 it can be seen as the human population is increasing the number of trees are stable and when the population reaches its peak there is drop in numbers of tress and with that human population also drops drastically to zero.This shows that humans are largely dependent on tress for their sustainability. And for the rats since they grow at a faster rate , a drastic increase in rate can be seen but they also die when trees count goes to 0. Almost similar result are shown in Fig3 for second model if it is assumed that humans eradicate rats but not in strongly manner. But for the model 3 there are some interesting things seen if rats are eradicated in first few initial days by the humans. If we look at dynamics over time of all state variables produced in Fig4, rapid eradication of rats causes the the fertilization of the trees is not affected, allowing trees to grow at their natural rate. In this model since the competition of Humans with the rats for utilizing trees is not present, the human also show logistic growth with population gradually reaching up to 6e7. The model in this case deviates away from the ecocide narrative seen in the earlier two models.

Another interesting thing to see of we plot the growth function of the trees defined in eq(4) dependent on rat population.



(a) Results when rats are not eradicated



(b) Results when rats are eradicated in intial days)

Figure 5: Effect on growth function of trees

From the above plot(a) it can be seen that the number of trees drastically falls when rats are presents but when rats die the growth rate increase and become stable after 200 days. But in Fig(b) when there are no rats the growth function increases instantly and become stable in initial days only.

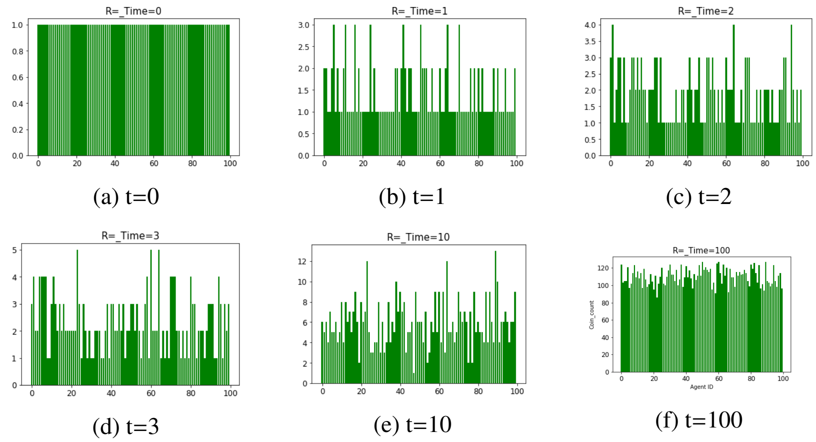
2 Coin exchange and the emergence of inequality

2.1 Solution 1

Python code file is attached in appendix.

2.2 Solution 2

For the initial number of agents n=100 and assuming that random selected agent donate only 1 coin to another random selected agent. The model was ran for different timestamps producing the histograms that represents the distribution of coins among agents.



(a) t=0 (b) t=1 (c) t=2

(d) t=3 (e) t=10 (f) t=100

Figure 6: Distribution of coins at different time-steps

2.3 Solution 3

The below figure represent the gini coefficient over time:

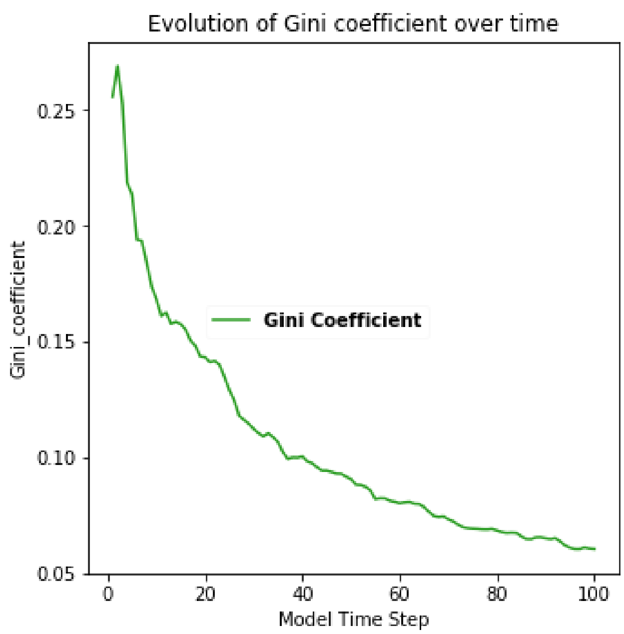


Figure 7: Gini coefficient

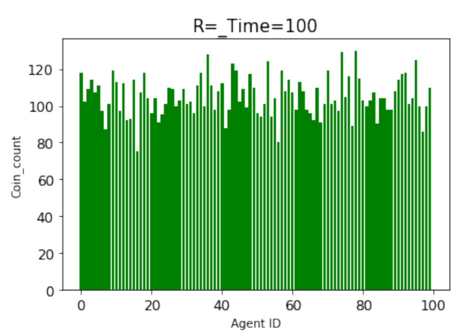
The Gini coefficient measures the inequality among values of a frequency distribution. A Gini coefficient of zero expresses perfect equality, where all values are the same. A Gini coefficient of one expresses maximal inequality among values. From the above figure it can be seen that in first few days days the Gini coefficient is around 0.25 and with increase in time the coefficient value gradually reduced to 0.05 which show as the people in society starts exchanging money and there is almost equality reached in the society.

2.4 Solution 4

From the above model it can be seen that there is not much inequality among the agents in the society.As the number of days increases the exchange happens in such a ways that everyone at the end has almost equal number of coins. It is interesting to see that how agents are randomly picked interacts among each other and the exchange of coins are done. This model does not represents how are society works, as in real life people get to choose who they want to give money or whom they want to interact with and that can be reason that we see inequality in our society.

2.5 Solution 5

The model was ran for different m value :5,10,100 where m represents initial number of coins with each agent in society. Below is the histogram distribution for 100 time steps and also Gini coefficient calculated with respect to different m values.

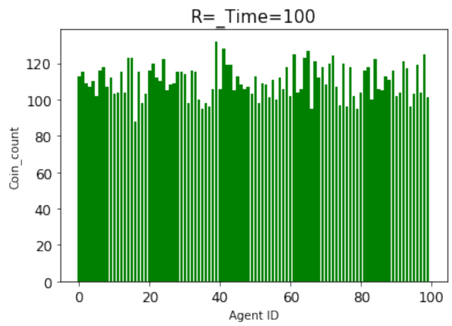


(a) Histogram of distribution of coins at m=5

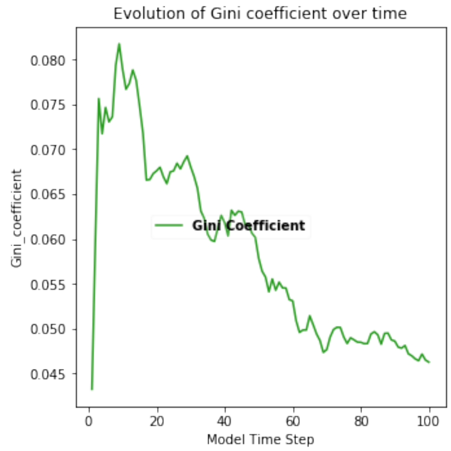
(b) Gini coefficient

Figure 8: Each agent has 5 coins initially

At m =5 :Initially the coefficient starts increasing showing showing the exchange in money and society is moving towards inequality. But,over the period of time it stabilises and there is less inequality in society showing that everyone at the end has almost equal amount of money.



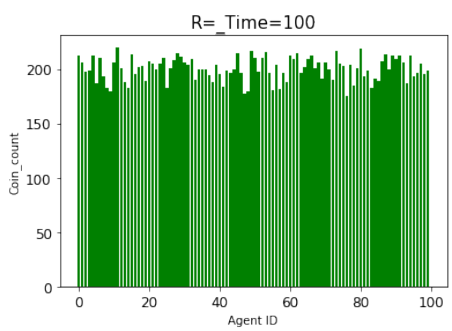
(a) Histogram of distribution of coins at m=10



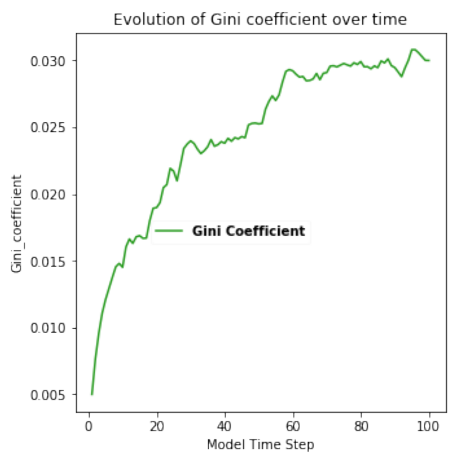
(b) Gini coefficient

Figure 9: Everyone has initially 10 coins

At m=10 the coefficient increases drastically initially but same trend from above plot is seen that the society at the end move towards equality among them.



(a) Histogram of distribution of coins at m=100



(b) Gini coefficient

Figure 10: Everyone has initially 100 coins

Something different is seen in this plot.As initially each agent has 100 coins the coefficient value is really low as everyone has equal number of money. But with the increase in period of time the coefficient value gradually start increases and showing more inequality in the society.

3 Latitude gradients of biodiversity

3.1 Solution 1

I have used the package Basemap which is an extension of matplotlib and i find easier in working with the same style and syntax of matplotlib.Further the package also supports using and plotting netCDF4 data types.The below plot represents the global sea surface temperature and at different geographic location:

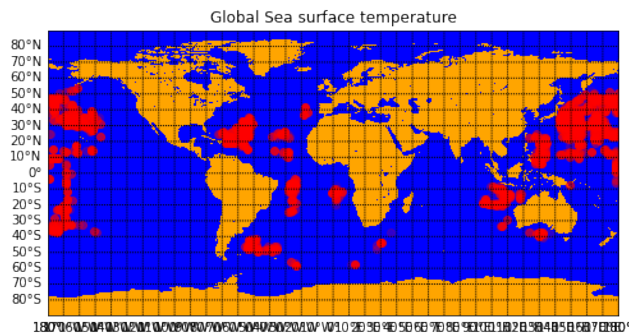


Figure 11: Global sea surface temperature

I could not make heat map plot for the temperature as i kept on getting errors (refer to code in appendix()). So i used marker that represents the temperature value and more darker the marker is, the higher the temperature is.

3.2 Solution 2

Using Random forest to predict the size diversity using sea surface departure as the only feature .We use sklearn package to resample the size to 100 and used random split to split the dataset into test and train. We ran random forest regressor model with following parameters :(n_estimators =500, oob_score=True , random_state =0) Then we determine the coefficient of determination which is about

Figure 12: Coefficient of determination

3.3 Solution 3

Below is the plot representing the global phytoplankton size diversity predicted using constructed RF model and annual climatological field of SST.

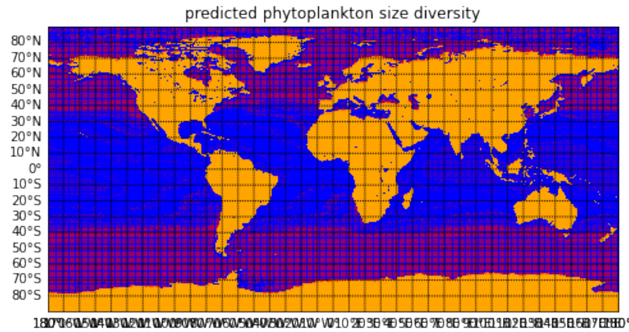


Figure 13: Global Phytoplankton size diversity

3.4 Solution 4

Below plot represents the annual average pythoplanbkton size diversity.

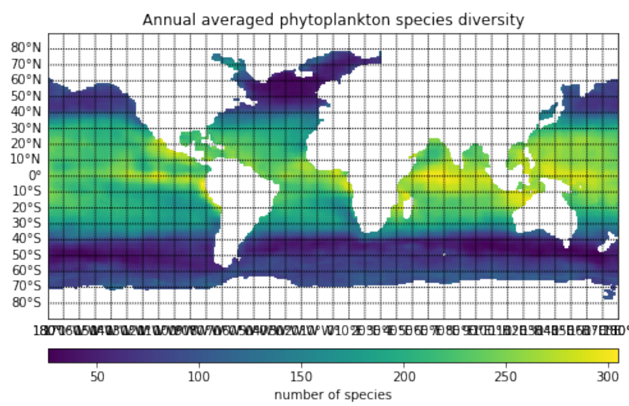


Figure 14: Annual average Phytoplankton size diversity

3.5 Solution 5

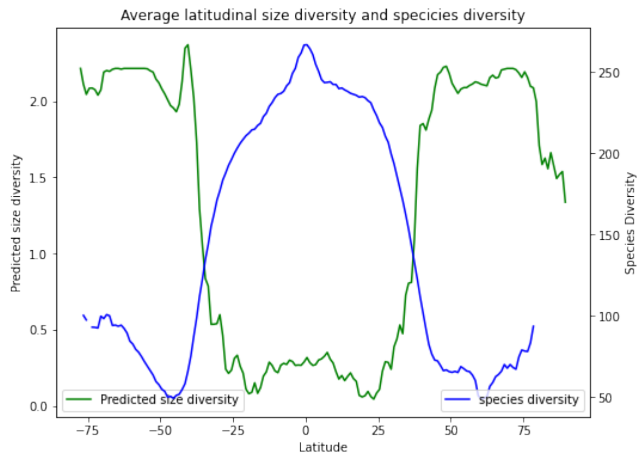


Figure 15: Average latitudinal diversity (size and species)

3.6 Solution 6a

Looking at our results (plots in task 4 and 5) the species diversity shows behaviour of being high in the equtorial region and low in the polar regions, While size diversity shows the opposite pattern. The pattern of having certain diversity high at particular latitudinal regions and lower in another, as seen for both the size and species diversity is the latitudinal diversity gradient. These changes according to our results are seen starrtung the range of 35-37 degrees South and North.

3.7 Solution 6c

From the above plots in task 4 and 5 we can see that higher species diversity are tend to be found mostly in tropical and sub-tropical region of the world.

3.8 Solution 6d

Phytoplankton communities with larger size diversity are expected to be found in areas with colder sea surface temperature i.e towards polar regions. This is supported by results of tasks task 3 and 5.

3.9 Solution 6e

The other environmental factors that can influence these latitudinal patterns can be a lot of environmental factors even simple factors like amount of Sunlight, wind patterns in the sea etc.